Discussion

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The fundamental basis for the formulation of a relativistically manifestly covariant quantum theory was given in the introductory chapter of this book. The thought experiment of Einstein, constituting two frames of reference, in relative inertial motion, for the generation and reception of light signals forms the basis for the construction of the special theory of relativity. Calling these frames F and F', according to this experiment, two signals emitted successively from F at, say, τ_1 and τ_2 are received in the frame F' at, respectively, τ_1' and τ_2', with the relation between them

$$\tau_2' - \tau_1' = \frac{\tau_2 - \tau_1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{11.1}$$

where v is the relative velocity of the two frames, and c is the velocity of light. The relation (11.1) follows, according to Lorentz, from the null result of the Michelson-Morley experiment. Einstein defines the *observed* difference as the time interval

$$\Delta t = \tau_2' - \tau_1'. \tag{11.2}$$

Therefore we see that the *Einstein time*, which transforms, along with the observed interval Δx between the places in the frame F. As observed in F', to provide the observed spacetime point (t', x') (understanding these variables as intervals), transforming according to the Lorentz transformation. These coordinates must be considered as *observables*, the outcome of a measurement. As our discussion of the gravitational redshift shows, this is true as well for the spacetime manifold of general relativity, for which it is remarkable that an assumed local diffeomorphism invariance of the physical laws provides a set of equations (the Einstein equations) which relate the *geometry* of such observable quantities to the energy momentum of the system.

In the Galilean (Newtonian) description of dynamics, the universal time t postulated by Newton provides a parameter for the description of the evolution of the state of a particle in phase space, $\mathbf{x}(t)$, $\mathbf{p}(t)$. Since, in the relativistic world, as we have argued, t is an observable on the same level as $\mathbf{x}$, the description of the evolution of the system requires the introduction of a parameter τ, admitting the description of

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a phase space $t(\tau)$, $\mathbf{x}(\tau)$ with $E(\tau)$, $\mathbf{p}(\tau)$. For this motion, Stueckelberg postulated the existence of a Hamiltonian for which this evolution follows a generalized set of Hamilton equations. In order to be able to treat the N body problem, Horwitz and Piron asserted that the parameter τ is universal, and therefore plays the role of the universal parameter of time postulated by Newton.

As for the nonrelativistic theory, one can then write a Schrödinger equation for a quantum wavefunction $\psi_\tau(x)$, a covariant function on spacetime in a Hilbert space $L^2(R^4)$, satisfying all the requirements of a full quantum theory.

Many of the properties of such a theory are described in these chapters, including bound state spectra, scattering theory and diffraction experiments constituting interference in time, such as the remarkable experiment performed by Lindner et al. The very high frequencies involved in such phenomena form an entrance into the developing field of attosecond physics.

As for the nonrelativistic quantum theory, the construction of the tensor product spaces leading to Fock space and second quantization is straightforward, and some of the properties of the resulting quantum fields are discussed. Although it has been shown by Andrew Bennett (and described here) that the lowest order correction to the electron magnetic moment can be computed without recourse to second quantization, since the first quantized theory can describe what appears to be particle creation and annihilation in the laboratory (as in Stueckelberg's original paper), there are clearly phenomena that can be associated with the creation and annihilation of *events*. The development of quantum statistical mechanics given here illustrates the utility of this concept in the description of relativistic many body systems.

Although we have discussed some applications of this framework to the geometrical approach to the dark matter problem of the galaxies of Milgrom, Bekenstein and Sanders, a general discussion of the application of the theory presented here to general relativity remains to be formulated.

The several phenomenological consequences of this theory, making contact with experiment in some important areas, as described here, with potential applications to general relativity and the emerging field of attosecond physics, provide a strong motivation for a continued effort to develop the theory.