A Finite Element Analysis to Validate the Rule-of-Mixtures for the Prediction of the Young's Modulus of Composites with Non-circular Anisotropic Fibres

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Abstract This paper considers the rule-of-mixtures in the context of the tensile modulus of unidirectional fibre reinforced polymer (FRP) composites made with fibres of irregular cross-section, having anisotropic mechanical properties. A finite element model is used to generate data for the determination of the tensile modulus of the FRP composite. A range of degrees of anisotropy are considered. The error in the predicted modulus is found to be small for irregular fibres.

Keywords Fibres · Elasticity · Finite element analysis (FEA) · Natural fibres

Introduction

The elastic modulus of a composite parallel to the fibre direction is generally predicted using the Rule of Mixture (RoM) (Daniel and Ishai 2005; Jones 1998; Summerscales et al. 2010a, 2013), Eq. 1.

$$E_c = V_f E_f + V_m E_m \tag{1}$$

where E_c is the composite modulus in fibre direction, E_f and E_m are the fibre and matrix modulus respectively and V_f and V_m are the fibre and matrix volume fraction.

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However, the RoM has a number of underlying assumptions:

(a) a perfect bond between the fibres and matrix,
(b) the fibres are parallel, continuous, homogenous, linear elastic and regularly spaced in the composite, have uniform cross-section along the length, and
(c) the matrix is homogenous, linear elastic and void free.

Despite all these assumptions, the RoM has been successfully used to predict the modulus of synthetic fibre reinforced composites (Hyer and Waas 2000; Hull and Clyne 1996), where the fibres have a regular cross-section shape (generally circular) and variation between fibre cross-sectional areas is low. However, the Cross-Sectional Area (CSA) of natural fibres is irregular (Virk et al. 2010a), exhibits greater variation in fibre-to-fibre cross-sectional area (Virk et al. 2010a) and can vary along the length of the fibre. Moreover, natural fibres have a high degree of anisotropy in their mechanical properties (Thomason 2009) and large variability in their mechanical properties (Summerscales et al. 2010a, 2011; Virk et al. 2009a). These variations are in addition to the above assumptions made to simplify the RoM model. A Finite Element Analysis (FEA) was carried out to predict the composite modulus of a unidirectional natural fibre (jute) reinforced polymer composite and the applicability of the RoM to the natural fibre composite was assessed. The deviation between FEA and RoM predictions was quantified.

Finite Element Analysis (FEA)

A unidirectional natural fibre composite structure with a fibre volume fraction of 20 % was modelled using finite element software to predict the modulus of the composite structure. The fibre cross-section shape and area were determined from micrographs of jute fibres (Fig. 1). The sample preparation used to obtain the micrographs is given in Virk et al. (2010a). The variation in the fibre cross-sectional area along the fibre length has been disregarded as only a small length (100 μm) of fibre was simulated in FEA (to reduce FEA model size). The fibres were assumed to be arranged in uniform square packing in the matrix as the simulated fibre volume fraction was low.

Typical experimental tensile stress-strain curves for the jute fibre are shown in Fig. 2 (Virk et al. 2009a). The fibres show elastic behaviour up to failure, therefore the fibres were modelled as linear elastic. The composite was assumed to be simulated below the glass transition temperature of the matrix therefore, the behaviour of the matrix is also assumed to be linear elastic. The material properties for the jute fibres [corrected for true fibre area (Summerscales et al. 2013; Virk et al. 2012)] and the epoxy matrix used for the FEA are given in Table 1 (Anon 2014; Thomason 2009; Virk et al. 2009a, b, 2011, 2012). The jute fibres are modelled as orthotropic with transverse isotropy. The material orientation direction for the anisotropic natural fibre FEA model is shown in Fig. 3. Direction '1' oriented parallel to the

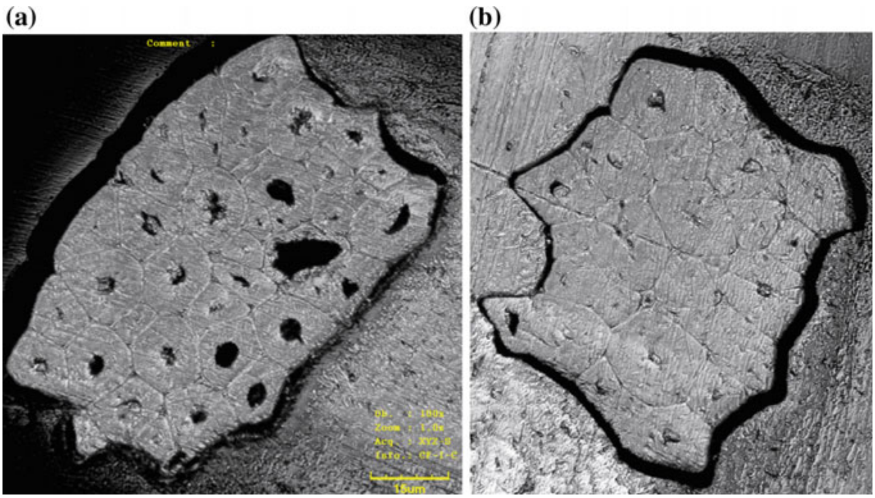

Fig. 1 Typical jute fibre cross-sections

Fig. 2 Typical jute fibre stress-strain curves

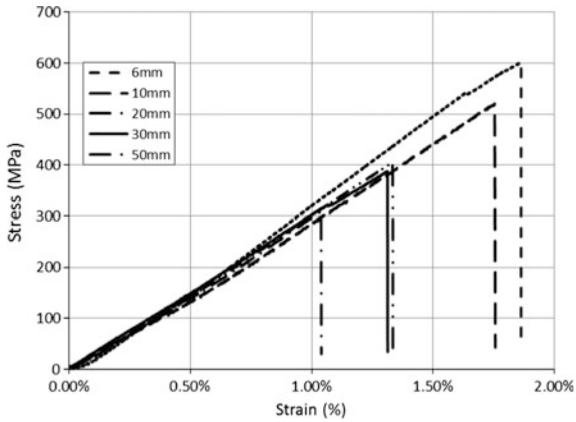

Table 1 Material properties for FEA model (MPa)

Material	E_1	E_2	E_3	ν_{12}	ν_{13}	ν_{23}	G_{12}	G_{13}	G_{23}
Jute	39,618	5500	5500	0.11	0.11	0.35	7124.1	7124.1	2037
Matrix	2650	–	–	0.35	–	–	–	–	–

Anon (2014), Thomason (2009), Virk et al. (2009a, b, 2011, 2012)

fibre principal axis (along the global Z axis) denotes the direction of E_1 fibre modulus. Fibre properties E_2 and E_3 are assumed radial symmetric, and hence numerically equal and are orientated along the radial (2 and 3) directions (global X and Y axes) respectively.

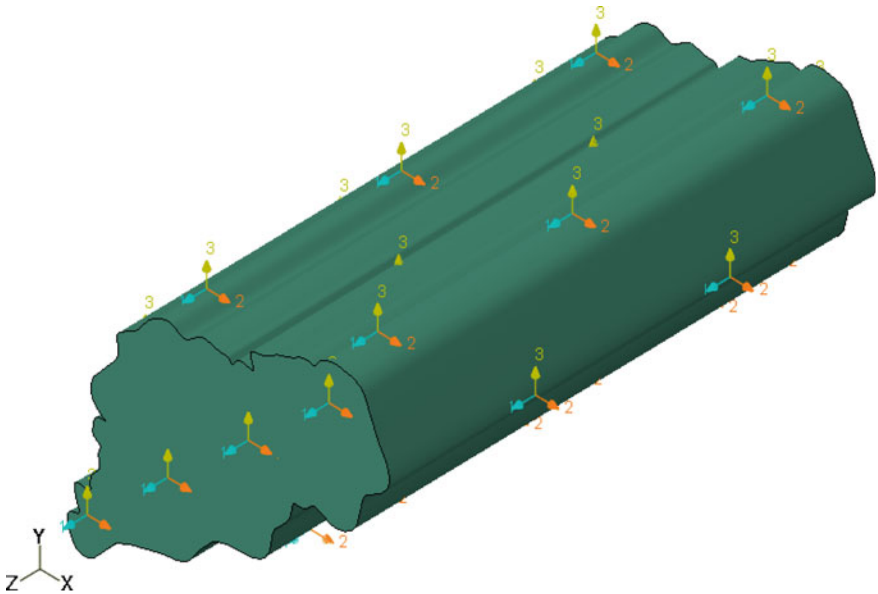

Fig. 3 Material orientation direction for the jute fibres

The interface between the fibre and the matrix was modelled using tie constraint i.e. the bond between the fibres and the matrix was assumed to be perfect. The composite representative volume was constrained using symmetrical boundary conditions along the direction normal to the faces of the cube. On each of the orthogonal axes (−X, −Y and −Z), symmetrical constraints were applied to only one face of the representative composite cube shown in Fig. 4. The composite was

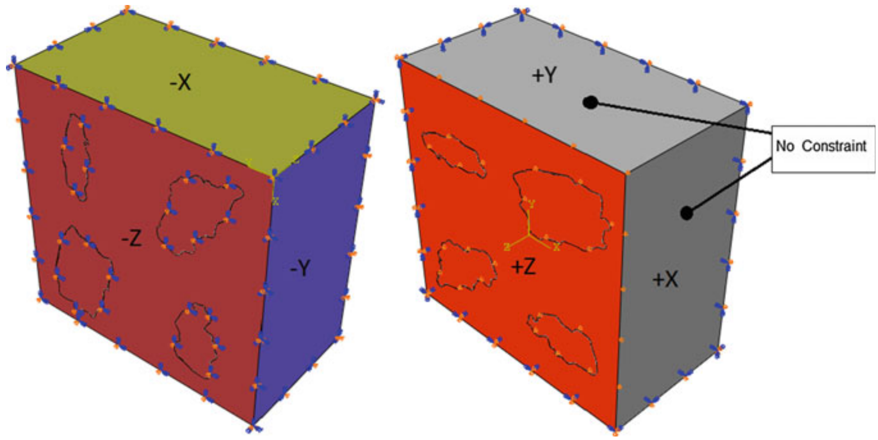

Fig. 4 Boundary condition definition for FEA model

loaded along the +Z axis (parallel to the fibre axis) with the prescribed displacement, the load was applied to the face opposite to the symmetric constrained face (−Z).

The fibres and the matrix were modelled using a mesh of 3D stress elements (Abaqus®C3D8R: an 8-node linear brick, reduced integration, hourglass control). In each case the mesh used ∼25,000 elements for the matrix and ∼4000 elements for each fibre as shown in Fig. 5.

An implicit solver was used to obtain the linear solution for the FE model. The computation time was 10 min on a Quad core 1.6 GHz computer. The reaction forces and the corresponding extension in the structure were recorded for each equilibrium iteration. These were used to calculate the average stress and strain respectively using the original composite area and fibre length. The effective modulus of the structure was calculated from the average stress and strain values generated from the forces and displacements in the FEA analysis

The FEA predicted the composite stiffness for a 20 % fibre volume fraction irregular fibre jute/epoxy composite to be 10.12 GPa. The corresponding RoM prediction (Virk et al. 2012) was 10.04 GPa. The discrepancy between the FEA and RoM predictions is 0.72 %. Thus it can be concluded that the RoM can be safely applied to predict the elastic modulus of composite reinforced by anisotropic natural fibres with irregular CSA.

To provide a baseline comparison, equivalent models were run with (a) circular cross-section fibres with equal diameters, and (b) circular cross-section fibres with varying fibre diameters (Fig. 6). The FEA derived composite modulus for circular cross-section fibres with equal and varying diameter was 10.02 and 10.01 GPa respectively. The small difference can be due to the circle being approximated by

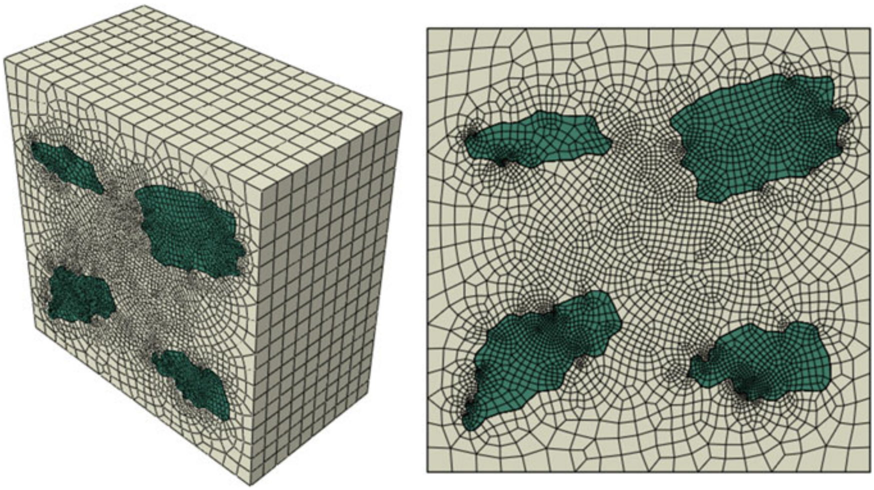

Fig. 5 FEA mesh for the natural fibre composite model

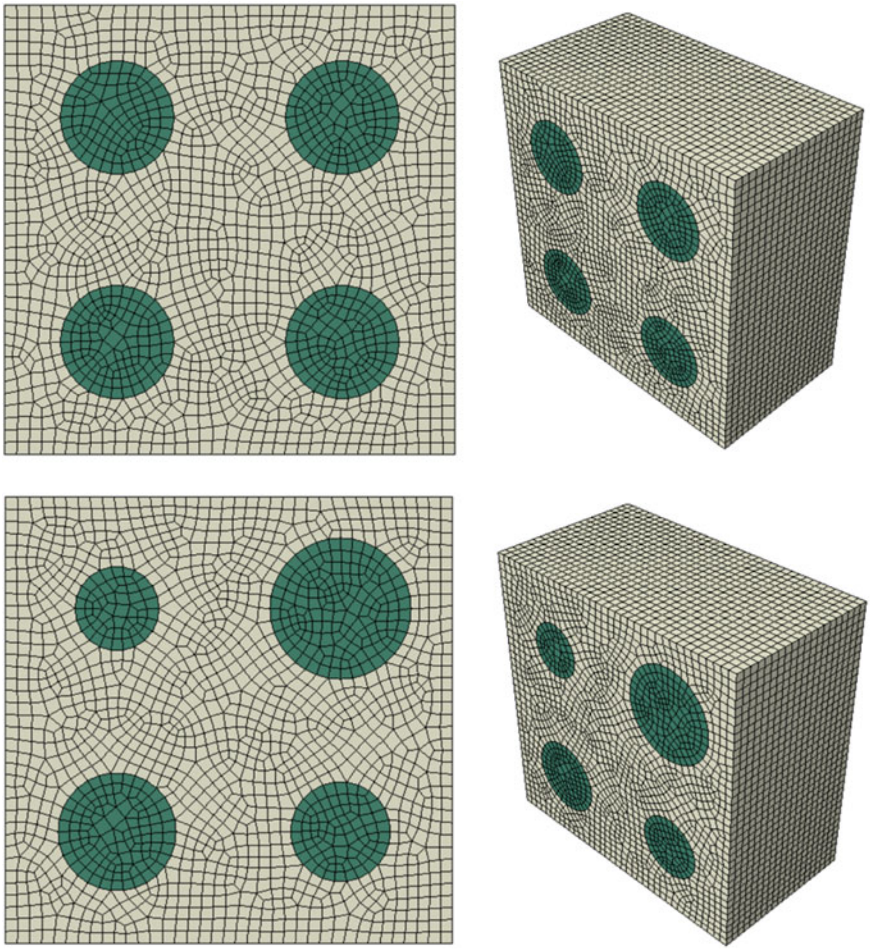

Fig. 6 FEA mesh for the circular fibre composite models

flat sided elements. Comparing FEA prediction to the RoM estimate, an error of −0.23 and −0.29 % was calculated for circular fibres with equal or varying diameters respectively.

Poisson's Ratio

It is not easy to accurately measure the Poisson's ratios of natural fibres. Therefore, to assess the effect of the Poisson's ratio on the predicted composite modulus a FEA study was carried out where the fibre Poisson's ratio, $\mathbf{v_{12}}$ was assumed to vary between 0.05 and 0.95. The first subscript '1' indicates the stimulus and the second subscript '2' response. The fibre material properties were varied such that the material stability requirements specified by the conditions of symmetry of compliances (Eqs. 2–3) and the Lemprière (1968) criteria for the limits on Poisson's ratios in orthotropic materials (Eqs. 4–5) were satisfied.

$$v_{ij}E_j = v_{ji}E_i \tag{2}$$

$$\left|v_{ij}\right| < \left(E_i/E_j\right)^{1/2} \tag{3}$$

$$E_1, E_2, E_3, G_{12}, G_{13}, G_{23} > 0 \tag{4}$$

$$1 - v_{12}v_{21} - v_{23}v_{32} - v_{31}v_{13} - 2v_{21}v_{32}v_{13} > 0 \tag{5}$$

The shear modulus of the fibre was predicted using Huber (1923) equation,

$$G_{ij} = \frac{\sqrt{E_iE_j}}{2\left[1 + \sqrt{v_{ij}v_{ji}}\right]} \tag{6}$$

The bulk modulus of the fibre was calculated using Eq. 7 (Summerscales 2000),

$$K_f = \frac{\sqrt[3]{E_1E_2E_3}}{3\left[1 - 2\sqrt[3]{v_{12}v_{31}v_{23}}\right]} \tag{7}$$

The analytical Eqs. 8 and 9 were used to predict the upper and lower bounds on the elastic modulus in the fibre direction in a unidirectional composite (Zweben 1994).

$$E_c(U) = V_fE_f + \left(1 - V_f\right)E_m + \frac{4V_f\left(1 - V_f\right)\left(v_f - v_m\right)^2}{V_f/K_m + \left(1 - V_f\right)/K_f + 1/G_f} \tag{8}$$

$$E_c(L) = V_fE_f + \left(1 - V_f\right)E_m + \frac{4V_f\left(1 - V_f\right)\left(v_f - v_m\right)^2}{V_f/K_m + \left(1 - V_f\right)/K_f + 1/G_m} \tag{9}$$

where E_c is the composite modulus in the fibre direction, U and L are upper and lower bound respectively, V_f is the fibre volume fraction, v_f is the axial Poisson's ratio of the fibre (v_{12}), v_{m} is the Poisson's ratio of the matrix, E is the axial modulus, G is the shear modulus, K is the bulk modulus and subscript f and m indicate the fibre and matrix respectively.

The fibre material properties used for the FEA study and Eqs. 8 and 9 are given in Table 2. The FEA models used for this study were same as detailed above i.e. irregular fibre cross-section and two equivalent models with round fibres, with (a) constant and (b) varying fibre diameter.

Table 2 Fibre properties for FEA model (MPa)

E_1	$E_2 = E_3$	$v_{12} = v_{13}$	$v_{21} = v_{31}$	v_{23}	$G_{12} = G_{13}$	G_{23}	K_f
39,618	5500	0.05	0.007	0.35	7246	2037	3930
39,618	5500	0.11	0.015	0.35	7090	2037	4253
39,618	5500	0.15	0.021	0.35	6990	2037	4459
39,618	5500	0.25	0.035	0.35	6752	2037	4984
39,618	5500	0.35	0.049	0.35	6529	2037	5554
39,618	5500	0.45	0.062	0.35	6321	2037	6196
39,618	5500	0.55	0.076	0.35	6125	2037	6941
39,618	5500	0.65	0.090	0.35	5942	2037	7827
39,618	5500	0.75	0.104	0.35	5769	2037	8906
39,618	5500	0.85	0.118	0.35	5605	2037	10,259
39,618	5500	0.95	0.132	0.35	5451	2037	12,013

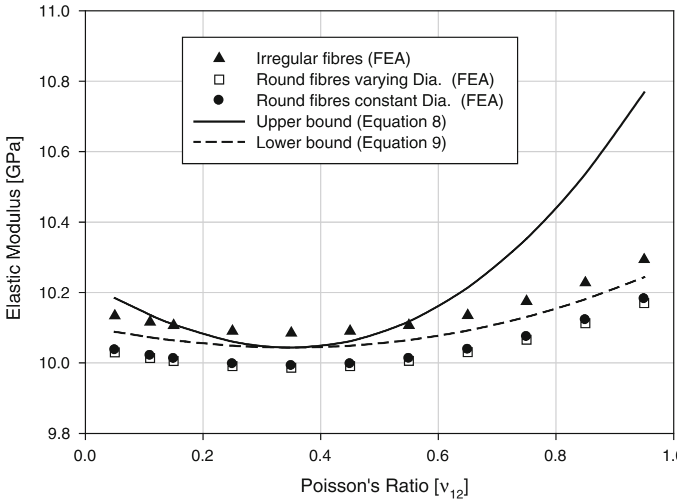

Fig. 7 Predicted composite modulus

The composite modulus predicted for different Poisson's ratio using FEA and Eqs. 8 and 9 at 20 % fibre volume fraction is shown in Fig. 7. For all the cases it was observed that with the change in the Poisson's ratio (from 0.05 to 0.95) the predicted minimum modulus occurs when the fibre axial (v_{12}) and the matrix Poisson's ratios are numerically equivalent. Note that the right hand term in Eqs. 8 and 9 goes to zero under this condition. The predicted composite modulus increases either side of this minimum. The FEA model predicts slightly higher modulus for the irregular fibres than for the circular fibres with varying or constant diameter. The difference between FEA predicted modulus for circular fibres with varying or constant diameter was small. The lower bound of the modulus predicted by Eq. 9

Table 3 Comparison of the errors arising in each of the FEA models

Poisson's ratio	Round fibres of constant diameter		Round fibres of varying diameter		Irregular cross-section fibres	
	Composite modulus (GPa)	Error (%)	Composite modulus (GPa)	Error (%)	Composite modulus (GPa)	Error (%)
0.05	10.04	−0.07	10.03	−0.14	10.13	0.90
0.11	10.02	−0.23	10.01	−0.29	10.12	0.72
0.15	10.01	−0.31	10.01	−0.38	10.11	0.63
0.25	10.00	−0.46	9.99	−0.52	10.09	0.47
0.35	9.99	−0.51	9.99	−0.57	10.08	0.41
0.45	10.00	−0.46	9.99	−0.52	10.09	0.47
0.55	10.01	−0.31	10.01	−0.37	10.11	0.63
0.65	10.04	−0.05	10.03	−0.13	10.14	0.91
0.75	10.07	0.31	10.07	0.23	10.18	1.31
0.85	10.12	0.78	10.11	0.69	10.23	1.83
0.95	10.18	1.38	10.17	1.26	10.29	2.49
Max	10.18	1.38	10.17	1.26	10.29	2.49
Mean	10.04	0.01	10.04	−0.07	10.14	0.98
Min	9.99	−0.51	9.99	−0.57	10.08	0.41

was able to capture the trend in FEA predicted modulus, but the upper bound (Eq. 9) only predicted the minimum and deviated strongly either side of this value

The discrepancy between the FEA and standard RoM (Eq. 1) predictions are given in Table 3. The difference between the two prediction methods (standard RoM and FEA) is less than 2.5 %. Thus it can be concluded that the variation in the Poisson's ratio has only small (potentially insignificant) effect on the predicted composite modulus.

Conclusions

The finite element analysis reported above shows that the RoM can be safely applied to predict the elastic modulus of composites reinforced by anisotropic natural fibres of non-circular cross-section. Further, variation in the anisotropy by changing Poisson's ratio only has minimal effect on the predicted composite modulus.

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