Allievi Lorenzo (1856–1941)

Marco Ceccarelli

Abstract Lorenzo Allievi is best known for the Water hammer solution, which he proposed in 1903. But he also contributed to Mechanism Design with the milestone work 'Cinematica delle Biella Piana' (Kinematics of Planar Couplers), published in 1895 as an original work from application of the Burmester Theory. His lifetime's professional activity cemented him as a Captain of Industry, as he experienced success in many Italian enterprises and organizations.

1 Biographical Notes

Lorenzo Allievi, Fig. 1, was born in Milan on 18 November 1856 and died in Rome on 30 October 1941.

He was the son of Francesca Bonacina Spini and Antonio Allievi, who was a Senator in the Italian Parliament in the recently established Italian Kingdom. Lorenzo started school in Como but when his father was appointed Senator in 1871, the family moved to Rome where he completed college and got an Engineering degree on 24 October 1879. His thesis on 'Internal equilibrium of metallic pylons according to elastic behavior', Fig. 2, was also published in 1882 in Rome and was circulated successfully in Italy, as indicated by the fact that it is stored in the libraries of several Italian Royal Schools of Engineering. He received a grant

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Fig. 1 Portrait of Lorenzo Allievi (1856–1941)

as a visiting scholar in Germany and was subsequently appointed to a temporary position at the Royal School of Engineering in Rome, where he mainly worked on studies of TMM. During this period he also put effort into other design problems, such as the Metro system in Rome and the railway line to Castelgandolfo.

On 31 August 1885 Lorenzo Allievi married Anna Brenna, who later gave him three children: Francesca, Raimondo, and Antonio.

In 1893 he left Rome to take the position of Director of the industrial enterprise 'Risanamento di Napoli' in Naples, where he promoted industrial development until 1901, after which he came back to Rome. There, he took up several positions in a number of industrial enterprises (Carburo Calcio, Risanamento della Romana Gas, Anglo-Romana, Terni, Romana Elettricità, Banca Commerciale, Meridionale di Elettricità, Electrochimica, Saline Eritreee), resulting in his appointment as President of the Association of User Electrical Companies. This success led to further appointments as President of the Industrial Union of Region Lazio and later Vice-President of the Italian Industrial Union.

Particularly interesting is his activity at the Elettrochimica company, for which he designed plant enlargements in Popoli, although primarily he studied problems in the plant at Papigno in Terni, where in 1902 a hydraulic pipe exploded, causing great damage to the structure. Following that, Allievi put considerable attention towards the study of perturbed motion of water in pipelines, working mainly at night after his daytime duties for the industrial companies. He often remained at home, surrounded by the smoke of his cigarettes, absorbed in the study of hydraulic phenomena in an attempt at rigorous formulation for design and operational purposes.

The study of Hydraulics was always of interest to him, even after he addressed the problems in the Papigno plants by solving the regulation of the Water hammer, as detailed in his first publication in 1902, reprinted in 1903. Figure 3 shows the

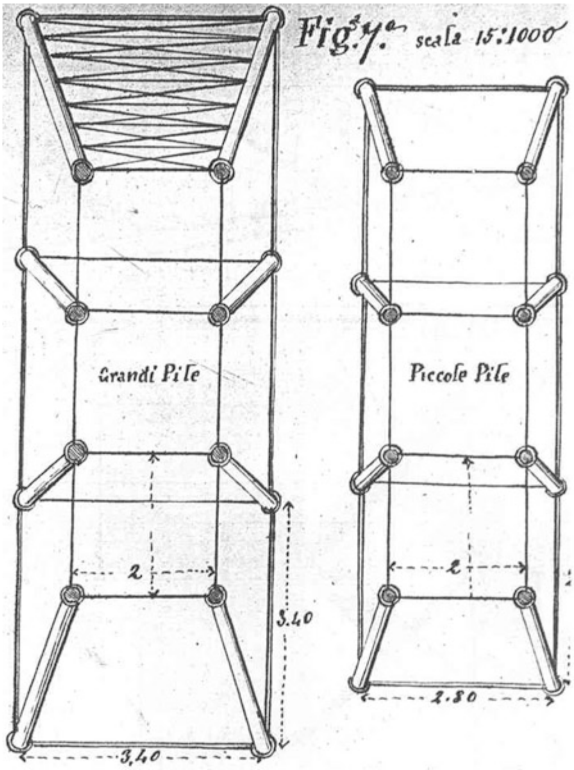


Fig. 2 Schemes of pylons in the thesis for an Engineering degree by Lorenzo Allievi (Courtesy of Mirta Lancellotti)

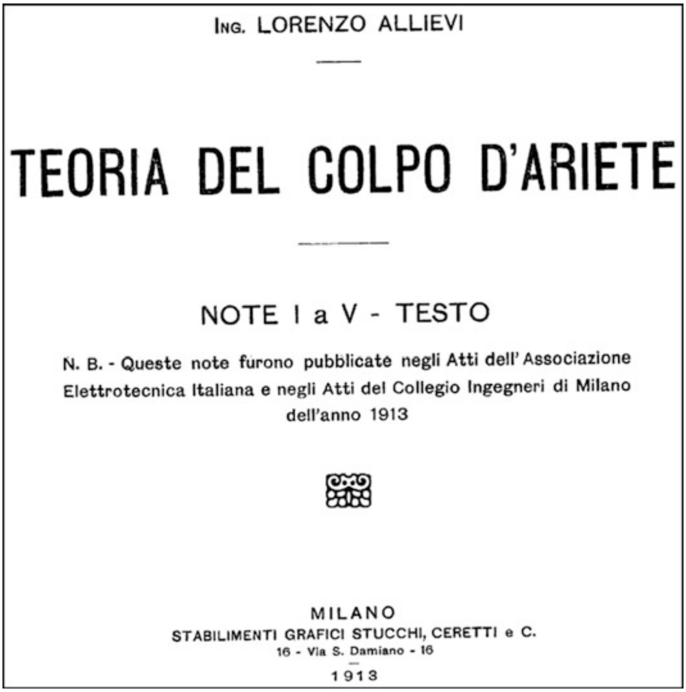
title page of the most widely distributed version from 1913. He continued to work on the theory of the Water hammer but never again considered problems of the Kinematics of mechanisms, the subject of his first scientific publication (Allievi 1895).

In his activity as an engineer and industrial manager, he also always paid attention to the satisfaction of the employers, since he considered the work in its entirety to be fundamental for achieving the scheduled goals of the company and the job being undertaken. Since he was also involved in economic aspects, Allievi addressed subjects of Finance in articles that were published later in 1918 in Rome in a volume entitled 'Spunti polemici di attualità'.

Despite all of this, he never neglected his family, to whom he dedicated attention and time, mainly in the holy day periods in Anzio (Fig. 4).

Lorenzo Allievi enjoyed success as both a professional engineer and industrial director. But the activity that brought him international fame was his scientific study of the Water hammer, which he addressed in several publications from 1902 until 1936 (Allievi 1902, 1913, 1932, 1933, 1934, 1936), a subject he was still investigating when he died in 1941.

In the hydraulic plant in Papigno, a large marble plaque stands as a monument to his contributions (Fig. 5) (ENEL 1996).



Ing. LORENZO ALLIEVI

TEORIA DEL COLPO D'ARIETE

NOTE I a V - TESTO

N. B. - Queste note furono pubblicate negli Atti dell'Associazione Elettrotecnica Italiana e negli Atti del Collegio Ingegneri di Milano dell'anno 1913

MILANO
STABILIMENTI GRAFICI STUCCHI, CERETTI e C.
16 - Via S. Damiano - 16
1913

Fig. 3 Title page of the 1913 publication of the theory of water hammer by Lorenzo Allievi

His approach, still known today as Allievi's Theory, gained him several prizes, both in Italy, such as the Jona Prize for Industrial Engineering achievements, and abroad, including immediate translations of his publications into French, German, and English. Significant is the Award that ASME, the American Society of Mechanical Engineers, gave him as a recipient for Honorary Membership in 1937 (ASME 1937), during a period of great international tensions before the Second World War.

Today, he is still honoured, his name having been bestowed upon technical schools and even streets in several Italian cities, particularly in Rome and Terni.

Several biographies have been written on Lorenzo Allievi from several viewpoints and at different dates, such as (Angelini 1992; Anonimus 1952; Marchetti 1941; Enciclopedia Italiana 1960; Marzolo 1942; Evangelisti 1956; Roger 1995; Roger Allievi 1980).

2 List of Main Works

Lorenzo Allievi published few works, since his primary activity was in Industrial Management. The most important ones, listed in the references with bibliographical data, are:

Fig. 4 Lorenzo Allievi with his grandchild AnneMarie in Rome during celebration of her first communion on 9 March 1929 (Courtesy of Mirta Lancellotti)

- Cinematica della biella piana (Kinematics of Planar Couplers), published in 1895.
- Teoria del colpo d'ariete (Theory of Water Hammer), first published in 1902 and then in several other more complete publications up to 1936.

3 Review of Main Works on Mechanism Design

In this chapter, the focus is on Mechanism Design, and therefore we will only address Allievi's book on Kinematics of Planar Couplers, whereas the presentation and historical significance of his work on the Water Hammer is celebrated in other

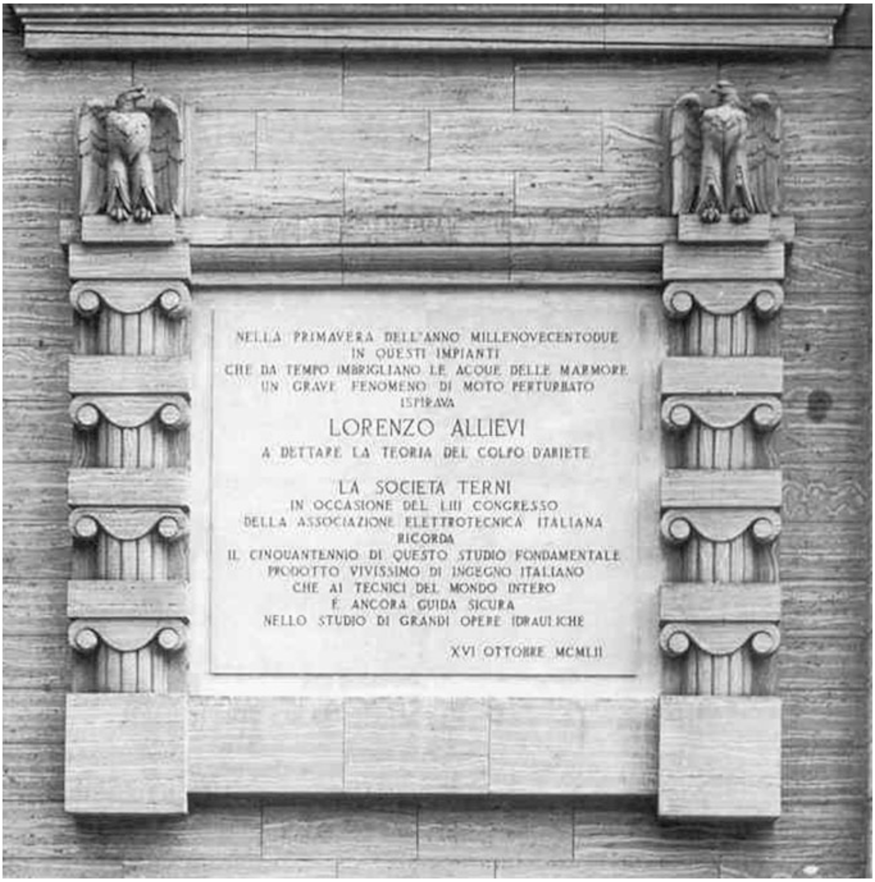
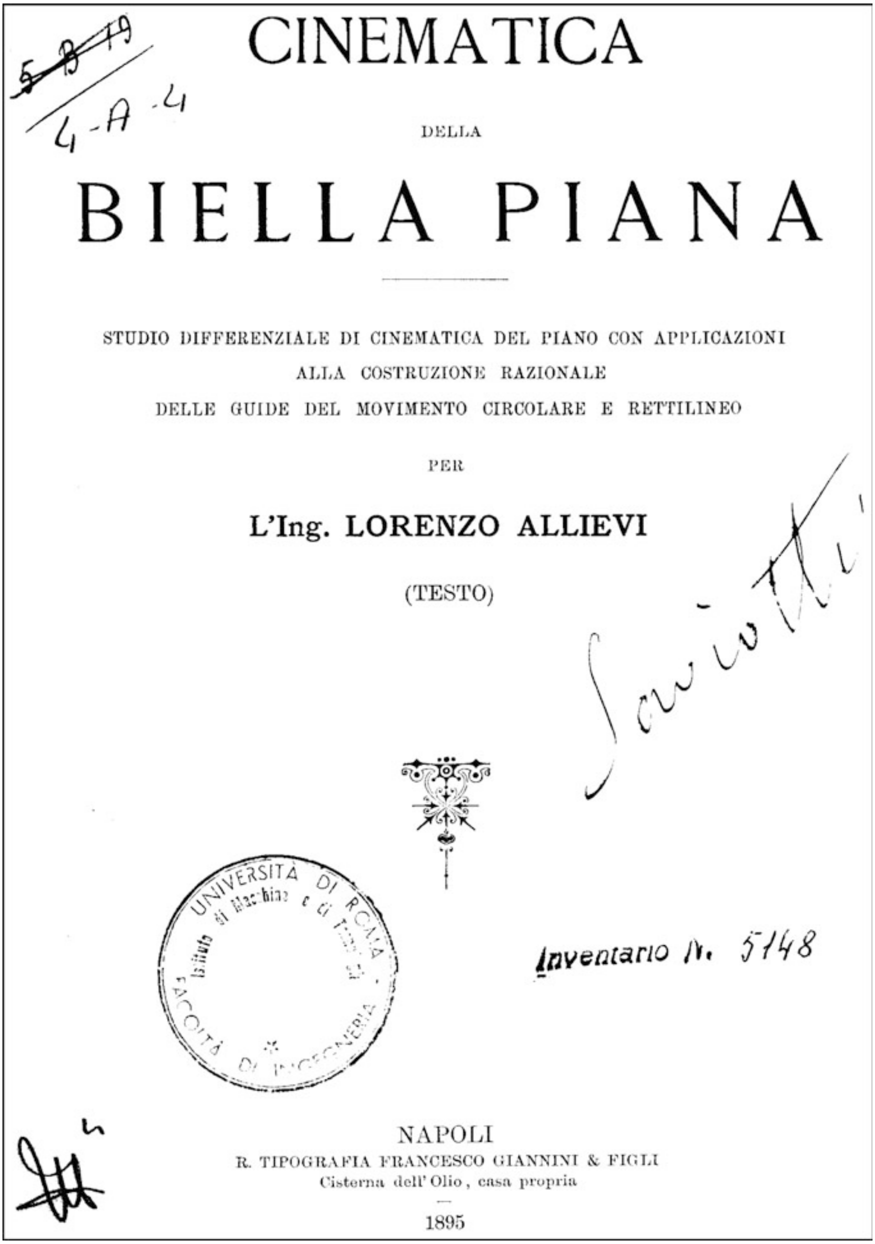


Fig. 5 The marble plaque acknowledging Allievi's contributions to the water hammer at Papigno plant in Terni

specific publications on the History of Hydraulic Engineering, like for example in (Anderson 2000).

Allievi wrote the treatise 'Cinematica della Biella Piana' (translated as Kinematics of Planar Couplers), Fig. 6, in Rome in 1892, most likely as a consequence of his experience in Germany, but he only published it in Naples in 1895 after he had already left his academic position. In several of Allievi's biographies, this treatise is considered to be a minor work and is often not even cited.

The treatise (Allievi 1895) is presented as a survey of Kinematics of planar motion as applied to mechanism design with a specific reference to the works (Burmester 1888 a,b) and (Schoenflies 1886) , as well as an original contribution by Allievi himself. Allievi refers to Burmester's book with the date 1886 (instead of 1888), since he probably got an early edition of the book during his stay in Germany.



CINEMATICA

DELLA

BIELLA PIANA

STUDIO DIFFERENZIALE DI CINEMATICA DEL PIANO CON APPLICAZIONI
ALLA COSTRUZIONE RAZIONALE
DELLE GUIDE DEL MOVIMENTO CIRCOLARE E RETTILINEO

PER

L'Ing. LORENZO ALLIEVI

(TESTO)

NAPOLI
R. TIPOGRAFIA FRANCESCO GIANNINI & FIGLI
Cisterna dell'Olio, casa propria

1895

Fig. 6 Title page of treatise "Cinematica della biella piana" by Lorenzo Allievi as published in 1895

The treatise by Allievi is organized into seven chapters: the first five chapters introduce a general theory and the last two chapters are related to the application of the theory in design solutions of mechanisms. In the preface, Allievi stresses the novelty of his work, both with theoretical arguments and design applications for a rational classification of mechanisms for planar motion and particularly for approximate straight-line and circular guides.

In the Introduction, a survey is presented on the correlation between points and lines as generators of geometric loci in planar motion. Characteristics of coupler curves are analyzed in terms of singularities through a mathematical characterization from Differential Geometry and a graphical characterization from Descriptive Geometry. This systematic analysis gives a complete classification of stationary singularities in coupler point trajectory that are called cusps (cuspidi in Italian), inflections (flessi in Italian), cuspidates (cuspidazioni in Italian, which are arcs due to p cusps with infinite curvature or very short cusps), undulations (ondulazioni in Italian, which are due to q inflections with long inflected trajectory or with zero curvature), falcates (falcate in Italian for the sickle shape, which are due to p = q as cusps with finite curvature and concavity of trajectory branches that are oriented in the same direction), and their iper-shapes as a function of the order p of cusp and degree q of inflection, as well as a function of their generation and shape. This classification is clearly summarized in the Table shown in Fig. 7. This classification is still a novel way to classify mechanisms in a very elegant and general way for planar mechanisms.

In the first chapter, there is a survey of theories on trajectory curvature; the circles of inflections and cusps are introduced; and an expression for curvature analysis is derived from a quadratic transformation that can be useful for a new synthetic classification of mechanisms for trajectory generation. Formulations are presented in simple expressions all throughout the treatise by means of synthetic methods that nicely mix approaches from Analytical Geometry and Descriptive Geometry.

Although the treatise is directed to four-bar linkages, Allievi approaches the generality of planar motion by also considering mechanisms that can be derived from four-bar linkages when their fixed and mobile joints are constrained on suitable trajectories by modelling different planar kinematic chains. Besides the common revolute and prismatic joints, he defined as head-cross (testa-croce in Italian) a joint with straight-line mobility when it is connected to a fixed joint that is located at infinity and as link-block (glifo in Italian) a joint whose center of motion is at infinity. He identifies six families of elementary mechanisms that are the basis of the study and are represented in Fig. 8, reproducing Figs. 10–13 of the treatise, namely four-bar linkages, slider-crank mechanisms, crank-slider mechanisms, slide-cross-head mechanisms, cross-sliders mechanisms, and the so-called Oldham Joint.

For each mechanism type, a simple graphical procedure is outlined to determine the circles of inflections and cusps, which are useful for computing the curvature of any point of the mobile plane through the Euler-Savary equation.

The second chapter deals with Kinematics of two infinitesimal movements. A calculus of the curvature variation gives a mathematical characterization of the

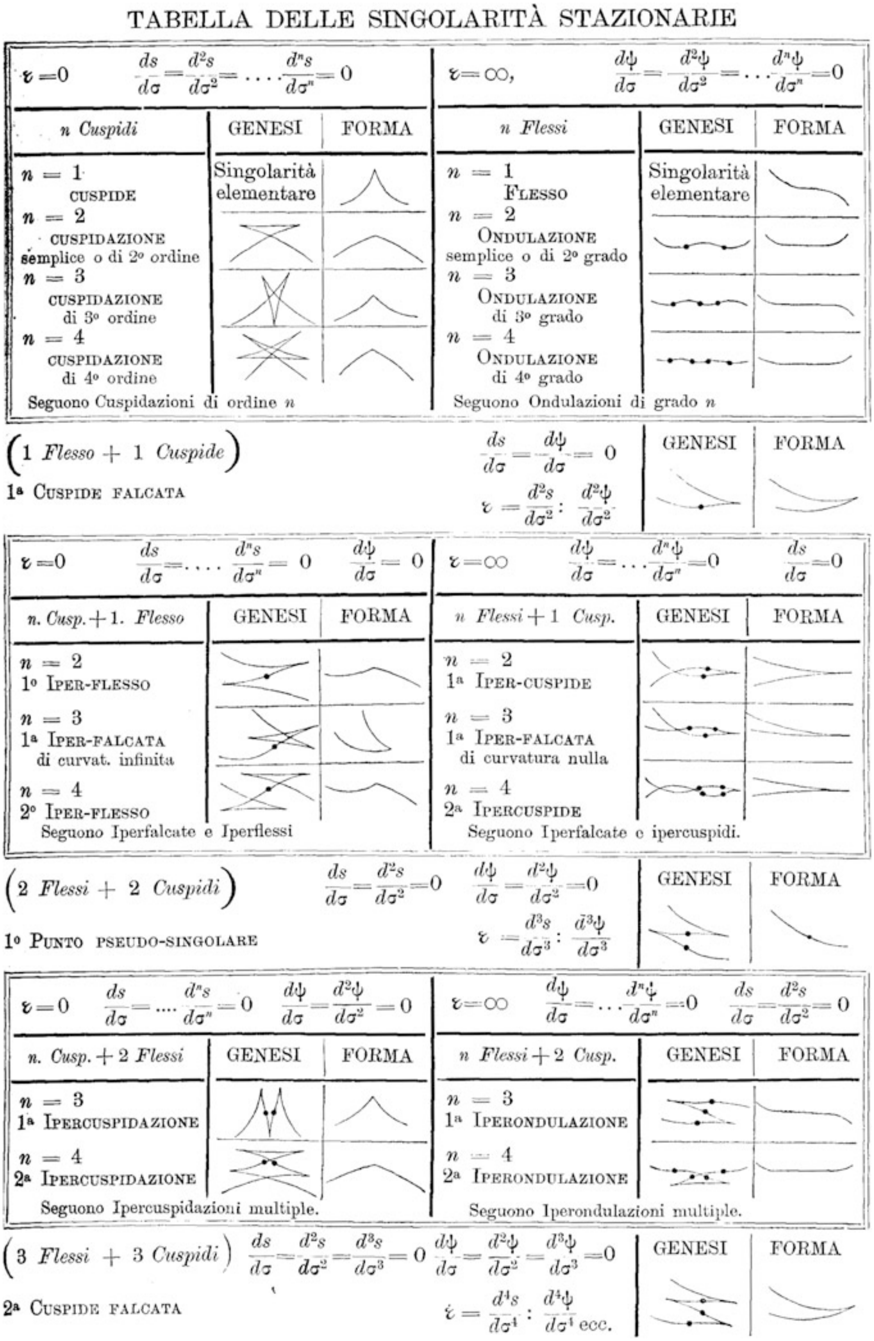


Fig. 7 Table summarizing stationary singularities in planar coupler curves from Allievi's treatise

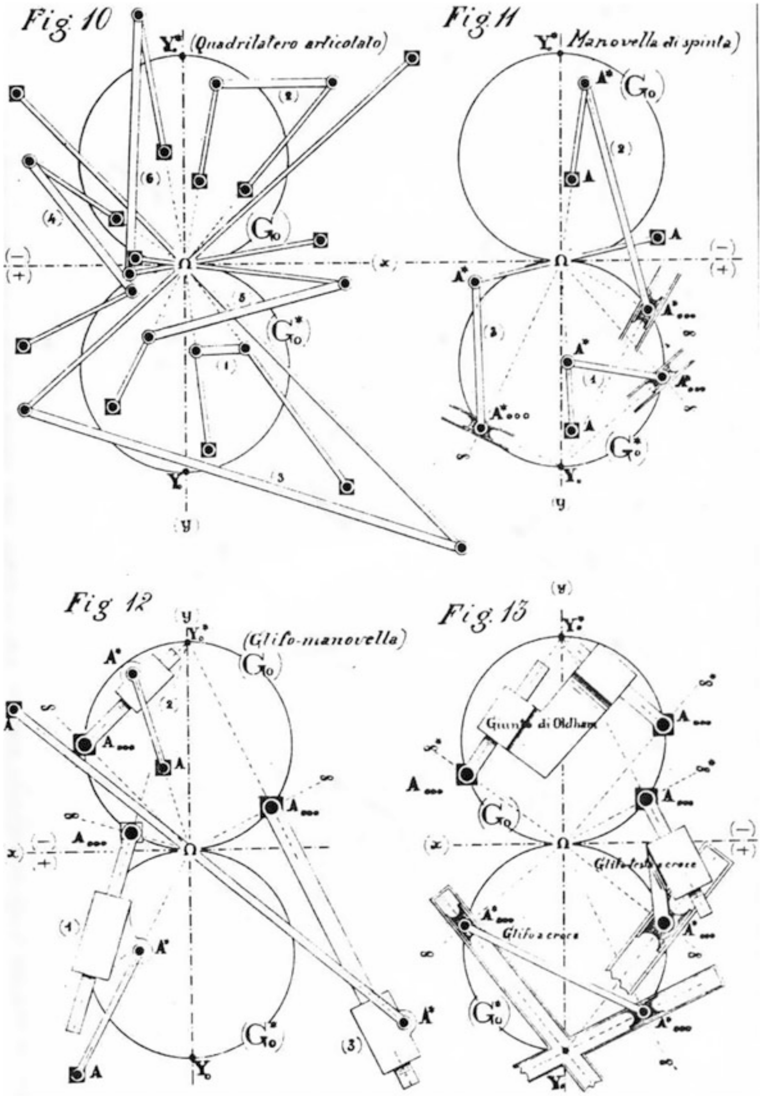


Fig. 8 Six elementary mechanisms for generation of planar coupler curves from Figs. 10 to 13 in Allievi's treatise

circles of inflections and cusps as loci of trajectory points with stationary curvature and of curvature centers of trajectory points with stationary curvature, respectively. The example in Fig. 9, reproducing Fig. 27 of the treatise, graphically illustrates such a characterization.

The loci of the points with stationary curvature in a fixed plane and a mobile plane can be expressed as the cubics in Eq. (12) of the treatise written as

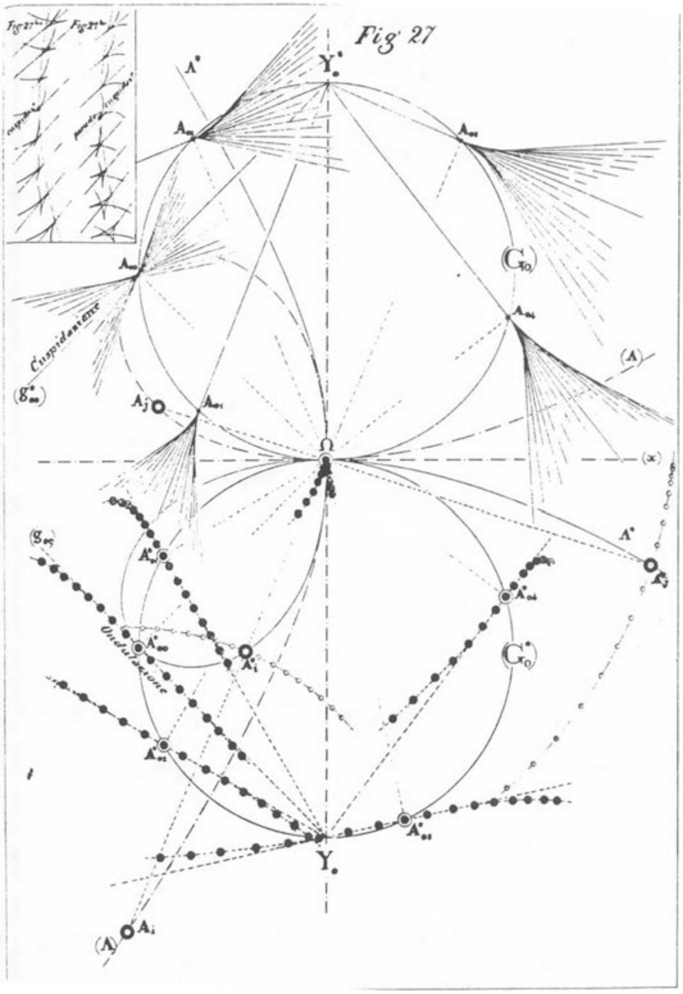


Fig. 9 Graphical representation of characteristics of points of inflection and cusp circles, from Fig. 27 in Allievi's treatise

$$\left(x^2+y^2\right)\left(\frac{1}{Ry}+\frac{1}{Sx}\right)=1$$
$$\left(x^{*2}+y^{*2}\right)\left(\frac{1}{Ry^*}+\frac{1}{Sx^*}\right)=1 \tag{1}$$

in which R and S, R* and S* are coefficients representing diameters of osculating circles in the acnode of the cubic, as shown in Fig. 10, which reproduces Fig. 22 of the treatise.

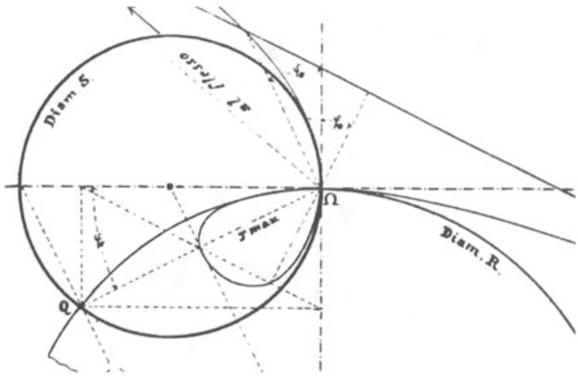


Fig. 10 Graphical interpretations of coefficients in the cubic of stationary curvature, from Fig. 22 in Allievi's treatise

In addition, manipulating the cubic expressions of these loci in Eq. (1), Allievi gives a proof and lemmas for kinematic characterizations of undulations and cuspidates that are correlated to the continuous motion, explaining: 'the loci of successive points of undulations and cuspidates are the loci of the successive intersections among the inflection circles and cusp circles, respectively, for successive motions' (pg. 43). Those mathematical arguments finally led to graphical procedures of a generation of the loci 'by means of the use of squares only' (pg. 48), as outlined in the construction of Fig. 17 of the treatise.

In the third chapter, Allievi extended the study to the case for three infinitesimal motions in order to characterize so-called pseudo-undulations and pseudo-cuspidates that are points with stationary curvature with multiple contacts with osculating circles. This characterization is obtained by discussing Eq. (20) up to its form (23) in the treatise, which are additional manipulations of Eq. (1), reproducing Eq. (12) of the treatise. In particular, in this short chapter, Allievi has shortened the original heavy treatment of instantaneous Kinematics by Burmester, extended by Schoenflies, to the case of continuous motion for determining the four cyclic points. In fact, Allievi outlines a handy procedure for graphical constructions by using analytic differentiation of (r–r*) with r and r* radii of polhodes, leading to Eq. (20) of the treatise.

In chapter four, degeneration of the loci of points with stationary curvature is discussed by using the cubic expression in Eq. (1) (Eq. (12) in the treatise) from chapter two. Degenerations into circles and straight-lines are analyzed through conditions on the cubic coefficients and corresponding kinematic relations for the motions they represent. Five classes of degenerated mechanisms are identified, as reported below.

In the first class, for $1/S = 0$, each cubic becomes a circle and a straight-line giving three series of mechanisms depending only on the location of their joints. In the first series, the relative location line of fixed joints gives only four-bar linkages, the cranks being convergent, crossed, or diverging. These mechanisms can show pseudo-undulations and pseudo-cuspidates or double undulations and double cuspidates. In the second series, with joints on circle and line, mechanism types are related to crack position giving four-bar linkages with two followers and slider-crank mechanisms, as shown in Fig. 32 of the treatise, with the possibility of having pseudo-undulations or pseudo-cuspidates expressed by a simplified expression of the cubic in the form of Eq. (31). The third series, with joints on a line only, is composed of mechanisms of the previous series at dead-lock configurations.

In the second class, with 1/S = 1/R = 0, a duality of series is identified as corresponding to the case in which a cubic degenerates into either the inflection circle or the cusp circle with a line joining their centers. In this class, there is a great variety of mechanisms with symmetric and asymmetric motion capability. Those mechanisms with symmetric motions are related to the possibility of having symmetrical motions of cyclic and paracyclic types. All the mechanisms can show several types of stationary singularities that are discussed with mathematical and graphical characterizations by using algebraic manipulations of Eq. (20) and illustrations from Figs. 34 to 45, which are then summarized in a synoptic view from pg. 92 to pg. 98 of the treatise.

A third class is identified by the condition 1/R = 0 or 1/R* = 0, which corresponds to the case in which one of the loci does not degenerate and the corresponding mechanism types are characterized as having pseudo-cuspidates and pseudo-undulations in the coupler curves. Those mechanisms are illustrated with their typical structures in Figs. 46 to 49 of the treatise.

In the fifth chapter, a fourth class of mechanism is introduced as deduced from degeneracy of the cubics due to the location of the instantaneous center of rotation at infinity. Therefore, possible mechanisms like four-bar linkages and slider-crank mechanisms are configured with parallel cranks, as shown in Figs. 52–54 in the treatise. The corresponding coupler curves are characterized by having iper-falcates and iper-undulations.

In the last two chapters, detailed analyses are reported for mechanisms with coupler curves for approximate circular and straight-line guides, respectively. The discussion is also focused on practical design with constraints using the proposed classification in classes and series. Practical solutions for design with very detailed graphical representations are shown in Figs. 56 to 107, referring to the last part of the treatise.

An example of the rich graphical details is shown in Fig. 11, which reproduces Fig. 89 of the treatise for a case of straight-line guide mechanism in the second class as an example in which the coupler curve of point Ω shows a falcate.

In particular, at the beginning of chapter six, the mathematical structure of synthesis problems is formulated and discussed in terms of available equations and conditions in order to make it possible to determine the eight design parameters that correspond to the coordinates of the four joints of a mechanism for planar motion. Allievi outlines that, in general, it is possible to design guide mechanisms for point trajectory up to the fourth order, and in the case of guiding two points, up to the third order so that the variety of stationary singularities in the proposed Table in Fig. 7 can be used to obtain suitable coupler curves.

As an example of the practical design-oriented approach of Allievi's treatise, the case of the Watt mechanism is shown in Fig. 12, reproducing Figs. 105–107 of the treatise.

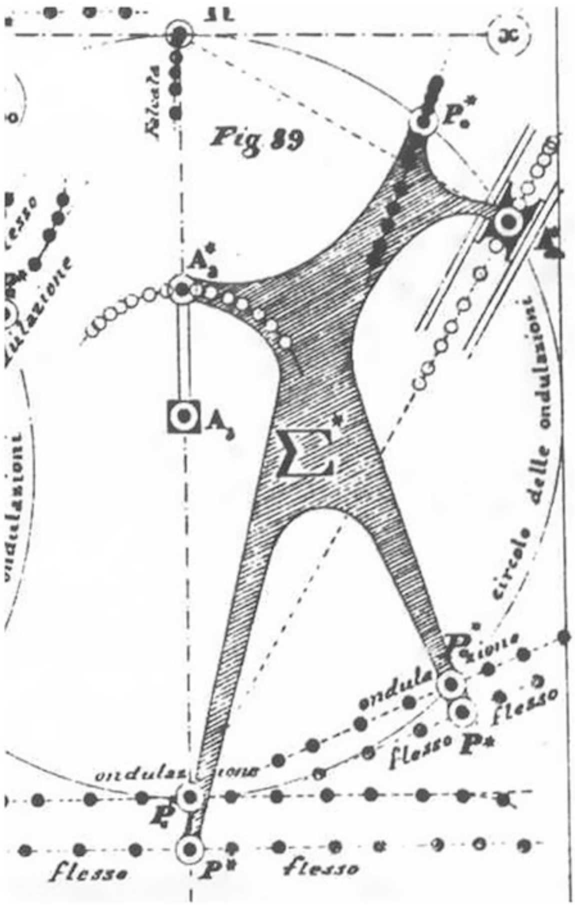


Fig. 11 An example of detailed design solutions for guide mechanisms from Fig. 89 in Allievi's treatise

The Watt mechanism of Fig. 106 is a mechanism whose coupler curve is used with three inflections coinciding at a point to give pseudo-undulations. The three inflections can be separated by means of enlarging or shortening the cranks, as stated by a proposed mathematical characterization of the corresponding mechanism series, and therefore it is possible to obtain an excursion of the approximate straight-line trajectory as long as required for a design application. In particular, considering Fig. 105 in Fig. 12 and referring to the table in Fig. 7, Allievi deduces the following proposition: 'once the inflections are separated, if the cranks are fixed with length $(m^2 + n^2)^{1/2}$, the two extremity inflections are located at a distance $\pm$ n from the central inflection, by being m the crank length and 4n the distance between them' (pg. 150). Similar considerations can be applied to a slider-crank mechanism of the fourth class to obtain the solution in Fig. 107 of the treatise as an efficient alternative to the Watt mechanism in Fig. 106, as shown in this Chapter's Fig. 12.

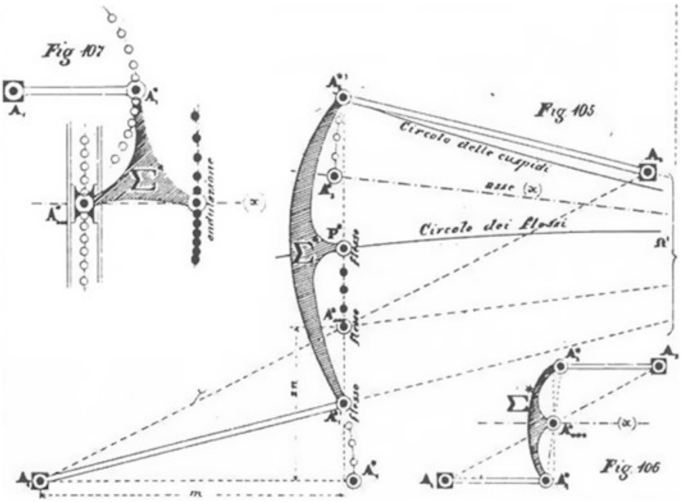


Fig. 12 Schemes and solutions for a practical approach to designing an approximate *straight-line* mechanism with a *Watt coupler curve* from Allievi's treatise

4 Modern Significance and Circulation

The work of Allievi can still be considered of modern significance and even of practical interest, both for investigation and application in Mechanism Design. In fact, the treatise on Kinematics of Planar Couplers is still known worldwide nowadays and is cited in research reports as a background or inspirational source.

After a period of oblivion, the work by Allievi has come to be considered some of the most brilliant results of the vigorous activity of Italian kinematicians in the 19th century, as pointed out in Ceccarelli (2000).

Just after its publication, the work circulated mainly within Italian circles, largely because of the language barrier. But it was soon forgotten for a long period, at least as an explicit reference. However, it was subsequently rediscovered, mainly because of international studies, and came once again to be appreciated as a significant contribution.

Since the 1940s, it has been considered as a basic reference, mainly in the German literature. For example, Richard De Jonge cited 'Cinematica della biella piana' with great emphasis in De Jonge (1940, 1943), and thus brought this reference to the attention of the U.S.A. As a result, Allievi's work was reconsidered and circulated all around the world, as indicated for example by the references to it in Hain (1967), Hunt (1978) and Nieto (1978).

The importance of Allievi's treatment has been recognized in its analytical developments, mainly for the derivation of the Burmester points in four-bar mechanisms, both from an historical viewpoint, as in (Nolle 1974), and as technical modern formulation in (Freudenstein and Sandor 1961). The work has garnered great attention and stimulated further investigations into the stationary points of the coupler points of coupler curves, even though it has not been explicitly cited.

Nowadays, 'Cinematica della biella piana' is still referenced from an historical viewpoint, as for example in (Angeles 1997, Ceccarelli 1999, 2001 and 2004). Emblematic of this renewed interest is the fact that it led to an anastatic reproduction of Allievi's treatise by CFR (FIAT Research Center) in 1999, (CFR 1999), an indication of the significance of Allievi's approach even in the modern field of industrial applications.

The work of Allievi has been influential in the development towards a modern discipline of Mechanism Design, as recognized in the edited book (Erdman 1993) celebrating Professor Freudenstein. Even today, it is considered an inspiration, as indicated in textbooks, for example, (McCharty 2000), in journal papers for example (Pennock 2008), and conference events, for example (Chicurel 2011).

5 Conclusions

Lorenzo Allievi was an engineering practitioner with a strong formational background in MMS and a life-long interest in the theory of mechanical engineering aspects. Allievi's book on Kinematics of Planar Couplers is a milestone work in modern Mechanism Kinematics with its rigorous theory, complete with applications in practical mechanism designs.

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