

# Karl Hoecken (1874–1962)

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**Abstract** After the death in 1922 of Wilhelm Hartmann, the direct successor of Franz Reuleaux in the “Lehrstuhl für Getriebelehre” (Chair of Kinematics) at the Technische Hochschule (TH) in Berlin-Charlottenburg, there was a succession of interim professors, until Hermann Alt from Dresden finally got the chair in 1939, again as full professor. Karl Hoecken belonged to this row of interims, serving from 1930 to 1934. Looking through Hoecken’s estate, we find some new aspects of the famous kinematic model collection established by Reuleaux. This paper also takes up four publications by Hoecken on mechanisms, models and their kinematic equations, and thus, gives a historical overview of Hoecken’s contribution to the field of kinematics at that time.

## 1 Biographical Notes

Because of the fact that the author never met Karl Hoecken personally and because the family name “Hoecken” is unknown now in Braunschweig, where the family members lived until the early ‘60s, it was difficult to trace exact data in the present day. Reliable sources were hidden in two big cardboard boxes that were handed over to the author by the supervisor of his own doctoral thesis, the late *Bekir Dizioğlu* (1920–2006). The boxes were mainly filled with copies of Hoecken’s publications and patent papers, many photos and lists of mechanism models, and partly unpublished documents, charts and sketches.

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Karl Hoecken was born on May 31, 1874 in Berlin. In 1946, after the end of World War II, he moved to Braunschweig with his family, his wife Anna and their two sons, Hermann and Richard. He continued to live in Braunschweig until his death in 1962 at the age of 88.

Hoecken studied land surveying at the Landwirtschaftliche Hochschule (Agricultural College) in Bonn-Poppelsdorf, where he passed his engineering examination in 1896. In 1903, he continued his studies, now in mathematics and physics, at the University of Bonn. In 1906, he became assistant lecturer for geodetics at TH Darmstadt, and then transferred to the same position at TH Berlin in 1908. After a quarrel with his professor one year later, Hoecken changed again and got a leading technical position as an instrument maker at the company “Optische Anstalt C. P. Goerz AG” in Berlin-Friedenau until 1914. This year was also the beginning of World War I. From 1914 to 1918, Hoecken was employed in the field of military technical equipment in the “Reichswehrministerium” in Berlin. Also thereafter, until 1928, Hoecken worked as a teacher and consultant in different ministry departments and co-operating companies in Berlin, Danzig and Amsterdam.

Between 1922 and 1928, there was no real holder of the chair of kinematics at TH Berlin. When *Franz Reuleaux* (1829–1905) had to leave the TH in 1896, *Wilhelm Hartmann* (1853–1922), one of his scholars, became his successor in the chair and at the same time curator of Reuleaux’s famous kinematic model collection, housed in the western part of the ground floor in the main building of the TH.

In 1930—at the age of 56—Karl Hoecken got his chance, becoming lecturer of kinematics and curator of the Reuleaux collection. He was active in the chair for only four years until 1934. Among Hoecken’s papers, there is—among other things—a map of the collection rooms (Fig. 1).

The area of the collection comprised 205 m<sup>2</sup>, including two rooms for the lecturer and one assistant. After Hoecken’s retirement and the short interim of *Rudolf Franke* (1870–1962), the chair of kinematics was finally transferred to *Hermann Alt* (1889–1954) from TH Dresden. Alt became full professor again in 1939, i.e., 43 years after Reuleaux’s forced retirement (Mauersberger 1988; Kerle 2011, 2012).

Hoecken tried a comeback as a civil employee and instructor in military departments of the then “Wehrmacht”. The author found documents in the archive of the library of TU Braunschweig which prove that in 1946 Hoecken—now living in Braunschweig—applied for a lecturer position in kinematics, but his request was declined.

## 2 List of Main Works

Following Hoecken’s educational and professional career chronologically, we can divide his main works, i.e., scientific papers and patents, into four categories (cf. Tables 1, 2):

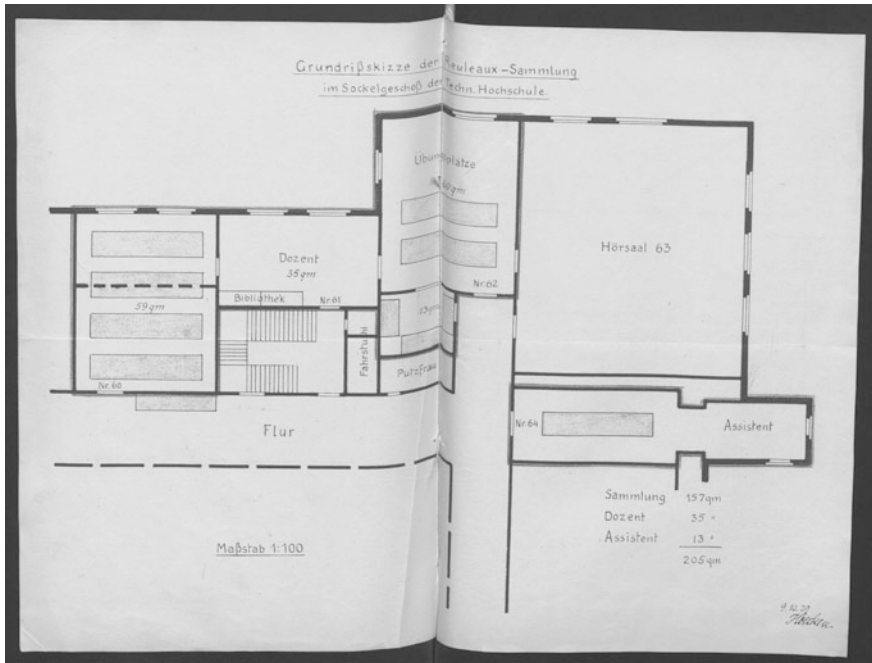
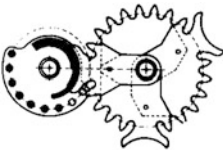


Fig. 1 Map showing the rooms of the Reuleaux collection in 1929

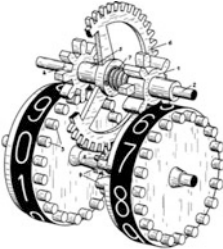
- Computing Mechanisms and Linkages (C1)
- Computing Machines (C2)
- Nomographics (Graphical Calculus) (N)
- Mechanism Theory or Kinematics (K).

The list of patents describes Hoecken’s industrial period when he was working at “Optische Anstalt C. P. Goerz AG”. The company was founded in 1886 by *Carl Paul Goerz* (1854–1923) and started off selling mathematical instruments. Soon, Goerz added photographic and astronomical devices to his sale supply. The company grew very fast and became famous for the development and manufacture of optical instruments for all purposes, including military. Eventually, Goerz had branch factories in New York, London, Paris, Vienna, St. Petersburg, Riga and Bratislava. In 1926, the Goerz company was merged with its biggest rival, the Zeiss Ikon AG company in Dresden and Jena (Zaun 2009).

**Table 1** List of Hoecken's main scientific publications

Year of publication	Category	Title of scientific publication
1909	C1	Neuer Rechenapparat zur Ermittlung der Produkte $s \cdot \sin a$ und $s \cdot \cos a$ (New calculating apparatus for determining the products $s \cdot \sin a$ and $s \cdot \cos a$ )
1911a	C1	Arithmetischer und trigonometrischer Universalrechenapparat nach Hamann (Arithmetical and trigonometrical universal calculating apparatus by Hamann)
1912	N	Über ein neues Instrument zur Ermittlung des Steuerwinkels und der resultierenden Geschwindigkeit bei Flugzeugen und über die Anwendung der Nomographie zur Lösung solcher Aufgaben (On a new instrument for determining the control angle and the resulting velocity of aeroplanes and on the application of nomographics for the solution of such tasks)
1913	C2	Die Rechenmaschinen von Pascal bis zur Gegenwart, unter besonderer Berücksichtigung der Multiplikationsmechanismen (The calculating machines from Pascal up to today with special consideration of multiplying mechanisms)
1916a	C1	Mechanismus zur automatischen Einstellung konjugierter Objekt- und Bildpunkte (Mechanism for automatic adjustment of conjugate object and image points)
1923/1924a	K	Berechnung der Räderstellung für Stirnräderwendegetriebe (Calculating the wheel positions of spur gears for positive as well as negative rotary direction of the output wheel)
1923/1924b	K	Zur Theorie der Wälzhebel und über einen daraus abgeleiteten Zahnradmechanismus (On the theory of profile lever pairs and on a gear mechanism derived from it)
1930a	K	Das exzentrisch angetriebene Räderknie zur Vermittlung ungleichförmiger Drehung mit Rast und Rückwärtsgang (The eccentrically driven gear train for generating non-uniform rotation with dwell and reverse motion)
1930b	K	Sternradgetriebe (Star-wheel mechanisms)
		
1930c	K	Die Erzeugung von Rasten durch Koppelgetriebe (Generating dwells with linkages)
1930d	N	Ermittlung zusammengehöriger Objekt- und Bildpunkte mittels des Kardanproblems (Determination of matching object and image points by means of cardanic motion)
1933a	K	Rechnerische Ermittlung eines Konchoidenlenkers (Determining the links of a conchoidal linkage)
1933b	C1	Ellipsenzeichner (Ellipses drawing device)
1938	K	Verfahren zum Verzahnen unrunder Räder (Method of cutting non-circular gearwheels)

**Table 2** List of Hoecken’s main patents

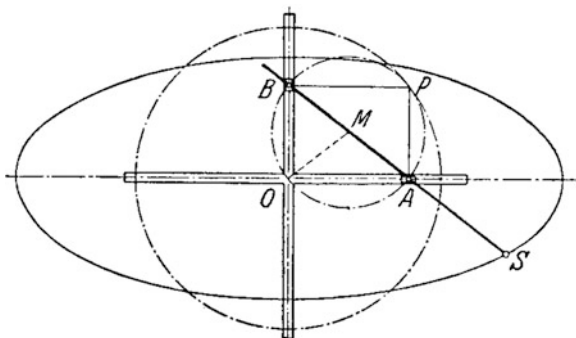
Year of publication	Category	Title of patent
1911b	C2	Addiervorrichtung mit Zahlenschiebern (Adding apparatus with number sliders)
1914	C2	Rechenmaschine mit Tastatur und durch eine Schere betätigtem Zahnstangensystem zum Antrieb des Zählwerks (Calculating machine with keyboard and a rack-and-pinion system actuated by a scissors-type device for driving the counter)
1916c	C1/C2	Zehnerschaltvorrichtung mit einem Umlaufgetriebe (Tens carrying apparatus with a planetary gear mechanism)
		
1916b	K	Schaltgetriebe zur Umwandlung einer ständigen Bewegung in eine absatzweise sich vollziehende (Mechanism for transforming uniform motion into intermittent motion)
1917b	C2	Zehnerschaltvorrichtung für Summierwerke (Tens carrying apparatus for adding machines)
1917c	K	Vorrichtung zur Übersetzung der Bewegung einer gleichförmig umlaufenden Welle ungleichförmig auf eine zweite (Apparatus for transforming uniform rotation of one shaft into non-uniform rotation of another shaft)
1919	C1	Ellipsenzeichner (Ellipses drawing device)
1921a	C2	Schaltwerke für Rechen- und Additionsmaschinen (Intermittent motion mechanisms for calculating and adding machines)

### 3 Review of Main Works on Mechanism Design

In the following, four examples of Hoecken’s main works on mechanism design are presented:

- Ellipses drawing device (“ellipsograph”)
- Six-link dwell linkage
- Conchoidal straight-line linkage
- Redtenbacher’s gear train.

**Fig. 2** Use of Cardanic movement generated by a double slider ( $A$  and  $B$ ) to draw ellipses ( $S$ )



### 3.1 Ellipses Drawing Device (“*ellipsograph*”)

In Hoecken’s time, a mathematical instrument for generating exact ellipses in technical drawings was very much in demand with engineering designers. A parametrical description of an ellipse is given by the equations  $x = a \cdot \cos \varphi$  and  $y = b \cdot \sin \varphi$ , with  $a$  and  $b \neq a$  being the half axes of the ellipse. The best known mechanism for generating ellipses is a perpendicular double-slider, which may also be replaced by two circular centrodes (Cardan circles or circles of *Geronimo Cardano*, 1501–1576) rolling on each other, with radii  $AB$  and  $AB/2$  respectively (Fig. 2). Each point on the straight line through points  $A$  and  $B$ —for example, point  $S$ —follows an ellipse with  $a = BS$  and  $b = AS$  (Hartmann 1891).

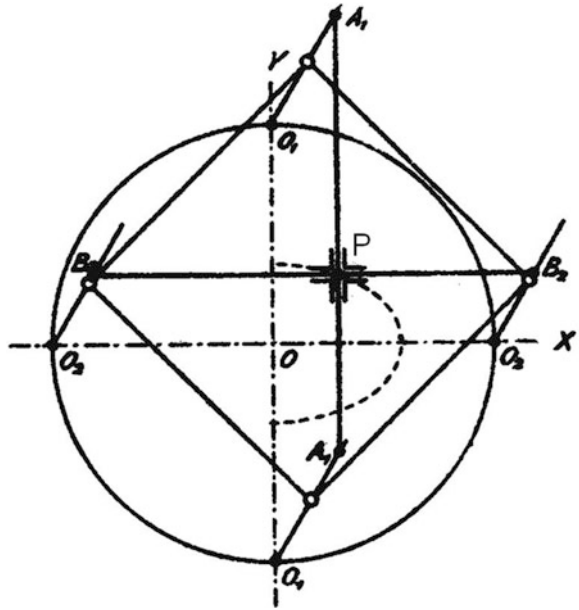
Hoecken created a special version of the double slider above, choosing a “pair-contraction” of the two pins in  $A$  and  $B$ , resulting in a perpendicular cross-slider with centre point  $P$  (drawing pencil) (Fig. 3) (Hoecken 1919).

The cross-slider is driven by a square frame that is coupled to and moved by two parallel-crank linkages of different lengths, thus giving adjustable half axes  $a = O_1A_1$  and  $b = O_2B_2$ , and moving the point  $P$  on an ellipse in the  $xy$ -plane, i.e.  $(x/a)^2 + (y/b)^2 = \cos^2 \varphi + \sin^2 \varphi = 1$ . The square frame follows a circular-parallel path in the  $xy$ -plane without rotation (Hoecken 1919; Kaiser 1919). Figure 4 shows a prototype of Hoecken’s “*ellipsograph*”.

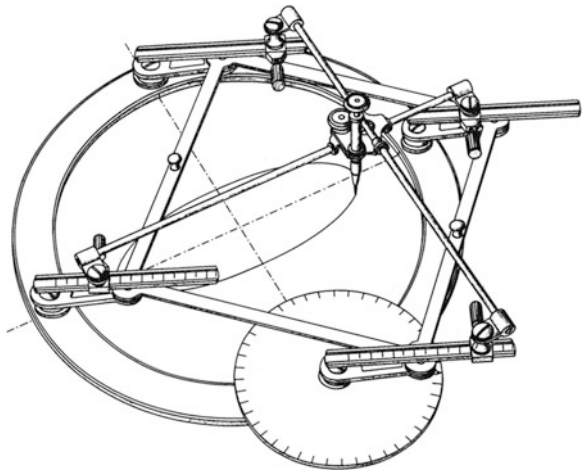
### 3.2 Six-Link Dwell Linkage

With dwell linkages having one degree of freedom, the output link temporarily comes to a standstill, while the input link moves continuously. The standstill or dwell position is characterized by zero values of the velocity and acceleration of the output link, related to the fixed link. In return positions, i.e., so-called “dead positions”, the velocity of the output link is also zero, but not its acceleration. The dwells generated by linkages are normally only approximate ones, i.e., the zero

**Fig. 3** “Pair-contraction” of A and B in P changes the double slider into a cross slider to draw ellipses

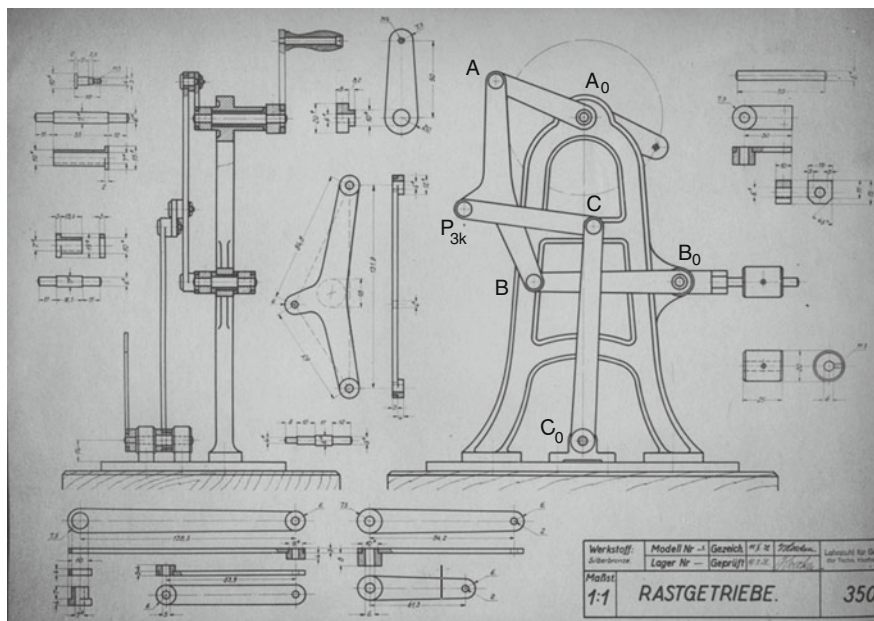


**Fig. 4** Design details of Hoecken’s “ellipsograph”



values of velocity and acceleration of the output link mentioned above are reached only approximately or on an average (Alt 1932a, b; Meyer zur Capellen 1963).

The best known structures, or so-called “kinematic chains”, of dwell linkages have six links and are of the type “Watt” and “Stephenson” (Hain 1980). In Hoecken’s estate, there is a technical drawing showing a four-link crank-rocker with a two-bar coupled to it. The drawing “Rastgetriebe” from February 11, 1932 reveals details of the links, their dimensions and even a counterweight on the opposite end of the rocker for static balancing (Fig. 5).



**Fig. 5** Technical drawing of Hoecken's six-link dwell linkage showing details for its manufacture

The dimensions of Hoecken's six-link dwell linkage are as follows (dimensions in mm length units):

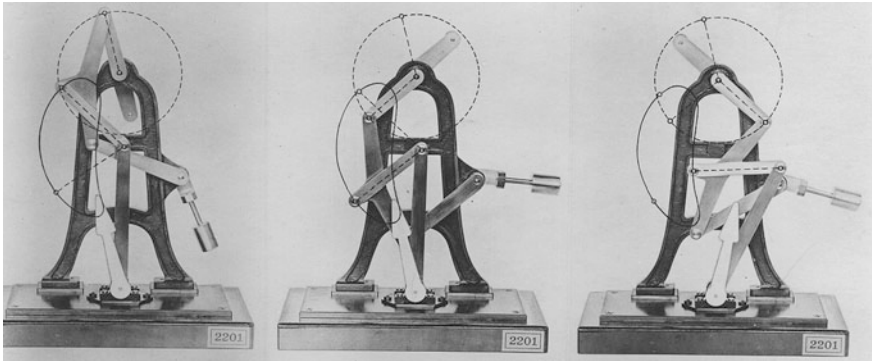
$$\begin{aligned}
 n_0 &= B_0C_0 \approx 124.0 & A_0C_0 &\approx 215.0 \\
 n_1 &= A_0B_0 \approx 127.0 & n_2 &= A_0A = 61.3 \\
 n_3 &= AB = 131.9 & n_4 &= B_0B = 94.2 \\
 n_5 &= CP_{3k} = 83.9 & n_6 &= C_0C = 139.5 \\
 AP_{3k} &= 84.8 & \angle P_{3k}AB &= 24.45^\circ
 \end{aligned}$$

It is very interesting to discover that Hoecken put his linkage onto a typical Reuleaux frame and assigned it the model number 2201. There is also a paper by Hoecken on dwell linkages (Hoecken 1930c), but it does not describe his model as shown in Fig. 6.

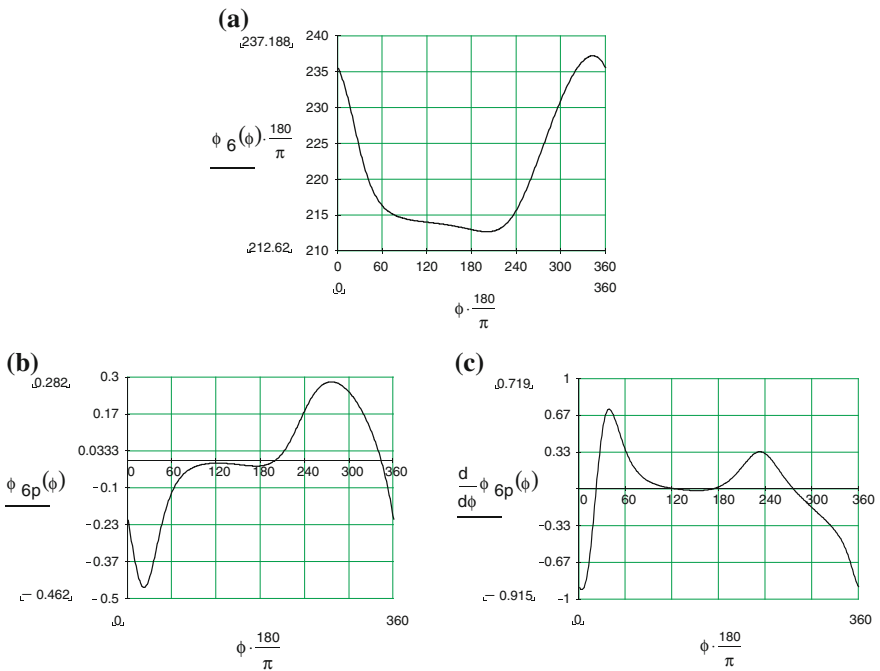
By means of the commercial program "Mathcad 11," a kinematic analysis of Hoecken's six-link dwell linkage can now be performed to find out the accuracy of Hoecken's approach, the results of which are presented in Fig. 7.

The approximate dwell of output link 6 ( $C_0C$ , Fig. 5) is realized within the range  $120^\circ \leq \varphi \leq 180^\circ$  of input link 2 ( $A_0A$ , Fig. 5). Within this range, the angle  $\varphi_6$  of the output link only falls from  $214^\circ$  to  $213^\circ$ , the angular velocity of the output link shows an absolute maximum deviation from zero of  $0.022 \text{ rad/s}$ , and that of the angular acceleration amounts to  $0.018 \text{ rad/s}^2$ .





**Fig. 6** Hoecken's six-link dwell linkage as a model mounted on a Reuleaux frame

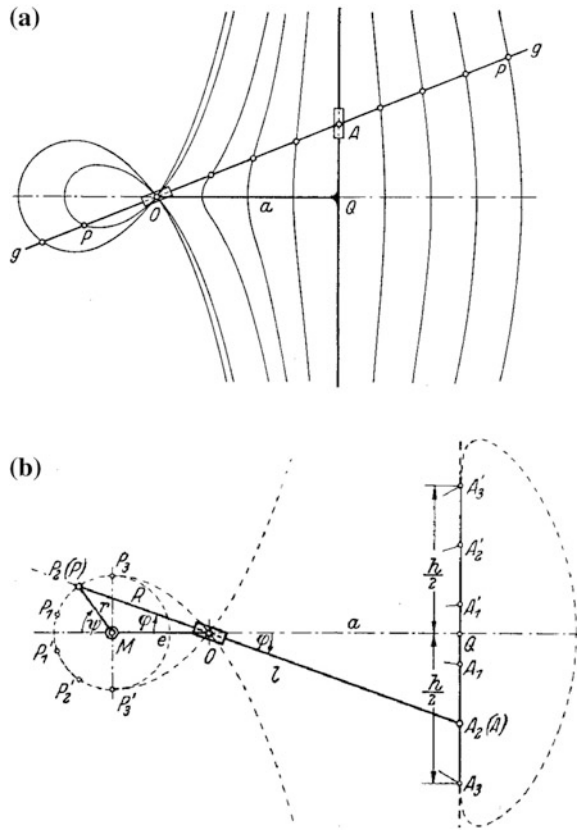


**Fig. 7** Kinematic analysis of Hoecken's six-link dwell linkage: Angle (a), angular velocity (b), and angular acceleration (c) of output link 6 versus crank angle  $\phi$  (input link 2)

### 3.3 Conchoidal Straight-Line Linkage

From the kinematic point of view, the conchoidal straight-line linkage is a four-link centric angle-slider linkage (Fig. 8a).

**Fig. 8** Centric angle-slider with conchoidal coupler curves **(a)** and a centric inverted slider-crank **(b)** which approximately replaces **(a)**

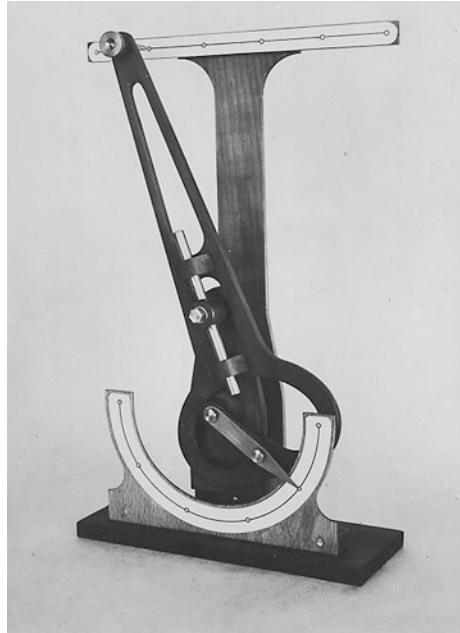


The coupler link, shown as the straight line  $gg$ , constantly slides through the fixed sliding point  $O$ , whereas the movable sliding point  $A$  is led along the fixed straight line through  $Q$  having the distance  $a$  from  $O$ . The sliding direction of point  $A$  is perpendicular to the symmetry axis passing through  $O$  and  $Q$ , the latter also being the horizontal axis  $x$  of a rectangular coordinate system.

Every point  $P$  on coupler straight line  $gg$  follows a conchoidal curve. In case  $P$  is located on the opposite side of  $OA$ , the conchoidal curve contains a closed crunode. If this crunode is partly similar to a circle, it may be approximately replaced by a circle arc with centre point  $M$  on the  $x$ -axis and radius  $r$  (Fig. 8b). Thus, a centric inverted slider-crank linkage is found as a substitute mechanism, where the slider in  $A$  can be dropped, because its path partly consists of an approximately straight line, perpendicular to the  $x$ -axis (Hoecken 1933a).

Hoecken equates the polar equation of the crunode with that of a circle around  $M$  and takes into account that all coupler curves of the inverted slider-crank are symmetric related to the  $x$ -axis. He determines the dimensions of this substitute mechanism, which approximately leads coupler point  $A$  along a straight line

**Fig. 9** Hoecken model of a conchoidal straight-line linkage



through six points in finite neighbouring order with a distance of  $h/5$  between two points each. Taking  $a = OQ = 100$  mm length units for given, Hoecken gets the following data:  $e = OM = 38.6378$ ,  $h = A_3A_3' = 120.7658$ ,  $l = AP = 161.9516$  and  $r = MP = 23.3341$ . Hoecken writes that the maximal deviation of the coupler curve within  $h = A_3A_3'$  from an exact straight line results in 0.0203. Moreover, Hoecken built a compact model in order to show the usefulness of his ideas and the validity of his calculations (Fig. 9).

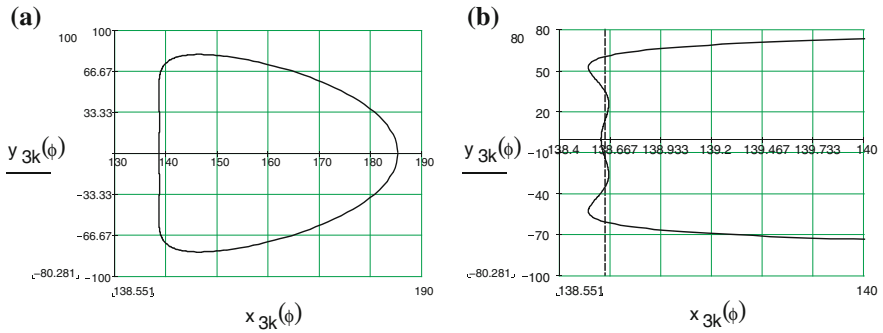
One should keep in mind that the mathematical aids in Hoecken’s time were limited and mainly consisted of tables of logarithmic and trigonometric functions. Slide rules were also in use, but not very efficient and/or precise. Solving Eq. (1), for example, namely

$$2el \cdot \cos^3 \varphi - (l^2 + 2ae + e^2 - r^2) \cdot \cos^2 \varphi + 2al \cdot \cos \varphi - a^2 = 0, \quad (1)$$

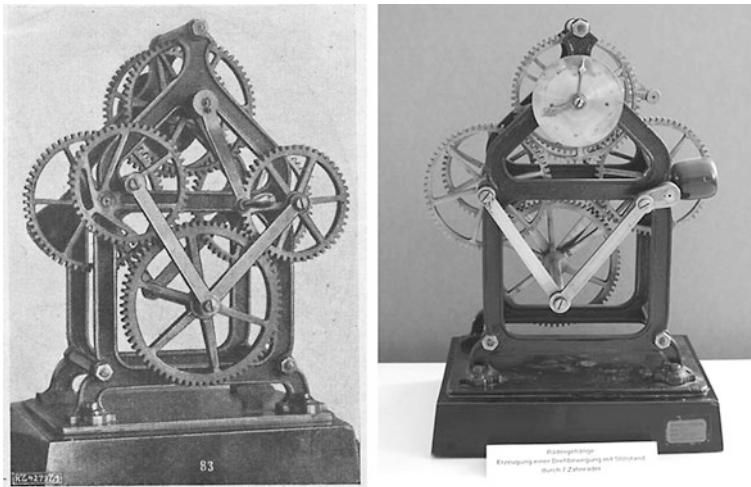
to calculate  $e$ ,  $l$ ,  $r$ , where three different angles  $\varphi = \angle AOQ$  are given above or below the x-axis (cf. Fig. 8b), requires multiple interpolation and iteration procedures to gain results with four digits after the decimal point.

By means of the commercial program “Mathcad 11, ” Hoecken’s calculations can be verified and his statements confirmed (Fig. 10).

In Fig. 10, the input crank angle  $\phi = 180^\circ - \omega$  is introduced, instead of  $\omega$  as in Fig. 8b. The deviation of only  $\pm 0.0203$  mm from  $x_{\text{ref}}$  is valid between  $122^\circ \leq \phi \leq 238^\circ$  crank angle.



**Fig. 10** Coupler curve of Hoecken's conchoidal straight-line linkage: **a** general view, **b** zoomed view around the reference vertical (broken) straight line  $x_{3k} = x_{\text{ref}} = a + e$



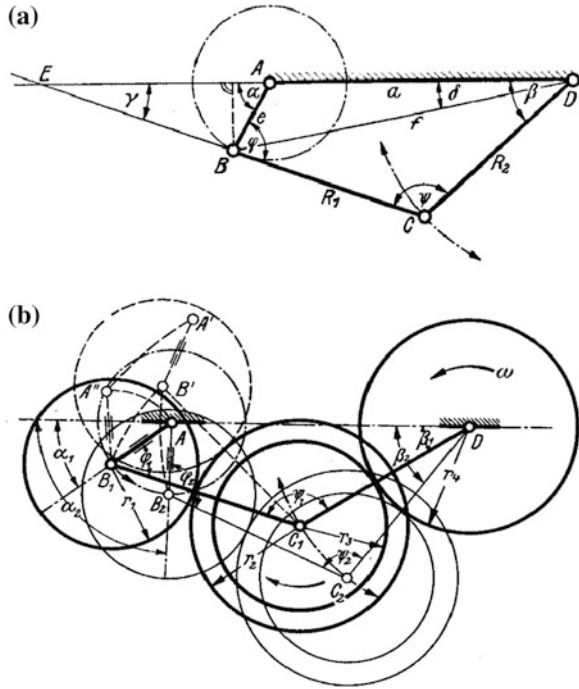
**Fig. 11** Redtenbacher's gear train, back view (left) and front view (right)

### 3.4 Redtenbacher's Gear Train

In 1926, Hoecken visited TH Karlsruhe. He found, among other things, the model of a gear train with seven gearwheels. The model belonged to the Redtenbacher model collection in Karlsruhe, but nobody could explain to Hoecken the purpose of this model (Hoecken 1930a).

Figure 11 (left) shows the back of the Redtenbacher gear train with model number 83 in a photo taken by Hoecken himself. The author is happy about the fact that there is a second photo on the internet (Moon 2011), now revealing the front of the model (Fig. 11, right). This photo was taken in 2001 or 2002 by Francis C. Moon from Cornell University in Ithaca, NY (USA), during a visit to

**Fig. 12** Hoecken’s sketch of Redtenbacher’s gear train: Basic four-bar linkage (a), compound gear train (b)



TH Karlsruhe. It is not necessary to write about the significance of *Ferdinand Jakob Redtenbacher* (1809–1863) in the evolution of mechanical engineering as a science in Germany, because there is a detailed and excellent description of his lifework, cf. (Wauer et al. 2009, 2010).

Hoecken explains that three of the seven gearwheels are unnecessary for the mechanism to function correctly, but only serve to make the input rotary axis with the driving crank coincident with the output rotary axis (“returning” mechanism). Both axes have metal pointers in the form of arrows for demonstration, similar to those on a clock.

The base mechanism is that of a crank-rocker ABCD (Fig. 12a) with four two-by-two mating gearwheels with their centres in B, C and D (Fig. 12b). The gearwheel centred in B forms the input crank (eccentric wheel) and rotates around A (input angle  $\alpha$ ). Depending on the dimensions of the crank-rocker and the radii  $r_1$  to  $r_4$  of the gearwheels, it is possible to vary the output angle  $\omega$  of gearwheel 4 related to the input angle  $\alpha$  of crank or gearwheel 1, i.e., the transmission ratio can be partly positive, zero (standstill of gearwheel 4) or negative (the “pilgrim’s step” motion of gearwheel 4), while crank 1 rotates continuously with constant angular velocity. The rotary motions of the gearwheels originate from the variable difference angles  $\varphi(\alpha) = \angle ABC$ ,  $\psi(\alpha) = \angle BCD$  and  $\beta(\alpha) = \angle ADC$  between the adjacent bars of length  $e-R_1$ ,  $R_1-R_2$  and  $a-R_2$ .

Hoecken used his own extensive geometric-mathematical equations to calculate the output angle  $\omega = \omega(\alpha, t)$  (time  $t$ ) and output angular velocity  $d\omega/dt = (d\omega/d\alpha) \cdot (d\alpha/dt)$  for given dimensions and given input quantities  $\alpha$  and  $d\alpha/dt \equiv 1$  rad/s of the gear train.

Hoecken's results are checked by using the commercial program "Mathcad 11" again. Given the dimensions  $a = AD$ ,  $e = AB$ ,  $R_1 = BC$ ,  $R_2 = CD$  of the crank-rocker and the radii  $r_1$  to  $r_4$  of the gearwheels, where  $r_1 + r_2 = R_1$  and  $r_3 + r_4 = R_2$  and setting  $r_1/r_2 = \mu$  and  $r_3/r_4 = \nu$ , we first make use of a set of kinematic equations for four-bar linkages and calculate the angles  $\varphi$ ,  $\psi$  and  $\beta$ . Then, we care for the wheel rotations and follow the equations

$$\omega_0 = \omega(\alpha = \alpha_0 = 0), \text{ etc. for } \varphi_0, \psi_0, \beta_0, \quad (2)$$

$$\omega_0 = (1 + \nu) \cdot \beta_0 + \nu \cdot (1 - \mu) \cdot (360^\circ - \varphi_0 + \alpha_0) = 0, \quad (3)$$

$$\omega(\alpha) = \omega_0 + \beta(\alpha) - \beta_0 - \nu \cdot [\psi(\alpha) - \psi_0] + \mu \cdot \nu \cdot [\varphi(\alpha) - \varphi_0]. \quad (4)$$

In order to demonstrate the effectiveness of these equations, we take Hoecken's example and set the lengths in mm of the four-bar links and the gearwheel radii as follows:  $a = 50$ ,  $e = 15$ ,  $R_1 = R_2 = R = 35$ . We distinguish between three cases concerning different gearwheel ratios, i.e., case I:  $r_1 = 25$ ,  $r_2 = 10$ ,  $r_3 = 20$ ,  $r_4 = 15$ ; case II:  $r_1 = 20$ ,  $r_2 = 15$ ,  $r_3 = r_1 = 20$ ,  $r_4 = r_2 = 15$ ; case III:  $r_1 = 10$ ,  $r_2 = r_3 = r = 25$ ,  $r_4 = r_1 = 10$  (Hoecken made a slight mistake when he wrote  $r_1 = r_4 = r = 25$ ). Case III is equivalent to the fact that there are only three instead of four gearwheels. The results are shown in Table 3.

The columns represent the output angle  $\omega = \omega(\alpha)$  and its derivatives  $d\omega/d\alpha$  and  $d^2\omega/d\alpha^2$ . Because of the fact that the input angular velocity is constant and of unit value, the derivative functions  $d\omega/d\alpha$  and  $d^2\omega/d\alpha^2$  are equivalent to the angular velocity and angular acceleration with respect to the output of gearwheel 4.

Case I reveals a continuous positive ratio between the output angle  $\omega$  and input angle  $\alpha$ , the output angular velocity varying within the positive range without reaching zero; in case II, there is a standstill of the output gearwheel momentarily at point  $\alpha = 285^\circ$ , a condition that Kurt Hain already named a "point dwell" position in coherence with a special six-bar linkage (Hain 1981); case III stands for a "pilgrim's step" motion of the output gearwheel in the range  $205^\circ \leq \alpha \leq 340^\circ$ , i.e., gearwheel 4 rotates partly in the opposite direction related to input gearwheel 1.

Some 17 years later and now living in Braunschweig at the age of 73, Hoecken again tackled the problem of finding the dimensions of a "pilgrim's step" geared four-bar linkage. Hoecken wanted to solve the complicated equations by means of nomographic charts. He typed all the pages and drew all the figures of a new publication, but he never published it. Hoecken's economic problems just after World War II were greater than his wish for publication. In Hoecken's estate, there is also a photo of a model of a four-bar with three gearwheels (Fig. 13).

**Table 3** Output motion of gearwheel 4 versus input crank angle in Redtenbacher’s gear train showing three different cases

<p><b>Case I</b> Positive ratio</p>	<p><b>Case II (Hain)</b> “Point dwell”</p>	<p><b>Case III (Hoecken)</b> “Pilgrim’s step”</p>

Hoecken set the length  $R_1$  of the coupler BC equal to the length  $R_2$  of the rocker CD, the radius of input gearwheel 1 equal to the radius of output gearwheel 4 and the radius of gearwheel 2 equal to that of gearwheel 3, this latter condition again being equivalent with the earlier case III with only three instead of four gearwheels, i.e.,

$$R_1 = r_1 + r_2 = r_3 + r_4 = R_2 = R, \tag{5}$$

$$r_1 = r_4 = r, \quad r_2 = r_3 = R - r. \tag{6}$$

There is an input range of width  $\alpha_2 - \alpha_1 = 2\delta$  of input gearwheel 1 or crank AB, where the output gearwheel 4 rotates in reverse direction to gearwheel 1, this output range being  $\omega_2 - \omega_1 = \varepsilon$ . For given  $r, R = r + r_2$ , and  $\delta$ , Hoecken found the following solutions for the lengths  $a$  (fixed link) and  $e$  (crank, eccentricity) of the four-bar linkage ABCD:

$$(a/R)^2 = 1 + [(R^2 - r^2)/(R^2 + r^2 \cdot \tan^2 \delta)]^{1/2}, \tag{7}$$

$$(e/R)^2 = 1 - [(R^2 - r^2)/(R^2 + r^2 \cdot \tan^2 \delta)]^{1/2}, \tag{8}$$

$$\varepsilon/2 = \delta - (R/r) \cdot \arctan[(r/R) \cdot \tan\delta]. \quad (9)$$

One possible solution of a proper nomographic chart for solving Eqs. (7) and (8) is shown in Fig. 13 (below), consisting of three straight lines. After having chosen, for example, two arbitrary values of  $2\delta$  and  $a/R$ —or  $e/R$ , respectively—on the two corresponding straight lines on the left and right side, these two values define a straight line themselves which intersects the third straight line in the middle at the solution points  $r/R$  or  $r_1/R$ , respectively.

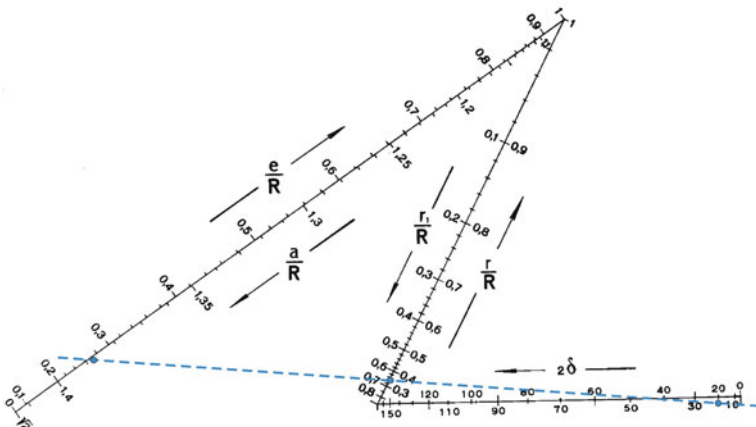
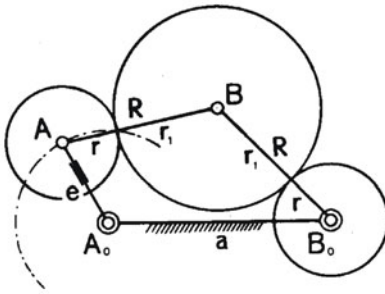
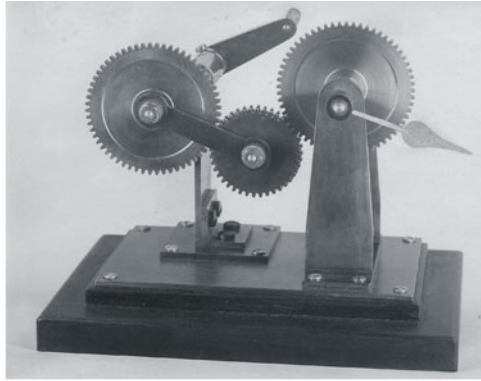
The nomographic chart in the figure additionally shows a broken line which represents the solution  $a/R = 1.387$  or  $e/R = 0.28$ , respectively, for given values of  $2\delta^\circ = 20$  and  $r/R = 0.38$ .

#### 4 On Hoecken's Role as One of the Successors of Franz Reuleaux in Berlin

When Hoecken took over the leadership of the “Lehrstuhl für Getriebelehre” at TH Berlin in the summer term of 1930, he offered two lectures to his students, “Ausgewählte Kapitel aus der Getriebelehre” (Selected Chapters of Kinematics) and “Einleitung in die Nomographie” (Introduction to Nomographics). Two years later, Hoecken added a lecture about “Die mechanischen Rechenmaschinen” (Mechanical Calculating Machines). Hoecken's lectures were not obligatory for graduate students. This fact is astonishing only at first glance; it was a leftover consequence of the quarrels Reuleaux had had many years earlier with some of his colleagues at TH Berlin. Later on in 1896, Reuleaux had to leave TH Berlin involuntarily. Moreover, in 1902—three years before Reuleaux's death—lectures on kinematics were even removed from the obligatory curricula for mechanical engineering students (Mauersberger 1988; Moon 2003). It is also meaningful that Wilhelm Hartmann, Reuleaux's follower, did not get the position of a full professor in kinematics. Hoecken wanted to base his lectures on those of Hartmann, and consequently Hoecken's lectures on kinematics were strongly related to the ideas and works of Reuleaux. Among Hoecken's papers is an announcement advising his students to visit the famous Reuleaux kinematic collection in order to learn more about kinematics. Hoecken's activities at TH Berlin ended as the winter term of 1934 was still ongoing; clear reasons for this sudden end have not been found. Some of Hoecken's personal notes from that time indicate that the social democrat Hoecken was unpopular with the national socialists in Berlin. On the other hand, there are also contemporary documents which supply evidence that Hoecken never missed any chance to quarrel with his colleagues and the university administration in order to get a better position and more money. In addition, Hoecken was already 59 years old by that time.

The much more interesting point as far as the kinematics community is concerned is this: Immediately after his appointment as lecturer and curator of the Reuleaux collection, Hoecken took inventory of it. He made a primary list on a





**Fig. 13** Redtenbacher's gear train reduced to three gearwheels ( $R_1 = R_2 = R$ ) and gearwheel radii  $r_1 = r_4 = r$

typewriter of all 795 items in the collection, but not every item named on this list was a mechanism, a mechanism element or a machine model.

Hoecken found a new numbering system for the models in the collection, differing strongly from that which Reuleaux, his model maker *Gustav Voigt* and

**Table 4** Hoecken mechanism models belonging to the Reuleaux collection

No.	Model name	No. in Hoecken's primary list
1	Keyboard with straight-line guide for computing machines	348
2	Double straight-line guide having opposite directions	347
3	Reversing mechanism with Oldham coupling	833
4	Flip ratchet mechanism	1764
5	Double rotating inverted slider-crank	1239
6	Drive of parallel shafts by means of a parallel linkage	2050
7	Seven-link spatial screw chain	2001
8	Spatial shaft coupling	
9	Spatial chain with 3 + 4 d.o.f.	
10	Spatial branched chain with 2 + 2 + 3 d.o.f.	
11	Angular straight-line guide	350
12	Star-wheel gear having internal gear meshing (driver bigger than star-wheel)	
13	Star-wheel gear having internal gear meshing (driver smaller than star-wheel)	
14	Kirsten propeller	
15	Device for cutting elliptical gearwheels	
16	Rolling pairs, one limited linearly	
17	Straight-line guide having opposite directions	
18	Spatial chain with 2 + 5 d.o.f.	
19	Eccentric conchoidal straight-line guide	
20	Intermittent mechanism having 120° intermittent angle	
21	Reversing mechanism with gearwheels	
22	Shifting mechanism	
23	Angular shaft coupling for constant rotary transmission	
24	Revolving mechanism	929/930
25	Cardanic gearwheels	349

his son *Otto Voigt* had introduced and used at the beginning (Voigt and Voigt 1907). On the occasion of the two industrial fairs in Leipzig in 1928 and 1929, there were also two mechanism model exhibitions to which German university institutes and industrial companies contributed. The models presented there were also compiled in two booklets (AWF and VDMA 1928, 1929) with new number stickers. Hartmann probably changed the numbering system of the model collection, perhaps because he wanted to merge the two lists “I. Verzeichnis” and “II. Verzeichnis” of the Voigt catalogue.

Hoecken wrote a second list on his typewriter that comprised 25 “genuine Hoecken models” (cf. Table 4). Hoecken pointed out that he himself had designed and manufactured all the models named on this second list. A closer inspection of this list reveals that 10 of the specified models were already part of the Reuleaux collection, while the other 15 models were new ones, and it seems that Hoecken added these 15 new models to the collection during the years of his leadership.

What's more, there is a third list by Hoecken, this one handwritten, which comprised items/models that

- were eliminated from the collection because they belonged to the group of auxiliary parts or were damaged,
- Hoecken himself had built, bought or got as gifts from industrial companies.

We also learn from this third list that Hoecken created a new group “20” of spatial mechanisms, consisting of 24 models. He assigned the name “Freiheits-grade” (degrees of freedom, d.o.f.) to this group.

All the three of Hoecken's lists are combined below (cf. Table 5). This table presents the state of the art of the kinematic Reuleaux collection by the end of 1934.

Some additional remarks are necessary:

- First column: There are 22 different groups of models; the content of group 21 is unknown.
- In the second column, all the models of the Reuleaux collection in Berlin are given following Hoecken's primary list numbers. Genuine Hoecken models included are written in italics and put in parentheses. The Reuleaux models that were exhibited on the occasion of the two industrial fairs in Leipzig are given separately below a broken line. The photo numbers attached to these latter models and published in (AWF and VDMA 1928, 1929) are put in parentheses.
- The fourth column contains the group names or model types. The matching Reuleaux-Voigt models listed in (Voigt and Voigt 1907) and described by Moon (2007, 2011) are written in italics and put in parentheses.
- The list reveals a total of at least 772 models in the Reuleaux collection, including 21 Hoecken models, as well as 17 small models made of sheet metal, also contributed by Hoecken. In addition, there were 15 genuine Hoecken models that Hoecken mentioned, but had not had the time to assign numbers to (cf. Table 4).

Owing to the fact that the Reuleaux collection contained models that were not invented or designed by Reuleaux himself, the question sometimes arises as to how to identify genuine Reuleaux models, those “designed by Reuleaux”. Neither the typical Reuleaux frame nor the stand of a model, as shown, for example, in Fig. 6, gives a secure answer. It makes sense to assume that the models at the beginning of the collection from 1870 were genuine Reuleaux models. But the best source for finding genuine Reuleaux models are the two lists in (Voigt and Voigt 1907). Here, some of the model names have supplements, such as “von Reuleaux” or “nach Reuleaux” or simply “Reuleaux”. Hoecken also added these supplements to his primary list.

In addition to his lists, Hoecken made a row of photos of the models in the famous kinematic Reuleaux collection (cf. Figs. 14 and 15).

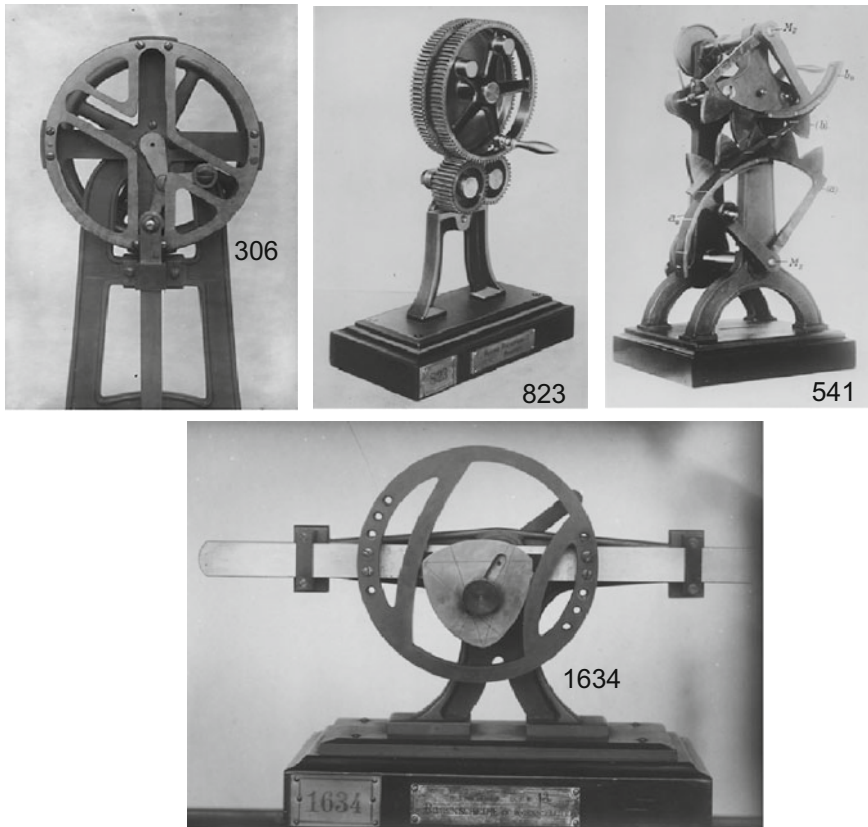
**Table 5** List of models of the Reuleaux collection in Berlin in 1934

Group number	Berlin model numbers (incl. "Hoecken models")	Number of models	Model types (incl. "Reuleaux-Voigt models")
	AWF/VDMA model numbers R ... (photo no.)		
0	001-009, 016-032, 040-043 (041)	30	Element pairs and four-link mechanisms (A1-A3, B1, B2, C1-C4, C6, D1-D4, D14, H1, H; 91)
1	101-106, 108, 109, 112, 114, 116, 117, 120-129, 132-135, 137-148, 152-156 ----- 148 (36), 155 (35)	43	Drawing auxiliary devices and mathematical instruments (B3, B4, D10, D12; 119-122)
2	201-203, 211-214, 221-224, 227	12	Simple machine tools (87, 88, 118, 123)
3	301-350 (347-350) ----- 307 (196)	50	Straight-line guide mechanisms (S1-S39)
4	401-434 ----- 423 (284)	34	Parallel guide mechanisms (T1-T14, U1, U2; 39-45)
5	501-510, 512, 513, 516-526, 528-531, 534, 535, 539-554 ----- 528 (219), 535 (194), 550 (28)	45	Mechanisms with gearwheels and friction wheels (C7-C9, Q1-Q8, W1-W6; 46-49, 52-54)
6	601-629 ----- 608 (171), 609 (172), 614 (279), 615 (162), 618 (164), 622 (169), 623 (283), 626 (281/282)	29	Chamber wheel mechanisms (F1-F6, H1-19; 67-71)
7	710-715, 726, 727, 731-733, 741-773, 781-788 ----- 765 (110), 768 (107)	52	Cycloidal rolling mechanisms (R1-R7)
8	801-836 (833) ----- 809 (237), 815 (241), 824 (113), 832 (239)	36	Gear trains and reversing mechanisms (G1-G7, M9, Y11, Y13, Y18; 60, 62, 63, 73-77, 84-86)
9	901-931 (929, 930)	31	Reversing and shifting mechanisms with gearwheels, friction wheels and belts (V14-V16, Y3, Y5-Y7, Y9, Y10, Y14-Y17, Y19, Y20)
10	1001-1017	17	Reversing mechanisms with planetary gearwheels (Y2; 56-59)

(continued)

**Table 5** (continued)

11	1101-1124, 1130 ----- 1108 (181), 1109 (4), 1112 (12), 1118 (3), 1119 (20)	25	Four-bar linkages and coupling mechanisms ( <i>D5, D7, D8; 72, 78, 79, 89, 90</i> )
12	1201-1239, 1241, 1242 (1239) ----- 1230 (193), 1241 (175), 1242 (176)	41	Crank mechanisms ( <i>C5, D6, D9, D11, D13, E1-E7, K2, K3</i> )
13	1301-1321, 1324, 1325, 1327 ----- 1307 (225), 1312 (222), 1313 (161), 1319 (224)	24	Screw mechanisms ( <i>M1-M8; 116</i> )
14	1401-1423	23	Belt mechanisms ( <i>Y1, VI-VI3; 80-83</i> )
15	1501-1503, 1507, 1509- 1514, 1516, 1517, 1520, 1550-1562, 1564-1567, 1570-1572 (1550) ----- 1513 (197)	33	Planetary gear trains and cycloidal rolling mechanisms ( <i>O1-O5; 92-105</i> )
16	1601-1657, 1659-1666, 1668, 1671 (1660) ----- 1603 (48), 1611 (52), 1613 (51), 1629 (50), 1644 (61)	67	Cam mechanisms with translating follower ( <i>L1-L6; 1-38</i> )
17	1701-1757, 1760, 1763- 1765, 1767-1783, 1785, 1786, 1788, 1789, 1792- 1799 (1764, 1776, 1792-1794) ----- 1721 (252), 1724 (140), 1726 (112), 1727 (145), 1737 (133), 1772 (134), 1779 (153), 1786 (37), 1789 (254)	90	Ratchet mechanisms ( <i>N1-N3, N6-N13, N15-22, N24-N26, X1-X4, X6-X12; 50, 51, 64-66, 117</i> )
18	1800-1833 (17), 1837 (1833 (17)) ----- 1827 (243), 1829 (47), 1830 (151), 1831 (45)	34 (+17)	Jointed couplings and coupling mechanisms ( <i>P1-P5, Z1-Z7; 55</i> )
19	1901-1913, 1915	14	Belt and rope trains ( <i>61, 106-115</i> )
20	2000, 2001-2008, 2012, 2015, 2016, 2025-2027, 2030-2032, 2035, 2038- 2040, 2050, 2051 (2001, 2031, 2039, 2050)	24	Mechanisms with more than one d.o.f. and spatial mechanisms
21			Unknown
22	2201 (2201)	1	Dwell mechanisms
<b>Total</b>	(21 (+17))	755 (+17)=772	



**Fig. 14** Hoecken photos showing genuine Reuleaux models: oblique double slider ellipse and straight-line linkage 306 (*S04*), two coupled spur-gear sets for reversed motion 823 (*G07*), involute gear tooth profile 541 (*Q04*), positive return cam mechanism 1634 (*L06*)

The capture of Fig. 14 also contains the model numbers (in italics) that are used in the Voigt catalogue. The four genuine Reuleaux models shown have different stands. With two of them, namely no. 823 and no. 1634, there are brass labels affixed to the model stands—perhaps another distinguishing mark of a genuine Reuleaux model.

## 5 On the Circulation of Works

Hoecken published the results of his research activities at the end of an era which, in Germany and Europe, is called the mechanization period. His profound knowledge of mathematics and the practical experience he got as an employee in

several industrial companies were ideal prerequisites for developing and presenting useful solutions for various mathematical and measuring instruments and mechanisms. In Hoecken's time, there were 3 existing German journals that were mainly esteemed by the German-speaking kinematic community:

1. Reuleaux-Mitteilungen—Archiv für Getriebetechnik
2. Zeitschrift für Instrumentenkunde
3. Zeitschrift des Vereins Deutscher Ingenieure (VDI).

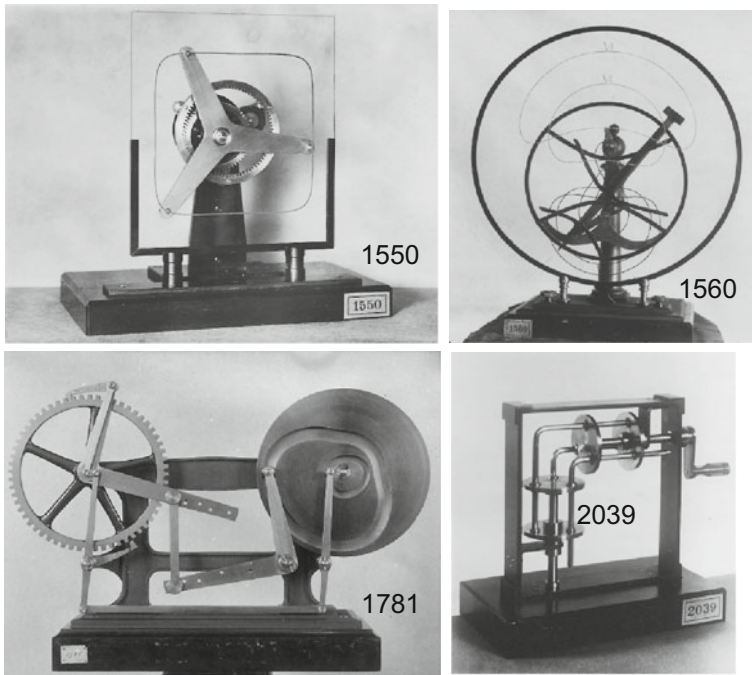
Planned publications were evaluated carefully by acknowledged experts in the field of kinematics. Hoecken's most fruitful years were those during the period of his leadership of the "Lehrstuhl für Getriebelehre" at TH Berlin from 1930 to 1934. In 1926, on the occasion of a conference on mechanisms at TH Dresden, Hoecken was invited to speak about straight-line linkages in front of 25 famous kinematicians, including Hermann Alt, Ludwig Burmester, Martin Grübler, Karl Kutzbach and Rudolf Hundhausen (Hoecken 1926). Additionally, Hoecken's works on straight-line linkages, star-wheel gears, dwell linkages, and gearwheel linkage combinations were spread in, among others, the following specialized books: Beyer (1931, 1953), Jahr and Knechtel (1930) and Hain (1967).

Hoecken's creative urge in the field of kinematics decreased considerably when he left TH Berlin in 1934.

## **6 Modern Interpretation of Main Contributions to Mechanism Design**

Hoecken lived and worked in a period when computers were based on mechanical elements, say, mechanisms and gears. They were taken as mechanical auxiliary devices, working slowly and able to calculate only through the use of the four fundamental operations of arithmetics. Trigonometric or logarithmic functions had to be replaced by proper numerical progressions. Hoecken's education and studies in the field of geodetics strongly helped him to overcome mathematical difficulties. His interest and practical talent for mathematical and optical instruments grew parallel to his employment in industrial companies. Following the list of his patents, we learn that Hoecken became an expert in mechanical calculating machines and computing mechanisms (Hoecken 1909, 1911a, 1913). Geodetics and his excellent knowledge of the mechanics of calculating machines formed the basis of Hoecken's access to kinematics. The combination of linkages with lower pairs (surface-contact) and gearwheels with higher pairs (curve-contact) attracted him especially. And indeed, the use of both types of mechanisms, i.e., geared linkages or linkages with profile links (cam type), makes kinematics more interesting and enlarges the spectrum of applications.

Graphical methods in kinematics (and statics) were familiar to Hoecken, but he did not use them. Instead, he preferred nomographical methods, i.e., the design of



**Fig. 15** Hoecken photos showing non-genuine Reuleaux models: hypocycloidal gearwheels for triple square tracing 1550 (Hoecken), four-bar linkage with replacing centrodes 1560 (Hartmann), ratchet mechanism with cam and gearwheel 1781 (Hundhausen) spatial rectangular shaft coupling 2039 (Hoecken)

charts or nomograms. Nomograms show mathematical equations in a graphical form, and thus, replace tedious numerical tables. Admittedly, nomograms are normally less precise than tables, but they have more clarity (Schwerdt 1924; Svoboda 1948). Perhaps Hoecken’s masterpiece in regard to using nomograms in kinematics is his paper on star-wheel mechanisms (Hoecken 1930b) with a triple set of the main design angles for the star-wheel (output) and its driver (input). It must be pointed out that Hoecken’s nomographical approach when evaluating analytical expressions in kinematics was also taken up later by Hermann Alt and Johannes Volmer in order to find the proper dimensions of four-bar linkages for given geometric conditions (Alt 1941; Volmer 1958).

Hoecken’s paper on a conchoidal straight-line linkage (Hoecken 1933a) was part of some straight-line linkage ideas which Hoecken presented in the course of lectures and national conferences on kinematics. Some straight-line linkages with special dimensions still have the supplement “nach Hoecken (by Hoecken)” today, and—for example—Hoecken’s fantastic “ellipsograph” (Hoecken 1933b) is even mentioned in a modern book of geometry (Glaeser 2005).



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