

# Chapter 1

## Light Distribution

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### Summary

This chapter describes (1) how light distributes within a leaf canopy and (2) light interception by leaves. Basic equations are shown so that readers can make light distribution models by themselves.

**Keywords** Leaf angle • Solar angle • Scattering • Direct and diffuse light • Leaf spatial distribution (regular, random, and cluster)

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## I. Incoming Radiation

### A. Its Total Value

The total flux of natural daylight at ground level is called Global Radiation. Outside the atmosphere the energy flux from the sun through one square meter, measured in a perpendicular position towards the solar rays, is about  $1370 \text{ W m}^{-2}$  (Monteith and Unsworth 1990). This number is called the solar ‘constant’ although it is not really a constant. Most of its variation is due to the eccentricity of the earth’s orbit around the sun which causes a regular annual variation of the flux of 3.3 % above and below its average value. The solar output itself has a much smaller variation, mainly due to sunspots.

The flux that reaches the earth surface is, of course, dominated by the day-night regime, but also it is strongly affected by absorption in the atmosphere. Absorption of ultraviolet radiation by ozone in the stratosphere weakens the flux by about 25 %. As a result, Global Radiation at ground level has a maximum of about  $1000 \text{ W m}^{-2}$  which should still be multiplied by the sine of solar height (or to the cosine of zenithal distance) to get the intensity at a horizontal ground surface. Cloudy conditions will further reduce the Global Radiation.

In contrast to the solar constant and the sine of solar height, atmospheric transmission cannot be accurately predicted. Generally it varies between about 0.2 and 0.7, but it may occasionally wander outside this range.

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*Abbreviations:*  $F$  – Fraction;  $h$  – Height;  $I_c$  – Absorbed flux per leaf area;  $I_d$  – Downward flux;  $I_u$  – Upward flux;  $k$  – Light extinction coefficient;  $k'$  – Light extinction coefficient for “black” leaves;  $L$  – Cumulative amount of leaf area per unit soil area;  $L_t$  – Total amount of leaf area per unit soil area; LAI – Leaf area index;  $N$  – Sky radiance;  $n$  – Layer; NIR – Near infrared radiation;  $p$  – Path width; PAR – Photosynthetically active radiation;  $r$  – Radius;  $S$  – Soil surface;  $T$  – Temperature in Kelvin; UOC – Uniform overcast sky; UV – Ultraviolet radiation;  $w$  – Width;  $\alpha$  – Azimuth;  $\beta$  – Solarelevation;  $\zeta$  – Angle between leaf normal and solar ray;  $\rho$  – Reflection coefficient;  $\lambda$  – Leaf angle;  $\sigma$  – Scattering coefficient;  $\tau$  – Transmission coefficient

Atmospheric transmission is really a climatic variable and it must therefore be continuously measured if we want to know its value at all times, and this is also the case in Global Radiation.

### B. Spectral Energy Distribution

The spectral properties of leaves permit us to work with only three major wavebands (McCree 1981; Gausman and Allen 1973; Gates 1980): Ultraviolet Radiation (UV) (about 4 %) between 350 and 400 nm, Photosynthetically Active Radiation (PAR) (about 50–60 %) between 400 and 700 nm and Near Infrared Radiation (NIR) (about 40 to 45 %) between 700 and 2000 nm (Fig. 1.1). Outside the atmosphere the fraction of PAR is equal to 0.368, which means a solar constant for PAR at  $504 \text{ W m}^{-2}$ . At ground level, the maximum level of PAR is reduced but not to the same extent as that of NIR and particularly of UV, resulting in a higher fraction of PAR at ground level than outside the atmosphere. Leaves absorb UV almost completely, but NIR is strongly scattered. The chlorophyll pigment, which is the most important pigment in leaves, is translucent to NIR. These facts enable a strong simplification in modelling radiation absorption in green crops. It is normally not needed to distinguish dozens of different waveband regions, but it is sufficient to just lump all PAR and lump all NIR radiation, so that we end up with only two major radiation

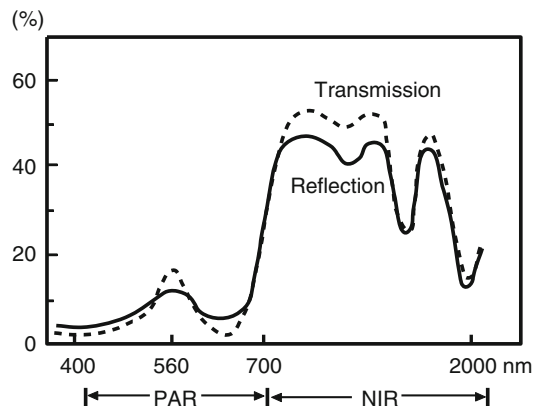


Fig. 1.1. Typical spectral distribution of reflection and transmission of a green leaf

bands in the solar spectrum. Light is the term that is usually reserved for visible radiation and it is practically identical to PAR. The most common leaf color is green, which means that leaves scatter more green light than blue and red light (Fig. 1.1). Radiation outside these boundaries is not visible nor does it have any photosynthetic effects but it still carries energy. As noted, NIR represents an energy flux of similar magnitude to that of visible radiation.

Remarkably, in non-succulent plants leaf reflection and transmission are of similar magnitude, both in the PAR and in the NIR region, although leaf color is stronger in transmission than in reflection. Unlike transmission, leaf reflection contains a colorless component of about 4 % of direct reflection on the cuticular surface. After passage through the cuticula the radiation is partly absorbed and scattered into all directions, both as transmitted and as reflected radiation.

Values for leaf reflection and leaf transmission are about 0.1 and 0.4 for PAR and NIR respectively. This means that leaf scattering is about 0.2 and 0.8 in PAR and NIR respectively for non-senescent leaves, though the former value may differ somewhat between and within species depending on e.g. leaf thickness, anatomy and chlorophyll content (Lambers et al. 1998).

### C. Directional Distribution

If light has a direction, what then is its direction? It is obvious for solar beams. The position of the sun at the sky can be precisely calculated, both in terms of solar elevation and azimuthal position (compass position), which is expressed as the angular distance with respect to the south, measured in clockwise direction. Therefore, in the northern hemisphere, north of the Tropics, the azimuth will be negative in the morning, pass through zero around noon and be positive at sunset. For the calculation of the solar azimuth and solar elevation, see Box 1.1. The algorithm, given in this Box, is also valid for the southern hemisphere and for the tropical regions (Goudriaan and Van Laar 1994).

#### Box 1.1: Solar Coordinates

Maximum solar declination is equal to the latitude of the tropics:

$$\delta_{\max} = 23.44\pi/180 \quad (\text{B1.1.1})$$

Sine and cosine of solar declination  $\delta$  depend on day number  $t_{\text{day}}$  counted from 1 January onwards:

$$\sin \delta = -\sin \delta_{\max} \cos \left[ \frac{2\pi(t_{\text{day}} + 10)}{365.24} \right] \quad (\text{B1.1.2a})$$

$$\cos \delta = \sqrt{1 - \sin^2 \delta} \quad (\text{B1.1.2b})$$

Intermediate variables  $a$  and  $b$  dependent on local latitude  $\lambda$  and solar declination  $\delta$

$$a = \sin \lambda \sin \delta \quad (\text{B1.1.3a})$$

$$b = \cos \lambda \cos \delta \quad (\text{B1.1.3b})$$

Daylength  $D_L$  expressed in hours:

$$D_L = 12 \left[ 1 + \frac{2a \sin \frac{a}{b}}{\pi} \right] \quad (\text{B1.1.4})$$

Sine and cosine of solar elevation  $\beta$ , dependent on solar time  $t_h$  (hr):

$$\sin \beta = a + b \cos \frac{2\pi(t_h - 12)}{24} \quad (\text{B1.1.5a})$$

$$\cos \beta = \sqrt{1 - \sin^2 \beta} \quad (\text{B1.1.5b})$$

Cosine of solar azimuth  $\alpha_s$ :

$$\cos \alpha_s = \frac{\sin \beta \sin \lambda - \sin \delta}{\cos \beta \cos \lambda} \quad (\text{B1.1.6a})$$

$$\alpha_s = -\arccos(\cos \alpha_s) \quad \text{if } t_h < 12 \quad (\text{B1.1.6b})$$

$$\alpha_s = \arccos(\cos \alpha_s) \quad \text{if } t_h > 12 \quad (\text{B1.1.6c})$$

The apparent size of the solar disc is very small, about half a degree in diameter, so that the solar rays are practically all parallel and we may consider the solar disc as a point source of light. Objects will then cast shadows with well-defined outlines. Diffuse light coming from other parts of the sky will enter the shaded areas and is additional to the light coming directly from the solar disc. Very small or narrow leaves, such as conifer needles, do not cast full shade on lower leaves if they are far enough apart. They are then too small to mask the solar disc completely. A rigid treatment of this partial shade (penumbra) is outside the scope of this study. A practical approximation is to consider part of the direct sunlight as diffuse radiation, in other words to increase the fraction of diffuse radiation.

Observed values of the fraction diffuse versus direct radiation are often not available, and one should then use statistical approximations, for instance on the basis of daily radiation sums (Spitters et al. 1986). The diffuse sky radiation can be measured by carefully shielding the sun using a small disc mounted on a thin bar. Ideally there should be no other shading objects that stand out above the horizon in order to make sure that the radiometer receives all radiation from the hemisphere of the sky except direct solar radiation. However, in this way we can only measure the total diffuse radiation, not how the radiance is distributed over the sky dome. As a first guess the radiance is often assumed to be uniform, the Uniform Overcast Sky (UOC) (Monteith and Unsworth 1990). In this situation azimuth does not matter. The contribution of a sky portion is proportional to the sine of its elevation (or cosine of its zenith angle) and to its size which is expressed in solid angles. In plane geometry the unit for an angle of a portion of a circle is a radian, defined as the ratio between the arc and its radius. Similarly, for an area on a sphere the unit for its size (its “solid angle” expressed in steradians, sr) is given by the ratio between the area on the sphere and the radius squared of this sphere. In this way the value of the solid angle is dimensionless and

does not depend on the size of the sphere itself.

#### D. Radiance and Irradiance

Irradiance is an energy flux that enters a surface and it is expressed in  $\text{J s}^{-1} \text{m}^{-2}$  or in  $\text{W m}^{-2}$ . Global Radiation, mentioned above, is an example. If it is confined to a spectral region, for instance UV, it is still an Irradiance, but with the specification UV Irradiance. Radiance refers to the brightness of the radiating surface and indicates the energy flux that is emitted per unit surface area per unit solid angle. For an isotropically radiating surface, the radiance is independent of the angle of view. The solid angle of a hemisphere is equal to  $2\pi$  sr. If the sky has a radiance with a value  $N \text{ W m}^{-2} \text{ sr}^{-1}$ , what will then be the corresponding irradiance on the ground surface? If we consider a small portion of the sky located at elevation  $\beta$  and azimuth  $\alpha$ , and with a very small height  $d\beta$  and width  $d\alpha$ , its solid angle will be  $\cos\beta d\alpha d\beta$ . Its contribution to the irradiance at the ground will be equal to  $N$  times  $\sin\beta \cos\beta d\alpha d\beta$ . The cosine factor arises from the fact that the sky portions will get narrower towards the zenith and the sine factor arises from the angle of incidence effect. Now, let us extend the small portion all over the azimuth from 0 to  $2\pi$ , because azimuth does not matter anyway. We then have an annular sky zone at elevation  $\alpha$  and with a solid angle  $2\pi \cos\beta d\beta$  and consequently contributing  $N2\pi \sin\beta \cos\beta d\beta$  to the ground irradiance. In order to find the total irradiance from a homogeneous dome with radiance  $N$  we have to integrate this value over a range for  $\beta$  from 0 to  $\pi/2$ . Elementary calculus shows that the result is equal to  $\pi N$ . This somewhat strange result reminds us to the fact that the unit of irradiance is  $\text{W m}^{-2}$  whereas the unit of radiance is  $\text{W m}^{-2} \text{ sr}^{-1}$ . To give an example, if the global radiation under a UOC is equal to  $100 \text{ W m}^{-2}$ , the radiance of the UOC is equal to  $31.83 \text{ W m}^{-2} \text{ sr}^{-1}$ . It is interesting to compare this value to that of the solar disc. The solar disc has a diameter of half a

degree. Its solid angle is therefore equal to  $0.0598 \cdot 10^{-3}$  sr (you can check this). Outside the atmosphere the solar irradiance is equal to  $1370 \text{ W m}^{-2}$  and therefore the radiance of the solar surface is equal  $22.9 \cdot 10^6 \text{ W m}^{-2} \text{ sr}^{-1}$ . Imagine the whole sky to be as bright as the naked sun, the irradiance would then be equal to  $72 \text{ MW m}^{-2}$  ! This value is also equal to the outgoing radiation flux at the solar surface. Using the equation for thermal black-body radiation ( $\sigma T^4$ , where  $T$  is temperature in Kelvin) we can calculate a thermal radiative temperature of  $5970 \text{ K}$ , not too far from the observed value of the temperature of the solar surface of  $5778 \text{ K}$  (p. 84 in Phillips 1992).

## II. Modelling Radiation in Leaf Canopies

We are going to develop a mathematical model for the absorption and scattering of radiation in leaf canopies. We will find expressions for the reflection and transmission by a leaf canopy as a whole, and also for the distribution of irradiation of individual leaves. Leaves differ in place and orientation and therefore their irradiation and absorption should be a function of their position in the leaf canopy and of their orientation towards the sun. These expressions for radiation absorption per leaf area are needed for the calculation of photosynthesis and transpiration, first for individual leaves, and by integration of the leaf rates also for the whole canopy. The expressions for leaf absorption will be continuous functions of canopy depth where the latter is usually defined as the cumulative amount of leaf area per unit soil area (Leaf Area Index, LAI) above a given point (Monsi and Saeki 1953). Because of the distinct character of direct light, the functions will be different for sunlit and shaded leaves.

We will now gradually derive such expressions, starting with a simple model situation of black horizontal leaves

### A. Black Horizontal Leaves

A thin layer of non-overlapping horizontal leaves with an area of  $\Delta L \text{ m}^2$  leaf per  $\text{m}^2$  of ground area will intercept a same fraction  $\Delta L$  of the incoming radiation so that the fraction transmitted will be equal to  $1 - \Delta L$ . If there are many layers of leaves, the positioning of leaves in each subsequent layer is supposed to be independent of that in other layers so that each subsequent layer will reduce the transmission by the same fraction  $1 - \Delta L$  (Fig. 1.2). If there are  $n$  such layers, one below the other, with leaf positioning at random, the downward flux just under layer  $n$  will be given by:

$$I_{d,n} = I_{d,0}(1 - \Delta L)^n \quad (1.1)$$

where  $I_{d,n}$  is the downward flux after  $n$  layers, and  $I_{d,0}$  is the downward flux above the crop.

Of course, the area of all leaf layers together is equal to  $n\Delta L$ . So far, we have considered leaves to be spaced in discrete layers but it is more realistic to assume that the leaves are homogeneously distributed. We can represent this situation by increasing the number of layers while proportionally reducing the area per layer, so that the total leaf area  $L$  is kept the same. We can write the number of layers  $n$  as the ratio  $L/\Delta L$ :

$$I_d = I_{d,0}(1 - \Delta L)^{\frac{L}{\Delta L}} \quad (1.2)$$

When  $\Delta L$  approaches zero (and the number of layers ( $n$ ) becomes very large), this expression becomes:

$$I_d = I_{d,0}\exp(-L) \quad (1.3)$$

Here, the depth ( $L$ ) in the canopy represents the leaf area above the level considered, varying between zero and the total LAI. The radiation flux that reaches the soil surface can be found by substituting the value of the LAI.

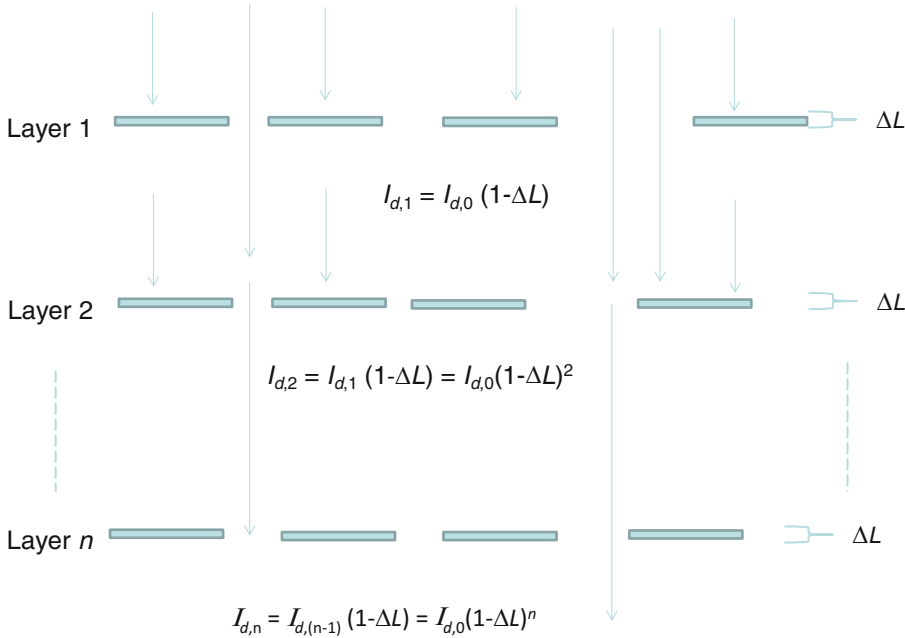


Fig. 1.2. Scheme of leaf layers and fluxes of downward and upward radiation in a model canopy with a horizontal leaf angle distribution

### B. Non-horizontal Leaves

It is an exception that all leaves are horizontal. The size of the shade cast on an underlying horizontal surface is normally not identical to the area of the shading leaf but it will differ by a factor  $k$ . For a direct beam and a single leaf orientation, the value of  $k$  will be equal to  $\cos(\zeta)/\sin(\beta)$  (Fig. 1.3), in which  $\zeta$  stands for the angle between the leaf normal and the solar rays and  $\beta$  stands for the angle between the horizon and the solar rays (“solar elevation”). By the same procedure as given earlier for horizontal leaves the equation for exponential extinction now becomes

$$I_d = I_{d,0} \exp(-kL) \quad (1.4)$$

Because of its function as a multiplier in the expression for extinction,  $k$  is generally called *extinction coefficient*. The theoretical extinction coefficient for a leaf canopy with leaves with different leaf orientations can be found by weighted addition of the values of  $\cos(\zeta)$  for the different leaves weighted

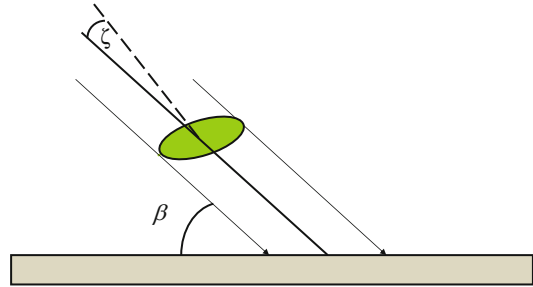


Fig. 1.3. A flat leaf, inclined with angle  $\zeta$  with respect to the solar rays. The sun has an elevation angle  $\beta$ . The size of the shade on the ground is equal to the size of the leaf multiplied by  $\cos \zeta / \sin \beta$

for their presence in the leaf angle distribution.

The exponential extinction equation is a powerful approximation of real-world radiation profiles in plant canopies. However, the conditions for which it was derived are never satisfied in practice. Therefore, empirical values of  $k$ , which can be obtained by fitting this equation to observed values of  $I_d$  versus  $L$ , may vary considerably in a range between 0.5 and 1. The physical meaning of  $k$  is the



average absorbed radiation per leaf area as a fraction of the downward radiation flux at the same canopy depth. For low solar elevation, the irradiation of the leaves may largely exceed the irradiation of a horizontal surface, so that the value of  $k$  may then be larger than unity, especially when the leaves have a dominant upright position. The description of leaf angle distribution and its effect follows next.

### C. Leaf Angle Distribution

The orientation of a single leaf can be specified by its normal, the vector perpendicular to the leaf surface, for which we need its inclination above the horizon and also its azimuth. For a horizontal leaf the normal points towards the zenith and no azimuth is needed, but in general we need the compass direction (azimuth) of the leaf normal as well, for instance West. Figure 1.3 is drawn such that the azimuthal direction of the leaf normal is precisely towards the sun, but this is in general not the case. Any orientation is possible and a full description of the leaf angle distribution consists of its statistical distribution over inclination and azimuth. We now make the important simplifying assumption that the leaves do not have an azimuthal preference, in other words they are not preferentially oriented towards a certain compass direction, even though they may have a fixed inclination with respect to the horizon. The second simplifying assumption is that their distribution of inclination is the same as that of the surface elements of a sphere. This distribution is known by different names that all mean the same thing: the spherical – the isotropic – and the random leaf angle distribution respectively (de Wit 1965; Ross 1981). In crop science it is customary to refer to the leaf angle by the inclination of its surface, not to that of its normal. Therefore the leaf angle  $\lambda$  of a vertically standing leaf is set at  $90^\circ$  ( $\pi/2$  radians), although its normal is in fact horizontal. Using this convention, the density distribution of a spherical leaf angle distribution is given by  $\cos(\lambda)$ , in accordance to the fact that

the density of the surface elements of a sphere per inclination angle decreases towards the top of the sphere.

When illuminated by a direct beam, a sphere intercepts an area that is identical to its largest cross-section, which is a quarter of the external sphere surface area. The total area intercepted by leaves with a spherical leaf angle distribution, however, is not a quarter but half of the leaf area because the leaves are initially considered not to shade each other. We could think of them as of the fragments of a shattered sphere that keep their original orientation wherever they may be. Now the ratio between the total leaf area and the corresponding shade area is identical to that of a hemisphere that is oriented towards the sun. At a solar elevation  $\beta$ , the shade that a hemisphere, precisely oriented towards the sun, casts on a horizontal ground surface, is equal to  $0.5/\sin(\beta)$  times its own area (Fig. 1.4). This ratio  $0.5/\sin(\beta)$  gives also the value of the extinction coefficient for black leaves with a spherical leaf angle distribution. It varies between 0.5 when the sun is in zenith, passing through 1 when the sun is at  $30^\circ$  elevation and getting infinitely large when the sun reaches the horizon. The canopies of several plant species have leaf angle distributions that are indeed approximately spherical (e.g. maize and sorghum), but other plant species (e.g. many grasses and Eucalyptus) tend to have more vertically inclined leaves whereas other species have more horizontal ones. The above concept of a sphere can be extended to an ellipsoid whose height to width ratio reflects the extent to which leaves are vertically inclined.

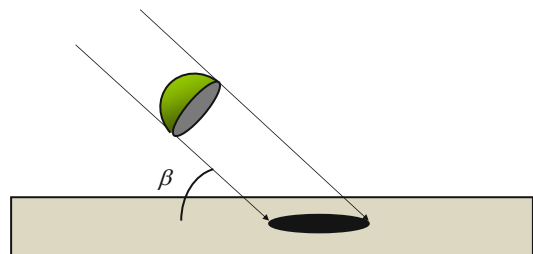


Fig. 1.4. For a spherical leaf angle distribution the relative size of the shade is equal to  $1/(2 \sin\beta)$

A description of that model can be found in Campbell (1990).

For purposes of calculation of canopy photosynthesis, a resolution of leaf angle distribution into just three equally spaced classes of leaf angle between 0 and 90° is sufficient (Goudriaan 1988).

#### D. Leaf Scattering and Canopy Reflection

Leaves are not black, which means that they reflect and transmit some of the radiation that they receive. The fraction that they reflect is called leaf reflection coefficient and the fraction that they transmit is called leaf transmission coefficient. Reflection and transmission together is called scattering. Conservation of energy requires that the sum of reflection, transmission and absorption is equal to incident radiation. When radiation is scattered by leaves it may be reabsorbed by other leaves but it may also disappear towards the sky or to the soil surface. Normally the radiation level decreases downward in a crop canopy. The soil will also receive some radiation and reflect some of it. The theory of the relation between leaf scattering and the resulting radiation profile in the canopy, and between the leaf scattering and the reflection coefficient of the canopy as a whole is based on the work of Kubelka and Munk (1931), who have written an analysis of radiation absorption and scattering in homogeneous substances such as paint. Their analysis can be applied to leaf canopies. In the most concise version of their analysis there is only one layer of leaves above a soil surface that has a reflection coefficient  $\rho_{\text{soil}}$ . The leaves are horizontal and non-overlapping, similar to the situation presented above for black leaves. They are all at precisely the same height so that they do not shade each other. Their total area is  $\Delta L$  m<sup>2</sup> leaf per m<sup>2</sup> of ground area. Reflection by leaves and soil and also transmission by leaves is assumed to be independent of the direction of the incident radiation. With these assumptions there are just two directions of radiation

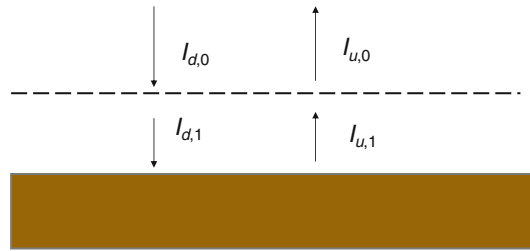


Fig. 1.5. A very sparse leaf canopy with only one layer above a soil surface. The incoming flux  $I_{d,0}$  is the starting point for the calculation of the transmitted and reflected fluxes

flux, one downward and one upward (the “two-stream simplification”) as shown in Fig. 1.5.

Out of the incoming downward radiation  $I_{d,0}$ , the leaves intercept a fraction  $k'\Delta L$ . The apostrophe is used to indicate the  $k$  value for non-scattering, thus for interception only. Subsequently a fraction  $\tau_{\text{leaf}}$  will be scattered downward and a fraction  $\rho_{\text{leaf}}$  will be scattered upward. Transmitted and scattered fluxes together constitute a downward flux  $I_{d,1}$  under the leaf layer, of which the underlying soil surface reflects a fraction  $\rho_{\text{soil}}$ . The upward flux  $I_{u,1}$  between the soil surface and the leaf layer is therefore given by  $\rho_{\text{soil}}$  times  $I_{d,1}$ . A fraction  $1 - k'\Delta L$  of this intermediate upward flux will pass the leaf layer immediately but a fraction  $\rho_{\text{leaf}}$  out of the complementary fraction  $k'\Delta L$  will be reflected back to the soil surface and likewise a fraction  $\tau_{\text{leaf}}$  will be transmitted towards the sky. The two downward and the two upward fluxes are related by equations that are now known as the Kubelka-Munk equations. Using our symbols we have for the intermediate fluxes  $I_{d,1}$  and  $I_{u,1}$ :

$$I_{d,1} = \left[1 - k'\Delta L(1 - \tau_{\text{leaf}})\right]I_{d,0} + k'\Delta L\rho_{\text{leaf}}I_{u,1} \quad (1.5a)$$

$$I_{u,1} = \rho_{\text{soil}}I_{d,1} \quad (1.5b)$$

These two linear equations with the two unknowns  $I_{d,1}$  and  $I_{u,1}$  can be solved by



algebraic elimination of the unknowns, which gives:

$$I_{d,1} = \frac{[1 - k' \Delta L (1 - \tau_{\text{leaf}})] I_{d,0}}{1 - k' \Delta L \rho_{\text{leaf}} \rho_{\text{soil}}} \quad (1.6a)$$

$$I_{u,1} = \rho_{\text{soil}} \frac{[1 - k' \Delta L (1 - \tau_{\text{leaf}})] I_{d,0}}{1 - k' \Delta L \rho_{\text{leaf}} \rho_{\text{soil}}} \quad (1.6b)$$

The upward flux  $I_{u,0}$  above the leaves is related to the intermediate upward flux and the incoming downward flux as:

$$I_{u,0} = [1 - k' \Delta L (1 - \tau_{\text{leaf}})] I_{u,1} + k' \Delta L \rho_{\text{leaf}} I_{d,0} \quad (1.7)$$

It can now be immediately expressed in the incoming downward flux alone by substitution of Eq. 1.6b:

$$I_{u,0} = \left\{ \frac{\rho_{\text{leaf}} [1 - k' \Delta L (1 - \tau_{\text{leaf}})]^2}{1 - k' \Delta L \rho_{\text{leaf}} \rho_{\text{soil}}} + k' \Delta L \rho_{\text{leaf}} \right\} I_{d,0} \quad (1.8)$$

The ratio  $I_{u,0}/I_{d,0}$  is the same as the reflection coefficient of the leaf-soil system, which we denote by  $\rho_{\text{system}}$ . We have now found the expression for the reflection coefficient of the leaf soil-system as a function of leaf and soil properties for the case of one single leaf layer above a bare soil surface:

$$\rho_{\text{system}} = \rho_{\text{soil}} \frac{\{1 - k' \Delta L (1 - \tau_{\text{leaf}})\}^2}{1 - k' \Delta L \rho_{\text{leaf}} \rho_{\text{soil}}} + k' \Delta L \rho_{\text{leaf}} \quad (1.9)$$

What about a canopy with a large LAI? In principle it is possible to add more layers, thus increasing the number of linear equations and solve them by matrix inversion, but this is cumbersome and it does not result in a neat expression. There is a better method.

### 1. The Reflection Coefficient of a Leaf Canopy with a Large Leaf Area Index

If we have a leaf canopy with a very large leaf area index, the soil underneath does not

have any effect on the reflection coefficient of the canopy. We may apply the expressions that we have just derived, to the top leaf layer of the large leaf canopy. This time however, the entire leaf-canopy has the same role as the soil surface in the earlier calculation, so that we can replace both  $\rho_{\text{soil}}$  and  $\rho_{\text{system}}$  by  $\rho_c$ :

$$\rho_{\text{canopy}} = \rho_c \frac{[1 - k' \Delta L (1 - \tau_{\text{leaf}})]^2}{1 - k' \Delta L \rho_{\text{leaf}} \rho_c} + k' \Delta L \rho_{\text{leaf}} \quad (1.10)$$

This expression results in a second order equation in  $\rho_c$ , which can of course be solved (see Eq. 1.21), but before doing so it is useful to realize that this equation is valid locally at any depth in the canopy as long as the effect of the soil surface can be neglected. In other words, well above the soil surface the ratio between upward and downward radiation fluxes is constant. This constancy of ratio will now be used.

### 2. Extinction of Radiation Within the Leaf Canopy

There must also be constancy of ratio between the fluxes going down or going up at either side of any layer with same size  $\Delta L$ , because there is no other difference in properties and environment of the layers than the magnitude of the fluxes. A constant ratio per leaf area index  $\Delta L$  is equivalent to exponential decrease of radiation at a constant value of extinction coefficient  $k$ . It should be remembered that this is only true under the condition that there is no variation in leaf optical properties. Exponential extinction was derived before for the case of black horizontal leaves (zero scattering). If the leaves do not absorb all radiation, the extinction coefficient  $k$  will be smaller than the value of  $k'$  for black leaves (and which is equal to unity for black horizontal leaves). What is the value now in terms of leaf reflection and absorption coefficient?

The expression can be found by solving the differential equations for upward and downward fluxes, but it is also possible to

use the expression for the net radiation flux, which is the difference between the downward and the upward flux. The absorption per leaf area is identical to the rate of decrease of the net flux per leaf area index at leaf level  $L$ . For the exponential function  $\exp(-kL)$  this rate of decrease is equal to its first derivative  $k \exp(-kL)$  times the net flux above the canopy, so that net absorbed radiation per leaf area is given by:

$$I_c = (I_d - I_u)k \exp(-kL) \quad (1.11)$$

As  $I_u$  is equal to  $I_d$  times  $\rho_c$ , this expression is identical to

$$I_c = I_d(1 - \rho_c)k \exp(-kL) \quad (1.12)$$

On the other hand, the absorption per leaf area must be equal to the sum of the radiation absorbed from above and the radiation absorbed from below. A fraction  $k'$  ( $1 - \tau_{\text{leaf}} - \rho_{\text{leaf}}$ ) will be absorbed out of each of these two fluxes. This leads to a second expression for the absorbed radiation per leaf area, which must of course be equal to the one above:

$$I_c = (1 + \rho_c)k'(1 - \tau_{\text{leaf}} - \rho_{\text{leaf}}) \exp(-kL)I_d \quad (1.13)$$

Combining the two expressions for the absorbed radiation per leaf area gives:

$$(1 - \rho_c)k = (1 + \rho_c)k'(1 - \tau_{\text{leaf}} - \rho_{\text{leaf}}) \quad (1.14)$$

A second relation between  $k$  and  $\rho_c$  can be found by using the upward scattering per leaf area at depth  $L$ , keeping in mind that the total reflected radiation consists of the contributions of upward scattering of all leaf layers together. This upward scattering per leaf layer at depth  $L$  is equal to  $k'(\tau_{\text{leaf}} I_{u,L} + \rho_{\text{leaf}} I_{d,L})$  or to  $k' I_{d,0} (\tau_{\text{leaf}} \rho_c + \rho_{\text{leaf}}) \exp(-kL)$ . Not all of the radiation that is scattered upward at depth  $L$  will reach the top of the crop canopy because of partial masking by leaves overhead. This

interception proceeds in the same way as the extinction of radiation by black leaves, because only the direct lines of visibility count. Therefore only a fraction  $\exp(-k'L)$  and not  $\exp(-kL)$  will escape to the top of the canopy. This means that the total reflected radiation must be equal to the integral of the product of upward scattering and this escape fraction, so that the crop canopy reflection coefficient is given by

$$\rho_c = \int_0^\infty (\tau_{\text{leaf}} \rho_c + \rho_{\text{leaf}}) k' \exp(-kL) \exp(-k'L) dL \quad (1.15)$$

or

$$\rho_c = \frac{\tau_{\text{leaf}} \rho_c + \rho_{\text{leaf}}}{k' + k} k' \quad (1.16)$$

The two Eqs. 1.14 and 1.16 relate  $k$  and  $\rho_c$  to the canopy property  $k'$  and the leaf properties  $\tau_{\text{leaf}}$  and  $\rho_{\text{leaf}}$ . Algebraic manipulation leads to the explicit expressions:

$$k = k' \sqrt{(1 - \tau_{\text{leaf}})^2 - \rho_{\text{leaf}}^2} \quad (1.17)$$

$$\rho_c = \frac{\rho_{\text{leaf}}}{k/k' + 1 - \tau_{\text{leaf}}} \quad (1.18)$$

and also to

$$k = k'(1 - \tau_{\text{leaf}}) \frac{1 - \rho_c^2}{1 + \rho_c^2} \quad (1.19)$$

In Sect. I.B, we have seen that leaf reflection and leaf transmission do not differ much, each being practically equal to half the scattering coefficient  $\sigma_{\text{leaf}}$ . The equations for  $k$  and  $\rho_c$  can then be further simplified to (Cowan 1968):

$$k = k' \sqrt{1 - \sigma_{\text{leaf}}} \quad (1.20)$$

$$\rho_c = \frac{1 - \sqrt{1 - \sigma_{\text{leaf}}}}{1 + \sqrt{1 - \sigma_{\text{leaf}}}} \quad (1.21)$$

The latter two equations occur frequently as building blocks in simulation models for

plant growth. This equation for  $\rho_c$  approaches  $\sigma_{\text{leaf}}/4$  (or  $\rho_{\text{leaf}}/2$ ) when the leaves have a small reflection and transmission coefficient. This is usually the case in the PAR region of the spectrum where reflection and transmission coefficient are about equal to 0.1. Therefore the reflection coefficient of a closed canopy in the PAR region is typically about 5 %. Note that this calculation does not consider the effect of solar elevation on  $\rho_c$  that occurs for non-horizontal leaf angle distributions. In Sect. IV under Eq. 1.34, I will show that  $\rho_c$  will then be larger than 5 % under low solar elevation and smaller under high solar elevation.

### III. Absorption of Radiation in Row Crops

#### A. Directional Distribution of Incoming Radiation

So far I have treated canopies as being horizontally homogenous. This may apply to uniformly sown crop stands or natural mono-species stands. Most vegetation however is horizontally heterogenous, there being different species or plants of the same species forming discontinuous canopies. Further in the book some of these cases will be treated e.g. mixed species vegetation (Chap. 14 Anten and Bastiaans 2016) and 3D architecture of plants (Chap. 8, Evers 2016). Here I discuss a specific but very common example of a discontinuous stand, the row crop.

The irradiance on a horizontal surface coming from a well-defined sky portion, for instance, a window, can be calculated by a double integration of sky radiance  $N(\alpha, \beta)$  over azimuth  $\alpha$  and elevation  $\beta$ , using the formula:

$$I = \frac{1}{\pi} \int N(\alpha, \beta) \sin \beta \cos \beta d\beta d\alpha \quad (1.22)$$

If the radiance  $N$  is the same for all directions (Uniform Overcast Sky),  $N$  can be used as a constant multiplier in the integration. In the

case that the boundaries of azimuth and elevation are independent, the integration is straightforward. To give an example, a horizontal circular window in a zenithal position at height  $h$  and with radius  $r$  is bounded at  $\arctan(h/r)$  for all azimuthal values, resulting in a relative irradiance of  $r^2/(h^2 + r^2)$ .

#### B. Row Crops

A row crop has heterogeneity, it has the leaves clustered into the rows. Due to clustering, row crops absorb less radiation as compared to a fully homogeneous crop. Radiation absorption is the primary factory that determines crop photosynthesis and therefore it is to be expected that a row crop has a smaller rate of photosynthesis than a homogeneous crop at the same LAI. It is our goal to quantify this reduction. We know that row crops are common practice in agriculture and it would therefore be strange if the reduction of photosynthesis is large. We investigate the reduction by considering model situations with increasing complexity.

##### 1. Infinite LAI, Black Leaves

At the soil surface on a path between rows, the sky is only partly visible (Fig. 1.6). The outline of a row is simplified to a rectangular hedge, at height  $h$ , width  $w$ , separated by path widths  $p$ . For simplicity, the crop field is considered infinitely large and so row length does not matter. It is illustrative to use orthographic projection to find the relative irradiance (Monteith and Unsworth 1990). In the orthographic projection the sky portion that is visible is first projected onto the imaginary dome of the upper hemisphere which is subsequently projected vertically downward onto its horizontal circular base. The relative size of the projected sky portion on the area of the base is equal to its relative contribution to the irradiance. The correction for the sine of incidence is automatically taken into account by the inclination of a piece on the sky dome when it is vertically projected. Some types of fish-eye lenses do indeed produce an orthographic projection, but this

Fig. 1.6. Cross-section through a simplified row crop. The angles  $\alpha_1$  and  $\alpha_2$  delimit the part of the sky that is visible from the given point at the soil surface

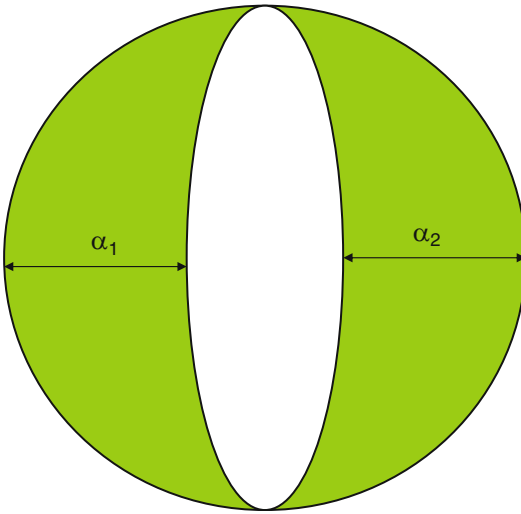
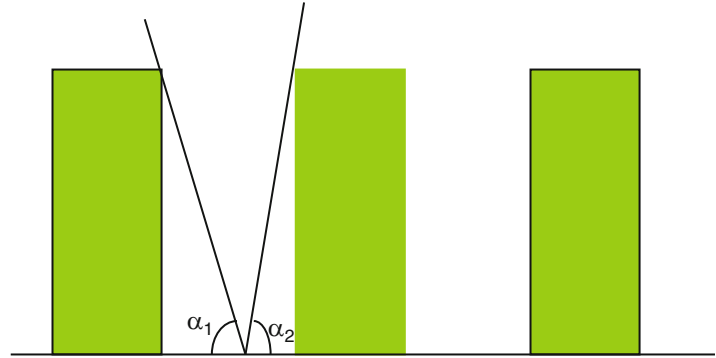


Fig. 1.7. The rows and the gap of the path as seen in a fish eye projection from the same point as in Fig. 1.6

type of projection does not retain the solid angle because vertical angular distances near the horizon will get compressed. In Fig. 1.7 the orthographic projection of the visible sky is drawn for a point halfway the path between the two adjacent rows.

The relative irradiance at such a point is given by the projected area as a fraction of the whole circle area. This fraction is identical to half the sum of the two cosine values of  $\alpha_1$  and  $\alpha_2$  in Fig. 1.6, which can be calculated from row height and lateral distance of the point. The fraction value will slightly vary from a maximum in the middle of the path to the minimum value at each side of the row.

Its average value over the path is found by analytical integration (Goudriaan 1977) and is equal to

$$I_{\text{path}} = \frac{\sqrt{h^2 + p^2} - h}{p} \quad (1.23)$$

In an extremely dense row crop with black leaves, this is all radiation that is transmitted. The complementary fraction is absorbed by the crop.

This model situation is unrealistic: the row structure is so dense that no radiation can penetrate to the soil surface under a row itself because it has infinite LAI and it is completely black. Yet, it is a useful starting point for the situation in which radiation does penetrate the rows.

## 2. Non-infinite LAI, Black Leaves

A precise geometric calculation is possible (Gijzen and Goudriaan 1989; Röhrig et al. 1999; Colaizzi et al. 2012) but it is rather complicated. It involves the distinction of the zones delimited by the row edges of many subsequent adjacent rows. Here, we will aim at a much faster approximation that is sufficiently accurate in practice and that also gives more insight.

To do so, we distinguish two configurations at the same averaged LAI between which the row crop is situated. The first one is the completely homogeneous crop that we have considered already. The

radiation level at the soil surface under this crop is given by:

$$S_{\text{homogeneous}} = \exp(-kL_t) \quad (1.24)$$

where  $L_t$  is the total LAI. In the other one all rows are pushed together so that we get a new homogeneous, but denser, crop at part of the total land area being equal to the sum of all row area. The local leaf area index,  $L_{t\text{comp}}$ , of this compressed crop is equal to the original LAI multiplied by the ratio  $(w + p)/w$ . The radiation level at the soil surface under this compressed crop is equal to:

$$S_{\text{comp}} = \exp(-kL_{t\text{comp}}) \quad (1.25)$$

In the actual row crop we have two types of soil surface, one right under row itself and one right under the path.

The radiation level at the soil surface of the path contains of course the unobstructed component  $I_{\text{path}}$  (Eq. 1.23) but in addition there is radiation penetrating through the rows adjacent to the path. The view factor for this radiation is complementary to  $I_{\text{path}}$ . As an approximation we assume that for laterally penetrating light the row crop is identical to a homogeneous crop, so that the fraction laterally penetrating is identical to  $S_{\text{homogeneous}}$  (as it turns out, this assumption causes a small systematic error to which I will come back right below Eq. 1.29).

The sum of the two components is now given by

$$S_{\text{path}} = I_{\text{path}} + (1 - I_{\text{path}})S_{\text{homogeneous}} \quad (1.26)$$

For the radiation level at the soil surface right under the row we follow a similar approach: it is also separated in two parts, one transmitted through the top of the row and one transmitted through the sides of the row. For the first component we need the view factor of the row  $I_{\text{row}}$ , in the same way as was done for the path (Fig. 1.6), but this time multiplied by extinction in the row. The extinction is calculated with the LAI value within the row, which is the same as that of

the compressed crop. For the radiation that enters laterally we use the LAI value of the homogeneous crop (averaged  $L_t$ ) multiplied by the view factor that is complementary to  $I_{\text{row}}$ . Combining the two components we find for the  $S_{\text{row}}$ :

$$S_{\text{row}} = I_{\text{row}}S_{\text{comp}} + (1 - I_{\text{row}})S_{\text{homogeneous}} \quad (1.27)$$

For the entire soil surface we use the weighted average of the two components for path and row:

$$S_{\text{soil}} = \frac{wS_{\text{row}} + pS_{\text{path}}}{w + p} \quad (1.28)$$

The fraction of radiation absorbed by the crop is the complement of the fraction of radiation absorbed by the soil surface:

$$F_{\text{crop}} = 1 - S_{\text{soil}} \quad (1.29)$$

However, one correction to the equation for  $S_{\text{path}}$  is still needed, because in the present form the radiation absorbed by the crop for near zero values of leaf area index deviates from  $kL$ . We know that the first derivative of absorbed radiation with respect to  $L$  at  $L = 0$  should be equal to  $k$  because there is no mutual shading in the limit transition to zero LAI. Algebraic analysis shows that this requirement is met if  $S_{\text{path}}$  is corrected by a small term in the lateral view factor:

$$S_{\text{path}} = I_{\text{path}} + \left[ \frac{1 - I_{\text{path}} - (1 - S_{\text{homogeneous}})}{I_{\text{path}} - I_{\text{row}}} \right] S_{\text{homogeneous}} \quad (1.30)$$

Apparently the lateral penetration of radiation in a row canopy to the bottom of the path is slightly different from that in a homogeneous canopy, which was assumed in the simpler equation. Usually the correction is small, but in a sparse row crop with narrow rows and wide paths it cannot be ignored. This correction is a price that must be paid for the brevity of the approximative equation.

### 3. Loss of Radiation due to Plant Arrangement in Rows

The equation for the radiation that is lost on the path (Eq. 1.23) is an upper limit to the relative loss. It shows that the losses will rarely exceed 30 % and usually be in the order of 10–20 %. In a homogeneous crop there is also transmission to the soil surface and in fact we should compare the result of Eq. 1.28 ( $S_{\text{soil}}$ ) with that of Eq. 1.24 ( $S_{\text{homogeneous}}$ ). The difference between these two values gets smaller as LAI gets smaller. For LAI = 0.5 the relative loss will only be about 5–10 %.

A general conclusion is that there only a moderate loss of intercepted radiation due to row cultivation as compared to that in a homogeneous crop.

The same conclusion probably applies to even more complicated plant arrangements, such as in tree nurseries (Pronk et al. 2003) in which small trees are regularly positioned between intersecting rows. A more sophisticated approach of a similar plant arrangement can be found in Röhrig et al. (1999).

## IV. Direct and Diffuse Light in Photosynthesis Modeling

Leaf photosynthesis is a curvi-linear function of radiation absorption that gets saturated (i.e., it levels off) at high light intensities. If we simply multiplied the average radiation absorption and quantum efficiency, we would end up with a gross overestimation of canopy photosynthesis. There are two major causes of variation of radiation absorption in a leaf canopy: first the exponential extinction with increasing canopy depth, and second the unevenness among sunlit and shaded leaves, even at the same height within the canopy. Thus we need to consider the distinction between direct beam radiation and diffuse radiation (both diffuse sky radiation and scattered light in the canopy) that is intercepted by all leaves. This section therefore deals with the

modeling of diffuse and direct radiation in relation to canopy photosynthesis.

With a completely overcast sky there is no direct radiation, and the radiation extinction in this situation is treated by assuming a single value for the extinction coefficient  $k$  for diffuse light, called  $k_{\text{dif}}$ , and likewise for canopy reflection,  $\rho_{\text{dif}}$ , usually with nominal values of 0.7 and 0.05 respectively. These values can be refined if needed. If we consider a leaf canopy with an LAI value that is small, (less than 1), we make a small error if we assume that all leaves have the same level of radiation absorption. A good estimate for this radiation absorption can be obtained by using Eq. 1.12, which gave the radiation absorption at canopy depth  $L$ . We should however be careful about the value for  $L$  that we are going to use. Clearly LAI itself is not correct: we then get the radiation absorption at the bottom of the canopy which is an underestimate for the average value. A better estimate is the one in the middle at one half of LAI, let us call it  $L_{\text{middle}}$ . The expression that we thus get for radiation absorption per leaf area is:

$$I_{c,\text{dif}} = I_{\text{dif}}(1 - \rho_{\text{dif}})k_{\text{dif}}\exp(-k_{\text{dif}}L_{\text{middle}}) \quad (1.31)$$

Energy conservation requires that total LAI ( $L_t$ ) times this quantity be identical to the difference in net radiation above and below the canopy. The expression for net radiation at any level  $L$  in the canopy is  $I_{\text{dif}}(1 - \rho_{\text{dif}})\exp(-k_{\text{dif}}L)$ , and so the total absorption is equal to  $I_{\text{dif}}(1 - \rho_{\text{dif}})(1 - \exp(-k_{\text{dif}}L_t))$ . The result of Equation 1.31 times LAI is not identical to this total absorption, but it is a very good approximation, getting better as LAI gets smaller. In mathematical terms, the first derivative of a function of  $x$  is approximated by its difference over  $dx$  divided by  $dx$ , as  $dx$  approaches zero. Check for yourself at, for instance,  $L_t = 0.4$  and see if you also get 0.23201 and 0.23125 times  $I_{\text{dif}}$ , using the nominal values of  $k$  and  $\rho$  at 0.7 and 0.05 respectively.



The reason to work with the leaf absorption level in the middle, rather than with the difference of net radiation above and below the layer divided by its layer size, is anticipation upon the numerical procedure of Gaussian integration that we will use later in a more general model for canopy photosynthesis.

The Gaussian integration procedure solves the problem that the above approximation begins to fall short as LAI gets larger. For instance, at  $L_t = 5$ , the balance method gives 0.921 of  $I_{\text{dif}}$ , whereas the midpoint method of Eq. 1.31 gives 0.578 of  $I_{\text{dif}}$  which is far too small. As the balance method gives the right result for the total radiation absorption, you may wonder why not just using this method of balancing above and below the leaf canopy. However, the problem we then get is that we overestimate canopy photosynthesis due to the mentioned saturation of leaf photosynthesis at high radiation levels. The best way out of this problem is to calculate leaf photosynthesis at a number of carefully chosen levels in the leaf canopy, using the radiation absorption in situ (see Box 1.2). Having chosen these points, the radiation absorption per leaf area is calculated at each point, subsequently the leaf photosynthesis that goes with it, and finally leaf photosynthesis over the whole canopy is found by integration. For reasons of balance checking it is sensible to do the integration for absorbed radiation as well, and compare it with the result of the balance method for net radiation.

**Box 1.2: Example of Calculation of Photosynthesis When There Is only Diffuse Radiation**

```
C Input data, normally available through
subroutine arguments:
C LAI = 3., REFH = 0.05, KDF = 0.7,
PARDF = 100., EFF = 0.011, AMAX = 1.
C Units mg(of CO2), m, s, J (of PAR),
C Gaussian numbers
DIMENSION WGAUSS(3), XGAUSS(3)
DATA IGAUSS/3/
DATA XGAUSS/0.112702,0.5,0.887298/
DATA WGAUSS/
0.277778,0.444444,0.277778/
```

```
C Canopy photosynthesis and absorbed
radiation are initialized at zero:
PHOT = 0.
RADABS = 0.
DO I = 1,IGAUSS
C Calculate the three Gaussian depths
LAIC = XGAUSS(I)*LAI
C Calculate absorbed radiation at that depth
VISDF = PARDF*(1. -REFH)
*KDF*EXP(-KDF*LAIC)
C Calculate the resulting rate of
photosynthesis
PHOTL = AMAX*(1. - EXP
(-EFF*VISDF/AMAX))
C Weighted addition
PHOT = PHOT + WGAUSS(I) *
PHOTL
RADABS = RADABS + WGAUSS
(I)*VISDF
ENDDO
C Calculate totals from the mean values:
PHOTT = PHOT*LAI
RADBST = RADABS*LAI
```

Up to this point we have dealt with canopy photosynthesis under an overcast sky with just diffuse radiation.

If the sun shines there is also a large unevenness in the distribution over sunlit and shaded leaves. You can appreciate this in a forest understory where sun flecks are considerably brighter than the rest of the understory. First the diffuse component in incoming radiation must be singled out (Spitters et al. 1986). The incoming diffuse radiation can be dealt with in the same way as above, but in addition all leaves will receive radiation originating from the direct component. As mentioned before, the sunlit leaves receive the direct radiation, but also all leaves, including the sunlit ones, will receive a small amount from scattering by directly illuminated leaves. This additionally scattered radiation is intercepted on its turn and it is decreasing both, as goes down and as it goes up into the canopy. This secondary illumination seems to burden us with a complicated iterative problem, but it is possible to implicitly take care of this iteration, in the same way as it was handled in Eq. 1.12 for absorbed radiation at the leaf level. The

coefficients for leaf scattering were already incorporated into canopy reflection and extinction by using Eqs. 1.20 and 1.21.

By setting apart the incoming diffuse component, the direct radiation component can be treated as if the rest of the sky is black. Direct radiation is intercepted and partly scattered, but no matter how large this scattering, it is lost from the direct beam. This means that the direct beam will follow an exponential extinction curve with a  $k$ -value that was derived earlier for black leaves ( $k = 0.5/\sin(\beta)$  for a spherical leaf angle distribution), which we will denote by  $k_{\text{dir}}$ . The fraction of sunlit leaves at canopy depth  $L$  is now given by  $\exp(-k_{\text{dir}}L)$  and it is identical to the frequency of sunflecks at that same level. Conversely, the fraction of shaded leaves is equal to its complement. The incidence angle of the direct beam is usually not perpendicular to a leaf (Fig. 1.3), which must be taken into account in the calculation of the radiation absorbed on a leaf area basis. As explained earlier and implied in the value of  $k_{\text{dir}}$ , for a spherical leaf angle distribution the average sine of incidence (or cosine to the leaf's normal) has a value of 0.5, irrespective of solar height, whereas the actual value of the sine of incidence for an individual leaf may vary between 0 and 1. In the procedure for calculating canopy photosynthesis this variation is taken into account by integration over the sine of incidence while allowing for its frequency distribution. Here again Gaussian integration will be used. The frequency distribution of leaf area with the sine of incidence is simple for a spherical leaf angle distribution: it is uniform between the values 0 and 1.

The effect of scattering needs special attention as it implies a redistribution of radiation from sunlit leaves to shaded leaves. The sunlit leaves absorb a fraction  $(1 - \sigma)$  of what is incident upon them, and then they scatter the fraction  $\sigma$  into all directions, which in turn is intercepted by both other sunlit leaves and by shaded leaves. We assume that there is no difference between sunlit and shaded leaves at the same canopy

height as far as the absorption of the scattered radiation is concerned.

The mean total absorption rate at depth  $L$ , averaged over both sunlit and shaded leaves is given by:

$$I_{\text{mean,dir}} = I_{\text{dir}}(1 - \rho_c)k\exp(-kL) \quad (1.32)$$

where  $k$  is the overall extinction coefficient given by  $k' \sqrt{1 - \sigma}$ . This average total absorption is the sum of absorption of direct radiation in sunlit leaves only and of scattered, diffused radiation in both sunlit and shaded leaves. The first term, the absorption rate of the direct radiation in the sunlit leaves only, must be equal to:

$$I_{c,\text{dir}} = I_{\text{dir}}(1 - \sigma)k'_{\text{dir}}\exp(-k'_{\text{dir}}L) \quad (1.33)$$

which is smaller than  $I_{\text{mean,direct}}$ . The difference between Eqs. 1.32 and 1.33, defined by  $I_{c,\text{sca}} = I_{\text{mean,direct}} - I_{\text{sunlit,direct}}$ , represents absorption of scattered radiation (secondary, but also tertiary, etc), which is the same for all leaves at that same height in the canopy, whether sunlit or shaded.

For PAR the scattering coefficient is about 0.2 which means that tertiary and higher order scattering can be ignored. A mathematical series development shows that the first order approximation of the difference  $I_{\text{mean,direct}} - I_{\text{sunlit,direct}}$  for a spherical leaf angle distribution is given by

$$I_{\text{sca}} = \sigma \left[ \frac{1}{2(1 + 2\sin\beta)} + \frac{L}{8\sin^2\beta} \right] \exp\left(-\frac{2L}{\sin\beta}\right) \quad (1.34)$$

The integral of this expression over the whole canopy from 0 to infinity is equal to  $\sigma (1 + 4\sin\beta)/(2 + 4\sin\beta)$ , which varies between  $\sigma/2$  for very small solar heights passing  $3\sigma/4$  at  $30^\circ$  solar height to  $5\sigma/6$  for a zenithal position of the sun. Because the primary scattering is equal to  $\sigma$  itself, the remainder must have escaped as radiation reflected by the canopy (at infinite LAI there is no absorption by the soil surface).

This means that the reflection coefficient of the canopy is equal to  $\sigma/(2 + 4 \sin\beta)$  which is a decreasing function of solar height. At  $30^\circ$  solar height this is equal to  $\sigma/4$ , the same as for a horizontal leaf angle distribution. Thus, for a lower sun most canopies will reflect a slightly larger fraction and for a higher sun they will reflect a slightly smaller fraction.

This “second-hand” absorbed radiation enhances total crop photosynthesis particularly because the shaded leaves will use it much more efficiently than the sunlit leaves. Finally the absorption from the diffuse sky radiation (see Eq. 1.31) (that we had temporarily set apart) must be added to get the total absorption rate of diffuse radiation,  $I_{c,sh}$ , which is common to sunlit and shaded leaves:

$$I_{c,sh} = I_{c,dif} + I_{c,sca} \quad (1.35)$$

The rate of photosynthesis of the shaded leaves at the canopy depth  $L$  can now be calculated. Their contribution to crop photosynthesis on a ground area basis still requires multiplication by the fraction shaded which is equal to  $1 - \exp(-k_{dir}L)$ .

To get the photosynthesis of the sunlit leaves the absorption rate of direct light should be added to the absorption rate for the shaded leaves that we have just obtained. On a ground area basis this absorption rate of direct light is given by Eq. 1.33. On a leaf area basis it is equal to (see also Fig. 1.3):

$$I_{c,su} = I_{dir}(1 - \sigma) \frac{\cos \zeta}{\sin \beta} \quad (1.36)$$

with  $I_{c,sh}$  the total absorption rate of diffuse radiation,  $\zeta$  the angle between the leaf normal and the solar beam and  $\beta$  the solar inclination angle. For horizontal leaves,  $\cos(\zeta)$  is equal to  $\sin(\beta)$  so that the photosynthesis rate

of the sunlit leaves can be immediately calculated, but for any other leaf angle distribution  $\cos(\zeta)$  varies and we need another integration over  $\cos(\zeta)$  that is nested within the integration over canopy depth. For a spherical leaf angle distribution (see also Fig. 1.4)  $\cos(\zeta)$  varies uniformly between 0 and 1, irrespective of solar height.

We can now choose a photosynthesis-light response curve in order to calculate the contribution of the sunlit leaves at the considered canopy depth  $L$ . For the often-used negative exponential function  $A_{max} (1 - \exp(-\epsilon I/A_{max}))$  an analytical solution exists for this case, and with  $I$  given by  $I_{dir} \cos(\zeta)$  it is equal to

$$A_{max}(1 + A_{max}/(\epsilon I_{dir}) (1 - \exp(-\epsilon I_{dir}/A_{max})) ) \quad (1.37)$$

with  $A_{max}$  the photosynthetic at saturating light and  $\epsilon$  the initial slope of the light response curve (i.e., the apparent quantum yield). Other equations can be used such as the non-rectangular hyperbola which also includes a curvature factor (Marshall and Biscoe 1980; see Chap. 9 Hikosaka et al. 2016). This expression could be used in a numerical model but there are two reasons not to do so: first there is a zero division for  $I_{dir} = 0$ , and second it excludes the use of another light response curve. It does, however, give a chance to check the numerical performance of the Gaussian integration procedure. For  $\epsilon I_{dir}/A_{max} = 1$  the analytical solution gives  $0.3678794 A_{max}$  whereas the numerical three-point approximation gives  $0.3678797 A_{max}$ , so that in this situation the Gaussian integration is extremely accurate.

A modelling example of the nested Do – loop is given in Box 1.3.

### Box 1.3: Example of Calculation of Canopy Photosynthesis When There Is also Direct Radiation

```

C Input data, normally available through
subroutine arguments:
C LAI = 3., KDF = 0.7,
PARDF = 100., PARDR = 200., EFF = 0.011
C AMAX = 1.,SIGMA = 0.2,
SINB = 0.7
C Units mg(of CO2), m, s, J (of PAR),
REFH = (1.-SQRT(1.-SIGMA))/
(1. + SQRT(1.-SIGMA))
REFS = REFH*2./(1. + 2.*SINB)
KBL = 0.5/SINB
KDRT = KBL*SQRT(1.-SIGMA)
C Gaussian numbers
DIMENSION WGAUSS(3), XGAUSS(3)
DATA IGAUSS/3/
DATA XGAUSS/0.112702,0.5,0.887298/
DATA WGAUSS/
0.277778,0.444444,0.277778/
C Canopy photosynthesis and absorbed
radiation are initialized at zero:
PHOT = 0.
VISABS = 0.
DO I = 1,IGAUSS
C Calculate the three Gaussian depths
LAIC = XGAUSS(I)*LAI
C Calculate absorbed radiation at that depth
VISDF = PARDF*(1. -REFH)
*KDF*EXP(-KDF*LAIC)
VIST = PARDR*(1.-REFS)
*KDRT*EXP(-KDRT*LAIC)
VISD = PARDR*KBL*EXP
(-KBL*LAIC)
VISSHD = VISDF + VIST-VISD
C Calculate the resulting rate of
photosynthesis
PHOSHD = AMAX*(1. - EXP(-
EFF*VISSHD/
AMAX))
VISPP = PARDR*(1.-SCP)/SINB
PHOSUN = 0.
VISLL = 0.
DO I2 = 1,IGAUSS
VISSUN = VISSHD +
VISPP*XGAUSS(I2)
PHOS = AMAX*(1.-EXP
(-EFF*VISSUN/AMAX))
PHOSUN = PHOSUN +
PHOS*WGAUSS(I2)
VISLL = VISLL +
VISSUN*WGAUSS(I2)
ENDDO
FSSLA = EXP(-KBL*LAIC)

```

```

PHOTL = FSSLA*PHOSUN+
(1.-FSSLA)*PHOSHD
VISL = FSSLA*VISLL+(1.-
FSSLA)*VISSHD
C Weighted addition
PHOT = PHOT + WGAUSS(I) *
PHOTL
VISABS = VISABS + WGAUSS(I)
*VISL
ENDDO
C Calculate totals from the mean values:
PHOTT = PHOT*LAI
VISABST = VISABS*LAI

```

Even the three-point Gaussian integration is not accurate enough for a leaf area index larger than 3. Therefore, it is advisable to use a 5-point method for larger leaf area indices.

The following table gives the Gaussian distances and weights for the 5-point method (see Lanczos 1957; Scheid 1968):

```

DATA IGAUSS/5/
DATA XGAUSS/0.0469101,0.2307534,0.5,
0.7692465,0.9530899/
DATA WGAUSS/0.1184635,0.2393144,
0.2844444,0.2393144,0.1184635/

```

## V. Conclusions and Prospects

Real plant canopies exhibit fuzzy features that are superimposed on the regular and smooth model skeleton which was used for the description of the canopies in this chapter. The idealizing model approach may be criticized for this reason, arguing that real canopies are very different. However, it will not be an easy task to show and prove to what extent the irregularities in real crops modify the outcomes provided by the approach given here. It will be an enormous effort to collect the statistical attributes of 3-D plant forms that are needed, let alone to properly include them in a model (see Chap. 8 Evers 2016).

Another criticism to the modelling approaches as presented in this chapter is that they tend to be unbalanced: too much emphasis on the mathematical analysis,

thereby suggesting an unjustified accuracy, and shying away from the harder to grasp biological and agricultural variability. For instance, how can we include and describe the spatial and temporal variations in photosynthetic leaf properties? This topic is treated in the Chap. 4 (Niinemets 2016) and 5 (Pons 2016) of this book.

Such criticisms tell us as modellers to be cautious. Our results are vulnerable, depending as they are on the underlying assumptions. This may be a weakness of the modelling approach, but at the same time this weakness turns into a strength when it gives us also the power to analyze the sensitivity of the model outputs to these same assumptions.

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