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## Transformation Optics

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**Abstract** We briefly reviewed the concept of transformation optics for designing functionalities. We also gave recent experimental examples from different areas of physics ranging from optics via mechanics to thermodynamics.

**Keywords** Transformation optics • Metamaterial • Invisibility cloaking • Conformal mapping • Anisotropy

### 2.1 Introduction

In physics, we often take a certain material distribution and aim at calculating the way light waves, mechanical waves, or other energy fluxes not even connected with waves emerge from this material distribution. A certain function may result. Let us call this the forward problem. In engineering, the problem is often rather the backward problem: A customer wants a certain function, e.g., he/she wishes that light behaves in a certain manner and the engineer must come up with a material distribution that leads to the requested behavior or function. Usually, such inverse problems are much more difficult to solve than the forward problem. In principle, with computers becoming ever more powerful, one can likely solve each and every such problem numerically in the future. No human brain or intuition will be required at all. Today, this is generally not possible yet, although specific examples have been given. In any case, however, numerical solutions are not entirely satisfactory. One would rather also get some intuitive understanding as to why a certain material distribution solves the problem. The ideas of transformation optics

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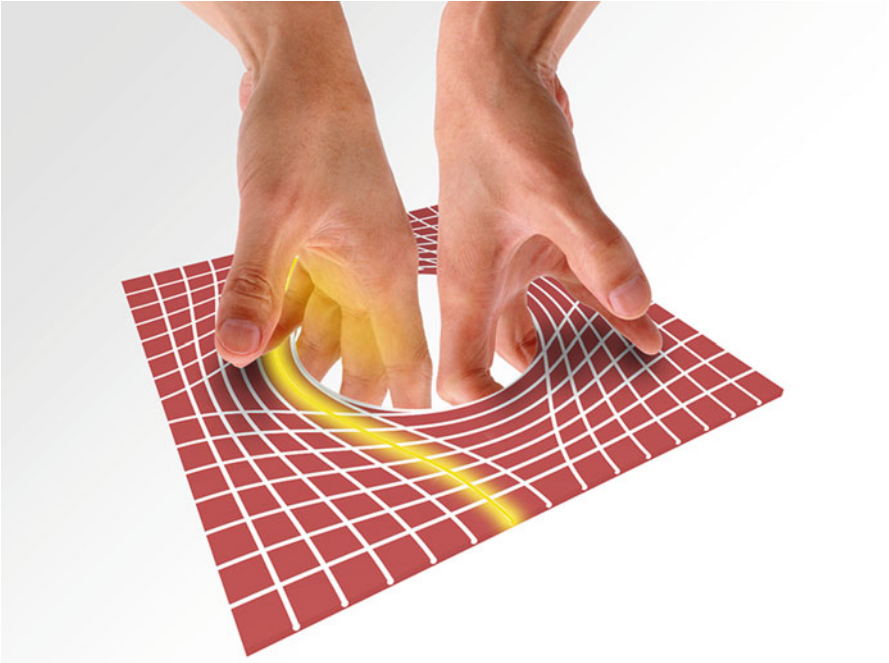
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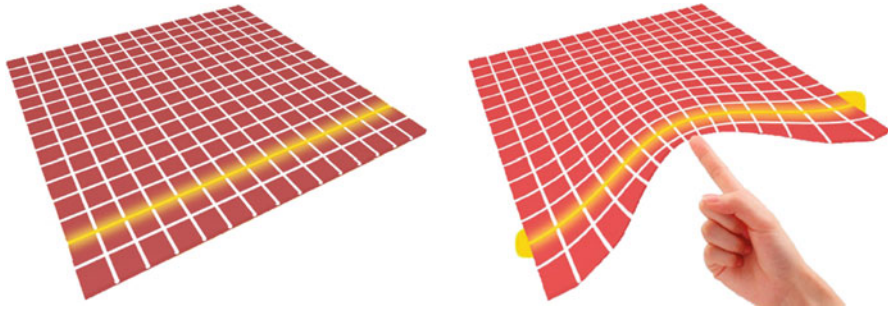
[1–3] to be outlined in the following section help us in solving the inverse problem by connecting wave propagation and/or energy fluxes with fictitious coordinate transformations, i.e., with the geometry of space.

## 2.2 Principle of Transformation Optics

Suppose you take the rubber sheet shown in Fig. 2.1 with a Cartesian grid drawn onto it [3]. An observer looking normal onto the rubber sheet will see an undistorted rectangular grid. Upon stretching the rubber sheet within the plane or by even pulling and distorting the rubber sheet in the third dimension (mathematically, by performing a coordinate transformation), the observer will see a distorted set of lines. Any of these lines represents the potential path of a light ray [3]. By appropriate pulling on the rubber sheet, essentially any light path can be tailored. For example, if we take a screw driver and punch a hole into the rubber sheet and then open up this hole to macroscopic size in Fig. 2.1, no grid line will pass this hole. Hence light does not enter the hole; we have created an invisibility cloak [1]. Any person within the hole cannot be seen and cannot look outside his/her hole either.



**Fig. 2.1** Transformation optics connects the geometry of space with propagation of waves or with energy fluxes in inhomogeneous materials (Taken from Ref. [3])



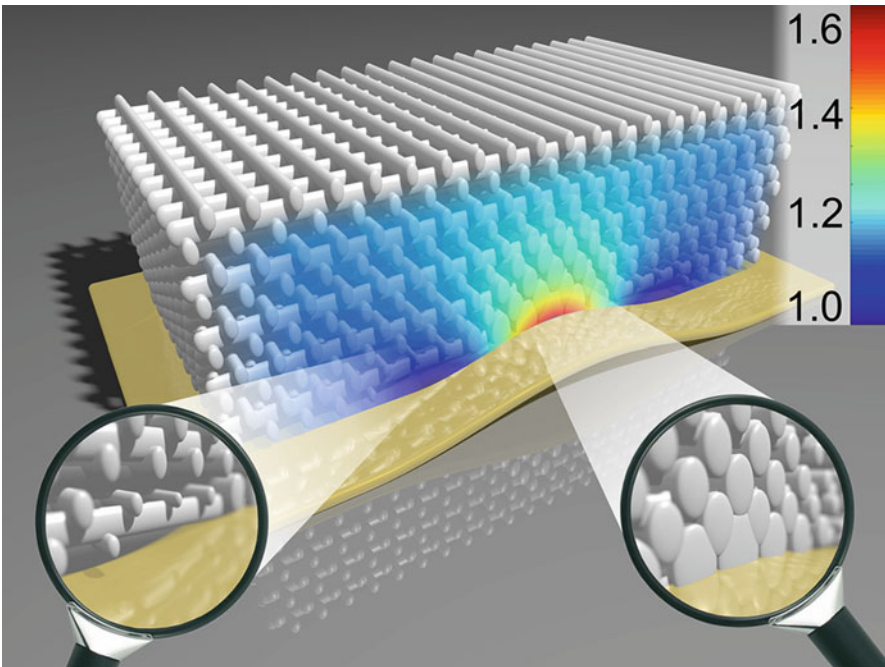
**Fig. 2.2** Illustration of the carpet-invisibility-cloak coordinate transformation (Figure provided by Michael S. Rill)

So far, however, the situation is purely fictitious. How can we map it onto an actual material distribution? Suppose the white lines in Fig. 2.1 are roads for cars in a city. All cars drive with the same constant velocity of, e.g., one line per second. For an observer watching the distorted image of the city on a computer screen, it appears as though cars are driving faster in regions where the distance between lines is stretched and slower where the distance between lines are compressed. The basic idea of transformation optics is to make this apparent or fictitious velocity real. This is possible by introducing a (meta-) material distribution leading to exactly that velocity distribution. For the situation depicted in Fig. 2.1, this means that the azimuthal component of the velocity needs to be larger than in the surrounding and that the radial component approaches zero towards the inner radius. The former aspect is somewhat problematic for electromagnetic waves as the azimuthal component needs to be faster than the vacuum speed of light. The phase velocity as well as the group velocity of light may well exceed the vacuum speed. However, the energy velocity cannot exceed the vacuum speed of light  $c_0$  as energy and mass are equivalent. According to special relativity, mass cannot propagate faster than  $c_0$ . For a non-dispersive medium, energy and phase velocity are identical. Thus, it is fundamentally impossible to build a non-dispersive invisibility cloak. It can at best operate in certain frequency region. This means, however, that the invisibility cloak can be revealed by a pulsed experiment.

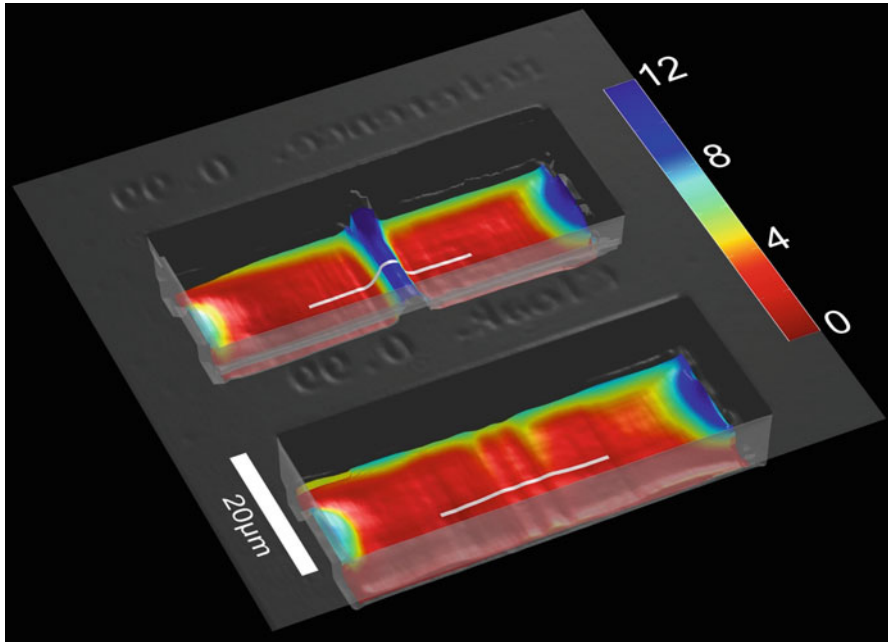
Figure 2.2 shows another example for a coordinate transformation leading to an invisibility cloak, the so-called carpet cloak [4]. Here, the used conformal transformation leads to a locally isotropic distortion of the grid. This means that a locally isotropic material is sufficient to mimic the transformation. Furthermore, the distortions are gentler and, hence, neither zeroes nor infinities occur in the resulting velocity distribution. For moderate bump height to bump width, the resulting refractive indices can stay moderate, e.g., in the interval  $[1, 1.5]$ .

## 2.3 Experimental Examples

The carpet cloak shown in Fig. 2.2 does not start from a single point (compare Fig. 2.1), but rather from the edge of a fictitious two-dimensional space. In optics, this edge corresponds to a mirror (or ground plane). Thus, an object can be hidden underneath the metallic carpet. To make the resulting bump in the carpet disappear (which can also be viewed as a particularly demanding example of aberration correction), a graded-index structure needs to be put on top of it. Transformation optics allows for designing this graded-index profile and metamaterials for realizing any refractive index. Corresponding experiments in three-dimensional space have recently been published at telecom [4] and at visible [5, 6] frequencies. A scheme of the fabricated sample is shown in Fig. 2.3, a measured phase image reflecting the time-of-flight in Fig. 2.4. Without cloak, the measured phase image simply resembles the geometrical shape of the bump. With cloak, the phase is nearly flat as expected for a flat metal mirror. The measured cloaking behavior is also broadband [4, 5].



**Fig. 2.3** Scheme of a three-dimensional optical carpet invisibility cloak composed of a polymer woodpile photonic crystal used in the long-wavelength limit. Underneath the gold carpet, objects can be hidden (After Refs. [4–6])



**Fig. 2.4** Measured phase images (false-color scale is in units of rad) on a reference structure (top) and on a carpet cloak structure (bottom) like schematically illustrated in Fig. 2.3. Measurements are taken at 700 nm wavelength (Taken from Ref. [6])

In optics, however, as pointed out above, realizing free-space (rather than carpet) broadband cloaks like shown in Fig. 2.1 is not compatible with special relativity, where the vacuum speed of light sets an upper scale. For other situations outside of electromagnetism, for which relevant velocities are far below the vacuum speed of light anyway, no such limitation exists and free-space cloaks can be realized experimentally. For example, mechanical cloaks for vibrations propagating in a thin plate or membrane have been demonstrated [7]. Here, the available off-resonant constituent materials also exhibit a velocity contrast of about 40, which is much larger than in visible optics. This helps to approximate the very low radial and very large azimuthal velocities discussed above.

Measured movies of flexural waves propagating in such structured media have shown excellent broadband free-space cloaking [7] (compare Fig. 2.1). Finally, we mention that the same construction principles can also be applied to heat conduction (or diffusion) in thermodynamics [8] – despite the fact that this type of energy transport is not connected with waves at all. Here, the heat conductivity takes the role of the wave velocity. Material contrast  $>1,000$  enables excellent “free-space” cloaking [8].

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