Gauge Fields and Flavor Oscillations

4

In this chapter we discuss the general formulation of gauge fields in the quantum theory, both abelian and nonabelian. A generalization of the elementary Stueckelberg diagram (Fig. 2.1), demonstrating a "classical" picture of pair annihilation and creation, provides a similar picture of a process involving two or more vertices (diagrams of this type appear in Feynman's paper in 1949 (Feynman 1949) with sharp instantaneous vertices). A single vertex, as in Stueckelberg's original diagram, in the presence of a nonabelian gauge field, can induce a flavor change on the particle line, resulting in a transition to an antiparticle with different identity. An even number of such transitions can result in flavor oscillations, such as in the simple case of neutrino oscillations. On the quark constituent level, such transitions can be associated with K, B or D meson oscillations as well. The construction of the Lorentz force acting on particles with abelian or nonabelian gauge will also be discussed, with results consistent with the assumptions for the semiclassical model. In view of our discussion of the previous chapter, it will also be shown that this picture could provide a fundamental mechanism for CP violation.

4.1 Abelian Gauge Fields

In his original paper Stueckelberg (1941) introduced the electromagnetic vector gauge fields, as we shall explain below, as compensation fields for the derivatives on the wave functions representing the four-momenta. For a Hamiltonian of the form (2.4), i.e.,

$$K = \frac{p^\mu p_\mu}{2M}, \tag{4.1}$$

for which the Stueckelberg-Schrödinger equation is

$$i\frac{\partial}{\partial \tau}\psi_\tau(x) = K\psi_\tau(x), \tag{4.2}$$

L.P. Horwitz, *Relativistic Quantum Mechanics*,
Fundamental Theories of Physics 180, DOI 10.1007/978-94-017-7261-7_4

one must introduce so-called compensation fields to retain the form of the equation when the wave function is modified by a (differentiable) phase function at every point. Thus, for

$$\psi(x)' = e^{ie\Lambda(x)}\psi(x), \tag{4.3}$$

the relation

$$(p^\mu - eA^\mu(x)')\psi(x)' = e^{ie\Lambda}(p^\mu - eA^\mu(x))\psi(x), \tag{4.4}$$

is satisfied if

$$A^\mu(x)' = A^\mu(x) + \partial^\mu\Lambda. \tag{4.5}$$

One sees that the gauge transformation induced on the compensation field is of the same form as the gauge transformations of the Maxwell potentials, and therefore this procedure may be thought of as an underlying theory for electromagnetism (Wu 1975). Stueckelberg (1941) noted that he was unable to explain the diagram of Fig. 2.1 with this form of the electromagnetic interaction. The reason is that the canonical velocity is

$$\dot{x}^\mu = \frac{p^\mu - eA^\mu}{M}, \tag{4.6}$$

so that

$$\dot{x}^\mu \dot{x}_\mu = -(\frac{ds}{d\tau})^2 = \frac{(p^\mu - eA^\mu)(p_\mu - eA_\mu)}{M^2}. \tag{4.7}$$

This expression is proportional to the conserved Hamiltonian (for a closed system), so that the proper time cannot go through zero. To avoid this difficulty, he added an extra force term in the equations of motion. However, this construction did not take into account the compensation field required for the τ derivative in the Stueckelberg-Schrödinger equation.

Applying the same procedure to the nonrelativistic Schrödinger equation, the t derivative in the equation requires a compensation field A^0 (in addition to the $\mathbf{A}$ fields compensating for the action of the derivative $-i\frac{\partial}{\partial\tau}$), thus providing the full set of Maxwell fields. Taking this requirement into account in the Stueckelberg-Schrödinger equation, we arrive at a *five dimensional generalization* of the Maxwell theory (Saad 1989; see also Wesson 2006). We furthermore recognize that since the gauge phase depends, in general, on τ, the compensation fields, which we shall denote by a_μ, a_5, must also depend on τ. We shall see that under integration over τ, i.e., the zero mode, the fields a_μ reduce to the usual Maxwell fields satisfying the usual Maxwell equations, and the a_5 field decouples. The more general theory therefore properly contains the Maxwell theory.

We first remark that a_5 and a_μ must transform under a gauge change according to

$$\begin{aligned} a_5(x,\tau)' &= a_5(x,\tau) + \frac{\partial\Lambda}{\partial\tau} \\ a_\mu(x,\tau)' &= a_\mu(x,\tau) + \frac{\partial\Lambda}{\partial x^\mu}, \end{aligned} \tag{4.8}$$

or, with $\alpha = (0, 1, 2, 3, 5)$, and $x^5 \equiv \tau$,

$$a_\alpha(x,\tau)' = a_\alpha(x,\tau) + \frac{\partial \Lambda}{\partial x^\alpha}. \tag{4.9}$$

The Stueckelberg-Schrödinger evolution operator in the presence of this $5D$ gauge field must therefore have, minimally, the form

$$i\frac{\partial \psi_\tau(x)}{\partial \tau} = \Big\{\frac{(p^\mu - e'a^\mu)(p_\mu - e'a_\mu)}{2M} - e'a^5(x)\Big\}\psi_\tau(x), \tag{4.10}$$

where e' is related to the Maxwell elementary charge e, as we shall see, by a dimensional scale factor.

One may extract from (4.10) the form for the corresponding classical Hamiltonian,

$$K = \frac{(p^\mu - e'a^\mu)(p_\mu - e'a_\mu)}{2M} - e'a^5(x). \tag{4.11}$$

In this form, the Stueckelberg line drawn in Fig. 2.1 is, in principle, realizable. If $-e'a^5$ reaches a value equal to K, these terms can cancel; at these points the proper time interval can pass through zero, and the semiclassical picture of pair annihilation becomes consistent in a simple way. We shall discuss an example of this mechanism in a semiclassical mechanism for neutrino oscillations to be given below.

It follows from the transformation laws (4.9) that the quantities (we use $\partial_\alpha \equiv \frac{\partial}{\partial x^\alpha}$)

$$f_{\alpha\beta}(x,\tau) = \partial_\alpha a_\beta - \partial_\beta a_\alpha \tag{4.12}$$

are gauge invariant, and may be considered, in analogy to the Maxwell case, as field strengths. To consider these quantities as *tensors* requires an additional, very strong assumption, i.e., that the five variables $\{x^\mu, x^5\} \equiv \{x^\alpha\}$, where $x^5 \equiv \tau$ transform together under some group such as $O(3,2)$ or $O(4,1)$. An examination of the field equations suggest that there may be such a symmetry, as one sees in the parallel derivation of the Maxwell equations from the gauge invariant nonrelativistic Schrödinger equation. For the latter, the explicit invariance which is evident in the homogeneous equations, that of the Lorentz group, had significant experimental evidence to justify such an assumption; at the present time there is some evidence for such a larger symmetry as $O(3,2)$ or $O(4,1)$, as we shall see in the discussion of the applications of the five dimensional generalization of Maxwell's theory below, but it is not yet definitive. We therefore do not assume, *a priori*, the full symmetry under $O(3,2)$ or $O(4,1)$. It is sufficient for our purposes to achieve manifest Lorentz covariance (and Poincaré symmetry for the equations of motion). We first demonstrate this argument with an analysis of the gauge theory for the nonrelativistic Schrödinger equation. Nevertheless, we shall refer to quantities such as $f_{\alpha\beta}(x,\tau)$ as *tensors* as a matter of notation.

The nonrelativistic fully gauge invariant Schrödinger equation is

$$i\frac{\partial}{\partial t}\psi_t(\mathbf{x}) = \frac{(\mathbf{p} - e\mathbf{A}(\mathbf{x},t))^2}{2M}\psi_t(\mathbf{x}) - eA_0; \tag{4.13}$$

the current $\mathbf{J}$ and the charge density $J^0 \equiv \rho$ satisfy the conservation law

$$\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0, \tag{4.14}$$

where

$$\mathbf{J} = \frac{ie}{2M}[\psi^*(\nabla - ie\mathbf{A})\psi - \psi(\nabla + ie\mathbf{A})\psi^*], \tag{4.15}$$

and

$$J^0 = \rho = e\psi^*\psi. \tag{4.16}$$

The inhomogeneous Maxwell field equations, written formally in terms of the four-vector indices, are (e.g. Jackson 1974; Landau 1951)

$$\partial_\nu F^{\mu\nu} = eJ^\mu; \tag{4.17}$$

they may be obtained from a Lagrangian providing the Schrödinger equation as a field equation, with a gauge invariant term proportional to $F_{\mu\nu}F^{\mu\nu}$, as we shall describe below in our discussion of the $5D$ fields.

There is clearly no linear coordinate transformation that can generate a linear combination of $\mathbf{J}$ and J^0, and therefore the relation (4.17) is not covariant. The relativistic covariance of the Maxwell equations, as discussed by Einstein (1905) is based on the assumption that the current is a covariant four vector. As we have seen, this does not hold for the gauge field construction based on the nonrelativistic Schrödinger equation.

For the relativistic case, Jackson (1974) has shown how one can construct a covariant four vector current from a sequence of elementary charged *events* in spacetime, which we shall refer to again below. It is, however, important to note that the *homogeneous* equations corresponding to (4.17), i.e., for $J^\mu = 0$, do reflect the Lorentz symmetry, suggesting that such a symmetry may indeed be a symmetry of the world. To realize this symmetry consistently, one must use a form of the quantum theory that gives rise to a covariant four current based, as we see above, on (4.10).

A simple set of field equations, providing second order derivatives of the potentials, is obtained by considering the Lagrangian density due to the field variables to be of the form $f_{\alpha\beta}f^{\alpha\beta}$, where we leave open for now the question of choosing a signature for raising and lowering the index of the fifth component. Writing a Lagrangian density for which setting the coefficient of the variation of ψ^* equal to zero gives the Stueckelberg-Schrödinger equation,[1] with this additional term for the gauge fields of the form

[1]Gottfried (1966) has pointed out that this procedure is not completely consistent since the Schrödinger wave ψ is not a mechanical quantity; it is, however, consistent for quantum field theory, and provides a convenient procedure to generate field equations for the first quantized theory under discussion here. The method is widely used as a heuristic tool (for example, Bjorken 1964).

$$\mathcal{L} = \frac{1}{2}\big(i\frac{\partial\psi}{\partial\tau}\psi^* - i\psi\frac{\partial\psi^*}{\partial\tau}\big) - \frac{1}{2M}\big[(p^\mu - e'a^\mu)\psi((p_\mu - e'a_\mu)\psi)^*\big] + e'(a^5\psi\psi^*) - \frac{\lambda}{4}f^{\alpha\beta}f_{\alpha\beta}, \tag{4.18}$$

where λ is, as we discuss below, an arbitrary real dimensional scale factor.

As for the $4D$ Maxwell fields, for which the Lagrangian does not contain $\partial A_0/\partial t$, the Lagrangian does not contain $\partial a_5/\partial\tau$, and therefore a full canonical quantization (as contrasted with path integral approaches such as Fadeev-Popov (Fadeev 1967)), which requires identification of a canonical momentum for the fields as the derivative of the Lagrangian density with respect to the time derivative of the field, is not easily accessible. Henneaux and Teitelboim (1992) and Haller (1972) have discussed methods for dealing with this problem; these methods have been applied by Horwitz and Shnerb (1993) to carry out the canonical quantization of the $5D$ fields (see Sect. 10.7). We just remark here that the three photon polarization states (in dimensionality the number of field components minus the two constraints due to Gauss's law and a gauge condition) may fall under the $O(2, 1)$ or $O(3)$ symmetry groups; as we discuss in Chap. 10, the two degrees of freedom of black body radiation is the result of the application of a second gauge condition on the asymptotic fields.

The variation of the potentials a^α in (4.18) then provides the field equations

$$\lambda\partial^\alpha f_{\beta\alpha} = j_\beta \tag{4.19}$$

where

$$j_\mu = \frac{ie'}{2M}\{(\partial_\mu - ie'a_\mu)\psi\psi^* - \psi((\partial_\mu - ie'a_\mu)\psi)^*\}, \tag{4.20}$$

and

$$j_5 = e'\psi\psi^* \equiv \rho_5. \tag{4.21}$$

As for the nonrelativistic gauge theory based on the Schrödinger equation, there is no coordinate transformation which can induce a linear combination of j_μ and j_5, and therefore these equations cannot be covariant under $O(4, 1)$ or $O(3, 2)$, although the homogeneous form of (4.19) for $j^\alpha = 0$ does admit such a higher symmetry.[2]

[2]If such a higher symmetry, such as $O(3, 2)$ or $O(4, 1)$ were to be found as a general property of particle kinematics, such as Lorentz covariance, in the framework of our present experimental knowledge, then a generalization of the Stueckelberg theory could be written with five momenta transforming under this group. The corresponding gauge fields would then be one dimension higher, to take into account the evolution of the system, and the resulting homogeneous field equations would appear to be $O(4, 2)$, $O(3, 3)$ or $O(5, 1)$ invariant. The corresponding theory of spin, as worked out in the previous chapter, would then rest on the method of Wigner applied to the stability group of a five-vector. In this book, we shall restrict our analysis to systems which are manifestly covariant on the level of the Lorentz group.

Furthermore, the current j^α satisfied, as follows from the Stueckelberg-Schrödinger equation, the conservation law

$$\partial_\alpha j^\alpha = 0 \tag{4.22}$$

In general, then, the current j^μ, cannot be the conserved Maxwell current (Saad 1989; see also Stueckelberg 1941). Writing Eq. (4.22) in the form

$$\partial_\mu j^\mu + \frac{\partial \rho}{\partial \tau} = 0 \tag{4.23}$$

suggests taking the integral over all τ (Stueckelerg 1941). If $\rho_\tau(x) \to 0$ for $\tau \to \pm\infty$, that is, that the expectation of the occurrence of events in a finite region of x^μ vanishes for large values of the evolution parameter (the physical system evolves out of the laboratory), then the second term vanishes under this integration, and one finds that

$$\partial_\mu J^\mu = 0, \tag{4.24}$$

where

$$J^\mu(x) = \int_{-\infty}^{+\infty} d\tau\, j_\tau^\mu(x) \tag{4.25}$$

can be identified as the Maxwell current (this procedure has been called "concatenation" (Horwitz 1982).

In his book on electrodynamics, Jackson (1974) provides a construction of a covariant current by starting with an elementary current element $e\dot{x}^\mu \delta^4(x - x(s))$, where s is considered to be some parameter along the worldline $x^\mu(s)$ of the moving charged event, say, the proper time. He then asserts that

$$J^\mu(x) = e \int ds \dot{x}^\mu \delta^4(x - x(s)) \tag{4.26}$$

is conserved by noting that

$$\begin{aligned} \partial_\mu J^\mu(x) &= e \int ds \dot{x}^\mu \partial_\mu \delta^4(x - x(s)) \\ &= -e \int \frac{d}{ds} \delta^4(x - x(s)), \end{aligned} \tag{4.27}$$

which vanishes if the worldline moves out of the range of the laboratory as $s \to \pm\infty$. The transition from (4.25) to (4.26) is achieved by noting the identity

$$-\frac{d}{ds}\delta^4(x - x(s)) = \dot{x}^\mu \partial_\mu \delta^4(x - x(s)); \tag{4.28}$$

this is, however, precisely the conservation law (2.21) for the case $\rho(x) = \delta^4(x - x')$ for a charged event at the point x'. It follows from Jackson's construction, as well as the argument leading to (4.25), that what is considered a "particle", in electromagnetism, but also in the probability theory associated with quantum mechanics, i.e. an object which satisfies a law of conserved current and charge (or probability density), corresponds to at least a large segment of a worldline (Land 1998), an essentially nonlocal object in the Minkowski space. The nonrelativistic Schrödinger equation has a locally defined conserved current; the bilinear density $\psi_{NR}(\mathbf{x})^*\psi_{NR}(\mathbf{x})$ contains the product of wave functions of precisely equal mass. As we have seen in Chap. 2, e.g. (2.27), the Stueckelberg wave function for a free particle evolves according to

$$\psi_\tau(x) = U(\tau)\psi(x) = \frac{1}{(2\pi)^2}\int d^4p e^{-i\frac{p^\mu p_\mu}{2M}\tau} e^{-ip^\mu x_\mu}\psi(p); \tag{4.29}$$

since $p^\mu p_\mu = -m^2$, the variable corresponding to the measured mass, the τ integration of the bilinear has the effect of reducing this form to an integral over a bilinear diagonal in the mass. Thus, the τ integration is associated with the retrieval of "particle" properties, as in our discussion of the Newton-Wigner problem in Chap. 2, and Nambu's (1950) reduction, by integrating the wave function over τ with a factor $e^{-i\frac{M\tau}{2}}$ with predetermined M of Feynman's formulation (Feynman 1950) of perturbation theory to the particle mass shell.

Turning to the field equations (4.19), we see that an integral over τ, assuming the asymptotic vanishing of the $f_{\mu 5}$ field in τ, results (for the μ component) in

$$\partial^\nu \int d\tau f_{\mu\nu}(x,\tau) = \int d\tau j_\mu(x,\tau); \tag{4.30}$$

the right hand side corresponds, as we have argued, to the conserved current of Maxwell, so that we may identify, from (4.12),

$$\int d\tau a_\mu(x,\tau) = A_\mu(x), \tag{4.31}$$

i.e., the Maxwell τ-independent field. Thus the Maxwell field emerges as the zero mode of the fields $a_\mu(x.\tau)$, which we have called the "pre-Maxwell" fields (Saad 1989). Due to the linearity of the field equations, the integral over the field equations (4.19) reduce precisely, as we have seen, to the standard Maxwell form (this remark does not hold, as we shall see, to the nonlinear equations of the nonabelian Yang-Mills fields).

The physical situation that we have described here corresponds to the emergence of the Maxwell fields from detection apparatus that intrinsically integrates over τ. It would appear that there is, according to this theory, a high frequency modulation of the Maxwell field that is not easily observable in apparatus available in laboratories at the present time. There has been some indirect evidence, in connection with the self-interaction problem, for the existence of the classical $5D$ fields in connection with an extensive investigation of the self-interaction problem (Aharonovich 2011). Furthermore, the fifth field, as we have pointed out above, can be responsible for the transition represented in Stueckelberg's diagram Fig. 2.1; it also plays an essential role in the neutrino oscillation model that we shall describe below.

Equation (4.31) implies that the dimensionality of the pre-Maxwell fields must, since the Maxwell fields A have dimensionality L^{-1}, be L^{-2}. Thus the charge that we have called e' must have dimensional L (p^μ has dimension L^{-1}). The gauge invariant field strengths then have dimension L^{-3}. The quadratic contribution of the field strengths to the Lagrangian, $f^{\alpha\beta}f_{\alpha\beta}$ then has dimension L^{-6}. Since the action is an integral of the Lagrangian density over $d\tau d^4x$, of dimension L^5, the quadratic field strength terms must have a dimensional factor λ. The current in the resulting field equations contains the factor e', and the derivatives of the field strength on the right emerge with a factor λ; thus we can identify

$$e = e'/\lambda \tag{4.32}$$

with the dimensionless Maxwell charge.

Assuming the analog of the Lorentz gauge for the five dimensional fields (4.12),

$$\partial^\alpha a_\alpha = 0, \tag{4.33}$$

the field equations (4.19) become

$$(-\partial_\tau^2 + \partial_t^2 - \nabla^2) a_\beta = j_\beta/\lambda, \tag{4.34}$$

where we have taken the $O(4, 1)$ signature for the fifth variable τ. Representing $a_\beta(x, \tau)$ in terms of its Fourier transform $a_\beta(x, s)$, with

$$a_\beta(x, \tau) = \int ds e^{-is\tau} a_\beta(x, s), \tag{4.35}$$

one obtains

$$(s^2 + \partial_t^2 - \nabla^2) a_\beta(x, s) = j_\beta(x, s)/\lambda, \tag{4.36}$$

providing a relation between the off-shell mass spectrum of the a_β field and the quantum mechanical current source. As we have pointed out earlier, the solutions of wave equations with a definite mass m have, according to Newton and Wigner, contain nonlocality of the order of $1/m$; thus (for application of their arguments, thinking of the field as the wave function of a quantum of the field) the massless particle would have a very large support. There is some difficulty in imagining the emission of a photon from an atom of the size 10^{-8} cm which instantaneously has infinite support. However, if the photon being emitted is far off shell, and has an effective mass s, as in the equations above, which is fairly large, the particle being emitted can have very small spatial support, undergoing a relaxation process asymptotically to a particle with very small, essentially zero, mass.

A similar argument can be applied to the photoelectric effect; the energy $\hbar\omega$ associated with a photon of frequency ω is absorbed by a metal plate, and an electron emitted with exactly this energy (minus the work function to free the electron). The contraction of the energy of a highly nonlocalized radiation field into the very small region occupied by the electron is often attributed to "collapse of the wave function", but this statement does not account for the physical mechanism (even "collapse" mechanisms require the construction of a model (Hughston 1996; see also Silman 2008). In this process, again one may think of the photon going far off mass shell to be able to be absorbed locally.

It has often been argued, moreover, that an experimental bound on the photon mass is provided by gauge invariance. This argument would, of course, provide a bound if the mass term in the field equations had some given constant value; then the shift of the vector potential by a gradient term, even if the gauge function satisfied a homogeneous d'Alembert type equation, would leave an extra term in the equation that would not vanish. However, as we have seen, the field equation contains a second derivative with respect to τ, and if the gauge function has a vanishing $5D$ d'Alembertian of $O(4, 1)$ or $O(3, 2)$ type, gauge invariance would be maintained.

Finally, we remark that the foliation due to spin-statistics in the framework of Wigner's theory of induced representations, although worked out for the four-vector gauge fields in the previous chapter, remains valid for the $O(3, 1)$ part of the 5D gauge fields; only a fifth scalar field must be added to the Hamilton constructed, as we shall see in the next section.

4.2 Nonabelian Gauge Fields and Neutrino Oscillations

In this section we extend the picture of Stueckelberg for pair annihilation in classical dynamics to a diagram with two (or more) maxima, such as shown in Fig. 4.1, in which the incoming line eventually continues to move in the positive direction of t. Recall that the diagram of Fig. 2.1 constitutes, in the simplest interpretation, in its application to electromagnetism, to particle-antiparticle annihilation. In the case of a system representing a higher symmetry group than the $U(1)$ of electromagnetism, the two branches of the curve can correspond to two different "flavors", i.e. two different types of particles, each corresponding to a component of a vector-valued wave function, such as the nucleon, containing both the neutron and the proton. Yang and Mills (1954) thought of the nucleon as represented by such a wave function with two components corresponding to the neutron and the proton, which have different charge but almost the same mass, as a doublet state. Such a wave function would support the action of a higher symmetry group, in this case $SU(2)$. In our discussion of the gauge transformation $\psi \to e^{i\Lambda}\psi$, one may use a two by two matrix for the exponent Λ; the resulting gauge compensation fields, which one might call b^α, again for $\alpha = 0, 1, 2, 3, 5$ would then be two by two matrices as well and noncommuting. The corresponding $SU(2)$ group is called "isotopic spin," or "isospin," since the transition between neutrons and protons is involved in the generation of isotopes in nuclear physics.

Such fields are called nonabelian gauge fields, and play an important role in modern gauge theories. The importance of such theories lies largely in the fact that fields corresponding to gauge groups obey Ward identities (Kaku 1993; Peskin 1995) that control the singularities generated by the quantized fields and admit the application of the renormalization program (Bogliubov 1959; 't Hooft 1971). In such a construction, the vertex of the Stueckelberg diagram can contain not just a transition to antiparticle, but to an antiparticle with a different identity; the transition is induced through an interaction with a field that can connect different components of the incoming and outgoing (in τ) wave function. The diagram of Fig. 4.1 can then corresponds to two such transitions, one at each vertex (such a diagram appears in Feynman's paper in 1949 (Feynman 1949) with the sharp vertices characteristic of a perturbation expansion), resulting in a change of components of the particle as it evolves in spacetime.

As a simple example of such a phenomenon, we discuss here the so-called neutrino oscillation. In the theory of weak and electromagnetic interactions of Glashow et al. (1967), the electron neutrino, observed, for example in neutron decay to proton, electron and (anti-)neutrino, and the μ-neutrino, observed in muon decay to electron neutrino and antineutrino, form a doublet under a group called "weak isospin". There is an additional type of neutrino, the τ neutrino, which occurs in the decay of the τ meson, produced, for example, in high energy e^+e^- collisions (Henley 2007). Even though the masses of the three types of neutrinos are quite different they may be thought of as a triplet, with a gauge group $SU(3)$ with good analogy to the $SU(3)$ of quantum chromodynamics. Since the τ neutrino appears to be much heavier, it is less likely to be involved in the neutrino oscillations, but certainly not ruled out. However, for simplicity, we shall restrict our attention here to the ν_e and ν_μ oscillations, although he same qualitative picture would be applicable if all three neutrinos were taken into account. The corresponding gauge fields are called W and Z, after the particle resonances thought to play an important role in the mediation of the weak and electromagnetic (along with the elctromagnetic potential field A) interactions.

In the flavor oscillations of the neutrino system, interactions with the vector bosons of the Glashow-Salam-Weinberg (GSW) theory (Weinberg 1967) which induce the transition can produce, as we have pointed out, pair annihilation-creation events. In the framework of Stueckelberg theory, pair annihilation and creation events can be correlated, as shown in Fig. 4.1, by following the world line.[3] The methods of Feynman's original paper, based on a spacetime picture (Feynman 1949), closely related to Stueckelberg's earlier formulation, would admit such a construction as well. An "on-shell" version of our Fig. 4.1 appears, with sharp vertices, in Feynman (1949).

It may be noted from this figure that there is a net decrease in the time interval, possibly very small, observed for the particle to travel a certain distance. One might expect that over a long distance of transmission (long baseline experiments), neutrinos, due to this oscillation phenomenon, might arrive earlier at their destination than predicted by light speed estimates. The most recent experiments have shown that the arrival times are consistent with light speed; in the most recent OPERA experiment (Acquafredda(2009), Adam (2013)) over the 732 km distance form CERN to Gran Sasso, an arrival time of $6.5 \pm 7.4\left\{{+8.3 \atop -8.0}\right\}$ ns less than light speed arrival is reported, certainly *consistent* with light speed. There is, however, some room in the distribution found for early arrival; it would require higher precision to rule out early arrival.

[3]The curve shown in Fig. 4.1 should be thought of as corresponding to the expectation values computed with the local density matrix associated with the gauge structured wave function of the neutrino beam.

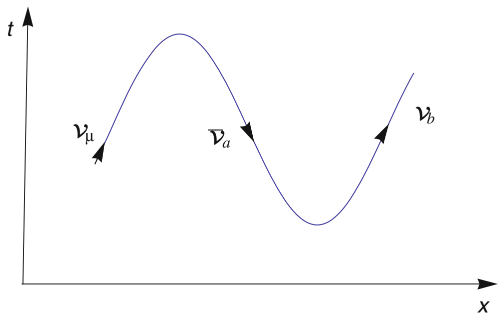

Fig. 4.1 Semiclassical neutrino oscillation

We remark that it has, however, been observed in the Supernova 1987a that the neutrinos arrive about 3 h before the light signal (Bahcall 1989). To show that a small advance in neutrino arrival times ("pull back" in time) could be consistent with this data as well, we make the following estimate.

An advanced arrival of the order of 6.5 ns in each 730 km (consistent with this data) would result in approximately 3×10^3 h early arrival. However, as we shall see below, the mechanism for the oscillations associated with such a "pull-back" involves the participation of the fifth field in an essential way, expected to fall off far from sources. One may estimate on the basis of a 3 h early arrival the range of effectiveness of the fifth field, assuming an advance of 6.5 ns in each 730 km where effective. A simple estimate yields about 30 parsec (pc), as an effective size of the supernova. The Sun is about 10^4 pc from the center of the galaxy, so an effective range of about 30 pc is not unreasonable. This argument is certainly not a proof of a "pull back"; it is meant to show that a small effect of this type could be consistent with the supernova 1987a data (see, moreover, further discussion in Bahcall (1989)).

Suppose, for example, that such oscillations can occur twice during the transit (Kayser 2004) from CERN to the Gran Sasso detectors, as in Fig. 4.2. The particles (and antiparticles) have almost everywhere propagation speed less than light velocity (except for the vertices, which we estimate, based on the Z, W lifetimes, to occur in about 10^{-22} s); it is clear from Fig. 4.2 that an early arrival would *not* imply, in this model, that the neutrinos travel faster than light speed. The effect noted by Glashow and Cohen (2011), indicating that Ĉerenkov radiation would be seen from faster than light neutrinos, would likely not be observed from the very short lived vertices, involving interaction with the W and Z fields, without sensitive detectors placed appropriately on the track. The neutrino arrivals detected at Gran Sasso appear to be almost certainly normal particles. The ICARUS detector (Acquafredda et al. 2009) records no γ's or e^+e^- pairs which would be expected from Ĉerenkov radiation from faster than light speed neutrinos (Cohen 2011).

A quantum mechanical counterpart of this model, in terms of Ehrenfest wave packets, is consistent with this conclusion. The derivation of the Landau-Peierls relation

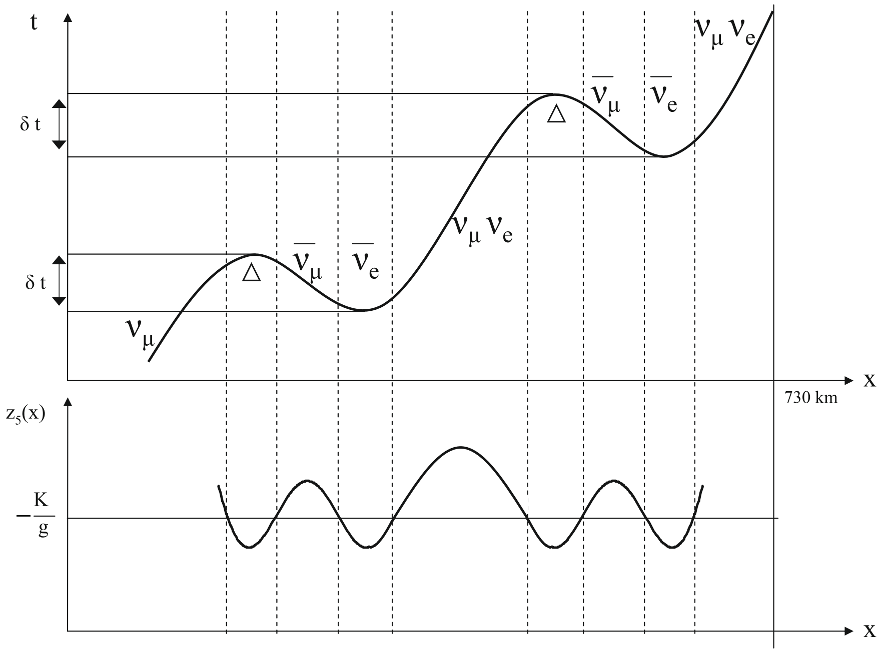

Fig. 4.2 Gauge condition for oscillations

$\Delta p \Delta t \geq \frac{\hbar}{2c}$ in the framework of the Stueckelberg theory discussed in Chap. 2, involves the assumption that the energy-momentum content of the propagating wave function contains predominantly components for which $\frac{p}{E} < 1$. Interactions, e.g., at the vertices of the curve in Fig. 4.2, can affect this distribution in such a way that, for some (small) interval of evolution, the wave packet can contain significant contributions to the expectation value of p/E much larger than unity, and thus the dispersion Δt in the Landau-Peierls relation can become very small without violating the uncertainty bound established by $< E/p >$. The interaction vertex may then be very sharp in t, admitting a precise manifestation of the deficit time intervals (as in the corresspponding Feynman diagram (Feynman (1948)).

The upper part of Fig. 4.2 shows schematically the orbit of a neutrino in spacetime during its transit, according to this theory, in which the first (annihilation) event results in the transition from a ν_μ to either a ν_μ or ν_e through interaction with a GSW boson (for this simple illustration we consider only the μ and e neutrinos, although there is no reason to exclude the τ neutrino) and the second (creation) event involves a transition from either of these states back to a ν_μ, ν_e state.

We now proceed to formulate the nonabelian gauge model; here, we call generically, the nonabelian gauge field z^α.

The gauge covariant form of the Stueckelberg Hamiltonian, valid for the non-Abelian case as well as for the Abelian, with coupling g to the 5D fields, is

$$K = \frac{(p^\mu - gz^\mu)(p_\mu - gz_\mu)}{2M} - gz^5(x), \tag{4.37}$$

where the z^μ fields are non-Abelian in the $SU(2)$ sector of the electroweak theory. Since, as we shall below, $\dot{x}^\mu$ is proportional to $p^\mu - gz^\mu$, the local expectation of the square of the "proper time" is proportional to that of the first term in the Hamiltonian. Therefore, see we see that the local expectation of z^5 must pass through that of the conserved value of $-K/g$ to admit passage of the orbit through the light cone. In the lower part of Fig. 4.2, we have sketched a form for a smooth z^5 wave (in expectation value) that would satisfy this condition. Such a wave can be easily constructed as the superposition of a few harmonic waves with different wavelengths (originating in the spectral density of the neutrino wave functions [see Eq. (4.49) below).

The occurrence of such a superposition can be understood from the point of view of the structure of the $5D$ GSW fields. Working in the context of the first quantized theory, where the functions ψ belong to a Hilbert space $L^2(x, d^4x) \otimes d$, with d the dimensionality of the gauge fields ($d = 2$ corresponds to the Yang-Mills case (Yang 1954) and the $SU(2)$ sector of the electroweak theory which we shall deal with here; our procedure for extracting the field equations and Lorentz force applies for any d), the field equations can be derived from the Lagrangian density (we consider the case of particles with spin in the next section)

$$\begin{aligned}\mathcal{L} = {} & \frac{1}{2}\mathrm{Tr}\big(i\frac{\partial\psi}{\partial\tau}\psi^\dagger - i\psi\frac{\partial\psi^\dagger}{\partial\tau}\big) \\ & - \frac{1}{2M}\mathrm{Tr}\big[(p^\mu - gz^\mu)\psi((p_\mu - gz_\mu)\psi)^\dagger\big] \\ & + g\mathrm{Tr}(z^5\psi\psi^\dagger) - \frac{\lambda}{4}\mathrm{Tr} f^{\alpha\beta} f_{\alpha\beta},\end{aligned} \tag{4.38}$$

where ψ is a vector valued function representing the algebraic action of the gauge field, and $\psi^\dagger$ is a 2-component (row) conjugate vector valued function; $\mathcal{L}$ is a local scalar function. The operation Tr corresponds to a trace over the algebraic indices of the fields; the dimensional parameter λ arises from the relation of these fields to the zero mode fields of the usual $4D$ theory (Yang 1954), as for the electromagnetic fields discussed above. For the variation of the field strengths we take δz^α to be general infinitesimal Hermitian algebra-valued functions. Extracting the coefficients of these variations, with the definition of the non-Abelian gauge invariant field strength tensor (Yang 1954)

$$f^{\alpha\beta} = \partial^\alpha z^\beta - \partial^\beta z^\alpha - ig[z^\alpha, z^\beta], \tag{4.39}$$

one obtains the field equations

$$\lambda\big[\partial^\alpha f_{\beta\alpha} - ig[z^\alpha, f_{\beta\alpha}]\big] = j_\beta \tag{4.40}$$

where

$$j_\mu = \frac{ig}{2M}\{(\partial_\mu - igz_\mu)\psi\psi^\dagger - \psi((\partial_\mu - igz_\mu)\psi)^\dagger\}, \tag{4.41}$$

and

$$j_5 = g\psi\psi^\dagger \equiv \rho_5. \tag{4.42}$$

Let us now impose, as done by Yang and Mills (1954), the subsidiary condition

$$\partial^\alpha z_\alpha = 0. \tag{4.43}$$

We then obtain from (4.40)

$$(-\partial_\tau^2 + \partial_t^2 - \nabla^2)z_\beta = j_\beta/\lambda + ig[z^\alpha, f_{\beta\alpha}], \tag{4.44}$$

where we have taken the $O(4,1)$ signature for the fifth variable τ. Representing $z_\beta(x,\tau)$ in terms of its Fourier transform $z_\beta(x,s)$, with

$$z_\beta(x,\tau) = \int ds e^{-is\tau} z_\beta(x,s), \tag{4.45}$$

one obtains

$$(s^2 + \partial_t^2 - \nabla^2)z_\beta(x,s) = j_\beta(x,s)/\lambda + ig\int d\tau e^{is\tau}[z^\alpha(x,\tau), f_{\beta\alpha}(x,\tau)], \tag{4.46}$$

providing a relation between the off-shell mass spectrum of the z_β field and the sources including the quantum mechanical current as well as the non-linear self-coupling of the fields.

Since the behavior of the z_5 field plays an essential role in the immediately applicable predictions of our model, consider the Eq. (4.46) for $\beta = 5$,

$$(s^2 + \partial_t^2 - \nabla^2)z_5(x,s) = j_5(x,s)/\lambda + ig\int d\tau e^{is\tau}[z^\nu(x,\tau), f_{5\nu}(x,\tau)]. \tag{4.47}$$

In a zeroth approximation, neglecting the nonlinear coupling term, we can study the equation

$$(s^2 + \partial_t{}^2 - \nabla^2)z_5(x,s) \cong j_5(x,s)/\lambda. \tag{4.48}$$

The source term is a convolution of the lepton wave functions in the Fourier space, so that

$$(s^2 + \partial_t{}^2 - \nabla^2)z_5(x,s) \cong \frac{g}{2\pi\lambda}\int ds'\psi(x,s')\psi^\dagger(x,s'-s). \tag{4.49}$$

The Fourier representation over s of the wave function corresponds to the set of probability amplitudes for finding the particle in the corresponding mass states; we expect these functions to peak in absolute value, in free motion, at the measured neutrino masses. There is therefore the possibility of several mass values contributing to the frequency of the spectrum of the z_5 field (the diagonal contributions contribute only to its zero mode, a massless radiative field of essentially zero measure). In order for the forces to give rise to a form for the z_5 field of the type illustrated in Fig. 4.2, there must be at least three peaks in the mass distribution of the wave functions, corresponding to three families of neutrinos. This condition has been noted in a somewhat different context (Nunokawa 2006) and in other studies (for example Refs. Fogli 1995; Bandyopadahyay 2002) discussing the three family structure).

We now turn to study the trajectories of the particles with non-Abelian gauge interactions to further check the consistency of our model. The Heisenberg equations of motion are associated with expectation values for which the classical motion is a good approximation if the wave packets are fairly well localized.

From the Hamiltonian (4.37) one obtains

$$\begin{aligned}\dot{x}^\lambda &= i[K, x^\lambda] \\ &= \frac{1}{M}(p^\lambda - gz^\lambda),\end{aligned} \tag{4.50}$$

of the same form as the classical result.

The second derivative is defined by

$$\ddot{x}^\lambda = i[K, \dot{x}^\lambda] + \frac{\partial \dot{x}^\lambda}{\partial \tau}, \tag{4.51}$$

where the last term is necessary because $\dot{x}^\lambda$ contains, according to (4.50), an explicit τ dependence which occurs in the fields z^λ. One then obtains (the Lorentz force for the non-Abelian case was also obtained, using an algebraic approach, in Land 1995)

$$\ddot{x}^\lambda = -\frac{g}{2M}\{\dot{x}^\mu, f^\lambda{}_\mu\} - \frac{g}{M} f^{5\lambda}. \tag{4.52}$$

Let us make here the crude approximation that was used in obtaining (4.48), i.e., neglecting the nonlinear coupling to the spacetime components of the field. Then, (4.52) becomes, for the time component,

$$\ddot{t} \cong -\frac{g}{M}\frac{\partial z^5}{\partial t}. \tag{4.53}$$

The rising z^5 field (Fig. 4.2), before the first passage through the light cone, would imply a negative curvature, as required. This consistency persists through the whole process.

We further note that

$$-\frac{ds^2}{d\tau^2} = \frac{2}{M}(K + gz^5), \tag{4.54}$$

so that

$$\frac{d}{d\tau}\frac{ds^2}{d\tau^2} = -\frac{2g}{M}\frac{dz^5}{d\tau}, \tag{4.55}$$

consistent as well with the form of Fig. 4.2.

4.3 The Hamiltonian for the Spin $\frac{1}{2}$ Neutrinos

The Lorentz force for the Abelian case with spin can be computed in exactly the same way from (3.32) with the additional term $-e'a_5$ as in (4.11). Note that the Lorentz force is not linear, so it cannot be mapped back to the Maxwell Lorentz force directly by concatenation.

Following the method of Chap. 3 for the non-Abelian case, we find a Hamiltonian of the form

$$K = \frac{1}{2M}(p - gz)_\mu (p - gz)^\mu - \frac{g}{2M} f_{\mu\nu} \Sigma_n{}^{\mu\nu} - gz^5, \tag{4.56}$$

where $\Sigma_n{}^{\mu\nu}$ is defined by (3.33).

Since γ^5 commutes with this Hamiltonian, there is a chiral decomposition (true for (3.32) as well), *independently of the mass of the neutrinos*, which admits the usual construction of the $SU(2) \times U(1)$ electroweak gauge theory. The $SU(2)$ sector that we discuss below would then apply to the left handed leptons. The asymptotic (free) solutions also admit a (foliated) helicity decomposition (Arshansky 1982).

We shall discuss the possibilities of *CP* violation provided by this structure below.

To compute the Lorentz force, as in (4.50), one obtains the particle velocity

$$\dot{x}^\lambda = i[K, x^\lambda] = \frac{1}{M}(p^\lambda - gz^\lambda). \tag{4.57}$$

For the second derivative, from (4.51) and (3.34), we obtain

$$\begin{aligned} \ddot{x}^\lambda = &-\frac{g}{2M}\{f^{\lambda\mu}, \dot{x}_\mu\} - \frac{g}{M} f^{5\lambda} \\ &+ \frac{g}{2M^2}\partial^\lambda f^n{}_{\mu\nu}\Sigma_n{}^{\mu\nu} + \frac{ig^2}{2M^2}[f^n{}_{\mu\nu}, z^\lambda]\Sigma_n{}^{\mu\nu}. \end{aligned} \tag{4.58}$$

The third term of (4.58) corresponds to a Stern-Gerlach type force. Note that we have included the subscript or superscript n to the quantities that are transverse in the foliation.

Under the assumption that the fields are not too rapidly varying, and again neglecting coupling to the spacetime components of the field z^α, we see that the acceleration of the time variable along the orbit may again be approximated by (4.53).

We are now in a position to write the Lagrangian for the full theory with spin. We take for the Lagrangian the form (4.38) with an additional term for the spin interaction and factors of $\gamma^0(\gamma \cdot n)$ to assure covariance, yielding under variation of $\psi^\dagger$ the Stueckelberg equation for ψ with Hamiltonian (4.56):

$$\begin{aligned} \mathcal{L}_n = &\frac{1}{2}\mathrm{Tr}\big(i\frac{\partial\psi}{\partial\tau}\bar{\psi} - i\psi\frac{\partial\bar{\psi}}{\partial\tau}\big)(\gamma\cdot n) \\ &- \frac{1}{2M}\mathrm{Tr}\big[(p^\mu - gz^\mu)\psi\overline{(p_\mu - gz_\mu)\psi)}(\gamma\cdot n)\big] \\ &+ g\mathrm{Tr}(z^5\psi\bar{\psi}(\gamma\cdot n)) - \frac{\lambda}{4}\mathrm{Tr} f^{\alpha\beta} f_{\alpha\beta} \\ &+ \frac{g}{2M}\mathrm{Tr}(f_{\mu\nu}\Sigma_n{}^{\mu\nu}\psi\bar{\psi}(\gamma\cdot n)). \end{aligned} \tag{4.59}$$

Defining j_α as in (4.41), (4.42), but with the factor $\gamma^0\gamma \cdot n$, required for covariance, i.e.,

$$j_{n\mu} = \frac{ig}{2M}\{(\partial_\mu - igz_\mu)\psi\bar{\psi} - \psi\overline{(\partial_\mu - igz_\mu)\psi)}\}(\gamma\cdot n), \tag{4.60}$$

and

$$j_{n5} = g\psi\bar{\psi}(\gamma \cdot n) \equiv \rho_n, \tag{4.61}$$

the variation of the Lagrangian with respect to the z-fields, where we have used the cyclic properties of matrices under the trace, yields, setting the coefficients of δz^ν, δz^5 equal to zero, the field equations

$$\lambda(\partial_\beta f^{5\beta} - ig[z_\beta, f^{5\beta}]) = \rho_n \tag{4.62}$$

and

$$\begin{aligned} &\lambda(\partial_\beta f^{\nu\beta} - ig[z_\beta, f^{\nu\beta}]) \\ &\qquad = j_n{}^\nu + \frac{g}{M}\Sigma_n{}^{\mu\nu}\{\partial_\mu\rho_n - ig[z_\mu, \rho_n]\}. \end{aligned} \tag{4.63}$$

Equation (4.63) corresponds to a Gordon type decomposition of the current, here projected into the foliation space (spacelike) orthogonal to n^μ. Note that the covariant derivative of ρ_n in the last term is also projected into the foliation space.

With the subsidiary condition $\partial^\beta z_\beta = 0$, as before, we may write the field equations as

$$\lambda(-\partial^\beta\partial_\beta z^5 - ig[z_\mu, f^{5\mu}]) = \rho_n \tag{4.64}$$

and

$$\lambda(-\partial^\beta\partial_\beta z^\nu - ig[z_\beta, f^{\nu\beta}]) = j_n{}^\nu + \frac{g}{M}\Sigma_n{}^{\mu\nu}\{\partial_\mu\rho_n - ig[z_\mu, \rho_n]\}. \tag{4.65}$$

Note that the spin coupling is not explicit in (4.65). Neglecting, as before, coupling to the spacetime components, one reaches the same conclusions for the approximate behavior of the z^5 field, i.e., as determined by Eq. (4.49) with $\psi^\dagger$ replaced by $\bar{\psi}\gamma \cdot n$. The latter reduces to the same expression for $n^\mu \to (-1, 0, 0, 0)$.

4.4 *CP* and *T* Conjugation

The association of this timelike vector with the spacelike surfaces used by Schwinger and Tomonaga (1948) for the quantization of field theories has been recently discussed (Horwitz 2013). These spacelike surfaces form the support of a complete set of commuting local observables on which the Hilbert space of states in constructed. It follows from the properties of the wave functions for a particle with spin, discussed in Chap. 3, that the *CPT* conjugate theory would be associated with the same spacelike surface, corresponding to $\pm n^\mu$. However, the *CP* conjugate, taking $\mathbf{n} \to -\mathbf{n}$ and $n^0 \to n^0$ refers to an entirely different spacelike surface (the time reversed states, for which $\mathbf{n} \to \mathbf{n}$ and $n^0 \to -n^0$ are associated with this spacelike surface as well, with reflected unit timelike vector). The equivalence of the physical processes described in these two frameworks would depend on the existence of an isometry (including both unitary and antiunitary transformations) changing the basis of the space from the set of local observables on the first spacelike surface to those defined on the conjugated surface as well as the equivalence of the physics evolving from it after the *CP* (or T) conjugation.

The spin coupling term in (4.56) contains the possibility of *CP* violation in generating a physics that is inequivalent on the new spacelike surface. The nonrelativistic quantum theory with Zeeman type $\boldsymbol{\sigma} \cdot \mathbf{H}$ coupling is, of course, not invariant under T conjugation. Precisely the same situation is true in the corresponding relativistic equation (4.56); as we have pointed out, in the special frame in which $n^\mu = (-1, 0, 0, 0)$, the matrices $\Sigma_n^{\mu\nu}$ reduce to Pauli matrices. Under Lorentz transformation they still generate the algebra of $SU(2)$ in a fundamental representation, and therefore still contain the imaginary unit. Therefore, the physical evolution on the *CP* conjugate spacelike surface is not, in general, equivalent to the original evolution. For this phenomenon to occur, it is necessary that there be present an $f_{\mu\nu}$ field. In addition to self-interaction effects, for which the intrinsic *CP* violation can be expected to cancel, the Stueckelberg oscillation diagram of Fig. 4.1 suggests the existence of fields present in the equations of motion of the second branch due to the proximity of the accelerated motion in the first branch, thus providing a fundamental mechanism for *CP* violation. A consequence of this structure is that the physics in the corresponding *CP* conjugated system of the quantum fields, evolving from the *CP* conjugate spacelike surface, could be inequivalent.

In this chapter, we have argued that, according to the derivation of the Landau-Peierls relation given in Arshansky (1985), the vertices of the neutrino-antineutrino transitions may be very sharp, and provide for a rather precise "pull back" of the time interval. Significantly higher precision than available in the present experiments would be necessary to see such an effect.

We have worked out the equations describing the Lorentz forces and the field equations of the corresponding ($5D$) non-Abelian gauge theory, with the help of Stueckelberg type Hamiltonians both for the spinless case and for the case of relativistic particles with spin in interaction with such a nonabelian gauge field, and have shown that the conclusions reached are, in lowest approximation, consistent with our simple model. We emphasize that, in the framework of the Stueckelberg model, the dynamics of the fifth gauge field, modulated by the particle mass spectrum contained in the wave function (as in Eq. (4.49), plays an essential role for the oscillation process.

The presence of spin, described in the relativistic framework of Wigner (1939), as in Arshansky (1982), Horwitz (2013), introduces a foliation in the Hilbert space and in the structure of the fields, both classical and quantum. Since, in Tomonaga-Schwinger quantization (Tomonaga 1948) of the fields, the spacelike surface constructed to define a complete set of local observables is characterized by being orthogonal to a timelike vector n of the foliation (Horwitz 2013), the actions of the discrete *CP* or T transformations change the basis for the construction of the Hilbert space to essentially different spacelike surfaces. Along with the form of the spin coupling term in (4.56), this suggests a model for *CP* or *T* violation on the first quantized level.

We furthermore remark that our model would be applicable to the *K*, *B* and *D* systems (Kayser 2004) as well, manifested by the quark gluon interactions in their substructure.