Logic, Epistemology, and the Unity of Science

Ken Akiba
Ali Abasnezhad *Editors*

Vague Objects and Vague Identity

New Essays on Ontic Vagueness

Springer

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LOGIC, EPISTEMOLOGY, AND THE UNITY OF SCIENCE

VOLUME 33

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New Essays on Ontic Vagueness

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Preface

Fuzzy boundaries of material objects such as stars, mountains, and furry animals; imprecise properties and relations such as baldness, tallness, love, and hate; and the gradually changing colors of the sky at the sunset – vagueness is all around us. But, traditionally, the source of vagueness has been attributed to the imprecision in our representational faculty – our perception, language, etc. According to the traditional view, the material world itself is crisp and precise, with all the sharp edges, but we experience vagueness because of lack of precision in our representation of the world. In this respect, vagueness is like the monster you thought you saw in the dark corner of the street. Something unreal.

This view of vagueness, however, is now being challenged. The new view, onticism, sets forth the idea that vagueness exists in the world itself instead of, or in addition to, our representation. Vagueness is real. Furthermore, even though traditionally people talk more often about vague *properties* such as tallness and baldness, according to onticism, there are also vague *objects* or vague *individuals* in the world, such as mountains and cats. This vagueness might be created by indeterminacy in quantum mechanics, or it might be created by the vagueness in the part-whole relation between larger objects and their smaller constituents, i.e., vagueness about which smaller objects (e.g., molecules) are part of the larger objects (e.g., cats) in question. Or it might be created by something else (especially in the case of vagueness in abstract or nonmaterial objects). Whatever the cause, according to onticism, there are vague objects in the world. So we do not have to deny either the existence or the fuzziness of such objects as mountains and cats.

The once-seminal idea of ontic vagueness had fallen into disrepute when Gareth Evans gave a negative answer to the title question of his famous paper, "Can There Be Vague Objects?" in 1978. Evans argued that if there were a vague material object with fuzzy boundaries, its identity ought to be vague; in particular, if we compare a putative vague object, say *a*, with its precise counterpart, *b*, another object which coincides with *a* except for *a*'s vague parts, it should be vague whether *a* is identical with *b*. But that is impossible, Evans contended, because *a* has the property *being definitely identical with a* whereas *b* does not have the property; so *a* and *b* must be definitely distinct, after all. Thus, the very idea of vague object is incoherent. Many people have given up on the possibility of ontic vagueness because of this simple but cogent argument.

Evans's argument, however, has proved to be not iron clad; indeed, far from it. Among other things, it relies on the unchallenged assumption that a vague object must have vague identity. Is this assumption really correct? Can't we make sense of the idea that vague objects can exist without vague identity? This question invites us to reflect on the very concepts of vague object and vague identity and their relations. Some people also think that a vague object must have vague existence: it must exist not entirely but partially. Whether these ideas are correct or not depends on how we use the relevant concepts. What, really, does it mean to say that an object is vague, that it has vague identity, or that it has vague existence? And under what interpretation, if any, does it make sense to say so? Finally, if those claims, or at least some of them, can indeed be made sense of, are they actually true in some cases?

The chapters in this volume address these and other questions pertaining to ontic vagueness. Some defend ontic vagueness; others argue against it. The topics they discuss are diverse, ranging from quantum indeterminacy to mereology to personal identity to phenomenal qualities to tolerant logic. Some chapters raise and answer such questions as "How should we count vague objects?" "How should we apply abstraction principles to vague properties?" and "Does linguistic vagueness, taken as a species of ontic vagueness, exist in the word-sense relation or the sense-reference relation?" All of the chapters included in this volume are new and are written specifically for this occasion. The manuscript of each chapter was anonymously refereed by at least one external expert on the subject. Many of the essays that were accepted were revised in response to the referees' comments. The whole book was refereed by a reviewer chosen by Springer.

We would like to thank all the contributors to the volume for their cooperation, patience, and timely submission of their manuscripts. Our utmost gratitude goes to all the anonymous referees, who agreed to review the submitted manuscripts on a very tight schedule without taking any credit. Their service to the discipline of philosophy and the philosophical community was incredible. We are also grateful to Springer, in particular, Christi Lue, Ties Nijssen, and the *Logic, Epistemology, and the Unity of Science* series editor Shahid Rahman, for the support and guidance for the project they have provided.

Richmond, VA, USA — Ken Akiba
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Chapter 1
Introduction

Ken Akiba

This book is an anthology of 15 new essays written on the topic of ontic (metaphysical or "worldly") vagueness. As a whole, it represents the state of the art in research in the field. The basic idea of ontic vagueness is that, at least in some cases, vague things (facts, states of affairs, etc.) are vague because of the way they themselves are and not because of the way we see them or think about them. Just as it is an objective fact that Michael is 200 cm tall, it is an objective fact that he is a tall man, and just as it is an objective fact that John is 178 cm tall, it is an objective fact that it is vague (or indeterminate) whether John is a tall man or not. Vagueness has been a very popular topic in philosophy especially over the last 30 years or so, and a number of collections of essays[1] have been published during that period. However, the idea of *ontic* vagueness has been rather ignored. This volume, for the very first time, puts together essays that are focused on the topic of ontic vagueness, whether they argue for it or against it.

Until around the 1990s, philosophical discussion on the nature of vagueness was dominated by a single theory – the semantic (or linguistic) view of vagueness, also called *semanticism* or *linguisticism*. According to semanticism, the world itself is perfectly crisp and precise. Vagueness does not exist in things themselves. It exists in our language, thought, and perception because of the vagueness in our *representation* of the world.

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[1]The most standard collections are Keefe and Smith (1997) and Graff and Williamson (2002). For the sake of readability, most references in this Introduction are relegated to the endnotes. Note that the works referred to are often merely samples selected from a much larger literature.

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K. Akiba and A. Abasnezhad (eds.), *Vague Objects and Vague Identity*, Logic, Epistemology, and the Unity of Science 33, DOI 10.1007/978-94-007-7978-5_1,

> Vagueness and precision alike are characteristics which can only belong to representations, of which language is an example. They have to do with the relation between a representation and that which it represents. Apart from representation, whether cognitive or mechanical, there can be no such thing as vagueness or precision; things are what they are, and there is an end of it. (Russell 1923, p. 85)

Thus, while it is an objective fact that Michael is 200 cm tall, there is no objective fact that he is a tall man, and while it is an objective fact that John is 178 cm tall, there is no objective fact about whether he is a tall man or not, or even about whether his being a tall man is vague or not. Unlike the precise properties *being 200 cm tall* and *being 178 cm tall*, there is in reality no vague property *being a tall man* or even, for example, *being a tall British male citizen*. (Russell would not have called even the former properties "precise" because there is no distinction between vague and precise properties in reality.) It is indeterminate whether the proposition "John is a tall man" is true or false only because it is semantically indeterminate which precise candidate property – *being a man at least 177 cm tall*, *being a man at least 178 cm tall*, ..., *being a man at least 180 cm tall*, etc. – is denoted by "is a tall man." (Needless to say, the actual increments ought to be much, much smaller; so, even though those candidate properties must be reasonable, or *admissible*, there still will be numerous candidate properties.) "Michael is a tall man" will be determinately true, however, because it will be true no matter which precise candidate property (or "precisification" or "sharpening") is denoted by "is a tall man," even though there is no property *being a tall man* in the world. This way, semanticism has made it possible for us to embrace a crisp ontology but accept as (determinately) true many propositions that involve vague concepts. This method of making vagueness-involving propositions true is called *supervaluation*,[2] and the theory that features supervaluation, by far the most popular brand of semanticism, is called *supervaluationism*.[3] In what follows, we often ignore the difference between semanticism and supervaluationism.

In the last 20 years, two other theories have emerged as strong contenders (although they existed even before then) – one is the epistemic view (or *epistemicism*), and the other is the ontic view (or *onticism*). Epistemicism agrees with semanticism that there is no vagueness in the world itself, but disagrees about the reason why, then, vagueness seems to be all around us. Unlike semanticism, epistemicism finds it in our ignorance about the world. Just as properties such as *being 200 cm tall* and *being 178 cm tall* exist, the property (in a thin, innocuous sense of "property") *being a tall man* exists, and it is a crisp, precise property. Presumably it coincides with one of the properties, *being a man at least 177 cm tall*, *being a man at least 178 cm tall*, ..., *being a man at least 180 cm tall*, etc. We just do not know – and cannot know, according to Williamson (1994) – which one it is. Vague predicates have sharp boundaries just as precise predicates do; the difference

[2] Fine (1975).

[3] Lewis (1993), Keefe (2000), among many other advocates.

is that we cannot know the boundaries of the vague predicates. Consequently, there is a fact of the matter whether John is a tall man or not, but we just cannot know what it is.

In contrast to semanticism and epistemicism, onticism – the subject of this anthology – holds that there are objectively vague facts (or states of affairs) in the world. *That John is a tall man* is such a state of affairs. The vague property *being a tall man* exists in reality, and Michael (definitely) possesses that property, while John neither definitely does nor definitely does not possess it.

As is already apparent from the above brief characterizations, onticism, on the one hand, embraces a larger ontology that includes vague things (vague facts, states of affairs, propositions, properties, individuals, etc.) along with crisp things. Onticism holds that the property *being a tall man* exists as a vague property, whereas semanticism maintains that it does not exist, and epistemicism maintains that even though it exists, it is indeed nothing but a crisp property. It is a major challenge to onticists to explain how such vague things can exist alongside crisp things. On the other hand, onticism can avoid explaining the gap between the world and us, alleged to exist by semanticism and epistemicism, whether it is a gap between the world and our representation of it or between the world and our knowledge of it. Epistemicism's claim that our usage of the term "is a tall man" determines its precise extension in a way inscrutable to us ourselves is often met with an incredulous stare, and the idea of *partial reference* to multiple precisifications, embraced by semanticism (supervaluationism, in particular), has been less criticized perhaps only because we do not even know what ordinary, or "whole," reference is (its physical nature, etc.).

With the emergence of epistemicism and onticism, it is now fair to think of the current debate over the nature of vagueness as a three-way battle between semanticism, epistemicism, and onticism (although other views, such as contextualism[4] and nihilism,[5] should not be forgotten). It is probably true that the majority of contemporary philosophers are still most sympathetic to semanticism. There are still many philosophers who express an utter disbelief in onticism. Some of them go even so far as to say that they cannot make sense of such an outrageous view. Michael Dummett once wrote:

> . . . the notion that things might actually *be* vague, as well as being vaguely described, is not properly intelligible. (Dummett 1975, p. 314)

David Lewis agreed:

> The only intelligible account of vagueness locates it in our thought and language. (Lewis 1986, p. 212)

> . . . I doubt that I have any correct conception of a vague object. (Lewis 1993, p. 27)

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[4]Raffman (1994), Graff (2000), and Shapiro (2006).

[5]Unger (1979) and Wheeler (1979).

Such disbelief in (or at least skepticism about) the intelligibility of onticism – let alone its correctness – is still shared by many philosophers. We should remember, however, that, contrary to the popular image, philosophers are also prone to fashion, as recent history shows. As Sorensen (2012) points out in a similar context, until about the 1970s almost all philosophers had believed that all necessity was linguistic (or *de dicto*), stemming from the meanings of the words involved. Most philosophers nowadays believe in *de re* modality, modality in things themselves, thanks to Lewis (1986),[6] as well as to Kripke (1980) and others. This example does not merely show that philosophers can change their minds 180°; it also points to a direction in which contemporary philosophy seems heading. In recent years, philosophers have become increasingly aware of the similarity between modality and ontic vagueness. Some[7] even consider ontic vagueness a sort of modality. They take a history lesson from the fact that we have been treating modality more and more realistically over the last half century and see their own view as part of this trend. Of course, their opponents may say that *that's* just a fashion. So it is not clear what the future holds for onticism. Still, it is by now an undeniable fact that onticism is taking a more and more significant part in the debate about the nature of vagueness, and that no one can ignore it as a mere curiosity any longer.

The rest of this Introduction will do three things. First, it will sketch, in two sections, three arguments for, and one argument against, onticism. Second, it will attempt to clarify often confusing uses of such terms as "ontic vagueness," "vague objects," "vague existence," and "vague identity" – their differences and relations. One group of popular arguments which purports to connect vague objects to vague identity will also be discussed. The third and final part briefly summarizes the chapters included in this volume. The purpose of this Introduction is merely to provide the reader with background knowledge sufficient for the reading of the chapters included; so no complete or detailed accounts of the issues touched on here should be expected.

1.1 Why Onticism?

We have not yet seen in the literature any *decisive* argument in favor of onticism, and it is possible that we never will. We have seen, however, a few arguments that may persuade some people to embrace onticism. Here, three such arguments will be mentioned. The first, *the argument from ordinary objects*, is well known to philosophers and is widely considered rather strong. The second, *the argument from semantic indeterminacy*, is lesser known, but it is perhaps almost as important as

[6] This is somewhat ironical because Lewis is very open-minded about modality and possible worlds but closed-minded about ontic vagueness, which, however, may prove to be a kind of modality, as we shall see below.

[7] Akiba (2000a, 2004), Morreau (2002), Williams (2008a, b), Barnes (2009, 2010), and Barnes and Williams (2011).

the first. The third, *the argument from quantum mechanics*, may be as convincing an argument for onticism as any argument can be, but the relevance of the argument may be questioned.

1.1.1 The Argument from Ordinary Objects

The first argument, *the argument from ordinary objects*, concerns mereological indeterminacy and vague objects. (The difference and similarity between vagueness and indeterminacy will be discussed later in Sect. 1.3; but, in the meantime, we shall ignore the difference and use the terms interchangeably.) One apparent species of ontic vagueness is the existence of vague objects. Vague objects, as we use the term here, are individual physical objects whose spatiotemporal boundaries are vague. Many – indeed, probably most – ordinary, so-called "medium-sized," physical objects around us, including rather small objects such as cells and molecules and rather large objects such as stars and planets, are vague objects in this sense. When one amoeba splits into two amoebas, it is indeterminate when the original amoeba ceases to exist and new amoebas start existing, or where the spatiotemporal boundaries of the amoebas are. When a cat is about to lose a hair or a human is about to lose an old skin cell, it is indeterminate when the hair/cell ceases to belong to the cat/human. It seems indeterminate exactly when a person starts and stops existing – this is related to the issues of abortion and euthanasia – and so on and on. Here, the common issue is that the mereological (or part-whole) relations between a larger physical body and its constituents seem indeterminate. Indeed, part of our concepts of *amoeba*, *cat*, *human*, *person*, etc., is that they are mereologically vague. So if we deny the existence of vague objects, we must deny the existence of most ordinary physical objects such as amoebas, cats, humans, persons, etc.

In fact, that's exactly what semanticists do, though they tend to downplay this point. Put baldly, semanticism is nihilism (or eliminativism) about ordinary medium-sized physical objects. Recall that the semanticists' project is that of making vague sentences apparently true without being committed to the existence of vague objects. But this seems to require a sort of disingenuous philosophical double-talk. At the commonsensical level, semanticists talk with the masses and say, for example, that the cat, Tibbles, is on the mat, whereas, at the philosophical level, they say that there is no (unique) thing to which "Tibbles" refers, and they demand that the first assertion be understood in terms of supervaluation, but not the second. It seems more reasonable for us to accept the real existence of ordinary physical objects such as cats and humans. But then we must accept the existence of vague objects.

Epistemicists will say in response that the ordinary medium-sized physical objects indeed do exist, but not as vague objects but as crisp objects. But this view is also hard to accept. It is unbelievable that our uses of terms such as "cat," "the sun," etc., determine the sharp boundaries of a cat, the sun, etc., in a way inscrutable to us. How, for instance, can our uses of the terms "the sun," "molecule," etc., possibly determine exactly when – down to a nanosecond – a gas molecule coming out of the sun is no longer a part of the sun? How can our uses of the terms "fetus" and "person" possibly determine exactly when – down to a nanosecond – a fetus becomes a person? It's not as if there were clear "joints" in nature that demarcate those things unbeknownst to us. So onticism seems to have an advantage over both semanticism and epistemicism on this issue.

1.1.2 The Argument from Semantic Indeterminacy

The second argument, *the argument from semantic indeterminacy*, is due to Merricks (2001), but will be put slightly differently here. This argument shows that supervaluationism ought to imply ontic vagueness in semantic relations unless epistemicism is correct. To recall, supervaluationism postulates partial references to various precisifications: it holds that a vague expression partially refers to one precisification, partially refers to another precisification, etc. But what is a partial reference? It's an indeterminate reference. So if epistemicism is incorrect and there are indeed partial references in reality, there ought to be ontic indeterminacy in reference relations. Put simply, language is a part of the world. But, then, it would be extremely surprising if ontic vagueness is confined to semantic relations. For instance, if reference relations are nothing but causal relations of some kind, as the causal theory of reference[8] maintains, there must be ontic vagueness in other causal relations. So supervaluationism actually seems to support onticism (unless, again, epistemicism is correct).

It is possible that deflationism[9] is correct and that there are no substantive reference relations in the world. But that does not give semanticism any relief. On the contrary, if there is no substantive reference in the world, there is no substantive partial reference, either. So if deflationism is correct, it seems extremely difficult, to say the least, to make sense of semanticism. Again, it seems we have to embrace onticism or epistemicism.

1.1.3 The Argument from Quantum Mechanics

The third argument for onticism is *the argument from quantum mechanics*. It simply points out the well-known fact that according to the currently orthodox Copenhagen interpretation of quantum mechanics, there is much indeterminacy in the world of fundamental particles. A physical system does not have a determinate state that uniquely determines the values of its measurable properties. Quantum mechanics can only give a certain probability distribution on the set of outcomes of the

[8]Kripke (1980).

[9]Horwich (1990).

measurements of observables. Assertions such as "particle p is in location x with momentum m" cannot have a determinate truth value. So the idea that everything is determined in the world is simply incorrect.

This argument seems to give as strong support for onticism as any argument can give, if onticism is understood very broadly, as the view that there is some indeterminacy in the world itself. One major problem with the argument, however, is that it is unclear how quantum indeterminacy is related – or unrelated – to the kind of vagueness most philosophers are interested in, such as that involved in tallness, baldness, heaps, people, and other medium-sized physical objects. No philosopher can deny quantum indeterminacy, but many might deny its relevance to the philosophical issues of vagueness. One common element in the kind of vagueness most philosophers are interested in seems to be that of human convention: vagueness seems to arise because of the way we humans demarcate and categorize things. That, of course, does not necessarily refute the existence of vagueness in reality (although it does seem to give an initial plausibility to semanticism). This conventional element seems absent in quantum indeterminacy. Thus, it seems possible that quantum indeterminacy is a different kind of indeterminacy than the kind most philosophers are interested in and calls for a different treatment.[10] Just as causal indeterminism, also a result of quantum indeterminacy, is widely considered irrelevant to the issue of free will, quantum indeterminacy may prove to be irrelevant to the most central philosophical issues of vagueness. At the very least, a further clarification of the relation between quantum indeterminacy and other kinds of indeterminacy/vagueness is called for.

1.2 Why Not Onticism? Evans's Argument

By far the most well-known and influential argument – perhaps the *only* well-known and influential argument – against onticism is Gareth Evans's argument, given in his one-page essay, titled "Can There Be Vague Objects?" (Evans 1978). Evans answers the question in the negative. It is now widely acknowledged that Evans's argument does not support his conclusion.

At the outset of his paper, Evans asserts (without argument) that if there are vague objects that have vague boundaries, then some identity statements involving the objects must be vague.

> It is sometimes said that the world might itself *be* vague. Rather than vagueness being a deficiency in our mode of describing the world, it would then be a necessary feature of any true description of it. It is also said that amongst the statements which may not have a determinate truth-value as a result of their vagueness are identity statements. Combining these two views we would arrive at the idea that the world might contain certain objects about which it is a *fact* that they have fuzzy boundaries. But is this idea coherent? (Evans 1978, p. 208)

[10]See Darby (2010) and Skow (2010) for the differences between quantum indeterminacy and other kinds of indeterminacy.

It cannot be, as the following argument shows. Therefore, Evans argues, there cannot be vague objects.

Read "∇p" as "it is indeterminate whether p." Suppose:

(1) $\nabla(a = b)$. It is indeterminate whether a is identical with b. (Assumption.)

Then,

(2) $\lambda x[\nabla(a = x)]b$. b has the property of being indeterminately identical with a. (From (1) by λ-abstraction.)[11]

However,

(3) $\neg\nabla(a = a)$. It is not indeterminate whether a is identical with a. (By the nature of self-identity.)

Thus,

(4) $\neg\lambda x[\nabla(a = x)]a$. a does not have the property of being indeterminately identical with a. (From (3) by λ-abstraction.)

Therefore,

(5) $\neg(a = b)$. a is not identical with b. (From (2) and (4) by the contrapositive of Leibniz's law.)

Evans states that (5) already contradicts the assumption (1),[12] but, furthermore, if we assume that the logic of "determinately," Δ, is S5 modal logic, where $\nabla p =_{df} \neg\Delta p \wedge \neg\Delta\neg p$,[13] then we can derive

(5′) $\Delta\neg(a = b)$. It is determinate that a is not identical with b.

This formally contradicts (1).

There have been a number of attempts to block this argument. The inferences from (1) to (2) and from (3) to (4) may be blocked if (1) and (3) are understood *de dicto*. This is exactly what semanticists such as Lewis (1988) propose to do:

[11]Generally, if "Pa" is a sentence "a is P," then the λ-abstraction "$\lambda x(Px)$" denotes the property *being P*. Then "$\lambda x(Px)b$" will mean "b has the property *being P*," or, simply, "b is P," that is, "Pb."

[12]Salmon (1981) gave essentially the same argument.

[13]Read this "it is indeterminate whether p or not-p if and only if it is not determinate that p and it is not determinate that not-p." Thus, unlike the necessity and possibility operators $\Box$ and $\Diamond$, Δ and ∇ are not duals; Δ corresponds to $\Box$, but ∇ does not correspond to $\Diamond$. ∇ corresponds to "contingently" in alethic modality.

from the claim that "a is identical with b" is indeterminate, it does not follow that there is a single object, b, which is indeterminately identical with a, because "b" can be partially referring to various precisifications. There are onticists[14] who agree with semanticists that the source of (1) is the referential indeterminacy involved in the names "a" and "b"; they contend, however, that this referential indeterminacy is created by ontic vagueness and not by semantic indecision on our part. These onticists also maintain that the inferences from (1) to (2) and from (3) to (4) should be blocked. Some other onticists[15] try to block the inference from (2) and (4) to (5), alleging that even though Leibniz's law itself holds, its contrapositive, that is, $\exists P(Pa \wedge \neg Pb) \rightarrow \neg(a = b)$ for any a and b, need not hold when vagueness is involved.

What people have become aware of in the last dozen years or so, however, is that the argument (1) through (5) need not be blocked at all. We can accept the argument – or, more specifically, the argument from one implicit disjunct $\neg\Delta(a = b)$ of (1) to (5) $\neg(a = b)$ – as a valid argument, essentially the contrapositive of Kripke's (1971) argument[16] for the necessity of identity, $a = b \rightarrow \Box(a = b)$, where $\Box$ is replaced with Δ.[17] The upshot is simply that identical things are determinately identical, or that there are no identical things that are only indeterminately identical. Evans somehow thought that this is contradictory, but it simply is not. Accepting it does not affect the main theses of onticism at all.

Certainly, if it turns out that the logic of "determinately" must be S5, then we can derive a formal contradiction from (1) (together with (3), which, however, has never been seriously challenged). Indeed, as demonstrated by Williamson (1996) and Hughes and Cresswell (1996, p. 314),[18] a logic of "determinately" weaker than S5 will still produce a contradiction as long as it is a *normal* modal system that includes axiom B: $p \rightarrow \Delta\neg\Delta\neg p$, that is, if p, then it is determinate that $\neg p$ is not determinately true. (We shall come back to this axiom in the next section, when the relation between vague objects and vague identity is discussed.) But, even if we accept such a logic, the above examination only shows that (1) cannot be true for any two objects a and b; that is, it only shows that there cannot be indeterminate identity.[19]

There have been other attempts to strengthen Evans's argument (1) through (5) to derive a formal contradiction. One of the most famous is to strengthen the premise (1) into (1′) $\Delta\nabla(a = b)$, that is, it is determinate that it is indeterminate whether $a = b$. Though a formal contradiction is derivable from this premise and (3), the

[14] Williams (2008a) and Barnes (2009).

[15] Parsons and Woodruff (1995) and Parsons (2000).

[16] Barcan (1947) and Marcus (1961) gave essentially the same argument.

[17] Williamson (1996) and Akiba (2000a, b, 2004). Wiggins (1986) presented his argument for the determinacy of identity this way.

[18] Kripke (1980, note 56) pointed this out earlier, without providing a proof.

[19] See Heck (1998) for a close logical analysis of Evans's argument.

argument only shows that ($1'$) cannot be true, and there is no good reason why onticism should be committed to ($1'$); so the argument fails to refute the existence of vague objects.

To summarize, Evans's argument at best shows that either S5 is too strong for the logic of "determinately" or there is no indeterminate (or vague) identity. It should be noted that most philosophers interested in vagueness – onticists and non-onticists alike – think that S5 is too strong for the logic of "determinately." Evans's assumption that vague objects must have vague identity also needs to be questioned. Some people dismiss this connection. We shall consider a little more about the relation between vague objects and vague identity in the next section. Evans's argument only shows that a brand of onticism that accepts both S5 logic and the existence of vague identity must block the argument in some way. Needless to say, a particular brand of onticism that does not accept both of these optional commitments may still block the argument, depending on its tenets; but, logically speaking, that is not necessary.

1.3 Ontic Vagueness: Vague Objects, Vague Existence, and Vague Identity

As is often the case in emerging fields, there is some confusion in our uses of certain key concepts pertaining to ontic vagueness. In some cases it is merely terminological, but in others it is more substantive. This section aims to clarify some of these concepts. One group of arguments that purport to connect two of the concepts, vague object and vague identity, will also be briefly discussed.

1.3.1 Vagueness vs. Indeterminacy

Many people make a distinction between *vagueness* and *indeterminacy*. The most common idea is that vagueness is the species of indeterminacy that is *sorites-susceptible* – that is, the kind of indeterminacy with which a sorites paradox can be constructed. That may be a reasonable idea, but it should be borne in mind that what is called indeterminacy may not itself be a uniform phenomenon, as was suggested earlier in connection with quantum indeterminacy. So some instances of indeterminacy may be closer to vagueness as we understand it than others. For instance, the indeterminacy often involved in *mathematical reduction* – for example, that involved in the question whether the number 2 is (reducible to) the Zermelo set $\{\{\varnothing\}\}$ or the von Neumann set $\{\varnothing,\{\varnothing\}\}$ – seems closer to vagueness than quantum indeterminacy is; indeed, it seems closer to vagueness than it is to quantum indeterminacy. This is perhaps because the conventional element, touched upon in the previous discussion on quantum indeterminacy, seems to be involved also in mathematical reduction. (Something similar can be said about the kind of indeterminacy to be discussed below in connection with vague identity.) So sometimes, sufficient care is called for. In many contexts, however, "vague" and "indeterminate" can be used interchangeably with little harm, as in the case of "vague" and "indeterminate identity."

1.3.2 Vague Objects vs. Vague Properties

Suppose that onticism is at least partially correct, and that there are vague states of affairs in the world. *That John is tall*, *that Michael is bald*, *that this collection of grains is a heap*, *that I am on Kilimanjaro*, and *that this hair is a part of my cat Tibbles* are such states of affairs if John is borderline-tall, Michael is borderline-bald, the collection of the grains is borderline-heap, I am near the border of Kilimanjaro, and the hair in question has fallen halfway off of Tibbles. But the next question is: wherein lies the vagueness? In particular, does the vagueness exist in the properties or in the objects involved? Another question is: is there a fact of the matter about wherein the vagueness lies? Or is it a matter of stipulation?

On the second question, there is reason to think that, indeed, there is no fact of the matter about wherein vagueness lies and that it is a matter of stipulation whether we ascribe vagueness to objects or properties. Suppose it is indeterminate whether John is tall. Then John is not determinately in the extension of "is tall," and he is not determinately outside of it. But from this what can we conclude? We cannot conclude either that John is vague with respect to tallness or that tallness is vague with respect to John. It is the *combination* of John and tallness that creates vagueness. In comparison, the combination of John and thinness and the combination of Michael and tallness do not create vagueness. The same can be said about any combination of individuals and properties, including spatiotemporal and mereological properties. So arbitrariness seems to be involved if we decide that John is a vague object or that tallness is a vague property.[20]

If this answer to the second question is correct, then it seems wisest to answer the first question by stipulating that it is properties that are vague, rather than individuals. The reason is that there are apparently vague *relations* too, such as *loving* and *being smarter than*, and if we ascribe vagueness to individuals, the problem will arise as to which of the relata vagueness should be ascribed to. This seems to complicate the matter unnecessarily. It seems simpler to ascribe vagueness to properties and relations instead.

A somewhat surprising consequence of this view is that what is usually considered to be a precise property may not be actually precise. Suppose, for instance, that Tibbles has about 153,407 hairs, but that a few hairs are now borderline-parts of Tibbles. Then the state of affairs *Tibbles has exactly 153,407 hairs* is vague. Thus,

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[20] See Akiba (2000a) and Williamson (2003) for relevant discussions.

on the present view, the property *having exactly 153,407 hairs* is a vague property. The same can be said about many other properties such as *being exactly 178.00 cm tall* and *containing exactly 1,000 grains*.

Even if the preceding points are correct and vagueness should be taken to reside in properties, we need not, however, eschew the use of the term "vague objects" if we use it with sufficient care. On the most common conception, a vague object is an individual whose part-whole relation or spatiotemporal relation with other objects is vague. Thus, in the above examples Kilimanjaro and the cat Tibbles may be said to be vague objects. Certainly not all vagueness is reducible to mereological or spatiotemporal vagueness, for, among other things, some abstract objects also involve vagueness; for instance, a number can be vaguely large or small in a given context. Insofar as it is acknowledged that mereological or spatiotemporal vagueness is just one among many other kinds of vagueness, talk of "vague objects" may be innocuous (even though this qualification seems often forgotten or ignored).

1.3.3 Vague Existence

Some people[21] also talk about vague existence. If that means only "the existence of vague objects," it is also innocuous. But sometimes people mean something more: sometimes people talk as if there are degrees of existence, and as if some things only partially exist and partially do not exist. No convincing example of vague existence in this sense, however, has ever been offered. In some cases, the vagueness of the temporal boundaries of physical objects such as persons and living creatures – that is, the vagueness of when they begin and when they end – is misleadingly called "vague existence" even though there is nothing vague about their four-dimensional existence. Vague existence might make sense if, for instance, the ontic indeterminacy of the existential assertion $\exists xPx$ did not stem from the ontic indeterminacy of its instances, Pa's. But usually the indeterminacy of $\exists xPx$, that is, that a P exists, is a direct consequence of the indeterminacy of Pa, that is, whether a is P, for some a. Here, the (full) existence of a is unquestioned. For instance, if it is indeterminate at one point of time whether a certain collection, a, of cells have already become a person, P, then it is indeterminate whether there is a person in that space-time region, R. This indeterminate existence of a person in R, however, is a direct consequence of the indeterminacy about whether the collection of cells is a person in R. But if that is the case, it is difficult to see why we should not instead make a more conservative claim that Pa is a vague state of affairs, or that P is a vague property, or that a is a vague object. It is difficult to see the point of using the term "vague existence" in such a case.

[21] van Inwagen (1990), Hawley (2002), and Smith (2005).

1.3.4 Vague Identity

Many onticists[22] maintain that the identity relation between individuals can be vague (or indeterminate). Many people, including those who are critical of vague objects,[23] think that the vagueness of identity follows from the existence of vague objects. Clearly Evans assumed that it does. Typically, when they make their arguments, they offer examples that involve a vague object and a crisp (or another vague) object in the same vicinity. The following is one such argument, originating in Shoemaker (1963).

Suppose Brown's brain and psychological states are somehow transplanted from his dying body into Robinson's body, erasing Robinson's original psychological states. Call the person who comes out of the surgery "Brownson." Thus, the post-transplant Brownson has the brain and psychology of the pre-transplant Brown and the body of the pre-transplant Robinson. It is indeterminate whether Brown survived the transplant surgery as Brownson (= the sum of the pre-transplant Brown and the post-transplant person). Brown is in that respect a vague object, whereas Brownson is a crisp object. In one scenario (or possible world), Brown did survive the transplant, in which case Brown = Brownson; in another scenario (possible world), he did not, in which case Brown $\neq$ Brownson. Thus, it is indeterminate whether Brown = Brownson. (Some might even deny the existence of Brownson in the second scenario, but that does not affect the conclusion.)

In another well-known example presented in Weatherson (2003), suppose that Kilimanjaro is a vague object, and that it is indeterminate whether the atom Sparky is a part of Kilimanjaro. Then it is indeterminate whether Kilimanjaro = Kilimanjaro+ (= the sum of Kilimanjaro and Sparky) because in one scenario, in which Sparky is a part of Kilimanjaro, Kilimanjaro = Kilimanjaro+, and in another scenario, in which Sparky is not a part of Kilimanjaro, Kilimanjaro $\neq$ Kilimanjaro+. (Again, the conclusion will not be affected even if Kilimanjaro+ does not even exist in the second scenario.)

While arguments of this sort are rather popular, they are not entirely convincing. As we have already seen in this Introduction, without doubt there is a significant similarity between vagueness and modality. So think of the following analogy. Following Gibbard (1975), suppose that Goliath is a statue and that Lumpl is a single piece of bronze of which the statue is made; suppose, furthermore, that Lumpl, as a single piece, is created and destroyed at the same time as Goliath. Suppose someone contends that Goliath is thus identical with Lumpl. One natural response to this contention, whether it is ultimately correct or not, is that they are

[22] van Inwagen (1990), Tye (1990), Lowe (1994), Parsons and Woodruff (1995), and Parsons (2000).

[23] The number of works discussing vague identity is rather large. See Williamson (2003, note 12) for references.

not identical but only coincident (or overlapping) throughout their existence, for they have different dispositional or modal properties; for instance, Goliath can but Lumpl cannot survive the loss of a small piece of the bronze, and Lumpl can but Goliath cannot survive the flattening of the statue. That is, Goliath and Lumpl are coincident (and observationally indistinguishable) in the actual world, but may not be in some other possible worlds. *Pace* Gibbard, most of us do not take an example like this to be an example of contingent identity; most of us take Goliath and Lumpl to be nonidentical, even in the actual world. But then, by analogy, why should we take Brown and Brownson (or Kilimanjaro and Kilimanjaro+) to be identical at least in one scenario (or possible world) and, thus, indeterminately identical as a whole? It seems more reasonable to say that Brown and Brownson (or Kilimanjaro and Kilimanjaro+) are coincident but not identical in one scenario, while they are neither coincident nor identical in another; thus, they are determinately distinct.

This response can be strengthened by the following consideration, if we assume that the accessibility relation among the possible worlds involved is symmetric, that is, if w_1 has access to w_2, w_2 has access to w_1. The symmetric accessibility relation supports the aforementioned axiom B: $p \to \Delta\neg\Delta\neg p$, included in modal systems like S5. Brown, a vague object, and Brownson, a crisp object, are not identical in the actual world, whereas Brown is identical with Brown. So Brown does and Brownson does not have the property *being identical with Brown*, that is, $\lambda x(x = \text{brown})$, in the actual world. But both scenarios (possible worlds) have access to the actual world because the actual world has access to both and the accessibility relation is symmetric. So, by (the contrapositive of) Leibniz's law, Brown $\neq$ Brownson in either scenario after all. Thus, they are determinately distinct. The same argument applies to Kilimanjaro and Kilimanjaro+. Of course, as we already know, a modified version of Evans's argument shows that indeterminate identity is inconsistent if the accessibility relation is symmetric. But the above account better explains why pairs like Brown and Brownson, or Kilimanjaro and Kilimanjaro+, cannot be indeterminately identical.[24]

The above response is by no means intended to be a decisive refutation of the popular arguments for vague identity. One might, for instance, deny the claim that coincident objects may not be identical, for various reasons.[25] Or one might deny the symmetry of the accessibility relation involved.[26] The response at the very least seems to show, however, that the popular arguments are far from establishing the claim that vague identity follows from the existence of vague objects, or even that vague identity is at all possible. It seems to be a very delicate issue to determine whether a certain state of affair should count as an instance of vague identity or something else. More work is called for on this matter. Generally, the reader should be very careful in interpreting how the authors are using certain key concepts, how

[24] See Williamson (2002) for a similar argument.

[25] One potential problem is that of *grounding* (i.e., what could possibly make coincident objects distinct?); see van Inwagen (1990), Heller (1990), and Zimmerman (1995).

[26] Field (2000).

the concepts are related to one another, and whether one author's use of a concept is or is not the same as another's, before she determines whether they successfully establish their points involving those concepts, including whether they successfully support or refute another author's contentions.

1.4 Summaries of the Chapters Included

1.4.1 Part I: Mereological Vagueness

As indicated above, the species of vagueness that is most often discussed as an instance of ontic vagueness is vagueness in mereological relations. This anthology, thus, begins with *Part I: Mereological Vagueness*, which contains two chapters that consider mereological vagueness from different viewpoints.

As its title indicates, in Chap. 2, "Mereological Indeterminacy: Metaphysical but Not Fundamental," Thomas Sattig defends ontic mereological indeterminacy, but argues that it is not a fundamental feature of the world; rather, it is only derivative. Sattig draws the distinction between *sortal-sensitive* and *sortal-abstract* perspectives on ordinary objects (e.g., regarding an object as a mountain vs. regarding it as a material mass) as well as the *formal* and *absolute* modes of predication that stem from these two perspectives. He then construes ordinary mereological indeterminacy as formal indeterminacy *de re*. On his double-layered picture, a mountain, for instance, is absolutely precise but formally vague because it hosts slightly different individual forms superimposed.

In Chap. 3, "A Linguistic Account of Mereological Vagueness," Maureen Donnelly works primarily within the semanticist framework and argues that the vagueness of ordinary mereological claims is typically due not to the imprecision of singular terms but to the imprecision of mereological terms such as "is part of." But she also compares her linguistic account with ontic alternatives and contemplates the possibility of hybrid theories, although she is, in the end, rather skeptical of their virtues.

1.4.2 Part II: Varieties of Ontic Vagueness

While mereological vagueness is no doubt the premier candidate for ontic vagueness, it is by no means the only one. The four chapters in *Part II: Varieties of Ontic Vagueness* deal with other candidates for ontic vagueness. In Chap. 4, "Vague Objects in Quantum Mechanics?" George Darby seeks to determine whether quantum particles can be thought of as vague objects and whether indeterminate identity of quantum particles can be made sense of. In particular, Darby carefully follows and dissects the series of exchanges that appeared in the journal *Analysis* following Lowe's (1994) paper on quantum indeterminacy. He concludes that there are cases of indeterminate identity of particles that are best described as "$\nabla(a = b)$" (see Sect. 1.2 above for the notation), where either "a" or "b" is referentially indeterminate because of ontic indeterminacy involving the particles. Darby also considers French and Krause's (1995, 2003, 2006) view that the concept of identity is not properly applicable to quantum particles, and wonders if it can be construed as an indeterminacy view of particle identity. But here, Darby's verdict is negative.

In Chap. 5, "Vague Persons," Kristie Miller defends what she calls "nontraditionalist" accounts of personal identity, according to which some disputes over personal identity are theoretically unresolvable because of the vagueness involved in the concept of personal identity. Miller wonders what the source of the vagueness is, and searches for an answer by launching an investigation into the three main theories of vagueness – semanticism, epistemicism, and onticism. In particular, Miller examines Donald Smith's (2007) argument against semanticism and epistemicism as applied to personal identity, and concludes that it does not hold up under scrutiny. Thus, she concludes, semanticist and epistemicist versions of nontraditionalism are still live options.

Phenomenal properties are often considered to be premier instances of vague properties, but their ontological status poses difficult questions. In Chap. 6, "Indiscriminable but Not Identical Looks: Non-value Phenomenal Predicates and Phenomenal Properties," Elisa Paganini argues against the deep-rooted assumption that indiscriminability between visual appearances is the same as the identity of the appearances. She does this by showing that indiscriminability of appearances is non-transitive while the identity of the appearances is transitive. In phenomenal sorites arguments, many premises of the sort, "if a looks red, and if a and b are indiscriminable, then b looks red," are accepted. But the correctness of such premises is questionable. Paganini agrees with the dominant opinion that, at least in the cases of non-phenomenal predicates, their vagueness is compatible with their coherence, but she insists on retaining, until proven incorrect, the Fregean idea that coherent phenomenal predicates cannot be vague. Phenomenal predicates can, thus, be ultimately incoherent and may lack references. Consequently, phenomenal properties may not exist.

In Chap. 7, "Attitudes, Supervaluations, and Vagueness in the World," Ángel Pinillos raises an interesting question. Suppose that the Fregean distinction between intension (sense) and extension (reference) is basically correct, and that a word expresses an intension, which in turn determines the word's extension. Also suppose that semanticism is basically correct and that a vague word refers to precise candidate objects (extensions) indeterminately. Assuming all this, where does semantic indeterminacy lie – in the word-sense relation or in the sense-object relation? Pinillos does not give an ultimate answer to this question, but considers the implications of, and problems with, each position. He contends that, on the one hand, if indeterminacy lies in the sense-object relation (i.e., a vague word determinately expresses an imprecise sense, which indeterminately picks out different precise candidate objects), vagueness can no longer be said to be a representational phenomenon, for it can be taken to exist in the objects as much as in the senses. On the other hand, if indeterminacy lies in the word-sense relation (i.e., a vague word indeterminately expresses various precise candidate intensions, each of which determinately picks out an object), a problem will arise as to how to analyze propositional attitude sentences in terms of supervaluation when they involve vagueness. Pinillos further argues that similar problems will arise even if one employs the Millian account of proper names or even if one embraces the ontic view that vague words refer to vague objects determinately. (The reader will notice that even though Pinillos's primary interest is in semantic vagueness, he treats it like a species of ontic vagueness involved in semantic relations; hence, the inclusion of the essay in *Part II*.)

1.4.3 Part III: Formal Issues

Part III: Formal Issues includes three chapters that deal with relatively formal issues that arise from ontic vagueness. In Chap. 8, "Boolean-Valued Sets as Vague Sets," Ken Akiba tries to revise fuzzy logic and fuzzy set theory by removing the feature that makes them susceptible to the so-called *problem of penumbral connections*. Fuzzy logic and fuzzy set theory assign degrees of truth between 0 and 1 to propositions and set membership, but they do it in such a way that the truth degree of a disjunction is equal to the highest of the truth degrees of its disjuncts. This produces a counterintuitive result that "John is either tall or not tall" will have truth degree 0.5 rather than 1 if "John is tall" has truth degree 0.5. Akiba proposes an alternative logic which makes the degree of truth come out as 1. The proposed logic is classical logic, whose semantic values, however, are not just truth and falsity or simple degrees between 0 and 1 but values in a general Boolean algebra. Akiba expands this idea for set theory by using the notion of Boolean-valued sets developed in Scott's and Solovay's Boolean-valued models of ZFC set theory.

What happens to familiar mathematical operations on collections if the collections in question are vague? In the next two chapters, Nicholas J. J. Smith and Stewart Shapiro deal with some specific instances of this general question – counting and cardinality for Smith and abstraction for Shapiro. In Chap. 9, "One Bald Man … Two Bald Men, … *Three* Bald Men—Aahh Aahh Aahh Aahh Aaaahhhh!" Smith considers how the notions of counting, ordering, and cardinality apply to (finite) fuzzy sets, whose membership relations, again, have degrees between 0 and 1. On Smith's conception, the counting of the members of a fuzzy set can be represented as a function that assigns to each of the members the numbers 1, …, n, weighted by its degree of membership. There have been various proposals on the notions of ordering in, and cardinality of, fuzzy sets. Smith neither makes any new proposal of his own nor decides which of the existing proposals is the most appropriate. However, he argues against the idea that we should approach the cardinalities of fuzzy sets via truth degrees of certain numerical formulas of a logical language (e.g., "$\exists x \forall y(Py \leftrightarrow y = x)$" for "there is exactly one P") because cardinality can and should be defined directly from the outputs of the counting process.

In a somewhat similar vein, Shapiro, in Chap. 10, "Vagueness and Abstraction," tries to answer the question: what will happen to abstraction principles if they involve vague properties instead of crisp ones? In abstraction principles such as "the direction of line l_1 is identical with the direction of line l_2 if and only if l_1 and l_2 are equi-directed" and "the number of Fs is identical with the number of Gs if and only if F and G are equi-numbered," abstract entities such as *directions* and *numbers* are abstracted from equivalence relations such as *being equi-directed* and *being equi-numbered*. But how about "the weight a is identical with the weight b if and only if a and b are equi-weighted," where a and b are weights of physical objects or people? The problem here is that unlike *being equi-directed* and *being equi-numbered*, *being equi-weighted* is not an equivalence relation because of vagueness; that is, it is possible for person p and person q to be equi-weighted and for person q and person r to be equi-weighted without p and r being equi-weighted. Shapiro develops an account of *quasi-abstract* objects such as *weights* within his contextualist framework.

1.4.4 Part IV: Ontic Supervaluationism

As was noted above, supervaluationism was originally a brand of semanticism. The most recent versions of onticism, however, defend onticism while accepting the basic idea of supervaluation. The two chapters in *Part IV: Ontic Supervaluationism* discuss such versions of onticism. In Chap. 11, "Vagueness in the World: A Supervaluationist Approach," Ali Abasnezhad and Davood Hosseini consider two existing versions of ontic supervaluationism – Williams's (2008a) and Akiba's (2004). On Williams's view (shared by Elizabeth Barnes), there is a single actual world, but it is ontically indeterminate which precisified world among many the actual world is, or there is a single Mt. Everest, but it is ontically indeterminate which precisification among many Mt. Everest is. On Akiba's view, the actual world is made of and divided into precisified worlds, and Mt. Everest is made of its precisifications. Abasnezhad and Hosseini are not satisfied with either view, however, and present an alternative view – the incomplete-object view. According to the incomplete-object view, there is a single Mt. Everest, but it is an incomplete object, and its precisifications are different possible ways of making Mt. Everest completely precise.

In Chap. 12, "What Could Vague Objects Possibly Be?," Dan López de Sa critically examines Barnes's and Williams's theory of ontic vagueness. Barnes and Williams characterize ontic vagueness as non-epistemic vagueness that would remain after the precisification of the relevant representational content. In the first half of his chapter, López de Sa argues that this negative characterization of ontic vagueness makes ontic vagueness no more intelligible to those who initially find it of dubious intelligibility. In the second half, he examines Barnes's and Williams's response to Evans's argument. In particular, he scrutinizes their view that vague objects would exclude the existence of precise objects that indeterminately overlap with them. López de Sa criticizes this view for making the notion of vague object even more obscure.

1.4.5 Part V: Vague Identity

Part V: Vague Identity, the final part of the volume, includes four chapters that concern issues surrounding vague identity. It begins with Brian Garrett's chapter, Chap. 13, "Some Comments on Evans's Proof." This is a short but close analysis of Evans's paper. Garrett concludes that Evans's argument fails, but he also expresses his own doubts about ontic vagueness.

The next two chapters, one by David B. Hershenov and the other by Benjamin L. Curtis and Harold W. Noonan, both attempt to show, in ways similar to that sketched in Sect. 1.3 above, that if a certain type of ontic vagueness exists, then vague identity must exist, too. The title of Hershenov's chapter, Chap. 14, "Vague Existence Implies Vague Identity," makes its central thesis plain. Hershenov's example of vague existence is a mountain being formed by the crashing of two tectonic plates. In such a case, it is at one point indeterminate whether the mountain exists, according to Hershenov.

In Chap. 15, "Castles Built on Clouds: Vague Identity and Vague Objects," Curtis and Noonan defend Evans's argument against some onticists' objections and conclude that there is no such thing as ontically vague identity. This in itself does not mean that there cannot be vague objects. However, Curtis and Noonan present their own argument for the claim that vague objects exist if and only if the identity of the objects is vague. Thus, they conclude, there cannot be vague objects. (The reader should take note, however, that Curtis and Noonan, following Williams (2008a), assume that the logic of vagueness ought to be S5 modal logic; without this assumption their conclusion does not follow.)

Finally, in Chap. 16, "Evans Tolerated," Elia Zardini presents two systems of *tolerant logic*, which place restrictions on the transitivity of logical consequence. In these systems, each inference step in a sorites series is valid, but the entire sorites argument is not, so the paradox does not arise. In one of the systems, Evans's argument (1) through (5) is not valid; in another, it is no longer problematic. Either way, Zardini argues, the argument is harmless. Zardini is well aware that Evans's argument is in itself nothing but the contrapositive of Kripke's argument for the necessity of identity and, thus, is harmless even it is valid. But he still considers his results important because they would preempt any potential strengthening of the argument.

It should be noted that the division of the chapters into five parts, as well as the order of the chapters in each part, is somewhat arbitrary. Some topics, especially Evans's argument and vague identity, are covered in various parts. Each chapter is self-contained and does not assume any prior knowledge of the other chapters; at the same time, however, the reader will benefit more from reading through the entire volume or a large portion of it at one time. The volume should enable the reader to survey the state of the art of the fast-growing field pertaining to ontic vagueness.[27]

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Part I
Mereological Vagueness

Chapter 2
Mereological Indeterminacy: Metaphysical but Not Fundamental

Thomas Sattig

2.1 Introduction

Suppose that mountain M is a massive collection of rocks deposited in layers. As a result of melting glacial ice, M gradually sheds rock mass; some rocks in the mountain's surface layer slowly become loose and slide off. In this process, several surfaces become equally good candidates to be the boundary of the mountain. A surface including a particular loose rock, *r*, is an equally good candidate to mark the boundary of the mountain as a surface excluding *r*. As a consequence, the rock attains a questionable status: it is indeterminate whether M has *r* as a part. M's mereological boundary is indeterminate.

This mereological indeterminacy claim has different readings, the *de dicto* and the *de re* reading. The two readings may be specified informally by using the colon to indicate the scope of the operator 'It is indeterminate whether':

De dicto It is indeterminate whether: M has *r* as a part.

De re M and the property of having *r* as a part are such that it is indeterminate whether: this object instantiates this property.

The difference is that on the *de dicto* reading it is indeterminate whether a certain description of the world is true, whereas on the *de re* reading it is indeterminate *of* a particular object and a particular property, whether the latter applies to the former.[1] Adopting a popular *façon de parler*, I shall say that if the *de re* reading of

[1] See Sainsbury (1989) and Williamson (2003) for the characterisation of claims of indeterminacy *de re* as having the form: for some object x and some property φ, it is indeterminate whether x instantiates φ.

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our claim of mereological indeterminacy is true, then M is a vague, or fuzzy, object. Mereological indeterminacy *de re* is naturally viewed as an instance of *metaphysical* indeterminacy, in the sense of being independent of conceptual, linguistic, or epistemic representation.

The main question of this chapter is whether such *de re* indeterminacy claims about ordinary objects' parts might be true—whether ordinary objects might be mereologically vague. To emphasise, the question is not whether any object could be indeterminate in any respect. The question specifically concerns the status of intuitive mereological indeterminacy claims about ordinary objects—that is, about objects falling under ordinary sortal concepts, such as the concept of a mountain.[2]

When philosophers contemplate the status of ordinary mereological indeterminacy, they typically juxtapose the following two positions:

(I) All ordinary mereological indeterminacy is merely *de dicto* and has its source in how we represent the world.
(II) Some ordinary mereological indeterminacy is *de re* and has its source in how the world is, independently of how we represent it.

According to the standard version of (I), mereological indeterminacy is linguistic in nature, arising from imprecision in how we refer to ordinary objects. The dominant brand of linguistic theory of indeterminacy is supervaluationism. In the case at hand, the supervaluationist recognises a cluster of massively overlapping aggregates of particles with different precise decompositions (at a given time), such that each of these aggregates is a candidate referent for the name 'M'. Mereological indeterminacy of M is then analysed by supervaluating over these candidates. It is indeterminate whether: M has *r* as a part, since it is true of some admissible precisification of 'M' that it has *r* as a part, but not true of all admissible precisifications of 'M'. The standard supervaluationist thus accepts the *de dicto* reading of our indeterminacy claim about M. But she rejects the *de re* reading, because it is not the case of M that it is indeterminate whether it has *r* as a part, as each candidate referent has a clear-cut decomposition. There are no vague mountains in this world.[3]

This view is problematic. But I shall not attack it or any other version of position (I) here.[4] I mention (I) only to set the stage for position (II), which will be the subject of the following discussion. As an instance of (II), ordinary mereological indeterminacy of mountains might be taken to be *de re*. Instead of viewing the name 'M' in our example as referring imprecisely to multiple, precise objects, one might view the name as referring precisely to a unique, vague object. In

[2]Some find the popular talk of vague objects dubious, on the grounds that an object is only ever vague, or indeterminate, in a certain respect. See, for example, Hawley (2002) and Williamson (2003). I share these doubts and emphasise that talk of vague objects will here be understood merely as loose and vivid talk. The serious notion in the background is that of indeterminacy *de re*.

[3]See Lewis (1993).

[4]For criticism of the supervaluationist account of mereological indeterminacy in the context of a discussion of the problem of the many, see Sattig (2013).

Sect. 2.2, I will sketch the standard version of mereological indeterminacy *de re* and subject it to criticism. According to this version, mereological indeterminacy is metaphysical in nature, and it is fundamental. As my aim is primarily constructive, the purpose of this section is not to refute the account. The point is rather to highlight worries that give sufficient reason to scout for alternatives. In Sect. 2.3, I shall develop a novel version of mereological indeterminacy *de re* and show that it avoids the problems for the standard version considered in Sect. 2.2. According to this account, ordinary mereological indeterminacy is metaphysical, in virtue of being representation-independent, but it is not fundamental.

2.2 Fundamental Indeterminacy *De Re*

Indeterminacy *de re* of mereological boundaries of ordinary objects is naturally viewed as an instance, possibly one of many instances, of *metaphysical* indeterminacy or of indeterminacy in the world, in the sense of being independent of conceptual, linguistic, or epistemic representation. If there are facts of indeterminacy *de re* about ordinary objects, then these objects really are indeterminate, independently of how we represent them. Of course, this characterisation of indeterminacy *de re* as metaphysical only says something about what the indeterminacy is not, namely, a consequence of an imprecise representation. A positive account of its nature is a different matter.

So what is the nature of mereological indeterminacy *de re*?[5] The standard view is to construe this indeterminacy, along with metaphysical indeterminacy in general, as *fundamental*, either in the sense that facts about such indeterminacy are not grounded in any more basic, indeterminacy-free facts or in the sense that the operator 'it is indeterminate whether' is perfectly natural that it 'carves nature at the joints'.[6] Friends of this view emphasise that while metaphysical indeterminacy cannot be analysed reductively, the notion can still be elucidated. It is indeterminate of mountain M whether: it has r as a part. This could be made intelligible by saying that reality itself has different precisifications, all of which are perfectly precise, including one in which M has r as a part and one in which M lacks r as a part. One way of developing this idea is to view metaphysical indeterminacy as a kind of modality, which concerns worlds that are precisificationally possible—in other words, which concerns multiple actualities.[7] In this framework, mereological indeterminacy may be explicated by supervaluating over the varying mereological profiles of a given object in different actual worlds—instead of supervaluating over the mereological profiles of different, overlapping

[5] For constructive discussion of metaphysical indeterminacy, see, *inter alia*, Akiba (2000, 2004), Barnes (2010), Barnes and Williams (2009, 2011), Morreau (2002), Parsons (2000), Rosen and Smith (2004), Skow (2010), Smith (2005), Williams (2008), and Williamson (2003).

[6] See, *inter alia*, Barnes and Williams (2011: 106).

[7] See, *inter alia*, Barnes and Williams (2011).

objects, as standard supervaluationism has it (see Sect. 2.1). It is indeterminate of M whether: it has *r* as a part iff there is a precisificationally possible world, an actuality, in which M has *r* as a part, and another precisificationally possible world, another actuality, in which M lacks *r* as a part.

The view of ordinary objects as having metaphysically indeterminate properties has been greeted with much resistance. Many, including Michael Dummett (1975), have found it unintelligible that there should be metaphysical indeterminacy. However, progress has been made on this front for defenders of metaphysical indeterminacy have offered ways of rendering such indeterminacy intelligible, as the modal approach mentioned above illustrates. If you understand the idea that an object can be at home in different precisificationally possible worlds and that it can vary in its mereological profile across these worlds, then you understand mereological indeterminacy *de re*.

Even if intelligibly glossed as arising from multiple precisifications of reality, many philosophers still refuse to admit fundamental metaphysical indeterminacy. And they may not base their attitude on arguments, because their real ground is the simple intuition that metaphysical indeterminacy, conceptualised as involving multiple actualities, is unbearably radical—for short, that it's crazy. This is a respectable attitude, comparable to Goodman's and Quine's motive for rejecting abstract entities: 'Fundamentally this refusal is based on a philosophical intuition that cannot be justified by an appeal to anything more ultimate' (Goodman and Quine 1947: 174).

More can be said, though. An objection to mereological indeterminacy *de re* that takes the form of an argument is a recent attack by Brian Weatherson.[8] Consider again our mountain M. To start the argument, assume for *reductio* that it is indeterminate of M whether rock *r* is a part of it. The second premise of the argument is that there is an object, M-minus, that determinately has all and only the parts of M that do not overlap with *r*; it is determinate of M-minus that: for all *x*, *x* is a part of it iff *x* is a part of M and *x* does not overlap with *r*. Intuitively, there is this object, rigidly designated by the name 'M-minus' across different precisificationally possible worlds, which is the mountain, M, without that particular rock. Note that this remainder of the mountain may itself have indeterminate parts.

Now let us say that an object *o* *coincides* with an object *o** at a time *t* just in case *o* and *o** occupy the same place at *t*. (Henceforth, I shall focus on objects at a particular time, and drop all temporal modifiers for presentational simplicity.) If M lacks rock *r* as a part, then M and M-minus share all their parts; and if they share all their parts, they coincide. Moreover, I shall assume that if M coincides

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[8] The original argument appears in Weatherson (2003: §4). In unpublished work, Weatherson has presented a second version of the argument that is meant to avoid a weakness in the first version, pointed out in Barnes and Williams (2009). The argument to be presented here is more or less Weatherson's second argument. For reasons of space, I shall be unable to address differences among versions and the interesting debate behind them. My aim is to sketch an account of vague ordinary objects that withstands the Weatherson attack in its most severe form—that is, even under the assumption that the Barnes-Williams-bug is fixable.

with M-minus, then M lacks rock *r* as a part. Hence, M coincides with M-minus iff M lacks rock *r* as a part. Since it is indeterminate of M whether *r* is a part of it, it is indeterminate of M and M-minus whether they coincide. The third premise of the argument is that coinciding objects are identical. This premise may be backed in various ways. I shall focus on the simplest way, namely, to appeal to the intuition that distinct objects cannot fit into the same place at the same time, that distinctness of coinciding objects leads to overcrowding.[9] For this reason it is plausible that if M coincides with M-minus, then M is identical with M-minus. Since the converse obviously holds as well, M coincides with M-minus iff M is identical with M-minus. Since it was established earlier that it is indeterminate of M and M-minus whether they coincide, it follows that it is indeterminate of M and M-minus whether they are identical. Importantly, this statement of indeterminate identity is *de re*. The final premise of the argument is that the well-known Evans-Salmon argument shows successfully that there can be no *de re* indeterminate identity, contrary to what was established by means of the first three premises. Roughly, M-minus has the property of being indeterminately identical with M. But M lacks that property. Hence, M and M-minus are distinct.[10]

The argument may be summarised as follows:

1. It is indeterminate of M whether: it has *r* as a part. [P1]
2. There is an object, M-minus, such that it is determinate of M-minus that: it has all and only the parts of M that do not overlap with *r*. [P2]
3. It is determinate of M and M-minus that: the former coincides with the latter iff the former lacks *r* as a part.
4. It is indeterminate of M and M-minus whether: the former coincides with the latter.
5. It is determinate of M and M-minus that: the former coincides with the latter iff the former is identical with the latter. [P3]
6. It is indeterminate of M and M-minus whether: the former is identical with the latter.
7. It is not indeterminate of M and M-minus whether: the former is identical with the latter. [P4]

This argument is an attempt at a *reductio* of the claim that M is a mereologically vague object, via the assumptions that there is an object, M-minus, that is composed of all of M except *r* (P2), that coinciding objects are identical (P3), and that Evans-Salmon-style reasoning establishes the incoherence of *de re* indeterminate

[9]A more complex reason for rejecting distinct coincidents is driven by the 'grounding problem'; see, *inter alia*, Bennett (2004).

[10]See Evans (1978) and Salmon (1981). As Lewis (1988) pointed out, semantically indeterminate identity statements are not the target of the Evans-Salmon argument, but only *de re* indeterminate identity statements. Note further that the distinctness of M and M-minus may also be supported without identity-involving properties. M has the property of having *r* as an indeterminate part. But M-minus lacks that property. Hence, M is distinct from M-minus.

identity (P4). I find each of these assumptions compelling, but I won't elaborate on their motivation.[11]

Summing up the considerations of this section, friends of fundamental indeterminacy *de re* face at least two worries. Many will judge the picture of fundamental metaphysical indeterminacy, conceptualised in terms of multiple actualities, a 'crazy ontology'. Furthermore, mereological indeterminacy *de re* raises what I shall call the *problem of indeterminate coincidence*. In what follows, I shall develop a picture of mereological indeterminacy *de re* that avoids these worries. According to this approach, the indeterminacy is metaphysical and yet derivative.

2.3 Derivative Indeterminacy *De Re*

Here is the rough picture. The account of ordinary mereological indeterminacy to be proposed rests on a quasi-hylomorphic ontology of ordinary objects as material objects with multiple, 'superimposed' individual forms. Each individual form of an ordinary object is, roughly, a total intrinsic profile. And all individual forms of an ordinary object differ minutely from each other—that is, the intrinsic profiles contain slightly different properties. These assumptions are consistent with the orthodox view that material objects are ultimately precise objects—that it is not fundamentally indeterminate of any material object and any property whether the former has the latter.

Given this ontology, I propose to construe ordinary mereological indeterminacy as *formal* indeterminacy *de re*. A mountain is formally indeterminate in its composition in virtue of having multiple mereological candidate-boundaries, where these candidate-boundaries are the boundaries encoded in different forms of the mountain. This indeterminacy in the mountain's composition is metaphysical, in that it does not have its source in representational imprecision, and it is nonfundamental, or derivative, in that it derives from perfectly precise facts about the composition of material objects.

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[11] As an attempt to block the argument, consider replacing P3 by the following weaker premise, P3′: [It is determinate of M and M-minus that: the former coincides with the latter] iff M is identical with M-minus. With P3′, P1 and P2 do not lead to (6) below and hence do not clash with (7). Intuitively, P3 has it that for any way *w* of making our world precise—for any precisificationally possible world *w*—if M and M-minus have the same mereological boundary according to *w*, then M is identical with M-minus according to *w*. P3′, in contrast, has it that if M and M-minus have the same mereological boundary according to *all* ways of making our world precise, then M is identical with M-minus.

While weakening P3 in this fashion should be acknowledged as a way of blocking the argument, the move will strike many as *ad hoc*. If worries about distinct coincidents are worries about overcrowding, then it is hard to see why these worries should be limited in the way of P3′. Intuitively, distinct complex objects cannot fit into the same place at any time in any nomologically possible world and in any precisificationally possible world, irrespective of their spatial relationship at other times and in other worlds. Overcrowding is a local concern.

To emphasise, this is not intended as an account of every instance of indeterminacy, not even of every instance of mereological indeterminacy, but merely as an account of certain familiar instances of mereological indeterminacy of ordinary objects. In what follows, this metaphysically harmless picture of vague ordinary objects will be developed in some detail and shown to provide a satisfactory answer to the problem of indeterminate coincidence.[12]

2.3.1 Ordinary Objects with Multiple Individual Forms

For any ordinary kind K (corresponding to the sortal concept of a K), there are specific properties (and relations) of material objects that partly realise, or ground, the kind. Suppose, for example, that material object *a* has properties that jointly realise the kind *mountain*. Among its mountain-realisers are not only its specific shape and its specific altitude but also the property of having a mereological and spatial boundary that is sufficiently contrasted from its environment. I shall call a K-realising boundary of a material object a *K-boundary*. Comparing a mountain-shaped aggregate of rocks covered in snow with a mountain-shaped aggregate completely enclosed in a bigger aggregate of rocks, the former has a mountain-boundary, while the latter does not. This idea is rough but fairly intuitive.[13]

Moreover, for any ordinary kind, K, a *K-state* of a material object is a conjunctive fact about the object, which is associated with the kind in virtue of its constituent K-realising properties (and that obtains at a particular time). A K-state of object *a* is the maximal conjunction of the facts that *a* has φ_1, that *a* has φ_2, ..., that *a* has φ_n, such that each φ_i is an intrinsic property of *a* or a property of *a* that realises K. A K-state is an instantaneous, intrinsic and K-realising profile of a material object. Some ordinary kinds are presumably completely realised by intrinsic properties of material objects, while others are partly realised by extrinsic as well as intrinsic properties. A mountain-state—in short, an *m-state*—of a material object *a* is a conjunctive fact that contains all intrinsic and mountainhood-realising properties of *a* (at a given time), including *a*'s mountain-boundary.

[12]This framework is developed in more detail and with further applications in Sattig (forthcoming). For an application of the framework to the problem of the many, see Sattig (2013).

[13]It is likely that the sortal *mountain* is semantically imprecise. If so, different precisifications of the sortal determine different clusters of mountain-realising properties. In particular, different precisifications specify different minimal degrees of boundary contrast and hence specify different sets of eligible mountain boundaries. While I claim that mereological indeterminacy as it occurs in the case of M does not have its source in the semantic imprecision of *mountain*, M may, in addition, be indeterminate in a way that does have its source in this semantic imprecision. The latter type of indeterminacy requires a separate treatment. As it will not play a role here, I shall assume that it is always a precise matter which properties realise which sortals or kinds.

K-states are instantiated by composite material objects. I shall make three metaphysical assumptions about these objects. First, composite material objects exist. Second, there is no fundamental metaphysical indeterminacy, and hence material objects, composite or not, are clear-cut. So it is not fundamentally indeterminate of any material object and any property whether the former has the latter. Third, composite material objects are mereological sums of material objects that overlap with a massive number of other composite material objects at any time, assuming mereological universalism.

On the assumption of mereological universalism, it seems plausible that given a mountain-shaped material object *a* with a certain mountain-boundary, there are many distinct material objects that massively overlap with *a* and that have more or less the same mountain-shape and mountain-boundary as *a*. Accordingly, any material object that is a subject of an m-state massively overlaps many other material objects that are also subjects of m-states with more or less the same intrinsic and realisation profiles. This holds for K-states in general. I shall say that when distinct K-states, for the same K, obtaining at the same time are that similar, then they are *superimposed*.

Next, let me introduce the notion of *hosting*. For any K-state *s*, such that a composite material object *a* is either the subject of *s* or has a proper part that is the subject of *s*, *a hosts s*. The relation of hosting between a complex material object and multiple K-states is less intimate than instantiation. But hosting is far from arbitrary. For all the K-states hosted by a material object lie within the object's spatial boundary. While not strictly the subject, the material object is the 'site' of these superimposed K-states.

Furthermore, for any range of massively overlapping material subjects of superimposed K-states—call these objects *K-objects*—there is, by the principle of mereological universalism, the fusion of all the massively overlapping K-objects, call this maximal fusion a *K-plus-object*. (Note that a K-plus-object may or may not be a K-object itself.) A K-plus-object hosts a plurality of superimposed K-states. In fact, a K-plus-object is the site of a maximal cluster of superimposed K-states.

With these assumptions about K-states and material objects in place, I shall characterise an *ordinary object* of kind K as a K-plus-object. An ordinary object of kind K thus hosts a plurality of superimposed K-states. All of these K-states lie within the material boundary of the object. In a hylomorphic spirit, I shall characterise a K-state hosted by an object of kind K as an *individual form* of that object. Ordinary objects are thus construed as having multiple individual forms. The multiple m-states hosted by a mountain are individual forms of this mountain. An m-state is a form of a mountain because it contains properties that realise mountainhood; and it is an individual form of a mountain because it is localised, a distribution of facts across a particular region of space (at a time). Notice that these different individual forms of an ordinary object do not reflect joints in nature: they are not needed to unify, to glue together, the parts of objects, which is a function forms are required to perform on Aristotelian conceptions.

On Aristotelian hylomorphism, an ordinary object could not have multiple forms. The point of the multiplication of forms in the present quasi-hylomorphic picture is a very different one.[14]

2.3.2 *Formal Indeterminacy De Re and Absolute Determinacy De Re*

Having paired ordinary objects with multitudes of individual forms, let us turn to ordinary statements of determinacy and indeterminacy about such objects. I shall begin with a distinction between two modes of predication that manifest different perspectives on ordinary objects. Then I shall draw a corresponding distinction between two notions of determinacy and indeterminacy *de re*.

We may conceive of ordinary objects from different *perspectives* in different contexts. These perspectives correspond to different psychological methods of individuating ordinary objects. We can adopt the *sortal-sensitive* perspective and think of an object as belonging to a specific ordinary kind. To do so is to recognise that the object's properties and relations realise that kind. We can also adopt the *sortal-abstract* perspective and think of an object in abstraction from any ordinary kinds to which it belongs. To do so is to think of the object just as a material object and to ignore which kinds (if any) its properties and relations realise. The sortal-sensitive perspective is the perspective of unreflective common sense which parses the world into mountains, trees, clouds, and other ordinary objects. The sortal-abstract perspective is the perspective of metaphysicians who ask what objects are really like, and who, accordingly, don't pay attention to sortal representation.

To a type of perspective on objects corresponds a *mode of predication*, a certain way of predicating a property (or relation) of an object. First, some terminology. By adopting the sortal-sensitive perspective on an ordinary object, a speaker employs the *formal* mode of predication when describing the object. By adopting the sortal-abstract perspective on an ordinary object, a speaker employs the *absolute* mode of predication when describing the object.[15]

How are these modes of predication represented syntactically? Consider a monadic predication 'o is F' about an ordinary object o (the extension to polyadic predications is straightforward). This predication may be read in two different ways, as an absolute predication and as a formal predication. If 'o is F' is read as an absolute predication, then it has the familiar logical form '$\mathrm{F}(o)$'. If 'o is F' is read as a formal predication, then it has the logical form '$\mathrm{F}(o)_{\mathrm{form}}$'. Henceforth, I shall specify these readings informally, as 'o is absolutely F' and 'o is formally F'.

[14]See, *inter alia*, Koslicki (2008) on Aristotelian and neo-Aristotelian hylomorphism about ordinary objects.

[15]This is a simplification of the distinction between three perspectives and corresponding modes of predication in Sattig (2010, forthcoming).

How do these modes of predication work semantically? The semantics of absolute predication will be taken as understood. Formal predication is a mode of predicating a property of material objects with individual forms. While predications in the absolute mode about an object *o* are made true by facts concerning which properties are instantiated by *o* itself, predications in the formal mode about *o* are made true by facts concerning which properties are contained in a given individual form of *o*. The formal mode requires the specification of an individual form for a predication to be evaluated for truth—that is, a formal predication is evaluated relative to a particular individual form of its material subject. Relative to an individual form *i* of an ordinary object *o*, *o* is formally F iff *i* contains the property of being F. For example, relative to individual form *i* of *o*, *o* formally has the property of having a certain object as a part iff *i* contains the property of having that object as a part. Given that an ordinary object, *o*, has multiple individual forms, the simple formal predication '*o* is formally F' is not truth-evaluable, since no particular individual form is specified relative to which the predication may be evaluated. It should be emphasised that the formal mode of predication does not correspond to a metaphysically basic mode of instantiating a property, in addition to the absolute mode of instantiation corresponding to the absolute mode of predication. Predications that are syntactically in the formal mode are made true by facts concerning the absolute instantiation of properties.[16]

Corresponding to the distinction between two modes of predication, absolute and formal, I shall distinguish between two notions of determinacy and indeterminacy, absolute and formal. I shall make two preliminary assumptions. First, to say that it is indeterminate whether *o* is F is to say that it is neither determinate that *o* is F nor determinate that *o* is not F. Second, 'it is determinate that' functions syntactically as a sentential operator. Corresponding to the absolute mode of predication, there is an operator of absolute determinacy: it is absolutely determinate that *o* is absolutely F—$\triangle(\mathrm{F}(o))$. Corresponding to the formal mode of predication there is an operator of formal determinacy: it is formally determinate that *o* is formally F—$\triangle_{\mathrm{form}}(\mathrm{F}(o)_{\mathrm{form}})$. Just as the different modes of predication are associated with different perspectives on the world of objects, so are the different notions of determinacy and indeterminacy. We can represent an object as belonging to a particular kind and ask whether it has an indeterminate formal boundary. Or we can abstract from any sortal representation of an object and ask whether it has an indeterminate absolute boundary.

The central notion for present purposes is that of formal indeterminacy *de re*. So I shall begin my explication here. Formal determinacy and indeterminacy *de re* are grounded in the multitude of an ordinary object's superimposed individual forms.

[16]This account of formal predications of properties may, with a bit of work, be extended to formal predications of relations. It will be assumed, however, that numerical identity can only be predicated absolutely. This simplifying assumption is made here because there is no need in the present context for formal predications of identity. For such predications, see Sattig (2010, forthcoming).

An individual form of an ordinary object *o* is a complex fact and hence contains properties (and relations). As we saw, a monadic predication in the formal mode, '*o* is formally F', is true relative to an individual form *i* of *o* just in case *i* contains the property of being F. Now, a monadic statement of formal determinacy *de re*, employing the formal mode of predication, 'It is formally determinate of *o* that: it is formally F', is true simpliciter just in case '*o* is formally F' is true relative to each individual form of *o*—that is, just in case each individual form of *o* contains the property of being F. Truth conditions of monadic statements of formal determinacy and indeterminacy *de re* may then be stated as follows:

(T1) It is formally determinate of *o* that: it is formally F iff each of *o*'s individual forms contains the property of being F.

It is formally determinate of *o* that: it is formally not F iff none of *o*'s individual forms contains the property of being F.

It is formally indeterminate of *o* whether: it is formally F iff some but not all of *o*'s individual forms contain the property of being F.

So statements of formal determinacy and indeterminacy *de re*, employing the formal mode of predication, are made true by facts concerning which properties are contained in which of the subject's many superimposed individual forms. The matching properties of an object *o*'s individual forms, those all individual forms contain, are *o*'s formally determinate properties. More importantly, the differing properties of *o*'s individual forms, those some but not all individual forms contain, are *o*'s formally indeterminate properties.

Statements of formal determinacy and indeterminacy about an object *o* may be sensitive to which sortal concept *o* is conceived under in a given context. One might hold that one and the same object may fall under different sortal concepts. One might hold, for example, that a statue is also a lump of clay and that a mountain is also an aggregate of rocks. If so, an object may be thought of under different sortal concepts in different contexts. The individual forms of an object of kind K are the complex K-realising states, the K-states, hosted by that object. Since different sortal concepts are realised by different properties, an object has different pluralities of individual forms relative to different sortal representations. Given that formal determinacy and indeterminacy *de re* are a matter of similarity and difference among an object's individual forms, which properties are formally determinate and which formally indeterminate may vary with respect to which sortal concept is in play in which context. While more could be said about this context sensitivity, I only mention it to set it aside. It won't play a role here.

It is obvious that formal indeterminacy is structurally similar to supervaluational indeterminacy. The standard supervaluationist account of 'It is indeterminate whether: *o* is F' supervaluates over the different candidate referents of '*o*' (Sect. 2.1). Recall also that the modal gloss of the fundamental account of 'It is indeterminate of *o* whether: it is F' supervaluates over the different qualitative profiles instantiated by *o* in various actual worlds (Sect. 2.2). By contrast, the present account of 'It is formally indeterminate of *o* whether: it is formally F' supervaluates over the different K-states, for some kind K, hosted by *o* in the unique actual world.

A well-known virtue of supervaluationism is that it preserves the logical truths of classical logic. Even if it is supervaluationally indeterminate whether *o* is F—because, say, the term '*o*' is imprecise and has multiple candidate referents—it is still supervaluationally determinate that either *o* is F or *o* is not F, because no matter which candidate referent is assigned to the term '*o*', this object is either F or fails to be F. Analogously, the account of formal indeterminacy preserves the classical tautologies. Even if it is formally indeterminate of *o* whether: it is formally F—because some but not all individual forms of *o* contain the property of being F—it is still formally determinate of *o* that: either it is formally F or it is formally not F, because each individual form of *o* either contains the property of being F or fails to do so.[17]

While this semantic picture takes care of ordinary, sortal-sensitive discourse that explicitly concerns the determinate and indeterminate properties of objects, there is a worry that the picture leaves apparently simple ordinary predications, such as 'M formally has at least mass *m*', standing in the rain. For given that mountain M has multiple individual forms and given that a formal predication can only be evaluated for truth relative to a specific individual form, such predications don't seem to be truth-evaluable. My response is that ordinary predications such as this one are truth-evaluable, on the assumption that they are implicitly modified by the formal-determinacy operator, yielding the explicit form 'It is formally determinate of M that: it formally has at least mass *m*'.

Formal indeterminacy *de re* is both metaphysical and derivative. It is metaphysical in the sense that it doesn't have its source in representational imprecision, such as imprecision of linguistic meaning. Statements of formal indeterminacy *de re* are made true by facts about ordinary objects, not by facts about representations of ordinary objects. While the standard supervaluationist truth conditions of indeterminacy claims concern different ways of specifying the semantic values of linguistic expressions and hence locate the indeterminacy in language, the present truth conditions of singular claims of indeterminacy locate the indeterminacy in reality, namely, in the differences among an ordinary object's multiple individual forms—just as the modal truth conditions of the fundamental account locate the indeterminacy in reality, namely, in the differences among an ordinary object's multiple qualitative profiles in different actualities. Furthermore, formal indeterminacy *de re* is nonfundamental, or derivative, in the sense that facts about such indeterminacy are grounded in more basic, indeterminacy-free facts about superimposed K-paths, and in the sense that superimposed K-paths, the individual forms of ordinary objects, do not, unlike Aristotelian forms, carve nature at the joints. Formal indeterminacy *de re* doesn't run deep.

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[17]The picture sketched here is merely the beginning of a reductive account of formal indeterminacy *de re*. One loose end is the problem of higher-order indeterminacy—the problem of whether the categorisation into determinate parts, determinate nonparts, and indeterminate parts of mountains may itself be indeterminate. An adequate discussion of this problem lies beyond the scope of this chapter.

This is the main part of the story. It remains to add a word about the semantics of statements of *absolute* determinacy *de re*. With the assumption in the background that material objects are precise, I shall give a deflationary account of the notion of absolute determinacy *de re*, according to which the operator ‘It is absolutely determinate that’ is redundant: for any ordinary object *o*,

(T2) It is absolutely determinate of *o* that: it is absolutely F iff *o* is absolutely F.

It follows that it cannot be true of any ordinary object *o* that it is neither absolutely determinate that *o* is absolutely F nor absolutely determinate that *o* is absolutely not F. That is, ordinary objects cannot be absolutely vague. While the availability of true ordinary claims of formal indeterminacy *de re* is our primary concern when analysing mereological indeterminacy of ordinary objects, the availability of true claims of absolute determinacy *de re* will come into play in response to the problem of indeterminate coincidence below.

2.3.3 Vague Ordinary Objects

We saw that as an alternative to construing indeterminate mereological boundaries of ordinary objects such as mountains as *de dicto* and as deriving from imprecision of our representational apparatus, such indeterminacy may be construed as *de re* and as arising independently of imprecision of representations of objects. While ordinary mereological indeterminacy *de re* is usually understood as fundamental indeterminacy *de re*, the present framework allows ordinary mereological indeterminacy *de re* to be understood as mere derivative indeterminacy *de re*. I shall now state my proposed analysis of the claim that

(IND) It is indeterminate of M whether: it has rock *r* as a part (at *t*)

and then point out advantages of this analysis over the rival discussed earlier.

First, it is plausible that (IND) manifests the sortal-sensitive perspective on objects. That is, in the contexts in which this claim is made M is conceived of as a mountain. Intuitively, it is indeterminate whether M has rock *r* as a part because different surfaces, some including *r*, some excluding *r*, are equally good candidates to be the boundary of the mountain. So let us ask further why we take the different surfaces to be equally good candidates to mark M’s boundary. The answer seems to be that each surface preserves what makes M a mountain. In the present terminology, each surface preserves M’s mountainhood-realising properties. Without sortal guidance, we would be unable to distinguish alternative boundaries of an object. Thus, our judgement that M has an indeterminate mereological boundary is sensitive to M’s being a mountain. Given that statements of indeterminacy *de re* manifesting the sortal-sensitive perspective employ the formal mode of predication and the corresponding formal notion of indeterminacy, our ordinary attribution of an indeterminate mereological boundary to M may be clarified as follows:

($\mathrm{IND}_{\mathrm{form}}$) It is formally indeterminate of M whether: it formally has rock *r* as a part (at *t*).[18]

It will now be shown that statement ($\mathrm{IND}_{\mathrm{form}}$) may be true in the present framework. We assumed earlier that material objects are fundamentally clear-cut and hence that it is fundamentally determinate of material objects which things they are absolutely composed of. In the case under discussion, there is an m-plus-object that massively overlaps with many m-objects—call one of these aggregates of atoms 'A'—and that, accordingly, hosts a cluster of superimposed m-states. By the ontology of ordinary objects stated above, the m-plus-object is a mountain—let it be M—that hosts a cluster of superimposed m-states, its individual forms. These individual forms are distributions of absolutely determinate facts across clear-cut material objects, namely, M and proper parts of M.

Let us assume, next, that one individual form of M includes the fact that M is composed of the *x*s, whereas another individual form of M includes the fact that A, a proper part of M, is composed of the *y*s, where the *x*s and the *y*s are distinct but overlap massively, in that rock *r* is one of the *x*s but not of the *y*s.[19] As a consequence of the foregoing specifications, M's multiple individual forms differ with respect to which mereological properties they contain. By truth conditions (T1) of statements of formal determinacy and indeterminacy *de re*, it is formally indeterminate whether M is formally composed of the *x*s or of the *y*s. In particular, it is formally indeterminate whether M formally has *r* as a part. Hence, ($\mathrm{IND}_{\mathrm{form}}$) is true. Since M's indeterminate formal boundary arises merely from mereological differences among its multiple individual forms, ($\mathrm{IND}_{\mathrm{form}}$) is compatible with the fact that it is absolutely determinate that M absolutely has rock *r* as a part. On this double-layered picture, M is a formally vague but absolutely precise object. What holds for M holds for other ordinary objects. Their indeterminate boundaries are derivative, the result of differences among their many superimposed forms, floating above the clear-cut boundaries of their underlying matter.

What speaks in favour of this derivative account of mereological indeterminacy *de re* in comparison with the fundamental account? First, those who are drawn to indeterminacy *de re* but oppose fundamental indeterminacy *de re* on the grounds that a picture of reality as having multiple precisifications is unacceptably radical should welcome an account of indeterminacy *de re* as derivative, as arising from a perfectly precise reality, just as orthodoxy conceives of it. This is an intuitive advantage. Moreover, those who recognise a distinction between fundamental and derivative facts should be restrictive about which facts are fundamental. They should accept the methodological principle that fundamental facts must not be multiplied

[18]More perspicuously, it is formally indeterminate *of* M and the property of having *r* as a part whether the former instantiates the latter (at *t*).

[19]For ease of exposition, I am here treating the properties of being composed of the *x*s and of having *r* as a part as complex monadic properties, ignoring individual forms of the *x*s and of *r*. Ultimately, the framework should be able to handle relational formal predications of parthood that are sensitive to the individual forms of all of its relata, see Sattig (forthcoming).

without necessity.[20] Accordingly, the proposed account of indeterminacy *de re* as derivative has a methodological edge over the standard account of indeterminacy *de re* as fundamental. Of course, which account is ultimately preferable depends on how they compare along other dimensions, as well.

Second, the framework offers a plausible response to the problem of indeterminate coincidence, which is not available to the fundamental account of mereological indeterminacy *de re*. The argument from indeterminate coincidence against vague objects was earlier summarised as follows:

1. It is indeterminate of M whether: it has *r* as a part. [P1]
2. There is an object, M-minus, such that it is determinate of M-minus that: it has all and only the parts of M that do not overlap with *r*. [P2]
3. It is determinate of M and M-minus that: the former coincides with the latter iff the former lacks *r* as a part.
4. It is indeterminate of M and M-minus whether: the former coincides with the latter.
5. It is determinate of M and M-minus that: the former coincides with the latter iff the former is identical with the latter. [P3]
6. It is indeterminate of M and M-minus whether: the former is identical with the latter.
7. It is not indeterminate of M and M-minus whether: the former is identical with the latter. [P4]

The proposed framework allows this argument to be blocked in the following way. As pointed out earlier, premise P1 manifests the sortal-sensitive perspective. We judge M's boundary to be unclear, because there are several boundaries that preserve what makes M a mountain. Accordingly, P1 is to be read as a formal predication:

P1* It is formally indeterminate of M whether: it formally has *r* as a part.

Premise P2 also manifests the sortal-sensitive perspective, since the boundary of M-minus is recognised relative to the boundary of M. We pick out M-minus as the object that is just like M, except for determinately lacking *r*. Let us call an object, such as M-minus, that is all of a given mountain except for one or more of its indeterminate parts, a *mountain**. In virtue of its sortal sensitivity, P2 is to be read as a formal predication:

P2* There is an object, M-minus, such that it is formally determinate of M-minus that: it formally has all and only the parts of M that do not overlap with *r*.

Premise P3 incorporates the principle that distinct objects cannot coincide, which owes its intuitive appeal to a worry about overcrowding. This is a metaphysical worry: that distinct objects cannot fit into the same place at the same time is

[20] Schaffer (2009: 361) suggests that a principle along these lines should replace Occam's Razor, according to which entities must not be multiplied without necessity.

supposed to be a truth about the world that is independent of how the world is represented. P3 thus manifests the sortal-abstract perspective on the world of objects, the perspective that cuts through sortal representations, and accordingly is to be read as an absolute predication:

P3* It is absolutely determinate of M and M-minus that: the former coincides absolutely with the latter iff the former is absolutely identical with the latter.

It is important that P3* is a claim about absolute coincidence, as opposed to formal coincidence.[21]

Premise P4 is driven by the Evans-Salmon argument, which is intended as an argument against metaphysically indeterminate identity. That is, the argument's conclusion is supposed to be a truth about the world that is independent of how the world is represented. P4 thus manifests the sortal-abstract perspective and is to be read as an absolute predication:

P4* It is not absolutely indeterminate of M and M-minus whether: the former is absolutely identical with the latter.[22]

If P1–P4 are read as P1*–P4*, then these premises are jointly consistent. According to the present ontology, an ordinary object of kind K is a K-plus-object, the fusion of all K-objects from a range of massively overlapping K-objects. M is thus an m-plus-object, and accordingly hosts multiple m-states—multiple individual forms. Some of these m-states contain the property of having *r* as a part; others do not contain this property. This makes P1* true.

Let us say, furthermore, that while an m-plus-object is the maximal fusion of all m-objects from a range of massively overlapping ones, an *m*-object* is any *nonmaximal* fusion of massively overlapping m-objects. And let a mountain* be an m*-object. Intuitively, a mountain* is all of a mountain except for one or more of its formally indeterminate parts. Now let M-minus be the fusion of all of the m-objects in M except for those that overlap with *r*. Accordingly, M-minus hosts all and only those individual forms of M that fail to contain the property of overlapping with *r*. This makes P2* true.

[21]The notion of coincidence was earlier introduced as follows: material objects x and y coincide $=_{df}$ for any place p, x occupies p iff y occupies p. Given the distinction between the absolute and the formal mode of predication, we get absolute versus formal coincidence: for any material objects x and y,

(C_{abs}) x coincides absolutely with $y =_{df}$ for any place p, x absolutely occupies p iff y absolutely occupies p.

(C_{form}) x coincides formally with $y =_{df}$ for any place p, x formally occupies p iff y formally occupies p.

[22]On the assumption that identity statements have only an absolute reading, this is the only sensible reading of P4 available. But even if a formal reading of identity statements were available, the absolute reading would be the intended one, because the Evans-Salmon argument is meant to be an argument against metaphysically indeterminate identity.

From P2* it follows that M coincides formally with M-minus iff M formally lacks *r* as a part. Since, by P1*, it is formally indeterminate of M whether: it formally has *r* as a part, it is then also formally indeterminate of M and M-minus: whether the former coincides formally with the latter. It does not follow, however, that it is absolutely indeterminate of M and M-minus whether: the former coincides absolutely with the latter. For on the assumption made in the previous section that material objects are metaphysically clear-cut, M and M-minus are absolutely distinct objects with slightly different absolute mereological and spatial boundaries. The indeterminate formal coincidence of M and M-minus is compatible with their determinate absolute noncoincidence. Accordingly, P1* and P2* do not yield absolutely indeterminate identity of M and M-minus via P3*. And so no clash with P4* occurs.

Let me sum up the foregoing discussion. The problem of indeterminate coincidence, directed against vague ordinary objects, rests on the metaphysical worry that it cannot be indeterminate of distinct objects whether they coincide. The combination of a derivative account of formal indeterminacy *de re* with the possibility of perspectival shift between formal and absolute claims of determinacy and indeterminacy *de re* allows us to endorse the absolute ban on indeterminate coincidence of distinct objects while still leaving room for indeterminate coincidence of another type. It is formally indeterminate of M and M-minus whether: they coincide formally. However, when M and M-minus are described absolutely, from the sortal-abstract perspective, then there are no indeterminate boundaries and hence no indeterminate coincidence.[23]

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Chapter 3
A Linguistic Account of Mereological Vagueness

Maureen Donnelly

3.1 Introduction

According to linguistic accounts of vagueness, any vagueness in our claims about the world is due to imprecision in our language. If our linguistic practices are too loose to fix unique interpretations for our terms, then certain claims may be true on some admissible interpretations of our language but false on others. Such claims are vague in the sense that they are neither definitely true nor definitely false. Proponents of linguistic accounts of vagueness generally trace vagueness in mereological claims to imprecision in singular terms. If the singular term "Y" lacks a unique referent, then "X is part of Y" may be true for some admissible assignments of referents to "Y" but false for others.[1]

The primary purpose of this chapter is to propose an alternative linguistic account of vague mereological claims. I suggest that in many cases the vagueness of ordinary mereological claims is due, not to imprecision in singular terms, but to imprecision in *mereological terms* such as the relational predicate "is part of." Though my account is not problem-free, I think it is preferable to the standard linguistic account because it preserves important ordinary intuitions about objects.

[1] Of course, the singular term "X" may lack a unique referent as well. Strictly speaking, the account which traces all mereological vagueness to imprecision in singular terms says that the sentence "X is part of Y" is vague iff, on some admissible interpretations of singular terms, the referent of "X" is a part of the referent of "Y," while on other admissible interpretations, the referent of "X" is not part of the referent of "Y." However, in typical parthood claims, variation in the referent of the first term (here, "X") would not seem to create any additional fluctuation in truth value beyond that already due to variation in the referent of the second term. Thus, I focus in my example parthood claims only on the purported imprecision of the second term.

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Moreover, my account is supported by the evident lack of rigor and specificity in our actual use of mereological vocabulary. I further suggest that my linguistic account of mereological vagueness might accommodate some intuitions motivating ontic accounts of vagueness while avoiding a troublesome commitment to indeterminacy in the world.

3.2 Vague Singular Terms

It is not hard to find examples of mereological claims that appear to be neither definitely true nor definitely false. The examples which have received the most attention in philosophical literature concern small objects in the process being incorporated into, or expelled from, larger objects. There seems to be no precise moment at which an atom processed by an organism's digestive system becomes part of that organism. Suppose my cat Tibbles drinks a bowl of milk containing the carbon atom *C*. Somewhat later at time *t*, *C* has been absorbed into Tibbles' blood stream, but is not yet incorporated into any of Tibbles' cells. Is *C* a part of Tibbles at *t* or is *C* instead merely located within Tibbles' body at *t*? There is no clear answer to this question.[2] A similar blurriness confronts us in the reverse direction, as small parts are gradually cut loose from an organism. Let H be a hair that is barely connected to Tibbles' skin at *t* and firmly embedded in my sweater shortly after *t*. The claim "H is part of Tibbles at *t*" seems to be neither definitely true nor definitely false—we could say that H is still part of Tibbles at *t* (since it is still touching his skin), but we might just as correctly say that it is not (since it is barely anchored to his body).[3]

Even if other types of objects are more stable than organisms, all largish material objects seem to stand in questionable parthood relations to smallish particles at their boundaries. As Lewis puts it "... all things are swarms of particles. There are always outlying particles, questionably parts of the thing, not definitely included and not definitely not included" (1993, pp. 164–165).[4] A tiny sliver of wood hangs from a scratched table leg. Is the sliver still part of the table? Are the bits of rust on a rusty nail still parts of the nail? There are no clear answers to these questions. Note further that mereological vagueness crops up not only in relations between tiny objects and medium-sized objects but also in relations among medium-sized or large objects. Are prosthetic devices—artificial heart valves, prosthetic limbs,

[2]van Inwagen discusses a similar example at (1990, pp. 94–96, 217–218) in support of his claim that the parthood relation is vague.

[3]This sort of example is a focus of Lewis (1993). See also Geach (1980, pp. 215–216) for an earlier discussion.

[4]See also Peter Unger's presentation of the problem of the many in Unger (1980).

silicon implants—parts of the bodies in which they are implanted?[5] Are boulders lying on (or embedded in) topsoil parts of the earth?

In all of these cases, it is not just that we lack a gut intuition about whether one thing is part of another. There is also no apparent way of uncovering determinate facts about the objects' mereological relations through empirical investigations or conceptual analysis. What could we possibly discover about biological processes or the nature of the human body that would settle the question of whether an artificial heart valve is part of the body in which it is implanted? No matter what advances we make in knowledge and understanding, any definite decision on the questionable parthood ascriptions would seem to amount to an arbitrary choice.

The standard linguistic account of vague mereological claims is that endorsed by David Lewis in his "Many, but almost one" (1993).[6] Lewis agrees that many parthood claims are neither definitely true nor definitely false. According to Lewis, vagueness in parthood claims is due to imprecision in singular terms. "H is part of Tibbles" is vague because the proper name "Tibbles" has multiple equally eligible referents, some of which have H as a part and others of which do not have H as a part. If we take "Tibbles" to refer to a mass of tissue which includes all loosely attached hairs, "H is part of Tibbles" comes out true. But we might just as correctly take "Tibbles" to refer to a slightly smaller cat-shaped object—one which excludes loosely attached hairs. On the latter interpretation, "H is part of Tibbles" would be false. Since our linguistic practices leave room for such divergent interpretations of "Tibbles," the sentence "H is part of Tibbles" is neither definitely true nor definitely false.

As has been roughly indicated above, given pretty much *any* ordinary singular term "Y" purportedly denoting a medium-sized or large material object, there is a vague parthood claim with "Y" in the second argument place. (Just take any term referring to a tiny object at the boundary of the referent(s) of "Y" as the first argument of the parthood claim.) Thus, for their account to succeed, proponents of the standard linguistic account of mereological vagueness must be committed to the following strong thesis.

(VST) Most ordinary singular terms (including proper names for animals and people) lack unique referents.

According to VST, not only do I fail to distinguish a single object when I refer to "my cat Tibbles," terms like "my dining room table," "my computer," and "the planet Earth" also lack unique referents. Even more surprisingly, given that many claims about the parts of a human body are vague, on the highly plausible assumption that each person has her body as a part, the VST account of mereological

[5]Notice that in the case of a prosthetic implant, unlike that of the digested carbon atom or the loose hair, the question is not *when* the implant becomes, or ceases to be, part of the organism, but whether it is ever part of the organism.

[6]Other proponents of the standard linguistic account of vague mereological claims include Mark Heller (1990) and Achille Varzi (2001).

vagueness requires that terms like "Hilary Clinton" and "Barack Obama" lack unique referents. When I make claims about myself, I fail to distinguish a unique subject of those claims.[7,8]

I think it is obvious that VST is extremely counterintuitive and difficult, if not impossible, to believe. I cannot see how to even think through (much less endorse) the proposal that many distinct objects are equally eligible referents of "Maureen Donnelly" or that when I describe myself as sitting in a chair and thinking about a philosophy problem, many overlapping Maureens describe the posture and ruminations of many overlapping Maureens. It seems impossible (even as an artificial exercise in philosophical contexts) to alter ordinary thought and speech practices to accommodate VST. How could we refer to ourselves in a way which does not imply that there is a singular object of self-reference?[9]

Besides this important objection to VST itself, I think there are at least two respects in which VST appears especially problematic in the context of the standard linguistic account of mereological vagueness. First, given the standard account's program of tracing any vagueness in parthood claims to imprecision in singular terms, it must follow that it is practically impossible for us to introduce any singular terms which *do* succeed in designating unique macroscopic objects. It would not help, for example, to replace the imprecise ordinary term "Tibbles" with the terms "Tibbles^{+}" and "Tibbles^{-}," where the former is supposed to designate a unique cat-like thing on Tibbles' mat which includes H as a part and the latter is supposed to designate a unique cat-like thing on Tibbles' mat which excludes H. For, according to the standard linguistic account, there is no *one* cat-like thing on Tibbles' mat

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[7] See Smith (2007) for an extended criticism this consequence of VST.

[8] Lewis and other proponents of VST may nonetheless hold that it is *supertrue*—that is, true on each admissible interpretations of our language—that exactly one object on the mat is Tibbles. See, for example, Lewis (1993, pp. 172–173). However, supertruth is not truth. Lewis himself points out that *within the supervaluationist framework*, we cannot even state the initial puzzle—that there appear to be many equally admissible ways delineating Tibbles' boundaries—much less Lewis' own VST-style solution to that puzzle. Thus, Lewis admits (as he must) that, supervaluationism aside, it is true that singular terms like "Tibbles" have multiple equally eligible referents. See, for example, Lewis (1993, p. 173): "Under the supervaluationist rule, it's right to say that there's only one cat, and so the candidates have unequal claim. Suspending the rule it's right to say the candidates have equal claim, and that all of them alike are not definitely not cats. Suspending the rule, it's even right to say that they are all cats!"

[9] I think it is helpful to compare VST to other philosophical claims which, though also highly unintuitive, are nonetheless much easier to accommodate than is VST. As one example, consider the "universal fusion principle" stating that, for any arbitrary plurality of objects, some object is composed of just that plurality of objects. Clearly the universal fusion principle counterintuitive—we normally assume that no object is composed of just Tibbles and the Empire State Building. Nonetheless, there seems to be no substantial barrier to altering ordinary thought and speech to accommodate the universal fusion principle. We can do so as long as we are willing to admit that, in addition to ordinary integral objects, there are also many scattered objects which we usually ignore. In contrast to the universal fusion principle, VST does not merely require that we countenance additional objects outside of the realm of ordinary concerns. VST implies that our thoughts and speech drastically misrepresent objects at the center of our practical lives.

having H as a part and no *one* cat-like thing on the mat lacking H as a part. Any number of hairs, cells, and microscopic particles must hover uncertainly at the boundary of what I take to be my cat Tibbles. Thus, according to Lewis, there are multitudes of candidate referents of "Tibbles" on Tibbles' mat—one for every way lumps of cat tissue might differ in excluding or including specific pluralities of borderline parts. If vagueness in mereological claims must always stem from imprecision in singular terms, then a singular term can succeed in designating a unique object only if its referent's parts are completely specified. But since it is practically impossible for us to specify all of the parts of any macroscopic object, it follows on the standard account that it is practically impossible for us to uniquely designate any macroscopic object.

A second, and related, complication of the VST account is that it requires that there are many more overlapping cat-like (and human-like, planet-like, ...) objects than we ordinarily assume. We have just seen that the VST account assumes a multitude of overlapping candidate referents of "Tibbles" on Tibbles' mat. On the assumption that any referent of "Tibbles" is a cat, the VST account implies either that there are very many overlapping *cats* on Tibbles' mat or, at least, that there are very many overlapping *cat-candidates* on the mat (where *x* is a cat-candidate iff *x* qualifies as a cat under some appropriate interpretation of the vague predicate "... is a cat"). The first alternative is obviously highly counterintuitive. Perhaps the second alternative may not seem so bad within a supervaluationist framework. It could still turn out to be *supertrue*—that is, true on all admissible interpretations of our language—that there is exactly one cat on Tibbles' mat. Lewis assumes that supervaluationism can to this extent validate many of our ordinary claims about the numbers of cats, people, tables, etc., in a given place.[10] However, even if it succeeds on its own terms, this sort of appeal to supervaluationism does not lessen the implausibility of Lewis' underlying claim that there are many *equally cat-like* things wherever we normally assume there is just one.[11] Lewis himself admits that, supervaluationism aside, there is a clear sense in which the proponent of his account must admit to multitudes of overlapping cats wherever there is any cat (1993, p. 174).

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[10]See Lewis (1993, pp. 172–173) and note 8 above. However, Lewis never explains what is supposed to be the basis of a restriction of admissible interpretations of our language to those which assign *exactly one* cat-like object on Tibbles' mat to the extension of the "... is a cat" predicate. Note that it is *not* a requirement of our concept of cathood that distinct cats cannot overlap—we allow that they do in cases of Siamese twins and can imagine fictional circumstances in which overlapping cats are commonplace. It is thus not clear what supports Lewis' assumption that any interpretation which assigns more than one of the overlapping Tibbles-candidates to the extension of "... is a cat" must be inadmissible.

[11]Toward the end of the next section, I briefly discuss the sort of alternative position advocated in Lowe (1982, 1995). Lowe allows that there are many cat-shaped *lumps of feline tissue* on Tibbles' mat, but (in opposition to Lewis) claims that there is exactly one (candidate) *cat* on Tibbles' mat. I take it that Lowe's position is much more intuitively acceptable than is Lewis'. The difficulty with Lewis' position lies not so much in his claim that there are many cat-shaped and cat-sized things on Tibbles' mat but in his claim that all of these many things equally qualify as cats.

Taken together, the unpalatable commitments of the VST-style account of mereological vagueness give us ample reason to look for an alternative account. In the next section I propose an alternative linguistic account which, though not without problems of its own, seems to me to be much more plausible than the VST account.

3.3 Vague Mereological Terms

I propose an account of vague mereological claims which rejects VST and relies instead on the following claim.

(VMT) Mereological predicates (including the parthood predicate) are imprecise in the sense that they are open to alternative interpretations assigning different (tuples of) objects to their extensions.

Throughout the remainder of this section, I assume that an interpretation of a one-place predicate assigns that predicate to a unary property with a *determinate* extension (i.e., a determinate set of objects instantiating that property), an interpretation of a two-place predicate assigns that predicate to a binary relation with a *determinate* extension (i.e., a determinate set of pairs of objects standing in that relation), and so on.[12] There are other possibilities. In Sect. 3.4 below, I briefly consider the possibility that some interpretations assign the parthood predicate to relations which lack determinate extensions.[13]

I take VMT to entail that there is no *one* parthood relation, not even a vague parthood relation which holds among objects to intermediate degrees. Rather, many different relations are equally eligible interpretations of the parthood predicate. Some of these candidate parthood relations hold between H and Tibbles (at *t*) and others do not. Some hold between prosthetic devices and the organisms to which they are attached. Other candidate parthood relations restrict an organism's parts to organic structures. In general, various candidate parthood relations differ from one another in the levels of attachment, functional integration, material continuity, and so on, characterizing the pairs of objects in their extensions.

There are two strong reasons for thinking that VMT is true. First, VMT lets us avoid the drastic claims of the VST account of mereological vagueness while still maintaining a purely linguistic account of vagueness. Even if the singular terms "X" and "Y" each have unique referents, the claim "X is part of Y" is neither definitely true nor definitely false if the pair <X, Y> is in the extension of some, but not all,

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[12] A terminological clarification: if interpretation *I* assigns relation **R** to the relational predicate *R* and the pair $<x, y>$ is in the extension of **R**, I will say that $<x, y>$ *is in the extension of R under interpretation I.*

[13] Alternatively, we might dispense with properties and relations altogether and just talk about predicates and the (determinate or fuzzy) extensions assigned to them under different interpretations.

interpretations of the parthood predicate. For example, granted that “Tibbles” and “H” each designate unique objects, the sentence “H is part of Tibbles” is vague if the pair <H, Tibbles> is included in the extension of the parthood predicate on some interpretations but not on others. Given VMT, we may reject VST’s sweeping allegation of imprecision in ordinary singular terms and instead trace mereological vagueness to imprecision in mereological terms.[14] The proponent of a VMT-style account may hold that cats, tables, planets, and persons are distributed through the world roughly as we have always assumed they are—not in overlapping multitudes but typically at positive distances from one another.

The second, and more important, reason for thinking that VMT is true is the fact that ordinary use of mereological vocabulary is remarkably spotty and loose—much more so than ordinary use of common nouns like “cat,” “table,” and “person” or of proper names like “Tibbles” and “Barack Obama.” Outside of philosophy, we don’t talk much about relations between parts and wholes except in specialized contexts—for example, auto repair or anatomy—where our interest in parthood is limited by practical goals. Your mechanic can tell you what parts are needed to keep your car running, but is unlikely to have answers to more esoteric questions about the parts of a car. (Is motor oil part of your car or is it merely located within the car? Is there a part of the car composed of just the steering wheel and carburetor?) Moreover, ordinary reasoning contexts generally involve only very minimal mereological reasoning—perhaps not much beyond occasional uses of rules linking the location or size of a part to that of the whole (e.g., any part of Tibbles is located where Tibbles is). I cannot see that ordinary usage supports any system of principles nearly as sophisticated as the formal mereologies studied in philosophy.[15]

Further, as philosophers like van Inwagen have pointed out, there seems to be no way of defining the parthood predicate in terms of nonmereological predicates (van Inwagen 1990). By contrast, we do agree upon at least a *rough* definition—in terms of genotype or of salient physical characteristics—of what it is for a thing to be a cat. Even if our definition of “cat” may leave some possible cases undecided, in practice we rarely disagree about where cats are in the world or how many cats are in a given place. But no analysis of the parthood predicate reveals whether it applies very many common cases, like those involving an organism and its loose hairs or an automobile and its motor oil. Nor is there any practical method for determining

[14]Of course, the proponent of the VMT-style account of mereological vagueness must admit that *some* singular terms lack unique referents. For example, “the nicest man in the room” may designate different men under different interpretations of “nicest.” Moreover, a proponent of VMT *could* hold that certain types of ordinary singular terms—for example, terms for geographical entities like mountains—never succeed in designating unique objects. Even with this more limited rejection of VST, VMT is required for a linguistic account of vague mereological claims involving whatever singular terms do succeed in picking out unique referents.

[15]See Simons (1987) for a discussion of alternative formal mereologies. The VMT account of mereological vagueness will be discussed in relation to *classical mereology* at the end of this section.

whether arbitrary objects qualify as "parts," as there is a method for determining whether arbitrary objects are approximately 8 ft apart. If we want to know whether two posts are approximately 8 ft apart, we can get a ruler and measure. What could we possibly do to settle the question of whether H is part of Tibbles?

Given the significant open-endedness of ordinary use of mereological terminology, unless there are overriding reasons for thinking otherwise, we should conclude that linguistic conventions fail to specify unique interpretations for mereological predicates. I cannot see that there are reasons for thinking that mereological vocabulary is any more precise than it seems to be and so conclude that VMT is true. This is not, however, to say that there are no restrictions whatsoever on admissible interpretations of mereological vocabulary. As with most vague terms, there are at least two important sources of restrictions on admissible interpretations of mereological predicates—exemplar cases and logical rules.

We assume that, however it is interpreted, certain paradigmatic pairs must be included in the extension of the parthood predicate—Tibbles' tail is part of Tibbles, the Australian land mass is part of the planet Earth, and a car's carburetor is part of the car. We further assume that, however it is interpreted, many other paradigmatic pairs are excluded from the extension of the parthood predicate—the Empire State Building is not part of Tibbles, Tibbles' tail is not part of me. But these paradigms still leave plenty of room for a wide range of interpretations that differ in whether they include or exclude the sorts of borderline parthood pairs discussed in this chapter.

In addition, ordinary usage does seem to underwrite some minimal logical rules for the parthood predicate. As stated above, I doubt that the commonsense rules are nearly so strong as those assumed in many philosophical discussions of mereology. But ordinary reasoning does at least link mereological predicates to shape, size, and location predicates.[16] An example mentioned above is the following rule linking parthood to location.

(Loc-In) x is part of $y \rightarrow x$ is located within y

Note that if all objects had precise (spatial or spatiotemporal) locations, then (Loc-In) might narrow the interpretation of the parthood predicate, at least in the sorts of boundary cases considered above. *If* H and Tibbles had precise spatial locations at t, then given (Loc-In) (and the assumption that the spatial inclusion relation among regions is precise), H should be part of Tibbles at t iff H's spatial location at t is included in Tibbles' spatial location at t.[17] However, I do not see

[16] See, for example, van Inwagen (1990, pp. 43–44) for additional rules along these lines.

[17] In fact, although (Loc-In) is relatively noncontroversial, we *might* interpret the parthood predicate in such a way that locational inclusion is necessary but *not sufficient* for parthood. For example, Lowe holds that a lump of clay constituting a statue does not have the statue—or the statue's arms, legs, etc.—as parts even though these objects are all located within the lump. See Lowe (2003). Doepke (1982) endorses a similar position. Thus, even if all material objects had precise locations, (Loc-In) would still not fix a *unique* interpretation for the parthood predicate, since it leaves room for variation among interpretations in which objects count as parts of wholes they are (spatially) included in.

why we should think that either Tibbles or H has precise locations. On the contrary, given the apparent vagueness in Tibbles' parts, it seems rather that Tibbles is *not* precisely located, at t or at any other time. What region counts as Tibbles' location depends on whether or not H is part of Tibbles. Since the parthood claim is vague, so too are claims about locational relations between H and Tibbles. Instead of eliminating candidate interpretations for the parthood predicate, rules like (Loc-In) require only that interpretations of the parthood predicate vary in coordination with interpretations of certain nonmereological predicates. The parthood predicate may be interpreted as either including or excluding the pair <H, Tibbles> from its extension as long as the "is located within" predicate is assigned a coordinated interpretation which preserves (Loc-In).

I take rules like (Loc-In) to express what Fine calls "penumbral connections" among the terms of our language (Fine 1975). In what follows, I will call a coordinated assignment of admissible interpretations to each term in our language, a "precisification" of the language. I assume that every precisification respects penumbral connections as well as exemplar cases.[18] To make it easier to discuss the coordinated interpretations for a fixed precisification, I introduce the following notational convention. For any precisification j and any n-place predicate *Pred*, I use the **bold** subscripted symbol $\mathbf{Pred}_j$ for the precise n-place relation which is the interpretation of *Pred* on j. For example, $\mathbf{part}_j$ is the (precise) binary relation which is the interpretation of the parthood predicate on precisification j and $\mathbf{located\ within}_j$ is the (precise) binary relation which is the interpretation of the location inclusion predicate on j. (Loc-In) requires that for any j, $\mathbf{part}_j$ is a subrelation of $\mathbf{located\ within}_j$ (i.e., any pair in the extension of the former relation is also in that of the latter).

Ordinary usage seems to support more complex rules than (Loc-In), but like (Loc-In), these rules only enforce a certain level of coordination between the interpretations of different predicates. To present additional examples, it is useful to introduce two further mereological predicates besides the parthood predicate. The fusion predicate and the disjoint decomposition predicates are defined as follows:

(D_{fus}) z is a fusion of the x's $=_{def}$ (i) each of the x's is a part of z and (ii) any part y of z has a part in common with at least one of the x's.

(D_{disc}) the x's are a disjoint decomposition of $z =_{def}$ (i) z is a fusion of the x's and (ii) no two of the x's have any common parts (i.e., the x's are pairwise disjoint).

Since each of the relational predicates above is defined in terms of the parthood predicate, their interpretations on a given precisification are entirely determined by that of the parthood predicate. For example, on any precisification j, Tibbles stands

[18] Thus, for example, <Tibbles' tail, Tibbles> is assigned to the extension of the parthood predicate on *every* precisification, <Tibbles' tail, the Empire State Building> is excluded from the extension of the parthood predicate on *every* precisification, and any precisification which assigns <H, Tibbles> to the extension of the parthood predicate also assigns <H, Tibbles> to the extension of the "located within" predicate.

in the **fusion**$_{\text{j}}$ relation to H and Tibbles iff H stands in the **part**$_{\text{j}}$ relation to Tibbles. (More generally, Tibbles stands in the **fusion**$_{\text{j}}$ relation to any plurality including H only if H stands in the **part**$_{\text{j}}$ relation to Tibbles.)

More complex reasoning rules link objects' shapes, sizes, and locations to those of the pluralities composing them. For example, each object's mass is the sum of the masses of its disjoint parts.

(Mass) if the *x*'s are a disjoint decomposition of *z*, then the mass of *z* is the sum of the separate masses of each of the *x*'s.[19]

(Mass) requires that interpretations of mass predicates (e.g., *weighs exactly 10 pounds*) vary in coordination with interpretations of mereological predicates. In general, we should expect that Tibbles is assigned slightly higher weights on precisifications which are more liberal in their interpretations of the parthood predicate (e.g., those which include among Tibbles' parts all loosely attached hairs) than on precisifications with stricter interpretations of the parthood predicate. Thus, for distinct precisifications *i* and *j*, Tibbles may instantiate the **weighs exactly 10 pounds**$_{\text{i}}$ property but not the distinct **weighs exactly 10 pounds**$_{\text{j}}$ property.

Even more complicated rules establish ties between an object's shape and the shapes of, and relations among, its parts. We should expect that Tibbles satisfies slightly different shape predicates depending on whether or not the parthood predicate is interpreted so as to include H among Tibbles' parts.

Thus, the VMT account requires not only that mereological terms receive different interpretations on different precisifications of our language but also that coordinated shape, size, and location predicates are interpreted differently on different precisifications. This may seem problematic. Intuitively, it seems that predicates like "weighs exactly 10 pounds" should be precise in a way that quintessentially vague predicates like "is heavy" are not. I address this issue below. Very roughly, my proposal is that the proponent of the VMT account may hold that predicates like "weighs exactly 10 pounds" have unique extensions over *subdomains of precise objects* and are in this sense "exact" in a way that prototypically vague predicates like "is heavy" are not.[20]

Before going any further, we will need a distinction between *vague objects* and *precise objects*. Note first that nothing in the VMT account requires that the parts of *all* objects vary over precisifications. To put this point more precisely, let us say that object *x* is a "borderline part" of object *y* just in case some, but not all, precisifications assign $<x, y>$ to the extension of the parthood predicate. In other words,

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[19]A caveat: although we ordinarily assume that (Mass) is generally true, it is not at all clear that it applies in case the *x*'s are extremely small. If each of the *x*'s is a minimal particle and *if* minimal particles have no mass, then *x*'s mass cannot be the sum of the separate masses of the *x*'s (unless of course *x* also has no mass). But we may at least assume that (Mass) holds in cases involving only objects with positive mass.

[20]See Rosen and Smith (2004) for a similar suggestion.

(D_{bord}) x is a borderline part of $y =_{def}$ there are precisifications i and j such that x stands in the $\mathbf{part}_i$ relation to y but does not stand in the $\mathbf{part}_j$ relation to y.[21]

Although I assume that ordinary objects like Tibbles and the planet Earth have many borderline parts (this is the motivation for the VMT account), nothing in the VMT account requires that *all* objects have borderline parts. Plausibly some objects—perhaps elementary particles and molecules—are assigned the same parts on any admissible interpretation of the parthood predicate.

(D_{prec}) x is a precise object $=_{def}$ x has no borderline parts.

It is tempting to simply define a "vague object" as any object which has borderline parts. However, I think this is not quite what we want. To see why, suppose there is a certain lump of matter M which on some, but not all, precisifications has Tibbles as a part. In other words, there are precisifications i and j such that Tibbles stands in the $\mathbf{part}_i$ relation to M but *not* in the $\mathbf{part}_j$ relation to M. This imprecision in M's parts need not stem from any imprecision in the particles composing M. Even if all precisifications assign the same particle parts to M, Tibbles may still be a borderline part of M since Tibbles himself is assigned different particle parts on different precisifications. Thus, if H stands in the $\mathbf{part}_j$ relation to Tibbles, but not in the $\mathbf{part}_i$ relation to Tibbles, then Tibbles may extend beyond M on precisification j even though he is included in M on precisification i and M is composed of the same particles on both precisifications. To leave room for a looser sense in which objects like M may be precise even though they have *some* borderline parts, I introduce a somewhat weaker predicate than that of (D_{prec}).

(D_{prec*}) y is a precise* object $=_{def}$ (i) for some precisification j, there are precise objects the x's such that y is a $\mathbf{fusion}_j$ of the x's and (ii) no precise object is a borderline part of y.

Taken together, clauses (i) and (ii) require that precise* objects are fusions of the same precise objects on all precisifications. Note that since any object is (trivially) a fusion of itself on all precisifications, any precise object is also a precise* object. Thus, the predicate defined in (D_{prec*}) is strictly weaker than that of (D_{prec}). Unlike the stronger predicate, the weaker predicate may apply to objects with borderline parts. If the lump M is composed of the same precise objects on every precisification, then M is a precise* object even if Tibbles is a borderline part of M.

A vague object y is any object which fails either (i) or (ii)—either y is not a fusion of precise parts on any precisification or y has different precise objects as parts on different precisifications.

(D_{vague}) y is a vague object $=_{def}$ y is not a precise* object.

[21]Note that unlike the parthood predicate, the borderline parthood predicate is not assigned different extensions on different (first-order) precisifications of our language. Nonetheless, the borderline parthood predicate may be vague if it is vague which interpretations of a language qualify as precisifications of that language (i.e., if the metalinguistic term "precisification" is vague). The same points apply to the predicates defined in (D_{prec}), (D_{prec*}), and (D_{vague}).

For example, Tibbles is presumably a vague object. Granted that on each precisification Tibbles is a fusion of *some* plurality of precise particles, he must have different precise objects as parts on different precisifications. For example, the particles in H are borderline parts of Tibbles.

It bears emphasis that, as a purely linguistic account of vagueness, the VMT account denies that *any* objects—including vague objects—stand in indeterminate relations or have indeterminate properties. The VMT account assumes that vague objects are just as determinate as precise (or precise*) objects. The difference is only that, unlike our descriptions of precise objects, our mereological descriptions of vague objects fail to exactly align with the actual structural features of vague objects. In particular, our loosely delineated parthood predicate fails to distinguish exactly one of the many different complex relations in which objects like organisms might stand to the smaller objects which are gradually involved in (or disconnected from) their biological processes. By contrast, a minimal particle cannot stand in complex relations to *smaller* objects, since there are no smaller objects. Over domains of minimal particles, the parthood predicate must be interpreted as the identity relation. There are no other options. Further, if there are composite objects (perhaps portions of matter) whose presence in the world is entirely determined by the *mere presence* of specific pluralities of particles,[22] then there would seem to be only one way of interpreting parthood as a relation between these composite objects and their underlying particles—it is the relation which holds between a particle p and a composite object x just in case p is among the precise plurality of particles whose combined presence (at a time or in a region) is necessary and sufficient for x's presence (at that time or in that region).

Though the VMT account does not require it, I will assume in what follows that there are actually precise objects—not every object in this world is vague. In particular, I will assume that microscopic particles are precise objects. Thus, I assume that any object which is composed of the same microscopic particles on every precisification is at least precise* (if not precise) and that any object which has a particle as a borderline part is vague.

The distinction between vague and precise objects may help address one liability of the VMT account of mereological vagueness. We have seen above that to preserve appropriate penumbral connections, the VMT account requires that interpretations of shape, size, and location predicates vary in coordination with interpretations of mereological predicates. Precisifications which assign different parts to Tibbles will generally also differ in their assignments of size, shape, and location to Tibbles. In some respects, this consequence of the VMT account is unsurprising and unproblematic. It is what we should expect given the imprecision in Tibbles' parts.

[22]Note that the presence of an organism, planet, or table is *not* determined by the *mere presence* of any particular plurality of particles. On the one hand, such objects are not tied to any specific plurality of particles (since they change parts over time). On the other hand, for an organism, planet, or table to be in the world, it is not sufficient that any particles merely exist. What is required is that certain kinds of particles stand in particular spatial or functional relations to one another.

Moreover, since all of Tibbles' borderline parts are extremely small in comparison with Tibbles himself, variation in Tibbles' size, shape, and location across different precisifications is negligible—much too small to make any practical difference in our ordinary rough assessments of size, shape, and location.

On the other hand, we normally assume that predicates like "weighs 10 pounds" or "is cubical" are precise in a way that prototypical vague predicates like "is heavy" or "is tall" are not. In fact, it may turn out that this intuitive distinction is confused. If microscopic particles lack determinate locations or masses (as quantum physics suggests), then even the *opponent* of VMT must admit that all shape, size, and location predicates lack precise extensions. I think it is open to the proponent of VMT to simply claim that intuitive distinctions between vague and precise predicates are confused. If this is a bullet for the proponent of the VMT account to bite, it is certainly easier to swallow than the VST-er's claim that "Barack Obama" lacks a unique referent. The VMT-er may explain our erroneous intuitions by pointing to the abovementioned practical inconsequence of imprecision in predicates like "weighs 10 pounds," as compared to the noticeable and important differences among alternative interpretations of predicates like "is heavy" or "is tall."

Another sort of response is open to the VMT-er whose ontology is expansive enough to include precise* objects of arbitrary sizes and shapes—perhaps portions of matter composed of precise pluralities of particles. This proponent of VMT may distinguish *exact* predicates as those whose interpretations do not vary over subdomains of *precise* objects*. Let us say that predicate *Pred* is exact iff either:

(i) any two precisifications assign the same extension to *Pred*; or
(ii) some precisification assigns some precise* object to *Pred*'s extension and any two precisifications assign the same precise* objects to *Pred*'s extension.

Disjunct (i) is satisfied by any predicate with a unique interpretation, even if that predicate does not apply to precise* objects. For example, *if* the predicate "is a person" applies to the same objects on all precisifications, then this predicate is exact whether or not its unique extension includes any precise* object. Disjunct (ii) may be satisfied by predicates which lack unique interpretations. The predicate "weighs 10 pounds" satisfies disjunct (ii) as long as it applies to some precise* object (perhaps a cat-sized lump of matter) and the same precise* objects are included in its extension on all precisifications. Given principle (Mass), this latter requirement should be satisfied as long as precise* objects are composed of precise objects with determinate positive weights—in this case, each precise* object's mass is fixed by the masses of its precise parts. Similarly, as long as precise objects have determinate locations and stand in determinate spatial (or spatiotemporal) relations, then shape and location predicates may be exact even if their applications to objects like Tibbles vary across precisifications. By contrast, predicates like "is heavy" and "is tall" are *not* exact since, if they apply to precise* objects at all, their application to precise* objects must vary across precisifications. A precise* lump of matter may qualify as heavy on one precisification but not on others.

Of course, the caveat is that even if there are precise* objects of arbitrary shapes and sizes, it is not certain that the particles composing these objects have determinate shapes, sizes, or locations. If so, then predicates like "weighs 10 pounds" or "is cubical" are not exact after all. But in this case, the proponent of the VMT account is no worse off than the proponent VST account, who must also countenance indeterminacy in shape, size, or location.

A separate liability of the VMT account of mereological vagueness is that it does not combine easily with the strong assumptions of classical mereology. Classical mereology may be axiomatized by the following five principles:

(P_{ref}) x is part of x (parthood is reflexive).
(P_{trans}) If x is part of y and y is part of z, then x is part of z (parthood is transitive).
(P_{antis}) If x is part of y and y is part of x, then $x = y$ (parthood is antisymmetric).
(P_{supp}) If x is not part of y, then x has a part which shares no part with y.
(P_{fusion}) For any objects the x's, there is some object which is a fusion of the x's.

It is logically consistent with the VMT account that mereological predicates satisfy all five of these principles on all precisifications. As in the case of principles like (Loc-In), if the axioms of classical mereology are satisfied on *all* precisifications, then they are all supertrue. To this extent the proponent of the VMT may endorse classical mereology, even though she holds that many claims about particular objects' parts are neither definitely true nor definitely false. She will hold, for example, that although it is indeterminate whether H is part of Tibbles, it is supertrue that *if* H is part of Tibbles, then any part of H is also a part of Tibbles.[23]

However, taken all together, the principles of classical mereology require awkward ontological commitments from the proponent of the VMT account. It is a consequence of the five principles above that for any objects the x's, *exactly one* object is a fusion of the x's. Thus, the VMT-er who endorses classical mereology must hold that

(*) for any precisification j and any plurality of objects the x's, *exactly one* object stands in the $\mathbf{fusion}_j$ relation to the x's.[24]

The awkwardness is that (*) seems to require that each vague ordinary object has a kind of double—a similar, but distinct, vague object which "swaps parts" with it on alternate precisifications. Suppose, for example, that particle y is a borderline part of Tibbles (y may be, e.g., a particle part of H). Let i and j be precisifications such that y stands in the $\mathbf{part}_i$ relation, but *not* in the $\mathbf{part}_j$ relation, to Tibbles. Let $x_1, \ldots x_n$ be particles composing the remainder of Tibbles on i. Tibbles stands in the $\mathbf{fusion}_i$ relation to $x_1, \ldots x_n, y$. Given (*), Tibbles is the *only* object standing in the $\mathbf{fusion}_i$

[23] See Donnelly (2009) for more details on the relation between classical mereology and a VMT-style account of mereological vagueness.

[24] Note that (*) does not require that the *same* object is the fusion of the x's on *all* precisifications. In other words, (*) does not require that some object z is such that for *every* precisification j, z stands in the $\mathbf{fusion}_j$ relation to the x's.

relation to $x_1, \ldots x_n, y$. Now consider the alternative precisification j, on which y does not count as a part of Tibbles. According to (*), a unique object T stands in the **fusion**$_\mathrm{j}$ relation to $x_1, \ldots x_n, y$. Note that T cannot be identical to Tibbles. Since y does *not* stand in the **part**$_\mathrm{j}$ relation to Tibbles, Tibbles cannot stand in the **fusion**$_\mathrm{j}$ relation to $x_1, \ldots x_n, y$ or to any other plurality of objects which includes y.[25] The problem is that it is not clear what T could be. T is similar enough to a cat to be composed on one precisification of the same particles which compose Tibbles on an alternate precisification. Thus, at least on precisification j, T is cat shaped and cat sized. Moreover, like Tibbles himself, T is a vague object. To see this, note that T cannot be a fusion of the same particles on *every* precisification. T cannot stand in the **fusion**$_\mathrm{i}$ relation to $x_1, \ldots x_n, y$ because, by assumption, Tibbles is the only object standing in that relation to $x_1, \ldots x_n, y$. If portions or lumps of matter are precise* objects (composed of the same particles on all precisifications), then T cannot be a portion or lump of matter—for example, a lump of cat tissue. But it is hard to see what other cat-shaped and cat-sized thing might be in Tibbles' vicinity.[26] It goes without saying that the proponent of the VMT account should *not* claim that T is another cat, or cat-candidate, since this would make her position just as problematic as the VST account.[27]

Perhaps the proponent of VMT can come up with some satisfactory account of objects like T. But given that a primary motivation for her position is to preserve commonsense intuitions about objects, I think she would do better to simply reject (*) and adopt weaker principles than those of classical mereology. After all, nonphilosophers do not assume that there is a fusion of any random plurality of objects. Thus, the proponent of VMT might reject (P_{fusion}), but retain the remaining four axioms of classical mereology. This weaker mereology does not require that there is a fusion of *every* plurality of objects, but does require that fusions are unique. For a fixed precisification i, if z and w stand in the **fusion**$_\mathrm{i}$ relation to the x's, then $z = w$. Given that Tibbles stands in **fusion**$_\mathrm{i}$ to $x_1, \ldots x_n$, y, no other object stands in this relation to $x_1, \ldots x_n$, y. But since $x_1, \ldots x_n$, y is not required to have a fusion on every precisification, there is now no reason to assume that some other object counts as a fusion of $x_1, \ldots x_n$, y on the precisifications at which Tibbles himself does not qualify as a fusion of these particles. Thus, the proponent of VMT who rejects (P_{fusion}) need not admit any mysterious Tibbles double and may even adopt a sparse ontology on which Tibbles is the only cat-sized and cat-shaped object on the mat.

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[25]Recall that according to the definition of the fusion predicate (D_{fus}), if z is a fusion of the x's, then each of the x's is a part of z.

[26]Note that, despite the similarities between T and Tibbles, there is no temptation to claim that their identity relations are indeterminate. T is definitely distinct from Tibbles since the two objects stand in different relations to the particle y.

[27]Of course, the problem described here compounded by the fact that Tibbles has a multitude of other borderline parts besides y. Thus, it seems that the VMT-er who endorses (*) must admit a multitude of other "Tibbles doubles" besides T.

Alternatively, the VMT-er may have reason to conclude that there are many more objects in the world than we ordinarily assume. One reason, considered above, is that a domain of precise* objects of arbitrary sizes and shapes might serve as a basis for a distinction between exact and inexact predicates. Thus, the proponent of the VMT account may claim that, besides vague ordinary objects like cats, planets, and human beings, there are also precise hunks of matter of arbitrary sizes and shapes. If so, he may endorse (P_{fusion}) but still avoid mysterious doubles like T by rejecting either the antisymmetry principle (P_{antis}) or the supplementation principle (P_{supp}). If either of the latter principles are dropped, then fusions need not be unique—for a fixed precisification i and a fixed plurality of objects the x's and $z \neq w$, both z and w may stand in $\mathbf{fusion}_i$ to the x's. Given this alternative weakening of classical mereology, the VMT-er might hold that for any plurality of particles the x's, some *precise** portion of matter counts as a fusion of the x's on all precisifications. In particular, he may hold that some precise* portion of matter M is such that for any precisification k, M stands in the $\mathbf{fusion}_k$ relation to the particles $x_1, \ldots x_n, y$. The weaker mereological principles do not require that Tibbles is identical to M, even though he also stands in the $\mathbf{fusion}_i$ relation to $x_1, \ldots x_n, y$. Indeed, Tibbles cannot be identical to M since, unlike M, Tibbles has borderline particle parts and is composed of different particles on different precisifications.[28] Note that this version of the VMT account agrees with the VST account that there are many cat-sized and cat-shaped objects on Tibbles' mat—for every collection of particles on the mat, a distinct precise* portion of matter is a fusion of those particles. But, unlike the VST account, the VMT account claims that Tibbles is definitely distinct from all other cat-sized and cat–shaped objects on the mat. Unlike the precise* hunks of matter, Tibbles is a vague object.[29,30]

On the whole, I think the VMT account of mereological vagueness stacks up quite well against its rival linguistic account of mereological vagueness. Granted the worst construal of the limitations and potential problems discussed above, the VMT-er's commitment to imprecision in shape, size, and location predicates and to somewhat weaker mereological principles is surely easier to accept than the VST-er's commitment to pervasive imprecision in singular terms and to multitudes

[28] Again, there is no reason to think that identity relations are indeterminate. Since M and Tibbles stand in different relations to the particles $x_1, \ldots x_n, y$ (M, but not Tibbles, stands in $\mathbf{fusion}_j$ to $x_1, \ldots x_n, y$), M and Tibbles are definitely distinct.

[29] See Lowe (1995) for a response to Lewis (1993) which endorses a position roughly along these lines. However, Lowe does not here explicitly endorse either a purely linguistic account of vagueness or the strong principle (P_{fusion}). He does, however, assume that there are many precise lumps of cat tissue in Tibbles' vicinity, each of which is definitely distinct from Tibbles.

[30] One additional advantage of the VMT account is that it offers more ontological flexibility than the VST account. Because the VST account requires multiple candidate referents for each ordinary singular term, the VST account is committed to an ontology which includes many overlapping complex objects wherever there is *any* complex object. As we have just seen, the VMT account is compatible with *either* a somewhat sparse ontology or an ontology abundant enough to include arbitrary portions of matter in addition to ordinary objects.

of overlapping cat-candidates and person-candidates. Moreover, as was noted above, it is not clear that the proponent of the VST account fairs any better on the first problem—the commitment to imprecision in shape, size, and location predicates. Whether he does or not depends on whether the building blocks of the world are in fact objects with determinate sizes and locations. Furthermore, many philosophers—including Doepke, Lowe, Simons, and van Inwagen—have rejected various principles of classical mereology, including (P_{fusion}) and (P_{supp}), for reasons independent of issues surrounding vagueness.[31]

Nonetheless, I suspect that there may be a lingering suspicion of VMT's claim that there is no unique parthood relation. Perhaps this is due to an assumption that no matter how loose and scanty are conventions surrounding mereological vocabulary, some one relation must in fact be the most eligible referent of our parthood predicate. I see no reason for thinking this is so. What could possibly make relation which holds between H and Tibbles (or between Fred's artificial heart valve and Fred) a more eligible referent of the parthood predicate than one which does not? Of course, from the perspective of the proponent of the VST account, these considerations are confused. For the VST-er, relations do not hold between H and Tibbles, but rather between the mereologically determinate objects which are the candidate referents of "H" and "Tibbles." On this view, parthood is the unique relation linking complex objects to the determinate pluralities of simpler objects composing them. But again, I cannot see what reasons we have to believe that any *one relation* provides this kind of precise link between complex objects and the simpler objects on which they, to varying degrees, depend. Rather, it seems to me that what counts as "composition" or "fusion" is vague. Mereological predicates are open to alternative interpretations across a spectrum of relations requiring different degrees of attachment, similarity, functional integration, and so on.

3.4 Other Accounts of Vagueness

The primary purpose of this chapter is to present the VMT account of mereological vagueness as an alternative to the better-known linguistic account which relies instead on VST. In this last section, I will much more briefly compare the VMT account to nonlinguistic accounts of mereological vagueness.

According to epistemic accounts of vagueness, apparently vague claims are in fact either definitely true or definitely false—they seem vague only because we have no way of knowing whether they are true or false.[32] Thus, the proponent of an epistemic account would say that parthood claims like "H is part of Tibbles" and "Fred's artificial heart valve is part of Fred" are either definitely true or definitely

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[31] See Doepke (1982), Lowe (1995, 2003) Simons (1987), and van Inwagen (1990).

[32] See Williamson (1994) for an influential recent defense of an epistemic account of vagueness.

false, even though there is no way for us to determine which such claims are true and which are false. The epistemic account assumes in particular that, despite its open-endedness, ordinary usage does manage to pin down a determinate interpretation for the parthood predicate. Any pair of objects, including <H, Tibbles> and <Fred's heart valve, Fred>, is either definitely included in, or definitely excluded from, the extension of the parthood predicate.

I have little to say about the epistemic account except that I do not see why we should believe it. It is true that proponents of the epistemic account may avoid the extreme counterintuitiveness of the VST account. It is consistent with the epistemic account that singular terms generally have unique referents and that ordinary objects are not usually present in overlapping multitudes. Nonetheless, the epistemic account's claim that there is a determinate moment at which a carbon atom becomes part of a cat or a new tire becomes part of a car is extremely counterintuitive. Moreover, as Rosanne Keefe points out, to be persuasive the epistemic account owes us some kind of explanation of the mechanism by which apparently vague terms are endowed with determinate extensions.[33] It is hard to see how such an explanation could go, especially in the case of a term so loosely regulated as is the parthood predicate.

Closer to the spirit of the VMT account are a variety of ontological accounts of vagueness. I take it that at least some of these accounts are, like the VMT account, motivated by the conviction that ordinary objects lack completely determinate parts. A variation on the purely linguistic VMT account of the previous section might embrace both linguistic imprecision and ontic indeterminacy—admitting *both* imprecision in mereological vocabulary and real indeterminacy in the instantiation of properties or relations, including whatever relations are admissible interpretations of the parthood predicate. However, unless there is a clear motivation for this sort of mixed account of vagueness, the purely linguistic version of the VMT account is preferable in that it avoids the metaphysical complications of the ontological accounts.

For the purposes of this brief discussion, I split ontological accounts of vagueness into two groups—accounts which posit individual vague properties or relations and accounts which trace vagueness to a kind of layered precisificational structure of the world as whole.

On the first type of account, it is assumed that certain properties or relations are vague in the sense that they may hold of (or between) objects to intermediate degrees. Notice that on this kind of ontic account, vagueness is attributed *not* primarily to the predicates of a language but rather to the properties and relations which the predicates denote. In the ontic accounts of mereological vagueness proposed by both Peter van Inwagen and Nicholas J. J. Smith, the parthood relation is represented as a mapping from pairs of objects (at times) to real numbers in the

[33] See Keefe (2000, pp. 62–84).

interval [0, 1].[34] Such an account of mereological vagueness differs from the VMT account in two important respects. First, it assumes that the ordinary predicate "is part of" has a *unique* interpretation—it denotes a specific vague relation. Second, it assumes that any pair of objects stands (at a time) in the unique parthood relation to a definite degree.

I find both assumptions problematic. Given the considerations raised at the beginning of the previous section, it is hard to see how ordinary usage could narrow admissible interpretations of the parthood predicate to a single relation, even a single vague relation. Why, for example, would the unique parthood relation be one that holds between prosthetic devices and the bodies in which they are implanted to intermediate degrees instead of a relation which restricts (borderline and determinate) parts of organisms to the parts of organic structures? It seems that either sort of interpretation of the parthood predicate is consistent with ordinary use of mereological predicates (though one may fit certain kinds of contexts better than others). Even more problematic is the assumption that the parthood relation holds to definite degrees. I cannot see why this assumption would be any less problematic than the assumption that objects have completely determinate parts. Given our intuition that sentences like "H is part of Tibbles" are neither definitely true nor definitely false, why would we think that sentences like "H is part of Tibbles to degree .782" or "H is part of Tibbles to a greater degree than that to which Fred's heart valve is part of Fred" are either definitely true or definitely false?

In addition, "intermediate degree" accounts are not well suited to accommodate higher-order vagueness—in particular, vagueness in cutoffs between borderline parts and determinate parts. If each object is part of Tibbles to a specific degree, then it is determinate which objects are parts of Tibbles to degrees between 0 and 1 and thus determinate which objects are borderline parts of Tibbles. Notice that the VMT account can accommodate higher-order vagueness by admitting that the metalinguistic term "precisification" is vague—different admissible interpretations of this term allow different ranges of interpretations of mereological terminology. It is indeterminate whether H is a borderline part of Tibbles if on certain very strict interpretations of "precisification", no precisification assigns <H, Tibbles> to the extension of the parthood predicate. (See note 21 above.)

I suggest that an "intermediate degree" account of mereological vagueness would be more persuasive if it were combined with VMT. In Sect. 3.3, I assumed that all candidate interpretations of the parthood predicate are precise—they either definitely hold, or definitely fail to hold, between any pair of objects. But VMT could be combined with an ontic account on which alternative interpretations of

[34]In fact, Smith (2005) uses two different parthood relations, only one of which (*concrete parthood*) holds to intermediate degrees. The other parthood relation (*notional parthood*) is a crisp relation. However, I take Smith's comments in Section 4 of his paper to indicate that he takes *only* the vague relation (concrete parthood) to correspond to the ordinary notion of parthood.

Also, Smith's concrete parthood relation is represented as a mapping that assigns a real number in [0, 1] to each pair of objects at a given time in a given *possible world*. I do not consider variation across possible worlds in this chapter.

the parthood predicate are vague relations. This sort of combined theory would avoid the awkward assumption that objects have their parts to definite degrees. For distinct precisifications *i* and *j*, H might stand in $\mathbf{part}_i$ to Tibbles to degree .782 and in $\mathbf{part}_j$ to Tibbles to degree .603. And Fred's heart valve might stand in $\mathbf{part}_i$ to Fred to degree .8, but in $\mathbf{part}_j$ to Fred to degree .5 (so it would be indefinite whether H is "more" a part of Tibbles than the artificial heart valve is a part of Fred). Further, the proponent of the combined account could, like the proponent of the purely linguistic VMT account, accommodate higher-order vagueness by citing imprecision in metalinguistic terms like "precisification."

We must wonder, however, to what extent such a mixed account would be an improvement on the purely linguistic version of the VMT account. I do not see that it is. The proposal that objects have their parts to varying degrees does seem appealing in contexts where we consider similar pairs of objects in terms of a single continuously varying feature. For example, let H* be a hair which is attached to Tibbles somewhat more firmly than is H. It is natural to think that H* is in some sense part of Tibbles to a higher degree than is H. But this sort of degree comparison does not seem appealing when applied to diverse types of objects pairs—for example, H and Tibbles, on the one hand, and Fred's artificial heart valve and Fred, on the other—especially when different sorts of features seem relevant to parthood ascriptions. Also, as proponents of linguistic accounts have pointed out, if desired, an assignment of degrees could be made *relative to a fixed finite set* ***P*** *of precisifications* by, for example, assigning to "H is part of Tibbles" the ratio of the cardinality of $\{j: j \in \boldsymbol{P}$ and H stands in $\mathbf{part}_j$ to Tibbles$\}$ to the cardinality of $\boldsymbol{P}$.[35] My tentative conclusion is that the metaphysical cost of admitting relations which hold of objects to intermediate degrees is not compensated by any particular advantages such relations offer an account of vague mereological claims.[36] Thus, I take the purely linguistic VMT account to be preferable both to van Inwagen's or Smith's original ontic accounts and to the mixed account which endorses VMT but allows that candidate parthood relations are vague.

A different kind of ontic account offers a more holistic picture of vagueness in the world. Instead of tracing vagueness to individual vague properties and relations, the second type of account proposes that the world as a whole may be represented through variations on Kripke models in which the multiple distinct worlds of the model correspond to precisificational layers, or dimensions, of the actual world. Different examples of this approach are developed in Akiba (2004) and Barnes and Williams (2010). On both accounts, the loose hair H may be part of Tibbles at some precisificational layers but not at others and thus neither determinately part of Tibbles nor determinately not part of Tibbles.

[35] See, for example, Keefe (2000).

[36] I leave open the possibility that vague properties and relations may be better suited for accounts of other forms of vagueness.

I do not have the space to discuss these holist accounts in detail—they are rather complicated in themselves and significantly different from one another. I confine myself to two very general comments on how this second kind of account of ontic vagueness relates to the VMT account.

First, given *any* linguistic account of vagueness (including the VMT account) and a fixed set $\boldsymbol{P}$ of precisifications, we may represent the actual world (at a given time) through a Kripke model in which each world w_i corresponds to a precisification $i \in \boldsymbol{P}$. A sentence *s* is true at w_i iff s is true of the actual world (at the fixed time) under precisification *i*. (Such models are a simplification of the more elaborately structured "precisification point" models developed by Fine in 1975.) For example, in VMT-friendly "precisification world" models, the sentence "H is part of Tibbles" is true at w_i iff H stands in the $\mathbf{part}_i$ relation to Tibbles. According to a linguistic account of vagueness, these alternate worlds correspond *only* to alternate ways of interpreting our language in relation to the actual world. They do not represent alternate precisificational dimensions of the actual world as in Akiba (2004), alternative indeterminately obtaining maximal states of affairs,[37] or any other sort of divided structure *in the world*. The central question for a comparison of a linguistic account of vagueness with such ontic accounts is whether, in addition to (or instead of) the multilayered metalinguistic structure resulting from alternate interpretations of our language, there is also some kind of multilayered precisificational structure in the world itself. I think that if we can get by without positing a complicated and elusive precisificational structure in the world, we should.

On the other hand, as with the "intermediate degree" accounts, VMT might be combined with one of the holistic ontic accounts. For example, one might hold that it is indeterminate which maximal state of affairs obtains, but claim that the alternate candidate states of affairs involve the precise relations which are alternate admissible interpretations of mereological predicates (and other vague vocabulary). In other words, a proponent of this combined view might admit that there is no unique interpretation of the parthood predicate, but claim that, even given a precise interpretation of this predicate, it may be indeterminate whether certain states of affairs involving the candidate parthood relation obtain. For example, given precisification *j*, it may be indeterminate whether the state of H's standing in the $\mathbf{part}_j$ relation to Tibbles obtains. I do not see any reason to definitely rule out such a mixed account. But again, it is not obvious exactly what advantage is to be gained through its more complicated metaphysical commitments. In particular, while it is clear what reasons we have for thinking that certain parthood claims are neither definitely true nor definitely false, it is not obvious why we should think it is indeterminate whether states involving precise relations like $\mathbf{part}_j$ obtain.

[37]By an "indeterminately obtaining" maximal state of affairs I mean a maximal state of affairs *S* such that it is indeterminate whether *S* obtains. If I understand correctly, the Kripke models introduced in Barnes and Williams (2010) are intended to represent alternative indeterminately obtaining (maximal) states of affairs.

3.5 Conclusion

It is sometimes assumed that the proponent of a purely linguistic account of vagueness must either deny that mereological claims are vague or endorse the counterintuitive claims of the VST account.[38] The primary purpose of this chapter has been to propose an alternative linguistic account of vague mereological claims. Because the VMT account traces mereological vagueness primarily to imprecision in mereological predicates, the proponent of the VMT account may reject the VST account's charge of pervasive imprecision in singular terms as well as its assumption that ordinary objects are present only in overlapping multitudes.

The VMT account is especially appealing in light of the open-endedness of the linguistic conventions surrounding mereological vocabulary. In actual practice, we use both singular terms and common nouns frequently and mostly agree on their proper application in our descriptions of the world. This is not so for mereological predicates. Ordinary use of mereological vocabulary is relatively rare and is usually confined by specific practical goals. Nothing in the ordinary use of mereological predicates determines how these terms might apply in many nonparadigmatic cases. In general, we seem to have much better reason to trace imprecision to mereological predicates than to ordinary singular terms and common nouns. The assumption behind the VMT account is that we do mostly succeed in precisely designating ordinary objects. It is in describing the structure of these objects that our vocabulary fails to distinguish among the many precise relations which equally satisfy whatever loose restrictions are embedded in our vague notion of parthood.

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[38] See, for example, Smith (2007) for an explicit statement of this assumption.

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Part II
Varieties of Ontic Vagueness

Chapter 4
Vague Objects in Quantum Mechanics?

George Darby

4.1 Introduction

There are some connections between the "vague objects" literature and some themes in the philosophy of quantum mechanics. In very general terms these have to do with considerations of identity and indiscernibility as they apply to quantum particles of the kind considered in French and Krause (2006). More specifically, a strand of the discussion of Evans's argument against vague identity concerns the claim that quantum particles present a counterexample to Evans (see Lowe 1994, 1999, 2001; Noonan 1995; Hawley 1998; Odrowaz-Sypniewska 2001). Recent theorists of "metaphysical indeterminacy" in general have also pointed to this kind of case as an example of a target for their analysis (Williams 2008a).

Darby (2010) argues that (a) if these examples of quantum particles are to relate to Evans's argument and the like, then they have to be understood as involving *vague* identity, as opposed to, say, the *meaninglessness* of identity for such particles, and that it is not clear that this is the case, and (b) there are also potential challenges to certain ways of thinking about metaphysical indeterminacy of property instantiation (as opposed to indeterminacy of identity specifically) that arises from the structure of the properties in quantum mechanics that are candidates for indeterminacy. Skow (2010) also raises issue (b). In this paper I would like to consider things from a different direction, in particular three things.

The treatments of the examples in the *Analysis* thread are somewhat different to the approach of philosophers of physics such as French and Krause. The first aim here, then, is to fill in some of the details. One of the things missing from the metaphysical account is that while there is a quite detailed discussion of the ins and outs of Evans's assumptions, there is little mention of the details of the physical

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states concerned. So I analyse the examples used by Lowe et al. with an eye on their physical realisations. Some of these can be validated, in particular Lowe's idea of "representational indeterminacy with an ontic source" (Lowe 2001, p. 242). I am less convinced that the other readings can be squared with a quantum-mechanical description.

This talk of a gap between physics and metaphysics suggests methodological points, so the second aim is to think about these in context. That gap might be taken as motivation for a more naturalistic metaphysics of the kind advocated by Ladyman et al. (2007): if the descriptions in the metaphysics literature don't square with physics, adopt a more purely physics-first approach. Others, such as French and McKenzie (2012), are more reconcilliatory: they suggest that analytic metaphysics may furnish machinery which they can harness to solve conceptual challenges that confront them *qua* philosophers of physics. One might then think that implicit in (1) is an answer to Ladyman and Ross: the way in which metaphysicians theorise about vague objects *can* in some cases be seen to fit with the right way of thinking about the physics, and one might take this to support French and McKenzie's outlook. But all the support in this example flows in one direction: from physics to metaphysics. It's not as though the metaphysical description is actually used to clarify the physical situation. This is especially so given that (a) the example was only a derivative kind of ontic indeterminacy and (b) it relies, as we will see, on an interpretation of quantum mechanics as involving metaphysical indeterminacy of another kind (indeterminacy of property instantiation rather than identity) that is itself problematic.

That being the case, I would, as a third aim, like to think about an instance where influence is reckoned to flow from metaphysics to physics. In particular, it is a case where metaphysics is said to constrain physics. As noted above, French and Krause (2006) discuss things in the ballpark of the vague objects literature. Indeed, this tradition is explicitly connected to Lowe's way of thinking in French and Krause (1995). There are some differences – for French and Krause, identity, rather than being vague, simply doesn't apply to quantum particles. But there are also similarities: while Evans's argument doesn't threaten French and Krause (because vagueness, rather than inapplicability, of identity is crucial for Evans), other arguments against vague objects do. In particular, Smith (2008) argues specifically against Krause's way of setting up a "quasi-set" theory of the kind used in French and Krause (1995).

In keeping with the naturalistic attitude, I argue that Smith is too conservative here: the constraints on understanding he advocates are driven by everyday examples such as clouds and the like. Since these are objects of a kind – "medium-sized dry goods" perhaps – to which identity clearly does at least sometimes apply, it is hard to motivate a reply to his argument that the denial of determinate identity makes no sense. But these considerations don't carry over to fundamental physics.

This is a reversal of the methodological point occurring elsewhere in the literature on metaphysical indeterminacy and physics. The theory of metaphysical indeterminacy advocated by Barnes and Williams (2011) appears to be at odds with certain interpretations of quantum mechanics, as is pointed out by Darby (2010) and Skow (2010). Skow in particular draws very specific conclusions: if Barnes and Williams' account of metaphysical indeterminacy fails to fit a possible interpretation of quantum mechanics, then that account is false. One might instead conclude: so much the worse for that particular interpretation of quantum mechanics.[1]

In the case of Smith, French and Krause, the objection is firmly that way round (perhaps because identity is so central): the interpretation of quantum mechanics *is* rejected on metaphysical grounds as incoherent; it is this that I argue that a naturalist oughtn't find that persuasive. There are, however, other worries about Krause's theory, which I'll take to motivate an alternative challenge along Smith's lines.

4.2 Vague Identity: Evans's Argument and the *Analysis* Thread

This section is concerned specifically with the question of whether *vague objects* are to be found in quantum mechanics in roughly the sense that is intended in the mainstream metaphysics literature. The focus will be on Evans's argument that ontically indeterminate identity is incoherent and on the idea that quantum particles, between which there is indeterminacy of identity, present a counterexample to Evans. The focus is therefore not so much on the general thesis that objects are ontically vague as on the apparent corollary that there might be ontically indeterminate identities.

4.2.1 Evans's Argument

Evans expresses indeterminacy using the sentential operator ∇ – 'it is indeterminate whether ϕ' is expressed by $\nabla\phi$. As is standard he considers ∇ a *modal* operator, suggesting at one point that its logic is S5 (which raises the question of whether that is true – see, for example, Pelletier (2006)).

Evans's point is that an absurd result can be derived from the assumption $\nabla(a = b)$, which a believer in ontic vagueness may be supposed to endorse:

1. $\nabla(a = b)$
   It is assumed to be indeterminate whether a and b are identical

[1]The account in question has to be a very particular one, after all: quantum mechanics apparently involves worldly indeterminacy; that is an interpretative problem for quantum mechanics to be solved either by giving a philosophical account of worldly indeterminacy or by coming up with another interpretation of quantum mechanics. Amongst interpretations of quantum mechanics that do require an understanding of worldly indeterminacy, some involve the particular kind of indeterminacy that Skow argues refutes the Barnes-Williams model, and some do not. At this point, why reject the Barnes-Williams model instead of concluding that those particular interpretations of quantum mechanics are indeed impossible?

2. $\hat{x}[\nabla(a = x)]b$
   From (1), by property abstraction, it follows that b has the property of being something whose identity to a is indeterminate.
3. $\sim\nabla(a = a)$
   Whether or not it is indeterminate whether a is identical to b, it is surely not indeterminate whether a is identical to a.
4. $\sim\hat{x}[\nabla(a = x)]a$
   From (3), a lacks the property of being something whose identity to a is indeterminate.
5. $\sim(a = b)$
   a and b are therefore *not* identical – this follows from (2) and (4) via a principle like $(a = b \wedge Fa) \rightarrow Fb$.

At the last step, Evans refers to "Leibniz's Law", which raises questions about exactly which law is involved, whether it contraposes, and so on. It is also unclear how to get to outright contradiction, as Evans suggests follows. But the existing conclusion seems to be enough to undermine vague identity – if indeterminate identities collapse to straightforward nonidentity, then there can be no such thing as the former.

Since it clearly *is* coherent to have vague statements of identity, because not all terms are precise, Evans's argument has to be restricted somehow to putative *ontic* vagueness only. On the reading of Lewis (1988), the ontic nature of the indeterminacy is to be captured by stipulating that 'a' and 'b' are *precise designators*. A precise designator is like a rigid designator, picking out the same objects on all ways of making the language precise – the point is that it is not just that 'Everest' picks out one object on one precisification and a slightly different object on another, which is obviously possible but obviously representational rather than ontic; instead even if 'Everest' is taken to pick out the same object in all precisifications, the idea is that certain statements may still be indeterminate in truth-value because Everest itself is vague.

4.2.2 The Connection with Quantum Mechanics

The line that I want to concentrate on here begins with Lowe (1994), who attempts to show that quantum mechanics provides a counterexample to Evans's argument. His example concerns an electron absorbed by an atom at some time, and the idea is that it is indeterminate whether that electron is identical to one later emitted. Lowe (2001, p. 241) contends that his description of the situation as one involving vague identity (i.e. one that is appropriately captured by $\nabla(a = b)$) is, despite the objections of Noonan (1995), Hawley (1998) and Odrowaz-Sypniewska (2001), *coherent*, and in Lowe (1994) he offers more than one possible diagnosis of the fault in Evans's argument.

The logical and metaphysical core of Lowe's escape from Evans's argument, aiming to establish the *coherence* of a description of vague objects, does not obviously depend on anything to do with quantum mechanics. Although Lowe resists applying it to a more everyday case (Lowe 1997, p. 89), it might be so used if required. We are not concerned here with the escape itself, so much as the motivational role of physics: if we find a case from quantum mechanics exemplifying what the believer in ontic vagueness intends by $\nabla(a = b)$, then we know that there is *something* wrong with Evans's argument. So the key question, if quantum mechanics is to support a thesis of worldly vagueness in the form of vague objects, must be whether such a description *is* supported by physics.

The idea that identity is *problematic* when it comes to fundamental particles is certainly found in the history of quantum mechanics and is treated in a formal theory by French and Krause, as above. Our concern here is whether either account really does give us vague objects in the sense that is intended, so to see whether either Lowe's, or French and Krause's examples provide clear-cut cases that are both supported by physics and count as indeterminacy of identity in Evans's sense.

To provide an example that can be captured by $\nabla(a = b)$ is to give a direct kind of answer to Evans, to claim that the assumption $\nabla(a = b)$, with a and b precise, is coherent, as evidenced by the example, so there must be something wrong with the argument. Alternatively, one might instead reply indirectly by rejecting the terms in which Evans frames the problem – principally, capturing the indeterminacy using the ∇ operator. Lowe canvasses this route, and although there is nothing about his proposed example that seems to fit any less well with the direct reply, in fact expresses a preference for the indirect one. It might be that that kind of indirect reply to Evans (which also fits the scheme of French and Krause) is better supported by physics. However, whether we find a direct reply to Evans in quantum mechanics seems an interesting question (the answer is certainly not *obviously* no), so it is the one I shall pursue here. In any case, I am not convinced that the required kind of indirect reply – that there *is* indeterminacy of identity in quantum mechanics but that it is *not* properly represented by the ∇ operator – is motivated by the physics either (see Sect. 4.3).

This is in keeping with the general naturalistic attitude to metaphysics: it is not enough that there be things in quantum mechanics that *could* be described as "vague objects". It is necessary that they be describable as instances of indeterminate identity, in Evans's sense. Or at least *roughly* Evans's sense – there is room for manoeuvre about whether the terms flanking the identity sign are precise designators or not; it is permissible for them not to be, as long as the referential indeterminacy has an ontic source (the reason it was thought that they should be precise was to ensure that the indeterminacy was not merely semantic, so if that is ensured by some other means, then all is well). Alternatively, it is enough to have an example of indeterminacy of identity, even if that is *not* to be captured by the ∇ operator. But it is important that the example be one of *indeterminate* identity.

4.2.3 A Minor Worry

In the case of indeterminacy of property instantiation, Darby (2010) worries about the difference between vagueness and mere indeterminacy – one might insist that genuine vagueness involves not just indeterminacy but sorites susceptibility or similar. That is in the case of vague predicates and property instantiation, one might or might not be able to squeeze the required feature out of, say, a bell-shaped wave function with the appropriate gradual increase in intensity, but the same seems to go for singular terms. Supposing that 'Everest' is indeterminate in reference between various objects, much the same things involving gradation can be done as can be done with the various candidate regions on the colour sphere for the reference of 'blue'.

Swapping vague singular terms for vague objects, there is a similar issue. In the kind of case envisaged by Evans (see the opening paragraph of Evans 1978), there are two objects, one or more of which has "fuzzy" boundaries, *because* of which there is an indeterminate identity. Instead of "$\text{Everest}_1 = \text{Everest}_2$" being indeterminate because it is undecided precisely which objects 'Everest_1' and 'Everest_2' pick out, it is supposed to be indeterminate because one or both of the things they pick out are in some way fuzzy. If we have an (even ontically) indeterminate identity, on the other hand, it might be that the source is completely different, and that seems to be the case in the quantum-mechanical examples – there is simply nothing close to a sorites series apparent there.

This does seem unanswerable – if you think that series-type features are required, or that talk of 'fuzzy boundaries' has to be appropriate to the example, then there is just no way that these quantum-mechanical cases will count, because they have nothing to do with series, just (allegedly) indeterminacy. However, the distinction between indeterminacy and vagueness is often ignored in the literature on vague objects, and I propose to do the same, following, for example, Sainsbury (1989) and Akiba (2004, p. 408) notes the difference but then proposes to use 'vagueness' and 'indeterminacy' interchangeably. The literature on Evans's argument focuses almost exclusively on indeterminacy, rather than vagueness (not unjustifiably, since indeterminacy of any kind in the world is remarkable).

In any case, indeterminacy will do to motivate a diagnosis of a fallacy in Evans's argument. If that is the goal, then it does not matter whether the source of the indeterminate identity is the objects' having fuzzy boundaries, or something quite different; either way, something must be wrong with the argument if we have a genuine example of $\nabla(a = b)$. And this worry, that we *only* have indeterminacy, is completely undercut by the second, that we do *not* have indeterminacy, and so the rest of this paper concerns the question of whether there are, in quantum mechanics, what may be thought of as cases of indeterminate identity. I will focus, more or less independently, first on Lowe's example and then on that of French and Krause. In each case, we might ask:

- Do we have an example of indeterminacy of identity, properly represented by $\nabla(a = b)$, where 'a' and 'b' are precise designators?

- Do we have an example of indeterminacy of identity, properly represented by $\nabla(a = b)$, where, while it may not be that 'a' and 'b' are precise designators, the indeterminacy does have an ontic source?
- Do we have an example of indeterminacy of identity, albeit one that is not properly represented by $\nabla(a = b)$?

4.2.4 Summary of the Analysis Thread

Lowe (1994) describes the following situation: an electron a is absorbed by an atom, and at some later time, an electron b is emitted. After absorption, a is 'entangled' with the other electrons in the outer shell of the atom. This makes it the case that the identity of each a with any of the electrons in the resulting negative ion is indeterminate. Before emission, b is one of the electrons in the ion. Therefore, it is indeterminate whether a is identical to b.

Lowe then offers a couple of diagnoses of the error in Evans's proof. One response is to reject the terms of Evans's argument, to say that the property of being an object whose identity to a is indeterminate is not a genuine property and that indeterminacy facts are not properly captured by ∇. Since the primary aim here is to engage as closely as possible with the literature on Evans's argument, where the use of that operator is standard, I will postpone consideration of this response to Sect. 4.3. Lowe too, while expressing a preference for this response, allows Evans's way of thinking and offers an alternative response: the property $\hat{x}[\nabla(x = a)]$ is supposed by Evans to differentiate a and b, since b is supposed to have the property while a lacks it. But since it is indeterminate whether a and b are identical, that property is not determinately distinct from the property $\hat{x}[\nabla(x = b)]$, which a *does* have, and so cannot after all differentiate a and b.

Noonan (1995) appears to accept Lowe's response to Evans's original argument but offers a new one, this time focusing on properties that do not involve identity. Since 'a' refers to the electron whose identity to the emitted electron is indeterminate, a has the property of being indeterminate as to emission; 'b' on the other hand refers to the emitted electron, so b cannot have the property of being indeterminate as to emission, then the rest of the argument is as before. (The argument Noonan actually sets out is for a different putative case of indeterminate identity and uses a more complicated Leibnizian principle than Evans's original; this seems to be a faithful representation of the key point, however):

1. $\nabla(a$ is emitted).
2. So $\hat{x}[\nabla(x \text{ is emitted})]a$.
3. But $\sim\nabla(b$ is emitted).
4. So $\sim\hat{x}[\nabla(x \text{ is emitted})]b$.
5. Therefore, $a \neq b$

Lowe's initial response to Noonan is that the properties being attributed to a and b must be treated as tensed and that:

> Granting that there is a property that is assignable to a in virtue of the fact that at t_1 it was indeterminate whether a had been emitted from the atom, we can nonetheless see that there is no reason to suppose that this property is determinately distinct from the property that is assignable to b in virtue of the fact that at t_0 it was indeterminate whether b was going to be emitted from the atom. (Lowe 1997, p. 91)

So Noonan's argument, properly spelled out as Lowe requires, becomes something like:

1. At t_1, $\nabla(a$ has been emitted).
2. So $\hat{x}[\text{at } t_1, \nabla(x \text{ has been emitted})]a$.
3. But at t_1, $\sim\nabla(b$ has been emitted).
4. So $\sim\hat{x}[\text{at } t_1, \nabla(x \text{ has been emitted})]b$.
5. Therefore, $a \neq b$.

However, the property $\hat{x}[\text{at } t_1, \nabla(x \text{ has been emitted})]$, which supposedly a has but b does not, is not determinately distinct from the property $\hat{x}[\text{at } t_0, \nabla(x \text{ will be emitted})]$, which both a and b do have, and cannot therefore differentiate between them.

Hawley (1998) denies that b has the property $\hat{x}[\text{at } t_0, \nabla(x \text{ will be emitted})]$. If b fails to have that property, then Lowe's response is unavailable. Accordingly, Lowe now considers the response that the term 'b' is imprecise; it is then "simply misconceived to ask whether b has a property which a lacks" (Lowe 1999, p. 329). That avoids all the threat of Evans-type arguments, which assume the terms to be precise. Of course it also threatens to undermine the ontic nature of the indeterminacy, but need not necessarily do so, as long as the indeterminacy has an ontic source.

To this, Odrowaz-Sypniewska (2001) raises the following objections:

1. If either of the terms 'a' and 'b' is imprecise, then there is no reason for thinking that the indeterminacy of identity is ontic.
2. If there are determinately two electrons, then they must be determinately distinct.
3. It is a consequence of Lowe's position that the electrons must be determinately qualitatively identical.

To which Lowe (2001) replies:

1. That there is referential indeterminacy in one of the terms 'a' and 'b' does not automatically mean that there is no ontic indeterminacy.
2. "Determinate countability and determinate identity do not necessarily go hand in hand" (p. 243)
3. There is a crucial difference between no property being assignable to one of the electrons *rather than* to the other, and no property being assignable to one of the electrons *and not* to the other; Lowe's distinction might be appropriately similar to the difference between *Determinately*(Fa) if and only if *Determinately*(Fb)

and *Determinately*(*Fa* if and only if *Fb*); only the latter would make the electrons determinately qualitatively identical.

Lowe (1998) gives a different response to Noonan. He says that one should consider the times at which a and b supposedly have the properties that are to distinguish them and that once that is done the problem disappears: for example (Lowe considers the property of indeterminacy as to absorption, but the strategy is the same), before absorption a has the property of being indeterminate as to emission, and after emission b lacks that property, but *before* absorption b has that property too, and after emission a lacks it. Now Noonan's argument ought to be something like:

1. At t_1, $\nabla(a$ has been emitted).
2. So at t_1, $\hat{x}[\nabla(x$ has been emitted)$]a$.
3. But at t_1, $\sim\nabla(b$ has been emitted).
4. So at t_1, $\sim\hat{x}[\nabla(x$ has been emitted)$]b$.
5. Therefore, $a \neq b$.

But Lowe regards (2) as false. This, says Lowe, allows us to regard 'b' as a precise designator at the time of emission, though not earlier (Lowe 2001, p. 244).

4.2.4.1 Two Options

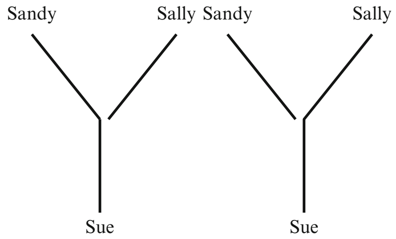
What I want to do now is to consider in a bit more detail what the account of the interactions of the particles is that underlies this example of indeterminacy of identity, in two different ways:

1. The indeterminacy of identity results from referential indeterminacy in one of the terms 'a' and 'b'. Lowe claims that that does *not*, however, mean that there is no ontic indeterminacy, while Odrowąż-Sypniewska demands reasons for thinking that the ontic explanation is appropriate. I think that such reasons can be given in the form of a picture where the indeterminacy of reference is perfectly transparent, where there is nevertheless ontic indeterminacy and which is supported by the physics. Although Lowe says that he was "only ever concerned to defend the coherence of the ontic explanation against the arguments of those who maintain that it is incoherent" (Lowe 2001, p. 242), it is surely desirable to see in detail how the referential indeterminacy surfaces, and so that is what I will try to do in Sect. 4.2.5. I am not quite sure that Lowe would accept this as an account of precisely his example, but it seems to me to fit his aim of referential indeterminacy with an ontic source.
2. It might be objected, however, that while there is indeterminacy of identity with an ontic source, in that picture the ontic source is not itself ontic indeterminacy of *identity*. There is therefore reason to prefer the alternative view, on which both 'a' and 'b' are precise, and so in Sect. 4.2.6, I will ask whether we can insist that there is no referential indeterminacy but still have indeterminate identity.

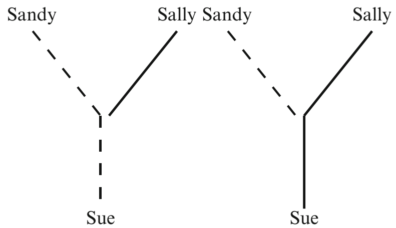
4.2.5 Option 1: One or Both of 'a' and 'b' Are Imprecise

It is in Lowe (1999) that Lowe suggests that the designator '*b*' is imprecise. Lowe envisages a pair of (at this point determinately distinct) electrons that become entangled and, therefore, become not determinately distinct. Lowe then suggests that "once the particles have become entangled, that they can never thereafter become determinately distinct again" (p. 328). He doesn't specify the states involved, but one might elaborate this in two ways. One way would be say that entanglement wipes out distinctness once and for all, and then the question is why that should be, the mechanism by which this happens. The other way, and what seems to be Lowe's intention, would be say that determinate distinctness comes and goes with entanglement, so particles in a non-entangled state of the form $c_1|\psi_1\rangle \otimes c_2|\psi_2\rangle$ are distinct, and it is then possible (it happens often, in fact) for their state to evolve to an entangled one of the kind $c_1|\psi_1\rangle \otimes |\psi_2\rangle - c_2|\psi_2\rangle \otimes |\psi_1\rangle$. Then the question would be whether their state can evolve back to an unentangled one, and the issue would be that this requires evolution of the state other than by the linear Schrödinger equation. But of course whether *that* happens depends on the details of a solution to the measurement problem, and so the viability of Lowe's story would depend very heavily on details of the interpretation. In the following example, I try to spell out Lowe's main idea without getting into such contentious details.

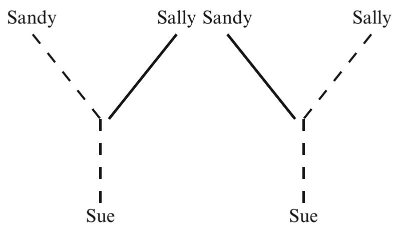
Williams (2008b) offers an account of metaphysical indeterminacy whereby more than one ersatz possible world may be actualised (or not determinately unactualised). It is then indeterminate whether p if there is an ersatz world that is, so to speak, a candidate for actuality and where p, and an ersatz world that is a candidate for actuality where $\sim p$. Here Williams is applying to indeterminacy of identity an account of metaphysical indeterminacy in general developed by Barnes and Williams (2011). Darby (2010) and Skow (2010) ask whether this account can be successfully married with an understanding of quantum mechanics according to which, for example, it is indeterminate whether Schrödinger's cat is dead or alive; the situation is not clear-cut (it depends on details of one's preferred solution to interpretive issues such as the measurement problem), but for present purposes we can accept that quantum mechanics supports indeterminacy of property instantiation as understood on the Barnes-Williams model and see how this generates indeterminate identity. Williams presents the following story. Sue the amoeba divides at some time, and one of the daughter cells, call it Sandy, moves off to the east; the other daughter cell, call it Sally, moves off to the west. Williams wants to capture the thought that it is indeterminate whether Sue, the original amoeba, survives as (is diachronically identical to) Sally or Sandy. He therefore invokes the following picture, in which there is a world where there is a surviving amoeba travelling west and a newly created amoeba travelling east and a world where there is a surviving amoeba travelling east and a newly created amoeba travelling west:



There are now at least two ways to think about the transworld identity facts in this situation. It might be that the eastbound amoeba in the first world is identical to the eastbound amoeba in the second world and that the westbound amoeba in the first world is identical to the westbound amoeba in the second world:

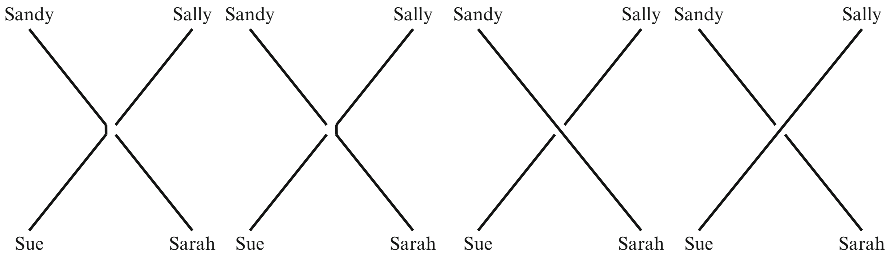


Or it might be that the newly created amoeba in the first world is identical to the newly created amoeba in the second world and that the surviving amoeba in the first world is identical to the surviving amoeba in the second world:



Working with the second way of fixing the transworld facts (as Williams does, though similar results are obtained either way round), there is referential indeterminacy: it is indeterminate whether the name "Sandy" refers to the surviving amoeba or the newly created one; since the name "Sue" determinately refers to the surviving amoeba, it is indeterminate whether Sue is identical to Sally. Although referential, the indeterminacy has an ontic source: it is ontically indeterminate whether an object (Sue) has a certain property (travelling east); the name "Sandy" is introduced via a description as having that property, and so it is indeterminate whether Sue = Sandy.

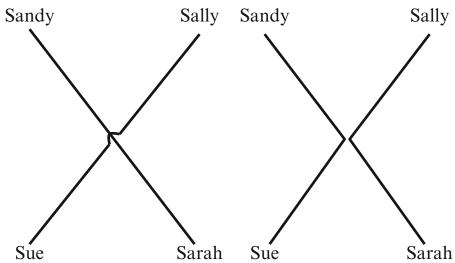
Now, I can see no reason why we cannot alter that picture as follows. This time, we have two amoebas, Sue and Sarah, that approach each other from the north and south. They then merge, and shortly afterwards there is a fission of the same kind as appears in Williams' example, with Sandy going off to the east, Sally to the west, as before. (This may or may not be entirely fictional – there are processes that go under the heading "cell fusion" – but in any case, it is not all that clear that the original splitting example is compelling either. A lot seems to hang on the symmetry of the situation there, such as whether original (token) DNA molecules are shared between the two daughters, which is plausibly indeterminacy of identity, or passed on to a particular one, which is most plausibly not indeterminacy. What matters of course is just the coherence of the account.) One way to picture this is:



In this picture, there are four worlds:

1. Sue survives as Sandy, Sarah ceases to exist, and Sally is created.
2. Sarah survives as Sally, Sue ceases to exist, and Sandy is created.
3. Sarah survives as Sandy, Sue ceases to exist, and Sally is created.
4. Sue survives as Sally, Sarah ceases to exist, and Sandy is created.

There is, however, another way of drawing the picture:

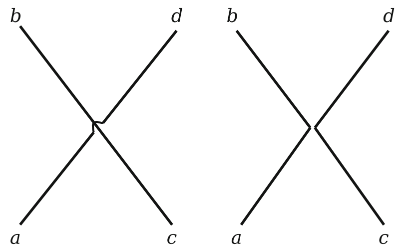


Here, there are two worlds:

1. Sue survives as Sandy, and Sarah survives as Sally.
2. Sue survives as Sally, and Sarah survives as Sandy.

The terms "Sally" and "Sandy" can be thought of as having their reference fixed via the description "the amoeba that travels west (east)", much as Lowe introduces the term "b" via the description "the emitted electron".

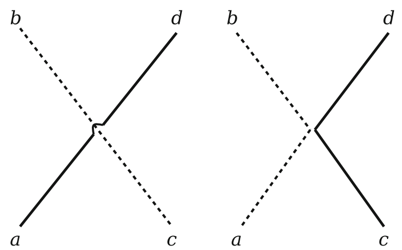
Now, we want to apply this to Lowe's example (he has an ion absorb an electron, a, and later emit one, b, such that it is indeterminate whether $a = b$): call the absorbed electron 'a', the emitted electron 'b', an electron already in the atom before absorption 'c', and an electron remaining in the atom after emission 'd'. Since this doesn't sound like a case of particles being created or destroyed, it looks better to go with the second picture. So we have:



In world 1 we have an absorbed-non-emitted electron and a non-absorbed-emitted electron. In world 2 we have an absorbed-emitted electron and a non-absorbed-non-emitted electron.

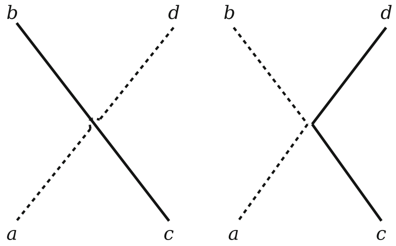
Here, it is indeterminate whether a is identical to b and whether c is identical to d. It is determinate that a and c and b and d, are not identical. It is indeterminate whether a is emitted, whether c is emitted, whether b is absorbed and whether d is absorbed. It is determinate that either a or c is emitted and that either b or d is absorbed. It is determinate that a is absorbed, that b is emitted, that c is not absorbed and that d is not emitted. This, including the indeterminacy of identity between a and b, appears even before fixing the transworld identity facts, which can be done in two ways:

- Say identity goes by emission/non-emission. So the absorbed-non-emitted electron of world 1 is identical to the non-absorbed-non-emitted electron of world 2, and the non-absorbed-emitted electron of world 1 is identical to the absorbed-emitted electron of world 2:



Here, "b" and "d" are referentially determinate; "a" and "c" are referentially indeterminate.

- Alternatively, say identity goes by absorption/non-absorption. So the absorbed-non-emitted electron of world 1 is identical to the absorbed-emitted electron of world 2, and the non-absorbed-emitted electron of world 1 is identical to the non-absorbed-non-emitted electron of world 2:



Here, "a" and "c" are referentially determinate; "b" and "d" are referentially indeterminate.

Either way, one or other of "a" and "b" is referentially indeterminate. So we have a case of vague identity – referential indeterminacy induced by ontic indeterminacy – similar to Williams' example. Although it does not depend on the transworld identity facts that it is indeterminate whether $a = b$, the source does so depend: in one case it is because 'b' refers precisely to something such that it is determinate that it is emitted but indeterminate whether it is absorbed while 'a' refers imprecisely to whatever is absorbed; in the other case it is because 'a' refers precisely to something such that it is determinate that it is absorbed but indeterminate whether it is emitted while 'b' refers imprecisely to whatever is emitted.

While this is, as in Lowe (1999), a case of an imprecise designator, there are a number of things that don't fit, which makes me wonder whether Lowe should accept it as a version of his account:

'a' here is perfectly precise. In Lowe (1999), where he suggests that 'b' is an imprecise designator, Lowe says that by contrast "the definite description 'the absorbed electron' does determinately pick out a unique entity, at least with respect to the time of absorption, though not with respect to any later time" (p. 329). That is not quite what we have here: 'b' is indeed imprecise, but 'a', fixed via the description 'the absorbed electron', is *always* precise.

Lowe stresses that his example essentially involves tense (cf. French and Krause, who find tense a red herring). As I think will emerge when we consider the physical description of the situation, tense is inessential here. It seems inessential too on Williams' picture: say there is a world where a is F and G and b is neither F nor G and a world where a is G but not F and b is F but not G. Then we have indeterminate identities like $\iota x Fx = \iota x Gx$.

One key feature of Williams' account is that it goes with Evans's modal operator understanding of indeterminacy, which Lowe expresses a preference against. Currently, however, we are assuming that the aim is to reply directly to Evans, in which case that understanding is essential.

Relatedly, a natural question is whether this kind of picture (ontic indeterminacy inducing referential indeterminacy) depends on the details of Williams' picture, in particular, on the ersatz worlds framework, or whether that is inessential. It seems to be inessential, since all that we need is the following: There are two objects present; it is ontically indeterminate which one is F, though it is determinate that precisely one is; 'a' determinately refers to a particular one of the objects (we can say that it is a 'precise designator', as long as that doesn't imply that we must have the worlds picture in mind); 'b' does not determinately refer to a particular one, but refers to whichever one is F (as when it is introduced via the description "the emitted electron"). Then 'b' doesn't determinately refer to the object that 'a' refers to and doesn't determinately *not* refer to the object that 'a' refers to; so it is indeterminate whether $a = b$. To put it another way, we have $\nabla(Fa)$, $D(Fb)$ and $D(\forall x \forall y(Fx \wedge Fy \rightarrow x = y))$, completely unmysteriously. It is unmysterious that $\nabla(Fa)$ since 'a' refers determinately to a particular object, but whichever object it refers to, it is indeterminate whether that object is F. It is unmysterious that $D(Fb)$ since 'b' refers to whichever object is F (so has to be unsettled in reference since which object is F is unsettled). It is unmysterious that $D(\forall x \forall y(Fx \wedge Fy \rightarrow x = y))$ since that just says that, definitely, no more than one thing is F. But now we can infer $\nabla(a = b)$. From

1. $D \sim (a = b)$
2. $D(Fb)$
3. $D(\forall x \forall y(Fx \wedge Fy \rightarrow x = y))$

we can infer $D(\sim Fa)$, and from

1. $D(a = b)$
2. $D(Fb)$

we can infer $D(Fa)$. So from $\nabla(Fa)$ we can infer $\nabla(a = b)$.

So however we understand ontic indeterminacy of property instantiation, we can use it to generate this kind of indeterminacy of identity, at least as long as indeterminacy is captured by the ∇ operator. (Note again that we are assuming that instantiation of properties in quantum mechanics may be thought of in that way, despite the concerns of Darby (2010) and Skow (2010)).

Lowe (2001), replying to Odrowaz-Sypniewska (2001), reiterates that he thinks that two things may be determinately two, and yet not determinately distinct. It therefore seems that he has to provide some account of how his conception of number differs from the standard one invoked by Odrowąż-Sypniewska. Here, however, that does not seem to be an issue: it is true that it is indeterminate whether the emitted electron and the absorbed electron are identical and that it is determinate that there are two electrons present, but it is indeterminate whether the things referred to by 'the emitted electron' and 'the absorbed electron' are two, so indeterminate number comes with indeterminate identity. If we focus instead on the electron described by the state to the left of the tensor product, and the electron described by the state to the right, that is, on the entities that we pay attention to when fixing transworld identity facts in Williams' picture, then it is both determinate that there are two and determinate that they are distinct.

Do the replies to Noonan and Hawley that Lowe gives still apply on this picture? Noonan's complaint was that *a* has the property of being indeterminate as to emission, whereas *b* does not have that property, which serves to differentiate them after all. *a* does indeed seem to have that property on this picture but, since '*b*' is imprecise, as Lowe says it is "simply misconceived to ask whether *b* has a property which *a* lacks" (Lowe 1999, p. 329). For what it's worth, what we in fact have here is one electron, definitely absorbed but indeterminate as to emission, and another definitely not absorbed but indeterminate as to emission. So whichever '*b*' refers to, it does in fact refer to an object about which it is indeterminate whether it was emitted. So just because it is determinate that *b* is emitted, it cannot be inferred that there is an object *b* that lacks the property of indeterminate emission.

As regards the reply to Hawley, the key point is that on this picture it is correct to say, of any electron, that it is indeterminate whether it is emitted. That goes for all times: considering the electron represented by the dotted line, it is indeterminate early on whether it will be emitted and indeterminate later whether it has been emitted. What of course is not indeterminate is whether *b* will be, or has been, emitted – that is always determinate, but '*b*' is imprecise.

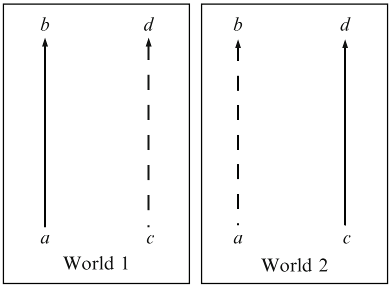
So, this looks like a coherent account of indeterminacy of identity, where the indeterminacy is semantic in the sense of being due to imprecision of one of the terms flanking the identity sign, but with an ontic source in the indeterminacy as to which objects instantiates which property. The key question now is whether the picture is permissible as an account of a real physical situation. I think that it is, and that there are at least three ways in which it may arise. They all rely on assuming that a system in a superposition manifests ontic indeterminacy of the 'indeterminate whether the cat is alive' kind.

4.2.5.1 Way 1

The typical example of an entangled state is of the form:

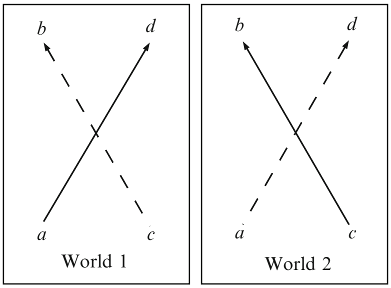
$$\frac{1}{\sqrt{2}}(|0\rangle|1\rangle + |1\rangle|0\rangle)$$

We might take $|0\rangle$ to be the state of an electron that travels from outside the atom, into the outer shell and back outside again and $|1\rangle$ to be the state of an electron that remains in the atom throughout. (Alternatively, $|0\rangle$ could be an initial state that will evolve as required from outside the atom to inside and then out again and similarly for $|1\rangle$.) Then we could draw a picture like Williams':

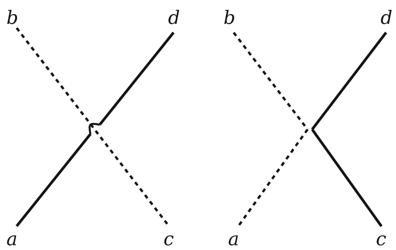


That, however, doesn't look like what we want: both 'a' and 'b' are indeterminate in reference, but in both worlds, they pick out the same thing, so $a = b$ is not indeterminate – it is definitely true.

Alternatively, $|2\rangle$ could be the (initial) state corresponding to an electron outside the atom that is absorbed and then remains in the atom, while $|3\rangle$ is the (initial) state corresponding to an electron that starts in the atom and is then emitted. Then the picture for the superposition $\frac{1}{\sqrt{2}}(|2\rangle|3\rangle + |3\rangle|2\rangle)$ would be



That doesn't look like what we want either: both 'a' and 'b' are indeterminate in reference, but in both worlds they pick out different things, so $a = b$ is not indeterminate – it is definitely false.



The picture we want, corresponds to no standard state of the form $\frac{1}{\sqrt{2}}(|0\rangle|1\rangle + |1\rangle|0\rangle)$. Instead we require a state of the form

$$\frac{1}{\sqrt{2}}(|0\rangle|1\rangle + |2\rangle|3\rangle)$$

Now we have a worry: if the particles in question are fermions, then the state ought to be anti-symmetric, and if they are bosons then it ought to be symmetric. The simplest way around this worry is to suppose that they are neither, but are macroscopic objects treated in a quantum-mechanical way – indeed, we might use the states of the original amoebas.

Note that here there is no need for particles being first entangled, and then not entangled, or vice-versa, as for Lowe – the whole picture assumes a superposition throughout.

4.2.5.2 Way 2

The second way is just the reverse of Lowe's description: Instead of particles that are first unentangled (and hence, for Lowe, determinately distinct), the same referential indeterminacy will be generated for particles that are first entangled and then not. So subscribe to an appropriate collapse interpretation, consider two particles in a singlet state $\frac{1}{\sqrt{2}}(|\uparrow_x\rangle|\downarrow_x\rangle - |\downarrow_x\rangle|\uparrow_x\rangle)$ at time t and introduce 'a' to refer to the particle that is spin-up in the x-direction *at time t*. Now it is indeterminate which of the particles 'a' refers to, since the two electrons are in a singlet state, and neither can be said to be spin-up or spin-down in any direction (again, given an appropriate interpretation of superpositions as involving indeterminacy of property instantiation). At time t' after the wave function collapses, we introduce the term 'b' to refer to the spin-up electron – the one found to be spin-up, the one that determinately *is* spin-up.

(Note that we cannot use Williams' view here, because we need to allow for things being determinate at one time that are not at another. This still counts as "ontically induced referential indeterminacy", however.)

4.2.5.3 Way 3

Both of the above examples introduced the terms like 'a' and 'b' via descriptions like 'the absorbed electron' and 'the emitted electron'. However, if we are using a framework like the 'labelled tensor product Hilbert space formalism' (Redhead and Teller 1992, p. 205), then one of those terms need not be so introduced. Instead, we could introduce the term 'a' to refer to the particle whose state is on the left of the tensor product, for example, in the singlet state $\frac{1}{\sqrt{2}}(|\uparrow_x\rangle \otimes |\downarrow_x\rangle + |\downarrow_x\rangle \otimes |\uparrow_x\rangle)$.

Then, if we introduce 'b' to refer to the particle that is spin-up in the x-direction, and if superpositions are understood as indeterminate, then it is straightforwardly indeterminate whether 'a' and 'b' refer to the same particle, since it is indeterminate which particle is spin-up in the x-direction, so indeterminate which particle 'b'

refers to. Now we have another way of getting referential indeterminacy from ontic indeterminacy of superposition. Again, in this example there is no need for particles moving in and out of an entangled state.

There might, however, be a suspicion here that 'b' is not indeterminate in reference, being supposed to refer to the particle that is spin-up in the x-direction, but refers to neither, since in a superposition *neither particle is truly spin-up* – a difference to Lowe's cases, where the description attaches to a particle that, for instance, *has* definitely been emitted. In the amoeba case, neither determinately went west, so the name introduced via the description "the westbound amoeba" is indeterminate in reference; there is, however, at least an amoeba-stage to attach reference to. This seems to depend on taking the particles to have an indeterminate spin state rather than simply no spin state at all. So again this depends on an interpretation of quantum mechanics such that there is genuine indeterminacy of property instantiation. Darby and Skow, who are otherwise skeptical of the full story, see the "indeterminate spin state" (rather than the "no spin state") reading as at least plausible and interestingly point to the same examples in the literature to back this up – see Skow (2010, Footnote 1) and Darby (2010, p. 235).

I conclude that if we take the route of Lowe (1999) and have either 'a' or 'b' imprecise, then there is a perfectly good quantum-mechanically motivated example of indeterminacy of identity that can be captured by $\nabla(a = b)$. But again, although the reference is indeterminate, and arose because of something non-representational, there is nothing essentially to do with *identity* involved here on the non-representational side.

It will be much better though if we *can* have a and b both precise designators. In the current picture there are determinately two, determinately distinct, electrons: there is no ontic indeterminacy of identity, merely referential indeterminacy and ontic indeterminacy of something else, which may not be enough to satisfy.

4.2.6 Option 2: Both 'a' and 'b' Are Precise

As noted above, there is something not quite satisfying about having 'b' an imprecise designator: while that indeterminacy of reference has an ontic source, it derives from indeterminacy of something other than identity (whether a particular particle travels through the left slit or the right slit or what the location of Sue is after fission). Everything about the identities of the *objects* in these cases is transparently determinate, hence the stipulation in the standard way of interpreting Evans's argument that the terms 'a' and 'b' must both be precise. Can we construct a similar picture to that in the previous section but with 'a' and 'b' precise?

We have Lowe's two escapes from Noonan's argument to employ: firstly, that the "property that is assignable to a in virtue of the fact that at t_1 it was indeterminate whether a had been emitted from the atom" (call this property P_1) is not determinately distinct from the "property that is assignable to b in virtue of the fact that at t_0 it was indeterminate whether b was going to be emitted from the atom" (call this property P_2). Noonan's claim was that a has property P_1 and that b does not have it and that therefore a and b are distinct, whereas Lowe thinks that P_1 cannot differentiate a and b because it is not determinately distinct from P_2, which b does have. But couldn't Noonan reply here that P_1 is the same property that is *not* assignable to b in virtue of the fact that at t_1 it was *not* indeterminate whether b had been emitted from the atom? Now since P_1 is a property that b *has*, and P_2 is a property that b *lacks*, P_1 and P_2 must be different properties? That would undermine their being not determinately distinct properties, and with it the claim that P_1 cannot differentiate between a and b.

Furthermore, Lowe's original (1994) reason for saying that the properties $\hat{x}[\nabla(x = b)]$ and $\hat{x}[\nabla(x = a)]$ are not determinately distinct, namely, that they differ only by permutation of objects that are themselves not determinately distinct, cannot apply here, since *these* properties are not identity involving. Indeed at first sight the property of being indeterminate as to future emission and the property of being indeterminate as to past emission do appear to be different.

Lowe's alternative (1998) response is that the property of being indeterminate as to absorption is at any time either held by both a and b (as is the case at the time t_2 of emission) or held by neither (at the time of absorption t_1). But what then is the identity that is indeterminate? The identity a-at-t_1 $=$ b-at-t_2 seems determinately false, since a-at-t_1 lacks, and b-at-t_2 has, the property of being indeterminate as to absorption. Likewise the identity a-at-t_2 $=$ b-at-t_1 seems determinately false, since a-at-t_2 has, and b-at-t_1 lacks, the property of being indeterminate as to absorption. So I do not see the indeterminacy of diachronic identity that Lowe seeks. Furthermore, at time t_1 of absorption there appear to be two determinately distinct electrons, one about to be absorbed and one already in the ion. So there doesn't seem to be any indeterminacy of identity at t_1. That leaves the time of emission t_2 (or the time just before it).

Lowe disowns the claim that "the entangled electrons are ontically indeterminately identical" (Lowe 2001, p. 243). That is because his example is explicitly diachronic: "there is no fact of the matter as to which of [the electrons in the entangled state during the time between absorption and emission] was the electron absorbed and which of them was the electron that was already contained in the helium ion prior to the event of absorption" (ibid.). There is no obvious reason, however, why Lowe should *not* claim that the ontic indeterminacy of identity pertains synchronically to the electrons in the entangled state.

But how should this be described? They are to enter an entangled state, perhaps:

$$\frac{1}{\sqrt{2}}(|\psi_1\rangle \otimes |\psi_2\rangle - |\psi_2\rangle \otimes |\psi_1\rangle)$$

There is no reason in principle why this state should not arise, in the way Lowe wants, as a result of evolution from a non-entangled state. Take a state $|\psi_0\rangle$ and operator describing the evolution of the system such that $|\psi_0\rangle \otimes |\psi_1\rangle$ will evolve to $|\psi_2\rangle \otimes |\psi_1\rangle$ and $|\psi_0\rangle \otimes |\psi_2\rangle$ will evolve to $|\psi_1\rangle \otimes |\psi_2\rangle$. Then the system can start in the non-entangled state

$$|\psi_0\rangle \otimes \frac{1}{\sqrt{2}}(|\psi_2\rangle - |\psi_1\rangle)$$

which can be equivalently written as

$$\frac{1}{\sqrt{2}}(|\psi_0\rangle \otimes |\psi_2\rangle - |\psi_0\rangle \otimes |\psi_1\rangle)$$

Then because $|\psi_0\rangle \otimes |\psi_1\rangle$ evolves to $|\psi_2\rangle \otimes |\psi_1\rangle$ and $|\psi_0\rangle \otimes |\psi_2\rangle$ evolves to $|\psi_1\rangle \otimes |\psi_2\rangle$ and because the operator describing the evolution is linear, that state will evolve to the entangled

$$\frac{1}{\sqrt{2}}(|\psi_1\rangle \otimes |\psi_2\rangle - |\psi_2\rangle \otimes |\psi_1\rangle)$$

(This is the way in which the state of a system to be measured becomes entangled with the state of the measuring apparatus, leading to the measurement problem (see, for example, Albert 1992, Chap. 4)).

The trouble with this description is that on the understanding of the formalism where the state on the left of the tensor product applies to one particle, and the state on the right applies to the other, it is already understood that there are two determinately distinct particles. There therefore seems to be no issue of identity, either at a time or over time.

If on the other hand the formalism is such that there are no such "particle labels" (the Fock space formalism mentioned by Redhead and Teller), then the understanding is one on which it *never* makes sense to think of the particles as being identical or distinct. This therefore seems to take us away from Lowe's description of the example and towards that of French and Krause, for whom quantum particles are entities of a kind to which identity simply does not apply.

4.3 Nonindividuality: French and Krause

We turn now to the departure from the main Analysis thread by French and Krause (1995). French and Krause's account of how quantum objects are vague objects differs from Lowe's example on a number of points, most importantly:

- The focus is on synchronic identity, or just identity simpliciter, whereas it is essential to Lowe that the issue is diachronic identity.
- It is not, as it is for Lowe, that quantum particles can be sometimes distinct, sometimes indeterminate in identity; instead (on the most important of the alternative views that French and Krause offer) they are of a kind to which identity simply does not apply. A corollary of this is that they are not self-identical either, as they

are for Lowe who (1994, p. 113) says that for there to be a determinate number of things requires that those things be determinately self-identical.

French and Krause identify two ways in which quantum vagueness may be understood: particles may be thought of as 'individuals', to which identity applies but which are 'veiled' by a system of non-supervenient relations, or as 'nonindividuals', to which identity simply does not apply.

4.3.1 Veiled Objects

Here, the source of vagueness, for French and Krause, is to be found in the relations that must hold between the particles in order to account for Bell-type correlations. These relations do not supervene on the intrinsic properties of the individual particles. The connection with vague identity is that the non-supervenient relations

> ... 'veil' the particles in such a way that we cannot assign the latter any determinately distinct identity-free properties. And this 'veiling' of the particles can be understood as giving rise to a form of ontic vagueness, in the following way: it is a central claim of the epistemic view of vagueness that 'vaguely described facts' supervene on 'precisely describable facts'... Such a claim cannot be maintained within the above context: the 'vaguely described fact' concerning the electron's identity in the situation envisaged by Lowe arises from its being in an entangled state. The properties represented by such states do not supervene on either the intrinsic properties of the particles or any set of hidden variables (this being ruled out by Bell's Theorem). Thus the above 'facts' involving entangled states do not supervene on any facts involving the intrinsic properties of the particles or hidden variables and we have an example of genuine ontic vagueness. (French and Krause 2003, pp. 104–105)

But how does the presence of these relations give us *indeterminacy of identity*? It seems to me that the 'vaguely described facts', such as which electron is the one that is spin-up, say, are not obviously facts about identity, and moreover that while there may be a failure of certain facts to supervene on facts about the intrinsic properties of the electrons, what *is* perfectly determinate in the supervenience basis, and what amounts to everything that is relevant to facts about *identity*, is that there are two distinct entangled electrons, between which hold certain relations. In this regard the situation (as far as determinacy of identity is concerned) seems not relevantly different from Black's two spheres. There, we would not be tempted to say that it was indeterminate whether one of those spheres was identical to the other – that is, not one of the usual examples in the vagueness literature.

There may be two ways of thinking about that case (one where there really are two spheres, one where there is just one sphere, since the 'two' are in fact identical), but that does not make for indeterminacy. If there are two indiscernible spheres, then there are two things each self-identical and neither identical to the other, so there is no indeterminacy of identity. There is another way of thinking about the situation, so there *is* an issue about identity here: instead of two spheres there might just be one sphere two miles from itself in a curved space (see French 1995), but again there is no indeterminacy of identity: there is determinately one self-identical sphere.

French and Krause point to the apparent differences between the spatiotemporal relations and the quantum-mechanical ones motivated by Bell's theorem. Do those differences undermine the comparison with the two-spheres case? I don't think that they do, in any way that counts in favour of indeterminacy of identity:

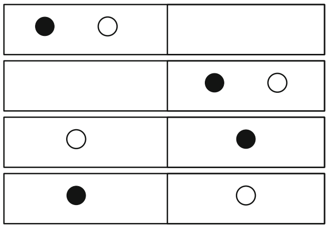
One difference is that the spatiotemporal relations are, and the quantum-mechanical one are not, *discriminating* (French and Krause 2003, p. 103). (The term is from Lewis 1986, p. 76). Indeed it might be that at our world and ones like it, inhabited by bosons and fermions but not by paraparticles, all particles in an entangled state are exactly alike with respect to their place in the corresponding relation. But whether or not quantum-mechanical or spatiotemporal relations *may* discriminate amongst particles, *neither* do so in the two-spheres case: in Black's world, both spheres are the same with respect to their place in the structure of spatiotemporal relations, so while those relations may be discriminating in the wider sense, they do not discriminate here. So it seems that if the lack of discrimination in the quantum case makes for indeterminacy of identity, then so it should in the two-spheres case. It does not in the two-spheres case, so the fact that the quantum relations are not discriminating does not, it seems, make for indeterminacy of identity.

French and Krause say that they are here operating under "the first of French and Redhead's metaphysical 'packages' according to which indistinguishable particles can still be regarded as individuals [for French and Krause, individuals are entities to which identity applies, as opposed to non-individuals, to which it does not]. Thus we are entitled to regard '*a*' and '*b*' as precise designators" French and Krause (2003, p. 103). But in fact the fact that the particles are individuals does not necessarily mean that we are entitled to regard '*a*' and '*b*' as precise designators: all of those cat-like objects on the mat are individuals in this sense, and yet "Tibbles" is not a precise designator. In something like the two-spheres case, it might be that it is *impossible* to fix precisely the names '*a*' to one and '*b*' to the other of the two spheres, but that would be merely representational indeterminacy. Could it be that this was representational indeterminacy with an ontic source, as in Sect. 4.2.5? Perhaps – any *description* that could fix reference to one sphere ("the big iron sphere", "the sphere two miles away from the big iron sphere"...) applies just as well to the other. But if so still the ontic source – the complete symmetry of the setup – is not *indeterminacy*. So this is even further from Lowe's intention.

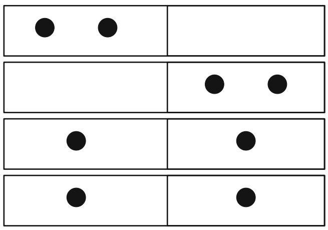
It seems then that in the case of two entangled particles which are considered individuals, there are still determinately two determinately distinct determinately self-identical particles – they may violate a principle of the identity of indiscernibles, but if they do that it is because they are (determinately) distinct. The preservation of PII by denying that these apparent counterexamples are individuals at all is the next, and separate, package that French and Krause consider.

4.3.2 Nonindividuals: Indeterminacy of Identity or Inapplicability?

The problems surrounding identity here stem from permutation symmetry. If we can distinguish the two particles, then there are four possibilities:

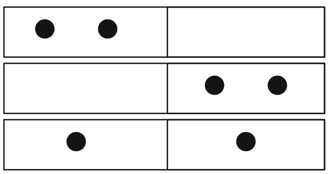


With classical particles, numerically distinct even if indistinguishable, there are still four possibilities:



That would suggest that if one asks something like “How many particles are in the left half of the box?”, the answer would be “two” a quarter of the time, “none” a quarter of the time and “one” half of the time. The *observed* statistics, however, are that each answer is found a third of the time.

This requires some kind of explanation. One strategy involves relations between the particles that explain the statistics; this strategy doesn’t get us into territory involving indeterminate identity, however. The other strategy, and the one that French and Krause take to drive talk of vague objects, is to say that the last two possibilities are not distinct, because it is meaningless to think of the particles as swapped around; so there are only three states:



Here then (glossing over details, of course) is an explanation of the observation that each of these possibilities occurs a third of the time.

The idea that this explanation is driven by meaninglessness of identity is important – French and Krause follow a tradition exemplified by Schrödinger's claim that "[I]t is beyond doubt that the question of 'sameness', of identity, really and truly has no meaning" (French and Krause 2006, p. 104). But does this mean that we have *indeterminate* identity?

The quote above from Schrödinger suggests something different from the linguistic case, where it is unclear whether two singular terms refer to the same or different objects, and different from the ontic analogue that most advocates of ontic vagueness picture (e.g. Akiba 2004; Barnes and Williams 2011), where the world is somehow unsettled between two ways it could be. Here, by contrast, there *is* no way that the world could be, such that the two particles are identical, nor any way such that they are distinct – the question is simply meaningless, not a matter of ontic unsettledness.

This shows up in the formal apparatus used by Evans, on the one hand, and French and Krause, on the other. In their framework, French and Krause have no place for Evans's ∇ operator but instead employ a "Schrödinger logic", a 2-sorted logic with *M-terms*, to which identity applies, and *m-terms*, to which it does not: If "a" and "b" are m-terms then "$a = b$" is not a well-formed formula. But this appears to be different from vagueness, or *indeterminacy* of identity, since if we are thinking of ∇ as a modal operator, as Evans and others do, then we would expect to use the usual formation rules for formulas in modal logic: the rules specify what the atomic formulas are, that we can negate, disjoin and quantify into formulas to get formulas, that if A is a formula then ∇A is a formula *and that nothing else is a formula*. But then if $a = b$ is not a formula, then $\nabla(a = b)$ is not a formula either. Since Evans's argument was also supposed to undermine the assumption of $\nabla(a = b)$, this can't be used as a reply to Evans. If the defender of ontic vagueness wants to prefix expressions with an indeterminacy operator of this kind, they require that those expressions be meaningful. So it is not clear that *indeterminacy*, as captured by the ∇ operator, is the idea at play here.

To compare with a standard linguistic example (from Lewis 1988), we know what it means to say "Princeton is identical to Princeton borough" – it is perfectly *meaningful* – we are just unsure whether it's true (in fact, we probably think it's unsettled, or indeterminate, because language doesn't draw those precise boundaries). What we don't count as unsettledness, or indeterminacy, is something like "borough Princeton to identical is Princeton". In the ontic case, perhaps what we want is that there is a way for the world to be that would be an $a = b$ way, but that it's just indeterminate whether the world is in that state. What we seem to have here is instead that there is no way for the world to be that would be an $a = b$ way, because that just doesn't make sense.

On this view (the 'nonindividuals' package where, unlike Lowe, we are looking for indeterminacy of identity synchronically, or in general, rather than specifically diachronically) then it looks like something more is needed to make the connection with metaphysical indeterminacy and the 'vague objects' debate. I don't think it can be done, but now consider three points that might be made in reply: it might be said that we do not *need* either $a = b$ or $\nabla(a = b)$ to be a formula to have an example of ontic vagueness; it is not necessarily true that, as I assumed, $\nabla(a = b)$ is not a formula if $a = b$ is not; and there might be an argument that we can get $\nabla(a = b)$ directly from the considerations that lead French and Krause to conclude that it is not well formed.

On the first thought, that ∇ is not required for vagueness, of course there are many ways of understanding 'vague objects', one of which might be precisely the *inapplicability* of identity, but that is not the understanding at issue in Evans's argument. There the identity is well formed, and indeterminate, not ill formed and inapplicable. So one who believes that there can be no vague objects on the grounds that Evans's argument is sound need not dispute the conclusion that quantum mechanics poses problems for thinking of particles as possessing identities; they can just deny that that conclusion has anything to do with their understanding of *indeterminacy*.

Of course it might be that that understanding of indeterminacy is too narrow. That the way to characterise ontic indeterminacy is not to use a sentential operator is the view of Pelletier (2006):

> ... vagueness *de re* – that is, vagueness inhering in an object – is not plausibly construed by *any* operator on sentences. To say that an *object* is vague is to say at least that some predicate neither applies nor doesn't apply to it; and this seems to call for some construal of sentences like Fa in a manner opposed to treating it first as meaningful and then prefixing an indeterminacy operator to it. (p. 415)

Similarly, Lowe, although he addresses Evans's argument, suggests that he would prefer not to be thinking about vague objects in Evans's terms but instead to say that:

> It is improper to express indeterminacy by a sentential operator like '∇', that is, it is wrong to treat the *lack* of any objective fact of the matter determining the truth or falsity of a sentence S_1 as itself being just at particular objective matter of fact capable of being reported by a true sentence S_2 obtainable from S_1 by a logical operation. (Lowe 1994, p. 112)

In the end of course it just depends on what is meant by 'indeterminacy'. Allowing that it need not involve an operator attached to a sentence, we have the question of how indeterminacy *should* be expressed, if not by the ∇ operator, and the key question: is it properly expressed by a sentence's *not being a formula*? Distinguish:

1. Expressibility by ∇ is not necessary for indeterminacy of identity.
2. Inapplicability of identity is sufficient for indeterminacy of identity.
3. Indeterminacy of identity is not necessary for vague objects.
4. Inapplicability of identity is sufficient for vague objects.

(1) seems to be the view of Lowe and Pelletier – they think that indeterminacy can be (ought to be) captured in some other way. (1) is surely true – there are other ways of thinking about vagueness that are in keeping with the mainstream vagueness literature but that do not employ an operator like Evans's – but does take us a little way from Evans's argument. But in any case (1) is not enough here. If the case is to have anything to do with *indeterminacy* at all, then (2) is needed.

Whether Schrödinger's view can be understood in terms of indeterminacy *instead of* inapplicability is not the point (that comes under the third idea below). The point is, holding fixed the understanding of Schrödinger's view as meaninglessness, is there a legitimate understanding of 'indeterminacy' as meaninglessness? That is suggested when Lowe writes that:

> ... there can be circumstances in which two or more [entities such as electrons] exist in an 'entangled' state, where all talk of numerical identity or diversity between those entities ceases to have meaning. (Lowe 1998, p. 70)

(Lowe thinks, however, that even in those circumstances, it is meaningful to talk of the identity of one of those entities with itself.) On a standard understanding, where perhaps some object fails to instantiate any determinate value of some determinable which nevertheless applies to it, meaninglessness doesn't seem to be it: it is indeterminate what the mass of Everest is, but it is not indeterminate what the mass of the number 7 is. Likewise the standard understanding seems to be that identity applies to vague objects, just not determinately. So the understanding of indeterminacy does seem to be nonstandard.

If we think that inapplicability does not deliver indeterminacy, then (3) is needed. (3) is no doubt true also, and in keeping with *some* understandings of vague objects, see, for example, Noonan:

> Everyone knows that Evans's argument against vague identity in-the-world doesn't show that there aren't vague objects ... Even if the argument succeeds all it proves is that every vague object is determinately distinct from every precise object and every other vague object. So it is consistent to hold both that there are vague objects and that the identity relation is precise.
>
> And indeed this seems to be the common-sense position. (Noonan 2004, p. 131)

But then we are already not in the business of replying to Evans's argument (we can take Noonan's option while conceding that the argument is sound). But (3) is not enough to ensure that in the present example we *do* have vague objects – the ways Noonan has in mind in which objects might be vague while avoiding Evans's argument have nothing to do with identity – and once we get to (4) my concern is that not only are we well beyond Evans's argument; we are also beyond what is meant by 'vague objects' in the metaphysics literature.

So I suspect that advocates of ontic vagueness may say 'thanks, but no thanks' to this example, denying that what we have in the case of quantum objects, on the Schrödinger-type view, counts as *indeterminacy* at all. But I'm not certain – there are perfectly legitimate understandings of 'indeterminacy' on which it counts. And perhaps Schrödinger's view can be reinterpreted to re-introduce Evans's ∇ operator. So let's acquiesce in Evans's way of thinking and ask the conditional question: if you insist on representing vague objects as Evans does, can you get a counterexample to Evans out of quantum mechanics?

The second thought, then, was that $\nabla\phi$ may be a formula even if ϕ is not. How would that be implemented? Of course in a certain language, "'p' is meaningless" is meaningful! But I can't see how this is going to help. To have ∇A abbreviate "'A'

is not a formula" is no good, since although that *is* a formula, it is not a formula of the language in which the assumption targeted by Evans is formulated.

No doubt it is *possible* to introduce the ∇ operator into the Scrödinger logic to make a place for it in the syntax and semantics so that it applies to non-formulas. We cannot of course just say

- If ϕ is a formula, then $\nabla\phi$ is a formula, and $\models \sim \nabla\phi$
- If ϕ is not a formula, then $\nabla\phi$ is a formula, and $\models \nabla\phi$

because now $\nabla \rightarrow\sim \wedge\phi \sim$is a formula and a logical truth, which surely isn't right. But something like

- If ϕ is a formula, then $\nabla\phi$ is a formula, and $\models \sim \nabla\phi$
- If a and b are m-terms, then $\nabla(a = b)$ is a formula, and $\models \nabla(a = b)$

might work. There are a number of questions that arise: what about compounds containing $a = b$? Are they formulas? What about ∇ prefixed to such compounds?

Generally, the indeterminacy operator ∇ functions like 'it is contingent whether' as a counterpart to a 'definitely' operator D which functions like 'necessarily'. What happens to the D operator here? On any acceptable understanding of indeterminacy, A is indeterminate if and only if $\sim A$ is, and that is true here (i.e. where we try to understand indeterminacy as non-well-formedness); if A is not a formula then nor is $\sim A$. But then all truth-functional compounds of ∇A and $\nabla \sim A$ are equivalent either to ∇A or to $\sim \nabla A$, neither of which captures what we meant by 'definitely', so there is no hope of the usual interdefinability. To put it another way, if $\nabla\phi$ says that ϕ is not a formula, what could D possibly mean such that $\sim D\phi \wedge \sim D \sim\phi$ and $\nabla\phi$ are equivalent?

In any case, even if something like this could be made to work, I can't see that it gets us any closer to the standard understanding of *indeterminacy* at issue in Evans's argument. Apart from anything else, it seems that for Evans $a = b$ *is* a formula: the key result, which Evans thinks conflicts with $\nabla(a = b)$, is $\sim (a = b)$. Compare again the quantum case with a standard example of linguistic vagueness: it is indeterminate whether Tibbles is blue, but 'Tibbles is blue' appears perfectly meaningful. The ∇ operator is just not introduced to capture meaninglessness, as has been done here.

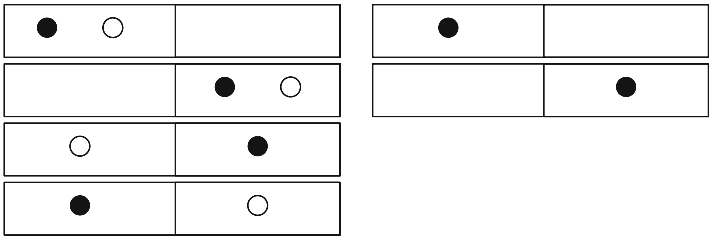
The final thought was that it might be claimed that French and Krause are wrong to treat identity in this way, that instead the correct way to understand the physics is to have the identity meaningful and indeterminate. In other words, the indeterminacy operator ∇ should be introduced as an alternative to the Schrödinger logic. Any identity statement is now well formed after all. But that seems ill motivated. The arguments:

1. If identity is applicable to particles, then there are four possible states for that particle pair.
2. There are not four, but three, states for the particle pair.
3. So identity is not applicable to particles.

and:

1. If identity of particles is determinate, then there are four possible states for that particle pair.
2. There are not four, but three, states for the particle pair.
3. So identity of particles is not determinate.

sound about equally plausible. But what really matters is whether the inapplicability of identity, or its indeterminacy, *explains* the statistics. And here the inapplicability view seems to apply in a way that indeterminacy does not. If we say that it is *indeterminate* whether the two particles are identical, then it sounds instead like we have not that third picture above, but *indeterminacy* between *two* pictures:



That is, it is indeterminate whether there are two particles, and four possibilities, or one particle, and two possibilities.

I cannot therefore see a motivation for replacing the view whereby identities are meaningless (i.e. French and Krause's view) with one where they are indeterminate (which is what we need to connect directly with Evans's argument), where 'indeterminate' is understood in the usual way as captured by the ∇ operator.

To conclude this section, at least three different kinds of reply to Evans are possible:

1. An indeterminate identity captured by $\nabla(a = b)$, where 'a' and 'b' are precise designators.
2. An indeterminate identity captured by $\nabla(a = b)$, where although at least one of 'a' and 'b' is imprecise, the imprecision has an ontic source.
3. An example of indeterminacy of identity not captured by $\nabla(a = b)$.

In Sect. 4.2.6 I have explained why I do not think that quantum mechanics motivates a reply of the first kind. In Sect. 4.3 I have explained why I am not sure that quantum mechanics motivates a reply of the third kind, principally because indeterminacy is not the same as inapplicability. So whether or not what we have is properly captured by $\nabla(a = b)$, it is not a case of indeterminacy of identity. That leaves the reply of the second kind. In Sect. 4.2.5 I have explained how it seems that if we make certain presuppositions about superpositions involving indeterminacy of property instantiation, then a reply of that kind looks straightforward; it is not clear, however, that it provides an example of ontic indeterminacy of *identity* in the strict sense – there is ontic indeterminacy resulting in indeterminacy of identity, but the indeterminacy of identity is not ontic but representational in nature.

4.4 Collections of Vague Objects

Up to now we've been considering how one might think about "single" vague objects in quantum mechanics. In this final section I discuss the idea of *collections* of vague objects. One reason for this is that French and Krause make the idea of a "quasi-set" of their nonindividuals central to their formal account – see French and Krause (2006, especially Chap. 7) and further formal work including Krause et al. (2005), Domenech and Holik (2007), Domenech et al. (2008), Arenhart and Krause (2009), French and Krause (2010), and Arenhart (2011, 2012a,b). Since the French-Krause approach is both motivated by considerations from physics and also comes with an accompanying formal theory, I'll focus on this, notwithstanding the concerns raised above that it's not *quite* what is usually meant in the literature on "vague objects".

The second reason for looking at the idea (and this connects to our meta-metaphysical interests) is that Smith (2008), who might be taken as representing non-naturalistic analytic metaphysics, has argued (1) that the only hope for understanding vague identity, indeed for understanding *anything*, is to be able to model it in set-theoretical terms (more specifically, what Smith calls "Chapter One set theory") (2) that although there *is* a (quasi-)set-theoretical approach to vague identity (that of Krause et al.), this goes no further than a set of axioms, in particular offering no adequate explanation of what the axioms are supposed to capture. He concludes on this basis that the prospects for understanding vague identity are dim.

4.4.1 Smith's Argument

Smith's argument starts like this:

> [A] necessary condition for *making clear sense* of a phenomenon is showing how the phenomenon may be modelled using standard set-theoretic tools.
> [...]
> Why is this so? Because when we show that some phenomenon can be modelled using standard set theory, we show that the phenomenon is unobjectionable *from a purely logical point of view*. When we know that some statements can be modelled in set theory, we know that they *have a model* in the logical sense, and thus at the very least are consistent. If a phenomenon is not even *logically* coherent, then there certainly will not be a good *naturalistic* account of it. (Smith 2008, pp. 2–3)

The link to identity then comes into play: "Identity has a special place in set-theoretic models: the identity facts are *built into* every such model. Each such model comes pre-equipped with a ‘factory-installed’ identity relation. Other relations can be added to the model, but the pre-installed identity relation cannot be removed” (Smith 2008, p. 3). Smith then gives a couple of examples: identity is already in the background when the axioms are given for a metric space (of course, since one of the axioms is the *identity of indiscernibles*).

Putting these together:

1. To make clear sense of something one must (at least) model it set-theoretically.
2. Vague identity cannot be modelled set-theoretically.
3. Therefore, we cannot make clear sense of vague identity.

(Smith 2008, p. 8)

Smith clarifies Premise 1: “Set theory” doesn’t mean ZFC. “By ‘set theory’ I mean something much more basic and fundamental, which I call ‘Chapter One’ set theory. This is the view explained in the first chapter of textbooks (and the first lecture of courses) in every area of mathematics. It is not so much a *theory* as a *way of thinking* with which we need to be inculcated if we are to understand any mathematical theory” (Smith 2008, p. 8). He then quotes some textbooks as saying that the idea of a set and its members is “primitive and well-understood”, “intuitively clear” and not in need of further analysis. (Note, however, that the first quote refers to those things that “Mathematics habitually deals with...”; of course one of French and Krause’s main points is that mathematics doesn’t *habitually* deal with nonindividuals...)

Smith now turns to the prospects for constructing an alternative to standard set theory. He first notes (p. 10) that “the burden of proof lies firmly with the friends of vague identity to actually produce a framework in which we can clearly think about vague identity.” Second, he notes that Krause et al. *have* attempted to produce such a theory, but then complains that they make no progress towards their goal. Since quasi-set theory is presented as an axiomatic theory, and since the axioms had better not be interpreted in standard set-theoretic terms (else genuine identity is there after all), “we need to have a new way of understanding them explained to us—or else we do not understand them at all. Yet in spite of their opening statements [...] Krause et al give no such explanation. From those opening statements, it sounded as though Krause et al would explain, from the ground up, a new framework for thinking about objects and collections in which we need not, and indeed must not, assume that whenever we have some objects, we have a complete and precise set of facts concerning the identity and nonidentity of those objects one with another. Yet they do not do this: they simply present a set of axioms.” (Smith 2008, p. 11)

4.4.2 *Set-Theoretic Modellability as Necessary Condition for Intelligibility?*

On the one hand, there is something right about the latter claim: quasi-set theory *is* in need of significant philosophical clarification. On the other hand, there is *far* more to the tradition, exemplified by French and Krause (2006), than simply a set of axioms. Moreover, I think that the attachment to set theory is an instance of conceptual conservatism that being naturalistically inclined, we should side with Ladyman and Ross in rejecting. Naturalists in general have reason to be skeptical about the idea of necessary (or indeed sufficient) conditions for intelligibility: there are plenty of things that philosophers have said are not comprehensible, that were later explained mathematically. Kant and non-Euclidean geometry would be the usual example. From this perspective the burden of proof is on the metaphysician to establish that something is and must be forever unintelligible.

As Ladyman and Ross point out, the kinds of intuitions that underwrite much of metaphysics and based on homely examples are not the kinds of intuitions that are going to guide us to the truth about the fundamental nature of reality: why should beings evolved to deal with medium-sized dry goods be expected to have concepts ready made to fit the "intuitively vertiginous conceptual space into which contemporary physics forces us" (Stanford et al. 2010)? Of course there is no guarantee that we will be able to cope in the vertiginous conceptual space, but claims of outright impossibility need proof.

Set-theoretic considerations, and in particular Chapter One Set Theory, cannot show this, because even the minimal apparatus Smith intends by Chapter One Set Theory really amounts to our everyday understanding of objects as distinct individuals. Since what is at issue is precisely whether quantum particles *can* be understood as distinct individuals, that set-theoretic modellability is a necessary condition of intelligibility is either false or question begging:

It appears to be false, since one can understand, say, love without being able to model it set-theoretically. Smith might reject the counterexample: we do not understand love, it is not at all intuitive, in fact every bit as vertiginous as modern physics. But even if we don't understand love, the reason for our misfortune is not, surely, that we are unable to model it set-theoretically. Indeed, the attempt to model it set-theoretically may be where we are going wrong. At this point he might protest that this is a bad example and isn't the kind of thing he intended: he didn't mean that *everything* intelligible can be modelled set-theoretically. He intended only that theories of *objects* can be modelled set-theoretically. And since you can't have a set-theoretic model of genuinely "vague identity", vague identity is unintelligible. But now we see that it's question begging: Krause never claims to be offering a theory of *objects*; that's the whole point – he thinks they're not the kind of thing that's modellable set-theoretically. So obviously the fact that he can't model them set-theoretically is neither here nor there.

4.4.2.1 Favourable Comparison with Quantum Logic

Smith reiterates the inevitability of precise identity:

> It is not that our current practice of making free appeal to a classical identity relation whenever we have a set of objects is a convention which is so firmly entrenched that we do not need to state it explicitly. Rather we have no alternative. This way of thinking is the

> only one we have – and will ever have – of thinking clearly about objects and collections of objects. We can think about other ways of thinking about identity, but we cannot think with them – we cannot accept that they are genuinely alternative ways of thinking about identity – for, as we have seen, the conception of sets of objects as pre-equipped with the classical identity relation is always operative in the background. (2008, p. 12)

Clearly if we are to learn to think of things as French and Krause intend, we have to learn a new way of thinking about objects and collections of objects. But does that mean we need a "genuinely alternative way of thinking about identity"? It depends on what you mean. No one is proposing to do without identity altogether or to deny that it applies to certain things. Krause's theory includes both sets, containing individuals to which identity applies, and quasi-sets, containing nonindividuals, to which it doesn't.

This invites a comparison with the treatment of (what we ordinarily think of as) the truth-functional connectives in certain traditions of "quantum logic". As Maudlin (2005) points out, you can't do away with, or supplant, the standard "or", since it's always there in the background. One might see a parallel: you can't do away with "=", because genuine identity is always there in the background. But the cases are disanalogous in a way that is favourable to Krause. Krause is not advocating *supplanting* identity, just *adding* to our ontology "entities" to which it does not apply. In the case of a logical connective, it is implausible to distinguish between "quantum propositions", to which quantum logic applies, and "classical propositions", to which classical logic applies. To do so would mean denying that some connective connects the things that are otherwise unproblematically connected by connectives. But the analogue for identity is exactly what Krause intends: there are some things to which identity applies, and some to which it doesn't.

4.4.2.2 Top-Down and Ground-Up Explanation

From the naturalist point of view, I'm not sure we should accede to the demand for ground-up explanation. It is not entirely clear, in fact, what that means. If it means starting from bedrock, which for Smith appears to *include* the ubiquity of regular identity, then of course our quest is hopeless. But then the demand is question begging. Instead, a naturalistic approach might note that there are more ways to make something like this intelligible than to square it with an intuitive or ingrained conceptual framework. After all, French and Krause's project is extremely ambitious, and we should expect only piecemeal progress from different angles, in which top-down considerations from physics will be as important as bottom-up logical and metaphysical constraints.

Part of what one might take top-down explanation to do is to provide a sense of the *goals* of the formalism. And French and Krause *do* tell us about the goals of the quasi-set theory. For example, there is to be a sense of *cardinality* attached to quasi-sets. One is to have a sense of number of these nonindividuals. And the motivations for this are top-down – connecting, for example, to physical features of the collections such as mass of an aggregate compared to mass of a "single" particle.

(Of course, talk of single or individual particles is not really legitimate here – this is just to give a *sense* of what they are trying to capture.) If top-down considerations help in articulating things that we *want* to conceptualise without identity, bottom-up logical and metaphysical considerations illuminate the kinds of things that identity *allows* us to conceptualise in something like set theory and how they might be understood by other means in various formal frameworks.

The claim that set-theoretic considerations show that sense *cannot* be made of vague identity seems too strong. In the end, though, of course Smith's skepticism is justified, albeit in a rather different form. In the final two sections I would like to consider two challenges, one concerning details of the quasi-set approach to cardinality and the other concerning the resources available in other ways of cashing out their formal theory, for French and Krause's project.

4.4.3 Cardinality and Identity

As noted above, part of the top-down explanation that French and Krause offer has to do with the idea of number, of cardinality attaching to quasi-sets. Introducing the mathematical apparatus in the book, they say:

> The basic idea is that in a quasi-set, there may exist elements for which the traditional concept of identity does not apply – these elements are called the 'm-atoms'. Thus, a quasi-set may have a cardinal but not an associated ordinal. The other elements can be regarded as standard elements of a set. Hence, the theory encompasses a copy of Zermelo-Frankel set theory with Urelement. . . (French and Krause 2006, p. 273)

The slogan "collections having a cardinal but not an ordinal", i.e. that it makes sense to have some *number* of nonindividuals, requires elaboration. Now, quasi-set theory *does* assign cardinals to quasi-sets – that is in the axioms. But how should we know if the theory has assigned the *right* cardinals? What could it even *mean* for it to assign the right cardinals? Why not just assign them all the cardinal **17**? Now in standard set theory (e.g. not quite Chapter One, but Chapter Three in Machover 1996), there is a good answer, which is presented something like this:

1. First say that two sets are *equinumerous* iff there is a bijection (total, 1:1, onto function) from one to the other.
2. Next, note that equinumerosity is an equivalence relation.
3. Now define cardinality such that $|A| = |B|$ iff A and B are equinumerous.
4. Note that there are several ways in which this might be done: take cardinality as primitive, so the cardinals are special abstract objects (Cantor); identify cardinals as equivalence classes modulo equinumerosity (Frege) (with the drawback that this makes some cardinals proper classes); identify a set's cardinal the least ordinal with which it is equinumerous (von Neumann); . . .
5. Note that in fact all of the following chapters could be rephrased in terms of functions between sets in the event that none of these candidates for the cardinal role are satisfactory.

6. Note that whatever way this is done, cardinals measure the size of sets.
7. Define ordering of cardinals so that $|A| \leq |B|$ iff there is a 1:1 function from A to B, define addition via union…

But this is doomed right from the start in quasi-set theory, because we lack identity; lacking identity means that we can't say what it is for a function to be 1:1. So we can't properly understand what (quasi-)cardinality is supposed to amount to. This is the kind of thing one might want explained "from the ground up". We need some way of understanding the function of number that isn't tied to identity.

Now, French and Krause note that there is work to be done here. An axiom asserts that each quasi-set has a cardinal, which does its usual job when only genuine sets of individuals are concerned:

> (Q19) The quasi-cardinal of a qset is a cardinal (defined in the 'classical part' of the theory) and coincides with its cardinal itself when this qset is a set:
>
> $$\forall_Q x \exists! y (Cd(y) \wedge y =_E qc(x) \wedge (Z(x) \rightarrow y =_E card(x)))$$
>
> (French and Krause 2006, p. 287)

In the cases where the quasi-set is not a set, that is, containing nonindividuals, they note that the usual strategy of identifying the set's cardinal with an associated ordinal is unavailable, and so they canvass some of the other alternatives under (4) above. They conclude that:

> Whichever option is chosen, it seems clear that the search for a more adequate definition of cardinal (and of course of 'counting') as far as quantum physics is concerned is something to be further investigated. (French and Krause 2006, p. 288)

But actually, the most pressing challenge isn't the first (what the cardinals *are*); it's the second (what they *do*). Say that the cardinals are *sui generis* entities if you must, but that won't help here. It's not the nature of the cardinals that matters, the things that are to play the role in the story above; it's the story itself and the role of identity in it. Answers to item (4) along the *sui generis* lines won't help, hence item (5); the other answers assume identity, so are unavailable.

Further work along these lines is ongoing. For example, Arenhart (2012a,b) discusses alternative ways of assigning cardinals to quasi-sets. If successful, this would go a very long way to answering objections such as Smith's. It would also bear on one of the points at issue between Odrowąż-Sypniewska and Lowe that determinate identity and determinate number do not necessarily go hand in hand.[2] However, various doubts occur. For example, (quasi-) cardinality is defined via the idea of quasi-functions (see e.g. Arenhart 2012b, pp. 441ff.) which in turn require "member of" and "forall". If those are available without commitment to identity of elements, then so is "subset", and one may then start to speak of *number* of subsets (the (quasi-) sets themselves being the kinds of thing to which identity applies) and

[2]Thanks for an anonymous referee for pointing this out, and for suggesting defences of quasi-set theory on which this section is based.

thereby begin to get a handle on cardinality of the quasi-sets. However, although one can appear to say things about all such objects without committing to identity, that is an illusion if, for example, quantification involves identity – see, for example, Dummett (1996). Moreover, even if one can coherently use quantification and membership without identity, one would still want a demonstration that the result is an account of something relevantly like cardinality. Finally, specific ideas used in the account suggest that these things have not really been divorced from identity. For example, the specification of quasi-functions involves talk of "*the* object z mapped by f to x" (Arenhart 2012b, p. 441, emphasis added), so a demonstration is needed that this is merely heuristic.

4.4.4 The Category of Quasi-sets?

Smith suggests that the argument against vague identity on the grounds that it is not intelligible set-theoretically is *more fundamental* than existing arguments like Evans's. I have argued above that the set-theoretic argument is question begging. What we need is another argument that is more fundamental than the set-theoretic one.

French and Krause want to talk about collections of nonindividuals, the quasi-sets. A collection, be it a set (a collection of precise objects, perhaps) or a quasi-set (a collection of nonindividuals or vague objects), is a mathematical object of a certain kind. Any coherent account of some domain of mathematical objects, be they vector spaces, or groups, or sets, or quasi-sets, can be formulated in category theory. A category consists of its objects $A, B, C, \ldots$, morphisms between them ($A \xrightarrow{f} B$, $B \xrightarrow{g} C$, $A \xrightarrow{h} B$, etc.) and associative composition of morphisms ($f \circ (g \circ h) = (f \circ g) \circ h$), with an identity morphism for each object (i.e. for each object, say B, a morphism $B \xrightarrow{1_B} B$ such that for any other morphisms $A \xrightarrow{f} B$ and $B \xrightarrow{g} C$, $1_B \circ f = f$ and $g \circ 1_B = g$). The specifics are then filled in via behaviour of the morphisms (e.g. via the existence of products and sums, which are themselves characterised by relations amongst morphisms).

So an alternative necessary condition for intelligibility is category-theoretic modellability: use that language to tell us about these mathematical objects that have less structure than a vector space, less structure than a group, less even than the minimal structure that a *set* has via the identity and diversity of its elements.

In one way, this sounds promising for French and Krause. In set theory, we can draw *internal diagrams*, and the internal diagram for an identity mapping will have to show each element mapped to itself and not another; an internal diagram for an isomorphism will have to show each element of the domain mapped to a unique element of the codomain; and so on, all of which relies on "genuine identity" of the elements. In category theory, by contrast, once the compulsory story for any category is given by specifying some objects (the quasi-sets in the case of the category **QSet**), some morphisms with composition and identity as above, the rest of the content is achieved by the use of *external* diagrams.

As an example of the resources available, French and Krause might try and exploit the fact that there are two distinct notions that happen to coincide with bijection in the category of sets. One is *isomorphism*, or "morphism having an inverse":

- An inverse of a morphism $A \xrightarrow{f} B$ is a morphism $B \xrightarrow{f^{-1}} A$ such that $f \circ f^{-1} = 1_B$ and $f^{-1} \circ f = 1_A$.

What French and Krause can say about this depends on what they say about 1_Q, of course.

The other notion is "*cancellation on the left and right*". An epimorphism is a morphism that cancels on the right:

- $A \xrightarrow{e} B$ is an epimorphism iff, for any object C and morphisms $B \xrightarrow{f} C$ and $B \xrightarrow{g} C$, if $f \circ e = g \circ e$ then $f = g$.

And a monomorphism is a morphism that cancels on the left:

- $B \xrightarrow{m} C$ is a monomorphism iff, for any object A and morphisms $A \xrightarrow{f} B$ and $A \xrightarrow{g} B$, if $m \circ f = m \circ g$ then $f = g$.

In the category of sets, the notions are equivalent: the isomorphisms are just the bijections, and to cancel on the left and the right entails having an inverse. (In *any* category, to be an isomorphism entails being a mono- and an epimorphism. In other categories, however, these come apart.) By carefully considering the options available, one might hope to illuminate the way in which a surrogate for bijection should behave while still being a surrogate and not the real thing.

If the story of **QSet** can be told like this, then headway might be made on questions about, for example, the way in which quasi-cardinality is to function. We might even have what Smith wants from a "ground-up" explanation.

At this stage even the $1_{\_}$ identity morphisms are not yet problematic: French and Krause can assert that for each qset Q, there exists a morphism $Q \xrightarrow{1_Q} Q$ such that for any other morphisms $P \xrightarrow{f} Q$ and $Q \xrightarrow{g} R$, $1_Q \circ f = f$ and $g \circ 1_Q = g$. And they can assert this without thereby telling us anything more about the internal structure of Q or indeed of 1_Q.

Of course, it is then encumbent on them to fill in the rest of the details – the burden of proof looks to be on French and Krause to provide this description, to describe **QSet** as a category and provide an account of how the morphisms behave. They will have to do this in such a way as to give a description of the category of quasi-sets that doesn't end up being simply the category of sets – **Set** also has a categorical description (e.g. as presented in Lawvere and McLarty 2005), and the description of **QSet** will have to avoid identifying the two, while still capturing the relevant ideas about, say, (quasi-)cardinality.

If this is not possible, if, at some point (perhaps when quasi-cardinality is introduced, perhaps sooner) **QSet** turns out to be **Set** after all, French and Krause might deny the need for identity morphisms. Then they would be committed to the idea that **QSet** is somehow less than a category (perhaps a *semi*category – "*quasi*-category" already means something else).

So, if a full explanation of quasi-cardinality is the first challenge, an account of the category of quasi-sets is the second. There are at least two things that French and Krause might say in response to this challenge: (1) Yes, **QSet** is a category; here is how the morphisms behave – you can see that the 1_{-} morphisms don't commit us to identity among the elements of the quasi-sets; here is how the morphisms' behaviour differs from that in **Set**; etc. (2) No, **QSet** is not a category, it is something else; here we see how, via an explanation of how the morphisms behave; etc...

These challenges link up: the problem seems to be that the quasi-sets have, as it were, even less structure than the minimal structure considered in the "simplest" category. That structure is just what is needed to make the usual sense of cardinality. But, from another perspective, one might again hope that a category-theoretic approach might *help* here: there are various category-theoretic tools which, since they can be stated in the language of external diagrams, need not wear their use of identity on their sleeve.

There is no obvious reason why the two challenges should line up precisely, though: it is not as though *any* category must come with the resources to capture cardinality – that appears as part of the detail of theories such as Lawvere's. So the possibilities include: (1) French and Krause can provide a description of **QSet** as a category, including the resources for an understanding of number, such that quasi-cardinality plays the required role but the category doesn't collapse into **Set**; (2) French and Krause can provide a description of **QSet** as a category, but cannot also include quasi-cardinality with sufficient structure to play the number role, without collapsing into **Set**; (3) French and Krause cannot provide a description of **QSet** as a category, because once they have spelled out the rest of the content of the theory beyond the basic definition of a category, it will turn out that the 1_{-} morphisms interact with the rest in such a way as to deliver the rest of set theory anyway, or at least to undermine the idea that the quasi-sets are collections of nonindividuals – this might motivate a more radical turn still, so that the Qsets don't even form a category, or it might sufficiently undermine the thought that quasi-sets can be treated as mathematical objects to cast doubt on the whole programme. For what its worth, (2) seems to be a natural suspicion, but of course this is speculation, pending further details.

4.5 Conclusion

In the present paper I have tried to articulate ways in which the stories given by Lowe et al. in the *Analysis* thread can indeed be given quantum-mechanical descriptions. In particular, it is relatively easy to find "referential indeterminacy with an ontic source", with two provisos: (1) this is still not *quite* how Lowe thinks of it (because the ontic source has nothing to do with identity), and (2) it relies on subscribing to a view whereby quantum mechanics involves metaphysical indeterminacy of property instantiation (doubts are raised about this also in Skow (2010)).

Returning to the French-Krause alternative, I looked at Smith's recent argument to the effect that French and Krause's project of constructing a theory of *collections* of vague objects cannot make sense. This seems overly conservative from a naturalistic perspective. I have sketched a couple of challenges that naturally arise, but, as far as I can see, whether they can be overcome is an open question.

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Chapter 5
Vague Persons

Kristie Miller

5.1 Introduction

It seems as though many of our ordinary singular referring terms fail to pick out an object that has precise boundaries (either spatial or temporal) and that many of our ordinary predicates fail to pick out a precise property.[1] Call these appearances *the phenomenon of vagueness.* This phenomenon is sometimes treated like an embarrassing illness to be accommodated as best as possible and borne with as little complaint as can be managed. Standardly, this accommodation takes the form of either pointing to a failure of knowledge of the precise meaning of our terms (epistemicism)[2] or pointing to the failure of our ordinary terms to have a precise meaning (semantic indecision)[3] or pointing to the presence of metaphysically vague objects or properties to which our terms determinately refer (ontic or metaphysical vagueness).[4]

With thanks to Sam Baron, David Braddon-Mitchell, Michael Duncan, Johann Harriman and James Norton for the helpful discussion of these issues.

[1] Or, if you prefer, many of our ordinary predicates appear to be such that their extension is not a precise set of objects.

[2] A paradigm defender of which is Williamson (1994).

[3] Defenders of which include Lewis (1986), Lewis (1993), Hyde (1997), Fine (1975), and Beall and Colyvan (2001).

[4] Defenders of such a view include Barnes and Williams (2011), Williams (2008) Smith (2005), and Akiba (2004).

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Among some in the personal identity literature, however, certain features of the phenomenon of vagueness have been embraced as ways of either accommodating or explaining away some of our more intractable intuitions about the identity of persons over time. Most especially, aspects of the phenomenon of vagueness have been marshalled in order to explain, or explain away, one particularly trenchant puzzle regarding personal identity: the fact that disputants typically continue to disagree about which events a person can survive despite the fact that (in at least some cases) they agree about all of the non-identity involving facts. Many, though by no means all, parties to the debate hold that there are no further facts not captured by the non-identity facts that could determine the persistence conditions of persons over time. The identity facts, whatever they are, are determined by the totality of non-identity facts. Call this the *no further fact hypothesis*. Parties who accept the no further fact hypothesis will naturally be puzzled by the resilience of certain personal identity debates. In the light of such acceptance, consider the following two tightly related claims:

(1) UNRESOLVABLE: Some personal identity disputes are unresolvable.
(2) NO-FAULT: Some personal identity disputes are no-fault disputes.

Those I call *traditionalists* hold that (1) and (2) are false. Moreover, they hold (1) and (2) do not even appear to be true. Traditionalists, such as (most typically) animalists and psychological continuity theorists, think that personal identity disputes are resolvable, at least in principle, because there is a fact of the matter as to which events entities like you and I can survive and that is determined by our persistence conditions as outlined by the correct account of personal identity—animalism or psychological continuity theory or whatever else is the correct account. This chapter will have little to say about traditionalists.

Those I call *non-traditionalists* accept at least one of (1) or (2). Unlike traditionalists, they also hold that each of (1) and (2) is intuitively compelling and thus that a rejection of either must be met with an explanation of the appearance of its truth. Non-traditionalists who accept both (1) *and* (2) are sometimes known as conventionalists.[5] Some conventionalists make room for (1) and (2) by holding that conventions (broadly understood to include psychological and social behaviours and dispositions including anticipation and future regard) are themselves proper constituents of persons[6] or partially determine the persistence conditions of persons.[7] Where these conventions differ between persons, the persistence conditions of those persons likewise differ. So sometimes each of a pair of disputing parties might be right about what they *themselves* would survive and such disputes are therefore (in some good sense) no-fault ones.[8] These versions of conventionalism will not be

[5] Olson (1997b) describes some such views as being relativist and offers a critique of views of this kind.

[6] See, for instance, Braddon-Mitchell and Miller (2004).

[7] See, for instance, Braddon-Mitchell and West (2001).

[8] For further discussion of these kinds of no-fault disputes, see Robinson (2004) and Miller (2013).

further discussed. Rather, this chapter will consider versions of conventionalism I have elsewhere called *weak conventionalism.*[9] These views typically appeal to semantic indecision to explain UNRESOLVABLE and NO-FAULT.[10]

Suppose that Jenny claims that person, P, survives event, E, at *t*, and John claims that P does not survive E at *t*. According to weak conventionalists, if this is a no-fault disagreement, it is because at least one of the terms is semantically undecided or, as I will sometimes say, semantically *imprecise*. Most usually the weak conventionalist will hold that it is the term 'person' (or the name 'P') that is imprecise. Thus, the weak conventionalist will hold that there are multiple equally eligible candidates to be the referent of 'P', some of which exist after *t* and some of which do not. That is, on some admissible precisification of 'P', there is no object that 'P' picks out after *t*, and on some admissible precisification, there is a unique object that 'P' picks out after *t*.[11] Thus, given the imprecision in 'P', neither Jenny nor John are at fault when they disagree. Moreover, Jenny and John's debate is unresolvable, at least if that debate is one in which they continue to use ordinary imprecise terms. There is the possibility of some kind of resolution, of course, since both Jenny and John could choose to subtly change the meaning of some of their terms to agreed ones that are semantically precise. They could then agree that P does indeed survive after *t*. But arguably that would be to subtly change the disagreement at issue, not resolve their initial disagreement.

There are other kinds of non-traditionalism that accept (1) or (2) but not both. Epistemicist non-traditionalists, for instance, accept (1) UNRESOLVABILITY but reject (2) NO-FAULT. Since they hold it is in principle impossible to know to what (at least some of) our terms determinately refer, they suppose that some disputes are in principle unresolvable. Nevertheless, such disputes are not no-fault disputes. At most one of the parties to a genuine dispute is correct since at most one of the parties is right about the meaning (and hence reference) of the terms involved.[12] Yet it is easy for the epistemicist to explain the attraction of (2). The parties might be epistemically faultless in that both have done everything that it is, in principle, possible to glean the correct view. Yet there is still space for fault on one or more parts of disputants if no amount of careful investigation can, in principle, reveal the truth.

Alternatively, metaphysical non-traditionalism, in at least some forms, accepts (2) but rejects (1). The metaphysical non-traditionalist contends that in some cases

[9]For discussion of the various kinds of conventionalism, see Miller (2009).

[10]A paradigm example of this is Sider (2001b).

[11]Where this is the case I will say that it is indeterminate whether P survives E at *t*. More generally, I will say that where 'P' is semantically imprecise such that there are candidates $O_1 \ldots O_n$ that are equally eligible to be the referent of 'P', then the reference of 'P' is indeterminate. Moreover, if some, but not all, of the candidates have feature F, then I will say it is indeterminate whether P is F. It is determinate that P is F iff every candidate to be the referent of 'P' is F. One could embrace semantic indecision without also accepting these claims about indeterminacy and determinacy but nothing of import to the paper hangs on such a choice.

[12]To say that at most one of the parties to the dispute is correct is not, of course, to say that that party is in any position to *know* that they are correct.

it is a metaphysically vague matter whether some person survives some event. Here, it is plausible to say that were the disputants to be in possession of the facts their dispute would be resolved. That is, no further disputation would make sense once both parties became aware of the relevant facts. Yet the metaphysical non-traditionalist can explain the appearance of the dispute being unresolvable. For it is easy, mistakenly, to suppose that the dispute is resolvable only if we can, in principle, determine whether the person in question survives, or fails to survive, the event in question. Yet if there is metaphysical vagueness, this is a mistake: some disputes will be resolved by pointing to metaphysically vague states of affairs. At least one version of metaphysical non-traditionalism rejects (1) but accepts (2). This version interprets the claims of disputing parties 'leniently', suggesting that they ought not be read as claims about what is determinately the case. Then when Jenny says that P survives E at *t*, and John says that P does not survive E at *t*, we are to suppose that what Jenny says is true just in case it is not determinately the case that P does not survive E at *t* and we are to suppose that what John says is true just in case it is not determinately the case that P does survive E at *t*. Construed this way, both Jenny and John speak the truth when they make their assertions. Arguably then, they are involved in a no-fault dispute.[13]

Non-traditional accounts of personal identity, therefore, make liberal use of some of the machinery of the standard accounts of the phenomenon of vagueness. In what follows I will first briefly suggest that metaphysical non-traditionalism is less appealing than either epistemicist non-traditionalism or weak conventionalist versions of non-traditionalism. This, however, is worrisome given that recently both semantic indecision and epistemicism have come under fire as responses to the phenomenon of vagueness on the grounds that they yield implausible outcomes with respect to personal identity. In light of this, Sects. 5.3 and 5.4 outline two different sorts of objections to these accounts of vagueness that arise from considerations of personal identity. If these arguments succeeded, they would be particularly devastating to non-traditionalists since in effect they attack the core claims of non-traditionalism but not the core claims of the three standard accounts of vagueness. This chapter will argue that these objections are misplaced. Finally, in Sect. 5.5 I consider some ancillary benefits of the resulting accounts of personal identity by attending to some features of diachronic prudence.

5.2 Why Not Metaphysical Non-traditionalism?

We have seen that there are various ways of being a non-traditionalist. In Sects. 5.3 and 5.4, I will consider some recent arguments against epistemicism and semantic indecision that are, without intending to be so, targeted arguments against

[13]One might contend that they are really not involved in any dispute at all in this case, since they are not truly disagreeing with one another. In that case one might say that (2) is strictly speaking false, but we can explain why it seems to be true.

epistemicist non-traditionalism and weak conventionalism. Before doing so, however, it is worth outlining some reasons why I take either of these two views to be preferable to metaphysical non-traditionalism given that this latter view is immune to these objections. I do not claim that these reasons are, in the end, definitive or that there are not possible responses on behalf of the metaphysical non-traditionalist; I suggest only that they provide good reasons to try to defend weak conventionalism and epistemicist non-traditionalism from the objections countenanced in Sects. 5.3 and 5.4.

Consider two different kinds of disputes that Jenny and John might have. One dispute proceeds as follows: Jenny contends that some person, P, persists until exactly temporal instant t and no longer, while John contends that P persists until $t + 1$ (some instants after t) and no longer. Jenny and John both agree about what sorts of events P can survive. P can't survive being rolled over by a combine harvester, but P can survive having his head replaced with a sophisticated pumpkin whose internal states are functionally analogous to those of P's current head. Call this a *type I* dispute.[14]

Another dispute goes as follows: Jenny contends that P is the sort of thing that can survive having his head replaced with a sophisticated pumpkin whose internal states are functionally analogous to those of his current head, and John demurs, contending that this would be death to P. Call this a *type II* dispute.

Now suppose that one wishes to make sense of UNRESOLVABLE and NO-FAULT by appealing to metaphysical non-traditionalism. Suppose one thinks Jenny and John have a no-fault dispute regarding whether P survives E at t. Moreover, suppose this is a type I dispute. Then the defender of metaphysical non-traditionalism can hold that P has metaphysically vague temporal borders. P determinately exists until $t-$, and determinately does not exist at $t+$, but it is metaphysically vague whether P exists at t. Or perhaps something, O, determinately exists at t, but what is metaphysically vague is whether O is P. This might seem like a plausible enough thing to say where throughout the relevant interval of time it is, for instance, a vague matter whether O has any conscious experience or indeterminate whether O is a person. Perhaps O is severely comatose with no hope of recovery. Thus it might be indeterminate which sortal kind O falls under and thus metaphysically vague whether O is P and thus vague whether P has survived or not. So let us grant that metaphysical non-traditionalism can make sense of type I disputes.

Now let us complicate matters. Suppose there is a series of stages—where, roughly, a stage is an instantaneous or short-lived object—united by relations of psychological continuity.[15] Call this the PC series, and the object composed of the stages of the series *PC*. Suppose there is a series of stages united by relations of

[14] Some aspects of the problem of the many, for instance, lend themselves to type I disputes.

[15] The following discussion is framed in terms of a four-dimensionalist, and largely perdurantist, view about persisting objects. I take it, however, that nothing in principle debars one from translating everything said here into terminology that is friendly to a three-dimensionalist perspective on persistence. (See, e.g. Miller 2005.)

human animalism. Call this the HA series, and call the object composed of this series *HA*. Now suppose that PC and HA coincide during some temporal interval, that is, during that interval they share all and only the same spatial parts. For simplicity, let us take coincidence to be a matter of the sharing of (maximal) temporal parts. Then stages correspond to maximal temporal parts of HA and PC and such stages coincide throughout interval T just in case they overlap during T.

The simplest case of a type II dispute occurs where there is an event, E, at *t*, such that HA survives that event but PC does not (or vice versa). Suppose, for instance, that Jenny holds that Hermione survives event E, and John maintains that she does not. Then, by analogy with type I disputes, we might explain this disagreement by noting that it is metaphysically vague of which stages Hermione is composed: the HA stages or the PC stages. Since these coincide before *t*, we do not notice such vagueness. After *t*, however, it is metaphysically vague whether Hermione has survived, since it is metaphysically vague whether she is composed of the HA stages after *t*. This is a paradigm type II dispute, since it centres around a disagreement about the kind of events Hermione can survive. The response just outlined is coherent. But it is starting to stretch plausibility. After all, it is not that the HA stages post *t* are (or need be) in any sort of state such that one is inclined to say that it is metaphysically vague whether or not they compose a person. That is, the HA stages after *t* might not in any sense be a borderline case of instantiating 'personhood'. Yet this account forces us to say that for many years after *t* (assuming HA continues to persist for many years), it is metaphysically vague whether Hermione has survived or not.

To take a more complicated case, suppose that PC and HA share the same temporal parts up until *t*. At *t* they diverge and HA and PC continue to exist for the next 10 years without sharing stages. Now suppose that Jenny and John are having a dispute that, while not a paradigmatic type II dispute, is clearly of type II origin. Jenny claims that at $t+$ Freddie is asleep in Bundanoon, while John claims that at $t+$ Freddie is awake in Sydney. We can explain this dispute by noting that it is metaphysically vague which sets of temporal parts compose Freddie: the set of HA stages or the set of PC stages. Suppose that at $t+$ an HA stage is asleep in Bundanoon and at that time a PC stage is awake in Sydney. Then since it is a vague matter of which parts Freddie is composed after *t*, there is no determinate fact of the matter regarding whether he is awake or asleep, in Bundanoon or in Sydney. But notice that, as in the previous case, this is now a fairly extreme kind of vagueness that we need to embrace. The defender of metaphysical non-traditionalism needs to say that there are 10 years worth of PC parts and 10 years worth of HA parts, and for that entire interval, it is metaphysically vague which of those parts are parts of Freddie. The problem is that there is not merely a single set of temporal parts united by what we might call a *proto*-PI relation—a relation such as that which unites HA or that which unites PC each of which is a prima facie candidate to be the relation that unites stages into a person—such that it is vague whether those parts are parts of P. Rather, we have two sets of temporal stages each united by proto-PI relations, extending throughout some significant temporal interval, such that for both series of stages it is vague whether those stages are parts of Freddie. But that makes Freddie a most peculiar sort of object after *t*. Indeed, the cost of making sense of the idea that

John and Jenny are engaged in a no-fault dispute about whether Freddie is awake or asleep is that Freddie turns out to be a massively metaphysically vague object with respect to his location, kind and most other properties.

This is not to say that metaphysical non-traditionalism is a non-starter. The non-traditionalist might think that at least some type II disputes are not disputes in which there is any metaphysical vagueness. But the appeal of non-traditionalist accounts is that they can accommodate, or ably explain away, the appearance of a range of disputes being no-fault. Metaphysical non-traditionalism does this by appeal to metaphysical vagueness. That means there is a range of type II disputes that other versions of non-traditionalism can accommodate (or explain away) as being no-fault disputes that the metaphysical non-traditionalist cannot. Metaphysical non-traditionalism is, therefore, less flexible than the other non-traditionalist accounts. Naturally enough, if one is independently motivated to say that some of these type II disputes neither appear to be unresolvable nor no-fault, then one will not consider this an objection to metaphysical non-traditionalism. Nevertheless, for those who do not share that intuition, this provides a reason to canvass recent objections to epistemic non-traditionalism and weak conventionalism with a view to determining whether those objections meet their mark.

5.3 Metaphysical Objections

What I will call *the metaphysical objection* is a perfectly general objection to semantic indecision. The contention, roughly, is that if some term, 'T', is semantically imprecise, then there must exist some number of candidate objects each of which is equally eligible to be the referent of 'T'. Depending on one's other metaphysical commitments, however, one might not in all cases be inclined to think that the relevant suite of candidate objects exists. Call this the *too-many-objects* version of the objection. While the objection is typically levelled at defenders of semantic indecision, it has recently been noted by Smith (2007) that an analogous objection lies in wait for the epistemicist. For while the epistemicist thinks that our terms have a determinate meaning and pick out a unique referent, they also suppose that we do not and cannot know which referent it is that they pick out. The epistemicist must, therefore, think that there is some range of candidate objects, each of which is, from the point of view of what we can know about the meaning of our terms, an equally good candidate to be the referent of our terms. Therein lies our ignorance. And that suggests that she too is committed to the existence of the self-same range of candidate objects.

Thus, the too-many-objects objection is a general objection to semantic indecision and epistemicism rather than a targeted objection to non-traditionalist accounts of personal identity. As such as I shall set it aside. There is, however, a personal identity-specific version of the too-many-objects objection. Weak conventionalists typically hold that there are a number of candidate objects, each of which is equally eligible to be the referent of imprecise terms like 'I' and 'person'. Therein lies the root of resilient personal identity disputes. Likewise, epistemicist non-traditionalists hold that there exists a number of candidate objects, any of which, for all we can know, might be the referent of our terms 'I' and 'person' although only one of which is in fact the referent of each of said terms. Either way, there must exist a plethora of conscious entities located in more or less the same place at the same time. Each of the entities in question must be conscious if it is to be either an eligible candidate to be the referent of 'I' or 'person' or if it is one of the sets of objects such that we cannot know which of those objects is the referent of 'I' or 'person'. But then we are committed to the existence of many equally person-like objects coexisting in the same location. Call this the *too-many-thinkers* objection.

There are (at least) two roads that potentially lead to many thinkers. Each is associated with either one of two distinct ways in which a term may be imprecise or one of two distinct ways in which our (in principle) knowledge fails to tell us which of a set of candidate objects is the one to which our term determinately refers. Consider semantic indecision first. It might be that a term, 'T', is imprecise in that 'T' fails uniquely to pick out an object, O, among a set of candidate objects, S, each member of which is of the same kind. Or it might be that a term, 'T', is imprecise in that 'T' fails to uniquely pick out an object, O, among a set of candidate objects, S, each member of which is of a different kind. In the former case, it is very clear what sort of the thing 'T' refers to, but it is unclear precisely which of many very similar things 'T' picks out. In the second case, though there are presumably relevant similarities between the members of S (otherwise they would not be candidates to be the referent of 'T'), the members of S are also substantially different in that they fall under different kinds. Call the former *exactitude indecision* and the latter *sortal indecision.* An analogous distinction arises for epistemicists. For epistemicists the relevant distinction is between two different kinds of, in principle, knowledge failure: what we might call *exactitude failure* and *sortal failure.* Exactitude failure occurs when, in principle, we cannot determine, for some term 'T', which of a set of very similar candidate objects of the same kind is the referent of 'T'. Sortal failure occurs when, in principle, we cannot determine, for some term, 'T', which of a set of candidate objects of different kinds is the referent of 'T'.

Exactitude indecision and exactitude failure, on the one hand, are associated with phenomena such as the various problems of the many. In such cases there appear to be a large number of equally good precise ways of drawing the borders of some object, O.[16] Appealing to exactitude indecision in this case tells us that there is a very large set of precise objects, each of which is of the same kind and which differ only minutely in their composition, such that it is undecided to which of those objects 'O' refers.[17] Appealing to exactitude failure tells us that the there is a very large set of precise objects, each of which is of the same kind and which differ only

[16]For more discussion of the problem of the many, see Weatherson (2003) Unger (1980), McKinnon (2002), Lowe (1982), and Lewis (1993).

[17]For this solution to the problem of the many, see Weatherson (2003), and for a contrary view, see McKinnon (2002).

minutely in their composition, such that, in principle, it is impossible to know to which of these objects ‘O’ refers.[18]

Sortal indecision and sortal failure, on the other hand, are associated with cases of coincidence. Suppose, for instance, HA and PC coincide for most of their existence. HA and PC fall under two distinct sortals. Sortal indecision occurs, with respect to the term ‘person’, if ‘person’ is undecided between referring to two equally eligible candidates that fall under distinct sortals: HA or PC.[19] Sortal failure occurs, with respect to the term ‘person’, if it is impossible to know to which candidates that fall under different sortals—HA or PC—‘person’ refers.

It is worth distinguishing these two kinds of indecision and epistemic failure since epistemicist non-traditionalists and weak conventionalists typically respond to UNRESOLVABLE and NO-FAULT by appealing to sortal indecision and sortal failure. Neither need also be committed to thinking that there is exactitude indecision or exactitude failure. Either could, for instance, hold that sortal terms like ‘person’ and ‘human animal’ are maximal and therefore that we know that they determinately pick out the maximal object, at any time, that falls under the sortal. Alternatively, either might hold that there exist metaphysically vague objects and that terms that appear to involve exactitude indecision or exactitude failure determinately refer to metaphysically vague objects. No large set of precise objects need be posited with respect to each semantically imprecise or epistemically opaque term, ‘T’, such that ‘T’ fails determinately to pick out one of those objects or such that we cannot know to which of those objects ‘T’ refers.

This is important, since it tells us that neither weak conventionalists nor epistemicist non-traditionalists need be committed to the existence of a *plethora* of very similar thinking things all located in roughly the same place. That does not, of course, entirely solve the problem. For even sortal indecision and sortal failure commit their adherents to some, rather smaller, number of candidate thinking things. Is there something puzzling about the existence of more than one thinker located in the *very* same place at the same time? Some certainly think so.[20]

Once again, it is worth disentangling two related worries that one can group under the too-many-thinkers objection. The first is really the *too-many-thoughts* objection. It is, roughly, the contention that there is something puzzling about the existence of multiple sets of thoughts occurring in the very same place and time, each set of which is the thoughts of a distinct object. The second is best thought of as the *whose-thoughts-are-whose* objection.[21] Here the concern is that if there were multiple sets of thoughts and objects having those thoughts, then it would be impossible to know whose thoughts were whose, and, moreover, it would be impossible for each of us to self-refer with personal pronouns or to know which object we pick out when we think about ourselves.

[18] See, for instance, Williamson (1994).

[19] See Sider (2001a, b) for a defence of this view.

[20] See, for instance, Olson (1997a) and Snowdon (1990). For a contrary view, see Noonan (1999).

[21] Olson (2002) makes both objections.

The too-many-thoughts objection is really a specific version of a more general issue that arises wherever there exist coinciding objects of different kinds. Suppose that a statue and a lump of clay coincide throughout some interval. The clay weighs 5 kg and the statue weighs 5 kg. So why don't they jointly weigh 10 kg during the interval in which they coincide? Why don't they jointly have four arms during that time, given that each has two arms?

The standard response to this worry, to which I have little to add, is to say that there is a single pair of arms (not two pairs) and a single weight of 5 kg (not two such weights) and that these are the arms and weight of the statue and the clay. They share the same weight and same arms because they are composed of the same matter. The weight is a joint weight and the arms are the arms of both the clay and the statue.

By analogy then, we can say that there is a single set of thoughts occurring whenever there are coinciding thinking objects. Thus, there is a single set of mental events, but those mental events are the events of each of the coinciding objects. More carefully, (framed in terms of a four-dimensionalist account of persistence) there is a single thing doing the thinking during the time of coincidence: the thing that is a temporal part of both of the coinciding objects and in virtue of which they coincide at that time. There is, therefore, a single set of mental events during that time. But since the thinking thing whose thoughts they are is a part of (at least) two persisting objects, those mental events get to count as the mental events of each of those objects. Thus, there are not two sets of mental events. There is a single set of 'shared' events: shared because the events are those of an object that is a proper part of two further objects. At the very least, if one thinks that there is no problem explaining why the lump and statue do not weigh 10 kg rather than 5, there seems no additional problem in explaining why there are not, in fact, too many thoughts. So far then, we have found no version of the metaphysical problem that specifically targets non-traditionalist accounts of personal identity. Perhaps, though, the whose-thoughts-are-whose objection is more pressing. It is to this objection that I now turn.

5.4 Whose Thoughts Are Whose?

Weak conventionalists typically suppose not just that terms like 'person' are semantically imprecise but also that the first-person pronoun is imprecise. Likewise, we can imagine that epistemicist non-traditionalists will hold that although the first-person pronoun determinately picks out a unique referent, it is impossible to know which referent that is. Thus, both are committed to thinking either that first-person reference is indeterminate (given our ordinary language) or that it is impossible to know its reference. Recently Smith (2007) has argued that either of these commitments is deeply implausible. In what follows I will outline Smith's argument as it pertains to semantic indecision and then reframe the argument as Smith intends it to apply to epistemicism. If Smith's argument succeeds against one or both of these views, then it shows that one (or both) of weak conventionalism or epistemic non-traditionalism are hopelessly flawed. After all, a defender of semantic indecision or epistemicism could simply respond to Smith's argument by denying that terms like 'I' and 'person' are semantically imprecise or that their exact referent is unknowable. But these last two claims lie at the heart of weak conventionalism (in the former case) and epistemic non-traditionalism (in the latter case). After laying out Smith's argument, I will suggest that Smith's argument against semantic indecision fails and that he offers no novel argument against epistemicism. In either case, non-traditionalist accounts of personal identity emerge unscathed.

Smith's argument proceeds as follows:

1. (Idr)[22] $\exists x$ definitely I am identical with x (assumption).
2. I am a composite material object (assumption of composite materialism).
3. Semantic indecision is the right way to understand vagueness (assumption).
4. The singular personal pronoun 'I' refers to a composite material object (from 2).
5. It is a vague matter whether I am composed of an even number of particles (plausible assumption for illustration).
6. There exists an x and a y such that x is composed of an even number of particles and y is composed of an odd number of particles and it is indeterminate whether 'I' refers to x or y (from 3 and 4).
7. It is not the case that $\exists x$ definitely I am identical with x (from 5).

(5) is supposed to be the sort of paradigm claim of which we want to use semantic indecision to make sense. Given what was said in the previous section, however, the weak conventionalist might not want to make sense of claims like (5) in terms of semantic indecision. Nevertheless, there will be some relevantly similar claims that she can switch for (5). Since Smith finds (Idr) unassailable he argues that we must either reject composite materialism, (2), or semantic indecision, (3). Since weak conventionalists typically endorse both claims, this is a problem. Why think (Idr) is true? Smith motivates (Idr) by noting that any assertion of 'I exist' is indubitably true.

Given that, there must be something with which it is definitely the case that I am identical. Moreover, he suggests, the denial of (Idr) is incoherent. To accept the denial of (Idr) is to accept that it is not the case that $\exists x$ definitely I am identical with x, which is equivalent to accepting the following:

CONTRA(Idr): $\forall x$ either it is definitely not the case that I am identical with x or it is indefinite whether I am identical with x.

Thus for any x, either 'I' definitely fails to refer to x or it is indefinite whether 'I' refers to x. But then, Smith notes, we are committed to the following sentence he calls Odd.

ODD: I exist and $\forall x$ either the use of 'I' in this sentence definitely fails to refer to x or it is indefinite whether the use of 'I' in this sentence refers to x.

[22] This is Smith's shorthand for the claim that follows.

The sentence is problematic because the universal quantifier ranges over everything and thus ranges over the use of ‘I’ in the sentence. Thus, if I utter ODD I assert that either I definitely fail to refer to myself with the use of ‘I’ in the sentence or it is indefinite whether I refer to myself with that use of ‘I’. So an assertion of ODD involves two features that are in tension. First, I seem to take myself to have successfully self-referred with my use of the first-person pronoun when I state that I exist. But then I seem to take that self-same pronoun either to definitely fail to refer to me or to be indefinite in whether it refers to me or not. But I cannot both take the personal pronoun to have self-referred and take it to have definitely failed to refer or to be indeterminate in whether it refers to me. Thus, concludes Smith, a rejection of (Idr) is incoherent.

What is going on here? Let’s replace the use of ‘I’ in ODD with a proper name: Todd. Suppose Sheila asserts the following at *t*:

*ODD**: Todd exists, and $\forall x$ either the use of ‘Todd’ in this sentence definitely fails to refer to x or it is indefinite whether the use of ‘Todd’ in this sentence refers to x.

Suppose ‘Todd’ is semantically imprecise. Then, plausibly, if Sheila asserts, at *t,* ‘Todd exists’ what she asserts is determinately true just in case for every four-dimensional object that is a candidate to be the referent of ‘Todd’, every one of those candidates has a temporal part present at *t*. Thus, there might be times at which it is not determinately the case that ‘Todd exists’ is true, namely, any time at which only some four-dimensional objects that are candidates to be the referent of the name have temporal parts present. But let us suppose that this is not so. It is determinately the case that Todd exists at *t*. So ‘Todd exists’ is true.

The next clause of ODD* tells us that every x either is such that ‘Todd’ fails to refer to it or is such that it is indeterminate whether ‘Todd’ refers to it. In effect that is to say that every x is either definitely not a referent of ‘Todd’ or is a candidate to be the referent of ‘Todd’ under some (but not all) precisifications of the name. So far, then, there seems nothing objectionable in Sheila uttering ODD* at *t.*

Suppose, however, that some x ranged over by the universal quantifier is Todd. Then, the second clause of ODD* says either that Todd is definitely not a referent of ‘Todd’ or that it is indeterminate whether ‘Todd’ refers to Todd. But that cannot be a sensible claim for Sheila to assert. We will return to Todd and Sheila shortly. First, let us see how the argument is supposed to be extended to be an objection to epistemicism.

Here, Smith suggests that in place of (Idr) we have (Icl), the claim that there is an x, such that *clearly* I am identical with x. Here, ‘clearly I am identical with x’ is to be read as the claim both that there is an x such that I am definitely identical with x and that it is clear, epistemically speaking, that I am identical with that x. Thus we have the following argument:

(i) (Icl) $\exists x$ clearly I am identical with x (assumption).
(ii) I am a composite material object (assumption of composite materialism).
(iii) Epistemicism is the right way to understand vagueness (assumption).

(iv) The singular personal pronoun 'I' refers to a composite material object (from ii).
(v) It is a vague matter whether I am composed of an even number of particles (plausible assumption).
(vi) There exists an x and a y such that x is composed of an even number of particles and y is composed of an odd number of particles and it unclear whether 'I' refers to x or y (from iv and v).
(vii) It is not the case that $\exists x$ clearly I am identical with x (from vi).

Smith takes it to be obvious that I am clearly identical to some x, namely, that I am clearly identical to myself and that 'I' clearly refers to me. Furthermore, a denial of (Icr) is taken to be problematic because it commits one to CONTRA(Icr).

CONTRA(Icr): $\forall x$ either clearly it is not the case that I am identical with x or it is unclear whether I am identical with x.

If CONTRA(Icr) is true then there exist multiple objects each of which is an equally, metaphysically speaking, good candidate to be the referent of 'I', such that it is *merely* semantic facts that determine that one, rather than the other, is the referent. This commits us to unacceptable metaphysical profligacy, says Smith, insofar as it requires that we think that there are multiple thinking things located at the same place. The worry, here, is not so much that such a view commits us to more things that we would like (that it is profligate per se) but rather that it implausibly commits us to the existence of such things: there simply are not many thinking things located in the same place at the same time.

One thing to say is that the argument against semantic indecision is stronger than that against epistemicism. The defence of (Icr) ultimately takes us back to the metaphysical argument already considered: the too-many-thinkers objection. Unlike the argument for (Idr), which offers us independent grounds (i.e. independent from any other more general arguments) to think that the first-person pronoun must be semantically precise, the argument for (Icr) does not offer us independent grounds to think that it must be clear to what the first-person pronoun refers. It does not, therefore, really introduce a new, independent, argument against epistemicism. Of course, one might simply thump the table and assert that it is implausible to suppose that for some x, it is unclear whether I am identical to x and that is why we should accept (Icr). But that would be question begging against the epistemicist.

So if there is a *new* argument here, it is really only an argument against semantic indecision. What ought the friend of semantic indecision to say? Here is a first-pass suggestion. She ought to reject (Idr) without embracing ODD or ODD*. Suppose, for simplicity, that the proponent of semantic indecision holds that the phenomenon of vagueness is to be fully accommodated without appeal to metaphysically vague objects. Then, every object within the domain of the universal quantifier is perfectly precise. By stipulation the name 'Todd' is semantically imprecise. This means that there is no unique object in the domain of quantification that answers to the name 'Todd': there are various objects, Todd_1, Todd_2, Todd_3 (let us suppose), each of which is a candidate to be the referent of the name under different precisifications.

So we cannot infer that there is a unique object—Todd—picked out by 'Todd' in the phrase 'Todd exists', such that we can find *that* object in our domain of quantification and then conclude that *it* is something that will either definitely fail to be the referent of the name 'Todd' or will be such that it is indefinite whether it is the referent of 'Todd'. No such object exists. Only Todd_1, Todd_2 and Todd_3 exist, and each of these is merely a candidate to be the referent of the name 'Todd'. If we substitute Todd_1 for x in ODD*, then we assert that 'Todd exists' and there is an x, Todd_1, such that either it is definitely the case that 'Todd' fails to refer to Todd_1 or it is indefinite whether 'Todd' refers to Todd_1. And that is precisely the outcome we desire since the latter clause is true.

With this in mind, let us turn back to the original sentence: ODD. Recall that the problem arises when we take the object, I, to be in the domain of precise objects and thus commit ourselves to saying either that 'I' definitely fails to refer to I or that it is indeterminate whether it refers to I. If we resolve this problem in the way we did with ODD*, we will say that it is illegitimate to suppose that I is in the domain of precise objects over which we can quantify. If 'I' is semantically imprecise, there are multiple candidates to be its referent. 'I exist' is determinately true when uttered at t if and only if every candidate to be the referent of 'I' has a temporal part present at t. Let us assume that to be true and return to the assumption in due course. Thus, although 'I exist' is determinately true at t, there is no unique, precise object, I, which is in the domain of quantification, such that *that* object either is definitely not the referent of 'I or is such that it is indeterminate whether it is the referent of 'I'. The precise objects in the domain of quantification are the various candidates to be the referents of 'I': I_1, I_2, I_3. And it is indeed indeterminate, of each of *those*, whether it is the referent of 'I'. It does not follow from that, however, that it is indeterminate whether 'I' refers to I.

Thus, it seems, Smith's argument is not in itself a threat to weak conventionalism. The weak conventionalist can point out that the argument illicitly infers from the truth of claims like 'I exist' to the existence of a unique, precise object, I, in the domain of quantification. Nevertheless, features of the argument suggest there is a problem. For there is, plausibly, a difference in status between an utterance of 'I exist' and one of 'Todd exists'.

As Smith notes, it is difficult to see how an utterance of 'I exist' could fail to be true. While we stipulated that 'Todd exists', uttered at t, is determinately true, it could fail to be so. Our suggestion, with respect to Todd, was that 'Todd exists', uttered at t, is determinately true just in case for every candidate to be the referent of 'Todd' those candidates have a temporal part present at t. But now consider 'I exist', uttered at t. Suppose (for illustration) that I am essentially a person and that 'person' is semantically imprecise. Suppose there are two kinds of candidates to be the referent of 'person': a four-dimensional human animal and a four-dimensional psychologically connected object. The two particular candidate objects are HA and PC. Now suppose that at t there exists a temporal part of PC but not of HA. We have represented the weak conventionalist as maintaining that terms like 'person' are semantically imprecise and that this imprecision can explain why it might be indeterminate whether a particular person exists at some time. But this seems inconsistent with the thought that, at *t,* whatever it is that utters 'I exist' will utter a truth. If it is persons who make first-person utterances, then it follows that the person *does* exist at *t* and that *that* person is, determinately, the referent of 'I' and that 'I' is not semantically imprecise. In our example, then, it follows that 'I' refers to PC and not HA and thus that PC is, determinately, the referent of 'person'.

Moreover, one might worry that if terms like 'person' are imprecise and 'I' is precise, then the sort of link that we usually expect to find between the notion of personhood and the first-person pronoun will be broken. Suppose 'I' is precise, and suppose every utterance of 'I' picks out a person at a time. Now suppose there is more than one candidate to be the referent of 'person', at *t*, and it is indeterminate to which of these candidates the term refers. Now suppose there is an utterance of 'I', at *t*, that is shared by all of those candidates. That utterance determinately picks out just one of the candidates as the referent of 'I'. But, arguably, if just one of the candidates is the referent of 'I', then they are not *equally* eligible to be the referent of 'person': one is more eligible, namely, the one picked out by 'I'. If that is so, then 'person' is not imprecise: it picks out the most eligible candidate. Perhaps there are ways to finesse the relationship between personhood and uses of the first-person pronoun. The point is just that taking 'I' to be precise raises further issues for the weak conventionalist.

This is not Smith's argument. But it is a potent one. What ought the weak conventionalist to say? Here is a suggestion. She could reject the claim that all terms refer to four-dimensional wholes. Instead she might say that some terms refer to temporal stages of wholes. One might, plausibly, think that phrases like 'person', 'human animal' and 'psychologically continuous object' pick out four-dimensional wholes. After all, it is a cornerstone of the literature about persons that one of the features that makes them important is that they are temporally extended: they are things that plan, reason, act, remember, anticipate, are held accountable, and so forth. Nothing that is instantaneous could be a person. At least, so many of us are inclined to think. Yet we clearly need to be able to quantify over shorter-lived stages of persons. Whole four-dimensional persons have radically inconsistent desire profiles. Whole four-dimensional persons are not the locus of planning or action. Short-lived objects have (one hopes) consistent preferences, and these are the objects that plan, reason and act. In effect, such short-lived objects plan and act for the benefit of other short-lived objects—those objects that are parts of one and the same four-dimensional whole. Call these short-lived stages of a person that are the locus of reason, planning and action *action-stages,* where reasoning and planning is taken to be a kind of action.

Then one might suggest that the first-person pronoun refers not to a four-dimensional whole but to an action-stage. How does this help? First, let us make a simplifying assumption that 'I' refers, determinately, to an action-stage. We will return to this assumption shortly. If that were the case, then the weak conventionalist would, like Smith, accept (Idr) and therefore reject the coherence of ODD. But in that event, how can she accommodate the semantic indecision of terms like 'person', much less provide an account of UNRESOLVABILITY and NO-FAULT?

Well if some terms, like 'I', refer to stages rather than wholes, we need to appeal to an alternative stage-theoretic semantics to make sense of ordinary claims involving those terms. This stage-theoretic semantics construes talk of an object, O, having been F, or going to be F, as being true just in case O has a *temporal* counterpart in the past that is F or a temporal counterpart in the future that is F.[23] Temporal counterpart relations are like modal counterpart relations in that various features of context determine which counterpart relation is the one that ought to be invoked in order to evaluate a particular claim. So, for instance, suppose that 'cow' were (surprisingly) a stage-theoretic term that picks out a cow-stage. Then 'this cow was once brown' is true if, roughly, there is a past temporal counterpart of the current cow-stage and that stage is brown. More carefully, since temporal counterpart relations are aplenty, the use of 'this cow' invokes a particular temporal counterpart relation: the gen-identity relation that, according to standard four-dimensionalism, unites cow-stages into a four-dimensional cow-whole. Thus, although there might be an object composed of this cow-stage plus a whole lot of chair-stages—call it cow-chair—the chair-stages are temporal counterparts of any stage of cow-chair picked out as such but are not temporal counterparts of any cow-stage picked out as such. For picking out a stage as a cow-stage rather than a cow-chair stage is part of the context that determines which temporal counterpart relation is the appropriate one. Thus 'this cow was once brown' comes out as true (assuming there is a past temporal counterpart that is brown) but 'this cow was once a chair' does not come out as true since none of the chair-stages are temporal counterparts of the cow-stage.

It is to the flexibility of the temporal counterpart relation that the weak conventionalist can appeal in explicating the sense in which 'I' is semantically undecided despite determinately referring to a unique action-stage. For utterances of 'I' might create contexts in which it is not determinate which temporal counterpart relation is the appropriate one relative to which to evaluate those utterances. If that is so, it will be indeterminate which set of stages are the temporal counterparts of a particular action-stage picked out by 'I'. Thus, it may be indeterminate whether utterances such as 'I was P' or 'I will be P' are true.

Suppose, for instance, that there exists both a human animal, HA, and a psychologically continuous object, PC. At t there exists an action-stage, A, which is a stage of both HA and PC. In the future there is a $t+$ stage which is a stage of PC but not of HA—call it $t+$ PC—and a stage which is a stage of HA but not PC—call it $t+$ HA. Suppose $t+$ PC has property, P, and $t+$ HA lacks P. Now suppose that at t A utters 'I will be P'. Suppose that utterance creates a context in which it is indeterminate whether temporal counterpart relation R or R* is being invoked. If R were, determinately, the appropriate relation, then $t+$ PC would be a temporal counterpart of A and 'I will be P' would be true. If R* were, determinately, the appropriate relation then $t+$ HA would be a temporal counterpart of A, and 'I will be P' would be false. But since there is no fact of the matter as to which temporal

[23] See Sider (2001a, b) and Hawley (2001) for an introduction to stage theoretic semantics. See also Rychter (2012) and Viebahn (2013) for further details.

counterpart relation is being invoked, when A utters the phrase ‘At $t+$ I will be P’ it is indeterminate whether that phrase is true.

Notice that this framework is quite flexible. It is an open possibility that ‘I am a human animal’ comes out as indeterminate. It will do so if ‘I’ does not fix a unique temporal counterpart relation such that there are (at least) two *candidate* temporal counterpart relations one of which is a relation that yields all and only human animal stages as temporal counterparts, and one of which yields all and only psychologically connected stages as temporal counterparts. Under these circumstances ‘I am a human animal’ will be indeterminate even if the current action-stage that utters the phrase is a temporal part of a human animal. ‘I will survive event E’ will likewise come out as indeterminate if ‘I’ fails to determinately fix a unique temporal counterpart relation and instead there are two candidate temporal counterpart relations, R and R*, such that given temporal counterpart relation R there exist no temporal counterparts after event E occurs and given temporal counterpart relation R* there exist temporal counterparts after E has occurred.

With this in mind, let us return to our earlier simplifying assumption that ‘I’ determinately refers to a particular action-stage. It is that assumption that allows the weak conventionalist to accept (Idr) and to explain how utterances of ‘I exist’ are indubitably true when uttered. If, however, the weak conventionalist supposes that there are no metaphysically vague objects, then it seems unlikely she will think that the semantics of ‘I’ are such that they uniquely pick out a particular short-lived stage among many overlapping short-lived stages of slightly different temporal extent. So let us see what happens if we relax that assumption. Then the conventionalist will think that ‘I’ is semantically undecided: it is undecided to which of the many short-lived stages it refers. Thus (Idr) will be rejected. Nevertheless, there is a ready account of why it is that utterances of ‘I exist’ are indubitably true: all of the candidate short-lived stages to be the referent of ‘I’ exist when the utterance is made. That is, all such stages have a temporal part present when the utterance is made, that is, in part, what makes that set of short-lived objects candidates to be the referent of the term. Short-lived objects with no temporal stage present when the utterance is made are not candidates to be its referent.

Thus we have, in effect, two levels of imprecision. We have standard semantic indecision with respect to ‘I’: indecision with respect to which of the many equally eligible candidate short-lived objects ‘I’ refers. We also have indecision with respect to temporal counterpart relations: we have contexts in which claims or utterances do not uniquely determine a particular temporal counterpart relation as being the appropriate one. As a result certain claims that are evaluated by appeal to temporal counterparts will come out as indeterminate.

This two-level picture somewhat complicates matters. For at least some range of utterances $U_1 \dots U_n$ involving the first-person pronoun (and perhaps all such utterances, though nothing the weak conventionalist says requires any such strong claim) and for *each* of the action-stage candidates to be the referent of ‘I’ as it is used in those utterances, there will be a set of temporal counterpart relations, $R_1 \dots R_n$, such that it is indeterminate which of those counterpart relations is the appropriate one. Almost certainly the very same set of temporal counterpart relations will be ‘candidate’ temporal counterpart relations for each of the candidate action-stages. After all, it is the same sorts of features that are both selecting which action-stages are candidates to be the referent of ‘I’ and which temporal counterparts ‘I’ invokes. Nevertheless, if action-stages that are candidates to be the referent of ‘I’ have associated with them, say, two temporal counterpart relations, R and R*, then each candidate will have associated with it two slightly different sets of temporal counterparts.

What, then, are we to say about ODD? Suppose there are only three candidate action-stages A_1, A_2, A_3 to be the referent of ‘I’ and two candidate temporal counterpart relations, R and R*. Suppose there is an utterance of ‘I exist’, at *t*, such that the referent of ‘I’ in that utterance does not uniquely pick out one of A_1, A_2 and A_3. We have already seen why that utterance comes out as determinately true (since each of the candidates has a temporal part present at the time of utterance). The next clause of ODD is as follows: for all *x*, either ‘I’ definitely does not refer to *x* or it is indeterminate whether ‘I’ refers to *x*. Then, following Smith, we are meant to substitute in the object, I, for *x*. But as I argued I Sect. 5.4, this is not a legitimate substitution: there is no I in the domain of quantification, there is only A_1, A_2 and A_3. But if we substitute in any of these for *x*, we get, in the case of A_1, either ‘I’ definitely does not refer to A_1 or it is indeterminate whether ‘I’ refers to A_1. And that is quite right. It is indefinite. It is indefinite even though it is indubitable that ‘I exist’ is true when uttered.

The weak conventionalist can, therefore, accommodate it being indubitably the case that ‘I exist’ is true when uttered, without thereby having to accept (Idr). And she can show why ODD is not the incoherent sentence that Smith claims, by noting that one can only substitute in for the variable, *x*, an object that falls within the domain of quantification and that does *not* include putative objects like I.

This is not to say that there aren’t other objections to weak conventionalism or epistemic non-traditionalism or that there aren’t broader objections to semantic indecision or epistemicism (or for that matter metaphysical vagueness). It is just to say that no special problems arise as a result of thinking about issues of personal identity. Moreover, those in the personal identity debate who avail themselves of some of the apparatus developed to make sense of the phenomena of vagueness are not automatically lumbered with all of the problems associated with that apparatus. That leaves views like weak conventionalism and epistemic non-traditionalism live views in the personal identity debate. As we will see in the following section, there are some interesting consequences to taking such views seriously.

5.5 Diachronic Prudence

The advantages of non-traditionalist views like weak conventionalism and epistemicist non-traditionalism are generally thought to lie in the way they can diagnose why certain disputes seem so resilient to resolution and the way they can make sense of the fact that disputing parties often seem to agree about everything except the right thing to say about personal identity. But once such accounts are available there are other uses to which they can profitably be put.

Another vexed issue, this time in the literature on prudence and reason, is whether, and when, future discounting is rational. Future discounting is, most generally, the discounting of value of future commodities or utility. At least some forms of future discounting, most notably the discounting of utility, are generally held to be irrational.[24] It is less clear whether the discounting of commodities is irrational and I will set that issue aside.[25] The discounting of future utility is irrational, the thought proceeds, because there are no grounds to prioritise the utility of a current person-stage at the cost of the utility of some later person-stage by effectively making utility-now worth more than utility-later.

Let us make two sets of distinctions that will be useful in thinking about diachronic prudence. Call person-stages that are parts of the same persisting person p-related person-stages. The first distinction then centres around whether (and when) it is ever rational for a person-stage (or set of stages) to weight differently the utility of stages to which it is p-related. That is, is it ever rational for a person-stage to weight its own utility differently to that of other p-related stages, and is it ever rational for a person-stage to differently weight the utility of two stages to which it is p-related. There are two views one might have. Call the first the *unequal weighting view*.[26] This is the view that it is (at least sometimes under some conditions) rational for a person-stage to differently weight the utility of stages to which it is p-related. Future discounting is one example of unequal weighting, but there are many other unequal weighting strategies. One could defend the rationality of inverse discounting: weighting the utility of current person-stages as less valuable than the utility of future person-stages. Or indeed there are infinitely many different strategies in between these two. The second view is what I will call the *equal weighting view*. This is the view that it is never rational for a person-stage or set of stages to differently weight the utility of p-related stages.

The second distinction centres around the issue of how to determine the relative wellbeing of a whole life. Let us distinguish two views. Call the first the *utility summation view*. This is the view that one's wellbeing—in the sense of one's life wellbeing—is to be determined by summing the utilities of one's person-stages. Here I use 'utility' broadly—it might be a measure of the hedonic pleasure of person-stages or a measure of the degree to which their preferences are satisfied or some other candidate account of utility. Suppose there is something called a utility curve that is the curve we get from plotting the utilities of each of the person-stages and joining up the dots. On this view two very different utility curves nevertheless yield the same wellbeing as long as the utilities thus plotted sum to the same amount of utility. Contrast the utility summation view with the *utility distribution*

[24] See, for instance, Broome (1994) and Parfit (1984).

[25] See Broome for a good discussion of these issues.

[26] This terminology is borrowed from David Braddon-Mitchell, in conversation.

view. This is the view that what matters in determining wellbeing is not just total utility, summed over person-stages, but the distribution of that utility across person-stages. This is the view that two very different utility curves that represent the same amount of total utility might nevertheless reflect different amounts of wellbeing for the persons whose utility curves they are.[27]

Notice that it is consistent with embracing the utility summation view that one thinks it rational for person-stages to have preferences about how utility is distributed across p-related stages. Faced with the prospect of two different utility curves, C_1 and C_2, each of which yields the same overall utility, one might allow that it is rational for a particular person-stage to prefer a utility curve that distributes more utility to its stage and less to some other stage than the reverse. That is, it might be rational for stages to be stage-selfish as long as their selfishness does not result in less utility summed across all of the stages than would otherwise have been the case.

On the other hand, if one thinks that wellbeing is a function of the amount of utility and how that utility is distributed, then one thinks that the very same amount of utility located at some locations in the utility curve is worth less, in terms of overall wellbeing, than utility located elsewhere. It makes sense, on such a view, to suppose it is rational for person-stages to differently weight the utility of p-related person-stages. If, for instance, the right utility curve is one in which utility had later in life contributes more to wellbeing than utility had earlier in life, then it makes sense for earlier person-stages to inversely discount utility if they wish to maximise wellbeing.

Those who advocate the utility distribution view typically do so because they reflect on what sorts of lives they take to be good, and perhaps also on empirical data about happiness and welfare, and conclude that the best lives are those with a particular sort of utility curve. For instance, one might think that a curve that generally goes up throughout life, rather than down, is better. I want to suggest, however, that even if one rejects any of these sorts of reasons to adopt the utility distribution view, there are independent reasons arising from the accounts of personal identity considered in this chapter. At the very least, I suggest, such accounts militate in favour of the view that the unequal treatment of person-stages is not always irrational because certain sorts of facts about the distribution of utility do matter in reasoning prudentially.

To see why, for simplicity, let us begin by assuming that wellbeing is simply the sum of utilities across person-stages. It has already been suggested that there is nothing obviously irrational in person-stage, P, at t, preferring utility curve C_1—which is associated with higher utility for P at t and lower utility for p-related P_1 at t_1—to utility curve C_2 which is associated with lower utility for P at t and higher utility for P_1 at t_1, where C_1 and C_2 are curves whose utilities jointly sum to the same amount. It does not, however, seem rational for P at t to prefer utility curve C_2 to C_3, if C_2s total utility is less than C_3s total utility, even if the C_2 curve affords P at t more utility than P_1 at t_1. In what follows, however, I argue that given certain views about the nature of personal identity, it does not look irrational for P at t to prefer

[27]See Velleman (1991) and Slote (1982) for a discussion of these issues.

utility curve C_2 to C_3. That can only be the case if P's wellbeing is not merely the result of summing utilities across stages, but rather is a function of both utility and distribution. As we will see, the sort of distributional facts I have in mind are facts about whether certain sets of person-stages are determinately temporal counterparts of the stage doing the prudential reasoning.

Suppose weak conventionalists are right about personal identity. Suppose further that that there exists a four-dimensional human animal, HA_1, and a four-dimensional psychologically connected object, PC_1. Suppose that the name 'Jonny' picks out a relatively short-lived object, call it J. It is indeterminate which other short-lived objects are temporal counterparts of J: the stages that are also the stages of HA_1 or the stages that are also stages of PC_1. Suppose that there will be an event that occurs at *t**. Now suppose that J is engaged in prudential reasoning about a time that occurs after *t**. Suppose HA_1 has stages that exist post *t**, but PC_1 does not. If it is indeterminate whether J's temporal counterparts are stages of HA_1 or of PC_1, then it is indeterminate whether Jonny will survive after *t**.

Now suppose that J is in a position to choose between two utility curves. One curve, C_1, distributes utility primarily over the temporal stages of HA_1 and PC_1 prior to *t** and distributes relatively little utility to the HA_1 stages post *t**. The other curve, C_2, distributes a good deal of utility to the HA_1 stages post *t** and somewhat less utility to the HA_1 and PC_1 stages prior to *t**. Let us suppose that the summed utility of the C_2 curve is greater than that of the C_1 curve. Thus HA_1 will have greater summed utility if C_2 is the one that comes to describe the individual utilities of the HA_1 stages than if C_1 is the curve that comes to describe those utilities. Stages of HA_1 ought, if they are rational, to prefer C_2 to C_1. But C_2 is such that a great deal of the utility in question accrues to the HA_1 stages after *t** and thus during a period in which it is indeterminate whether those stages are temporal counterparts of J or not. Thus if J is reasoning about which curve to prefer, it does not, *prima facie,* seem irrational for him to prefer C_1 to C_2, at least for some possible differences in value between the two different summed utilities associated with each curve.

It does not seem irrational for J to prefer C_1 to $C_{2.}$ How can that be given that one curve represents greater total utility than the other? The answer is that it is not irrational for Jonny to care about the distribution of utility across person-stages in cases where it is indeterminate whether some of the person-stages in question are parts of him or not. More specifically, if there is a set of stages, S, and a set of stages S*, such that it is indeterminate whether the stages in S are temporal counterparts of J, and it is determinate that the stages in S* are temporal counterparts of J, then J should, or, at least rationally, can, prefer that utility is distributed over the stages in S* rather than S. Not only can J prefer that utility is distributed over the stages in S* rather than S, but J can prefer a utility cure that distributes utility to stages in S* rather than S to one that distributes utility over the stages in S rather than S*, even if the former curve represents less total utility than the latter. That is because J's overall wellbeing is maximised by having a distribution of the former kind rather than the latter kind. This distributional principle does not prioritise any particular utility curve or any particular shape to a life. So it does not support many of the specific utility distribution proposals that are defended. But it does support it not being irrational for J to discount the utility of stages, such as the HA_1 stages, that exist post t^* given that it is indeterminate whether these are stages of Jonny.

Exactly how this discounting should proceed is unclear and for the purposes of this chapter does not entirely matter: so long as some kind of discounting is not irrational. But suppose (as in our example) there are two candidate sets of stages to be the temporal counterparts of J: the HA_1 stages and the PC_1 stages. Each set of stages is equally eligible to be the set of stages that are temporal counterparts of J. Thus, it is indeterminate whether Jonny survives after t^*: that much is certain. Since there are two sets of candidate stages and Jonny survives according to one precisification of 'Jonny' and does not survive according to the other precisification of 'Jonny', perhaps J ought to discount the utility of the HA_1 stages post t^* by 50 %. That is to say, perhaps J ought to conclude that the utility of the post t^* HA_1 stages contributes half as much wellbeing to Jonny as it contributes to the wellbeing of HA_1. Then the total wellbeing value of the C_2 curve may very well be less than the total wellbeing value of the C_1 curve thus explaining why he might be rational to choose that curve. At any rate, regardless of the details it is plausible that Jonny would not be irrational to discount the utility of the post t^* HA_1 stages at some rate or other, and thus that there is some distribution of utilities across the C_1 and C_2 curves such that even though the summed utilities in the C_2 curve are greater than the C_1 curve, Jonny would not be irrational to prefer the C_1 curve. The very same considerations arise if rather than being semantically imprecise, 'Jonny' is precise but the name picks out a metaphysically vague object such that it is indeterminate whether the HA_1 stages are stages of Jonny. Then it will not be irrational for Jonny to choose C_2 over C_1 for strictly analogous reasons to those just outlined.

Now, you might think that non-traditionalism is not what is doing the work here. Suppose that there are, in principle, knowable facts of the matter regarding to which temporal stages 'Jenny' refers. Jenny, however, is ignorant of some of these facts: she does not know, for instance, how long she will live. Suppose Jenny is considering two possible courses of action, one of which, A_1, will distribute a lot of utility to later temporal stages of Jenny if such stages exist, and the other of which, A_2, will distribute most of its utility of over earlier temporal stages of Jenny. Suppose A_1 would result in greater amounts of total utility than A_2 on the assumption that Jenny lives long enough. But now suppose Jenny has 60 % credence that she will not live long enough. Then, it does not seem irrational for her to decide to perform A_2 rather than A_1. That seems like a case in which Jenny is permissibly risk averse. Moreover, one might think that this case is really just like that of Jonny in that Jenny is not irrational to choose a utility curve that does not proffer the most summed utility.

That, however, is a mistake. J is choosing between two ways of distributing utility across the person-stages in some set, S. It is determinately the case that all of the members of S do, did or will exist. What is undecided is how utility ought to be distributed across those stages. To put it another way, J can, in principle, know all of the facts that *can* be known regarding the reference of 'Jonny' and regarding possible distributions of utility over (in this case) some set of future stages, and it still not be irrational for him to prefer C_1 to C_2. Consider, by contrast, Jenny. There is no set, S*, of person-stages such that it is determinately the case that those stages do, did or will exist and such that it is not irrational for Jenny to choose a utility curve that is not maximising. Suppose that Jenny knows all of the facts that can be known regarding the reference of ‘Jenny’ and regarding possible distributions of utility over some set of future stages. One possibility is that Jenny knows that she will live to a ripe old age, since she knows there are elderly future stages that are picked out by ‘Jenny’. She knows, in effect, that the relevant set of person-stages are those in set S^. Given that S^ is the relevant set of person-stages, it seems straightforwardly true that Jenny should prefer curve C_3 to C_4, since Jenny determinately survives into old age and the C_3 curve gives her greater utility. The other possibility is that Jenny knows that she does not live into old age, since she knows that there are no elderly future stages picked out by ‘Jenny’. Then she knows, in effect, that set S** of person-stages is the one to consider. Given the person-stages in S**, however, it seems straightforward that Jenny ought to prefer curve C_4, since summed over the person-stages in S**, that curve provides greater utility than the C_3 curve. Either way, it is plausible that Jenny ought simply to choose that curve that offers her the greater amount of summed utility over the *appropriate* set of person-stages, namely, the set of person-stages that include all and only the stages of her. The difference between Jenny and Jonny, then, is that under circumstances of full knowledge Jenny will be irrational to choose a curve that does not maximise utility. Jonny, however, will not always be irrational to do so.

With that in mind let us move on to consider epistemic non-traditionalism. Let us suppose that we have a scenario that is structurally just like that of Jonny: Phoebe. It is, in principle, unknowable whether Phoebe is composed of HA_2 stages or PC_2 stages since, although ‘Phoebe’ determinately refers, it is, in principle, impossible to know to which set of stages it refers. There is an event at *t** such that HA_2 stages exist post *t**, but PC_2 stages do not. Prior to *t**, some P-stage of Phoebe is reasoning about the future. It is, in principle, impossible for the P-stage to know whether Phoebe survives post *t**.

The scenario just outlined is relevantly similar to that of Jonny in that the P-stage can know, prior to *t**, every piece of information that is knowable and still not know whether Phoebe survives, in just the same way that the J can know all the knowable information and not know whether Jonny survives. The scenario is relevantly similar to that of Jenny in that there is a fact of the matter as to whether Phoebe survives just as there is a fact of the matter as to whether Jenny survives. So would the P-stage of Phoebe be irrational to discount the utility of the post *t** HA_2 stages? Here is one thought. If the J-stage is fully epistemically apprised, she will think it equally likely that ‘Phoebe’ refers to the HA_2 stages as to the PC_2 stages and therefore she ought to think it 50 % likely that Phoebe survives after *t**. In that case the J-stage ought to discount the utility of the HA_2 stages post *t** since if Phoebe does not survive post *t**, then the utility accruing to the HA_2 stages is not hers.

Alternatively, here is a contrary thought. It is either determinately the case that Phoebe survives after *t** or that she does not. If she does not survive, then there is nothing to reason about, prudentially, after *t**: she has no post *t** interests. But if she does survive after *t** then she survives, and determinately so, as the HA_2 stages. So in reasoning about her interests post *t**, the only stages to be taken into consideration are the HA_2 stages. If, for instance, Phoebe knew that certain choices now would make the HA_2 stages post t^* experience terrible suffering, then since if she survives she *will* experience that terrible suffering and determinately so, she ought to take the full weight of that suffering on board when making decisions about the future. Thus Phoebe ought not discount the utility (or disutility) of the post t^* HA_2 stages.

At this point it is worth taking stock. It has not been the contention of this chapter that under certain circumstances failing to discount is not rational. The contention has been the much more modest one that under those circumstances discounting is not irrational. Perhaps it is not irrational for Phoebe not to discount the utility of the post t^* HA_2 stages. Equally though, it does not seem to be irrational for her to discount those stages. Certainly if Phoebe is an epistemic non-traditionalist she ought to think the following: either determinately I survive or determinately I fail to survive event E. If determinately I survive, then I survive as the HA_2 stages, and if I survive as the HA_2 stages, it will be better for me, overall, if I have chosen a utility cure that maximises utility over all of the HA_2 stages including those post t^*. So it will be the case that I *ought* to have chosen curve C_2. I had reason to choose curve C_2 all along. If I do not survive after t^*, then it will be the case that I ought to discount the utility of the post t^* HA_2 stages 100 %, since they are not stages of me. In such an event it will turn out that I had reason, all along, to choose curve C_1 since that curve maximises summed utility over the stages that are parts of me. That we can agree on. It does not seem irrational for the P-stage to further reason: since I do not know, and cannot know, which of these outcomes is the one that will come to pass, it is rational for me to discount the utility of the post t^* HA_2 stages.

5.6 Conclusion

Accounts such as weak conventionalism are able to respond to objections that suggest that if anything is determinately the case, it is facts about personal identity and, most particularly, facts about first-person reference. Once we see this, we see that these accounts open up the possibility of thinking differently about diachronic reasoning: they open up space to defend the utility distribution view without needing to accept what some might see as recherché claims about the shape of a good life. Given that such accounts are also, arguably, able to make good sense of the resilience of some of the personal identity disputes, they are accounts that are well worth further consideration.

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Chapter 6
Indiscriminable but Not Identical Looks: Non-vague Phenomenal Predicates and Phenomenal Properties

Elisa Paganini

Two numerically distinct objects occupying different regions of space are obviously discriminable to a person who observes them both at a single instant of time: at least the observer will recognize that one occupies a spatial region that the other does not. But in what way are two numerically distinct objects indiscriminable in look? It is tempting to say that in order for two objects to be indiscriminable in look, a sort of *identity* is relevant: the identity concerned here is not numerical identity between objects, but a *second-order* identity between properties instantiated by numerically distinct objects. For example, someone might want to affirm that for two twins to be indiscriminable in look, having identical intrinsic properties is both a necessary and sufficient condition (i.e., they have the same shape nose, the same hair color, the same hairstyle).

As a matter of fact this characterization is inadequate, as two objects may share all their intrinsic properties, but their looks may still be perfectly distinguishable, for example, because one is in shadow while the other is illuminated or because one is next to us and therefore completely visible, while the other is a bit farther away and therefore its appearance is a bit blurry. One may try to meet these objections by adopting the following definition of indiscriminability between visual appearances:

(IND*) Two objects are indiscriminable in look to an observer if and only if the objects have the same intrinsic properties and the observer sees them at the same time, at the same distance, and equally illuminated.

It is well known that even (IND*) fails to characterize indiscriminability between the visual appearances of things. As a matter of fact, two objects with the same intrinsic properties and observed at the same time, at the same distance, and equally illuminated may nonetheless have discriminable looks. Consider, for example, the

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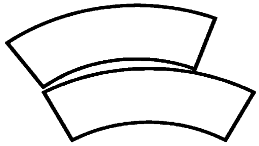
so-called Jastrow illusion represented below: the lower figure looks larger than the one above, but they have the same intrinsic properties and they are observed at the same time, at the same distance, and equally illuminated.



It is also useful to note that things may have very different intrinsic properties, while still being indiscriminable in look to an observer. Suppose, for example, that you perceive two vases in front of you and they are indiscriminable to you[1]; moreover, you see them at the same distance from you and equally illuminated. In fact, you may discover afterwards that one of them is made of plastic, while the other is porcelain. They had very different intrinsic properties, but they were indiscriminable to you.

It is therefore evident that (IND*) cannot be a good definition of indiscriminability between visual appearances. There may nevertheless be a tendency to characterize indiscriminability of appearances in terms of a kind of *second-order identity*: identity between appearances themselves. Before considering this characterization of indiscriminability, it may be useful to introduce some terminological considerations: I will use "identity of appearances," "identity in looks," "having the same (visual) phenomenal properties," and "looking the same" interchangeably. I will use "looking the same" more frequently, because I find it more natural to say that two objects look the same, instead of saying that two objects' looks are identical, or that there is identity between two objects' appearances, or that two objects have the same (visual) phenomenal properties. Now that these terminological considerations have been presented, the characterization of indiscriminability between visual appearances under consideration may be expressed as follows:

(IND) Two objects are indiscriminable in look to an observer O at a particular instant of time *T* if and only if they look the same to the observer O at that particular instant of time *T*.

(IND) is deep-rooted in our common sense and is considered preferable to the previous definitions of indiscriminability. Let us first consider why the previous definitions are excluded: it is commonly acknowledged that appearance and reality are two distinct realms, each of them with its own coherence. Complete correspondence between intrinsic properties of objects (which constitute reality) and the way things appear is therefore to be excluded, as was assumed by the previous definitions. (IND) is considered to be better because it does not establish a correlation between

[1]From now on, when I write that objects are indiscriminable, I mean that they are indiscriminable in look, and when I write about indiscriminability, I mean to consider indiscriminability between visual appearances.

appearance and reality, but it concerns instead reliability of appearances themselves; in particular it concerns the reliability of the relation of indiscriminability between appearances (or looks). As long as there is no empirical means of comparing the look of objects apart from relations of indiscriminability and discriminability, these relations are considered to be the only way to establish identity or diversity between looks. Therefore, indiscriminability and identity between looks are commonly considered to be equivalent and this grants the common sense acceptance of (IND).

My aim in this work is to argue that this assumption is false because (IND) may be false.[2] In the possible situation in which (IND) is false – a situation which may be actual, as far as is known – either the appearances of two objects may be indiscriminable to an observer at a particular instant of time even though they do not look the same to the observer at that instant of time or they may look the same to the observer at that time without being indiscriminable in look to her at that time.[3] And there being at least a possible situation in which indiscriminability and identity between looks are not co-extensional, it follows that there is no equivalence between them. More specifically, "being indiscriminable in look" is a predicate concerning the epistemic attitudes of an observer when confronted with distinct appearances, and it will be argued that such a relation – at least under certain conditions – may be non-transitive, while "looking the same" is a predicate concerning a metaphysical relation of identity between appearances independently of what is actually acknowledged by the observer and this second relation will be assumed to be transitive. It therefore follows that the two relations are not equivalent and therefore (IND) may be false.

[2]It is useful to distinguish any objection to (IND) from Goodman's (1951) claim that indiscriminability between qualia ("matching qualia" in his terms) is not identity between qualia. Goodman's claim concerns qualia and a quale is defined as a "presented quality" or as a "quality of a presentation of a thing" (p. 96); a quale is therefore an abstract entity. (IND) concerns instead the appearances of things acknowledged by an observer at a particular time, i.e., instantiated phenomenal properties. Other philosophers – following Goodman – argued for the difference between indiscriminability and identity of abstract phenomenal qualities; see, for example, Dummett (1975), Wright (1975), Peacocke (1981, p. 125), Parikh (1983), and Linsky (1984).

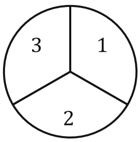
[3]Mills (2002) seems to argue against the left to right implication in (IND), that is, he claims "that pairwise indistinguishability does not entail sameness of phenomenal appearances" (p. 392). He considers two line drawings with seven small differences and claims that "even if I never succeed in distinguishing the two drawings – even if they are pairwise indistinguishable – still it is perfectly intelligible to suppose that they look different to me" (p. 392). I believe that even if I never succeed in distinguishing the two drawings, they may still be distinguishable to me (it may even be physically possible for me to distinguish the two drawings). In my opinion, Mills' observation shows that *actually non-distinguished* drawings may look different to an observer, but does not support the stronger thesis that *indistinguishable* drawings may look different to an observer (i.e., he does not succeed in giving a counterexample to the left to right implication in (IND)).

To repeat the idea in more schematic terms, the argument I will present for the possible falsity of (IND)[4] is divided into two parts. In the first part, it is argued that indiscriminability may be non-transitive, while the second demonstrates that non-transitive indiscriminability is inconsistent with (IND). It will therefore be deduced that (IND) may be false.

It may be useful to say how I am going to organize my work. In the next section I will argue that indiscriminability between appearances may be non-transitive (Sect. 6.1). Then I will present a second argument for the same thesis (Sect. 6.2). As far as I know, no one has presented the first argument, while some arguments similar to the second are actually present in the literature. I will therefore try to explain the main differences between my second argument and some more recent ones (Sect. 6.3). Then I will argue that non-transitive indiscriminability between visual appearances is inconsistent with (IND) and therefore that (IND) may be false (Sect. 6.4). I will conclude my presentation by claiming that the possible falsity of (IND) has important consequences for the analysis of phenomenal predicates and so-called phenomenal sorites (Sect. 6.5).

6.1 Non-transitive Indiscriminability: The First Argument

Before arguing that indiscriminability may be non-transitive, it may be useful to say what it is for indiscriminability to be non-transitive. Let us suppose we are looking at a round-shaped figure like the following one:



Imagine moreover that the three slices are colored. The point to consider is whether it may be the case that, at a single instant of time, the following sentences correctly describe the way the three slices appear to a single observer:

– Slice 1 is indiscriminable (in look) from slice 2.
– Slice 2 is indiscriminable (in look) from slice 3.
– Slice 1 is discriminable (in look) from slice 3.

If this were the case at least on one occasion, then indiscriminability would be non-transitive.[5] It is now time to consider the argument supporting this possibility.

[4]I am indebted to Alfredo Tomasetta for helping me to stress the modal character of my argument.

[5]An interesting question is: what is it like to have such an experience? I suppose that the observer will see a difference in shade, for example, at the border between slice 1 and slice 3 but does not acknowledge any other difference in shade for the rest of the figure.

In order to argue for this possibility, I maintain that a certain relation between physical properties and our perceptual abilities is possibly instantiated. In particular, supposing that an observer is presented simultaneously with different stimuli, let the relation under consideration be between the physical properties characterizing the stimuli and the actual experiences had by the observer of the stimuli.

It may be useful to consider a simple example in which a relation between physical properties and perceptual abilities is commonly presumed to hold and then to take into account the relation I propose to be possible. Suppose that you are presented with two black sticks lying half a meter in front of you on a white background, suppose moreover that their appearance is indiscriminable to you. But if you then go nearer to them and measure them with a ruler, you may discover that one of them is 20 cm long, while the other is 20.1 cm long. This would not be a surprise: we commonly accept that our perceptual abilities are not so fine-grained as to detect any slight difference in the physical bases which elicit our perceptions. But in order to grant reliability to our perceptual abilities, we assume that if the difference in length between the two sticks is greater than a certain amount *N* (let us say, 2 cm), then a normally gifted perceiver will be able to acknowledge the difference in the situation previously delineated.

Having these simple considerations in mind, let us consider the relation we expect between certain physical characteristics and our perceptual abilities. We tend to suppose that

(DIFF^S) If the difference between the length of the two sticks is greater than *N*, then the appearances of the two sticks are not indiscriminable (i.e., they are discriminable) to a normal perceiver in the conditions specified.

And, by contraposition, we have that

(DIFF*^{S}) If the appearances of two sticks are indiscriminable to a normal perceiver in the conditions specified, then the difference between the length of the two sticks is smaller than or equal to *N*.

Let us now reconsider the case of a person who observes the round-shaped figure presented at the beginning of this section. I maintain that it is possible for the following correlation to hold at least in certain situations and depending on certain physiological traits of the observer:

(DIFF) If the difference between the physical bases of the looks of two slices is greater than a certain amount *N* at time *T*, then the two slices are not indiscriminable (i.e., they are discriminable) to the observer O at time *T*.[6]

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[6]As a referee observed, the reader may wonder why I do not assume (DIFF) to be true. In order to follow my reasoning, reconsider the previously presented case of the two indiscriminable vases, one porcelain and the other plastic. In such a case, the difference between the physical bases of the look of the two vases is certainly greater than any reasonable threshold, but they are indiscriminable anyway. Therefore, it is not always true that if the difference between the physical bases of certain looks is greater than a certain threshold, the looks are discriminable. Having in mind examples like this one, I maintain that (DIFF) is not always granted to be true, but it is still possible that (DIFF)

By contraposition,[7] this assumption is equivalent to the following one:

(DIFF*) If two slices are indiscriminable to the observer O at time T, then the difference between the physical bases of the looks of the two slices is smaller than or equal to amount N at time T.

If this correlation is possible at least in a particular case, non-transitivity of indiscriminability is possible too. In order to grasp this, let us now reconsider the case of an observer of the round-shaped figure depicted at the beginning of this section and imagine that the above implications (i.e., (DIFF) and (DIFF*)) hold for this observer at a specific time T. Let us suppose moreover that *slice 1 is indiscriminable from slice 2* and that *slice 2 is indiscriminable from slice 3* to observer O at time T. By (DIFF*), we can deduce that the difference between the physical bases of the look of slice 1 and the look of slice 2 is smaller than or equal to N and that the same can be deduced concerning the difference between the physical bases of the look of slice 2 and the look of slice 3. But these observations are compatible with the fact that the difference between the physical bases of the look of slice 1 and the look of slice 3 is greater than N at time T (supposing their being equal to the sum of the two previously mentioned differences) and if this is the case, by (DIFF), we may conclude that *slice 1 and slice 3 are discriminable* (or not indiscriminable) to the observer O at time T. And non-transitive indiscriminability is therefore to be acknowledged in the situation delineated.

Let me try now to express the same argument in more schematic terms. Let us assume the following conventions:

"x", "y", and "z" are variables ranging over slices.

"s_1", "s_2", and "s_3" are individual constants which refer, respectively, to slice 1, slice 2, and slice 3.

"$\mathrm{Diff}_{>N}$" is a binary predicate constant denoting the relation "The difference between the physical bases of the look of ... and the look of ... is greater than amount N at time T."

"I" is a binary predicate constant denoting the relation "... and ... are indiscriminable in look to observer O at time T."

Now, we can rewrite the argument in the following schematic way[8]:

holds at least in a restricted range of cases, analogous to the previously mentioned case of the black sticks on a white background.

[7] I am assuming bivalent classical logic in this work.

[8] The adopted rules are universal elimination (∀E), modus ponens (MP), conjunction introduction (∧I), and existential introduction (∃I).

1	$\forall x \forall y\ (\mathrm{Diff}_{>N}(x,y) \rightarrow \neg \mathrm{I}x,y)$	1	(DIFF) assumption
2	$\mathrm{Is}_1,\mathrm{s}_2$	2	Assumption
3	$\mathrm{Is}_2,\mathrm{s}_3$	3	Assumption
4	$\mathrm{Diff}_{>N}(\mathrm{s}_1,\mathrm{s}_3)$	4	Assumption
5	$\mathrm{Diff}_{>N}(\mathrm{s}_1,\mathrm{s}_3) \rightarrow \neg \mathrm{Is}_1,\mathrm{s}_3$	1	$\forall$E from 1
6	$\neg \mathrm{Is}_1,\mathrm{s}_3$	1,4	MP from 4 and 5
7	$\mathrm{Is}_1,\mathrm{s}_2 \wedge \mathrm{Is}_2,\mathrm{s}_3 \wedge \neg \mathrm{Is}_1,\mathrm{s}_3$	1, 2, 3, 4	$\wedge$I from 2, 3, and 6
8	$\exists x \exists y \exists z(\mathrm{I}x,y \wedge \mathrm{I}y, z \wedge \neg \mathrm{I}x,z)$	1, 2, 3, 4	$\exists$I from 7

The conclusion at 8 states that indiscriminability is non-transitive: there are three slices such that the first and the second, like the second and the third, are indiscriminable in look to an observer at a particular instant of time, but the first and the third are not. As long as assumptions 1, 2, 3, and 4 are expected to be possibly instantiated in certain situations, it follows that non-transitive indiscriminability (i.e., conclusion 8) is possibly instantiated.

6.2 Non-transitive Indiscriminability: The Specular Argument

There is a second argument which may be used to argue for the possible non-transitivity of indiscriminability and which is – in a certain sense – specular to the one already considered. This second argument is not completely novel, and the expert reader may find some similarity between it and others already present in the literature.[9] In order to make the presentation more straightforward, I present my argument and then, in the next section, I will present the main difference between my argument and some more recent ones. This second argument assumes as possible that – at least in a certain situation and depending on the perceptual abilities of an observer O – the converse of (DIFF*) holds,[10] that is,

(DIFF*–CONV) If the difference between the physical bases of the looks of two slices is smaller than or equal to amount *N* at time *T*, then the two slices are indiscriminable to observer O at time *T*.

[9] It is very difficult for me to retrace the history of this type of argument as requested by an anonymous referee. I think that something similar to this argument inspired Goodman's (1951) distinction between indiscriminability ("matching" in his terms) and identity of qualia. Analogies with my argument may be found in arguments or reflections in Dummett (1975), Wright (1975), and Linsky (1984). As will be clear in Sect. 6.3, when writing up this argument I particularly had in mind specific arguments in Fara (2001), Chuard (2010), and Keefe (2011).

[10] As a referee observed, the reader may wonder why I assume (DIFF*–CONV) to be possibly true in a restricted range of situations and not simply true. As will be clear in Sect. 6.3, Fara (2001) argued that an assumption similar to (DIFF*–CONV) is not true. But her argument does not exclude that (DIFF*–CONV) may be true at least in a restricted range of cases. See also footnote 18.

By contraposition, (DIFF*–CONV) is equivalent to the converse of (DIFF), that is, it is equivalent to

(DIFF–CONV) If two slices are not indiscriminable (i.e., they are discriminable) to observer O at time T, then the difference between the physical bases of their looks is greater than a certain amount N at time T.

Now, let us consider a possible situation such that (DIFF*–CONV) and (DIFF–CONV) hold for an observer O of the round-shaped figure presented at the beginning of Sect. 6.1 and that *the looks of slices 1 and 3 are discriminable* to observer O at time T. By (DIFF–CONV), we may deduce that the difference between the physical bases of such looks is greater than amount N. This assumption is compatible with the fact that there is a third slice (i.e., slice 2) such that the difference between the physical bases of its look and the physical bases of the other two slices' looks is smaller than or equal to N. If this is the case, by (DIFF*–CONV) we may deduce that *the look of slice 2 is indiscriminable from the look of slice 1 and from the look of slice 3* to observer O at time T. In such a possible case indiscriminability is non-transitive.

It may be useful to present the new argument schematically. Let us add the following to the conventions above:

"$\mathrm{Diff}_{\leq N}$" is a binary predicate constant denoting the relation "The difference between the physical bases of the look of ... and the look of ... is smaller than or equal to amount N at time T."

Using the same rules as before, the argument is the following:

1	$\forall x \forall y\, (\mathrm{Diff}_{\leq N}(x,y) \rightarrow \mathrm{I}x,y)$	1	(DIFF*–CONV) assumption
2	$\mathrm{Diff}_{\leq N}\, (s_1,s_2)$	2	Assumption
3	$\mathrm{Diff}_{\leq N}\, (s_2,s_3)$	3	Assumption
4	$\neg \mathrm{I}s_1,s_3$	4	Assumption
5	$\mathrm{Diff}_{\leq N}(s_1,s_2) \rightarrow \mathrm{I}s_1,s_2$	1	$\forall$E from 1
6	$\mathrm{Diff}_{\leq N}(s_2,s_3) \rightarrow \mathrm{I}s_2,s_3$	1	$\forall$E from 1
7	$\mathrm{I}s_1,s_2$	1, 2	MP from 2 and 5
8	$\mathrm{I}s_2,s_3$	1, 3	MP from 3 and 6
9	$\mathrm{I}s_1,s_2 \wedge \mathrm{I}s_2,s_3 \wedge \neg \mathrm{I}s_1,s_3$	1, 2, 3, 4	$\wedge$I from 4, 7, and 8
10	$\exists x \exists y \exists z (\mathrm{I}x,y \wedge \mathrm{I}y,z \wedge \neg \mathrm{I}x,z)$	1, 2, 3, 4	$\exists$I from 9

Now, the argument presented in this section and the argument presented in the previous one are both intended to demonstrate that indiscriminability may be non-transitive. They are both grounded on an assumption considered as metaphysically possible: in particular, the first argument is grounded on the metaphysical possibility of (DIFF) holding in a restricted range of cases, while the second is grounded on the metaphysical possibility of (DIFF*–CONV) holding in a restricted range of cases. The conclusion of both arguments is therefore that it is metaphysically possible that indiscriminability is non-transitive. Notwithstanding the specular character of the two arguments, it is worth noting that they are not equivalent as long as (DIFF) and

(DIFF*–CONV) are not equivalent and there may be a possible situation in which one of them is true while the other is false. I maintain that (DIFF) and (DIFF*–CONV) are each metaphysically possible in a restricted range of cases and therefore I claim that both arguments show that indiscriminability between appearances may be non-transitive.

6.3 Confronting Alternative Approaches

The two arguments I have presented in the previous sections have a modal character: they are intended to show the metaphysical possibility – and, as far as is known, the actual eventuality – of non-transitive indiscriminability. Other arguments discussed in the literature do not have a modal character: they are intended to show that indiscriminability is non-transitive. Notwithstanding the difference between the modal character of my arguments and the non-modal character of the other arguments offered in the literature, the second argument I have presented is similar to others considered.[11] It is therefore useful to compare my own argument to some more recent ones in order to explain why I consider it important to give mine a modal character.

In particular, an argument very similar to the second I considered may be found in Fara (2001). The main difference is that while I assume (DIFF*–CONV) to be metaphysically possible, at least in a restricted range of cases, the argument she considers makes an assumption similar to (DIFF*–CONV) as being true in every actual situation and for any normally gifted observer of our world. Fara (2001) expresses the crucial assumption as follows: "For some sufficiently slight amount of change (in the *physical bases of* colour, sound, position, etc.), when we perceive an object for the entirety of an interval during which it changes by less than that amount, we will perceive it as not having changed at all."[12]

Fara claims this premise to be false and therefore the entire argument for non-transitivity to be unsound. She argues that this premise entails that "within a certain limited domain, our experiences are invariably veridical"[13]; in particular it entails that whenever two stimuli are characterized by identical physical bases, we perceive

[11] Some philosophers have offered reasons for non-transitive indiscriminability which are not my present concern because they are not similar to the arguments I have presented here. Among such lines of reasoning, it is worth mentioning an argument from phenomenal continua presented by Wright (1975) and discussed by Fara (2001) and an argument from inexact representation offered by Hellie (2005) and discussed by Pelling (2008).

[12] Fara (2001), p. 917, note 15 – emphasis in the original. It may be observed that the assumption proposed by Fara presupposes that the physical bases which elicit the appearances of an object in an interval are circumscribed to the object itself and do not extend to the environment in which it is positioned. This assumption does not seem to be justified. I am indebted to Andrea Bonomi and Clotilde Calabi for this interesting objection.

[13] Fara (2001), p. 918.

no difference between the corresponding appearances. But Fara claims that this is not true, for there are examples of perceptual illusions such that different appearances correspond to identical physical bases of stimuli; for example, "a motionless object could appear to be quivering; a constant note could sound to be pulsating."[14]

Chuard (2010) and Keefe (2011) wish to maintain that indiscriminability is non-transitive and, in order to argue for that, they try to overcome Fara's objection to the above premise. But in my opinion, their strategies are not convincing.

Keefe (2011) explicitly mimics Fara's assumption in the antecedent, while she changes the consequent. She actually writes, "For some sufficiently slight amount of change in colour, or in the physical basis of colour, when we perceive an object for the entirety of an interval during which it changes by less than that amount, *we cannot successfully discriminate between the way it is before the change and the way it is afterwards*."[15] This means that when the difference between the physical bases of two colored patches is below a certain threshold, either we discriminate unsuccessfully between the corresponding appearances or we do not discriminate. But notice that there is no way to conclude from this assumption that indiscriminability is not transitive. Suppose that the difference between the physical bases of the appearances of slice 1 and slice 2 is below the threshold, and so is the difference between the physical bases of the appearances of slice 2 and 3; moreover, suppose that slice 1 and 3 are discriminable to us. It does not follow that indiscriminability is not transitive as the assumptions are compatible with all patches being mutually discriminated by us (even if discrimination is not always successful in Keefe's terms).

Chuard (2010) assumes instead that "there are limitations L on the way in which the human visual system represents colours such that, for any two coloured objects x and y, where the chromatic difference δ between x and y is less than L but more than zero, *veridical* visual experiences of x and y represent x and y in the same way with respect to their colour."[16] And according to Chuard, "if an experience is veridical, it is not a case of illusion or hallucination."[17] Chuard adopts the notion of "veridical experience" which is a committal philosophical notion as is well known at least since Descartes' (1641) challenge of the Evil Demon. It would be too much to demand that Chuard gives a definition of such a notion, but it would be desirable for him to use it in an acceptable way. Unfortunately, his use of the notion goes against any intuitive idea of what a veridical experience is. According to him, a veridical experience is characterized (at least in the conditions specified) by the fact that we *do not perceive the changes* characterizing our stimuli, while we tend to believe that a veridical experience allows us to perceive the changes characterizing the stimuli. The fact that our perceptual apparatus is limited does not justify our

[14]Fara (2001), p. 919.

[15]Keefe (2011), p. 335, emphasis in the original.

[16]Chuard (2010), p. 180, my emphasis.

[17]Chuard (2010), p. 179, note 32.

veridical experiences not allowing us to acknowledge changes in the stimuli; it would better justify our experiences not always being veridical.

Let me sum up what has been presented so far. Fara considers an argument in favor of non-transitive indiscriminability, but she argues that one premise of the argument is false and therefore the argument unsound. Keefe and Chuard tried to restore the premise with expedients I do not find convincing. In my opinion, the premise criticized by Fara should not be restored: I believe that the counterexamples given by Fara are convincing and that the alternatives proposed by Keefe and Chuard are not persuasive. Therefore, I propose a different strategy: Fara convincingly argues that a relevant premise of her argument is not true, I argue instead that she does not exclude it being possible in a restricted range of cases. In particular, Fara considers and criticizes a premise assumed to be *true in the actual world* and to be *generally applied* to any situation and to any normally gifted human being. I assume instead a premise (i.e., (DIFF*–CONV)) to be *true in a possible world* (not excluding the actual world) and to *apply in an at least restricted range of circumstances*.[18] In other words, I maintain that it is metaphysically possible and, as far as is known, an actual eventuality that there is a restricted range of cases such that if the difference between the physical bases of visual appearances is below or equal to a threshold N at time T,[19] then the corresponding visual appearances are indistinguishable to a specific observer at that particular instant of time T.

In the same way, when I presented my first argument I claimed that (DIFF) is a crucial premise of the argument. But I did not claim that it actually holds in any situation; I assumed instead that it is metaphysically possible and, as far as is known, an actual eventuality that there is (at least) a restricted range of cases in which (DIFF) holds. As I said, I maintain that (DIFF) and (DIFF*–CONV) are both metaphysically possible in a restricted range of cases. And therefore as it is possible for there to be a setting in which either of them is true, it is possible there is a setting in which indiscriminability is non-transitive.

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[18]It may be not clear why I assume (DIFF*–CONV) to be possible *in a restricted range of circumstances*. The reason has to do with the fact that it is quite plausible to assume that (DIFF*–CONV) is not always true in the actual world: there may be stimuli, characterized by no difference in the physical bases, yet producing appearances distinguishable to an observer, as Fara (2001) observes (see quotation referred to in footnote 14). Notwithstanding this, it may be the case that at least in certain circumstances (DIFF*–CONV) holds. I am not arguing that (DIFF*–CONV) is true in a restricted range of cases, I am just maintaining that it is *metaphysically possible* that (DIFF*–CONV) is true in a restricted range of cases, without excluding that this restricted metaphysical possibility should actually occur. And the metaphysical possibility that (DIFF*–CONV) is true in a restricted range of cases is itself sufficient to make it possible that indiscriminability is non-transitive.

[19]A marginal difference between my argument and any of the others considered is the following: in the antecedent of the crucial implication, the others assume that the difference between the physical bases of two perceptual appearances is *below* a certain threshold, while in the antecedent of (DIFF*–CONV), I assume that the difference is *either below or equal* to the threshold. The reason for my assumption is that I want to stress the fact that (DIFF*–CONV) is the converse of (DIFF*). Nothing relevant for my claim depends on this difference between my presentation of the argument and the others.

6.4 Indiscriminable Versus Identical Looks

My main target is to argue that (IND) may be false. As I have already explained, my argument is in two steps and each step of the argument is intended to demonstrate a thesis. The first step shows that indiscriminability may be non-transitive. Let us express it schematically as follows:

1. $\Diamond$ (NON-TR)

The second step of the argument shows that non-transitive indiscriminability (i.e., (NON-TR)) and (IND) are inconsistent. And, as long as two inconsistent claims are necessarily inconsistent, we may say that the second step shows that (NON-TR) and (IND) are necessarily inconsistent.

2. $\Box$ [(NON-TR) and (IND) are inconsistent]

It follows from these two theses that (IND) may be false.

3. $\Diamond$ NON (IND)

My main aim in this section is to show that (NON-TR) and (IND) are inconsistent, that is, to argue for the second step of the argument. Let us first consider the idea in informal terms.

Let us suppose that indiscriminability between appearances to observer O at time T is *non-transitive* (as (NON-TR) says). Moreover, assuming identity between appearances to O at T to be *transitive*, it follows that indiscriminability between appearances to O at T is not equivalent to identity between them, contrary to what (IND) says. And therefore if (NON-TR) holds, then NON (IND) follows.

The contrapositive is evident: suppose that (IND) holds, that is, that indiscriminability between appearances to O at T is equivalent to their being identical to O at T. As long as identity is assumed to be transitive, we should conclude that indiscriminability to O at T is transitive too. Therefore, if (IND) holds, then indiscriminability is transitive (i.e., NON (NON-TR)).

Now, it may be useful to present the argument in a more detailed way. The thesis that indiscriminability is non-transitive is an existential claim: it says that there are at least three objects such that the first and the second, like the second and the third, are mutually indiscriminable in look to O at T, but the first and the third are instead discriminable (or non-indiscriminable) in look to O at T. As we have already considered, this may be expressed schematically as follows:

(NON-TR) $\exists x \exists y \exists z(\mathrm{I}x,y \wedge \mathrm{I}y,z \wedge \neg \mathrm{I}x,z)$

It is now time to express (IND) schematically, that is,

(IND) Two objects are indiscriminable in look to an observer O at a particular instant of time T if and only if they look the same to the observer O at that particular instant of time T.

In order to do this, what is required is a predicate variable with a universe of discourse appropriately restricted, that is,

"X" is a variable that ranges over predicates applying to slices and qualifying their look to observer O at a particular instant of time T.

We may therefore express (IND) schematically as follows[20]:

(IND) $\forall x \forall y\ (\mathrm{I}x,y \leftrightarrow \forall X\ (Xx \leftrightarrow Xy))$

Given that quantification over a predicate variable is introduced, it may be useful to state explicitly the rule that has been adopted on this, that is, Generalized Transitivity of the Biconditional (GTB), according to which from $\ulcorner \forall X(Xa \leftrightarrow Xb) \urcorner$ and $\ulcorner \forall X(Xb \leftrightarrow Xc) \urcorner$, it follows $\ulcorner \forall X(Xa \leftrightarrow Xc) \urcorner$ (where "a", "b", and "c" are meta-variables ranging over individual constants).[21]

The argument may therefore be schematically expressed as follows:

1	$\forall x \forall y\ (\mathrm{I}x,y \leftrightarrow \forall X\ (Xx \leftrightarrow Xy))$	1	(IND) assumption
2	$\exists x \exists y \exists z(\mathrm{I}x,y \wedge \mathrm{I}y,z \wedge \neg \mathrm{I}x,z)$	2	(NON-TR) assumption
3	$\mathrm{I}s_1,s_2 \wedge \mathrm{I}s_2,s_3 \wedge \neg \mathrm{I}s_1,s_3$	3	Assumption
4	$\mathrm{I}s_1,s_2$	3	$\wedge$E from 3
5	$\mathrm{I}s_2,s_3$	3	$\wedge$E from 3
6	$\neg \mathrm{I}s_1,s_3$	3	$\wedge$E from 3
7	$\mathrm{I}s_1,s_2 \leftrightarrow \forall X\ (Xs_1 \leftrightarrow Xs_2)$	1	$\forall$E from 1
8	$\mathrm{I}s_2,s_3 \leftrightarrow \forall X\ (Xs_2 \leftrightarrow Xs_3)$	1	$\forall$E from 1
9	$\forall X\ (Xs_1 \leftrightarrow Xs_2)$	1, 3	MP from 4 and 7
10	$\forall X\ (Xs_2 \leftrightarrow Xs_3)$	1, 3	MP from 5 and 8
11	$\forall X\ (Xs_1 \leftrightarrow Xs_3)$	1, 3	GTB from 9 and 10
12	$\mathrm{I}s_1,s_3 \leftrightarrow \forall X\ (Xs_1 \leftrightarrow Xs_3)$	1	$\forall$E from 1
13	$\mathrm{I}s_1,s_3$	1, 3	MP from 11 and 12
14	Contradiction	1, 3	From 6 and 13
15	Contradiction	1, 2	$\exists$E from 2 and [3–14]

This argument shows that (NON-TR) and (IND) are inconsistent. And if they are inconsistent, they are necessarily inconsistent. The second step of my argument is therefore demonstrated. Given that the first step of the argument shows it is possible that (NON-TR) holds, it follows that it is possible that (IND) is false.

Let us consider what it is for (IND) to be possibly false. It may be the case that indiscriminability between visual appearances to an observer at a particular instant of time is not equivalent to second-order identity between visual appearances to that observer at that instant of time. In other words, either it is possible for two visual

[20]I am indebted to Sandro Zucchi for giving some helpful advice concerning this schematic translation.

[21]Moreover, in order to introduce the argument, some new rules, besides the ones already adopted, should be introduced. They are existential elimination ($\exists$E) and conjunction elimination ($\wedge$E).

appearances to be indiscriminable to an observer at an instant of time without being identical at that time or it is possible for two visual appearances to be identical at an instant of time without being indiscriminable to the observer at that time.

The possible falsity of (IND) has important consequences for the analysis of so-called phenomenal sorites paradoxes. This is what will be dealt with in the next section.

6.5 Phenomenal Sorites and the Possibility of Indiscriminable but Not Identical Looks

As is well known, there is much debate over what vagueness is: there is no shared exhaustive definition of it.[22] When we consider vagueness for phenomenal predicates, things seem to be even more difficult. If we limit ourselves to observing what philosophers do when considering vagueness and phenomenal predicates, it is evident that the main concern is with such predicates' coherence.[23] It is therefore worth considering whether there is a relation between the coherence (or incoherence) of such predicates and their vagueness: if there is such a relation, then it may help in characterizing vagueness for such expressions.

The philosopher who actually pointed out a connection between vagueness and incoherence of linguistic expressions was Frege. According to him, "the use of vague expressions is fundamentally incoherent"[24]; once Frege's idea is adopted, it follows that if predicates are vague, then they will be incoherent, if instead they are coherent, they will not be vague. It is now worth evaluating whether Frege's idea may be maintained: I will argue that while Frege's idea is now much discredited for non-phenomenal predicates, it is still unchallenged for phenomenal ones.

Let us start by considering non-phenomenal predicates. Frege's idea is considered by most philosophers to be clearly wrong because it is commonly accepted that there may be coherent vague predicates. When vagueness is under scrutiny, the sorites argument puts the coherence of the predicates involved to the test: if it is demonstrated that at least one of the sorites premises is not true, the coherence of the expressions involved in the argument is restored. And most theories of vagueness just argue that one of the premises of the sorites is not true.[25] But

[22]See, for example, the different definitions offered by Greenough (2003), Eklund (2005), Smith (2005), and Weatherson (2010).

[23]By "phenomenal predicates' coherence," I mean that the rules governing the use of phenomenal predicates allow such uses to be coherent. This is the "governing view" on the coherence of vague predicates according to Wright (1975, pp. 326–27). I use the shorter expression "phenomenal predicates' coherence" for the sake of simplicity and because it is now quite widespread.

[24]Dummett (1975), p. 316; Frege's idea is very much stressed also by Wright (1975).

[25]Nihilists are an exception to the general strategy (see, e.g., Unger 1979; Wheeler 1979). They seem to accept Frege's idea that "the use of vague expressions is fundamentally incoherent." As they are not a challenge to Frege's idea, let me ignore them in the present chapter.

the fact that the sorites argument is not sound does not exclude vagueness: it is commonly agreed that explaining why a non-true premise of the sorites has been prima facie mistakenly accepted is a way of saying what vagueness is. As is well known, the most common strategies are of two types: (1) it may be argued that we mistakenly accept prima facie one of the sorites premises because we fail to notice our epistemic limits and that such limits characterize vagueness[26] or (2) it may be argued that we mistakenly accept prima facie one of the sorites premises because we fail to notice the under-specification of the semantic rules and that such semantic under-specification characterizes vagueness.[27] It is therefore clear that the coherence of certain linguistic expressions is compatible with their vagueness when non-phenomenal predicates are considered.

In the case of phenomenal predicates, as far as I know, no philosopher has ever tried to say what the vagueness of a coherent phenomenal predicate may be. And the reason may depend on the fact that once it is argued that a sorites argument involving such predicates is not sound, no characterization of vagueness seems available. The most common strategy assumed in order to solve the phenomenal sorites paradox is to argue that one of the premises is false because of a misunderstanding of the notion of indiscriminability. Now, if we accept the diagnosis, we can say that we mistakenly accepted prima facie one of the sorites premises because we misunderstood the notion of indiscriminability. But we cannot say that such misunderstanding characterizes vagueness: the reason is that when we notice the misunderstanding, the misunderstanding disappears.

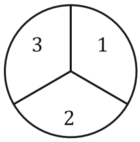
This is different from the sorites applied to non-phenomenal predicates: supposing we agree to the diagnosis according to which we mistakenly accepted a sorites premise because of epistemic limits, such limits do not disappear when noticed, they are still there and it may be maintained that such limits characterize vagueness. In the same way, supposing that we agree to the diagnosis according to which we mistakenly accepted a sorites premise because we overlooked the under-specification of semantic rules, such semantic under-specification does not disappear when noticed, it is still there and it may be maintained that such semantic under-specification characterizes vagueness. If instead we admit that we mistakenly accepted one of the phenomenal sorites premises because we misunderstood the notion of indiscriminability, the misunderstanding disappears once noticed and no space for a characterization of vagueness for phenomenal predicates remains. I do not know of any other strategy for solving the phenomenal sorites paradox compatibly with a characterization of vagueness for coherent phenomenal predicates. Therefore, until the challenge of characterizing vagueness of coherent phenomenal predicates has been overcome, I maintain the Fregean idea that coherent phenomenal predicates are not vague.

[26]This is argued by the supporter of an epistemic theory of vagueness like Sorensen (1988) and Williamson (1994).

[27]This is argued by a supporter of a semantic theory of vagueness like Fine (1975).

It is now interesting to consider why philosophers are concerned with the coherence of phenomenal predicates. The reason is quite evident: if phenomenal predicates were incoherent, we would be talking nonsense when talking about our phenomenal experience; and as far as we use phenomenal predicates to talk about phenomenal properties, we would have to conclude that such properties simply do not exist. Even if there are philosophers who accept this conclusion,[28] others do not and I am particularly concerned with these latter philosophers who want to save the coherence of phenomenal predicates and the ontological legitimacy of phenomenal properties. Before presenting the debate dividing these latter philosophers, let me first summarize the phenomenal sorites paradox.

The phenomenal sorites paradox is grounded on the possibility of a situation like the following. Let us reconsider the round-shaped figure presented at the beginning of Sect. 6.1, that is,



The situation under consideration is such that the following sentences correctly describe what an observer O may acknowledge at time *T*:

(a) Slice 1 is indiscriminable (in look) from slice 2.
(b) Slice 2 is indiscriminable (in look) from slice 3.
(c) Slice 1 looks a specific shade of red R.
(d) Slice 3 does not look the specific shade of red R.

The phenomenal sorites argument shows that the above situation is paradoxical. The paradox depends on the fact that the following assumptions are accepted:

1. Slice 1 looks the specific shade of red R to observer O at time *T*.
2. Slice 1 is indiscriminable in look from slice 2, and slice 2 is indiscriminable in look from slice 3 to observer O at time *T*.
3. For any two slices and any shade of a specific color a slice may look to O at *T*, if two slices are indiscriminable in look to O at *T* and one of them looks a specific shade of a specific color to O at *T*, the other must look the same.

And from these assumptions it is possible to conclude:

4. Slice 3 looks the specific shade of red R to O at *T*.

[28]Most notably, Armstrong (1968, p. 219) argued against "sensory items," Dummett (1975) maintained that there are no "phenomenal qualities as traditionally understood," and Wright (1975) and Peacocke (1981) claimed that there is not a coherent semantic for observational predicates.

But conclusion (4) is the obvious negation of (d) which is supposed to describe what is acknowledged by observer O at time *T*.

Let us now consider the two main alternatives for restoring coherence explored by philosophers when confronting this philosophical paradox. One option is to say that one of the sorites premises is simply false because of a misunderstanding of the notion of indiscriminability (i.e., it has not been realized that indiscriminability is not equivalent to identity between looks)[29] and therefore the situation which gave rise to the sorites reasoning is not paradoxical, nor are phenomenal predicates incoherent. The alternative strategy is to argue that there is no misunderstanding on the notion of indiscriminability, that all the premises of the sorites argument are true, but that the situation which gives rise to the paradox is impossible, and therefore it is reasonable to maintain that phenomenal predicates are coherent.[30]

The two alternative strategies have actually been claimed for using different arguments and different counterarguments. I have not considered all such arguments in this chapter. I have limited myself to two arguments for non-transitivity[31] and I have argued that in both cases the premises may be true in at least a restricted range of cases and therefore that transitivity may be non-transitive and that (IND) may be false, that is, that it is possible that indiscriminability is different from identity between looks. I will argue in what follows that if it is possible that (IND) is false, it is possible also that premise (3) of the phenomenal sorites is false too. This approach is a bit different from the first option above: I do not claim that one of the phenomenal sorites premises is false, I argue that one of the premises (premise (3)) *may be* false. But the fact that one of the premises may be false is itself sufficient to avoid the sorites conclusion and to allow coherence for phenomenal predicates. To be more precise, once acknowledged that one of the sorites premises may (metaphysically speaking) be false and that this premise may (epistemically speaking) be false in the actual world, it is as well to maintain reserves about this premise, and the coherence of phenomenal predicates is not put into question by an argument with such an ungrounded premise. In order to be challenged by the sorites argument, something more is required: the dubious premise (i.e., premise (3)) should be demonstrated to be true, but such a demonstration has not yet been given. It follows that the sorites argument is not a challenge for anyone who recognizes that it is both metaphysically and epistemically possible that indiscriminability is not transitive.

In order to follow my argument it is useful to note that the phenomenal sorites paradox may be expressed in schematic terms. Adopting the conventions already

[29]See, for example, Koons (1994) who distinguishes between "observational indistinguishability" and "absolute indistinguishability," Mills (2002) who argues that "pairwise indistinguishability does not *generally* entail that two things 'look the same'" (p. 392), Hellie (2005) who argues that "there can be indiscriminability without sameness of representation" (p. 485), and Pelling (2008) who offers his argument for non-transitivity of perceptual indiscriminability. See also Chuard (2010) and Keefe (2011) for arguing in favor of non-transitive indiscriminability.

[30]See, for example, Jackson and Pinkerton (1973), Raffman (2000), and Fara (2001).

[31]In footnote 11 some different argumentative strategies in favor of (and against) non-transitive indiscriminability are mentioned.

presented together with the predicate constant "P" to be interpreted as "... looks the specific shade of red R to O at *T*," the schematic formulation is the following:

(1) Ps_1
(2) $Is_1s_2 \wedge Is_2s_3$
(3) $\forall x \forall y \forall X\ ((Ix,y \wedge Xx) \rightarrow Xy)$

From these premises it clearly follows:

(4) Ps_3

Now, I claim that from the possible falsity of (IND), it follows that premise (3) of the phenomenal sorites may be false too. This is evident once it is shown that the falsity of (IND) entails the falsity of premise (3) of the phenomenal sorites argument. The schematic argument is as follows:[32]

1	$\neg\forall x\forall y\ (Ix,y \leftrightarrow \forall X\ (Xx \leftrightarrow Xy))$	1	Assumption NON-(IND)
2	$\neg\forall x\forall y\ (Ix,y \rightarrow \forall X\ (Xx \rightarrow Xy))$	1	$\leftrightarrow$E from 1
3	$\neg\forall x\forall y\forall X\ (Ix,y \rightarrow (Xx \rightarrow Xy))$	1	log. cons. from 2[33]
4	$\neg\forall x\forall y\forall X\ \neg(Ix,y \wedge \neg(Xx \rightarrow Xy))$	1	$\rightarrow/\wedge$ log. eq. from 3
5	$\neg\forall x\forall y\forall X\ \neg(Ix,y \wedge (Xx \wedge \neg Xy))$	1	$\rightarrow/\wedge$ log. eq. from 4
6	$\neg\forall x\forall y\forall X\ \neg((Ix,y \wedge Xx) \wedge \neg Xy)$	1	$\wedge$ associativity from 5
7	$\neg\forall x\forall y\forall X\ ((Ix,y \wedge Xx) \rightarrow Xy)$	1	$\wedge/\rightarrow$ log. eq. from 6

[32]The argument assumes rules of logical equivalence between formulas with different connectives, a logical consequence demonstrated in footnote 33, biconditional elimination ($\leftrightarrow$E), and associativity of conjunction ($\wedge$ associativity).

[33]Proof by contradiction that 3 is a logical consequence of 2:

1	$\neg\forall x\forall y\ (Ix,\ y \rightarrow \forall X\ (Xx \rightarrow Xy))$	1	Assumption (2)
2	$\forall x\forall y\forall X\ (Ix,\ y \rightarrow (Xx \rightarrow Xy))$	2	Assumption
3	$\exists x\exists y\neg(Ix,\ y \rightarrow \forall X\ (Xx \rightarrow Xy))$	1	$\exists/\forall$ log. eq. from 1
4	$\neg(Is_1,s_2 \rightarrow \forall X\ (Xs_1 \rightarrow Xs_2))$	4	Assumption
5	$(Is_1,s_2 \wedge \neg\forall X\ (Xs_1 \rightarrow Xs_2))$	4	$\rightarrow/\wedge$ log. eq. from 4
6	Is_1,s_2	4	$\wedge$E from 5
7	$\neg\forall X\ (Xs_1 \rightarrow Xs_2)$	4	$\wedge$E from 5
8	$\exists X\neg(Xs_1 \rightarrow Xs_2)$	4	$\exists/\forall$ log. eq. from 7
9	$\neg(Ps_1 \rightarrow Ps_2)$	9	Assumption
10	$(Is_1,s_2 \rightarrow (Ps_1 \rightarrow Ps_2))$	2	$\forall$E from 2
11	$Ps_1 \rightarrow Ps_2$	2, 4	MP from 6 and 10
12	Contradiction	2, 4, 9	From 9 and 11
13	Contradiction	2, 4	$\exists$E from 8 and [9–12]
14	Contradiction	1, 2	$\exists$E from 3 and [4–13]
15	$\neg\forall x\forall y\forall X\ (Ix,y \rightarrow (Xx \rightarrow Xy))$	1	$\neg$I from 2 and 14

I am indebted to Andrea Bonomi for highlighting this logical consequence to me and to Sandro Zucchi for helping me with this proof.

The conclusion at 7 is the negation of premise (3) of the phenomenal sorites argument. As long as (IND) may be false, it follows that premise (3) of the phenomenal sorites may be false too. Whoever is inclined to accept that indiscriminability may be non-transitive cannot safely assume premise (3), and she therefore avoids the paradoxical conclusion of the phenomenal sorites. This is an interesting result, unless indiscriminability between visual appearances is shown to be necessarily or at least actually transitive.[34]

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Chapter 7
Attitudes, Supervaluations, and Vagueness in the World

Ángel Pinillos

7.1 Introduction

It is well known that extensions cannot exhaust the meaning of linguistic expressions. Thus, "creature with a heart" and "creature with a kidney" may have the same extension but differ in meaning. Intensions are thought to be the elements of meaning which account for this difference. Intensions may be construed in different ways. For example, they may be treated as mental particulars (ideas), mathematical objects (sets of possible worlds), or sui generis platonic modes of presentations (Fregean senses). Intensions are also thought to determine extensions. But should we also say they *represent* their extensions? If they do, then since words also represent their extensions, we may wonder about which type of representation fact is more fundamental. Is the fact that the name "Venus" represents Venus more fundamental than the fact that the intension of "Venus" represents Venus? Or is it the other way around? A third possibility is that the representational facts are independent of each other.

A related issue arises when we start thinking about intensions and vagueness. A common view in philosophy is that vagueness has its source in the representational facts.[1] If intensions represent extensions, we may now wonder whether it is words or intensions that are responsible for vagueness. For example, when philosophers talk about "bald" being vague, is this vagueness located in the word or in its intension? I find this question to be philosophically interesting for at least two reasons. First, following Frege, it might be thought that words but not intensions

[1] Russell (1923).

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are mind-dependent entities. If so, then the answer to our question will be relevant to the metaphysics of vagueness. In fact, I will argue that if we treat intensions as "imprecise" Fregean senses, we can no longer justifiably say that vagueness is a representational phenomenon. Second, intensions and extensions play distinct roles in the semantics of natural languages. So we would expect semantic theories that are sensitive to vagueness to make different predictions depending whether vagueness is located in words or in their intensions. In particular, I will argue that if we treat intensions as "precise" Fregean senses, supervaluational approaches will fail. I end by constructing an argument against supervaluational approaches which depends on anti-Fregean assumptions.

7.2 Imprecise and Precise Fregean Senses

According to a Fregean theory of linguistic representation, each meaningful word expresses a sense, which in turn uniquely picks out (refers to) an object in the world (if it picks out anything at all).[2] Words refer, but reference is semantically mediated by senses. There are at least three different ways that vagueness may arise on this picture.[3] First, it may be that a word fails to determinately express sense S^1, fails to determinately express sense S^2, and so on for all the senses, but it indeterminately expresses sense S^1, it indeterminately expresses sense S^2, and so on for some cluster of senses that are distinct but closely related to each other. Second, it may be that a sense fails to determinately refer to object O^1, fails to determinately refer to object O^2, and so on for all objects. But the sense may indeterminately refer to object O^1, and indeterminately refer to object O^2, and so on for a cluster of objects that are distinct but closely related. Third, a sense may pick out an object in the world (either determinately or indeterminately) that is itself vague.

Here is some terminology. I will call the first view "vagueness in the word-sense relation." I will call the second view "vagueness in the sense-object relation." The third view is the "vague object" view. When there is vagueness of the second kind, we will call the senses at issue "imprecise."[4] When there is no vagueness of the second kind, the sense in question will be called "precise."

Of course, the three types of vagueness are not mutually exclusive. In a case of rampant vagueness, all three could be in play. For instance, a word may

[2] I use "object" liberally to apply to the referent of our words including properties, relations, and their extensions.

[3] I want to put aside Frege's own views on the question of vagueness. Frege (1892) seems to think that concepts without determinate extensions don't have *Bedeutung*. Commentators have pointed out that since the *Bedeutung* of a sentence is a truth-value, all sentences containing vague terms would lack a truth-value on Frege's view. This is a costly conclusion. See Puryear (2013) and the references therein for further discussion.

[4] Jc Beall (2010) discusses "vague intensions" but treats them as having empty extensions (like Frege did). This is a costly move.

indeterminately express a certain sense and that sense may indeterminately refer to an object which is itself vague. In this chapter, we will mostly put aside the vague object view though we will return to it at the very end.

There are a number of issues that arise about how words, senses, and objects interact to give rise to vagueness. Many of these I won't discuss here. For example, suppose I have a fuzzy recollection of uncle Joe from my childhood. I call him "my crazy uncle" (let us suppose this expression works like a name, not a description). It is indeterminate whether my recollection of him involves him driving a blue pickup truck, and it is indeterminate whether it involves him driving an orange pickup truck. On the precise sense view, there may be precise senses indeterminately expressed by "my crazy uncle," corresponding to the sharpenings of the recollection (where a resolution is made about the color of the pickup truck). Suppose uncle Joe's truck was actually blue. One of the senses refers to Uncle Joe and the other refers to no one. But what should we say about the reference of the linguistic expression "my crazy uncle"? Should we say that "my crazy uncle" indeterminately refers to Uncle Joe? Or maybe it does determinately refer to him after all. What is the semantic mechanism which ensures this?

Consider a different case. Suppose instead that my recollection is confused and has as its source two different uncles: uncle Joe and uncle Pepe. The recollection is an amalgamation of two different experiences: Uncle Joe drives a blue truck, but uncle Pepe drives an orange truck. Should we say that "my crazy uncle" determinately expresses an imprecise sense which in turn indeterminately refers to uncle Joe and indeterminately refers to uncle Pepe? Or should we say "my crazy uncle" indeterminately expresses a precise sense involving a blue truck and it indeterminately expresses a precise sense involving an orange truck but these senses determinately pick out different men? Or perhaps we should go for some third option.

In what follows, we will not discuss these sorts of fuzzy recollections. Instead, we will focus on the sort of vagueness associated with what is known as "the problem of the many." Following David Lewis, I will assume "Venus" does not refer to a vague planet. There are no vague planets. Instead, there are precise overlapping objects. "Venus" indeterminately refers to *X*, "Venus" indeterminately refers to *Y*, etc., where *X*, *Y*, ... are each distinct, precise (down to the last molecule), and overlapping Venus-like planet-like objects.[5] Let Adam be some atom such that, as we would say more or less pre-theoretically, it is indeterminate whether it is part of Venus. Let us call a precise Venus-like object which contains Adam as a part, "Venus-Adam-Yes." And let us call the object which is just like Venus-Adam-Yes except without Adam as a part, "Venus-Adam-No." "Venus" is vague because it indeterminately refers to Venus-Adam-Yes and it indeterminately refers to Venus-Adam-No.

[5]Lewis (1993) holds that each precise object is a planet (which predicts there are countless planets in the vicinity of Venus). An alternative view says that each precise object is indeterminately a planet. I will assume the latter view.

For the Fregean, the facts just mentioned about "Venus" do not tell us yet how vagueness arises. We could be faced with vagueness in the word-sense relation or vagueness in the sense-object relation. In the first case, the word "Venus" may indeterminately express one sense and it may indeterminately express a distinct sense. Yet, the senses determinately refer to precise objects, like Venus-Adam-Yes and Venus-Adam-No. In the second case, where in addition the first view is denied, "Venus" will determinately express a unique sense. However, this sense will indeterminately refer to Venus-Adam-Yes, and it will indeterminately refer to Venus-Adam-No. Finally, we could also have a case where both types of vagueness are at play.

To simplify things, we will not consider situations where both types of vagueness are at play. Now, since we are currently also ignoring vague objects, we will only be talking about vagueness as it arises from either precise or imprecise senses. The basic data that "Venus" indeterminately refers can be explained in at least two distinct ways by invoking either precise or imprecise senses. Which way should the Fregean go? The answer will impact the viability of the thesis that vagueness inheres in representations. And later, we will show that it will impact the viability of supervaluationism. Before we get to that, we do some necessary stage setting.

7.3 Vagueness in the World

Consider the following worry about the various claims about where vagueness is located. If it is vague whether term *t* picks out *o*, why should we locate the vagueness in *t* and not *o*. Aren't they both equally culpable? Representational vagueness needs at least two things: the thing doing the representing and the objects being (vaguely) represented. So why do some philosophers want to blame the thing representing and not the thing being represented? How can we make sense of the thesis that vagueness is located in representational reality or is a representational phenomenon?

A possible answer is that vagueness in the world is incoherent.[6] So by process of elimination, vagueness is located in our representations of the world and not in the world itself. However, a number of philosophers have made a good case that vagueness in the world is not an incoherent notion.[7] Let us try something else.

We begin by focusing on natural language. Assume for now that vagueness is a linguistic phenomenon. One reason to think that vagueness is in language is that linguistic forms represent reality. And it is, to a certain extent, within our power to develop more precise languages for this purpose (in the simplest case, by inventing new words). As we create more precise languages, vagueness disappears. This suggests that language is the source of vagueness or that vagueness depends on language.

[6] See, for example, the Evans-Salmon argument against *de re* vague identity: Evans (1978) and Salmon (1982). See also Pinillos (2003).

[7] Akiba (2004), Barnes (2009), and Barnes and Williams (2010).

To see this, imagine a world where there is just one meaningful word left in the universe, "bald" (with the same meaning it has in our world). The following may seem plausible, at least for those who think vagueness is a linguistic phenomenon.

(*) If there were no meaningful word "bald," there would be no vagueness.

That is, assuming vagueness is linguistic, if we got rid of the last word we would get rid of vagueness. So (*) seems plausible on this view. But now consider the analogous claim about nonlinguistic reality and vagueness (make the same assumption that "bald" is the last meaningful word).

(**) If there were no bald people (the referent of "bald"), there would be no vagueness.

(**) is false. Clearly, "bald" does not cease to be vague if the bald people die off or if a potent hair tonic is made widely available. The putative asymmetry in truth-values with (*) and (**) then would help us understand why vagueness is a linguistic phenomenon (or why vagueness is in "bald," not bald people). The asymmetry, if true, gives us a reason to locate vagueness in language. A related point is this: Constructing pairs of sentences such as (*) and (**) gives us a test for determining the source of vagueness.

Some might worry that the proper referent for "bald" is not the set of bald people but rather the property of being bald. So (**) should be replaced by (***):

(***) If there were no property of being bald, there would be no vagueness.

Does this derail our test for determining the source of vagueness? I do not think so. Suppose the property of being bald is pleonastic and gotten on the cheap – simply arising from meaningful uses of "bald." In that case, (***) would be true if vagueness was linguistic. Since if there were no property of being bald, there would be no meaningful word "bald" – and so there would be no vagueness left in the universe (recall that we are assuming "bald" is the last word in the universe). But all this shows that (***) cannot be used as a sentence to test whether vagueness comes from the nonlinguistic realm. The defender of the view that baldness is a pleonastic property should, perhaps, revert back to (**) for the test which yields the desired result.

On the other hand, suppose that the property of being bald is not linguistic. In that case, it is natural to use (***) in the test. And if indeed (***) is true, then the test would not predict that vagueness is linguistic (since the truth-values for the sentences would be the same). But this is as it should be. Perhaps, talk of the "property of being bald," understood as being nonlinguistic, should be eschewed by the defender of vagueness as a linguistic phenomenon. Presumably, the existence of such a property would refute the vagueness-in-language thesis.

Not all human-made representations are linguistic in nature. Some of them, like pictorial representations, may admit of vagueness. Bertrand Russell, for example, held that vagueness is essentially representational and not in the (nonrepresentational) world: "All vagueness in language and thought is essentially analogous to this vagueness which may exist in a photograph."[8] On this view, (*) is false if the world also contains typical pictorial representations. To meet this concern, we can stipulate in our thought experiment that "bald" is not only the last meaningful word but also the last agent-constructed representation (which includes natural languages, pictures, photographs, etc.).

Another concern is that mental states are vague. Our beliefs, desires, and fears may very well be vague. This should not be surprising since the contents of mental states are described using natural, yet vague language. For example, the sentence "I hope that I will become bald" describes a mental state whose content is specified using vague language ("bald"). Now, contentful mental states have concepts as constituents.[9] Concepts represent things in the world. If mental states are vague, then so are concepts.

We began by considering the thesis that vagueness is a linguistic phenomenon. The considerations just raised reveal that it is more plausible to hold that vagueness is a mind-dependent representation (language, pictures, concepts, mental states, etc.) phenomenon. We can then modify our proposed test and accompanying thought experiment to cover these mind-dependent representations: Suppose that *X* is the last mind-dependent representation in the universe, then the following should be acceptable to the defender of the view that vagueness is a phenomenon of mind-dependent representations:

(*)′ If *X* didn't exist, there would be no vagueness.

So long as we pick an *X* that has an extension/referent whose members have contingent existence, then the following is false:

(**)′ If the members of the extension/referent of *X* didn't exist, there would be no vagueness.

The reason it is false, again, is that vague predicates can have empty extensions. The example of "bald" above verifies this. Now, the alleged truth of (*)′ and the alleged falsehood of (**)′ would help explain how we can justifiably say that vagueness is a phenomenon of mind-dependent representations and not the mind-independent world.

The reader might find the approach taken so far to be strange. Why not consider the following pair of counterfactuals evaluated in a normal circumstance and forget about evaluating sentences in a situation where there is but one mind-dependent representation left?

(T*) If there were no mind-dependent representations, there would be no vagueness.

(T**) If the members of the extensions of all mind-dependent representations didn't exist, then there would be no vagueness.

[8] Russell (1923, p. 154).

[9] I am using "concept" here the way psychologists use it, to denote a mental entity not an abstract object or universal.

It might be thought that (T*) being true and (T**) being false are sufficient to help explain the thesis that vagueness is located in our representations. Though (T*) seems plausible, (T**) may not be false. The number 2 is in the extension of "2," but 2 exists necessarily. So there is no world in which the referent of "2" doesn't exist. (T**) is therefore true (though vacuously so). So the pair (T*) and (T**) would not give us a good criterion for detecting the dependency of vagueness on language (and representations).

In this section, we have developed a criterion for locating vagueness. If we are wondering whether vagueness is located in a type of representation, imagine there is one token representation left (*X*) of that type. Now consider the counterfactuals "if *X* did not exist, vagueness wouldn't exist" and also "if the extension of '*X*' did not exist, vagueness wouldn't exist." If the first counterfactual is true and the second false, then we have some good reason to think that vagueness is a representational phenomenon.

7.4 Imprecise Senses and Vagueness in the World

According to Frege, senses have the following two features. First, they are representations. Second, they are mind-independent and eternal entities.[10] Let us look at these features in detail.

First, Fregean senses are modes of presentation of objects in the world. For example, Venus may present itself a certain way to me, as the evening planet which appears in such and such location. And it may appear to me in a different way, as the morning planet which appears in such and such a location. These modes of presentation are two distinct senses of Venus. Sense determines reference. So the referent of a sense is the object which is uniquely "described" by the sense or mode of presentation. Fregean senses are therefore representational entities.

Second, senses are not psychological entities. They are mind-independent objects. This requirement is natural if we follow Frege in accepting that (a) senses constitute propositions, (b) propositions are the primary bearers of truth, and (c) there are mind-independent truths. If there are mind-independent truths, then at least some propositions (and hence some senses – their constituents) will be mind-independent. If we assume that senses are metaphysically uniform, then the consideration generalizes to all propositions and senses.[11] A further claim is that senses are eternal (necessary objects). This might be thought to follow from the idea that there are necessary truths since each sense is a constituent of some

[10] Frege (1956).

[11] See Bealer (1998) and Schiffer (2003) for arguments in support of the mind-independence of propositions.

necessary proposition.[12] If a proposition is true in all possible worlds, then it exists in all possible worlds.[13]

Now recall the distinction we made between vagueness in the word-sense relation and vagueness in the sense-object relation (where the simplifying assumption is made that these categories are mutually exclusive). On the first view, a word like "Venus" will indeterminately express a "precise" sense (more than one, normally). This sense, since it is precise, will determinately refer to a unique object if it refers to anything at all. On the second view, a word will determinately express a unique "imprecise" sense. This sense will indeterminately refer to things (out there in the world).[14]

If we accept the imprecise sense view, we lose the ability to explain why vagueness is a representational phenomenon. Recall that the original puzzle was that since claims like "it is vague whether t picks out o" involve a representation and a thing represented, it is hard to say why vagueness should have its source in representations. As a possible solution to this puzzle, I proposed that the truth of $(*)'$ and the falsehood of $(**)'$ can help explain this (purported) fact. However, this idea is not available for the defender of imprecise senses. Since senses exist necessarily, there is no possible world where there is just one sense in existence or where they cease to exist. Hence the analogues of $(*)'$ would not be true (or if true, merely vacuously true):

(S*) If X (where X is the last sense in existence) were to no longer exist, vagueness would cease to exist.

(ST*) If senses ceased to exist, vagueness would no longer exist.

On some semantic approaches, counterfactuals with impossible antecedents like these are true (vacuously). This does not help us with the problem, however, since on this semantic approach, the following sentence would also be (vacuously) true: "If senses (and all other representations) ceased to exist, vagueness would still exist." The defender of vagueness as a representational phenomenon cannot help himself to the (vacuous) truth of (S*) to bolster his position any more than a defender of vagueness in nonrepresentational reality can use the truth of the sentence we just mentioned to defend his view.

I do not think that those who accept imprecise senses have the resources to explain why vagueness should be located in representations and not the (nonrepresentational) world. On the Fregean picture, there are worldly objects and there are senses. Senses imprecisely describe the world. But I do not see any grounds

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[12]Taking any sense, it will be a constituent of some proposition P. But now this proposition will be a constituent of some necessary proposition which is the disjunction of P and some truth of mathematics.

[13]There is a lot of literature challenging this last assertion. See King (2007) for a discussion challenging this last line of reasoning.

[14]Recall that we are making the simplifying assumption that the options are mutually exclusive and that there are no vague objects.

to say that senses as opposed to the "world" are the source of vagueness. We can say senses fail to determinately pick out objects, but we can just as easily say that objects fail to be determinately represented by the senses. Of course, the same point can be made in the other direction. We also lose the ability to say that vagueness is a nonrepresentational phenomenon. The point is not merely epistemic. It looks like there is nothing in reality which can locate vagueness in Fregean senses as opposed to the objects they imprecisely denote.

7.5 Imprecise Senses and the Attitudes

Suppose there are imprecise senses, then there will be imprecise propositions constituted by imprecise senses. One such proposition will be the one expressed by "Venus is a planet." Consider the following *de dicto* attribution assumed to be (determinately) true:

(B) Venus is a planet and Jones believes it.

According to orthodoxy, (B) is true just in case (1) Venus is a planet and (2) Jones has a belief with the content which is the proposition expressed by "Venus is a planet."[15] We just saw that this proposition, however, is an imprecise proposition. But how can it be true? If one of its propositional constituents is an imprecise sense, the sentence does not have determinate truth conditions (recall that we are ignoring vague properties and objects for now). So what is the semantic mechanism which accounts for its truth?

The method of supervaluations can help here.[16] According to a standard construal, a sentence is true if it is true on all admissible sharpenings of its terms. We suppose that a sentence that contains a vague term can be made sharper or more precise in various ways. For example, "Venus is a planet" can be made sharper by construing the predicate "is a planet" to be a predicate which determinately has Venus-Adam-Yes but determinately does not have Venus-Adam-No in its extension. We say the sharpenings have to be admissible because not any sharpening counts as a proper candidate to be the extension of "planet." For instance, an admissible sharpening of "moon" should not overlap with an admissible sharpening of "planet." Sharpenings must also be coordinated. Although some admissible sharpening of "Venus" will include Venus-Adam-Yes and some admissible sharpening of "planet" will fail to include Venus-Adam-Yes as a member, no admissible sharpenings of the entire sentence "Venus is a planet" will assign "Venus" a referent which is not also a member of the extension assigned to "Planet." This ensures that the sentence is true in every admissible sharpening, thus yielding the intuitively correct

[15] As a simplifying assumption, I ignore tense.

[16] Fine (1975).

result.[17] A chief advantage of the supervaluational approach is that it allows us to preserve classical logic. For example, the logical truth “Venus-Adam-Yes is a planet or Venus-Adam-Yes is not a planet” will be true on every sharpening even if the disjuncts are indeterminate. This will happen because on any sharpening of “Planet,” either Venus-Adam-Yes will be in its corresponding extension or it won't, thus yielding the desired result.

Notice that this approach does not require sharpenings to precise senses (we made no appeal to precise senses). So there is a case to be made, on grounds of parsimony, that if you accept imprecise senses and the method of supervaluations, you ought to reject vagueness in the word-sense relation. Apparently, such an account would do no work on a semantic framework. Vagueness is located wholly in the sense-object relation, or so one might argue.

There is another reason for the Fregean to reject vagueness in the word-sense relation. I sketch this idea here. According to Frege, propositions (made up of senses) are thoughts that can be communicated from person to person. It is an important feature of language that it can make this possible. It is beneficial to me, for example, that if I have a belief that there is a lion by those weeds, I can communicate this very thought to my kin by using the words “there is a lion by those weeds.” A language that achieves this result would serve an important purpose. This suggests that there is a tight connection between language and thought. This in turn suggests that natural language sentences (in contexts) determinately express thoughts. In contrast, it is less important that there be a tight connection between thought and reality. It is less important that our thoughts capture reality precisely. Since we are limited beings that need to categorize and represent with speed, imprecision in our representation of reality would not at all be surprising. Returning to our example, it is not very important that by “those weeds” I mean to describe an area of vegetation with quantum precision. If this line of reasoning is on the right track, we wouldn't expect vagueness to be located in the language-thought(sense) nexus. Rather, we would expect it to be located in the thought(sense)-object nexus. So we would expect our words to determinately express imprecise senses.

A third reason to accept imprecise senses is that it simply does not seem like we indeterminately express precise senses. Venus presents itself to me in a certain way. On the Fregean account, this presentation is a sense for Venus. However, if senses were precise, there would be a cluster of senses (each of which determinately refers to a precise planet-like object in the vicinity of Venus) each of which I indeterminately grasp. But when I introspect my thoughts of Venus, I fail to detect the indeterminate grasping of precise senses. Although it is somewhat intuitive to think that “Venus” indeterminately picks out precise planet-like objects, the idea that “Venus” indeterminately expresses a cluster of precise modes of presentations is much more obscure.

[17]The coordination may extend outside the boundaries of a sentence to an entire discourse or even between a mental state and a report as Brian Weatherson (2003) has pointed out. Weatherson's paper is a response to Schiffer's (1998) criticism of supervaluations. For a related response, see Keefe (2010).

So far, we have explored the imprecise sense view (vagueness in the sense-object relation) and made the points that (1) on this view, vagueness cannot be said to be a representational phenomenon (as opposed to the nonrepresentational world) and (2) the position, together with a supervaluation semantics, provides no obvious difficulties for the semantics of attitude ascriptions.

The imprecise sense view we have been defending so far gives way to imprecise propositions (since senses constitute propositions). Stephen Schiffer (1998) has argued against imprecise propositions by focusing on *de re* ascriptions.[18] Schiffer considers a use of "Al said Ben was there" (where "there" denotes a location Al referred to) and argues that the proposition expressed by the complement clause cannot be imprecise. He argued that if it were imprecise, "there" would have to refer to some vague or imprecise location. But Schiffer does not see what a vague or imprecise location would be. Hence, Schiffer concludes that the proposition cannot be imprecise.[19] In order for his argument to go through, Stephen Schiffer must be assuming that the ordinary referent of the demonstrative "there" is what is contributed to the proposition expressed by the complement clause. Though this is an assumption that is widespread in the philosophy of language and has a lot to recommend it, it is an extremely anti-Fregean assumption. For Frege, senses and not ordinary objects constitute propositions. Since our present focus is to see how vagueness fits in with a Fregean theory of senses and language, we can put aside Schiffer's worries for now.[20]

7.6 Precise Senses and Supervaluations

We move away from imprecise senses and consider their precise construal. Using supervaluation semantics, the precise sense approach can also account for the truth of (B). Interestingly, the approach does not make the same commitments as the imprecise sense approach discussed earlier. According to that first account, "it" in (B) refers to the unique (imprecise) proposition determinately expressed by "Venus is a planet." This is not so for the precise sense approach. "It" has many admissible sharpenings, each being a precise proposition indeterminately expressed by "Venus is a planet."

[18] Schiffer (1998).

[19] Schiffer calls these propositions "vague" not "imprecise."

[20] Garcia-Carpintero (2000, 2010) addresses Schiffer's argument by invoking neo-Fregean machinery. According to his view, "there" and other expression taking on a *de re* interpretation in attitude contexts do not contribute their ordinary referents to the proposition expressed by that-clause. Instead, other representational entities may appear in the proposition expressed.

Stephen Schiffer (1998, 2000a, b) developed an interesting argument that tells against this proposal.[21] The supervaluation approach predicts that if "Jones believes that Venus is a planet" is true, it is true in all precisifications. According to Schiffer, this entails that the sentence will be true only if Jones believes each precisification of "that Venus is a planet." So if the sentence is true, Jones will believe a number of precise propositions. But according to Schiffer's way of thinking, it is false that Jones believes those precise propositions (in the normal case when the ordinary attribution is true). Hence, the supervaluation approach which precisifies that-clauses in mental state ascriptions is mistaken.

Rosanna Keefe and Brian Weatherson have responded to Schiffer.[22] They deny that Jones believes all the sharpenings of "that Venus is a planet." The key here is that coordination of a sharpening will hold not only between word tokens within a discourse but also between word tokens and elements in the mental realm.

To illustrate this, consider a simple model connecting belief attributions and mental states. Suppose that an agent believes some proposition P just in case his "belief box" contains a language of thought sentence S which expresses P. And an attribution "A believes N" is true just in case A believes the proposition expressed by "N."[23] Let us assume that "Jones believes that Venus is a planet" is true. Now, according to the supervaluational framework, "Jones believes that Venus is a planet" is true just in case it is true in all admissible sharpenings. Suppose that admissible sharpening S_1 assigns precise proposition P_1 to "that Venus is a planet." But, and here is the key, S_1 also assigns language of thought sentences in Jones' belief box precise propositions. Assuming that Jones' belief box contains a language of thought sentence Venus is a planet, S_1 also assigns it P_1. According to our simple model, then "Jones believes that Venus is a planet" will be true with respect to admissible sharpening S_1 since S_1 will assign P_1 to both "that Venus is a planet" and the corresponding language of thought sentence in Jones' belief box. Similar reasoning gets us that the belief attribution is true with respect to other admissible sharpenings. Hence, we get the desired result that the sentence is true with respect to all admissible sharpenings.

Now let us see how this construal can handle Schiffer's objection. It is not true that Jones believes all admissible sharpening of "that Venus is a planet" or even one of them. Adding symbols to the object language corresponding to the sharpenings, it fails to be true that "Jones believes P_1" is true with respect to all sharpenings. The sentence is true with respect to S_1, but not true with respect to the other admissible sharpenings. Consider a sharpening S_2 distinct from S_1. According to S_2, the language of thought sentence in Jones' belief box will have P_2 as a content (distinct from P_1). Since P_2 is distinct from P_1, then according to

[21] Schiffer's argument focuses on the "says" relation and I focus on "belief." What follows is an adaptation of Schiffer's argument to suit our purposes. One difference is that Schiffer does not focus on Fregean propositions but his argument carries over.

[22] Weatherson (2003) and Keefe (2010).

[23] For simplicity, I ignore tense and context sensitivity.

our simple model, the attribution "Jones believes P_1" won't be true with respect to sharpening S_2. And since it won't be true with respect to all sharpenings, it won't be true simpliciter. Hence, Schiffer's prediction that the agent will believe many distinct sharp propositions is not true.

A feature of this response that was not explicitly endorsed by Weatherson and Keefe is that the sharpenings will involve nontrivial sharpenings of the verb "belief." I think it is natural to think there will be such sharpenings. To see this, note that sentences evaluated with respect to any sharpening will be bivalent. According to sharpening S_1, "Jones believes that P_1" is true. But according to some other sharpening, that sentence will be false. If it were true with respect to all sharpenings, then the sentence "Jones believes that P_1" would be true simpliciter which concedes Schiffer his point (since the reasoning generalizes to belief attributions concerning other precise propositions). So there is some sharpening S_n in which "Jones believes P_1" is not true. This means it is false since we are assuming bivalence for sharpenings. We can now see that the extension of "believes" has to vary with the sharpenings. According to S_1 "believes" will have $<$Jones, $P_1>$ in its extension. But according to S_n, "believes" will not have $<$Jones, $P_1>$ in its extension. This just means that the sharpenings of attitude ascriptions involving vague language in the complement clause will also invoke sharpenings of the attitude verb.

Appealing to this insight, we can give a straightforward response to Schiffer. The supervaluation defender can respond by claiming that in "Jones believes that Venus is a planet," "believes" is also vague. Although it may not be true that Jones believes every sharpening of the that-clause, it will be true that for each sharpening there is a sharpening of "believes" such that Jones stands in that relation to a precise proposition. The ordinary attribution can be true without requiring that there is a sharp proposition such that Jones believes it.

Let us recap. We were interested investigating the precise sense view in light of a supervaluational approach applied to ordinary mental state attributions. We noted a possible problem with such an approach (raised by Stephen Schiffer). We looked at Keefe's and Weatherson's response to Schaffer. I argued that the best way of making sense of this response is by positing vagueness in "believes."

I will assume from now on that if we adopt the precise sense approach with supervaluations, "belief" and other attitude verbs will be vague. On a simplified semantics for *de dicto* attitude ascriptions, each admissible sharpening of the mental state verb will be a set of triples containing a sharpening of the subject, a sharp proposition indeterminately expressed by the that-clause, and a time.[24] What is interesting about this conception is that the expression "Jones' belief that Venus is a planet" will indeterminately refer to several closely related mental states. This should not be too surprising, however, since beliefs are individuated by their contents. But note that the imprecise sense approach does not have this feature: The content of a belief will normally just be an imprecise proposition. There won't be a

[24] See also Hawthorne (2005).

need to say that "believes" and other mental state words are vague (at least to same degree as we would need to on the precise proposition view).

In what follows, I want to develop a new problem for supervaluations and mental state ascriptions on the assumption that that-clauses get sharpened to precise propositions (complex senses). Consider this case:

> Let "Venus$_1$," "Venus$_2$," ... be nonvague names for precise overlapping planet-like objects in the vicinity of Venus (as indicated earlier). Suppose that Jones acquires these names and sincerely assents to "Venus$_1$ is not a planet," "Venus$_2$ is not a planet" .... Jones, for instance, might believe that Venus is a vague object and that sharp objects like Venus$_1$ and Venus$_2$ are not planets. He may be wrong, but it is not impossible that he has those beliefs. We can also assume that the senses of "Venus$_1$," "Venus$_2$," and so on correspond to the natural sharpenings of "Venus" on the precise sense with supervaluations.

Now the following are true on their *de dicto* readings:

(B$_1$) Jones believes Venus$_1$ is a not a planet.
(B$_2$) Jones believes Venus$_2$ is not a planet.
...
(B$_n$) Jones believes Venus$_n$ is not a planet.

The evidence for this is that Jones would assert the sentences embedded in the that-clauses. Note also that Jones is a competent user of the words in these sentences. He may have the wrong philosophical view about vagueness, but this does not mean that he is not competent with the names for these objects. Now consider the true sentence:

(B*) Jones believes that Venus is a planet.

According to the precise sense approach with supervaluations, (B*) is true just in case it is true in all admissible sharpenings. These sentences capture the admissible sharpenings and so are predicted to all be true on their *de dicto* readings (we are ignoring here the sharpening of "Jones" and just focusing on "believes," "Venus," and "planet"):

(SB$_I$) Jones believes$_I$ that Venus$_I$ is a planet$_I$.
(SB$_2$) Jones believes$_2$ that Venus$_2$ is a planet$_2$.
...
(SB$_n$) Jones believes$_n$ that Venus$_n$ is a planet$_n$.

Recall that sharpenings have to be coordinated and the distributions of indices here reflect this. For example, the sharpening of "planet" dubbed "planet$_1$" has Venus$_1$ in its extension but not Venus$_2$.

For any i, the truth of (SBi) is not compatible with the truth of (Bi). Consider (B$_1$) which we are assuming to be true. (B$_1$) is true on all admissible sharpenings which will include (B$_{1a}$):

(B$_{1a}$) Jones believes$_1$ that Venus$_1$ is not a planet$_1$.

This sentence, I submit, is incompatible with (SB$_1$). On a Fregean view, a rational and reflective person cannot have the beliefs attributed to Jones by (B$_{1a}$) and (SB$_1$). This follows from the Fregean constraint on senses. Frege developed his theory to explain how a rational person may have incompatible thoughts about the same object and not recognize them as conflicting. For instance, a rational person might think the evening planet does not appear in the morning but also believe that the morning planet does appear in the morning. Since the evening planet is just the morning planet, how can a rational person have these beliefs? Frege's solution is that the agent is thinking of the evening planet/morning planet through two distinct senses (modes of presentation). One sense corresponds to the description "the evening planet" and the other to the description "the morning planet." In fact, the beliefs are constituted by these distinct senses.

In addition, Fregean senses are "transparent." That is, if a rational agent grasps the same sense twice, he is in a position to recognize that the senses are about the same object. Frege needed this to account for the a prioricity and triviality of tautologies of the form "$a = a$." The same sense appears twice in the same proposition and it is this fact, together with the meaning of "$=$," which explains a prioricity and triviality. We may articulate transparency schematically as follows (beliefs are understood *de dicto*):

(Trans) If a rational and reflective agent believes that *a is F* and believes that *b is not F*, then it is not a priori for him that $a = b$.

An instance of (Trans) applied to our case would be:

(TransJones) If Jones believes that Venus_1 is a planet and believes Venus_1 is not a planet, then it is not a priori for him that $\text{Venus}_1 = \text{Venus}_1$.

Now, this principle contains vague terms, including "believes" and "planet," so the principle should be true in all admissible sharpenings, including the following:

(TransJones_1) If Jones believes_1 that Venus_1 is a planet_1 and believes_1 that Venus_1 is not a planet_1, then it is not a priori for him that $\text{Venus}_1 = \text{Venus}_1$.

Now given that (SB_1) and (B_{1a}) are true, the antecedent of this conditional is true. It follows that "it is not a priori for Jones that $\text{Venus}_1 = \text{Venus}_1$." But clearly this sentence is not true since the occurrences of "Venus_1" express the same senses. Something has gone wrong.

What this reasoning shows is that either the precise sense view or the method of supervaluations is on the wrong track. For the Fregean, this should be an interesting result. Supervaluationism is the leading semantic approach to vagueness. On the other hand, we saw that giving up on the precise sense view and adopting the imprecise sense view leads to a problem about articulating why vagueness is to be located in representational reality.

The argument we just gave can be generalized and can be recast without appealing to Fregean senses. It goes through under the much weaker and simpler assumption that (Trans) holds, which may be thought to have some independent plausibility. Hence, the argument may be rehashed as one which shows that (Trans) is inconsistent with supervaluations and precise propositions.

7.7 Millianism, Supervaluations, and Knowledge

The argument I just gave can be avoided by giving up on Fregean senses. Suppose you accept Millianism, which holds that the meaning of a proper name is exhausted by its referent. On that view and under some natural assumptions, the truths of (SB_1) and (B_{1a}) are compatible. In effect, "Venus$_1$" in those sentences must be read *de re* (or what they semantically express must be a *de re* claim) and so there is no problem with (SB_1) and (B_1). On their *de re* readings, they become "Venus$_1$ is such that Jones believes of it that it is a planet$_1$" and "Venus$_1$ is such that Jones believes of it that it is not a planet$_1$." These sentences are not incompatible. So it looks like the Millian can avoid the problem raised for those who accept supervaluations and precise senses.

I now want to develop a puzzle for supervaluations that does not assume Fregeanism. I assume the content of a name in an attitude ascription is just its customary referent (a thesis usually attributed to Millianism).[25] As before, we let "Venus$_1$," "Venus$_2$," and so on correspond to sharpenings of "Venus." Coordinated with these sharpenings are sharpenings of "planet": "planet$_1$," "planet$_2$," and so on. Now, assume Jones is a rational and reflective person and that (K) is true:

(K) Jones knows that Venus is a planet.

Since (K) is true and it has vague terms, it is true in all sharpenings:

(K_1) Jones knows$_1$ that Venus$_1$ is a planet$_1$.
(K_2) Jones knows$_2$ that Venus$_2$ is a planet$_2$.
. . . .
(K_n) Jones knows$_n$ that Venus$_n$ is a planet$_n$.

I will argue that each of (K_1) ... (K_n) is false. We begin by noting that the following is false:

(K_1*) Jones knows that Venus$_1$ is a planet.

I will give two arguments for why (K_1*) is false. This is enough to create a problem for the supervaluation approach. This is because if (K_1*) is false, then so is each admissible sharpening, including (K_1), contrary to what the supervaluational approach recommends.

So here are the two arguments for why (K_1*) is false. First, recall from our case that Jones will reject the sentence "Venus$_1$ is a planet." And we even saw earlier that he believed instead that Venus$_1$ was not a planet (Jones might reasonably believe that sharp objects aren't planets). In fact, I see no positive reason to think that Jones does have that knowledge except that supervaluationism is committed to such a claim. I conclude that he does not know that Venus$_1$ is a planet.

The second argument against (K_1*) relies on the intuitive idea that borderline cases can't be known – it is false that they are known.[26] For example, let Harry be a

[25] Roy Sorensen (2000) develops a different sort of puzzle for direct reference and supervaluations.
[26] See Williamson (1994, ch. 8).

borderline case of "bald." No one can know that Harry is bald. "Venus$_1$ is a planet" is also a borderline case.[27] So it is false that it is known. Hence (K_1*) is false.

Arguably, knowledge requires a margin of safety. For example, if you know somebody who has n hairs is bald, then somebody with $n + 1$ hairs will be bald.[28] Our powers of discrimination aren't powerful enough to give us knowledge at the boundaries. Similar remarks about "Venus" apply here. Let Venus$_2$ be just like Venus$_1$ except for the addition of a single atom. This suggests the following margin of error principle:

(KJ) If Jones knows that Venus$_1$ is a planet, then Venus$_2$ is a planet.

Now, since this is true, then it is true in all sharpenings. So consider the following sharpening which is true by hypothesis:

(KJ_1) If Jones knows$_1$ that Venus$_1$ is a planet$_1$, then Venus$_2$ is a planet$_1$.

But the consequent of (KJ_1) is false by the stipulations of "Venus$_2$" and "planet$_1$." By modus tollens, the antecedent is false. But the antecedent is just (K_1) which is supposed to be true under supervaluational lights given that (K) is true. Since (K) is obviously true, I blame the supervaluation machinery. So Millianism, or more specifically the view that names have their customary referents in attitude contexts, is incompatible with the method of supervaluations.

7.8 Last Thoughts

Let us take stock. I have argued that if you are a Fregean who thinks there are imprecise senses (vagueness in the sense-object relation), you can't say how vagueness is a representational phenomenon. A Fregean who is a foe of vagueness in the nonrepresentational world may wish to adopt precise senses (vagueness in the word-sense relation only). But we saw that this approach cannot be combined with a supervaluation framework. In addition, we saw that there was a problem with supervaluations and attitudes even if we drop Fregeanism and adopt a Millian perspective.

Now consider the possibility that there are vague objects. There is just one vague object, Venus. If you are Fregean, you may hold that "Venus" determinately expresses a unique sense. And the sense of "Venus" determinately picks out that one vague object Venus. On this approach, some of the worries I mentioned in this chapter need not come up. There is no need to make a decision about whether there is vagueness in the word-sense relation or vagueness in the sense-world relation. The vague object defender may hold that this is a false dichotomy and so the problems

[27] Discussions of margins of error in vagueness almost always concern sorites series. The case of Venus does not obviously admit of a sorites series but I see no reason why the same ideas cannot apply here.

[28] See Williamson (2007). However, the claim is derived from a more basic one.

that arose for the precise sense view won't arise. There won't be any obvious need to invoke sharpenings of "Venus" to account for the knowledge ascriptions. Typically, sharpenings and supervaluations are invoked when there is vagueness in language or representations. So the problems we saw with knowledge ascriptions won't obviously appear for the vague object view. Unfortunately, the worries about the attitudes will resurface for the vague object defender if they adopt a supervaluation approach. Vagueness in the world defenders, Akiba (2004), Barnes and Williams (2010) have done so. These theorists will face the difficulties I have raised here. It is better for vagueness in the world defenders to reject a supervaluational semantics.

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Part III
Formal Issues

Chapter 8
Boolean-Valued Sets as Vague Sets

Ken Akiba

8.1 Introduction

The *ontic* view of vagueness (or *onticism* about vagueness) is the view that the world itself contains vagueness and that that's the (or at least *one*) source of vagueness existing in our language and thought, including perception. This is in contrast to the *semantic* view (or *semanticism*) and the *epistemic* view (or *epistemicism*), both of which hold that the world itself is crisp and precise. They locate the source of vagueness somewhere else. According to semanticism, vagueness exists only in our language and thought, in their semantic relations to the world in particular. According to epistemicism, vagueness is a kind of ignorance on our part; sometimes *we* cannot draw sharp borderlines in things around us because *we* do not know where the borderlines are, and not because the borderlines do not exist – they do, unbeknownst to us. This chapter investigates the plausibility of onticism on its own merit, independently of the plausibility of the other two views. We shall thus set aside semanticism and epistemicism in this chapter.

Under the umbrella of onticism, various more specific opinions can be maintained. Onticism can mean, for instance, the existence of vague individuals, vague states of affairs, or vague identity. But one thing it definitely can mean is the existence of vague *properties. Being bald*, *being tall*, and *being on Mt. Everest*, for instance, can be such properties. If you are attracted to onticism about vagueness, you may want to uphold the view that there are vague properties. And that is the view I am interested in upholding. But what is the best way to do that?

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As I see it, the best way to uphold the existence of *crisp* properties is to defend the existence of (the standard, crisp) *sets*, which are considered to be the extensional surrogates of the crisp properties. Analogously, the best way to uphold the existence of *vague* properties ought to be to defend the existence of *vague sets*, the extensional surrogates of the vague properties. But what should vague sets be like?

Some years ago, I set forth a *modal* conception of vague sets and ontic vagueness in general (see Akiba 2000a, 2004). The original supervaluationism, introduced by Fine (1975) and others, postulates possible-world-like entities called *precisifications* (or *sharpenings*) and assumes that a vague predicate such as "is bald," "is tall," or "is on Mt. Everest" denotes different crisp sets in different precisifications, each of which is an admissible extension of the predicate; for instance, "is tall" may denote the set of people at least 5 ft 10 in. tall in one precisification, the set of people at least 5 ft 11 in. tall in another precisification, and the set of people at least 6 ft tall in yet another precisification. This popular theory seemed to be able to deal with vague predicates without assuming anything ontically vague in the world; thus, it was thought that the theory supported semanticism. In response, I argued that the theory could easily be turned into an ontic version, according to which a vague predicate denotes an ontically vague set, which is a transworld (or trans-precisification) object existing across different precisifications (or what I called "precisified worlds"), coinciding with different crisp sets in different precisified worlds; for instance, "is tall" denotes a vague set which coincides with the set of people at least 5 ft 10 in. tall in one precisified world, coincides with the set of people at least 5 ft 11 in. tall in another precisified world, and coincides with the set of people at least 6 ft tall in yet another precisified world. It had been widely assumed by then (see, e.g., Evans 1978; Lewis 1988; Parsons 2000) that supervaluationism supports semanticism, whereas the logic of onticism ought to be a many-valued logic. I dismissed this prevalent assumption, arguing that supervaluationism is neutral with respect to the debate between semanticism and onticism about which theory is the better theory of vagueness. Similar ontic theories of vagueness have since been set forth by some others, such as Morreau (2002), Williams (2008), Barnes (2010), and Barnes and Williams (2011).

I still think that my basic argument was correct. I doubt, however, that the argument has convinced many people – if any – of the existence of vague sets. The main source of this ineffectiveness is the heavy machinery the theory calls for: it asks us to believe in "precisified worlds" that divide the whole universe, as well as the transworld sets that exist across them.[1] If vague sets are to be believed in, they

[1]It should be emphasized, however, that semantic supervaluationism already accepts precisified worlds, whether they are called "precisified worlds" or "precisifications." A version of it that employs the transworld identity theory of individuals also accepts the existence of trans-precisificational objects. The only difference between this version of semantic supervaluationism and ontic supervaluationism I advocate is whether precisifications differ only with respect to semantic facts (i.e., what word refers to which precise object) or also with respect to non-semantic facts (i.e., what trans-precisificational object coincides with which precise object). Thus, the charge

ought to be much more intuitive than that! Ideally, the theory of vague sets ought to be like this.

In the standard set theory, the set membership relation $\in$ between a set s and an object a is binary: either "$a \in s$ (a is a member of s)" is true (or has value 1) or it is false (or has value 0). The prospective vague set theory ought to be modeled after the standard set theory and similar in structure, except that the set membership relation ought to be many-valued (or graded): "$a \in s$" may have a value between 0 and 1. If we can formalize such a theory and make sense of vague sets, we can support the existence of vague properties with it in the same way in which we can support the existence of crisp properties with the standard set theory.

One may think that we already do have such a set theory. *Fuzzy set theory*, proposed by Zadeh (1965) in the 1960s and steadily developed since then, postulates fuzzy sets whose set membership has degrees. I agree that was a good start. However, fuzzy set theory is degree-functional: generally, the truth degree of a compound proposition is determined by the truth degrees of its component propositions, and, specifically, the degree of one object's being a member of a set is determined by the degrees of the object's being members of sets of which the set is the intersection, union, or complement. This is a very problematic feature many philosophers do not want to accept. So in this chapter I would like to try to remove this feature from fuzzy set theory. In the resulting theory, called *Boolean-valued set theory*, the values of the set-membership relation have not a *linear* but a *Boolean* structure. Whether this theory should still be considered a version of fuzzy set theory or not is a mere terminological issue of little significance that I shall leave to the reader to decide on, though I myself am inclined to think that it is different enough from the standard fuzzy set theory to be understood as a separate theory.

More specifically, in the next section the original fuzzy set theory is sketched along with the standard, crisp set theory, and the sense in which fuzzy set theory is a generalization of crisp set theory is clarified. Then, in Sect. 8.3, the problematic feature of fuzzy set theory, that is, its degree-functionality, is pointed out in the form of the problem of penumbral connections. In Sect. 8.4, Boolean-valued set theory, which is not degree-functional (though it may still be considered *truth*-functional or *value*-functional) and is thus free of the problem of penumbral connections, is presented as another generalization of crisp set theory. In Sect. 8.5, Boolean-valued set theory is related to Boolean-valued models of ZFC set theory, introduced by Scott and Solovay in the 1960s. This chapter concludes with a brief discussion of some of the potential philosophical problems for Boolean-valued set theory. The main aim of this chapter is to introduce Boolean-valued sets as vague sets, together with some semi-technical details but setting aside most philosophical questions. For that reason, the sorites paradox, a very popular topic in the philosophy of vagueness, will not be discussed in this chapter.

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often made against my brand of ontic supervaluationism that it assumes a larger ontology than semantic supervaluationism, is totally off the mark.

8.2 Crisp and Fuzzy Sets

In the standard, crisp set theory, a set s is determined by its *characteristic function*:

(1) $$C_s : U \to \{0,1\},$$

where U is the universe of discourse. This function gives value 1 if its argument, a ($\in U$), is in the set s, and value 0 if it is not. We define, for any a,

(2) $$[\![a \in s]\!]_C = C_s(a);$$

that is, $[\![a \in s]\!]_C$ is the value of the sentence "$a \in s$" given by C. We can also define the values given to the *complement* of a, $\overline{a}$, the *intersection* of a and b, $a \cap b$, and the *union* of a and b, $a \cup b$, as follows: for any x,

(3) $$\begin{aligned} [\![x \in \overline{a}]\!]_C &= 1 - [\![x \in a]\!]_C; \\ [\![x \in (a \cap b)]\!]_C &= \min([\![x \in a]\!]_C, [\![x \in b]\!]_C); \\ [\![x \in (a \cup b)]\!]_C &= \max([\![x \in a]\!]_C, [\![x \in b]\!]_C). \end{aligned}$$

In fuzzy set theory, a fuzzy set s is determined by its *membership function*:

(4) $$M_s : U \to [0,1].$$

This function gives a real number in the interval [0,1] to any a ($\in U$) as its degree of membership: if a is completely in the set s, then the degree is 1, if a is completely outside of s, then the degree is 0, and if a is only partially in s, then a takes a degree between 0 and 1. In the way analogous to that for crisp sets, we define, for any a,

(5) $$[\![a \in s]\!]_M = M_s(a);$$

that is, $[\![a \in s]\!]_M$ is the value of the sentence "$a \in s$" given by M. The values given to the complement, the intersection, and the union are defined in the same way as those for crisp sets: for any x,

(6) $$\begin{aligned} [\![x \in \overline{a}]\!]_M &= 1 - [\![x \in a]\!]_M; \\ [\![x \in (a \cap b)]\!]_M &= \min([\![x \in a]\!]_M, [\![x \in b]\!]_M); \\ [\![x \in (a \cup b)]\!]_M &= \max([\![x \in a]\!]_M, [\![x \in b]\!]_M). \end{aligned}$$

The only difference is that in the case of fuzzy sets, these values may be any real number in [0,1].

In the case of both crisp and fuzzy sets, we can broaden our perspective and take the above definitions of complement, intersection, and union as consequences of crisp and fuzzy *logics*, respectively.[2] In both logics, for any propositions p and q,

[2] An infinite-valued logic called *standard Łukasiewicz logic* L_1 (or $L_{\aleph_1}$) is often used as the basis of fuzzy logic. Klir and Yuan (1995, Chap. 8) provides a useful account of fuzzy logic and its connection to fuzzy set theory.

$$\begin{aligned} [\![\neg p]\!] &= 1 - [\![p]\!]; \\ [\![p \wedge q]\!] &= \min([\![p]\!], [\![q]\!]); \\ [\![p \vee q]\!] &= \max([\![p]\!], [\![q]\!]). \end{aligned} \tag{7}$$

Suppose (7) holds. Then, since $x \in \overline{a} \iff \neg(x \in a)$, $x \in (a \cap b) \iff x \in a \wedge x \in b$, and $x \in (a \cup b) \iff x \in a \vee x \in b$, setting aside the subscripts, (3) and (6) follows. Thus, ultimately, the difference between crisp logic/set theory and fuzzy logic/set theory may be taken to be about whether atomic propositions take only values 0 and 1 or any real number in [0,1]. The valuations to atomic propositions can vary from interpretation to interpretation; so it is more accurate to write $[\![p]\!]_V$ where we wrote $[\![p]\!]$ above, where V is a valuation (or an interpretation) replacing and generalizing the previous C and M. Hence, in summary, the essential part of crisp logic/set theory and fuzzy logic/set theory may be presented in common form as follows:

$$\begin{aligned} [\![\neg p]\!]_V &= 1 - [\![p]\!]_V; \\ [\![p \wedge q]\!]_V &= \min([\![p]\!]_V, [\![q]\!]_V); \\ [\![p \vee q]\!]_V &= \max([\![p]\!]_V, [\![q]\!]_V); \\ [\![x \in \overline{a}]\!]_V &= 1 - [\![x \in a]\!]_V; \\ [\![x \in (a \cap b)]\!]_V &= \min([\![x \in a]\!]_V, [\![x \in b]\!]_V); \\ [\![x \in (a \cup b)]\!]_V &= \max([\![x \in a]\!]_V, [\![x \in b]\!]_V). \end{aligned} \tag{8}$$

"Crisp logic/set theory" is mouthful, so in what follows we use the term "crisp set theory" to include also crisp logic; analogously, we use the term "fuzzy set theory" to include fuzzy logic. Then, again, the only difference between crisp set theory and fuzzy set theory in this sense is that the latter takes any real value in [0,1], whereas the former takes only 0 and 1. In this respect, crisp set theory can be taken as a special case of fuzzy set theory; or, conversely, fuzzy set theory can be taken as a generalization of crisp set theory.[3]

8.3 The Problem of Penumbral Connections

Since the 1960s, when it was first proposed by Lotfi Zadeh, fuzzy set theory has gained much popularity in sciences and engineering; but it remains rather unpopular in philosophy as a solution to the philosophical problems of vagueness.[4] There are perhaps several reasons why, but here I only mention two.

[3] See Smith (2008a) for more details about this point.

[4] Klir and Yuan (1995) is a standard textbook of fuzzy logic and set theory. Goguen (1969), Lakoff (1973), and Machina (1976) use modified versions of fuzzy set theory in their philosophical accounts of vagueness. Smith (2008a) is a more recent attempt to defend fuzzy set theory.

For one thing – the critics argue – even if we accept degrees in set membership, it is unclear at what point the degree should all of a sudden become less than 1 (or more than 0). For instance, take $D = \{$the surface points on earth$\}$ and consider the property *being on Mt. Everest*. According to fuzzy set theory, this property extensionally denotes a fuzzy set, say e, of surface points on earth and, certainly, $[\![\text{the peak of Mt. Everest} \in e]\!]_V = 1$ on any reasonable valuation V. Presumably, in the closest neighborhood of the peak the degree is 1. But as we descend away from the peak, there is a point (or a circle) where the degree all of sudden becomes less than 1. But where is it, and how can we make sense of such a sharp borderline? If there is such a sharp borderline, then, after all, epistemicists seem correct in insisting that everything is crisp in reality.

My reaction to this argument is that even though it may be a good argument against onticism in general, it does not explain why fuzzy set theory is (or should be) even less popular than other versions of onticism, such as ontic supervaluationism mentioned in Sect. 8.1. Most non-degree-theoretic versions of onticism still assume the existence of higher-order vagueness and employ the "determinately" (or "definitely") operator. A certain geographical point on earth's surface may be $\underbrace{\text{determinately}\ldots\text{determinately}}_{n \text{ times}}$ on Mt. Everest but not $\underbrace{\text{determinately}\ldots\text{determinately}}_{n+1 \text{ times}}$ so. But then there must be a sharp borderline between *being* $\underbrace{\textit{determinately}\ldots\textit{determinately}}_{n+1\ \textit{times}}$ *on Mt. Everest* and *not being* $\underbrace{\textit{determinately}\ldots\textit{determinately}}_{n+1\ \textit{times}}$ *on Mt. Everest*. But where is it, and how can there be such a borderline? There is no good answer. So essentially the same problem seems to haunt the other versions of onticism.

I agree with many critics of fuzzy set theory, however, that there is a second problem which is specific to the theory. That's *the problem of penumbral connections*. It is the problem Fine (1975) pointed out to motivate supervaluationism (since supervaluationism is free of the problem). There are in fact two versions of the problem, one involving logical connections such as *tall* and *not tall* and the other involving nonlogical, conceptual connections such as *tall* and *short*, but here the logical version is presented. (Presumably the nonlogical version is reducible to the logical version with the addition of appropriate meaning postulates.) The source of the problem is that fuzzy logic, though it is many-valued, is still *degree-functional*: in fuzzy logic the degree of a compound sentence is determined solely by the degrees of its constituents regardless of their contents. (The feature just called *degree-functional* is usually called *truth-functional*; however, as we shall see, this term is not quite appropriate. Williamson (1994, pp. 135–138) uses the term *degree-functional*.) For instance, suppose $[\![\text{Adam is tall}]\!]_V = 0.5$ and $[\![\text{Adam is fat}]\!]_V = 0.5$ on some appropriate valuation V. Then, since $[\![\text{Adam is not tall}]\!]_V = 1 - [\![\text{Adam is tall}]\!]_V = 0.5$, $[\![\text{Adam is tall or not tall}]\!]_V = 0.5$, just like $[\![\text{Adam is tall or fat}]\!]_V = 0.5$.

Many philosophers find this odd. Surely, everybody, determinately, has the property *being tall or not tall*, regardless of his/her height, so $[\![\text{Adam is tall or not tall}]\!]_V$ should be 1. Analogously, $[\![\text{Adam is tall and not tall}]\!]_V = 0.5$ in fuzzy logic, but it should be 0, for nobody, determinately, has the property *being tall and not tall*.

The widely held view that the degree-functionality is really an undesirable feature has recently been challenged. Smith (2008a), for instance, defends the degree-functionalist theories against the above argument by pointing out that ordinary people's linguistic intuition on this matter is at best ambivalent.[5] I myself think that the argument has much force, but my reason has little to do with what ordinary people think. Rather, the reason is that I myself cannot form a coherent idea about what vagueness is like if it does not sustain logical penumbral connections.

Suppose there is a piece of paper in front of you. The left end of the paper is completely black, the right end is completely white, and there is a gradual change of tone in the middle. An ant is walking from right to left, left to right. Suppose that at the center, $[\![\text{this is black}]\!]_V = [\![\text{this is not black}]\!]_V = 0.5$ on some appropriate V. It is reasonable to assume that $[\![\text{this is black}]\!]_V = 1$ at the left end, and that $[\![\text{this is not black}]\!]_V = 1$ at the right end.[6] But then it follows that $[\![\text{this is black or not black}]\!]_V = 1$ at both ends but only 0.5 at the center. This seems possible only if the paper becomes more black faster than it becomes less nonblack as the ant walks toward the left end, and only if the paper becomes more nonblack faster than it becomes less black as the ant walks toward the right end. If the rate of the paper's becoming more black (nonblack) is the same as the rate of its becoming less nonblack (black), as it seems it should be, then $[\![\text{this is black or not black}]\!]_V$ should be constant. An analogous point applies to $[\![\text{this is black and not black}]\!]_V$.

Admittedly, this is hardly a knockdown argument; it is just an explanation of why I do not understand degree-functional vagueness. I am also open to the possibility that degree-functional vagueness may be a species of vagueness there is that I currently don't see. But at the very least, the above argument, or something like it, seems to give us a sufficient *motivation* for pursuing revision of fuzzy set theory that satisfies the following equations:

$$
\begin{aligned}
[\![p \wedge \neg p]\!]_V &= 0; \\
[\![p \vee \neg p]\!]_V &= 1; \\
[\![x \in (a \cap \overline{a})]\!]_V &= 0; \\
[\![x \in (a \cup \overline{a})]\!]_V &= 1.
\end{aligned}
\tag{9}
$$

[5]Experimental support of this claim is given in, e.g., Bonini et al. (1999), Alxatib and Pelletier (2011), and Serchuk et al. (2011).

[6]At the end of the day, I think we should give up this assumption, as I shall argue in the final section. It *is* a reasonable assumption, however, and whether we should give it up at the end of the day or not really does not affect the current argument. The argument holds up insofar as the value of the disjunction is not constant at the center, right end, and left end.

These will be the desiderata for the rest of this chapter. (In fact, since $x \in (a \cap \overline{a}) \iff x \in a \wedge x \in \overline{a} \iff x \in a \wedge \neg(x \in a)$ and $x \in (a \cup \overline{a}) \iff x \in a \vee x \in \overline{a} \iff x \in a \vee \neg(x \in a)$, the second two desiderata follow from the first two; so the important ones are the first two.) How, then, should we revise fuzzy logic to satisfy these desiderata?

8.4 Boolean-Valued Logic and Boolean-Valued Sets

My answer to this question is based on the following observation.[7] The basic idea behind fuzzy set theory is that we can generalize crisp set theory by allowing propositions and set membership to have values not just 0 and 1 but the numbers in between. However, fuzzy set theory's way of generalizing crisp set theory is not the only natural way of generalizing it. There is at least one other, equally natural, way.

Suppose B is some Boolean lattice (or Boolean algebra) such that

(10) $$B = \langle D^B, \wedge, \vee, \neg, 0, 1 \rangle,$$

where D^B is the domain of B; $\wedge$, $\vee$, and $\neg$ are the greatest lower bound (glb, or infimum), the least upper bound (lub, or supremum), and the complement associated to B; and 0 and 1 are the bottom and the top elements of B. (Perhaps the more common symbols for the glb, lub, and complement are $\cdot$, $+$, and $-$ or $*$; but, as you will see shortly, I have reasons to use $\wedge$, $\vee$, and $\neg$ despite their potential ambiguity.) Also assume the membership function B_s for a set s,

(11) $$B_s : U \to B,$$

where U, again, is the universe of discourse; and define

(12) $$[\![a \in s]\!]_B = B_s(a).$$

[7] Another possible way to satisfy the desiderata is Edgington's (1992, 1997). Her idea is to use the standard probability calculus to measure the degree of vagueness; that is,

$$\begin{aligned} [\![\neg p]\!]_V &= 1 - [\![p]\!]_V; \\ [\![p \wedge q]\!]_V &= [\![p]\!]_V \times [\![q \text{ given } p]\!]_V; \\ [\![p \vee q]\!]_V &= [\![p]\!]_V + [\![q]\!]_V - [\![p \wedge q]\!]_V. \end{aligned}$$

Edgington contends that just as credence and objective chance may share the same logic even though they are conceptually different (cf. Lewis 1980), the degrees of truth (or what she calls "verity") pertaining to vagueness may also follow the same logic even though it is different from either credence or objective chance. Discussion of Edgington's view is beyond the scope of this paper.

Then the following may be considered a generalization of crisp set theory:

$$
\begin{aligned}
[\![\neg p]\!]_B &= \neg\, [\![p]\!]_B;\\
[\![p \wedge q]\!]_B &= [\![p]\!]_B \wedge [\![q]\!]_B;\\
[\![p \vee q]\!]_B &= [\![p]\!]_B \vee [\![q]\!]_B;\\
[\![x \in \overline{a}]\!]_B &= \neg\, [\![x \in a]\!]_B;\\
[\![x \in (a \cap b)]\!]_B &= [\![x \in a]\!]_B \wedge [\![x \in b]\!]_B;\\
[\![x \in (a \cup b)]\!]_B &= [\![x \in a]\!]_B \vee [\![x \in b]\!]_B.
\end{aligned} \tag{13}
$$

We also assume $[\![p \vee \neg p]\!]_B = 1$ and $[\![p \wedge \neg p]\!] = 0$. Note that here $\neg$, $\wedge$, and $\vee$ are used differently in the object language inside $[\![\cdot]\!]_B$ and the metalanguage outside $[\![\cdot]\!]_B$; in the object language they are the negation, conjunction, and disjunction of propositions,[8] whereas in the metalanguage, they are the complement, glb, and lub of the relevant values. I use the same symbols, however, in order to emphasize that despite this difference, they behave exactly the same way, as we shall see immediately. (13) holds for crisp set theory if B is the two-element Boolean algebra $B2$, where $D^{B2} = \{0,1\}$. If $B = B2$, there is no substantive difference between (13) and (8). The difference will arise, however, if we generalize (13) by allowing more general Boolean lattices for B (or, more precisely, more general *nondegenerate* Boolean lattices, i.e., lattices with at least two members; one-membered, or degenerate, lattices will be set aside entirely for the rest of this chapter).

If B is a more general Boolean lattice, the ordering relation $\leq$ that determines the lattice is only partial and not total or linear; so there is no general way of assigning numbers between 0 and 1 meaningfully[9] even though the bottom and the top of the

[8]For the sake of simplicity we shall ignore in this paper the universal and existential quantifiers in the object language. To include those quantifiers, we have to require that the relevant Boolean lattice B be *complete*. Generally, a complete lattice G is a lattice that has the glb and lub for *any* subset of D^G whether it is finite or infinite. Then we can have

$$
\begin{aligned}
[\![\forall x\, \phi(x)]\!]_B &= \bigwedge_{a \in U} [\![\phi(a)]\!]_B;\\
[\![\exists x\, \phi(x)]\!]_B &= \bigvee_{a \in U} [\![\phi(a)]\!]_B.
\end{aligned}
$$

[9]It's not as if there is absolutely no way of assigning numbers between 0 and 1 that conforms to the ordering relation $\leq$. For instance, Zhang (1982) came up with one way, in which the paths in the lattice structure and the locations in the paths are given binary interpretations, which are then translated into real numbers in [0,1]. However, on Zhang's assignment, if, for instance, $[\![p]\!]_B = 5/8$ and $[\![q]\!]_B = 3/8$, then even though $[\![p]\!]_B \wedge \neg[\![p]\!]_B = 0$ and $[\![p]\!]_B \vee \neg[\![p]\!]_B = 1$, $[\![p]\!]_B \wedge [\![q]\!]_B = 1/8$ and $[\![p]\!]_B \vee [\![q]\!]_B = 7/8$. These are hardly the values one would expect or can make sense of. As this example illustrates, while it is possible for us to assign numbers between 0 and 1, such an assignment lacks intuitive meaning.

lattice are 0 and 1. On a brighter side, the many-valued logic that results from the definition of logical consequence,[10]

(14) $$\Gamma \vdash q \iff \bigwedge_{p\in\Gamma} [\![p]\!]_B \leq [\![q]\!]_B \text{ for any } B,$$

is entirely classical (in the deductive sense). Or, to put it the other way around, classical propositional logic is sound and complete with respect to the general Boolean semantics (see Rasiowa and Sikorski 1963). Needless to say, classical propositional logic is sound and complete with respect to the two-element Boolean semantics, but the important point here is that (14) can be retained even if B is not just a two-element Boolean algebra but *any* Boolean algebra; so the common characterization of classical logic as a *bivalent* logic is inaccurate.[11] As a consequence of the above assignment rules, $[\![p \wedge \neg p]\!]_B = [\![p]\!]_B \wedge \neg [\![p]\!]_B = 0$ and $[\![p \vee \neg p]\!]_B = [\![p]\!]_B \vee \neg [\![p]\!]_B = 1$ on any B; so this generalization of crisp set theory satisfies the desiderata (9).

As you can see in (14), an argument from the set Γ of premises to the conclusion q is valid if and only if on any value assignment B, the value of q is at least as high as the glb of the values of the premises. This implies that if the values of all the premises are 1, the value of the conclusion must also be 1, but it says much more than that; it maintains *the nonreduction (or nondecrease) of total values*.

In this chapter, we will not introduce the "determinately" operator Δ into the object language for the sake of simplicity. The absence of the "determinately" operator is not as significant as it is if the semantics is the ordinary bivalent one; "determinately" is used often as a substitute for many values, as you can see in our discussion in the last section involving higher-order vagueness, but we have little need for the operator given our many-valued semantics. For the same reason, fuzzy logic usually does not contain the "determinately" operator. However, we *can* introduce the "determinately" operator into the object language if we want to, by the following definition:

(15) $$[\![\Delta p]\!]_B = 1 \text{ if } [\![p]\!]_B = 1;$$
$$[\![\Delta p]\!]_B = 0 \text{ if } [\![p]\!]_B \neq 1.$$

Unlike in most other logics of "determinately" so far proposed, $p \vdash \Delta p$ does not generally hold in this logic, for if $0 < [\![p]\!]_B < 1$, $[\![\Delta p]\!]_B = 0$, so $[\![p]\!]_B \not\leq [\![\Delta p]\!]_B$. (The key here is that in this logic, the consequence relation $\vdash$ is defined as the nonreduction of values; if it were defined instead as *the preservation of truth*

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[10]Or, in a multiple-conclusion formulation such as that of the classical sequent calculus,

$$\Gamma \vdash \Delta \iff \bigwedge_{p\in\Gamma} [\![p]\!]_B \leq \bigvee_{q\in\Delta} [\![q]\!]_B \text{ for any } B.$$

[11]Needless to say, classical logic *can* be defined semantically, as the bivalent logic. But, then, classical logic, thus defined, is deductively no different from the logic defined by the general Boolean-valued semantics.

(or value 1), as it is in most other logics, then $p \vdash \Delta p$ would hold. These two notions of logical consequence correspond to what Williamson (1994) calls *local* and *global validity*.) This is good news because even in the other logics $\neg\Delta p \nvdash \neg p$ and $\nvdash p \rightarrow \Delta p$, for instance; so they have to give up some classical meta-inference rules such as contraposition and the deduction theorem. The present logic, in contrast, does not have to give up those rules, since $p \nvdash \Delta p$. We will not, however, consider the "determinately" operator in this chapter any further.

The logic proposed here, though it is not *degree*-functional, may still be considered *truth*-functional because the value of a compound sentence is still determined by the values of its constituents. In this regard, *value-functional* may be the more accurate, less ambiguous term, though *truth-functional* is perfectly fine insofar as truth values are identified with general Boolean values and not with just truth and falsity. This logic does not avoid the problem of penumbral connections by dropping functionality (or compositionality) from its semantics. As you can see in (13), the semantics is still functional; it's just that it assigns more appropriate values to compound sentences. For instance, to use the previous example, if $[\![\text{Adam is tall}]\!]_B = p$ and $[\![\text{Adam is fat}]\!]_B = q$, then $[\![\text{Adam is tall or not tall}]\!]_B = p \vee \neg p = 1$, whereas $[\![\text{Adam is tall or fat}]\!]_B = p \vee q$. Again, this shows that the oft-used term *truth-functional* should be used more carefully.

Let's call the present logic *Boolean-valued logic*. Just as fuzzy logic can be expanded into fuzzy set theory in the broad sense, Boolean-valued logic can be expanded into *Boolean-valued set theory* to include Boolean-valued sets of individuals. For any Boolean lattice B, a *B-valued set* is a function from the members of the universe of discourse U to the members of B. In what follows we shall often drop the prefix and simply call "set" where "B-valued set," with specific B, is more accurate. What the last three equations in (13) state is that for any $x \in U$ and any sets a and b, if the value of x's being a member of a is $p \in D^B$ and the value of x's being a member of b is $q \in D^B$, then the values of x's being a member of $\overline{a}$, $a \cap b$, and $a \cup b$ are $\neg p$ (the complement of p), $p \wedge q$, and $p \vee q$ (the glb and lub of $\{p, q\}$), respectively. Again, these follow from the first three equations in (13) together with the definitions of complement, intersection, and union.

Doubtless, it is a somewhat disappointing fact about Boolean-valued set theory that the values we assign to propositions and set membership cannot generally be linearly ordered (thus, that they cannot be *degrees*). However, it is a price we have to pay to satisfy our desiderata.[12] Besides, it is not necessarily a bad thing for the values not to be able to be linearly ordered; it is actually realistic. For many vague properties, such as *being bald*, *being smart*, and *being a heap*, are *multidimensionally vectored*. It is not one thing, such as the number of hairs on the skull, that determines whether someone is bald, but there are many elements that contribute to one's baldness: in addition to the number, the density, the distribution, the length, and perhaps even the color of the hairs seem to be contributing factors. And, for instance, if skull 1 < skull 2 on the numbers of hairs but skull 1 > skull 2

[12]Unless you embrace Edgington's aforementioned view.

on even distribution, then there may not be a fact of the matter whether skull 1 is more bald or less bald than skull 2. In many cases all we can say seems to be that *other things being equal*, a skull that has fewer hairs is more bald than a skull that has more hairs – that is, the contributing factors may be *incommensurable*. Thus, there is reason to think that the partial orders Boolean-valued set theory gives to propositions and set membership are more realistic representations of vague properties than the linear orders fuzzy set theory gives.[13]

Weatherson (2005) also embraces the general Boolean-valued semantics for the logic of vagueness[14] and discusses the following problem, originally raised by Jonathan Schaffer: for any proposition p and any Boolean algebra B, $[\![p]\!]_B$ and $[\![\neg p]\!]_B$ are incomparable, that is, $[\![p]\!]_B \not\leq [\![\neg p]\!]_B$ and $[\![\neg p]\!]_B \not\leq [\![p]\!]_B$, unless $[\![p]\!]_B = 0$ or 1. This is because if $[\![p]\!]_B \leq [\![\neg p]\!]_B$, then $[\![p \to \neg p]\!]_B = [\![\neg p]\!]_B = 1$; so $[\![p]\!]_B$ must be 0 – analogously for the other case. This means, for instance, that $[\![\text{Adam is tall}]\!]_B$ and $[\![\text{Adam is not tall}]\!]_B$ are not comparable if Adam is borderline tall, even though one may want to say that Adam is more tall than not; that is, the statement "Adam is more tall than not" cannot be interpreted as $[\![\text{Adam is not tall}]\!]_B < [\![\text{Adam is tall}]\!]_B$. This result, however, is not as bad as it may seem initially, for if we could say of each person if he is more tall than not, then we could draw a sharp borderline between *tall* and *not tall*. The result, thus, is as expected. Here it seems that, along with Weatherson, we should simply accept the consequences.[15]

8.5 Boolean-Valued Set Theory

The theory just presented can be generalized to include not just sets of individuals (or "urelements") but also sets of sets of urelements, sets of sets of sets of urelements, etc. In fact, that is exactly what is done in the so-called *Boolean-valued models* of Zermelo-Fraenkel-Choice (ZFC) set theory, which were introduced in the 1960s by Dana Scott and Robert Solovay in order to reformulate Paul Cohen's independence proof of the continuum hypothesis (CH) from ZFC set theory (see Cohen 1966 and Bell 2005). In an ordinary model of ZFC, any sentence written in the language of ZFC set theory has a definite truth value, that is, value 0 or 1. What Scott and Solovay discovered is that we can assign to such a sentence a value in a

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[13]Williamson (1994) and Keefe (1998, 2000, Chap. 5) also criticize fuzzy set theory for its linear value assignments. Goguen (1969), even though he is generally sympathetic to fuzzy set theory, also considers this feature a shortcoming of the theory and develops a system in which values are only partially ordered. Goguen's theory is rather different from the present proposal; most importantly, his general framework is not Boolean, though he accepts Boolean structures as special cases. We will not discuss Goguen's theory in this paper. See Williamson (1994, pp. 131–135) for a brief critical examination of Goguen's theory and other similar theories.

[14]Weatherson's general framework is similar to this chapter's, although his main concern is with introducing "$\leq$" (or "$<$," "truer than") in the object language.

[15]I would like to thank Richard Zach for pressing this point.

general Boolean algebra instead. Furthermore, from an ordinary model of ZFC, we can construct a Boolean-valued model in which all the axioms of ZFC receive value 1 as well as ¬CH. That is Scott's and Solovay's way of showing that the continuum hypothesis is independent from ZFC set theory. In fact, in informal accounts of their work (such as Chow 2008), this process is often characterized as "fuzzifying" the concept of set (even though such accounts are also often quick to add that the resulting "fuzzy sets" are not the same as those in fuzzy set theory).

The subsequent account of Boolean-valued set theory follows Zhang's (1980, 1982, 1983) account,[16] which in turn follows Bell's (2005, originally 1977) authoritative exposition, but adds urelements in the universe of discourse. The universe of discourse is defined inductively thus:

$$U_0 = \{x : Urelement(x) \vee x = \varnothing\};$$

(16) $$U_\alpha = U_0 \cup \left\{x : Function(x) \wedge range(x) \subseteq B \wedge \exists \xi < \alpha \left[domain(x) \subseteq U_\xi\right]\right\};$$

$$U = \{x : \exists \alpha \, [x \in U_\alpha]\}.$$

Intuitively, (16) states that we start with U_0, the set consisting of the urelements and the empty set $\varnothing$; we then expand U_0 into U_1 by adding all the (total and partial) functions from U_0 into B; and we repeat basically the same process to the transfinite level. In what follows, $rank(x)$ is the smallest α such that $x \in U_\alpha$.

Because ZFC set theory contains only $\in$ and $=$ as primitive predicates, it will be sufficient for the fuzzification of ZFC set theory with urelements if we give inductive definitions of $[\![x \in y]\!]_B$ and $[\![x = y]\!]_B$. This is done as follows:

(A) If $rank(x) = rank(y) = 0$,

(a) $[\![x \in y]\!]_B = 0$.

(b) $$[\![x = y]\!]_B = \begin{cases} 1 & \text{if } x \text{ and } y \text{ are identical;} \\ 0 & \text{otherwise.} \end{cases}$$

(B) If $rank(x) > 0$ or $rank(y) > 0$,

(c) $$[\![x \in y]\!]_B = \begin{cases} 0 & \text{if } rank(y) = 0; \\ \bigvee_{z \in dom(y)} (y(z) \wedge [\![z = x]\!]_B) & \text{otherwise.} \end{cases}$$

(d) $$[\![x = y]\!]_B = \begin{cases} 0 & \text{if } x \text{ or } y \text{ is a urelement;} \\ \bigwedge_{z \in dom(x)} \neg (x(z)) & \text{if } y = \varnothing; \\ \bigwedge_{z \in dom(y)} \neg (y(z)) & \text{if } x = \varnothing; \\ \bigwedge_{z \in dom(x)} (x(z) \to [\![z \in y]\!]_B) \wedge \bigwedge_{z \in dom(y)} (y(z) \to [\![z \in x]\!]_B) & \text{otherwise,} \end{cases}$$

[16] As far as I know, Zhang is the first researcher to point out the significant connections between fuzzy set theory and Boolean-valued set theory.

where $p \to q = \neg p \vee q$. Setting urelements aside, Bell shows that for any Boolean algebra B constructed in an ordinary model of ZFC, the Boolean-valued model consisting of the B-valued sets gives value 1 to all the axioms of ZFC on the above interpretation of set membership and set identity as well as the interpretation of the logical connectives given in the last section.

Note that by (b), for any distinct individuals a and b, $[\![a = b]\!]_B = 0$; so, by (c), for any individual a and any set s of individuals, $[\![a \in s]\!]_B = s(a)$. So, as far as individuals and their sets are concerned, this theory is no different from that presented in the last section and, thus, is a genuine generalization of it.

To better understand what is stated above, let us consider one example. Let $U_0 = \{\varnothing, a\}$ and $D^B = D^{B4} = \{0, v, \neg v, 1\}$, where $0 \le v \le 1$, $0 \le \neg v \le 1$, $v \not\le \neg v$, and $\neg v \not\le v$. The truth tables for the negation, conjunction, disjunction, and conditional are as follows:

p	$\neg p$
1	0
v	$\neg v$
$\neg v$	v
0	1

$\wedge$	1	v	$\neg v$	0
1	1	v	$\neg v$	0
v	v	v	0	0
$\neg v$	$\neg v$	0	$\neg v$	0
0	0	0	0	0

$\vee$	1	v	$\neg v$	0
1	1	1	1	1
v	1	v	1	v
$\neg v$	1	1	$\neg v$	$\neg v$
0	1	v	$\neg v$	0

$\to$	1	v	$\neg v$	0
1	1	v	$\neg v$	0
v	1	1	$\neg v$	$\neg v$
$\neg v$	1	v	1	v
0	1	1	1	1

Intuitively, while the two-element Boolean algebra $B2$ divides all propositions into true and false propositions, the four-element Boolean algebra $B4$ divides all propositions into necessarily true, contingent, and necessarily false propositions. For the sake of simplicity, let us ignore the partial functions from U_0 to D^{B4} and focus on the total functions. Then there will be sixteen sets at rank 1:

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}	s_{12}	s_{13}	s_{14}	s_{15}	s_{16}
$\varnothing$	0	0	0	0	v	v	v	v	$\neg v$	$\neg v$	$\neg v$	$\neg v$	1	1	1	1
a	0	v	$\neg v$	1	0	v	$\neg v$	1	0	v	$\neg v$	1	0	v	$\neg v$	1

Then from (d) above we can determine the values of the identity relations among the objects of ranks 0 and 1. For instance,

- $[\![s_1 = a]\!]_B = 0$.
- $[\![s_1 = \varnothing\,]\!]_B = \neg 0 \wedge \neg 0 = 1 \wedge 1 = 1$.

- $[\![s_2 = \varnothing\,]\!]_B = \neg 0 \wedge \neg v = 1 \wedge \neg v = \neg v$.
- $[\![s_1 = s_8]\!]_B = (0 \to v) \wedge (0 \to 1) \wedge (v \to 0) \wedge (1 \to 0) = 1 \wedge 1 \wedge \neg v \wedge 0 = 0$.
- $[\![s_4 = s_8]\!]_B = (0 \to v) \wedge (1 \to 1) \wedge (v \to 0) \wedge (1 \to 1) = 1 \wedge 1 \wedge \neg v \wedge 1 = \neg v$.
- $[\![s_8 = s_8]\!]_B = (v \to v) \wedge (1 \to 1) \wedge (v \to v) \wedge (1 \to 1) = 1 \wedge 1 \wedge 1 \wedge 1 = 1$.
- $[\![s_8 = s_9]\!]_B = (v \to \neg v) \wedge (1 \to 0) \wedge (\neg v \to v) \wedge (0 \to 1) = \neg v \wedge 0 \wedge v \wedge 1 = 0$.
- $[\![s_7 = s_{10}]\!]_B = (v \to \neg v) \wedge (\neg v \to v) \wedge (\neg v \to v) \wedge (v \to \neg v) = \neg v \wedge v \wedge v \wedge \neg v = 0$.
- $[\![s_9 = s_{11}]\!]_B = (\neg v \to \neg v) \wedge (0 \to \neg v) \wedge (\neg v \to \neg v) \wedge (\neg v \to 0) = 1 \wedge 1 \wedge 1 \wedge v = v$.
- $[\![s_1 = s_{16}]\!]_B = (0 \to 1) \wedge (0 \to 1) \wedge (1 \to 0) \wedge (1 \to 0) = 1 \wedge 1 \wedge 0 \wedge 0 = 0$.

We now can determine $[\![x \in y]\!]_B$ with (c), when x is rank 1 and y is rank 2. The value is determined not just by x and y but every object z in the domain of y and its relations to x and y. Specifically, the second equation in (c) asserts that $[\![x \in y]\!]_B$ is the lub of $y(z) \wedge [\![z = x]\!]_B$ for any $z <$ rank 2. Once the values of the membership relations between the rank <2 objects and the rank 2 sets are determined, then we can in turn determine the values of the identity relations among the rank ≤ 2 objects. The last equation of (d) asserts that $[\![x = y]\!]_B$ is the glb of $x(z) \to [\![z \in y]\!]_B$ for any z in the domain of x and $y(z) \to [\![z \in x]\!]_B$ for any z in the domain of y. Obviously this is a fuzzification of the usual claim that for two sets x and y, $x = y \iff \forall z(z \in x \leftrightarrow z \in y)$. We can determine $[\![x \in y]\!]_B$ and $[\![x = y]\!]_B$ for higher ranks in the same way, going back and forth between (c) and (d). As you can easily imagine, things become very complicated very quickly as we move to higher and higher ranks. In sum, (c) and (d) can be understood as a joint inductive definition of set membership and identity.

As is clear from the above account, Boolean-valued set theory holds that not only set membership but also identity among sets is vague. This is reasonable because, as we have just seen, identity of sets is determined by their membership relations. Furthermore, identity defined above is reflexive, symmetric, and transitive, and identical things in the above sense are substitutable salva veritate in any proposition in ZFC set theory (Bell 2005, Theorem 1.17).[17] Thus, we may say that Boolean-valued set theory upholds not only the idea of vague sets but also that of vague identity, though it does not support the idea of vague identity among individuals. Since vague sets are extensional surrogates of vague properties, we may conclude that the identities of vague properties are also vague.

[17]One might still contend that $=$ in Boolean-valued set theory is not really identity but something else. (I myself argued in Akiba (2000a, b, 2004) that what is often considered vague identity between vague individuals is not really identity but mere coincidence.) The issue of vague (or indeterminate) identity is without doubt an extremely delicate one. The root problem is that we, collectively, do not seem to have a clear idea of what vague identity ought to be. Still, I am inclined to think that vagueness of $=$ here is as good a candidate of vague identity as anything can be. But a full defense of this claim is beyond the scope of this paper. See Williamson (1996, 2002) and Smith (2008b) for arguments for the unintelligibility and nonexistence of vague identity, and see Akiba (MS) for my defense of vague identity. Thanks to the anonymous reviewer of this chapter for raising this issue.

Though, admittedly, the details are complicated, if we want to fuzzify the entire set theory based on classical logic, this is at least one way to do it. In fact, despite its success in applications, fuzzy set theory is usually not extended to higher ranks, and when it is (see, e.g., Gottwald 1979, 2006), it becomes equally – if not more – complicated. Therefore, the complication in Boolean-valued set theory should not count against the theory.

8.6 Conclusion

This chapter was an attempt to radically revise fuzzy set theory by removing its degree-functionality in the face of the problem of penumbral connections. The result was Boolean-valued set theory, which assigns Boolean-structured values to propositions, set membership, and set identity. The logic of the theory is classical, but, perhaps somewhat paradoxically, its semantics is many-valued though not degree-theoretic.

I submit that Boolean-valued sets, and not fuzzy sets, are the vague sets we have been looking for as the denotations of vague predicates such as "is tall" and "is bald." Consequently, "is tall and not tall" and "is bald and not bald" may denote the empty set, and "is tall or not tall" and "is bald or not bald" may denote the set of all individuals.

As far as I know, this is the first time when Boolean-valued sets are discussed by philosophers, let alone identified as vague sets; so in this chapter I have focused on introducing the basics of the theory, setting further philosophical questions aside. I consider this chapter a success if it succeeded in raising philosophers' awareness about the existence of Boolean-valued sets, even if it did nothing else. In concluding this chapter, however, I would like to briefly address three relatively philosophical questions:

(i) Does the present theory imply that in the world of sets, vague sets (i.e., Boolean-valued sets) are the fundamental sets and that crisp sets are in fact just special instances of vague sets?
(ii) Which Boolean lattice structure should we use to determine the values of propositions, etc.? How do we decide?
(iii) What is the relation between the conception of vague set advanced in this chapter and the modal conception I advocated before? Is there any relation?

I shall address these questions in order.

(i) Does the present theory imply that in the world of sets, vague sets (i.e., Boolean-valued sets) are the fundamental sets and that crisp sets are in fact just special instances of vague sets?

No, it doesn't. If you look at Scott's and Solovay's original Boolean-valued models, they are constructed *in crisp set theory*: everything in U_0 and D^B are crisp, and Boolean-valued sets are built out of them. So Boolean-valued sets do not give us any reason to believe that the world of sets is ultimately vague. It may be possible – indeed, it is interesting to see if it is possible – to start with Boolean-valued sets and define a crisp set, say s, as a special Boolean-valued set with $[\![x \in s]\!]_B = 1$ for any crisp object x (including set), but that's not what has been done here.

An analogy here is that the physical world may[18] ultimately be crisp: all strings, fundamental particles, etc., may have only crisp properties. Still, it may be possible for larger-sized physical objects such as people, cats, and mountains, which are composed of the smaller, crisp elements, to have vague boundaries and properties because of vagueness in composition (see, e.g., Sattig 2014). Similarly, there may be vague sets even if the world of sets is ultimately crisp.

(ii) Which Boolean lattice structure should we use to determine the values of propositions, etc.? How do we decide?

I have been talking about Boolean-valued sets in general, but this is misleading – or so one may contend – because there is no such thing as a Boolean-valued set; all there is is a B-valued set with a specific B. The values of propositions, p, $a \in b$, etc. – that is, $[\![p]\!]_B$, $[\![a \in b]\!]_B$, etc. – may differ depending on what Boolean structure B is selected. How should we deal with this relativity? Which structures among others should we use in particular cases?

My response to this question is that I don't have a good answer, but that that does not put the present theory to a competitive disadvantage, for this relativity (or indeterminacy) is common to most (or perhaps all) theories of vagueness. I have never seen a theory of vagueness which implies, for instance, that anybody 6 ft tall ought to be a tall person. All we can do is to provide a *framework* so that others can choose specific models in the framework for specific purposes. Fuzzy set theory is no exception. Just as Boolean-valued set theory has B as a parameter, fuzzy theory has M (or more generally, V) as a parameter, as we saw. So I do not see any particular problem for Boolean-valued set theory in this respect.

Incidentally, no matter what Boolean lattice structure we choose in order to evaluate propositions, I think it is a mistake to assign 1 to any contingently true proposition and 0 to any contingently false proposition; 1 should be assigned only to logically or metaphysically necessary propositions and 0 to logically or metaphysically impossible propositions, as is suggested by the way 1 $(= p \vee \neg p)$ and 0 $(= p \wedge \neg p)$ are defined. For instance, even if Michael Jordan has absolutely no hair on his skull, the value of "Michael Jordan is bald" should be ever so slightly less than 1, whereas the value of "Michael Jordan is either bald or not bald" should be 1. Even for the place on earth farthest from Mt. Everest, say P, the value of "P is on Mt. Everest" should be ever so slightly higher than 0, whereas the value of "P is both on Mt. Everest and not on Mt. Everest" should be 0. If we give value 1 or 0 to contingent propositions, the troubling question mentioned earlier in Sect. 8.3, "Where in the series of similar propositions does the value all of sudden become less

[18]Of course, it may not be, because of quantum indeterminacy, for instance. For quantum indeterminacy, see Darby (2014) and the papers cited therein.

than 1 (or higher than 0), and why?" will arise. (The analogous question "Where in the series of similar propositions does the value all of sudden become more/less than v, and why?" seems less threatening if v is an intermediate value, for the assignment of the specific value v is often somewhat arbitrary anyway; it would not matter if we assigned a value slightly higher or lower than v. In contrast, the values 0 and 1, the bottom and the top of the lattice, are structurally distinctive and special in this regard.)

(iii) What is the relation between the conception of vague set advanced in this chapter and the modal conception I advocated before? Is there any relation?

A couple of things need to be said about the relation between Boolean-valued models and modality. There is no doubt that Boolean-valued models tacitly involve modality. On the one hand, the simplest Boolean-valued models are the two-element models, assigning 0 or 1 to every proposition. On the other hand, the most complex models are those that assign different, unrelated values to different atomic propositions (or, more precisely, for any two atomic propositions p and q, $[\![p]\!]_B \not\leq [\![q]\!]_B$ and $[\![q]\!]_B \not\leq [\![p]\!]_B$). And there are other models in between. The two-element models differentiate propositions very coarsely, ignoring their contents and focusing only on their truth and falsity, whereas the most complex models differentiate propositions very finely, in accordance with their contents. The models in between differentiate propositions more finely than the two-element models but less finely than the most complex ones. The four-element model we considered in the last section, for instance, differentiates necessarily true, contingent, and necessarily false propositions. That is, while the two-element Boolean-valued models are what we call *extensional*, the other models are *intensional* to various degrees. This is an extremely important feature of Boolean-valued models; they can deal with intensionality – and, thus, modality – without resorting to any apparatus involving possible worlds. Thus, despite my initial motivation to find extensional substitutes for vague properties, Boolean-valued sets turn out to be not so extensional, after all.

However, we also *can* interpret Boolean-valued semantics in terms of possible worlds semantics and vice versa.[19] Consider all precisifications (or possible worlds). Identify $[\![p]\!]_B$ with the set of precisifications in which p is true. Then, assuming that the relevant accessibility relation is an equivalence relation like that of S5, the power set of the sets of precisifications constitutes a Boolean algebra B, where the relevant partial order $\leq$ is the subset relation $\subseteq$ among the sets of precisifications. The converse reduction can also be proved possible with the use of the Stone representation theorem. Thus, Boolean-valued semantics and supervaluational semantics are equivalent. Understood this way, the Boolean solution of the problem of penumbral connections presented in Sect. 8.4 above proves to be essentially the same as the supervaluational solution.

[19]I owe much to the anonymous reviewer for seeing this point.

Boolean-valued logic, at first sight, seemed ontologically innocuous. In contrast, supervaluational semantics, with its postulation of precisifications, whether they are understood in the semanticist or onticist fashion, at first seemed more ontologically involved. The above results seem to suggest, however, that their ontological commitments may in the end not be so different – indeed, they may be one and the same. Whether this is good news for supervaluational semantics or bad news for Boolean-valued logic may depend on one's perspective. Given the naturalness of Boolean-valued logic, however, there is reason to think that this is a piece of evidence that the supervaluational structure is not alien to our world, and that even what I call "precisified worlds" should not be dismissed out of hand.

Unfortunately, the above simple correspondence between Boolean-valued semantics and supervaluational semantics does not hold as is if the relevant accessibility relation among precisifications is not an equivalence relation but a more restricted relation such as a partial order. In that case, $\leq$ may not be identifiable with $\subseteq$. We cannot get into details in this chapter about the exact nature and logic of $\leq$; the interested reader should consult Bell (2005, Chap. 2).

There are some other issues to deal with concerning Boolean-valued set theory. A natural one is a comparison with fuzzy set theory. As was argued in this chapter, Boolean-valued sets are not degree-functional and, thus, are free of the problem of penumbral connections. On the other hand, the fact that fuzzy sets have *degrees*, not just *values*, of membership is doubtless in some respects an attractive feature that Boolean-valued sets do not have. So the question may still arise about the comparative virtues of the two theories as theories of vagueness. Here the answer does not have to be *either-or*; it is possible that one theory captures one feature or species of vagueness and can be used for certain purposes, while the other captures another feature or species and can be used for other purposes. Or some hybrid system – a cross between the two theories – may be possible (see Zhang 1980, 1982, 1983). We should be open-minded about various possibilities.

Another question to be asked is about the ontological status of Boolean-valued sets. In particular, the question may remain as to whether Boolean-valued set theory really endorses the worldly existence of vague sets, or whether another interpretation – semantic, epistemic, etc. – of the theory is possible. These philosophical issues, however, must be discussed somewhere else. This chapter aimed to introduce to the reader the idea of Boolean-valued sets and the view that they should be considered vague sets, the putative denotations of vague predicates. I hope it was a success.[20]

[20] I would like to thank the anonymous reviewer of this chapter for valuable comments. His/her comments on the relation between the possible worlds semantics and Boolean-valued semantics were particularly helpful. I also would like to thank for comments Nick Smith, Richard Zach, and the audience at the Society for Exact Philosophy 41st annual meeting, May 24, 2013, Université de Montréal, where a part of this chapter was presented.

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Chapter 9
One Bald Man . . . Two Bald Men . . . *Three* Bald Men—Aahh Aahh Aahh Aahh Aaaahhhh!

Nicholas J.J. Smith

9.1 Introduction

There is a familiar connection between *counting*, *ordering* and *cardinality*. When we have counted the elements of a collection—let's say, for the sake of example, a collection of brides, brothers, dwarves or wonders of the world—one, two, three, four, five, six, seven—we have achieved *two* things. First, we have *ordered* the collection: we have put its elements into an ordering from first through to seventh (viz. the order in which we counted them). Second, we have determined *how many* things there are in the collection—that is, the cardinality of the collection: this is given (when we count in the standard way, as in the example above) by the last number we state (in this case, seven).

In sum, when we have a (finite) set or collection of objects, there is a process we can perform on the (elements of the) set: counting. When we have performed this process, we get two things: an ordering of the elements of the set and an answer to the question how many things are in the set.

This connection between counting, ordering and cardinality is standard fare[1] in the case of classical or 'crisp' collections of objects—collections where there is never any vagueness or indeterminacy regarding whether a given object is in a given collection. When it comes to vague collections, however, the connection has not been kept in central focus in the literature. There have been numerous proposals for answering the question as to *how many* objects there are in a vague collection—that

[1]For example, it is reviewed on the first page of a recent handbook article on set theory (Bagaria 2008, 616).

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is, what is its cardinality—but these proposals concerning cardinality have tended to be discussed in isolation from the issues of counting and ordering.

In this paper, rather than focussing directly on the cardinality question for vaguely defined collections, I want to begin with the question of how to count vague collections. The aim will be to find a natural generalisation to the vague case of the familiar process of counting precise collections, which then—as in the classical case—yields both a notion of ordering and a notion of cardinality for vague collections. I shall not be proposing any new notions of cardinality for vague collections. What we shall see, however, is that only some of the existing notions mesh nicely with the conception of counting to be introduced here. I take it that a potential for coherence with an overall package of concepts analogous to the familiar classical package—counting, ordering and cardinality—is a mark in favour of a given notion of cardinality.

The paper proceeds as follows. Section 9.2 reviews the standard set-theoretic reconstruction of the classical story—outlined above in an informal way—of counting, ordering and cardinality. Section 9.3 introduces vaguely defined collections. In Sect. 9.4, I tell an informal story about counting vague collections, which is intended to generalise the classical story; in Sect. 9.5, I reconstruct this story in set-theoretic terms. In Sect. 9.6, I examine how this picture of counting fits with possible notions of *ordering* vague collections. In Sect. 9.7, I turn to cardinality: the various subsections of Sect. 9.7 look at existing proposals concerning the cardinality of vague collections and explore whether these proposals fit nicely with the story about counting presented in Sects. 9.4 and 9.5.[2]

9.2 Ordinals and Cardinals

Note that in the standard story of the connection between counting, ordering and cardinality, the numbers we recite when we count—one, two, three...—play two different roles: they can function as *ordinals*, which specify the position in an ordering of the objects to which they are assigned (first, second, third,...), and they can function as *cardinals*, which specify how many things there are in a collection (one, two, three,...).

The familiar story is standardly made more precise in the following way. Consider the following sequence of sets, where the first set is the empty set $\emptyset$ and each subsequent set is the set containing all the earlier members of the sequence:

[2] A word of explanation concerning my title: it is a reference to the Count, a character from the television show *Sesame Street*. He loved to count things—and when he had finished doing so, would laugh maniacally (Aahh Aahh Aahh Aahh Aaaahhhh!) to the accompaniment of thunder and lightning.

$$\begin{gathered}\emptyset \\ \{\emptyset\} \\ \{\emptyset, \{\emptyset\}\} \\ \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\} \\ \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}, \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}\} \\ \vdots\end{gathered}$$

(A piece of terminology that we shall use later: ω is the infinite set containing all, and only, the sets in the sequence just given.) Following von Neumann, the natural numbers $0, 1, 2, \ldots$ can be identified with the objects (sets) in this sequence: 0 is $\emptyset$, 1 is $\{\emptyset\}$ and so on:

$$\begin{aligned}0 &: \quad \emptyset \\ 1 &: \quad \{\emptyset\} \\ 2 &: \quad \{\emptyset, \{\emptyset\}\} \\ 3 &: \quad \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\} \\ 4 &: \quad \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}, \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}\} \\ &\vdots\end{aligned}$$

Note that we can then also write

$$\begin{aligned}0 &: \quad \emptyset \\ 1 &: \quad \{0\} \\ 2 &: \quad \{0, 1\} \\ 3 &: \quad \{0, 1, 2\} \\ 4 &: \quad \{0, 1, 2, 3\} \\ &\vdots\end{aligned}$$

It can now be seen clearly that the familiar ordering relation $<$ on the natural numbers simply becomes the membership relation $\in$.

We now have the objects that we use for counting (i.e. that we recite, in order, when we count): the natural numbers. Counting itself proceeds as follows. Informally, when we count a collection, we consider (point to, touch) its members in turn, without missing any and without repeating any. Each time we consider an object, we say a natural number, beginning with 1 and then proceeding in order:

2, 3, etc. The formal analogue of counting is a bijection between the set being counted and one of the natural numbers defined above.[3] For example, suppose we are counting dwarves: one, two, three, four, five, six, seven. The analogue of this is a bijection between the set of dwarves and the number 7, that is, the set $\{0, 1, 2, 3, 4, 5, 6\}$. The fact that we count every dwarf (missing none) corresponds to the function from the set of dwarves to 7 being total (or if we are thinking of the function as being from 7 to the set of dwarves, it corresponds to the fact that it is onto); the fact that we do not count any dwarf more than once corresponds to its being a function (or if we are thinking of the function as being from 7 to the set of dwarves, it corresponds to the fact that it is one-one).

Informally, counting yields an ordering of the set being counted and a cardinality for that set. In the formal reconstruction, this comes out as follows. If there is a bijection between the set of dwarves and the number 7, then that number *just is* the cardinal number of that set. As for ordering, the natural numbers come in a standard, familiar order: $0, 1, 2, \ldots$. As we have remarked, their formal analogues also come in a corresponding order, given by the set membership relation. Now suppose we consider each number not simply as a set—as we do when we think of it as a cardinal number—but as an ordered set: a set together with the ordering relation given by $\in$. Then, given a bijection between a number and a set, we get a corresponding ordering of that set. When we think of our numbers in this way—as ordered sets—they become ordinals.

Note the difference between a particular *ordering* of a set and its corresponding *ordinal*. There are many different ways of counting the dwarves—first Bashful, then Doc, then Dopey, Grumpy, Happy, Sleepy and finally Sneezy; or Grumpy first, then Sleepy, Sneezy, Doc, Dopey, Happy and then Bashful last; etc. Each of these is represented by a *different* bijection between the set of dwarves and the number 7. But when we abstract away from the particular identities of the objects in the ordering, and just look at the *type* of ordering we get, we see that we get the same type of ordering each time: one object, then another, then another, then another, then another, then another and finally another—seven things in a row. An *ordinal* represents an order *type*. So the multiple different orderings of the set of dwarves all correspond to the same ordinal, 7.[4]

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[3]A function $f : S \to T$ is said to be *total* if it satisfies the condition that every member of S gets sent to some member of T; *onto* (aka surjective, a surjection) if it satisfies the condition that every member of T gets hit at least once; and *one-one* (aka one-to-one, into, injective, an injection) if no member of T gets hit more than once. A *bijection* (aka correspondence) is a function that is total, onto and one-one. If there is a bijection f from S to T, then there is a bijection (the inverse of f) from T to S; hence it is common to talk nonspecifically of a bijection *between* S and T.

[4]An ordinal, as Cantor (1915) put it, results from a single act of abstraction: we ignore the particular identity of each object in the set and simply look at the order in which these objects appear; a cardinal results from a double act of abstraction, in which we ignore both the particular identity of each object in the set and the order in which these objects appear, paying attention only to the number of objects in the set.

Note also that in the informal story, 0 plays no role—whereas in the set-theoretic reconstruction, it does. In the informal story, we count $1, 2, 3, \ldots$—starting at 1—and the cardinal number of the set we are counting is the last number stated. In the set-theoretic reconstruction, n is the set $\{0, 1, 2, \ldots, n-1\}$, which has n elements: but not the numbers $1 \ldots n$, rather the numbers $0 \ldots n-1$. The counting process is represented as a bijection between the set being counted and a number n. The bijection associates the first element in the set (i.e. first in the ordering generated by the counting process) with 0 (not with 1) and the last with $n-1$ (not with n). The cardinal number of the set is then the set of all the numbers associated with objects in the set—which is the number after the last one associated with an object in the set, rather than the last one itself. One might therefore think that if we wish to speak strictly correctly, we need to say—for example—that the set-theoretic story reconstructs, not the standard counting procedure itself but an equally good alternative procedure that starts from 0 (instead of 1) and assigns as cardinal the first number not stated (rather than the last number stated). We shall not enter into these sorts of issues here, as they would be a distraction in the present context. For our purposes it will be best to speak simply of the set-theoretic story as a reconstruction of the familiar counting process (which starts from 1)—leaving it to readers who regard any of our formulations as strictly speaking incorrect to reword them mentally to their own satisfaction.

Summing up: In the formal version of the familiar story, we have a sequence of sets. They play the role of numbers. If we think of them simply as sets, they are cardinal numbers; if we think of them furthermore as ordered (by set membership), they become ordinal numbers. Counting a set and getting the answer n corresponds to the existence of a bijection between that set and the number n. Such a bijection yields two things: an ordering of the set (transferred from the ordering of n, when we consider it as an ordinal) and an answer to the question how many things are in that set (n itself, when we think of it as a cardinal).

The classical story just told extends from crisp finite collections to crisp infinite collections in a standard way (as explained in any introductory work on set theory). In this paper we wish to generalise in a different direction: we shall consider only finite sets—but sets whose membership is not precisely defined.

9.3 Vaguely Defined Collections

Given a predicate P, we can (try to) count the set of Ps. How to proceed when P is vague? For example, suppose that there are 20 men in the room: 10 professional basketball players, 6 professional jockeys and 4 more or less borderline cases of tallness. How to count the tall men in the room? Obviously, we count each of the basketball players and none of the jockeys—but what about the borderline tall men? It is unclear whether we should count them or not.

A similar problem arises if we suppose that the identity relation can be vague.[5] For example, suppose that Jane, who is $5'8''$, and Emma, who is $6'1''$, work at moon base 9. One morning, Emma teletransports to base 7 for the day, returning that evening to base 9. Suppose we wish to count the persons of $6'$ or more in height who were present in base 9 that day. '$6'$ or more in height' is a precise predicate—and yet we face a similar problem to the one we face when we wish to count the tall men: it is unclear whether the Emma who steps out of the teletransporter in the evening is a distinct person from the Emma who stepped into the teletransporter in the morning; hence, having counted Emma in the morning, it is unclear whether (in addition) to count Emma in the evening.[6]

Of course, if there can be cases of the two sorts just described, then there can also be hybrid cases, involving both vague identity and vague predicates—for example, counting the tall persons in base 9 on a given day.

I have argued elsewhere that ultimately sense cannot be made of the idea of vague identity (Smith 2008a). I shall therefore focus entirely on cases of counting collections whose vagueness arises from the vagueness of some predicate used to define the set: for example, the tall men, the bald men, the heavy suitcases, the long walks and so on. I have also argued elsewhere that vague predicates should be analysed in terms of degrees of truth—in particular, using fuzzy sets (Smith 2008b). I shall therefore carry out my discussion of vaguely defined collections in terms of fuzzy sets. Nevertheless, much of what I say could be applied, mutatis mutandis, both to other approaches to vagueness and to counting issues arising from vague identity. Therefore, I shall often talk generally of 'vague sets', 'vague collections' and so on, rather than specifically of 'fuzzy sets'—even though at all points at which rigour is required, the precise technical development will be in terms of fuzzy sets. This paper is intended as a general contribution to the literature on counting and cardinality in the presence of vagueness, illustrated in terms of one particular source of vagueness (vague predication, not vague identity) modelled in one particular way (using fuzzy sets). Some readers may take vague identity seriously, or they may model vague predication using machinery other than fuzzy sets: most of what I say should still be relevant to such readers.

Some terminology: $[0, 1]$ is the closed real unit interval, comprising all the real numbers between 0 and 1 inclusive, that is, all real numbers x with $0 \le x \le 1$. $(0, 1]$ is the set of all real numbers x with $0 < x \le 1$, and $[0, 1)$ is the set of all real numbers x with $0 \le x < 1$. A fuzzy subset S of some background set M is a function from M to $[0, 1]$; the number assigned to $x \in M$ represents x's degree of membership in S. The set of all things in M assigned a value strictly greater than 0 is called the *support* of S, here denoted S_*. The set of all things in M assigned the

[5]See, for example, Parsons (2000, Sect. 8.1).

[6]The teletransporter is playing the role of a disrupter. Readers who do not like the example should substitute their favourite case from the personal identity literature of a disruptive process where it is unclear whether the person who enters the process is the same as the person who exits the process.

value 1 is called the *core* (or kernel) of S, here denoted S^*. Note that the support and the core of a fuzzy set are both crisp sets.

Throughout—as already foreshadowed—we shall restrict our attention to fuzzy sets whose support is a finite set. (Note that the background set M need not be finite.) In the classical case, things get really interesting when we move beyond the realm of finite sets; as we shall see, in the vague case things are already rather interesting in the finite case.

9.4 Counting: The Informal Story

Suppose then that we wish to count, say, the tall men in the room. Of course we cannot simply count them in the usual way: the familiar procedure of intoning $1, 2, 3, \ldots$ as we go through the members of the set—being sure not to miss any nor to count any twice—simply 'crashes' if we get to an object such that it is unclear whether or not that object is in the set. If it is in, we count it; if it is out, we do not—but the familiar procedure assumes everything is in or out, and hence, it breaks down when we confront a set with elements that are to some degree in and to some degree out. Of course we can count—in the familiar way—any precise sets in the vicinity: the set of men who are greater than $6'$ in height, the set of men who are members of the set of tall men to a degree greater than 0.5 and so on. But the issue here is whether we can go further. Can we generalise the familiar procedure of counting the members of a crisp set, to the case of vague sets?

I can think of only one natural, satisfactory way of extending the usual counting procedure. In the classical procedure, we intone the counting numbers in turn: $1, 2, 3, \ldots$. We assign one number to each object that is in the set—and no number to any object that is not in the set. Thus, the tagging of objects with numbers is an on/off matter: objects that are in the set get tagged with a number and those that are not in do not get tagged. The degree of tagging—the strength of the glue with which the tag is affixed to the object, so to speak—matches the degree of membership of the object tagged in the set being counted: it is 'full on' or 'full off'.

In the new context of vague sets, objects can be completely in a set (in it to degree 1) and they can be completely out of a set (in it to degree 0), and objects can also be in a set to any intermediate degree. Maintaining the idea that the degree of tagging of an object should match the degree of membership of the object tagged in the set being counted, we now tag objects to various degrees. That is, we attach numbers to objects—but some are attached more firmly than others. So, the counting procedure is this. Go through the members of the set to be counted, intoning the counting numbers in turn: $1, 2, 3, \ldots$. For each object we come to, the degree of attachment of the tag (i.e. counting number) to the object matches the degree of membership of that object in the set being counted. We can think of this degree of attachment as being expressed by confidence—or loudness, or what have you—of intonation. If we come to an object that is fully in the set, we intone the next number with full confidence; if we come to an object that is not in the set at all, we do not intone anything (we save the next number for the next object that is in the set to some non-zero degree); if we come to an object that is in the set to an intermediate degree, we intone the next counting number with a degree of hesitation—or at a volume, or whatever—that matches the degree of membership of that object in the set being counted.

For example, suppose that Allison, Bridget, Caroline, Diana, Eleanor, Frances, Greta and Hazel (and no-one else) are in a room. Suppose that their degrees of membership in the set of tall persons in the room are as follows (where x/y denotes the degree x of membership of the person with initial y):

$$1/a, \quad 0.5/b, \quad 0.8/c, \quad 1/d, \quad 0/e, \quad 0.2/f, \quad 0.9/g, \quad 0.3/h$$

Then, we might count the members of the set of tall persons in the room as follows (where the table is to be read this way: looking at the person named in the left column, we intone the number in the middle column with the degree of hesitation given in the right column; or this way: to the person named in the left column, we attach—with the degree of attachment given in the right column—the number given in the middle column):

Allison	1	1
Bridget	2	0.5
Caroline	3	0.8
Diana	4	1
Frances	5	0.2
Greta	6	0.9
Hazel	7	0.3

Note that Eleanor does not get assigned any number—not even to a tiny degree—because her degree of membership in the set being counted is 0.

Here's another representation of this counting process, where this time the strength of attachment of number to person is indicated by the density of the type in which the number is written (with the idea being that 1 is written in 100 % black ink, 2 in 50 % greyscale, 3 in 80 % greyscale and so on down to 7 in 30 % greyscale):

Allison	1
Bridget	2
Caroline	3
Diana	4
Frances	5
Greta	6
Hazel	7

Of course, there is no reason why we should count the members of the set in the order we just did. An equally good way of counting would be the following:

Bridget	1	0.5
Caroline	2	0.8
Allison	3	1
Diana	4	1
Greta	5	0.9
Hazel	6	0.3
Frances	7	0.2

Bridget	1
Caroline	2
Allison	3
Diana	4
Greta	5
Hazel	6
Frances	7

as would the following:

Diana	1	1
Hazel	2	0.3
Frances	3	0.2
Bridget	4	0.5
Caroline	5	0.8
Greta	6	0.9
Allison	7	1

Diana	1
Hazel	2
Frances	3
Bridget	4
Caroline	5
Greta	6
Allison	7

and so on. Note that for each element of the set, the number assigned to that element need not remain the same across different ways of counting the set—but the degree to which its number (whatever number it is) is assigned does remain the same: it corresponds to the degree of membership of that element in the set.[7]

9.5 Counting: The Formal Reconstruction

In the set-theoretic reconstruction of the standard picture of counting a crisp finite set, the counting process is represented by a bijection between the set S being counted and a natural number—which is seen as a set of objects. The standard ordering on this natural number (i.e. on the elements of the set with which this number is identified) yields (via the bijection) an ordering of the set being counted. The natural number—together with the standard ordering of its elements—plays

[7] I said that much in this paper could be applied, mutatis mutandis, both to approaches to vagueness that do not employ fuzzy sets and to counting issues arising from vague identity. The story that I have just told about counting vague collections extends in an obvious way to any treatment of vagueness wherein the extension of a vague predicate can be modelled as a function from the domain of discourse to a set of membership values—for example, supervaluationist (or subvaluationist) treatments and treatments employing a many-valued or gappy (or glutty) logic. For the case of vague identity, the extent to which the next counting number is attached to the next object in the set should reflect both the extent to which that object is a member of the set and the extent to which it is distinct from all other objects in the set.

the role of an ordinal (denoted $\bar{S}$). It also—when considered by itself, without the ordering of its elements—plays the role of a cardinal (denoted $\bar{\bar{S}}$).[8]

We want to follow a similar line of thought in relation to vague collections. In the previous section we told a story about counting the members of a vaguely defined set. Our first task now is to give a more precise reconstruction of this story in set-theoretic terms.

The process of counting the members of a fuzzy set S can be represented by a function from S_* to $(0, 1] \times \omega$ satisfying the following conditions[9]:

1. The function is total
   (Everything is counted.)
2. The function is one-one
   (Two different things are never conflated and counted as one.)
3. Each element of $\bar{\bar{S}}_*$ appears *exactly once* in the image of the function[10]
   (The image of the function, for a given set S_*, is a set of pairs; the idea here is that if we look at all the second elements of these pairs, each element of $\bar{\bar{S}}_*$ appears exactly once. This captures the idea that we go through the elements of the support of S one by one, assigning successive numbers to them—just as in the classical story; the only difference, which we get to below, is that the association of each number is now a matter of degree.)
4. For each object x in S_*, the first element of the pair to which x is mapped by the function is the same as x's degree of membership in S
   (This captures the idea that as we count the elements of the support of S, the degree to which we associate the next counting number with the next object considered is the same as that object's degree of membership in S.)

This function assigns to each $x \in S_*$ a pair of things: the second element in the pair is a counting number (of the ordinary classical sort); the first element represents the degree to which that counting number is attached to x.

Let's refer to each member of $(0, 1] \times \omega$—i.e. each pair (x, n) whose first element x is a real in $(0, 1]$ and whose second element n is a natural number—as a *weighted* number, or more specifically a weighted version of the number n. We can then describe the present proposal as follows: we represent the process of counting a vague set as a function that assigns to each element of the support of that set a weighted version of one of the numbers $1 \ldots n$, where n is the number of elements in the support; furthermore, the function assigns these numbers in such a way that a weighted version of each of the numbers $1 \ldots n$ gets assigned to some element of the support, no two elements get assigned a weighted version of the same number,

[8]The notation is Cantor's. Each bar represents an act of abstraction: one for an ordinal, two for a cardinal (see Footnote 4 above).

[9]The symbol $\times$ represents the Cartesian product. $S \times T$ is the set of all ordered pairs whose first element is a member of the set S and whose second element is a member of the set T.

[10]Recall that $\bar{\bar{S}}_*$ is the number of elements in the support of S—and we may think of this number as a set.

and the weighting on n in the weighted number assigned to a is precisely the degree of membership of a in S.

If we look back at the tables in the previous section, we can now see them as pictures of counting functions of the sort just described.

9.6 Ordering

In the classical story, the process of counting the objects in a set yields an ordering of the set: the order in which we count the elements. In the set-theoretic reconstruction, the counting process is represented by a bijection between the set S being counted and a natural number. This natural number—thought of as a set—comes with a natural ordering. This ordering then yields—via the bijection—an ordering of the set being counted.

Can we tell a similar story in the vague case? We have represented the process of counting a vague set S as a function which assigns to each element of the support of S a pair. The first element of the pair is a real number; the second is a natural number. If there is a natural way of ordering these pairs, it will yield (via the counting function—which is total and one-one) an ordering of S_*.

It seems to me that there are two natural orderings on the pairs. (This is typical: where, in the classical case, there is one natural option, there are usually multiple equally natural options when we move to the fuzzy case.) The first ordering puts $(x_1, y_1)<(x_2, y_2)$ iff $y_1<y_2$. (The ys are natural numbers, and the most recent occurrence of $<$ denotes the standard ordering on the natural numbers.) The resulting ordering of S_* is the one that simply takes the members in the order we count them—ignoring any differences in the degrees to which successive counting numbers are attached to these objects. (Note that the ordering of the pairs ignores the xs altogether.)

The second ordering puts $(x_1, y_1) < (x_2, y_2)$ iff either $x_1 > x_2$, or $x_1 = x_2$ and $y_1 < y_2$. The resulting ordering of S_* is the one that goes through the members in order of degree of membership—starting with the degree 1 members, if there are any, and then working down. Where there are multiple elements with the same degree of membership, they are ordered in the order in which they were counted.[11]

Both of these options result in a crisp, linear ordering of S_*. The order types of these orderings are simply classical ordinals—and just as in the classical (finite) case, the order in which we count the elements of a set does not affect the resulting ordinal. No matter what order we count it in, and no matter whether we take the first or the second option just discussed, the ordinal associated with a fuzzy set will simply be the classical ordinal associated with its support. In other words, there are different options regarding the order in which we put the objects in the set—but the

[11]Of course there is also a reverse version of this ordering, where we begin with the lowest degree members and work up.

resulting order type will, in the cases discussed so far, simply be 'n objects in a row', where n is the number of objects in the support.

Another kind of possibility would be to look for a fuzzy ordering of S_*: that is, a mapping from $S_* \times S_*$ to $[0, 1]$ (rather than to $\{0, 1\}$, as in the case of a crisp ordering). Presumably, one would want the degree to which x comes before y to be a function of both x's and y's degrees of membership in S and the order in which they were counted—that is, a function of both which counting numbers are assigned to objects when we count the set and the strengths of those assignments. We shall not explore the options here any further in this paper. Suffice it to note that orderings of this kind could be derived from the process of counting a vague set, modelled in the way suggested here—and our concern is to preserve the *connection* between the notions of counting and ordering, rather than to explore the options regarding ordering in detail.

9.7 Cardinality

There are numerous options in the literature regarding the notion of the cardinality of a vague set—that is, numerous proposals for how to answer the question as to *how many* things there are in a vague set. Our concern here is that the answer to the cardinality question should flow from the output of the counting process: once we have counted a vague set, we should have sufficient resources in hand to answer the 'how many?' question.

This is how things go in the classical case. In the informal version of the story, the counting process consists in tagging each object in the set with a number. The output of this process is a list of numbers: $1 \ldots n$. The cardinality of the set is then the last of these numbers. In the set-theoretic reconstruction, the counting process is represented by a (bijective) function between the set S being counted and some natural number n (thought of as a set). The output of this process—the image of the set being counted under this function—is a set of numbers/sets $0 \ldots n - 1$. This set of numbers—which is itself the number n—is then the cardinality of the set being counted.

Turning to the vague case, in the informal version of the story, the counting process consists in tagging each object in the set with a number—with the strength of attachment of the tag matching the level of membership of the object being tagged in the set being counted. The output of this process is a list of numbers, $1 \ldots n$, with each number said in a softer or louder voice—or written in a lighter or darker shade of grey. In the set-theoretic reconstruction, the counting process is represented by a function (satisfying certain constraints) from the support S_* of the fuzzy set S being counted to pairs of reals in $(0, 1]$ and natural numbers. The output of this process—the image of the support under this function—is a set of pairs of reals in $(0, 1]$ and natural numbers. The idea now is that we should be able to derive the cardinality of S from this set of pairs—from the set of pairs that we get as output when we count S. We should not have to return to S itself, nor draw on any other sources of information. Just looking at the list of numbers, written in varying shades of grey, should be enough to answer the question as to how many objects there are in the fuzzy set.

Here is a straightforward idea. The cardinality of a crisp set S is simply the set that gathers together the values of the counting function that we get when we take members of S as input: $0, 1, 2, \ldots, n-1$ for some n. Now when we count a fuzzy set S, the values of the counting function that we get when we take members of S_* as input are pairs: $(x_0, 0)$, $(x_1, 1)$, $(x_2, 2)$, ..., $(x_{n-1}, n-1)$ for some n, where the x_is are reals in $(0, 1]$. Such a set of pairs determines a fuzzy subset of n (where n is conceived as the set containing $0, \ldots, n-1$): the fuzzy subset that assigns as degree of membership to each member of n, the number with which it is paired in the list of outputs. So, can we not take this fuzzy subset of n to be the cardinality of S?

We cannot: because if we count S again in a different order, we will (in general) get a *different* fuzzy subset of (the same natural number) n. (If, on one way of counting, 1 is assigned to a degree 0.8 member of S, then 1 will be a degree 0.8 member of the resulting fuzzy subset of n; if, on another way of counting, 1 is assigned to a degree 0.3 member of S, then 1 will be a degree 0.3 member of the resulting fuzzy subset of n; and so on.) Yet it is a fundamental constraint on the notion of cardinality that simply changing the *order* in which we count the elements of a set should not change the answer we get as to *how many* objects there are in the set.[12]

At this point, rather than trying to make up new proposals regarding the cardinality of vague collections, we shall turn to the numerous proposals already in the literature and ask whether these proposals fit with the account of counting given above. We shall not consider every proposal that has been made; rather, we shall consider some proposals that play a prominent role in the current literature on this topic.[13]

9.7.1 Cardinalities as Natural Numbers

The first class of proposals holds that the *form* of the answer to the question 'How many objects are in the set?' should be a natural number—in the vague case as well as the classical case. The natural proposals in this area are as follows. The cardinality of a fuzzy set S is the (classical) cardinality of:

1. The support of S
2. The core of S
3. S^x, where S^x is the (crisp) set of all elements whose degree of membership in S is strictly greater than x, for some specified threshold $x \in [0, 1)$

[12] Recall Cantor's second act of abstraction.

[13] My judgements regarding prominence in the literature have been heavily influenced by Wygralak (2003), which readers should consult for further details of—and bibliographical references regarding—the views discussed in Sects. 9.7.1–9.7.3.

4. S_x, where S_x is the (crisp) set of all elements whose degree of membership in S is greater than or equal to x, for some specified threshold $x \in (0, 1]$

Obviously, cardinalities of all these sorts can readily be extracted from the output of the process of counting S in the way presented above. To find the cardinality in sense 1, we count up (in the classical way) *all* the weighted numbers in the output of the process of counting the vague set S. To find the cardinality in sense 2, we count up the weighted numbers whose weight is 1—that is, the numbers written in 100 % black ink. To find the cardinality in sense 3, we count up the weighted numbers whose weight exceeds x—that is, the numbers written at a level of greyscale darker than x % and so on.

9.7.2 Cardinalities as Real Numbers

The second class of proposals holds that the form of the answer to the question 'How many objects are in the set?' should be a single number—but a nonnegative real number, not necessarily a nonnegative natural number (as in the classical case). The most natural proposal here is that the cardinality of S is the sum, over all x in the support of S, of the degree of membership of x in S. This is called the *sigma count* of S, denoted $sc(S)$:

$$sc(S) = \sum_{x \in S_*} S(x)$$

(Here, $S(x)$ denotes the degree of membership of x in S—i.e. the value assigned to x by S, when we think of S as a function from some background set to [0, 1].) So when we are counting up the bald men, a degree 1 bald man adds 1 to the count, a degree 0.3 bald man adds 0.3 to the count and in general a degree x bald man adds x to the count.[14]

Obviously, the sigma count of S can be extracted from the results of counting the members of S in the way presented above. The output of the counting process is a bunch of weighted numbers; to get the sigma count, we simply add the weights on these numbers.

9.7.3 Cardinalities as Fuzzy Sets of Natural Numbers

The first class of proposals held that the form of the answer to the question 'How many objects are in the set?' should be a natural number—in the vague case as well

[14]Compare the way that universities count students for certain purposes: a full-time student adds 1 to the count; a half-time student adds 0.5 to the count; and so on. (Thanks to David Braddon-Mitchell for this example.)

as the classical case. The second class of proposals generalised in one direction—maintaining that the cardinality should be a single number, but not requiring that it be a natural number. The third class of proposals generalises in a different direction, holding that the cardinality of a fuzzy set should be a fuzzy set of natural numbers, rather than a single such number.

One proposal along these lines is as follows. For each natural number n, we ask 'What is the highest level at which we can set the membership threshold x, such that the number of things that are in S to a degree of at least x is at least n?' The answer—a real in [0, 1]—is the degree of membership of n in the fuzzy set of natural numbers that constitutes (on this proposal) the cardinality of S. More precisely, the cardinality of S is a fuzzy subset of the set $\mathbb{N} = \{0, 1, 2, \ldots\}$ of natural numbers—that is, a function $l : \mathbb{N} \to [0, 1]$—defined as follows. For each $n \in \mathbb{N}$:

$$l(n) = \sup\{x \in (0, 1] : \bar{\bar{S}}_x \geq n\}$$

Note that if there is no positive threshold x such that at least n things are in S to degree x or more, then $l(n) = 0$.

Recall the example of the fuzzy set of tall persons described in Sect. 9.4, with degrees of membership as follows (where x/y denotes the degree x of membership of person y):

$$1/a, \quad 0.5/b, \quad 0.8/c, \quad 1/d, \quad 0/e, \quad 0.2/f, \quad 0.9/g, \quad 0.3/h$$

The cardinality of this fuzzy set—on the present proposal—is the following fuzzy subset of $\mathbb{N}$ (where x/n denotes the degree x of membership of the number n):

$$1/0,\ 1/1,\ 1/2,\ 0.9/3,\ 0.8/4,\ 0.5/5,\ 0.3/6,\ 0.2/7,\ 0/8,\ 0/9,\ 0/10,\ \ldots$$

The cardinality of S in this sense is readily recoverable from the output of the process of counting S in the way presented above. The output of the counting process is a bunch of weighted numbers. To get the cardinality, we write out the *weights* in nondecreasing order (including any repetitions)—in the present example:

$$1, \quad 1, \quad 0.9, \quad 0.8, \quad 0.5, \quad 0.3, \quad 0.2$$

(Note that we do not write 0 at the end of this list, because when we count we get a weighted number for each member of the *support* of S—i.e. each thing that is a member of S to some non-zero degree.) The cardinality that we seek is a fuzzy subset of $\mathbb{N}$—a function that assigns a degree of membership to each $n \in \mathbb{N}$. For $n = 0$, the degree of membership of n is 1. For n greater than 0, and less than or equal to the number of things in the list of weights that we just wrote out, the degree of membership of n is simply the nth weight in the list. For all larger n, the degree of membership of n is 0.

The cardinality proposal that we just looked at sees the cardinality of S as a fuzzy subset of $\mathbb{N}$, where the degree of membership of n in this fuzzy subset is a measure of the truth of the claim that there are *at least* n things in S. A second proposal replaces 'at least' here with 'at most'. On this proposal, the cardinality of S is a function $m : \mathbb{N} \to [0, 1]$ defined as follows:

$$m(n) = 1 - l(n + 1)$$

A third proposal replaces 'at least' in the first proposal with 'exactly'. On this proposal, the cardinality of S is a function $e : \mathbb{N} \to [0, 1]$ defined as follows:

$$e(n) = \min\{l(n), m(n)\}$$

As cardinality in the sense of l can be extracted from the output of the process of counting a vague set, evidently so can cardinality in the senses of m and e.

9.7.4 Cardinalities via Logical Formulas

It is well known that for any finite n and any predicate P, there are formulas of first-order logic that are true in exactly those (classical) models in which the extension of P contains exactly n things. There are different recipes for constructing such numerical formulas. For example—Recipe 1—we can represent 'There are exactly n Ps' as the conjunction of 'There are at least n Ps' and 'There are at most n Ps', where the 'at least' claims are rendered as follows:

1. $\exists x Px$
2. $\exists x \exists y (Px \wedge Py \wedge x \neq y)$
3. $\exists x \exists y \exists z (Px \wedge Py \wedge Pz \wedge x \neq y \wedge x \neq z \wedge y \neq z)$

$\vdots$

and the 'at most' claims are rendered as follows:

1. $\forall x \forall y ((Px \wedge Py) \to x = y)$
2. $\forall x \forall y \forall z ((Px \wedge Py \wedge Pz) \to (x = y \vee x = z \vee y = z))$
3. $\forall x \forall y \forall z \forall w ((Px \wedge Py \wedge Pz \wedge Pw) \to (x = y \vee x = z \vee x = w \vee y = z \vee y = w \vee z = w))$

$\vdots$

Recipe 2 is just like Recipe 1 except that the 'at most' claims are rendered as follows:

1. $\neg \exists x \exists y (Px \wedge Py \wedge x \neq y)$
2. $\neg \exists x \exists y \exists z (Px \wedge Py \wedge Pz \wedge x \neq y \wedge x \neq z \wedge y \neq z)$

3. $\neg\exists x\exists y\exists z\exists w(Px \wedge Py \wedge Pz \wedge Pw \wedge x \neq y \wedge x \neq z \wedge x \neq w \wedge y \neq z \wedge y \neq w \wedge z \neq w)$

$\vdots$

That is, 'There are at most n Ps' is the negation of 'There are at least $n + 1$ Ps' (as rendered above). Recipe 3 does not represent 'There are exactly n Ps' as the conjunction of 'There are at least n Ps' and 'There are at most n Ps', but simply renders the 'exactly' claims as follows:

1. $\exists x\forall y(Py \leftrightarrow y = x)$
2. $\exists x\exists y(x \neq y \wedge \forall z(Pz \leftrightarrow (z = x \vee z = y)))$
3. $\exists x\exists y\exists z(x \neq y \wedge x \neq z \wedge y \neq z \wedge \forall w(Pw \leftrightarrow (w = x \vee w = y \vee w = z)))$

$\vdots$

There are further options besides these three.[15]

Parsons's approach to the issue of counting in the presence of vagueness is as follows (Parsons 2000). If we want to know how many Ps there are—where either P is a vague predicate or indeterminacy of identity is involved or both—we consider each of the numerical formulas in turn and assess its truth value. So, in our example of the fuzzy set of tall persons with degrees of membership as follows:

$$1/a, \quad 0.5/b, \quad 0.8/c, \quad 1/d, \quad 0/e, \quad 0.2/f, \quad 0.9/g, \quad 0.3/h$$

what we do is consider each numerical formula—'There is exactly one P', 'There are exactly two Ps', etc.—and determine its degree of truth relative to a model in which P is assigned as extension the fuzzy set just described.[16]

There are two ways of interpreting what is going on here. The first way—which I take to be the sort of thing Parsons has in mind—is that the possible answers to the cardinality question (i.e. 'How many tall persons are there in the room?') are natural numbers. But it may not be that a unique answer is correct and all others incorrect. Various answers—various numerical formulas—may each have a non-zero degree of truth.[17] On this interpretation, the present approach does not yield a single object as cardinal number of the set of tall persons: it just yields an assessment (in the form of a degree of truth) of each possible answer. This is unsatisfactory: our goal is to extract a cardinality from the output of the counting process—not to deny that there

[15]For more details on the foregoing material see, e.g. Smith (2012, Sect. 13.5).

[16]Parsons does not work with fuzzy sets or degrees of truth. Here and below I adapt his ideas to the present context, in which we use fuzzy sets to model vagueness.

[17]See, for example, Parsons (2000, p. 135): "It follows that the question of how many persons there are all told has no correct answer. ... in this case it seems clear that these are the right things to say: any answer less than two or more than three is wrong, and either "two" or "three" is such that it is indeterminate whether it is correct" and p. 136: "It appears that in this case any answer less than one or more than three is definitely wrong, but the answers "one", "two", or "three" should all have indeterminate truth-value."

is such a thing as 'the cardinality' of a vague set. But of course there is a second way of developing the present idea: we take the cardinality of a fuzzy set S to be a fuzzy subset of $\mathbb{N}$: the one that assigns as degree of membership to each $n \in \mathbb{N}$ the degree of truth of the nth numerical formula on a model on which P has S as its extension.

Evidently, once we move beyond the classical framework, numerical formulas constructed according to different recipes—which are classically equivalent—need not remain equivalent. Thus, we shall get different versions of the present story—different cardinalities for vague sets—depending on which recipe we pick for constructing our numerical formulas and depending on the truth conditions that we adopt for the logical operators in the new nonclassical setting.

Our concern here is with whether the cardinality of a vague set can be recovered from the output of the process of counting that set. So, if we have counted a vague set S, and have to hand the output of the counting process—a list of weighted numbers—can we reconstruct the truth values of the numerical formulas on a model on which the predicate P has the set S as its extension? Note that we do not have the set S itself to hand—we have only the list of weighted numbers. But of course this list allows us to reconstruct how many things are in the support of S and their degrees of membership in S—and so, given certain assumptions about how the model theory is supposed to work in the new vague context, we can indeed reconstruct the truth values of the numerical formulas.

But now the question arises: why should we want to go this long way around—via the numerical formulas (and furthermore settling on a particular choice of recipe for constructing them and a particular set of truth conditions for the logical operators)—rather than simply extracting the desired cardinality (fuzzy set of natural numbers) directly from the output of the counting process? (E.g. note that if we define the truth conditions for negation and conjunction as follows—where $|\alpha|$ is the degree of truth of the formula α:

$$|\neg\alpha| = 1 - |\alpha|$$
$$|\alpha \wedge \beta| = \min\{|\alpha|, |\beta|\}$$

and the truth condition for the existential quantifier in terms of sup, and if we construct our numerical formulas according to Recipe 2, then the cardinality that we arrive at for a given fuzzy set by going via the numerical formulas will turn out to be the same as cardinality in sense e of Sect. 9.7.3.) There is only one possible reason: we might think that this route, while lengthy, is *conceptually* correct. That is, we might think that there is some special relationship between the numerical formulas and questions of cardinality—a connection that it is important to retain. This seems to be what Parsons thinks. He refers to the numerical formulas as *analyses* of cardinality claims and writes "These analyses are natural hypotheses about the meaning of cardinality claims" (Parsons 2000, 139). Hyde, who follows Parsons's approach to cardinality issues in the context of vagueness, also refers to the numerical formulas as *analyses* of claims of the form 'There are exactly n Ps' (Hyde 2008, 171). However, I think that this is the wrong attitude to numerical formulas. The fact that for any finite n and any predicate P there are formulas of first-order logic that are true in exactly those (classical) models in which the extension of P contains exactly n things is not properly seen as a fundamental fact about what it means for there to be n Ps. It is a fact about the expressive power of (classical) first-order logic. It is a useful fact—but if it did not hold, that would not reflect badly on the concept of cardinality: it would reflect badly on the logic. We would still know exactly what it means for there to be n Ps—it would just be something that we could not express in a logical formula. Consider the claim 'There are finitely many Ps'. It is well known that we cannot construct a formula—or even a set of formulas—such that on every model on which that formula—or all the formulas in that set—is true, the extension of P is a finite set. This does not threaten our understanding of the notion of finitude. It simply means that first-order logic lacks the power to express certain claims.

Given that the numerical formulas do not have any special connection to the concept of cardinality—they do not enshrine the very notion of cardinality—there would seem to be no good reason for approaching the issue of cardinality in the context of vagueness along the roundabout route via the truth values of numerical formulas. Simpler and better, it seems, to define cardinality directly from the outputs of the counting process—for example, in the ways that cardinality in the senses of l, m and e were defined in Sect. 9.7.3.

9.8 Conclusion

My concern in this paper has not been to add to the many existing proposals in the literature concerning the cardinality of vague collections, but to bring some order to the landscape—specifically, by bringing into focus the connection between the notions of counting, ordering and cardinality—a connection that is central in the classical case. I proposed a method for counting vague collections and discussed the relationships between this method and various notions of ordering for vague sets. Turning then to the notion of cardinality, we saw that not all existing views concerning how we should answer the question as to how many things there are in a vague collection fit equally well with my proposal about how to count the members of such a collection. In particular, the idea that we should approach cardinality via certain formulas of a logical language—which has been quite influential in the recent philosophical literature—seems to me to be less attractive than other existing proposals.[18]

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Chapter 10
Vagueness and Abstraction

Stewart Shapiro

This chapter is based on part of Chapter 6 of my Vagueness in context (Shapiro 2006).

10.1 Abstraction

An abstraction principle is any proposition in the form

$$\text{(ABS)}\ \forall a \forall b\, (\Sigma(a) = \Sigma(b) \equiv E\,(a,b))\,,$$

where a and b are variables of a given type (typically individual objects or properties/sets of objects); Σ is a higher-order operator, denoting a function from items of the given type to objects; and E is a relation over items of the given type. In what follows, I will usually omit the initial universal quantifiers.

Abstraction principles have come in for special attention in the philosophy of mathematics, at least in some circles. In his development of logicism, Gottlob Frege (1884, 1893) employed three such. One of them, used for illustration, comes from geometry:

> The direction of l_1 is identical to the direction of l_2 if and only if l_1 is parallel to (or identical with) l_2.

A second Fregean abstraction principle is now called *Hume's principle*:

$$(\#F = \#G) \equiv (F \approx G)\,,$$

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where "#" is a cardinal-number operator, on concepts (or properties, or sets), and "$F \approx G$" is an abbreviation of the second-order statement that there is a one-to-one relation mapping the F's onto the G's.

Frege's third abstraction principle is his ill-fated Basic Law V:

$$(\mathrm{E}F = \mathrm{E}G) \equiv \forall x\,(Fx \equiv Gx)\,,$$

stating that for all properties F, G, the extension of F is identical to the extension of G, if and only if every F is a G and every G is an F.

There is some dispute concerning the status of abstraction principles. The neo-Fregeans hold that at least some abstraction principles are akin to implicit definitions, true by stipulation and knowable a priori (see, e.g., Hale and Wright 2001). The good abstraction principles are thus analytic, or all but analytic. We need not engage that matter here. Whatever their epistemological status, abstraction principles are common fare in mathematics. Much of abstract algebra revolves around them.

Notice that in any standard, non-free logic, it *follows* from (ABS) that the embedded relation E is an equivalence relation: it is reflexive, symmetric, and transitive. This is because identity is an equivalence. For example, $\Sigma(a) = \Sigma(a)$ is a truth of logic, an axiom of identity. So it follows from (ABS) that, for all a, $E(a,a)$. So the relation E is reflexive. The symmetry and transitivity of E similarly follow from the symmetry and transitivity of identity, respectively. So a proposition in the form (ABS) is true only if the relation E is an equivalence.

Basic Law V notwithstanding, most abstraction principles are foundationally unproblematic. The mathematician can simply identify the abstracts with the equivalence classes, assuming that there aren't "too many" such classes (as with Basic Law V). There is a potential problem when the equivalence classes are themselves not sets, but even then, the abstracts can be identified with a representative from each equivalence class (invoking the axiom of choice if necessary).

The serenity and clarity of pure mathematics is compromised when the messy physical universe gets involved, and we try to apply abstraction principles to the physical world (or "the real world," as some might say). Inevitably, vagueness rears its head. Consider a shelf that contains two t-shirts. One of them is red and the other has a color that is borderline between red and orange. Let T be the property (or predicate) of being a red t-shirt on that shelf. What is the number of T? Is it 1 or 2; or is it some other thing, a vague number indeterminate between 1 and 2? What is the mathematics for these vague numbers?

We won't worry about vague numbers here; we have other fish to fry. In ordinary, nonmathematical discourse, concerning ordinary physical objects, we sometimes invoke what look like abstraction principles (in the form (ABS)). For example, we speak of the weights of individuals, indicating whether those are the same or different. We might say that Harry has the same weight as Sarah, but not as Joe. This seems to involve the following, which we may call the *Weight Principle*:

(W) The weight a is identical to the weight of b if and only if a and b are equi-weighted, where *a* and *b* are variables ranging over physical objects or people. The notion of "equi-weight" can be defined in terms of a pan balance, or else it can be taken as primitive. If the Weight Principle (W) is coherent, then *weights* are abstract objects, but they are abstracts *of* physical objects. Let us call such things *quasi-abstract objects*.[1]

Here are some more abstraction principles, yielding, as quasi-abstract objects, heights, income groups, colors, and meanings:

> The height of *a* is identical to the height of *b* if and only if the top of their heads are level when they stand back to back on flat ground.
>
> The income group of *a* is identical to the income group of *b*, if and only if *a* and *b* have the same annual income.
>
> The color of a patch *a* is identical to the color of a patch *b* if and only if they are indistinguishable in color.
>
> The meaning of a lexical item *a* is identical to the meaning of lexical item *b* if and only if *a* is synonymous with *b*.

What does an instance of, say, the Weight Principle (W) mean? When we say that the weight of *a* is identical to the weight of *b*, we might mean that they have *exactly* the same weight. In this sense, the identity is false if *a* is .0000000001 g heavier than *b*—assuming that differences this small make physical sense. Although we are not usually this strict when talking about weights (perhaps never), this reading gives the best chance for the relation on the right-hand side of (W)—being equi-weighted—to be an equivalence relation, as required by the literal interpretation of (W) (using a standard logic). Something analogous holds for the other abstraction principles. We'd have to hold that two people have the same annual income only if their incomes match to the penny (and we are clear as to what counts as income), and we would need a better definition of color beyond indistinguishability. We can set aside the corresponding issues concerning synonymy.

There are problems with interpreting these abstraction principles in this strict manner. First, with respect to (W), what can we make of differences in weight that are so small that quantum indeterminacy becomes a factor? Is it even physically possible for weights to be completely precise? Even if we ignore quantum factors, there are issues concerning what counts as part of a physical object or a person. Suppose that two people *a* and *b* are identical (molecule for molecule),[2] except that *a* has a small piece of a toenail that is loosely attached, and about to fall off, or a dead skin cell that is all but dislodged. Prima facie, it is indeterminate whether the piece of nail or the skin cell is part of *a* or not (see Morreau 2002). If the nail or cell

[1] Charles Parsons (1990) coined the term "quasi-concrete" for abstract objects (or properties) that have concrete instances. A typical example is the type shared by all tokens of a 12-point Times Roman letter "e." In a sense, the present quasi-abstract objects are also quasi-concrete.

[2] I suppose we are assuming that like molecules have exactly the same weight, whether or not this makes sense in light of what is known at the quantum level.

is not part of *a*, then the two have the same weight, but otherwise they do not, since the piece of nail or the cell makes *a* a bit heavier.

Suppose that these matters can be resolved and that it is possible to read "equi-weighted" as with absolute precision. Interpreting the Weight Principle (W) in this way would result in a massive error theory concerning natural language. Clearly, ordinary speakers often say that two objects, or two people, have the same weight, even though their weights differ, albeit negligibly in the relevant context. Think of two professional boxers who differ by about half a pound or two 2-lb dumbbells in the same set that differ by less than .1 ounce or even two 1-g weights in a carefully calibrated set used for important business transactions. On the proposed interpretation of the Weight Principle, just about all such pronouncements of equi-weight are false (or at best indeterminate). A theorist can turn to pragmatics to give an account of the use of quasi-abstract objects, but essentially the same issues would arise if we try to get rigorous about this pragmatics.

Clearly, when speaking of weights, ordinary speakers of ordinary language are not strict. When one says that the weight of a person *a* is the same as the weight of a person *b*, she means that they are *roughly* the same weight, depending on the standards that are in effect in the given conversation. Such is the case with most ordinary-language abstractions, such as the above principles concerning heights, income groups, and colors (if not synonymy). However, the relation of being roughly the same weight, or of having roughly the same height, or of having roughly the same income, or of being roughly the same color is not transitive.[3] It is an easy exercise to construct sorites series for those terms. Each member of the series is roughly the same as its neighbors, but the first is clearly not roughly the same as the last.

Since "roughly the same" weight, say, is not an equivalence relation, our principle (W) should fail as an abstraction. The same goes for most of the other abstraction principles, all attempting to characterize quasi-abstract objects. So it should not be possible to speak of weights, heights, income groups, and colors as (quasi-abstract) *objects*.

Nevertheless, we *do* speak of heights, weights, income groups, and colors with ease, all the time, and we *seem* to know what we are doing, if not what we mean. The issue here is reminiscent of Peter Unger's (1975, 65–68) argument that nothing (or hardly anything) is flat. The analogous conclusion here would be that no two distinct objects have the same weight or have the same height. Indeed, no single person has the same height or weight from one moment to the next. Michael Dummett (1975) argues that talk about vague things is incoherent, due to sorites. I take it that it would be better to avoid such conclusions. I think we can.

[3]Delia Graff (2001) challenges the nontransitivity in the case of colors. If she is right, then we can drop this example. There are plenty of others. Shapiro (2014) floats the possibility that, in effect, synonymy is not transitive. See also Field (2009). This would bring meanings into the present focus.

David Lewis (1979) concedes that, strictly speaking, Unger is right about flatness, but there is a looser sense in which statements like "Kansas is flat" are *true enough*. Presumably, the same goes for statements concerning the identity of heights, weights, colors, and income groups. Something along these lines can be developed with full rigor. In the next section, I'll sketch the account of vagueness developed in my *Vagueness in context* (2006). After that, I'll show how it handles the notion of a quasi-abstract object, more or less in stride. We will then ponder the fallout for ontology.

10.2 Vagueness

It is widely (but perhaps not universally) agreed that vague terms, especially gradable adjectives, are context-sensitive. The extensions of such terms depend on a comparison class, a paradigm or contrasting case, or something like that. A person who is tall for a professional jockey would be downright short for a grown woman in Pennsylvania; a person can be rich for a philosophy professor and poor, if not destitute, among the members of a given country club; and a man can be bald vis-à-vis Jerry Garcia and hairy vis-à-vis Terry Bradshaw.

Call the comparison class, paradigm cases, etc. associated with the use of a vague term its *external context* (following Diana Raffman 1994, 1996). It is also widely agreed that vagueness remains even after these external contextual factors are fixed. For example, the predicates tall-for-a-professional-jockey and bald-vis-à-vis Jerry Garcia are vague. There are sorites series for such predicates.

The account of vagueness developed in Shapiro (2006) introduces other, "internal" contextual factors that arise in the course of conversations involving vague terms, conversations in which, to repeat, external contextual factors are held fixed. What follows is a sketch of the view.

Let F be a monadic (possibly complex) predicate in a natural language. Vann McGee and Brian McLaughlin (1994, §2) introduce a technical term "definitely" as follows:

> ... to say that an object a is definitely an F means that the thoughts and practices of speakers of the language determine conditions of application for ... F, and the facts about a determine that these conditions are met.

I count external contextual factors, such as comparison class and paradigm or contrasting cases, as part of what thoughts and practices have fixed, concerning the use of a vague term. So, for example, a man who is 6 ft 2 in. tall is *definitely* short for an NBA player, especially a forward. But other, internal contextual shifts do not count as part of what "thoughts and practices have fixed" (and here, perhaps, I move away from McGee and McLaughlin).[4]

[4]Shapiro (2006) employs the term "determinately" for what McGee and McLaughlin call "definitely."

McGee and McLaughlin insist that the "definiteness" operator is not, or may not be, compositional. A sentence Φ can fail to be definitely true without $\neg\Phi$ being definitely true. For McGee and McLaughlin, an object a is a *borderline* case of a predicate F if Fa is "unsettled," that is, if a is not definitely an F and a is not definitely a non-F. The idea is that with borderline cases, nothing in the thoughts and practices of speakers of the languages—including whatever it is about such practices that fixes meaning and external context—and nothing in the realm of nonlinguistic facts determines whether a is an F or a non-F.

The main operative thesis of my account is that, in some circumstances, speakers can go either way in the borderline region of a vague predicate, without compromising their competence with the term and without sinning against any nonlinguistic facts. Unsettled entails open. Suppose that Jack is borderline bald and that, in the course of a conversation, someone says that Jack is bald. Suppose also that this remark goes unchallenged in the conversation. Then, in that context, Jack *is* bald, and it is true that Jack is bald (etc.). Of course, speakers can retract this pronouncement later, and it is not binding on any future conversations about Jack (or anyone else). In other conversational contexts, the very same Jack is not bald (even though the number and arrangement of his hairs has not changed).

I call this the *open texture* of vague terms, roughly after a similar thesis due to Friedrich Waismann (1945).[5] Crispin Wright (1987, 244) seems to endorse an open-texture thesis: "Borderline cases are ... cases about which competent speakers are allowed to differ." And Mark Sainsbury (1990, §9): "Given the nature of boundarylessness, semantics give freedom. There is some number of minutes such that the nature of the concept of a person, together with the nature of the world, makes it neither mandatory nor impermissible to apply the concept to a foetus of that age in minutes." Similar views are developed by Soames (1999, Chapter 7), Kamp (1981), Gaifman (2010), Raffman (1994, 1996), and Graff (2000).

It is not that anything goes, at any time, within the borderline region. That would make the use of vague predicates considerably less useful. The open-texture thesis is that *in some circumstances*, a competent speaker can, in fact, go either way without offending against the meaning of the terms, the nonlinguistic facts, and the like. Suppose that a is a borderline case of a vague predicate P. It is not true that the rules for language use allow a speaker to assert Pa in any situation whatsoever. For example, one is not free to assert Pa if one has just asserted (and does not retract) $\neg Pa$. This would offend against logic (dialetheism notwithstanding). Similarly, and perhaps more controversially, one is not normally free to assert Pa if one has just

[5]My use of the phrase "open texture" is not quite Waismann's (1945). He writes, "Vagueness should be distinguished from *open texture*. A word which is actually used in a fluctuating way (such as 'heap' or 'pink') is said to be vague; a term like 'gold', though its actual use may not be vague, is non-exhaustive or of an open texture in that we can never fill up all the possible gaps though which a doubt may seep in. Open texture, then, is something like *possibility of vagueness*."

asserted (and does not retract) $\neg Pa'$, where a' is only a little different from a. That would offend against the nature of vague predicates, either a penumbral connection or a principle of tolerance (see below).

In short, if a sentence Φ is definite, then one can correctly assert it at any time and can never correctly deny it, and if $\neg\Phi$ is definite, then one can correctly deny Φ at any time and can never assert it. If Φ is unsettled, then it depends on the conversational situation one is in. The details of the project concern the kinematics of vague terms in conversations.

It is part of the foregoing account that each vague predicate is response-dependent or, better, judgment-dependent, at least in its borderline area. The extension of a vague predicate in a given context is, in part, a function of the judgments of competent speakers concerning this extension. For at least some objects—those in the borderline region—an item falls in the extension (or anti-extension) of a vague predicate only if it is competently judged to be so under appropriate circumstances (see Shapiro (2006), especially §8 of Chapter 1).

Crispin Wright (1976, §2) defines a predicate F to be *tolerant* with respect to a concept φ, "if there is ... some positive degree of change in respect of φ insufficient ever to affect the justice with which F applies to a particular case". Vague predicates, or at least some vague predicates, are said to be tolerant. Wright's language suggests that the issue concerns the proper *application* of a predicate, or perhaps proper *judgment*. What are the rules for how vague predicates figure in proper judgments?

Consider the predicate "rich," which is arguably tolerant with respect to income. Suppose that two people p, p' differ only marginally in their income, say by a few dollars. Wright's principle seems to say that if someone competently judges p to be rich, then she must judge p' to be rich too, and vice versa—either both are rich or neither are. Similarly, the predicate "red" is tolerant with respect to small, or at least indistinguishable, differences in color. If two colored patches q, q' are visually indistinguishable and if someone judges q to be red, she must judge q' to be red as well.

As plausible as this interpretation of tolerance may appear, it leads to absurdity in the now-familiar manner, thanks to the presence of sorites series. Marginal differences can add up to a difference that is substantial enough to "affect the justice with which" the predicate applies. Rather than concede that vague predicates are incoherent, I propose the following *principle of tolerance*:

> Suppose that two objects a, a' in the field of P differ only marginally in the relevant respect (on which P is tolerant). Then if one competently judges a to have P, then she cannot competently judge a' in any other manner.

This, I suggest, is the key to the consistent deployment of vague predicates. Suppose, again, that p and p' differ only marginally in their income. It is compatible with this latter principle of tolerance that someone judges p to be rich and leaves the richness state of p' unjudged (one way or the other). Note, however, that it *does* violate tolerance, in the present sense, if a subject judges p to be rich and *decides* to leave p' unjudged. The decision to leave the case unjudged is itself a decision, and a judgment (as paradoxical as this sounds). To decide to leave p' unjudged is to judge p' different from p. The claim here is that the subject does not violate tolerance if he has not considered the state of p'.

My principle of tolerance demands that if a person competently judges p to be rich and is then asked to judge p' and *does not change his mind* about p, then he must judge p' to be rich. Suppose that our subject judges p to be rich and we then force a judgment about p'. He can satisfy tolerance by judging p' to be rich, but he can also satisfy tolerance by judging p' to be not rich (or in some borderline category or in no category) *and retracting his previous judgment* that p is rich. I suggest that in the cases of interest, the meaning of the word "rich" and the semantic and nonsemantic facts allow this option.

This is not to say that tolerance can never be violated. There are some situations where it must be, and a speaker is not incompetent just because she violates tolerance in such a situation. To follow an example from Sainsbury (1990), consider a hardware store that has red paints on one shelf and orange paints on another. The proprietor would not be judged incompetent concerning English, or otherwise in error, if no one can distinguish the last jar on the "orange" shelf from the first one on the "red" shelf.

In general, tolerance must be violated whenever one is required to judge every member of a sorites series. I take the goal to be to show how vague predicates can be deployed, consistently, in situations in which tolerance is in force.

To articulate the view further, we need a mechanism to track the features of an ongoing conversation.[6] This is provided by Lewis's influential "Scorekeeping in a Language Game" (1979). The *conversational score* is a local version of common knowledge. During a conversation, the score contains the assumptions, presuppositions, and other items implicitly or explicitly agreed to. For example, the conversational score contains the range of quantifiers like "everyone" and the relevant comparison class, paradigm cases, and/or contrasting cases for predicates like "tall" (European entrepreneurs, professional basketball players, etc.). The score also contains propositions that have been (implicitly or explicitly) agreed to and are not up for dispute or discussion, at least for the moment.

The conversational score is a sort of running database, a theoretical posit to help make sense of conversational flow. The score is continually updated, in that items are put on the score and, notably here, removed from it in the course of a conversation. Items get removed when the topic changes, when what is agreed to or some presupposition comes into question, or when some of the participants publicly change their minds about items that are on the score.

[6] Shapiro (2006) also contains a model-theoretic development, but that is omitted here in the interest of brevity. The system is similar to the Kripke semantics for intuitionistic logic (a variation on the modal logic S4), except that, at each node of each frame, vague terms have both an extension and an anti-extension. A node represents a way that vague terms can competently be deployed in a conversation, including one in which a tolerance principle is in force.

Lewis (1979, 345) delimits some features of the conversational score:

(1) . . . the components of a conversational score at a given stage are abstract entities. They may not be numbers, but they are other set-theoretic constructs: sets of presupposed propositions, boundaries between permissible and impermissible courses of action, or the like.
(2) What play is correct depends on the score. Sentences depend for their truth value, or for their acceptability in other respects, on the components of the conversational score at the stage of the conversation when they are uttered . . . [T]he constituents of an uttered sentence—subsentences, names, predicates, etc.—may depend on the score for their intension or extension.
(3) Score evolves in a more-or-less rule-governed way. There are rules that specify the kinematics of score . . . .
(4) The conversationalists may conform to directives, or may simply desire, that they strive to steer components of the conversational score in certain directions . . . .
(5) To the extent that conversational score is determined, given the history of the conversation and the rules that specify its kinematics, these rules can be regarded as constitutive rules akin to definitions.

Since conversations tend to be cooperative, "*rules of accommodation* . . . figure prominently among the rules governing the kinematics of conversational score" ((1979, 347), my emphasis). The idea is that the conversational score tends to evolve in such a way that, other things equal, whatever is said will be construed as to count as correct, if this is possible. The score will be updated to make this so. To be sure, cooperation is "not inevitable, but only a tendency," as Lewis puts it. And, of course, presuppositions can be challenged and retracted later.

Recall our premise of open texture: borderline cases of vague predicates can go either way in some conversational contexts. Once a borderline case is "resolved" in the course of a conversation, that information goes on the conversational score and will remain on the score unless it is implicitly or explicitly retracted. The plan is to sketch a model of how that happens.

Consider a conversational variant of Terence Horgan's (1994) "forced march" sorites. Suppose we have 2,000 cards lined up in a row. The first is a clear green, and the last is a clear yellow. After the first, each card differs from the one before by being only slightly less green and slightly more yellow. Let us assume that the difference between adjacent cards is imperceptible, at least via casual observation.

Now suppose that the participants in a conversation start asking themselves about the color each card in the series, starting with the first, and suppose that we insist on a communal verdict in each case. As each question in the form "Is card *n* green?" is put, they are to provide an answer of "yes" or "no."

This insistence on bivalence is only a convenience. The situation would be essentially the same if we gave them the option to answer "borderline green," "unsettled," "no fact of the matter," etc. It would also be the same if we gave them the option to go silent. To paraphrase Diana Raffman (1994, 41, n. 1), it isn't merely that there is tolerance between (definite) cases of greenness and (definite) cases of yellowness. There is tolerance between greenness "and *any* other category—even a 'borderline' category". Or Wright (1976, §1): "no sharp distinction may be drawn between cases where it is definitely correct to apply [a vague] predicate and cases of *any* other sort." What matters for present purposes is that the participants in our conversation must answer (or refuse to answer) by consensus—whatever the allowed answers may be. We do not allow them to stop the march by simply failing to agree on a verdict, unless we count that failure as itself a verdict.

Being competent speakers of English and assuming normal eyesight and lighting conditions, the conversationalists all agree that the first card is green, that the second is green, and so on for a while. Eventually they will move into the borderline area and encounter cards whose color state is "unsettled," as defined above. Again, the thoughts and practices in using the language have established truth conditions for statements about these colors. In the borderline region, the nonlinguistic facts do not determine that these truth conditions are met. Nevertheless, the conversationalists in this exercise will probably continue to call the cards green as they move through the borderline area—for a bit. If they call card n green, they will probably call card $n + 1$ green as well, since by hypothesis, they cannot really tell the colors of the two cards apart.

By the lights of my account of vagueness, this is all fine, so far as their language competence goes. Given open texture, borderline cases can go either way (without offending against meaning and the facts), and the participants in this conversation are just going one way rather than the other as they work their way into the borderline region from this direction. It does not matter whether they realize that they can go either way with these borderline cases. As they start to consider the borderline cases, they continue to see the cards as green—for a while, anyway. This puts propositions like "card 904 is green" and "card 905 is green" on the conversational score (assuming that those are borderline cases).

Since the participants in the exercise are competent speakers of English, with more or less normal eyesight, we can be sure that they will not go through the entire series and call the last card, #2000, green. That way lies gross perceptual error or linguistic incompetence. At some point, enough of them will demur, and the consensus on "this card is green" will break down. Given the "forced march" instruction, they will eventually agree to call one of the cards yellow, or not green. There is no particular card on which they must shift, but competence demands that they switch somewhere. Suppose this first happens with card 1025 (and assume that card 1025 is borderline green and borderline yellow).

Of course, when the conversationalists say that #1025 is yellow, they did not contradict themselves. They had no previous verdict on that card. My proposal here is they need not have violated a tolerance principle either. In declaring card 1025 to be yellow, they can implicitly *deny* that card 1024 is green, and so "card 1024 is green" is *removed* from the conversational score. It is similar to what happens when any presupposition or assertion is challenged (or contradicted) in the course of a conversation.

Just as "card 1024 is green" comes off the score, so does "card 1023 is green," and similarly for a few more of their recent pronouncements (again assuming that tolerance is in force). There is no need to set a precise boundary as to how many sentences are removed from the score once they jump. Exactly what is and what is not on the conversational score at any given time is itself a vague matter.

I would think that borderline cases of "what is on the score" will not interfere with a conversation. If a question arises about a specific case—say #1020—the participants can ask each other about that card, and so the status of that case can be made explicit.

Suppose we now take the conversationalists back down the series. In particular, suppose that we ask them about card 1024 again, after reminding them that they just called 1025 yellow. I speculate that they would (or at least might) retract that judgment, saying that #1024 is yellow (and thus put "card 1024 is yellow" on the score). Suppose that we then ask them about #1023. They would retract their previous judgment there as well. Then we can ask about #1022. Of course, they will not move all the way back down the series and end up calling the first card yellow. At some point, they will jump again and declare a certain card to be green. Suppose it is at 997. This again will result in the removal of certain items from the conversational score, such as the statement that 998 is yellow. We can then go back up the series, asking them about card 998 and card 999. In general, they might move backward and forward through the borderline area. Tolerance is enforced at every stage, by removing judgments from the conversational score whenever a jump occurs. Raffman calls this phenomenon "hysteresis," after a well-studied feature of physical systems.

10.3 Quasi-abstract Objects and Identity

Let us now turn to quasi-abstract objects, introduced by principles in the form (ABS):

$$\forall a \forall b \, (\Sigma(a) = \Sigma(b) \equiv E\,(a,b)) \, ,$$

The interesting cases are those where a potential sorites series demonstrates that the embedded relation E is not transitive and, indeed, vague. We will focus on the exemplar we called the Weight Principle above, as naturally interpreted.

> The weight a is identical to the weight of b if and only if a and b are roughly equi-weighted,

where a and b are variables ranging over physical objects or people. The abstract objects yielded by this are *weights*. Anything said here will apply directly to other abstraction principles. Examples above include heights, income groups, and possibly colors and meanings.

Imagine a sorites series consisting of 60,001 people. The first weighs 40 kg, and for each n in the series, except the last, person n weighs one gram less than person $n+1$. So the last person weighs 100 kg. Fix the external context in such a way that the first person is extremely light and the last is hefty (and so definitely not light). Clearly, the first person is not roughly equi-weighted with the last.

Now assemble a bunch of conversationalists who start discussing the weight status of members of the series. They ask themselves whether person 1 is roughly equi-weighted with person 2. Surely yes. Then they ask if person 1 is roughly equi-weighted with person 3. Surely yes. Their weights differ by only 2 g. They continue in order, asking if person 1 is roughly equi-weighted with person *n*. For a time, they will answer "yes." Eventually, they will enter the borderline area of this binary relation or, equivalently, the borderline area of the monadic predicate/property of being roughly equi-weighted with the first person in the series.[7] As they do, certain propositions like "person 4,001 is roughly equi-weighted with person 1" go on the conversational score.

Of course, the conversationalists will not go through the entire series and end up saying that the last person is roughly equi-weighted with the first. That would reveal incompetence, either with the terms in question or with perception. At some point, they will jump and explicitly deny that a certain person in the lineup is roughly equi-weighted with person 1. Suppose that first happens with #6,200. Assuming that tolerance remains in force, at that point some items get removed from the conversational score, such as the statement that #6,199 is roughly equi-weighted with person 1. We then take the conversationalists back down the series, asking if #6,200 is roughly equi-weighted with #6,199, and then, after an affirmative answer, asking if #6,200 is roughly equi-weighted with #6,198. Eventually, they will jump, and for the second time, items will be removed from the score.

One might think that a principle of tolerance for this sorites series would be something like this: if two people are roughly equi-weighted and a third person's weight differs from one of them by at most one gram, then that person is roughly equi-weighted with each of the first two. As usual, this leads to paradox, given that there are two folks in the series—the first and the last—who are not roughly equi-weighted with each other. To follow the above theme, a better principle of tolerance is this:

> If someone judges two people to be roughly equi-weighted and a third person's weight differs from one of them by at most one gram, then she cannot judge that the third person as anything other than roughly equi-weighted with each of the first two.

If the judge is forced to rule on the third person, vis-à-vis the other two, she can satisfy tolerance by either declaring the third person to be roughly equi-weighted with each of the first two or retracting her original statement that the first two are roughly equi-weighted with each other.

So far, nothing is new here. We have just applied the foregoing account of vagueness to the binary predicate of being roughly equi-weighted. In particular, we have not yet talked about quasi-abstract *objects* nor about vague *identity* statements. That matter will flow from the Weight Principle.

So let us introduce *weights*, as quasi-abstract objects, into the picture. For each object or person *x*, let *Wx* be the weight of *x*. Suppose that the pair <#1,#4,500>

[7] Recall that we are holding external contextual factors, like comparison class and the like, fixed.

is in the borderline area of the relation of being roughly equi-weighted. Then the statement $W\#1 = W\#4{,}500$ is not definitely true nor is it definitely false. When a group of conversationalists say that #1 and #4,500 are roughly equi-weighted, they implicitly (or perhaps explicitly) assign them the same weight. So once the judgment is made, it is true that $W\#1 = W\#4{,}500$ (in that context). In another context, it might be true that $W\#1 \neq W\#4{,}500$.

In other words, the acceptance of propositions like the Weight Principle seems to require that statements in the form of identities are themselves subject to indeterminacy and open texture (given the foregoing account of vagueness). So we have to deal with ontological issues. What, exactly, *are* weights, income groups, heights, colors, meanings, etc.? Is it even coherent that *identity* is a vague relation?

In a much discussed one-page note, Gareth Evans (1978) argues that the very notion of indeterminate identity is incoherent. We can adapt the reasoning to the present case. Suppose, as above, that it is indeterminate whether $W\#1 = W\#4{,}500$. Then $W\#4{,}500$ has the property of "being neither definitely identical to $W\#1$ nor definitely distinct from $W\#1$". Call this property θ. Clearly, $W\#1$ *is* definitely identical to $W\#1$ if anything is, and so $W\#1$ does not have the property θ. So $W\#4{,}500$ has a property that is not shared by $W\#1$, namely, θ. So, by (the contrapositive of) Leibniz's law on the indiscernibility of identicals, we conclude that $W\#4{,}500 \neq W\#1$. These two weights are (definitely) distinct after all, in every context.

In a short follow-up article, Lewis (1988) points out that the Evans argument does not undermine the possibility of indeterminate *statements* of identity nor was it intended to. We can illustrate the point with a simple ambiguity. Suppose that, in the domain of a given conversational context, there are two people named "Bob," say Bob Jones and Bob Smith. And suppose also that there is only one Smith in the domain. Then the statement that Bob is identical to Smith is indefinite. But the blame for this indefiniteness lies with the denotation relation. In particular, it is indeterminate which person the name "Bob" denotes. Clearly, situations like this are not ruled out by the Evans argument.

According to Lewis, what *is* ruled out by the Evans argument are indeterminate *identities*. There can be no statements in the form $m = n$ in which m determinately denotes exactly one thing and n determinately denotes exactly one thing, and those (two?) things are themselves neither definitely identical to each other nor definitely distinct from each other.

There are several options for interpreting the Weight Principle and the other principles that deliver quasi-abstract objects. Only some of these run up against the Evans argument. I take it as a desideratum that our ordinary talk of things like weights, income groups, heights, colors, and meanings is coherent and that at least some of it is literally true. I will only explore alternatives that sanction this, Unger (1975) and Dummett (1975) notwithstanding.

10.3.1 Weights as Dynamic Groups

The first option is somewhat conservative. Think of weights as things that, like sets, have members. Call them weight groups. So if *a* is equi-weighted with *b*, then *b* is a member of the weight group of *a*, and *a* is a member of the weight group of *b*. In the above terminology, the term *Wa* stands for the weight group of *a*. And so we might write $b \in Wa$ and $a \in Wb$. The proposal here is to think of a weight group as a *dynamic* set, the sort of thing that can gain and lose members over time. Other dynamic sets are the US Senate and the roster of the Cleveland Cavaliers.[8] Hilary Clinton used to be a member of the Senate, but she is no longer; LeBron James used to be a member of the Cavaliers, but he is no longer (alas).

Under this interpretation, weight groups gain and lose members as borderline cases of the relation on the right-hand side of the Weight Principle are called or retracted in the course of a conversation. So, for example, in the above scenario, when it is declared that #6,199 has roughly the same weight as #1, then #6,199 goes into the weight group of #1 (and #1 goes into the weight group of #6,199). When that statement is implicitly retracted a moment later, #6,199 is removed from the weight group of #1 (and vice versa). In a sense, weight groups are quasi-equivalence classes.

On this reading, the statement on the left-hand side of the Weight Principle is not actually a statement of identity between objects—weight groups in this case. That is, $Wa = Wb$ does not say that the weight group of *a* and the weight group of *b* are the very same *weight group*. Rather, it says that the two groups have the same members at the contextually indicated time.

This represents a natural distinction concerning dynamic sets. Suppose, for example, that at a certain point in time, the members of a certain softball team, the Prawns, happen to coincide with the members of a certain Senate subcommittee. Then it would be true, in a sense, that the Prawns are the Senate subcommittee, even though these are different dynamic sets. The members of these two dynamic sets were different in the past and are likely to be different in the future. But, right now, the dynamic sets are the same, not in the sense that they are the same dynamic set, but that they have the same members at that point in time.

So our first proposal for interpreting the Weight Principle is that, in any given context, the weight of a given person is the set of people that are (competently) judged to be of about the same weight *in that context*. Let *a* be a person. As the context changes, the members of the weight group of *a* change, but it remains the same weight group throughout, just as the one and only Supreme Court changes its members over time (sometimes for better, sometimes for worse). The same goes for heights, income groups, and the like (even meanings, as per Shapiro 2014).

Consider a collection of people as described in the above sorites series. In any conversational context in which one case of equi-weight has been called, there will

[8] See Uzquiano (2004) for an illuminating account of dynamic sets.

be at least two people whose weights differ by a single gram such that one of them is not in the weight group of the other. Otherwise, we would have to conclude that #1 and #8,000,001 are in the same weight group in every context—and we don't want *that*.

Admittedly, it is counterintuitive that two people whose weights differ by a single gram are not in the same weight group in every context. How much difference can a single gram make? But such is vagueness and such is sorites. I submit that this feature of the view does not strain intuitions that much. If a principle of tolerance is in force, it requires that if the weights of two persons differ by one gram, then they are never in *different* weight groups. In other words, if the two people both get classified at the same time, they must be classified together. The key to maintaining tolerance is that not everyone gets classified. This is similar to our conclusion, in the previous section, that it is not possible to classify every member of a sorites series as either green or yellow at the same time, on pain of violating tolerance. But we can insist that we never (or rarely) classify close cases differently.

To sum up this option, the Weight Principle is false if we read its left-hand side as an identity between weight groups. Indeed, the weight group of a person a is not the very same weight group as that of b unless they have *exactly* the same weight—whatever that might mean. A better reading of the Weight Principle is to take $Wa = Wb$ as a statement that the weight groups have the same members in the context at hand. More precisely, the principle says that if a and b are roughly equi-weighted in a given context, then their respective weight groups have the same members in that context.

Since, on this reading of the Weight Principle, we do not have a genuine identity on the left-hand side, we do not encounter indeterminate identity statements, much less indeterminate identities. Consequently, we do not come up against the conclusion of the Evans argument. And we do not have an uncomfortable view on ontology to deal with, unless dynamic sets themselves cause metaphysical discomfort.

10.3.2 Weights as Bona Fide Objects

A more radical way to interpret the Weight Principle is to take its left-hand side as invoking a genuine identity between the weights themselves. Given the vagueness of the relation on the right, it follows that for some pairs of people, a and b, it is not definite that the weight of a is identical to the weight of b, nor is it definite that the weight a is distinct from the weight of b. We have vague identity *statements*, and perhaps even vague *identities*.

As noted in the previous section, it is part of the foregoing account that each vague predicate is response-dependent or, better, judgment-dependent, at least in its borderline area. The extension of a vague predicate in a given context is, in part, a function of the judgments of competent speakers concerning this extension. If there is no judgment (one way or the other), then there is no fact of the matter as to where the object falls. This holds, at least, for objects in the borderline area of the predicate.[9]

The present reading of the Weight Principle extends the judgment dependence to identity statements, and possibly also to ontology. The thesis is that what identities hold in a given context is, at least in part, a function of judgments that have been made, explicitly or implicitly, in that context. Suppose, for example, that in the course of a conversation, someone says that a man of 65 kg is roughly equi-weighted with a woman of 70 kg. This establishes a weight that includes these two people, and this same weight includes everyone whose weight is in between them. This information goes on the conversational score. Suppose that, a few minutes later, someone else in the conversation notes that a person of 73 kg is roughly equi-weighted with the 70 kg woman. This explicitly extends the above weight. It now stretches (at least) from 65 kg to (at least) 73 kg. But now suppose that later on, the conversationalists identify the latter weight 73 kg with someone of 77 kg, *and* they agree 77 kg is *not* roughly equi-weighted with the first man, who is a paltry 65 kg. At this point, the conversationalists identify the 73 and 77 kg folks, but distinguish those two from 65 kg. The conversational score is thereby updated to include this information, retracting the statement that 65 kg is roughly equi-weighted with 73 kg. This retraction is analogous to a jump in the forced march sorites treated above. There, the jump results in changes to the extensions and anti-extensions of predicates. Here, the jump results in a change in identity statements, and possibly also a change in ontology. Before the jump the conversationalists had a single weight in their sights, after the jump they have two.

As with forced march sorites, conversationalists cannot call every weight without violating tolerance. Indeed, if they had to pronounce on ever pair of people in the series, whether or not they are roughly equi-weighted, then the conversationalists would have to either identify all of the weights, which is patently absurd (given the presumed external context) or else they would have to distinguish two weights that differ by only one gram, thus violating tolerance.

On the present option, as the conversationalists make and change various assignments of rough equi-weight, the ontology of quasi-abstract objects changes to fit their pronouncements, with the conversational score keeping track of which identifications have been made.

What are we to say about weights of those people in the series that have not been called at a given moment? For example, suppose that person *a* is 54 kg and person *b* is 85 kg. What is the status of the weight of *a* and the weight of *b* at the moment in question, during the above conversation?

There are two theoretical options. One is to hold that the relevant weights do not exist at the time. On this view, throughout the above-discussed conversation, there is no such thing as the weight of *a* and no such thing as the weight of *b*. Of course,

[9] An extreme version, dubbed the "Copenhagen view of vagueness" in Chapter 5 of Shapiro (2006), is that this holds for *every* object in the field of the predicate.

the conversationalists could go on to discuss the weights of one or both those folks, in which case the requisite weights would be created (and, if necessary, adjustments made to other items on the conversational score). But as it happens, they do not discuss those weights, and so the weights do not exist during that conversation.

This, of course, is counterintuitive. In the foregoing account of vagueness, competent speakers are masters of the *extensions* of vague predicates, at least in their borderline areas. On the present option of present view, competent speakers are also in control of *ontology*, at least for quasi-abstract objects introduced by vague abstraction principles. In other words, on the present interpretative option, the judgments of competent speakers determine which objects exist at any given moment in a conversation.

As counterintuitive as it may be, this interpretive option is not threatened by the Evans argument. On the present perspective, there are no indeterminate *identities*, or at least none between *existing* quasi-abstract objects. As a conversation proceeds, weights come into existence and go out of existence, depending on the decisions of the participants. But whenever weights exist at a given point in a conversation, any pair of them are definitely identical or definitely distinct *at that point*. In some contexts, a given *statement* of identity between weights may lack a truth-value, but that is because at least one of the groups does not exist in that context. We'd need a free logic to develop this option further.

I presume that the indiscernibility of identicals does not apply to objects that do not exist. If the weight of *a* does not exist at a given moment, it is not the case that this very weight has the property of not existing at the time, nor does it have the property of being indeterminately identical to the weight of *b*, for any *b*. There simply is no "it" to have or lack properties.

A second, perhaps more radical interpretive option is to hold that all of the possible weights exist, all the time. Every person, and every material object, has a weight. Actually, this seems kind of obvious. It also seems obvious that we do not create or destroy objects just by talking. However, on such a view, some identities between weights lack truth-values. For example, throughout the above conversation, it is neither true nor false that the weight of someone 54 kg is identical to the weight of someone of 65 kg. Indeed, throughout the above conversation, it is neither true nor false that the weight of a 54 kg person is identical to that of someone who weighs a single gram more or a single gram less. So long as tolerance is in force, at least some identities between weights must lack truth-values. Indeed, some identities between the weights of objects or people that differ by only one gram must lack truth-values. It seems that there is no option that has no counterintuitive features, or at least no option that I can think of.

Notice, however, that on the present option, it is nevertheless true that for each object or person *a*, the weight of *a* is identical to the weight of *a*. Also, at the end of the conversation, it is true that the weight of our 54 kg person is *distinct* from that of 73 kg, since the first is lighter than 65 kg, and that weight is explicitly distinguished from 73 kg.

On this final interpretive option, we do not hold that competent speakers are masters of which quasi-abstract objects like weights exist. All weights exist.

But here speakers are in charge of what identities hold between and among quasi-abstract objects. So speakers sometimes competently determine *how many* weights are under discussion. Suppose we are talking about the weights of three different people. How many weights are there? In some contexts, there may be one, in others two, and in still others three. In yet other contexts, it is indeterminate how many weights there are.

So the present interpretive option allows that there are indefinitely many weights—exactly how many is a function of the context. Moreover, we *do* have genuinely indeterminate *identities*. So this option runs afoul of the conclusion of the Evans argument.

I conclude with a brief sketch of how a resistance to the Evans argument might be motivated on our present option. A full defense would take us too far afield of both the scope of this chapter and my own competence and interests.[10]

I presume that it is not an option to simply reject the Leibniz principle of the indiscernibility of identicals. At the very least, the indiscernibility principle borders on being analytic of the notion of identity. Intuitively, someone who denies the principle of the indiscernibility changes the subject; she is not talking about *identity*.

The key move, I suggest, is to insist that predicates like "is determinately identical to the weight of *a*" do not pick out properties, or at least they to not pick out properties that are relevant to the Leibniz principle. One question-begging way to proceed is via a modus tollens on the Evans argument. Since, on the present view, there are indeterminate identities, it follows that the predicates in question, namely, of being indeterminately identical to, are not appropriate to the Leibniz principle. By itself, this move has the flavor of Lakatosian monster barring. It is quite ad hoc and unmotivated.

We can do better. What we need here is a (independently) motivated reason to exclude the determinacy predicates used in the Evans argument. I submit that the present account of vagueness supplies such a reason.

It is clear that not every *predicate* counts toward the indiscernibility of identicals. For example, Karl may know that the evening star is actually a planet and not know that the morning star is a planet. So the evening star has, and morning star lacks, the "property" of being known by Karl to be a planet. Surely, on any sensible view, this does not entail that the evening star is not identical to the morning star. Presumably the inference fails because "Karl knows that x is a planet" is not the right sort of predicate. It does not represent a property.

A predicate P is said to be "intensional" if one cannot always substitute co-referring terms without changing the truth-value or, in other words, if Pa and $a = b$

[10] Terence Parsons and Peter Woodruff (1995) defend the coherence of indeterminate identities. They point out that the Evans argument invokes the *contrapositive* of the Leibniz principle of the indiscernibility of identicals and that in a three-valued system, the contrapositive of a valid inference need not be valid. Moreover, they suggest that predicates that invoke indeterminacy need not express properties. Parsons and Woodruff also provide a nice model, in a crisp, bivalent metalanguage, to illustrate the coherence of vague identity. Shapiro (2006, Chapter 6) provides another.

do not entail Pb. The predicate "Karl knows that x is a planet" is one such. The proposal is that the identity of indiscernibles need not hold of intensional predicates. An advocate of the strict reading of the Weight Principle would argue that "being determinately identical to" is similarly disqualified: it is intensional.

Recall that, on the present account, vague predicates are judgment-dependent, at least in their borderline regions. What counts as satisfying a given vague predicate is sometimes determined by the pronouncements of competent judges. As such, all vague predicates are intensional, at least in part. How a given person will judge a given case depends on how the case is presented to him or her. Suppose, for example, that someone makes a competent judgment in the form $\Phi(a)$, and suppose that a is (definitely) identical to b. It does not follow that the person could, or would, judge that $\Phi(b)$, unless, of course, she knows that $a = b$. Since vague statements are intensional, it is reasonable to exclude such predicates from the indiscernibility of identicals. And via the contrapositive of abstraction principles like the Weight Principle, this intensionality extends to statements of the identity of quasi-abstract objects like weights, heights, and income groups.

To sum up, then, there are at least two ways of reading abstractions like the Weight Principle. One of them interprets weights to be dynamic sets, and so weights change their members over time. In this case, we do not even have indeterminate statements of identity. The indeterminacy only applies to statements that certain weight groups have the same or different members at various times. A second option embraces weights as quasi-abstract objects. This splits into two further options. On the first, weights are created and destroyed in the course of conversations, along with identities between them, as judgments of equi-weight are made and retracted. Here, too, there is no threat from the Evans argument. The other interpretive option takes all weights to exist, all the time. What changes are identities between the various weights. This option does run afoul of the Evans argument, at least prima facie. The best response, I think, is to point out that statements of identity, on this option, are themselves intensional, due to the judgment dependence of the vague relation of being roughly equi-weighted. This saves the identity of indiscernibles.

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Part IV
Ontic Supervaluationism

Chapter 11
Vagueness in the World: A Supervaluationist Approach

Ali Abasnezhad and Davood Hosseini

11.1 Introduction

Russell once said that "Vagueness and precision alike are characteristics which can only belong to a representation, of which language is an example. They have to do with the relation between a representation and that which it represents. Apart from representation, whether cognitive or mechanical, there can be no such thing as vagueness or precision; things are what they are, and there is an end of it" (1923). In other words, expressions like *ontological vagueness* (and even ontological precision) are category mistakes and therefore make no sense. Half a century later, as the view began to emerge that some vagueness could be intelligibly ascribed to the world, Dummett opined that "It is not apparently absurd to suppose ... that the physical world is in itself such that the most precise description of it that even omniscience would yield might yet involve the use of expressions having some degree of vagueness" (1981). Evans had already questioned the idea of worldly vagueness, arguing that it was inconsistent (1978). A quarter of a century later, however, many papers had been written in defense of the intelligibility and possibility of this once nonsense and then contradictory claim. This chapter is one of them, albeit from a different viewpoint.

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The aim of this chapter is twofold: firstly, to consider the intelligibility of vague ontology,[1] and secondly, to discuss the most famous argument against vague ontology, i.e., Evans' argument. To this end, in Sect. 11.2 we present some commonsensical considerations on vague ontology. In Sect. 11.3, we look at some supervaluationist theories and the kind of vague ontology they accommodate before arguing that this kind of ontology is inconsistent with commonsense. Section 11.4 outlines a version of supervaluationism that can accommodate the commonsensical considerations discussed in Sect. 11.2. The outlined view is an ontological variation of Kit Fine's characterization of vagueness as incompleteness (1975). It will be argued that the view is more effective than other versions of supervaluationism at accommodating commonsensical and metaphysical considerations. In the following section, we consider vague identity. Based on the view outlined in Sect. 11.4, there may be vague identity in some special cases. In Sect. 11.6 we focus on Evans' argument and argue that it can be blocked in the first step.[2]

11.2 Vague Ontology and Commonsense

We live in a world full of entities such as mountains, deserts, trees, and rocks. However, these sorts of objects are not like cubes, spheres, etc., in the geometrical sense in that they do not occupy a precise region of a three-dimensional space. For example, there is no precise spatial boundary for Everest. If one descends Everest step by step, from its peak, it will not be determinate at which step one actually leaves the mountain. As Tye (1996) noted: "[i]t is part and parcel of our everyday, commonsense view of the world that there are such things as mountains, deserts, and clouds. It is also part and parcel of our commonsense view that these items are not perfectly precise, that they have fuzzy boundaries."

Although vague ontology may seem intelligible from a commonsensical point of view, it is a controversial matter from a theoretical point of view. Indeed, the possibility of a systematic conception was the target of Russell's and Evans' doubts. The aim of this chapter is to propose one such conception.

To begin with, let us clarify what vague ontology is. Here, we are concerned only with vague objects and vague properties (including relations). In the

[1]A note on terminology: In this chapter "vagueness" and "indeterminacy" will be used interchangeably, both being substitutable by "vagueness-related indeterminacy." Of course, there may be indeterminacies which are not vagueness related, such as indeterminacies caused by open future. A list can be found in Williams (2008b). Here, however, we are only concerned with vagueness-related indeterminacy.

[2]Throughout this chapter we will take a conservative approach to logical issues. Since the proposed view is a version of supervaluation, it is safe to assume that classical logic can be preserved in some way or other. As such, the present proposal can be considered independent of logic-related issues. For discussions see Williamson (1994), Keefe (2000), Akiba (2004), Williams (2008c), and Barnes (2010).

abovementioned case of Everest, what concerns us is the fact that it lacks a precise spatial boundary. Indeed, many vague objects have no precise temporal boundary, either. And there may be many other kinds of indeterminacies associated with vague objects. Here, it is not our intention to characterize vague objects. Among the various indeterminacies, the lack of a precise spatial boundary seems to be a common feature of ordinary vague objects. Consequently, we will focus primarily on vague objects which lack a precise spatial boundary.

The second ontological category that we are concerned with is property. Everest is a gray mountain.[3] Naively, this means that the object, Everest, has the color gray. Gray is a property of the object, but not a precise one. Not every colored thing is either determinately gray or determinately not gray. There are (or at least there might be) borderline cases of gray, i.e., something which is not determinately gray and not determinately nongray. The indeterminacy associated with the color gray is not due to the vagueness of Everest, or of any other possessor of the property. Even if everything were precise, the color gray would be vague, since there might be a precise object which is not determinately gray and not determinately nongray. Gray is a typical example of a vague property. In general, then, and in rough terms, a property is vague iff there is (or there might be) an object that neither determinately has nor determinately lacks that property, and the indeterminacy is not due to the vagueness of the object.

Of course, it is controversial how best to characterize vague objects, vague properties, and, in general, ontic vagueness. Here, the rough characterizations mentioned above are adequate for the purposes of this chapter. Thus, we overlook issues concerning definition, such as those in Barnes (2010), Rosen and Smith (2004), and Tye (1990). Another reason for avoiding a definition is that the proposal that will be defended is a form of supervaluationism. Thus, wherever the idea of precisification makes sense, this chapter's proposal can be applied.[4]

Another important consideration is the relationship between our vague language and the vague world (the actual world that contains vague objects and vague properties). Naïvely, language associates vague names and vague predicates with vague objects and vague properties in the world. What do we mean when we speak about vague objects, e.g., when we speak about Everest? This question is implicitly addressed in the considerations mentioned earlier. Explicitly, when we speak about Everest, we are referring to a specific mountain in the world, located in a specific place on Earth. When we say, "Everest is a gray mountain," what is being said by the use of the sentence, and regularly understood from it, is that a specific object in the world has a specific property. In other words, we are using

[3] It does not matter what its color is actually. It may be brown, black, or any other color. Take your choice.

[4] As we mentioned at the outset, we are concerned with spatial boundary vagueness. One main exception, however, will be Geach's Paradox, below, in which the parthood relation plays a role. This is not a fatal deviation, since the idea of precisification makes sense there and thus the present proposal works.

the sentence, commonsensically, to firstly refer precisely to a vague object, and then ascribe precisely a vague property to that object. Therefore, it is plausible to interpret commonsense such that it presupposes that vague singular terms (at least some of them) refer to vague objects in the world, and vague predicates (at least some of them) express vague properties in the world.

These considerations on vague ontology, and the relationship between vague language and the vague world, will form the basis of the remaining parts of this chapter, the main purpose of which is a systematic conception of these intuitive ideas.

11.3 Supervaluations and Ontic Vagueness

Supervaluationism, as a theory for vagueness, is based on the works of Fine (1975) and Lewis (1970, 1993), who explain vagueness as a linguistic phenomenon. In this section, we focus on Lewis's version. The idea can be best explained by the use of examples of vague predicates. It is not the case that everyone is determinately bald or determinately not bald. There are men for whom it is not determinate whether they are bald or not bald. They are borderline cases of the predicate "bald." Certainly, we can precisify this predicate in many ways. We can conventionally suppose the boundary of application of "bald" is n, for any n in the area of indeterminacy, and introduce a predicate "baldn" as follows: anyone who has less than or equal to n hairs on his scalp is baldn, otherwise is not. These predicates have classical extensions and these extensions are equally good to be considered as the extension of bald. Now, it is obvious that a man is determinately (not) bald iff he is (not) baldn for any n in the area of indeterminacy.[5]

Having discussed what is required for predicates to be considered vague, let us now consider vague names. When is a name (or generally a singular term) vague? A reductive approach is one strategy that may be adopted for dealing with vague names. Thus, a name a is vague iff there is a name b such that the statement "$a = b$" is of indeterminate truth-value. Another reductive strategy consists in replacing any vague name with a vague predicate. These are Fine's suggestions (1975). In fact, the vagueness of names need not be reduced to the vagueness of other kinds of expressions. Indeed, it is better to be neutral on the possibility of reduction and define explicitly what is required for a name to be considered vague. Lewis (1993) gives a typical supervaluationist picture of vague names. An example can illustrate the idea.

It is obvious that there is some vagueness associated with "Everest." With a delineation d on the border of Everest in hand, a name "Everestd" can be introduced as follows: any point on Earth is on Everestd iff it is inside the delineation d. Any

[5]Not all precisifications are admissible, only those which are consistent with determinate facts. For more on this see Fine (1975).

name "Everestd" thus introduced behaves like "Everest" in terms of determinacy, i.e., any point which is determinately (not) on Everest is (not) on Everestd. Now, any point is determinately (not) on Everest iff it is (not) on any Everestd. The same applies for any vague name.

Now, let us consider what happens, from the supervaluationist point of view, when we say something about Everest, e.g., "Everest is gray." As before, we focus mainly on Lewis's version (1993) in the following. According to this theory, there are many objects that equally qualify as the referent of the name "Everest." These objects are referents of "Everestds." When we speak of Everest, in fact, it is indeterminate which of these objects we are speaking of. Our usage of language is such that it does not fix any of these objects as *the* referent of the name. In general, the picture drawn by supervaluationists is that the vagueness of singular terms in language is due to some kind of indeterminacy of the reference relation. From a class of objects that are all equally good enough to qualify as a referent of the name, it is undetermined which object is the actual referent of the name.

This theory, nevertheless, is in conflict with the commonsense view, as described above. According to the commonsense view, when we speak of Everest, we are actually speaking of something, but something which is vague, i.e., has fuzzy boundaries. On the other hand, according to the supervaluationist view, when we speak of Everest, it is indeterminate which precise object (which Everestd) we are speaking of. The difference is a matter of scope, i.e., the scope of the indeterminacy operator. In the commonsense view, the scope is narrow; whereas in the supervaluationist view, the scope is wide. In a more precise formulation:

> Commonsense view: "Everest" refers to an object which is vague (and there is no vagueness as to which vague object this is).
> Supervaluationist view: It is vague which object "Everest" refers to.

Let us call the latter kind of vagueness, *referential indeterminacy* or *referential vagueness*, since it ascribes vagueness to the reference relation. And let us call the former – the kind of vagueness conveyed in the commonsense view – *worldly indeterminacy* or *worldly vagueness*, because it ascribes vagueness to an object in the world.[6]

This distinction raises the following question: which of these two notions of vagueness is *ontic*? It seems that worldly vagueness is straightforwardly ontic, since it refers to vagueness as attributed to objects in the world, but what about referential vagueness? Initially, supervaluationism was developed as a linguistic theory of vagueness, and it has been presented as such in previous paragraphs. More precisely, Lewis (1986) says, "[t]he only intelligible account of vagueness locates it in our thoughts and language. The reason it's vague where the outback begins is not that there is this thing, the outback, with imprecise borders; rather there are many things, with different borders, and nobody has been fool enough to try to enforce a choice of one of them as the official referent of the word 'outback.'"

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[6] This terminology matches that of Akiba (2004), but not necessarily that of others such as Williams (2008b).

However, over time the theory has also been used to explain ontic vagueness. Suppose the vagueness of singular terms is referential. A question still remains open: what is the source of this vagueness? In other words, why does this kind of vagueness occur? According to Fine and Lewis, the source of such vagueness is language. However, there is another option: the source is the world itself. Instead of saying that there are many objects and it is indeterminate which of them is the referent of "Everest," this alternative says that the world itself is indeterminate between many states. Indeed, there is only one world around us, but it is indeterminate which one – from a set of possible worlds – it is. Williams (2008a) and Barnes (2010) are defenders of this position. In summarizing his idea, Williams says, "There is just a single reality that includes ourselves and our surroundings. Nevertheless, most plausible theories of modality and possible worlds leave room for this single reality to be represented by a multitude of possible worlds that 'correspond with' reality to exactly the same extent" (2008a).

A similarity between supervaluations and possible worlds can be used to clarify how referential ontic vagueness is supposed to work. It is a well-known similarity: since each precisification is a classical model, it can be considered as a possible world. Referential ontic vagueness takes this similarity between supervaluations and possible worlds at face value and claims that there is some vagueness-related possibility. There is a range of possible worlds – vagueness-related possible worlds – and it is indeterminate which of these possible worlds the actual world is. In the cases of Everest and being bald the difference can be illustrated. According to the linguistic conception of supervaluations (that of Lewis), there are many regions on Earth – referents of "Everestds" – and it is not determinate which of these the referent of "Everest" is. And there are many sets – extensions of "baldns" – and it is indeterminate which of them the extension of "bald" is. From the ontic point of view (that of Williams), there are many possible objects – Everestds – and it is indeterminate which of them Everest is. And there are many possible properties – baldn-nesses – and it is indeterminate which of them the property baldness is.[7]

So, one can conclude that referential vagueness and worldly vagueness can both be seen as different kinds of ontic vagueness. However, they are not equivalent. One is in accordance with commonsense, the other not. The upshot is that if we want to model ontic vagueness *commonsensically*, modeling referential vagueness will not be related. Is supervaluationist technique responsible for this failure? Not necessarily. The idea of precisification seems to be a move in the right direction. Whenever there is vagueness there can be various precisifications. The question is what is the relationship between an object (a property) and its precisifications? According to ontic supervaluationists, they are possible objects that the object in question *may be*. If they are right, vagueness turns out to be referential. Fortunately this is not the only possible answer. One other possible answer is that precisifications are ways in which the object in question *may be precisified*. This is in accordance

[7]Here it is assumed, for the sake of example, that baldness is a property in the world.

with our commonsensical view that ontic vagueness is of the worldly vagueness kind. This will be our leading idea in the next section.

11.4 Outline of the Idea

We are now in a position to outline an approach to understanding the intelligibility of the commonsense view of ontic vagueness (i.e., worldly vagueness) through the use of supervaluations. As a starting point, a contrast between Fine and Lewis may be useful. Fine (1975) characterized vagueness (of a predicate, a sentence, etc.) as incompleteness of meaning and precisifications as “ways of making it [meaning] completely precise.” In the semantics of a vague language he assumes a “base point,” which corresponds to the actual meaning of the vague (incomplete) language, and a range of “limit” points, corresponding to “potential” meaning of the vague language. Each of these limit points is a way of making the meaning of expressions of the language completely precise. For a vague language, none of these base points are identical to any of the limit points. Especially, no base point is indeterminate between limit points.[8]

Fine’s semantic for vague language may be best understood in contrast with Lewis’s. The case of predicates is a good example. Consider “bald” and “baldn” as presented above. For Lewis, the extension of “bald” is indeterminate between extensions of “baldns.” Equally, any extension of “baldn” is one of the sets that the extension of “bald” *may be*. On the other hand, for Fine, the extension of “bald” is in the base point, and extensions of “baldns” are in the limit points. Each extension is in one of the limit points. So, the extension of “bald” is something different from those of “baldns.” Indeed, each extension in a limit point is a precisification of the extension in the base point. Equally, any extension of “baldn” is a way in which the extension of “bald” *may be precisified*. In sum, for Lewis, precisification is a matter of “choice.” For Fine, on the other hand, it is a matter of “precisification.”[9]

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[8] In Fine’s semantics there are intermediate points between a base point and any limit point, too. However, these intermediate points do not play a role in what has been stated here.

[9] There are sentences in Fine’s paper that may be considered as contrary to what has been said in this paragraph and following. One such sentence is, “Vagueness is ambiguity on a grand and systematic scale.” It is tempting to say that this kind of unification of vagueness and ambiguity makes Fine’s view more like Lewis’s than what is suggested in this chapter. In fact, this is not the case. The evidence from his paper, a couple of paragraphs after that quote, is this: “Ambiguity is like the super-imposition of several pictures, vagueness like an unfinished picture, with marginal notes for completion. One can say that a super-imposed picture is realistic if each of its disentanglements are; and one can say that an unfinished picture is realistic if each of its completions are. But even if disentanglements and completions match one for one, how we see the pictures will be quite different.” Fine’s metaphor of the unfinished picture is much like the metaphor of delineation stated above.

Of course, Fine and Lewis are linguistic supervaluationists.[10] Nonetheless, their semantics can be reread from an ontological point of view. An ontological rereading of Lewis's semantics justifies referential ontic vagueness. If the semantic view is that the extension of the predicate "bald" is indeterminate between extensions of predicates "baldns," the ontic view turns out to be that the property baldness is indeterminate between properties baldn-nesses. Therefore, an ontic rereading of Lewis will result in something akin to Williams's view (2008a). On the other hand, a rereading of Fine's semantics from an ontic stance justifies a kind of worldly vagueness. If, based on the semantic view, extensions of predicates "baldns" are ways in which the predicate "bald" can be made completely precise, the parallel ontic view will be that properties baldn-nesses are ways in which the property baldness can be made completely precise. This ontic rereading of Fine is the spirit of the present proposal.

Fine restricted his work to the case of predicates and does not apply this strategy directly to names. As mentioned above, he suggested reducing the vagueness of names to the vagueness of predicates (including identity). The present proposal, instead, seeks to extend Fine's idea to vague names. Thus, the vagueness of a name turns out to be a matter of the imprecision of its *meaning*. If a version of the direct reference theory of names is true, it can be said that the vagueness of a name is a matter of the imprecision of its *referent*. The meaning of a name *can be made completely precise*, i.e., there may be precisifications of the referent of the name.

An example may clarify the idea. The name "Everest" precisely, not vaguely, refers to an object which does not have a precise boundary. The vagueness is in the object itself: it has a fuzzy (spatial) boundary. This (vague) object might have many precisifications. Any precisification of the mountain is something without a fuzzy boundary: were the precisification an actual object, every point which is determinately (not) on Everest in the actual world would (not) be on the precisification; and any point for which it is not determinate whether it is on Everest in the actual world or not would be either determinately on the precisification or not.[11, 12]

These considerations can be generalized to the actual world. Suppose the actual world is with its vague objects and properties. Insofar as there are indeterminacies in the actual world, the actual world can be seen as a *vague* world. A precisification of the actual world is a possible world such that, if it were actual world, all determinate truths (falsities) of the actual world would be determinately true (false) in that precisification and all indeterminacies of the actual world would be determinate in

[10] In footnote 10 of 1975, Fine says that he is not sure that all material and mathematical entities are precise. However, he did not develop a theory about vague objects. So, when we say that Fine is a linguistic theorist on vagueness, we are referring to his supervaluationist view, not what his personal view might be.

[11] Not all precisifications are admissible. There are penumbral connections that any precisification should satisfy.

[12] Actually, mountains are not precise objects, but it is by no means obvious that there could not be precise mountains, and our view does not entail that there could not be such precise mountains. Thanks to an anonymous referee who brought this point to our attention.

the precisificational world, some way or other.[13] Now, the structural similarity with Fine's semantics is as follows: the actual world in the present proposal corresponds to the base point in Fine's semantics, and the precisificational worlds correspond to its limit points. The relationship between the base point and the limit points is the same in both: any limit point is a precisification of the base point. However, in Fine's semantics, this is a semantic claim, and in the present framework this is an ontic claim.[14]

Applying these ideas to Geach's paradox may illustrate the proposal. Let Tibbles be the only cat on the mat. And let $h_1, \ldots, h_{1,000}$ be Tibbles's loose hairs. It is indeterminate whether h_i ($i = 1, \ldots, 1{,}000$) is a part of Tibbles or not. Name the object on the mat, the object on the mat minus h_1, the object on the mat minus h_1 and $h_2, \ldots,$ the object on the mat minus $h_1, \ldots, h_{1,000}$, respectively, $p_0, p_1, \ldots, p_{1,000}$ ("p" for "precisification"). It seems, then, that if one of the p_is is a cat, any other should be as well. So, either there is no cat (none of p_is is a cat) or there are 1,001 cats (all of them are cats). Intuitively, this is implausible: there is only *one* cat on the mat. These conflicting intuitions raise a series of questions: How many cats are there? Is there a cat at all? Is there only one cat? Which of these p_is is a cat? According to the present proposal, these questions have natural answers. Of course, there is only one cat, named Tibbles. Tibbles is a vague object, an object with a fuzzy boundary. Each p_i is a possible precisification of Tibbles. None of p_is is a cat, in the ordinary sense, because cats usually do not have a precise boundary. Therefore, there is no conflict.

Here, it may seem that Lewis's objection against the worldly view of vagueness has some force. He argues that the ontology of vague objects is "unparsimonious and unnecessary" (1993). Anything that the vague ontology view explains, his semantic view can explain, too. So, it is unnecessary to suppose vague objects if we are already committed to precisifications. His explanation goes something like this: there is one cat on the mat, but it is indeterminate which of the p_is the cat on the mat is. Therefore, we have all that we need without commitment to vague objects. Thus, it is unnecessary to add vague objects to the ontology.

There can be three lines of responses to this. First, notice that Lewis himself accepts a multitude of objects in his ontology, namely, p_is. So, as Akiba (2004) said, "If we are happy to countenance thousands of objects on the mat anyway, what could be so wrong about countenancing just one more object—namely, the vague object Tibbles?"

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[13]Here, an indeterminate world should not be identified with a world which is indeterminate considered as a whole. As Rosen and Smith (2004) argued, a world can be determinate as a whole even if it contains some indeterminacy. Rather, an indeterminate world, here, is just a world which contains some indeterminacies.

[14]The main idea does not entail that precisificational worlds be *precise* partial worlds. There may be higher-order vagueness, since being a precisificational world may be vague, too. Our proposal is not a reductive theory of ontic vagueness. Therefore, our view is not in conflict with the intuitive idea that vague objects do not divide the world into three precise regions. Thanks to an anonymous referee who brought this point to our attention.

Second and more important, it is controversial whether Lewis's semantic view can explain everything that its rivals, namely, worldly vagueness views, can explain. Of course, the semantic view is committed only to one cat, but that is not all. There are two other intuitions that the semantic view fails to accommodate: first, the cat is not a precise object; it does not have a precise spatial boundary. Second, this vague object is the referent of the name "Tibbles." In a precise phrase, the kind of vagueness we are concerned with here is not referential but worldly. As mentioned before – a couple of paragraphs earlier – the proposal presented in this chapter can accommodate these intuitions. On the other hand, Lewis's semantic view characterizes vagueness as referential, according to which all of these intuitions are literally false. So, there are intuitions – supported by commonsense – that the semantic view cannot accommodate and which the present proposal, like any other worldly vagueness view, can. These considerations make the worldly vagueness view more palatable than Lewis's semantic view.

Third and most important, according to the present proposal, there are vagueness-related possibilities. Precisifications are not actual objects. Instead, they are possible objects that can be considered as precisifications of the actual object. For example, each Everestd is a possible object which can be considered as a precisification of Everest. In a precise statement:

(1) It is possible that there are some precise objects which are precisifications of Everest.

These possible precise objects are Everestds. There is no need to assume that Everestds are actual objects. Indeed, there is no need to assume.

(2) There are some precise objects that may be precisifications of Everest.

The present proposal is only committed to (1) rather than (2). And it is controversial whether (1) entails (2). Indeed, "if (1) then (2)" is equivalent to an instance of the restricted Barcan formula in modal logic: $\Diamond\exists x(\mathrm{F}x \wedge \mathrm{G}x)\rightarrow\exists x(\mathrm{F}x \wedge \Diamond\mathrm{G}x)$. There are, however, many intuitive counterexamples to the restricted Barcan formula: it is plausible that possibly some bread can satiate anyone; but it is hard to accept that some bread possibly can satiate anyone. It is safe to accept (1) and deny (2). So, we can assume that precisifications are only possible objects, not actual ones.

In fact, what is true about precisifications are some counterfactual conditionals. Any precisification of Everest is such that the following is true of it: Were it an actual object, any point which is (not) actually on Everest would (not) be on it. It is not our intention here to go into the details of a theory of counterfactuals. Perhaps the best theory of counterfactuals uses possible world semantics or not. Perhaps a possible world semantics uses the notion of possible object or not. These issues are not important here. What is important for the present proposal is that it need not presume the existence of precisifications in the actual world.

One might object here that even if this line of argument is sound, there may be other reasons for accepting precisifications as actual objects, such as arguments from unrestricted mereological composition. Interestingly, Lewis himself is one of those who argued in this way. In 1986, he says, "I claim that mereological composition is unrestricted: ... whenever there are some things, no matter how disparate and unrelated, there is something composed of just those things" (p. 211). And the reason for such an unintuitive claim, he believes, is that "[i]t is a vague matter whether a given class satisfies our intuitive *desiderata* for composition" (p. 212). It is not the purpose of this chapter to assess Lewis's argument for unrestricted mereological composition. Rather, the related problem, as far as the objection goes, is that whenever one is committed to unrestricted mereological composition, one is already committed to precisifications, and the claim that they are not actual is unwarranted.

An obvious response is at hand: there is no logical connection between the problem of composition and the problem of fuzzy boundary. The former is about the part-whole relation and asks whether any collection of material objects forms a whole of which they are its parts. On the other hand, the latter is about the region that an object occupies and asks whether that region is precise or not. Maybe one accepts that there are vague objects without accepting that mereological composition is unrestricted. Insofar as the present issue is vague objects and fuzzy boundaries, considerations about composition are irrelevant.

Therefore, although the present proposal is committed to vague objects, there is no need for it to be committed to precisifications as actual objects: either they are merely possible objects or there is no need to assume them as existent.[15] That is, properly understood, the present proposal is much more parsimonious than Lewis's semantic view. None of the precise objects which Lewis's semantic is committed to is in the ontology of the present proposal. And if being parsimonious is a criterion in virtue of which one should prefer a theory over another, the present worldly proposal should be preferred.

Before closing this section, a contrast may help clarify the present proposal. The present proposal is not the only one that uses the supervaluation technique to model worldly vagueness. Akiba (2004) also attempts to do this. Here is a rough exposition. It is intuitive that the material world has three spatial dimensions. By analogy, four-dimensionalists may claim that there is another dimension, a temporal dimension, such that each material object extends over temporal, three-dimensional worlds. Again by analogy, possible-world realists may claim that there is a fifth dimension, a modal dimension, such that any material object extends over modal, four-dimensional worlds. Now, by analogy, Akiba (2004) suggests that "the entire world has another dimension: the precisificational dimension." Then, "[a]nalogously, every material object extends over precisified worlds." A vague object, then, is "a sort of 'worm' or 'sausage'" and any precisification is a "slice" of it. He dubbed this view of ontic vagueness the "modal view."

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[15]Here, Lewis might object that, based on his view, the best theory of counterfactuals must assume possible objects and based on his theory of modality, any possible object exists on a par with actual objects. So, since precisifications are possible, they exist. And the semantic view turns out to be more parsimonious again. Of course, he can argue this line. The objection, however, may have some force only if his theory of modality is true. Currently, his theory of modality is hard to swallow.

Akiba's view can be better understood in contrast with Williams (2008a) and the present proposal, concerning the relation between a vague object and its precisifications. Williams' view is that a vague object is something indeterminate between its precisifications. The view held in the present proposal is that it is an object which has many possible precisifications. Akiba's view is that it is, in some sense, an aggregation of its precisifications. For example, the first considers Everest as *indeterminate* between Everestds; the second considers Everest as an object for which there *can be complete precisifications* as Everestds; the third considers Everest as a worm whose *slices* are Everestds. Therefore, Akiba's proposal, like the present proposal and unlike that of Williams, is a kind of worldly view.

These three views are also different with respect to ontological commitment. The ontic referential view (like the semantic view) is only committed to one sort of object in its ontology of vague objects, i.e., precisifications. Vague objects are among these precisifications, though it is indeterminate which of them they are. The present view, too, assumes only one sort of object in its ontology of vague objects, i.e., vague objects. All precisifications are, at most, merely possible. On the other hand, the modal view is committed not only to vague objects but also to their precisifications: any precisification is as real as the vague object of which it is a precisification.

Together with these contrasts, there are a few considerations that support the claim that the present proposal is in a better position to accommodate ontic vagueness than its rivals. First and minor: as far as vague ontology is considered, the present view, like the referential view, is committed to only one category of objects, i.e., vague objects. It is also committed to much fewer objects than the other two views, because there is no precisification in the ontology; precisifications are, at most, merely possible. In short, if parsimoniousness is considered as a criterion for judging the value of an ontology, the present view should be preferred over its two rivals.

Second and more substantial: the main and common problem with the ontic and semantic referential views, as discussed above, was that they could not accommodate commonsense. They could not accept the existence of ordinary vague objects. Akiba, himself, admitted this objection when he said that based on "Lewis's ontology ... there is no cat on the mat, and there are indeed no ordinary objects in the world" (2004, p. 419). Now, the modal view, of course, does not have this problem, since it is a variety of worldly views. The main problem with modal view, rather, is that it is associated with a highly substantial metaphysics that is hard to swallow. At least, it is as unbelievable as its temporal and modal origins. Notice that this is not to be considered as a knockdown argument against the modal view, since unbelievability does not entail falsity. The point, rather, is that, as far as commonsense and Geach's paradox are concerned, there is no need for such a substantial metaphysics. The present view can accommodate commonsensical considerations and resolve the paradox, with a much "lighter" metaphysics. Therefore, it seems that the present proposal is in a better position to accommodate ontic vagueness.

So far, we have outlined an idea according to which vagueness lies not in the reference relation between language and the world, but rather in the world itself. In the following section we provide some considerations on vague identity. And in the final section we turn to Evans's argument.

11.5 Some Notes on Vague Identity

Is there any such thing as genuine vague identity? Regardless of the answer, it is not the question under consideration in this chapter. As in the previous sections, in which the question was whether vague ontology is intelligible, a more relevant question here is: is vague identity intelligible? This question can be answered affirmatively if there is an example of vague identity that can be explained worldly. Is there any such example?

In the referential view of ontic vagueness, any vague object is indeterminately identical to its precisifications. For example, it is indeterminate whether Everest is identical to Everestd. However, in the outline of the idea, it has been implied that vague objects and their precisifications are not identical, since (1) Everest is a vague object with a fuzzy boundary and the other is not, and (2) Everest is an object in the actual world and its precisifications are at most merely possible objects. Thus, Everest and Everestd are not identical. In fact, they are definitely not identical. Such is the case in any worldly view. If vagueness is worldly, no vague object can be identical with any precise one, since one is vague and the other is not. Therefore, no vague object is identical with its precisifications.

If vague objects and their precisifications do not provide material for vague identity, are there any plausible examples? Akiba (2004) argued that there is no genuine notion of vague identity according to his modal view: "The crucial point is that the modal view makes a distinction between identity and coincidence .... What may be indeterminate if [a name] refers to a vague object, according to the modal view, is a statement of coincidence, not identity. Vague objects do not have vague identity. Thus the vague object Mt. Everest coincides with different precise areas in different precisified worlds; it is not identical with any of them. Again, coincidence is relative to worlds, and coincidence in a world is indiscernibility in that world, whereas identity implies indiscernibility simpliciter, indiscernibility throughout the worlds" (2004). The idea seems to be that if two objects coincide in every precisificational world, they are identical, otherwise nonidentical. So, there is no room for indeterminacy of identity. Of course, this line of argument has some degree of plausibility. But this is only the case if one agrees with Akiba that there is a precisificational dimension and that the precisifications are, somehow, slices of vague objects. The argument, nevertheless, has no force outside of his theory.[16]

[16]However, it does reveal an important point: there is no necessary entailment from vague object to vague identity.

Is there any way that vague identity can be accepted based on the proposal presented in this chapter? It seems that there is. Suppose *a* and *b* are vague objects. Also, suppose some of their precisifications are identical and some not. What could one say about their identity? In fact, they are such that they can be precisified as the same object and also as different objects. Thus, facts in the actual world do not determine whether they should be considered as the same. It seems highly plausible and intuitive to say that it is indeterminate whether they are, in fact, identical. An example has already been provided by Shoemaker (1984): "If … the vagueness of the term 'building' makes it indeterminate whether Alpha Hall and Beta Hall should count as parts of the same building (they are connected by a rather flimsy covered walkway) it may be indeterminate whether the identity statement 'the building in which Smith is lecturing is the same as the building in which Jones is lecturing' is true, if Smith is lecturing in Alpha Hall and Jones is lecturing in Beta Hall" (Quoted from Cook 1986).

Of course, the vagueness of identity in this example can be explained in various ways. Referential semantic supervaluationists may say that there is a semantic indecision.[17] On the other hand, referential ontic supervaluationists may say that there are many possible worlds that account for reality. Akiba would say that they are not identical. From the standpoint of the present proposal, as a kind of worldly vagueness, one could say that the two names refer to vague objects and it is indeterminate whether they are identical. It does not matter which of these options is preferable in this example. The referential view may be right, or it may not. What matters here is that the type of example presented by Shoemaker is the only type that potentially can be explained as worldly vagueness of identity. The common feature of the type is that the terms flanking the identity sign are vague. This intuitively justifies the assertion that worldly vagueness plays a role in the indeterminacy of identity. This is all that is needed to defend the intelligibility of vague identity based on a view that is committed to vagueness in the world.

Now, since the present proposal admits vague identity, inevitably we must confront it with Evans' argument, which is what we do in the next section.

11.6 Evans' Argument

Up until this point, the aim of this chapter was to propose a systematic conception of vague ontology. Here, we turn to the second objective of this chapter: to defend vague ontology against the charge of inconsistency. The most famous argument against vague ontology is provided by Evans (1978). In the first part of his paper, Evans argues that there is a connection between vague identity and vague object. He then goes on to argue against the consistency of vague identity. We will not go

[17] This is Shoemaker's own preference as ascribed in Cook (1986).

into details of the considerations presented in the first part of his paper, since we saw in the previous section that the modal view is a counterexample. Also, even if Evans is right and there is such connection, if the second part of his argument against vague identity fails, the whole argument against vague ontology will lose its supposed force. Thus, in the following paragraphs, it will be argued that proponents of vague ontology are not obligated to accept Evans' conclusion.

Let us begin with a review of the argument. In arguing against vague identity, Evans assumes, for the sake of contradiction, that there is some vague identity: "Let 'a' and 'b' be singular terms such that the sentence '$a = b$'" is of indeterminate truth-value, and let us allow for the expression of the idea of indeterminacy by the sentential operator "∇." Then we have:

(1) $\nabla(a = b)$

(1) reports *a* fact about *b* which we may express by ascribing to it the property "$\lambda x[\nabla(x = a)]$"

(2) $\lambda x[\nabla(x = a)]b$

But we have

(3) $\sim \nabla(a = a)$

and hence,

(4) $\sim \lambda x[\nabla(x = a)]a$

But by Leibniz's Law, we may derive from (2) and (4):

(5) $\sim (a = b)$

contradicting the assumption, with which we began, that the identity statement "$a = b$" is of indeterminate truth value"[18] (1978).

There are many options available to proponents of ontic vagueness to avoid the inconsistency, because, in the argument, there are premises other than indeterminacy of identity, as well as a few logical steps that can be rejected. Here only one of them – which is related to the proposed worldly view of vagueness – will be considered: the inference from (1) to (2).

Rejecting the first step of the argument is not in itself a new idea. Indeed, it is the natural choice of referential vagueness theorists such as Lewis (1988) and Williams (2008a). The idea is that since there may be no unique object which is the referent of the name "b," one cannot infer from "it is indeterminate whether $a = b$" that "b is such that is has the property being indeterminately identical with a."[19] With the proposal outlined above, however, this is not the way out of the inconsistency

[18] After this stage, Evans complements his argument with a modal part. We are not concerned with it here, since the argument, as it will be argued, fails before it.

[19] This is essentially what Lewis (1988), Garrett (1988), Noonan (1982), Thomason (1982), Rasmussen (1986), Williams (2008a), Barnes (2009), and possibly many others have in mind.

forced by Evans' argument, since it works only if vagueness is referential and this is not the preferred view in this chapter. So, let us see how the proposed worldly view can reject the inference from (1) to (2).

In his line of argumentation from (1) to (2), Evans claims, without any further justification, that "(1) reports *a* fact about *b* which we may express by ascribing to it the property '$\lambda x[\nabla(x = a)]$.'" Why? (1) only reports that it is indeterminate whether *a* is identical with *b*. From this, one can only conclude that it is indeterminate whether *b* has the property being identical with *a*, not that *b* has the property indeterminately being identical with *a*. Formally, $\nabla(a = b)$ only entails $\nabla(\lambda x[x = a]b)$. It does not entail $\lambda x[\nabla(x = a)]b$. The adherent of vagueness in the world is not obligated to assume that if *b* indeterminately has the property being identical with *a*, there must be a higher-level property, i.e., the property indeterminately being identical with *a*, that *b* has.

Therefore, Evans' argument can be blocked in a natural way: if there is indeterminacy, there will be no fact of the matter. This is in accordance with the view that is proposed and defended in this chapter, since if there is an indeterminate identity, there should be a possible way that they can be precisified as the same object, and a possible way that they can be precisified as different objects. So, based on determinate facts in the actual world, there is no fact that determines their identity or nonidentity: it is indeterminate whether they are identical. Thus, the proposed view of vagueness is immune from inconsistency as far as Evans' argument is concerned.[20]

11.7 Conclusion

A commonsensical perspective on the world implies that (1) there are vague objects in the world, (2) such vague objects possess some vague properties, and (3), in general, the world we live in is vague. It has been proposed that such a naïve perspective can be conceived systematically. The proposed view is an ontic reading of vagueness as incompleteness. The view adopts a worldly view of vagueness that uses supervaluationist technique: precisifications of a vague object are possible objects that, were they actual, every determinate fact about the vague object would be true of them. It has been contrasted with the referential and worldly views in the supervaluationist framework and argued that it is more effective at accommodating metaphysical and commonsensical considerations. There is a natural way to understand what it is for an identity to be vague: simply, there is no fact of the matter. Hence, Evans' argument fails in the first step.

[20]This objection to Evans's argument may seem similar Lowe's (1994). Even if there are formal similarities between them, the main difference is that in Lowe (1994), the counter-example is based on a physical theory, but here, it is commonsensically motivated.

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Chapter 12
What Could Vague Objects Possibly Be?

Dan López de Sa

Nothing. Or at least, this is what some prominent defenders of alternative conceptions of the nature of vagueness seem to hold:

> The only intelligible account of vagueness locates it in our thought and language. The reason it's vague where the outback begins is not that there's this thing, the outback, with imprecise borders; rather there are many things, with different borders, and nobody has been fool enough to try to enforce a choice of one of them as the official referent of the word 'outback.' Vagueness is semantic indecision. (Lewis 1986, 213)

According to the view of vagueness as semantic indecision, whatever it is that in the thoughts, experiences, and practices of language users determines the meaning of vague expressions, it fails to determine any one single entity as referent from a given range of equally natural ("precise") candidates. According to a view where there is vagueness in rebus, by contrast, some objects can themselves be vague. And this is the view that is often accused of lacking appropriate motivation—and even intelligibility:

> [The dualism] of vague objects and their precisifications is unparsimonious and unnecessary . . . Semantic indecision will suffice to explain the phenomenon of vagueness. We need no vague objects. Further, I doubt that I have any correct conception of a vague object. (Lewis 1993, 27)

In recent work, Elizabeth Barnes and Robbie Williams (Barnes 2010; Barnes and Williams 2011a; Williams 2008 *inter alia*) have attempted to vindicate the

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intelligibility of the notion and to provide a conception of what vague objects could possibly be.[1]

The most salient objection to the possibility of vague objects is Evans's. Suppose that there being vagueness in rebus requires that there be indeterminate identity statements whose indeterminacy does not result from semantic indecision in any of the constituent expressions. Then Evans's infamous argument (1978) seems to show that the view would in fact be contradictory, provided that substantial—controversial, plausible—background assumptions are made. Most responses take one of the following forms: to deny that ontic vagueness requires that there be relevant indeterminate identity statements or to reject some of the background assumptions needed for Evans's argument to go through. Williams (2008) claims that even if all these assumptions are in place, one can resist the argument by exploiting the indeterminacy in reference that some expressions may exhibit due to ontic vagueness as opposed to semantic indecision.

In what follows I argue that negative characterizations of vague objects such as Barnes's (2010) are satisfactory in so far as they go, but precisely due to their negative nature, they do not suffice to vindicate the intelligibility of the notion characterized (Sect. 12.1). Furthermore, and following Eklund (2011), I argue that the arguments of Barnes and Williams (2011a) do not advance the dialectical situation with respect to this (Sect. 12.2), and I offer some reasons to find their rejoinder (Barnes and Williams 2011b) wanting (Sect. 12.3).

In the second half of the chapter, I summarize Evans's argument and its significance (Sect. 12.4). I then voice some concerns about the particular response offered by Williams (2008) (Sect. 12.5), and I explore a more general worry in connection with the discussion in Barnes and Williams (2009) (Sect. 12.6).

I conclude that those initially sympathetic to the traditional misgivings regarding the intelligibility of the notion should remain unmoved: there still seems to be nothing that vague objects could possibly be.

12.1 Negative Characterizations of Ontic Vagueness

According to the view of vagueness as semantic indecision, the source of the vagueness in statements involving the outback is the semantic indeterminacy of the word "outback" between various, equally natural, "precise" candidate referents.

What could it possibly be for *the outback itself* to be vague? One natural, straightforward thought seems to be: for the vagueness in some of the statements in

[1]A note on terminology. I will be focusing on the issue concerning vague objects and will accordingly speak of *ontic* vagueness and vagueness *in rebus*. Arguably, the issue of *metaphysical* vagueness might be more general, if the vagueness in question could have other metaphysical sources. Also, I will be taking *vagueness* to be a particular variety of the more general phenomenon of *indeterminacy*, characteristically manifested in sorites-susceptibility. See below for further discussion concerning ontic vagueness vs. metaphysical indeterminacy.

question *not* to have its source in such semantic indecision, but rather in something *else*—"the outback itself."

Suitably generalized, this seems to constitute a common working characterization of ontic vagueness: vagueness that does *not* issue from semantic indecision.[2]

Consider Katherine Hawley's:

> When I say that the indeterminacy of some utterance is ontic I will mean that the indeterminacy is not a consequence of semantic indecision in the component terms of the utterance. (Hawley 2001, 105)

This is, in essence, very similar to the proposal advanced by Barnes herself:

> Sentence S is ontically vague iff: were all representational content precisified, there is an admissible precisification of S such that according to that precisification the sentence would still be non-epistemically indeterminate in a way that is Sorites-susceptible. (Barnes 2010, 604)[3]

As Hawley and Barnes would themselves admit, these are just *negative* characterizations of vagueness in rebus. That doesn't make them unsuitable as characterizations, of course. But by themselves they don't suffice to demonstrate the intelligibility of the notion to those who doubt there is something vague objects could possibly be.

To illustrate, consider the following. Suppose one is (reasonably) skeptical as to whether the notion of a married bachelor is ultimately intelligible. In order to nonetheless discuss this notion, the following negative characterization might well do as a working definition:

> A man is a *married bachelor* iff he is a bachelor but *not* unmarried.

But adequate as this definition may be for the stated purpose, it quite clearly does not suffice to vindicate the intelligibility of the notion characterized. If one lacked any coherent notion of a married bachelor—as one certainly should—such a negative characterization would not provide one. There is nothing that a married bachelor could possibly be—the notion of a married bachelor is ultimately unintelligible, even given an appropriate negative characterization.

Likewise for the case at hand. The charge leveled by skeptics like Lewis against the intelligibility of ontic vagueness seems to be of precisely the same nature. Perhaps there is a deflated sense of understanding in which one can understand what it would be for an object to be vague, just as one can understand what it would be for a bachelor to be married. After all, "There are vague objects" and "There are married bachelors" are relevantly different from "There are xajsoif achndwxadw" and "There are slithy toves." This is arguably related to the availability of negative characterizations of the sort under consideration. But there is also clearly a

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[2]Nor from (vagueness-characteristic) ignorance. See discussion below.

[3]Barnes (2010, fn. 25) suggests that her proposal improves on Hawley's by allowing "for the possibility of 'mixed cases'—cases where the vagueness in question is, e.g., a mixture of semantic and ontic." As I see it, Hawley's proposal can be read, or reinterpreted, as offering a characterization of what it would be for vagueness to be *at least partly* ontic.

sense in which the notion of a married bachelor nonetheless remains ultimately unintelligible. And this is the sense in which skeptics like Lewis contend—as I read them—that the notion of a vague object is also ultimately unintelligible. Thus, the existence of appropriate negative characterizations of vagueness in rebus is compatible with the notion lacking ultimate intelligibility.

As indicated, this much seems to be common ground, since neither Hawley nor Barnes suggests otherwise. Indeed, Barnes and Williams explicitly concede this point:

> The definition [of (Barnes 2010)] isn't meant to make sense of metaphysical vagueness or render it 'kosher'—you can agree that the definition is extensionally adequate while still being strongly skeptical about the very idea of metaphysical vagueness. (Barnes and Williams 2011b, fn. 8)

Thus, negative characterizations of ontic vagueness, adequate as they may be for certain purposes, do not suffice per se to vindicate the intelligibility of the notion.

12.2 Ontic Vagueness Versus "Metaphysical Ambiguity"

On the face of it, however, the very same point seems to apply to the characterization offered by Barnes and Williams themselves—contrary, this time, to what they *do* seem to suggest:

> One common complaint among those sceptical of metaphysical indeterminacy is that they cannot *understand* the notion, or that they suspect it *makes no sense* . . . [We] argue that our opponents *can* make sense of what we're saying . . . All parties should admit that they have a grasp on a generic notion of indefiniteness (and related notions) as deployed in ordinary speech, and used informally in philosophy. This generic concept of indefiniteness is arguably all one needs to have a working understanding of our target notion. In particular, using it we can formulate the following biconditional:
>
>> it is metaphysically indeterminate whether p iff (1) it is indefinite whether p, and (2) the source of this indefiniteness is the non-representational world. (Barnes and Williams 2011a, 108)

It is not immediately obvious how this is supposed to relevantly differ from the previous sort of characterization. In the parody case, people certainly have a grasp of a generic concept of bachelorhood (and related notions) as deployed in ordinary speech, which can be involved in a negative characterization of the notion of a married bachelor along the lines suggested. As alluded to, perhaps there is an attenuated sense of understanding in which we can understand the characterized notion, but there is also a clear sense in which it is ultimately unintelligible. More to the point, all parties should indeed admit that they have a grasp of a generic notion of indefiniteness (and related notions) as deployed in ordinary speech and used informally in philosophy. But as we have just seen, negative characterizations of metaphysical vagueness, such as those submitted by Hawley and Barnes, do not suffice to demonstrate the intelligibility of this notion to those skeptical of it—as

Barnes and Williams themselves explicitly admit. So how is Barnes and Williams's proposal supposed to be relevantly different?

That it is *not* relevantly different is, in my view, forcefully argued by Matti Eklund via the following analogy—the inspiration for my example of married bachelors:

> [(Barnes and Williams 2011a)] argue that we possess a generic notion of indeterminacy . . . Metaphysical indeterminacy is then just indeterminacy with a metaphysical source, and hence is neither semantic nor epistemic. Perhaps a way to represent Barnes and Williams's stance is: "What part of metaphysically indeterminate don't you understand? Surely you understand 'indeterminate,' for you have a generic notion of indeterminacy. And surely you understand what it is for the source of a phenomenon to be metaphysical."
>
> But suppose a theorist—BW*—proposes that the world is *metaphysically ambiguous* (or *metaphysically context-sensitive*, etc.). When, bewildered, we ask what that is supposed to mean, BW* defends the intelligibility of metaphysical ambiguity by saying that surely we know what it is for something to be ambiguous and surely we know what it is for the source of a phenomenon to be metaphysical. So—what part of metaphysical ambiguity don't we understand? BW*'s defense of his idea shouldn't convince. Why should we be any more convinced by what Barnes and Williams say? (Eklund 2011, 152)

There is indeed a sense in which all parties should admit, as Barnes and Williams contend, that they have a grasp of a generic notion of indefiniteness (and related notions) as deployed in ordinary speech and used informally in philosophy. In that sense, clearly people also have a grasp of generic notions of ambiguity and context-sensitivity (and related notions) as deployed in ordinary speech and used informally in philosophy. Now, ambiguity and context-sensitivity are clearly semantic phenomena and have their sources in features of representational reality. One can negatively characterize a notion via one of these generic notions and contend that its subject has a different, metaphysical source, in the envisaged manner. But appropriate as this may be for some purposes, it would fail to vindicate the ultimate intelligibility of the notion negatively characterized in this way—be that the notion of metaphysical ambiguity or metaphysical context-sensitivity. Saying that metaphysical ambiguity is ambiguity with a metaphysical as opposed to a semantic source may be good enough in so far as it goes, but ultimately it fails to make sense of what it would be for things to be metaphysically ambiguous.

Likewise, again, for the case at hand. According to skeptics such as Lewis—as I read them—vagueness is a semantic phenomenon in exactly the same sense in which ambiguity and context-sensitivity are. Hence the charge that ontic vagueness is ultimately unintelligible. Saying that metaphysical vagueness is vagueness with a metaphysical as opposed to a semantic source may be good enough in so far as it goes—but ultimately it fails to make sense of what it would be for things to be metaphysically vague.

12.3 Ontic Vagueness Versus Metaphysical Indeterminacy

In their rejoinder to Eklund, Barnes and Williams seem to agree that there should be something in their proposal to relevantly distinguish it from Eklund's parody:

> surely if our defense allows you to make sense of metaphysical ambiguity then something's gone wrong. (Barnes and Williams 2011b, 175)

But what would this be? In other words, why would Barnes and Williams think that their (2011a) proposal succeeds as an intelligibility-vindicating project while the similar-sounding one of Barnes (2010), as they admit, would not?

I think that the key to answering this is hinted at in a remark they make almost in passing:

> Here is one [model out of many that could be provided for the relationship between the generic concept and the more specific ones] we find congenial: we should characterize generic indefiniteness via its conceptual or functional role, consisting perhaps of the characteristic attitudes and hedged responses that indefiniteness prompts. Anything that fulfills this conceptual role will (prima facie) count as a case of indefiniteness. But, like any role-functional concept, it's open that the concept can be multiply realized. (Barnes and Williams 2011a, 110)

Now, if a given generic concept is "functional" in the relevant sense, and allows for "multiple realization" in the relevant sense, then arguably negatively characterizing a notion by claiming that it is "realized" some way different from another will go at least some way towards vindicating its intelligibility. Such multiple realizability is absent in the contrast cases of married bachelors and metaphysical ambiguity.[4] And the claim that the generic concept of indefiniteness (or any similar concept) is multiply realizable was absent from the merely negative characterizations that, as conceded, didn't suffice to vindicate the intelligibility of metaphysical vagueness. Thus, I take it, this may turn out to be the crucial feature which according to Barnes and Williams makes their proposal suitably different—in which case the envisaged model is not only one that they find "congenial" but rather one they are actually committed to, if their proposal is to improve on the previous negative characterizations in the respect that concerns us here.[5]

Thus *if* vagueness were like this—a phenomenon that could in principle occur in ways other than as a result of semantic indecision—*then* the envisaged negative characterization would have some claim to vindicate the intelligibility of ontic vagueness, after all.

Now, should we believe that vagueness is indeed like this? Some may think that we should—as witnessed by the availability (and, in certain circles, even popularity) of views about the nature of vagueness alternative to that of vagueness as semantic indecision—notably, *epistemicism*. However, I think otherwise, for at least the following two reasons.

First, as argued by Eklund, it is controversial, in a sense that clearly matters for the present point, whether epistemicism actually accounts for what we took to be

[4]This is compatible with there being a sense in which (generic) ambiguity can be syntactically or lexically "realized."

[5]The accuracy of this attribution is perhaps reinforced by their remark: "*so long as* something along these lines ultimately works, our use of the generic notion in characterizing [metaphysical indeterminacy] will be legitimate" (2011a, 110, my emphasis).

the relevant phenomenon, rather than attempting to explain appearances away by invoking a certain characteristic *alternative*, epistemic phenomenon:

> it is far from obvious that the notion of epistemic indeterminacy is in good standing, or that it is another species of the genus indeterminacy. To be sure, epistemicists writing about vagueness tend to argue that they do not deny that vagueness is bound up with indeterminacy, but rather only understand the indeterminacy to be epistemic in nature, a matter of ignorance. But despite the epistemicists' protests, one may well think that what the epistemicist view really involves is that what we mistake for genuine indeterminacy is merely a certain kind of ignorance. If so, Barnes and Williams aren't better off than BW* is: in each case we have a phenomenon whose uncontroversial instances are semantic, and someone who claims that the phenomenon also has 'metaphysical' instances.[6] (Eklund 2011, 153)

Second, and as also argued by Eklund, the charge of unintelligibility leveled by skeptics such as Lewis concerns specifically ontic *vagueness*, as opposed to the more general case of metaphysical *indeterminacy*. Clearly indeterminacy is a more general phenomenon than that of vagueness—the latter characteristically manifested in *soriticality*—as witnessed by the arguable indeterminacy of "mass," "$\sqrt{-1}$," and the continuum hypothesis.[7] Maybe some issues concerning quantum physics or the open future provide reasons to think that some indeterminacy has a metaphysical source. But what skeptics such as Lewis contend is that the only intelligible account *of the indeterminacy that is vagueness* locates it in our thought and language. With respect to vagueness, the suggestion that we have a grasp of the notion ultimately independent of semantic indecision seems to be, the skeptic would claim, in no better standing than the corresponding suggestions concerning ambiguity and context-sensitivity.

With this in mind, let's consider Barnes and Williams's (2011b) rejoinder, with "vagueness" substituted for "indeterminacy":

> [E]ven if it turned out that, in fact, all [vagueness] is semantic indecision, we'd still have a generic *concept* of [vagueness]. We'd have this generic concept unless it turned out that it's *analytic* of [vagueness] that all [vagueness] is semantic indecision. That [vagueness] is *analytically* semantic seems much less plausible than the (still controversial) claim that all [vagueness] is semantic. (Barnes and Williams 2011b, 174)

[6] Barnes and Williams seem partly sensitive to some such possible misgivings and offer to accommodate them terminologically by reserving *indeterminacy* for the non-epistemic instances of a more general phenomenon of *indefiniteness*. Given this stipulation, it is certainly the case that epistemicism offers an account of indefiniteness, if not of indeterminacy. But due to the controversy just alluded to, it is controversial that the general phenomenon is not, after all, *disjunctive* in nature—see Eklund (2011, 153–4). (Alternatively, if one insists that "indefiniteness" is stipulated to stand for the "pre-theoretic," nondisjunctive phenomenon that is present in cases of semantic indecision (Barnes and Williams 2011b, 174–5), then the controversy does not allow one just to assume that there is epistemic indefiniteness proper, as opposed to characteristically epistemic alternative phenomena.)

[7] Vagueness can be characteristically manifested in soriticality even if some expressions turn out to be vague but not relevantly soritical: see Weatherson's "few children for an academic," or cases of nondegree combinatorial vagueness.

Now, there might be a sense in which it is "controversial" that all vagueness is semantic and perhaps even more so that this is "analytically" so—after all, there are competent philosophers who deny it. But this doesn't seem to matter for the present point. Barnes and Williams seem to concede this, as they add in a footnote:

> That [vagueness] isn't obviously semantic doesn't *rule out* its being analytically semantic (assuming that there can be non-obvious necessities)... (2011b, fn. 3)

But they continue:

> ...but an argument is required for why we should posit such non-obvious analyticity. (2011b, fn. 3)

On the face of it, however, this just seems to get the dialectical structure back to front—leaving aside epistemological worries about nonobvious analyticity. For remember that the task Barnes and Williams set themselves, as quoted above, is to *argue* that their opponents "can make sense" of the notion of ontic vagueness. But the latter remark amounts to a request that *the skeptics* justify their position. Barnes and Williams echo Lewis's point that

> any competent philosopher who does not understand something will take care not to understand anything else whereby it might be explained. (Lewis 1986, 203, fn. 5)

But crucially, in the present case nothing *else* whereby the notion might be explained has been provided yet. Rather, Barnes and Williams's rejoinder consists in the claim that *if* the skeptics' reasons for their skepticism turn out to be misguided, *then* we would have the materials for turning a negative characterization into something that could be used to vindicate the intelligibility of the characterized notion. Fair enough. But this point is quite dialectically ineffective against the skeptics themselves, I daresay.

12.4 Evans Against Indeterminate Identity

As I said, the most salient objection against the possibility of vague objects is Evans's. In a recent paper, Williams (2008) argues that even if the two main sets of assumptions are in place—concerning the significance of indeterminate identity vis-à-vis the issue as to whether there could be vague objects, and the underlying logic—one can resist the argument by exploiting the indeterminacy in reference that some expressions may exhibit due to ontic vagueness as opposed to semantic indecision.[8]

Evans suggested that vagueness in rebus would require that there be relevant indeterminate identity statements. This is why his argument against the latter bears on the issue of whether there could be vague objects.

[8] See Barnes (2009) for her own alternative, counterpart-theoretic response to Evans.

Sometimes people have contended that there may be ways for objects to be vague other than by figuring somehow in relevant indeterminate identity statements. Maybe vagueness in rebus consists in its being "metaphysically" indeterminate whether a given object instantiates a given property, or in its being "metaphysically" indeterminate whether the object exists, or in its being "metaphysically" indeterminate whether it has another given object as a part.

As Williams observes, however, there seem to be powerful arguments that these cases of metaphysical indeterminacy lead to indeterminacy in identity statements, provided again that substantial—controversial, plausible—background assumptions are also made, notably classical logic and extensional mereology.

Suppose, for instance, that it is indeterminate whether object a instantiates property F. Then it seems it would also be indeterminate whether the state of affairs of a's being F exists, or whether universal F itself exists, if a is the only candidate instance (adapted from Barnes 2005). Or suppose that it is indeterminate whether something a exists, and let b be the fusion of everything whatsoever and c be the fusion of everything whatsoever that is not identical to a. Then it seems it would be indeterminate whether b is identical to c (adapted from Hawley 2002). So indeterminacy in instantiation seems to lead to indeterminacy in existence, and indeterminacy in existence seems to lead to indeterminacy in identity statements. Or suppose it is indeterminate whether a has c as part, and let $a + c$ be the sum of a and c. Then it seems it would be indeterminate whether a is identical to $a + c$ (adapted from Weatherson 2003). So indeterminacy in parthood seems to lead to indeterminacy in identity statements.

Maybe one can resist these arguments, but it is not clear exactly how. In any case, there seems to be something going for the claim that vagueness in rebus requires indeterminate identity statements whose indeterminacy does not result from semantic indecision in any of the constituent expressions.

But there is a well-known difficulty in allowing for indeterminacy in such statements. As Williams aptly puts it, the core of Evans's argument against indeterminate identity statements is "disarming in its simplicity" (2008, 135):

(1) It is indeterminate whether a is identical to b.
(2) a has the property of being indeterminately identical to b.
(3) It is not indeterminate whether b is identical to b.
(4) b does not have the property of being indeterminately identical to b.
(5) Therefore, a is not identical to b.

(3) is taken to be self-evident, from (1) to (2) is property abstraction, from (3) to (4) is either a "generalized" form thereof or the contrapositive of property instantiation, and from (2) and (4) to (5) is the contrapositive of the indiscernibility of identicals.

As subsequent discussion has made clear (Noonan 1982; Lewis 1988), for the argument to go through, "*a*" and "*b*" must be determinate in reference. Otherwise, the step from (1) to (2) or from (3) to (4) would be blocked. This is why the argument allows for indeterminate identity *statements*, provided that some of the expressions are indeterminate in reference.

(Consider the following analogy:

(1′) It is contingent that *a* is identical to *b*.
(2′) *a* has the property of being contingently identical to *b*.
(3′) It is not contingent whether *b* is identical to *b*.
(4′) *b* does not have the property of being contingently identical to *b*.
(5′) Therefore, *a* is not identical to *b*.

For the argument to go through, "*a*" and "*b*" must be rigid. Otherwise, the step from (1′) to (2′) or from (3′) to (4′) would be blocked. That is why the argument allows for contingent identity statements, provided that some of the expressions are flexible.)

As Williams remarks, there are a number of substantial assumptions in the background, notably, classical logic; the legitimacy of ("abundant") property talk (or at least, of plural readings thereof along the lines of "*a* is one of the things that are indeterminately identical with *b*"); the indiscernibility of identicals; and, if a contradiction is to be derived, that if something is indeterminate, then it is determinate that it is indeterminate. Most responses to Evans's argument reject some of these. Williams's main point is that the argument can be resisted *even if one grants* all these other assumptions.

12.5 Semantic Indecision Versus Indeterminate Reference?

Vagueness in rebus requires indeterminate identity statements whose indeterminacy does not result from *semantic indecision* in any of the constituent expressions (or so we can assume). Evans's argument allows for indeterminate identity statements, provided that some of the constituent expressions are *indeterminate in reference*. What if the indeterminacy in reference of an expression could have a source other than semantic indecision? Williams's response to Evans consists in attempting to exploit a mismatch between these two notions:

> The stock examples of referentially indeterminate words ('mass,' 'the square root of minus one' and 'Kilimanjaro') are presented as cases where our reference-fixing procedures fail to isolate one amongst a range of candidate referents. Perhaps, even, there is some obstacle in principle to our doing so. These are cases of referential indeterminacy *in virtue of* semantic indecision.
>
> A different kind of case is possible. These are where as language-users we have done our part of the task, but because of worldly indeterminacy, we do not secure a determinate referent. (2008, 147)

The framework provided by Williams is this. Assume ersatzism about possible worlds, and let them be maximal precise world-properties:

> If reality is vague, then presumably it is vague which precise world property is instantiated. I shall define '*w* corresponds to reality' as '*w* is not determinately uninstantiated,' and I shall say that the world is an *actuality* when it corresponds to reality in this sense . . . Given all this, ontic indeterminacy surfaces in *multiple worlds' being actual* . . . A sentence will be true (*simpliciter*) if and only if it is true relative to all the actual worlds. (2008, 149)

According to Williams, this ontic indeterminacy gives rise to a kind of referential indeterminacy that protects indeterminate identity statements from Evans's result:

> Suppose a particular amoeba, Sue, splits into two 'daughter' amoebas, Sally and Sandy. After the fission, Sally wanders off to the west and Sandy to the east. What I shall defend is a description of the fission as one where
>
> (i) Sue survives past the fission.
> (ii) It is indeterminate whether Sue survives as Sally or as Sandy.
> (iii) This indeterminacy is a matter of ontic unsettledness, rather than of semantic indecision or epistemic limitations.
>
> Two relevant candidates to 'correspond' with [reality], therefore, are
>
> (a) a world where Sue survives as the amoeba who wanders off to the west after the fission (i.e., Sue survives under the name 'Sandy'); a new, distinct amoeba, Sally, is created at the fission and wanders off to the east.
> (b) a world where Sue survives as the amoeba who wanders off to the east after the fission (i.e., Sue survives under the name 'Sally'); a new, distinct amoeba, Sandy, is created at the fission and wanders off to the west.
>
> . . . The name 'Sue' suffers no referential indeterminacy. In each case, it refers to the surviving amoeba . . . Metaphysically, I am supposing that it is indeterminate where the surviving amoeba Sue is after the fission. Since I have introduced the names 'Sandy' and 'Sally' (in part) by pointing to an amoeba at a certain location, this ontic indeterminacy *induces* referential indeterminacy. The (ontologically based) referential indeterminacy produces a vague identity statement. 'Sue = Sandy' is true at one actual world, but false at another. So, overall, it is indeterminate in status. (2008, 151)

As presented, Williams's multiple actualities should be among other people's possible worlds. As such, they should be "metaphysically admissible," as it were. Let me explain. Suppose (as seems to me most plausible) that amoebas cannot survive fission—so that, in particular, Sue dies in the process of begetting two numerically distinct daughters: Sally, who wanders off to the west, and Sandy, who wanders off to the east. If amoebas are like this, then all the expressions are plausibly determinate in reference, and all (nontrivial) identity statements determinate in status (and false).

This is why in my view the example might not be the most felicitous. But never mind that: other entities do seem to have persistence conditions that allow for fission. Let's just assume amoebas are like them. On this assumption, it seems to me, the two scenarios considered still fall short of being possible worlds and, therefore, they cannot be "multiple actualities" in Williams's sense. For most plausibly, or so it seems to me, if amoebas could survive fission, then, in each case, all the daughters would be surviving amoebas—so "the" mother turns out to be several overlapping amoebas. How this should be described in more detail will depend on one's views on how things persist.[9] Maybe we have two temporally overlapping perduring worms. Or if amoebas survive fission and endure, then maybe we have two series of "mereologically essentialist" enduring *entia successiva* sharing coordinates prior to fission. Or maybe we have two enduring living organisms sharing the matter that constitutes them prior to fission. Or . . . In each case, there are two surviving amoebas. Given that "Sally" is stipulated to refer to the amoeba who wanders off to the west after the fission and "Sandy" the one who wanders off to the east, both expressions seem to be determinate in reference after all. What about "Sue"? Most plausibly, it seems to me, and assuming survival, "Sue" exhibits semantic indecision between Sally and Sandy, so we would not have a case that satisfies the description given by (i)–(iii) above.

Suppose, however, that we were told that "Sue" determinately refers to a vague object and that it is "metaphysically unsettled" whether this is Sally or Sandy. Now, one would think, the two "precise" overlapping things also exist—for more on this, see the discussion below. I have suggested that these are determinately referred to by "Sally" and "Sandy." If so, then it would be indeterminate whether Sue is identical to Sally, although both "Sue" and "Sally" are determinate in reference, against Evans's result. Suppose, however, that the stipulations on "Sally" and "Sandy" exclude this. Let me then introduce new expressions "Sally*" and "Sandy*" to determinately refer to the two envisaged overlapping "precise" entities. It is indeterminate whether Sue is identical to Sally*. But now both "Sue" and "Sally*" are determinate in reference, again against Evans's result.[10]

Perhaps Williams's thought is that if Evans's result is to be avoided, the supposition that "Sue" determinately refers to a vague object, and that it is "metaphysically unsettled" whether this is Sally or Sandy, needs to *exclude* the existence of the two "precise" overlapping things I have claimed would also exist, given that these could be determinately referred to, either by "Sally" and "Sandy" or by "Sally*" and "Sandy*". This points to a more general worry, which arises in a more general context.

[9]Or perhaps on one's choice of how to *word* claims regarding the persistence of things—if recent meta-metaphysical *indifferentism* with respect to the allegedly competing views on persistence turns out to be well taken.

[10]It could be said that I am implicitly assuming a "many" solution as opposed to a so-called "supervaluationist" solution to the problem of the many—both of which are compatible with the view of vagueness as semantic indecision. I defend this solution in López de Sa (2013). In any case, this seems to be a further complication of the amoeba example that is absent from more general cases concerning "thing" or "object," like the mereological example considered below.

12.6 "Precise" Entities Against Ontic Vagueness

Williams offers his amoeba example just for illustrative purposes, as the strategy generalizes to cover other interesting cases, notably in connection with vagueness in statements about parthood. A more general worry arises in the more general context. Let me focus on Barnes and Williams's (2009) response to Weatherson (2003).[11]

Suppose that it is indeterminate whether a certain electron Sparky is part of Mount Kilimanjaro, and assume that "is part of" is precise. As we saw above, assuming classical logic and extensional mereology, this seems to entail that it is indeterminate whether Kilimanjaro is identical to Kilimanjaro+, the sum of Kilimanjaro and Sparky. There is no problem for the view of vagueness as semantic indecision here, provided that "Kilimanjaro" is vague and Kilimanjaro+ is one of the many candidate referents. But the alternative view that "Kilimanjaro" determinately refers to a vague object seems to face Evans's result again, assuming that "Kilimanjaro+" determinately refers to the sum of Kilimanjaro and Sparky and hence is determinate in reference.

Or does it? Barnes and Williams point out that "Kilimanjaro+" could be indeterminate in reference for reasons other than semantic indecision—and compatibly with the stipulation, so that determinately Kilimanjaro+ is the sum of Kilimanjaro and Sparky:

> Here is one ontology of vague objects that makes 'K+' referentially indeterminate. There are two vague objects, *K*, and *dual-K*. For each x aside from Sparky, it is determinate that x is a part of K iff it is a part of dual-K. And further, Sparky is such that, determinately, it is part of K iff it is not part of dual-K . . .[12] Recall the way that 'K+' was introduced: to pick out that thing which was the sum of K and s. Granted the ontology just sketched, it will be determinate that something meets that description: it will be either K or dual-K, but which one it is turns on which contains s, and that is an indeterminate matter. Hence *it will be indeterminate which vague object 'K+' refers to*. (Barnes and Williams 2009, 181–2)

If those were the only relevant objects that existed, then there would be nothing that is determinately the sum of Kilimanjaro and Sparky—although determinately there is something that is the sum of Kilimanjaro and Sparky. But *surely* there are further things—in particular something that determinately has exactly the parts of Kilimanjaro together with Sparky. As before, one could contend this was the thing determinately referred to by "Kilimanjaro+"—so it is indeterminate whether Kilimanjaro is identical to Kilimanjaro+, against Evans's result. But never mind that; let's just introduce the new expression "Kilimanjaro+*" to determinately refer to it. Now, not only is it indeterminate whether Kilimanjaro is identical to Kilimanjaro+; it is also indeterminate whether Kilimanjaro is identical to Kilimanjaro+*. But both "Kilimanjaro" and "Kilimanjaro+*" are determinate in reference, again against Evans's result.

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[11] I am indebted here to Williams for a discussion of an earlier presentation of this argument on his blog *Theories'n Things* in July 2007.

[12] A further assumption, omitted here, will concern us below.

I just said that *surely* there are such further things, but Barnes and Williams's additional assumption is precisely that there are *not*:

> There are no objects which determinately have exactly the parts of K apart from s, and there are no objects which determinately have exactly the parts of K together with s. (Barnes and Williams 2009, 181)

As they conclude, given the other background assumptions,

> we cannot have a vague object *together with* a plenitude of precise objects. (Barnes and Williams 2009, 183)

But they continue:

> We suggest that here we reach stand-off: one who believes in a plenitude of precise objects has the resources to argue against the existence of a vague object "floating on top." But equally, one who believes in a plenitude of vague objects has the resources to argue against the existence of a precise object "floating underneath." Nothing we have as yet seen tells us whether the axe should fall on the vague objects, or on the precise objects. (Barnes and Williams 2009, 183)

As we saw, however, the notion of a vague object is not in as good standing as that of a "precise" object—contrary to what this verdict of a standoff suggests. That the existence of vague objects would *exclude* (given the assumptions) the existence of the "precise" ones may very well be a further reason to reject the notion of ontic vagueness as ultimately unintelligible.

12.7 Conclusion

Admittedly, the situation is dialectically delicate. Suppose the project of vindicating the intelligibility of the notion of ontic vagueness had succeeded, contrary to what I have suggested. Suppose that one worried whether, even then, Evans's argument provided a further independent objection to the possibility of intelligibly vague objects. *Then* exploiting the ways in which expressions could be indeterminate in reference as a result of such ontic vagueness as opposed to semantic indecision may well provide ingenious, unexpected responses to Evans's argument, which, unlike many other responses, succeed in granting the main controversial assumptions of the argument. This is by all means a remarkable and significant result.

I have argued, however, that those who doubt the intelligibility of the notion of ontic vagueness have not been given reasons to abandon their skepticism. To the extent that they regard Evans's argument as a further manifestation of the unintelligibility of the notion, they should judge the envisaged response, though ingenious, to be ultimately ineffective.[13]

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[13]Earlier versions were presented at the *Arché*, *LOGOS*, and *PERSP Metaphysics* Seminars, and in a *Vagueness Workshop* in Charmey. Thanks to participants in these events and to Ross Cameron, Pablo Cobreros, Aurélien Darbellay, Manuel García-Carpintero, John Hawthorne, John Horden,

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Part V
Vague Identity

Chapter 13
Some Comments on Evans's Proof

Brian Garrett

Gareth Evans's 'Can there be vague objects?' (Evans 1978) must surely win a medal for the shortest article to have generated the largest secondary literature. It is, however, a strange and problematic article, and, at many points, it is unclear why Evans proceeded in the way he did. Nonetheless there is gold in what Evans has to say. Properly reconstructed, we can secure Evans's desired conclusions: a cogent proof of the definiteness of identity and consequent vindication of one sense in which there cannot be vague objects.

Evans's paper is divided into three paragraphs. The first paragraph is informal and gives Evans's gloss on the idea of what it would be for the world to be vague and, more specifically, to contain vague objects. The second paragraph presents his formal proof of the determinacy of identity (i.e. of the impossibility of vague identity). The third paragraph provides, according to Evans, a supplement necessary to complete the proof. Evans does not explicitly say how he thinks the determinacy of identity implies the impossibility of vague objects, but it is plain enough what bridging principle he must have had in mind.

13.1 The First Paragraph

Evans writes:

> It is sometimes said that the world might itself *be* vague. Rather than vagueness being a deficiency in our mode of describing the world, it would then be a necessary feature of any true description of it. It is also said that amongst the statements which may not have a determinate truth value as a result of their vagueness are identity statements. Combining

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> these two views we would arrive at the idea that the world might contain certain objects about which it is a *fact* that they have fuzzy boundaries. But is this idea coherent?

There is clearly cognitive motion in this paragraph. But what exactly is the chain of reasoning? The thesis that the world itself is vague is taken to imply that vagueness is a necessary feature of any true description of reality.[1] Evans then mentions the view, held by some, that there may be identity statements which are indeterminate in truth value as a result of their vagueness. He next says that combining this view with the view that the world itself might be vague we arrive at the idea that there might be '... objects about which it is a *fact* that they have fuzzy boundaries.' This idea he wants to show to be incoherent.

How do these two views lead to this idea? Presumably the train of thought is this: if there were vague identity statements, the world itself would be vague, and vague in a quite specific way, namely, by containing objects about which it is a fact that they have fuzzy boundaries. Such objects are vague objects. Hence, there can be vague objects if and only if there can be vague identity statements. The point of the subsequent proof is to show that there cannot be vague identity statements and hence show that the world cannot be vague by virtue of containing vague objects.

One complication here concerns how Evans understands vague objects. Evans doesn't use the phrase 'vague objects' in his paper. Instead he talks of '... objects about which it is a *fact* that they have fuzzy boundaries.' We can call these fuzzy objects, i.e. objects whose spatial and/or temporal boundaries are fuzzy. So, it seems, for Evans vague objects are fuzzy objects. But how do fuzzy objects, so understood, connect up with the issue of whether there can be vague identity statements – as they must if Evans's paper is to make sense? Well, if X is a fuzzy object then there will be a range of terms 'A', 'B', 'C', etc., denoting objects which are plausible candidates for being X, such that it's vague whether $X = A$, vague whether $X = B$, etc.

A further complication concerns what Evans means by 'object'. One familiar use of 'object' contrasts objects (concrete or abstract) with, e.g. properties, events, processes, tropes, and facts. Another use of 'object' is more generic (as in 'object of thought' or 'object of reference'), meaning an entity which can belong to any ontological category. It seems that Evans must mean 'object' in the second sense. For identity statements can involve entities of any category, e.g. Tully = Cicero, water = H_2O, the waving of my hand = my greeting my friend, and the property of goodness = the property of maximising happiness. The essential vagueness of any such statement, by Evans's way of thinking, would imply that the world is vague, in virtue of containing vague entities of the relevant kind.

However, if we have captured the intent of Evans's first paragraph, a problem emerges that undermines the point and cogency of Evans's subsequent proof. For it seems plain (a) that there are vague identity statements, so any proof to the contrary

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[1] In his second sentence Evans implies that if the world is not vague, then vagueness is a 'deficiency' in our description of the world. This is tendentious. Those who deny the possibility of worldly vagueness do not have to see vagueness as any kind of linguistic deficiency (e.g. epistemicists such as Tim Williamson).

must be fallacious and (b) that such statements do not imply that the world is vague.[2] Thus imagine a sequence of pens, ranging in colour from black to white, such that all and only adjacent pens match exactly in colour. We have thus a seamless transition from black to white. Now consider the statement that my favourite pen is the first white pen in the sequence. This is a vague identity statement, yet – it is natural to think – it is vague not because reality is vague, but because it is vague which pen the definite description 'the first white pen in the sequence' picks out. Its being vague which pen the description picks out does not imply that it picks out something vague. That just looks like a fallacy.

It might be replied that this is a bad example. If definite descriptions are not singular terms, then 'my favourite pen is the first white pen in the sequence' is not an identity sentence, and so cannot express a vague identity statement. But the same point can be made using proper names. Consider a ship made of 100 planks. Call this ship 'Argon'. Suppose that 50 planks are quickly removed and replaced. Call the resulting ship, 'Boris'. Suppose we judge it to be vague whether Argon = Boris. For all Evans has said, this identity statement may be vague just because it is vague whether the name 'Argon' refers to Boris (and whether 'Boris' refers to Argon). Such trans-temporal referential indeterminacy implies no vagueness in reality.

With these points in mind, let us turn our attention to the proof in the second paragraph.

13.2 The Second Paragraph

> Let 'a' and 'b' be singular terms such that the sentence '$a = b$' is of indeterminate truth value, and let us allow for the expression of the idea of indeterminacy by the sentential operator '∇'.
> Then we have:
> (1) $\nabla(a = b)$
> (1) reports a fact about b which we may express by ascribing to it the property '$\widehat{x}[\nabla(x = a)]$':
> (2) $\widehat{x}\,[\nabla\,(x = a)]\,b$
> But we have
> (3) $\sim\nabla(a = a)$
> and hence,
> (4) $\sim\widehat{x}\,[\nabla\,(x = a)]\,a$
> But by Leibniz's Law, we may derive from (2) and (4):
> (5) $\sim(a = b)$
> contradicting the assumption, with which we began, that the identity statement '$a = b$' is of indeterminate truth value.[3]

[2]D Lewis (1988) suggests that Evans intended his proof to be fallacious. But it is hard to square this interpretation with the text.

[3]Apart from some comments in his final paragraph, Evans says nothing about the logical properties of 'Indefinitely' (∇). I assume that Evans intends ∇ to indicate absence of truth and falsity. So '∇P' means 'it's indefinite whether P' and is true just if P is neither true nor false. This assumption

In this paragraph Evans switches from talking about statements to talking about sentences, but I take this to be innocuous. Much more important is the fact that Evans lets '*a*' and '*b*' be singular terms. We have just seen that many English sentences of type-(1) are true – e.g. ∇ (Argon = Boris).

Further, in any case in which it is vague which object '*b*' singles out, the move from (1) to (2) will be invalid. If no object is determinately picked out by '*b*', there is no object of which one can go on to say 'it is vague whether *it* is *a*'. Evans's proof fails at its first step.[4]

This might seem bad news for Evans's proof, as indeed it is. But I think there is a sound argument in the offing. It is an argument for the determinacy of identity as a thesis in metaphysics, and (rightly) has nothing essentially to do with natural language identity sentences, statements or singular terms. To see the argument, let us remind ourselves of Saul Kripke's famous proof of the necessity of identity (Kripke 1971, p. 136).

13.3 Kripke's Proof

Kripke's proof of the necessity of identity is formulated using variables (not singular terms) and runs as follows:

(1) $(x)(y)\ ((x = y) \supset (\mathrm{F}x \supset \mathrm{F}y))$
(2) $(x) \Box (x = x)$
(3) $(x)(y)\ (x = y) \supset (\Box\ (x = x) \supset \Box\ (x = y))$
(4) $(x)(y)\ ((x = y) \supset \Box\ (x = y))$

Unlike Quine and others who regard quantified modal logic as incoherent, I regard this as a sound proof. From Leibniz's Law (1), together with (2) and (3) (the latter itself an instance of that law), we can validly infer (4).

I take (4) – the thesis of the necessity of identity – to be a thesis in metaphysics, not the philosophy of language. It tells us that it is impossible for the relation of identity to hold contingently. Since it uses variables and not singular terms, (4) has no direct implications for how we should think of natural language identity sentences. It does not, e.g. imply that all true identity sentences are necessarily true.

Kripke noticed that there happens to be a corollary of (4) in natural languages. In the case of, e.g. proper names 'A' and 'B', if A = B then necessarily A = B. That is, proper names are rigid designators. But the rigidity of proper names is not

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allows us to make sense of Evans's proof in a straightforward way. In that case, however, Evans's proof and the (1a)–(4a) proof below exclude indeterminate identity even when the source of the indeterminacy is something other than vagueness.

[4]Some also question premise (3). I have heard the reaction: 'But if **a** were a vague object, it would be indeterminate whether **a** = **a**'. However, I think (3) is hard to deny, and I agree with David Wiggins's response (Wiggins 1986, p. 175): even if vague, **a** is exactly the right object to mate with **a** in order to ensure a perfect case of identity.

implied by (4). (4) would still have been true even if Frege had been right about the semantics of proper names.

Now Evans could have presented an argument for the determinacy of identity exactly parallel to Kripke's argument.[5] If, following Evans in his third paragraph, we let 'Δ' indicate 'definitely', we can construct the following proof:

(1a) $(x)(y)\ ((x = y) \supset (\mathrm{F}x \supset \mathrm{F}y))$
(2a) $(x)\ \Delta\ (x = x)$
(3a) $(x)(y)\ (x = y) \supset (\Delta\ (x = x) \supset \Delta\ (x = y))$
(4a) $(x)(y)\ ((x = y) \supset \Delta\ (x = y))$

Here we have a proof of the determinacy (or definiteness) of identity. (4a) implies that it can never be an indeterminate matter whether the relation of numerical identity obtains. Hence, it is impossible for two objects to be such that it is indeterminate whether they are identical. Since (1a)–(3a) are all necessary truths, we can infer the necessary truth of (4a), and hence infer $\Box \sim (\exists x)(\exists y)\ \nabla\ (x = y)$.[6]

The thesis of the determinacy of identity does not imply that all identity sentences are determinate in truth value. What (4a) does imply, however, is that if an identity sentence is vague, that must be because one or both of its singular terms is an *imprecise designator*, where a designator is imprecise just if it is – or might be – vague which object it picks out.

Hence, if '$\mathrm{A} = \mathrm{B}$' is vague, that can only be because of the referential indeterminacy of one or both of 'A' and 'B'. The relation of identity cannot be a source of vagueness. Thus we get the following result. Just as Kripke holds that, for any pair of rigid designators, 'A' and 'B', if $\mathrm{A} = \mathrm{B}$, then necessarily $\mathrm{A} = \mathrm{B}$, so a defender of the definiteness of identity holds that, for any pair of precise designators, 'C' and 'D', if $\mathrm{C} = \mathrm{D}$, then definitely $\mathrm{C} = \mathrm{D}$.

Are there any precise designators in natural language? These are designators such that it is never vague which object they single out. Designators of objects which can undergo change seem to be ruled out.[7] Perhaps the only examples are names of abstract objects such as numbers (e.g. '2') or names of simple (partless) objects, if

[5]In a number of articles, Ken Akiba (2000a, b, 2004) has argued that Evans's proof is nothing but Kripke's proof of the necessity of identity in contrapositive.

[6]Why? If (4a) is true, it cannot have an indeterminate antecedent and false consequent. A case in which '$x = y$' is indeterminate would be a case in which (4a) had an indeterminate antecedent and a false consequent. Since (4a) is true, it follows that '$x = y$' can never be indeterminate in truth value.

[7]Even gerrymandered designators of concrete objects such as 'A-at-t1' do not count as precise. In cases of multiple personality or monstrous two-headed births, for example, it may well be indeterminate whether A at t1 = B at t1. What of an apparent diachronic or cross-temporal identity sentence such as 'C-at-t3 = D-at-t4'? On the four-dimensional view of ordinary continuants, the canonical truth condition of this sentence is, C-at-t3 is part of the same four-dimensional entity (ship, person, etc.) as D-at-t4. If it is vague whether C-at-t3 = D-at-t4, and both singular terms are precise, then the vagueness must lie with the part/whole relation (not with identity). But I take it there is nothing paradoxical in the possibility: $(\exists x)(\exists y)(\exists z)\ \nabla$ (x and y are both parts of z).

such there be. Thus, e.g. the identity sentences ‘$2^2 = 4$’ and ‘simple A = simple B’ are never indeterminate in truth value due to vagueness.

In sum, I am happy to regard the (1a)–(4a) proof as sound, and I think it gives Evans the conclusion he was after: a conclusion about the relation of identity and not (or not primarily) about identity sentences or statements.

13.4 The Third Paragraph

The third paragraph is puzzling, even if we grant, for the sake of argument, that Evans established (5). The paragraph runs:

If ‘Indefinitely’ and its dual ‘Definitely’ (‘Δ’) generate a modal logic as strong as S5, (1)–(4) and, presumably, Leibniz’s Law, may each be strengthened with a ‘Definitely’ prefix, enabling us to derive

(5′) $\Delta \sim (a = b)$
which is straightforwardly inconsistent with (1).

At the end of his second paragraph, Evans states that (5) ($\sim(a = b)$) contradicts (1) ($\nabla(a = b)$). But, in that case, the supplementation in the final paragraph is unnecessary.[8] If (1) implies (5), and (5) implies not (1), then (1) implies its own negation, and so must be false.

Admittedly, there is a way of reading (5) on which it does not contradict (1). If the negation in (5) is weak negation (i.e. ∼P is true if and only if P is either false or indeterminate), then (5) does not contradict (1). But there is no reason to think that the negation in (5) is anything other than strong negation (i.e. ∼P is true if and only if P is false). Indeed, since the negations in (3) and (4) are strong, presumably the negation in (5) is so too.

In addition, Evans suggests that ∇ and Δ are duals (analogous to ◊ and □, respectively) and that ∇ and Δ will generate a logic as strong as the modal system S5. Now S5 includes as axioms: $\Box P \rightarrow P$ and $\Diamond P \rightarrow \Box\Diamond P$. The analogous Evansian axioms would thus be $\Delta P \rightarrow P$ and $\nabla P \rightarrow \Delta\nabla P$.

The first axiom requires reading ΔP as ‘definite that P’ rather than ‘definite whether P’, for only the former implies P. But under that reading that ∇ and Δ are not duals. For example, $\sim\Delta P$ is clearly not equivalent to $\nabla \sim P$ (e.g. the former is consistent with $\Delta \sim P$ whilst the latter is not).

Second, on either reading of ‘definitely’, the axiom $\nabla A \rightarrow \Delta\nabla A$ is implausible since it conflicts with the possibility of higher-order vagueness. Just as, in the pen example above, there is no sharp dividing line between the black pens and the white pens, so there is no sharp dividing line between the black pens and the borderline (grey) pens. It can thus be vague whether some pen is a borderline case or not. If A exhibits second-order vagueness, then ‘∇A’ will be indeterminate in truth

[8] See Pelletier (1984).

value whilst '$\Delta\nabla A$' will be false. In that case, Evans's second axiom cannot be true (assuming that a true conditional cannot have an indeterminate antecedent and false consequent).

Finally, we have as theorems of S5: $P \rightarrow \Diamond P$ and $\Box P \rightarrow \Diamond P$. But obviously no logic of vagueness would want to endorse: $P \rightarrow \nabla P$ or $\Delta P \rightarrow \nabla P$. The logic of vagueness will not generate a logic as strong as S5.

13.5 Conclusion

I have tried to highlight some of the problems with Evans's article, but I have also indicated how, in my opinion, a proper defence of the determinacy of identity should run. How does the Kripke-inspired proof bear on the issue of vague objects? Presumably the thought would be that if it is impossible that $(\exists x)(\exists y) \nabla (x = y)$, then it follows that there cannot be objects such that it is vague which objects they are. This, it seems, yields one sense in which the world cannot contain vague objects.

What of the claim that, e.g. Everest is a vague object, in virtue of its lack of any precise spatial boundary? My inclination is to see the vagueness here as due to the vagueness of the sortal concept, *mountain*, and not to the world. If 'A', 'B', 'C', etc., denote mountainous regions in the vicinity of Everest, then it will be vague whether Everest = A, vague whether Everest = B, and so on. But, given the (1a)–(4a) proof, this must be due to the referential indeterminacy of one or more of the singular terms involved, and not to any vagueness in identity or objects.[9] Believers in ontic vagueness mistake features of our language and concepts for features of the world.[10]

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[9] Does the imprecision of 'Everest' mean, e.g. that it is not determinately true that Hillary climbed Everest? No; that is determinately true because it is determinately true that Hillary climbed A, determinately true that Hillary climbed B, and so on.

[10] Thanks to Peter Roeper, Daniel Nolan and an audience at ANU in October 2012 for useful comments. (After writing this chapter, I came across Barnes (2009) which presents an interesting counterpart-theoretic treatment of the determinacy/indeterminacy operators, according to which Evans's proof is invalid. Barnes's article requires careful study.)

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Chapter 14
Vague Existence Implies Vague Identity

David B. Hershenov

Most philosophers understand vagueness as due to semantic indecision. "On the vagueness-in-language approach, the world is fact-rich while the language is a work in progress. This is vagueness for fuzzy speakers in an exact world" (Salmon 2010). Add to their ranks the epistemicists who understand vagueness to be due to a "special kind of irremediable ignorance of borderline cases" (Williamson 2003) and the total will far exceed the number of philosophers who defend *de re* vagueness.[1] The latter maintain that vagueness is independent of how we know or represent the world, there being no fact of the matter about the way that the world itself is. "On the vagueness-in-the-world approach, the language is a finished product while the world is factually impoverished. This is vagueness for exact speakers in a fuzzy world" (2010). Of those philosophers who do defend *de re* or worldly vagueness, more are sympathetic to the view that the world contains vaguely existing objects than vaguely identical objects.[2] Such philosophers will accept that the world contains objects that neither determinately exist nor determinately fail to exist but will deny that there are any worldly objects that are vaguely identical to each other.[3]

[1] I am not claiming that these theories exhaust the range of options for dealing with vagueness nor that worldly vagueness can't occur alongside vagueness in language. See Sorensen (2012) for a survey of the options. See Merricks (2001) for a claim that there is not a distinct kind of vagueness due to semantic indecision.

[2] van Inwagen (1990), Parsons and Woodruff (1997), and Parsons (2001) are the exceptions, both defending vague existence and vague identity.

[3] Akiba (2004), Baker (2007, 121–41), Morreau (2002), Salmon (2010), Hershenov (2001), Smith (2005), and Tye (1990, 556; 2003, 154–63) accept vague existence, but all seven reject vague identity. Although I didn't assert it, I assumed that accepting vague existence didn't commit me to vague identity when I was writing my 2001 paper. Obviously, I no longer assume it.

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I am going to take issue with the claim that one can accept *de re* vague existence without *de re* vague identity. Whether we should accept such vague existence and vague identity is another matter – I think we shouldn't – but defending that claim is not my main concern here. I do record my attraction to some well-known arguments of Salmon and Evans against vague identity and add a little at the end of this chapter about how it seems impossible to wrap one's mind around the idea of beings that are indeterminately identical sort of possessing the same thoughts and feelings. However, my thesis is that one can't allow vague existence without accepting vague identity – at least if one rejects that there can be spatially coincident entities of the *same* kind. Thus I will show that far more philosophers are implicitly committed to vague identity than explicitly so committed. If they are committed to vague identity, then they also ought to accept vague identity. But if vague identity is impossible, philosophers should reject vague existence as well. And a surprising consequence is that if there is no vague identity, then the charge of arbitrariness leveled against epistemicism becomes less weighty. Arguments against vague identity (modulo independently reasonable principles) will entail there aren't vaguely existing entities or even determinately existing objects that indeterminately possess some parts.

14.1 Vague Objects: Vague Parts, Vague Existence, and Vague Identity

There are philosophers who claim the notion of vague object is unintelligible. They can't make sense of what it could be for an object to be vague rather than the vagueness residing in our ways of knowing or representing objects. Others will claim to understand reality containing vague objects as long as that doesn't involve any objects vaguely existing. Still other philosophers might think the notion of a vague object is intelligible, but there aren't any, in fact, it is impossible for there to be any such objects. They may find the concept intelligible, that is, they can understand it, but still insist it is incoherent, that is, it doesn't cohere with other truths they hold. And we will see that there are philosophers who think that it is even misleading to speak of vague objects and suggest we instead talk of vague states of affairs. I am concerned in this section to distinguish the different notions of vague objects, show how they are intelligible, and explain why some people think there are some kinds of worldly vagueness and not others. This will all serve to lay the groundwork for my claim in the next section that there can't be vaguely existing entities unless it is possible for there to be vaguely identical entities. If vague identity is impossible, then so is vague existence. Moreover, if there is no vague identity and vague existence, then there can't even be determinately existing things of which there is no fact of the matter whether they have some object as a part.

There is a need to first get clear about the distinctions between (1) determinately existing objects that vaguely possess parts, (2) objects whose existence is vague, and

(3) objects whose identity is vague.[4] An example of the first would be a rock at the base of a mountain that was borderline between being part of the mountain and part of the valley. An example of vague existence could be found in the distant past when that mountain was being formed and it was indeterminate whether after the crashing together of tectonic plates that the crumpled earth had been pushed sufficiently upwards enough to compose a mountain. A case of indeterminate identity could occur where it was vague whether there were two overlapping mountains or one mountain with two peaks.[5] All of the above may be called *vague objects.*

There are philosophers who just ignore distinctions between vague part possession, vague existence, and vague identity. Still another group of philosophers are split between whether any of the three instances of vagueness entail the others. Often those who claim vaguely existing entities entail vaguely identical entities don't fill in the details about why this is so. So their opponents are compelled to show that they can be kept apart. I will argue that their opponents are wrong, at least if they deny that the commitment to the possibility of vaguely existing entities entails commitment to the possibility of vaguely identical entities. If there are at times vaguely existing objects, it needn't be that they overlap and are vaguely identical to any other. However, the manner in which they retain and lose parts that makes their vague existence possible could be extended to scenarios where vague identity would be forced upon them.[6] And if there are objects with vague parts, they need not vaguely exist. However, it is possible that each can end up so because the relationship that allows determinate objects to possess some parts vaguely just differs in degree from when the objects vaguely exist. So if vague identity is incoherent, then we should avoid positing vague existence and even the apparently harmless determinately existing objects that posses just a few other parts in a vague manner.

Some philosophers (Evans 1978; Salmon 1981, 2002) offer proofs against the possibility of vague identity while others state that they find the notion of vague existence unintelligible (Lewis 1986, 212; Russell 1923; Dummett 1975, 314).[7]

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[4]See Williams (2008) for a survey of the different senses of vague object.

[5]I will explain below why I don't use the more common example of vague identity of there being various precise chunks of matter that are equally good candidates for being a mountain whose boundaries are vague.

[6]This assumes the world is one in which there is more than one composite object. But even in such a world, the only composite object would still have the possibility of being vaguely identical to another object.

[7]Hudson says "it is obvious that the world couldn't be vague but it is overkill to describe it as incoherent" for he understands the ontological indeterminate reading of it is indeterminate whether x and y are identical at T but states "I fail to see how it is possible that the indeterminacy in question could be anything other than epistemic or buried in some vague singular referring expression . . . " (2005, 11). Katherine Hawley thinks vague identity is wrong but coherent. Or more cautiously, she says "I will argue that there are good reasons to suppose that there is no ontic indeterminacy in identity over time, but I will not argue that the very idea is incoherent" (2001, 118–19). Evans asks about the notion of objects with fuzzy boundaries "is this idea coherent?" and then gives a proof that there can't be any vaguely identical objects which he takes to answer his question in the

Those who can't make sense of vague existence don't always say what aspect of it is unintelligible. Perhaps the entire phenomenon of vague existence makes so little sense to them that they can't distinguish the problematic from unproblematic aspects. Philosophers report that the mind boggles at the prospect of an object being somewhat there. Michael Morreau says he can't make sense of something not being *fully present*. Maybe what comes to mind is the cinema's spooky portrayal of an object fading away when it is being teletransported or annihilated by a science fiction ray gun. The object is portrayed on the screen as becoming lighter and hazier and then vanishing. Morreau actually compares the idea of not being fully present to the way a beam of light can have different intensities, writing, "Now, this idea is really mysterious. How can something neither quite be nor not be there? Must we imagine the presence of vague objects is somehow a matter of degree, like the intensity of a beam of light?" (2002, 336).

Morreau expresses bewilderment in a way which suggests that the problem of vague existence involves imagining an area with just individual things and no *determinate* composition. He writes, "So one might think, if ever there is no fact of the matter whether one thing is a part of another, that must be because there is no matter of fact whether the questionable part and some other things compose to make it up. But then one might think, there can be no matter of fact whether something, the composition of the questionable part and these other things, exists" (2002, 336). Morreau claims we shouldn't worry about that because we can endorse vague objects and unrestricted composition. Wherever there are any things, simple particles or not, they will compose something larger, they just may not compose an ordinary object like a cloud or a table. So Morreau denies that vague objects means vague composition. "There is no need, for such thinking is mistaken. In fact there can be vague objects though composition is precise. Imagine a vague cloud that has as its questionable part a wisp of water vapor on the edge. Suppose that, largely overlapping with this cloud, there is a collection of water droplets – cloud-minor – that is just like the cloud except that the wisp of vapor is a definite nonpart of it. Since the wisp is a questionable part of cloud, we can suppose that there is no fact of the matter that the wisp and cloud-minor make up a cloud. The crucial point is that they can still compose to make up *something* ... Composition is completely unrestricted, and nothing has any sort of shady presence" (2002, 336–37).

Morreau appeals to constitution to account for the vagueness. Objects are not vague by being vaguely *identical* to the quantities of matter. They are *constituted* by quantities of bronze or tissue. Objects like cats can have vague boundaries if there is no fact of the matter which quantity of tissue constitutes them, but that doesn't make it a case of vague identity since constitution is distinct from identity (2002, 342). Neither the quantities nor identity is vague. So regardless of whether a lump

negative. But he understands the idea of objects with fuzzy boundaries well enough to show why there can't be any such thing. Salmon avoids speaking of *vague objects* for somewhat idiosyncratic reasons, but thinks vague existence is plausible, just not vague identity. The latter he describes as "semantically incoherent" (2010). See the discussion below.

constitutes a statue or a piece of wood constitutes a bench, there will exist at least that entity which may constitute another object. It will be vague whether one thing constitutes another in a certain area but there will still be something composite there.

It seems that Morreau believes that the abovementioned wisp and cloud minor determinately compose a non-cloud and that determinate entity constitutes a vaguely existing cloud sort which is composed of cloud minor and the wisp. The latter is how I interpret his above claim "that there is no fact of the matter that the wisp and cloud minor make up a cloud." I must admit that I don't find constituted entities that vaguely exist to be any less problematic than vaguely existing non-constituted composite entities. The latter would involve a region where smaller objects vaguely compose a larger object and don't determinately compose anything else. So to whatever extent vague composition is mysterious, "a shady presence," constitution, and unrestricted composition don't render it any less mysterious. Sure, there'll always be an object that the Xs compose, so there is no area with simples that is without any determinate composite, but there is still the additional existential vagueness of whether there is anything that is constituted. Morreau's example seems to be an instance of the vague existence of a cloud even if the same wisp and cloud minor determinately compose something else, a non-cloud. Keep in mind that constitution theory has the constituted object still made out of the same parts as the constituting object. Imagine an aggregate (composed) of wood molecules constitutes a vaguely existing piece of wood. There does not seem to be any difference in intelligibility between the view that there is an aggregate that constitutes a vaguely existing piece of wood and the approach wherein there is no aggregate but just a plurality of things (simples) that vaguely compose a piece of wood. In the latter there is vague composition without constitution. In the former there is both vague constitution and composition. Or consider a determinately existing piece of wood that wasn't carved enough to determinately constitute a table, further whittling needed for a determinate table to emerge. Thus the table doesn't determinately exist and it doesn't determinately not exist. So the atoms that determinately compose the piece of wood indeterminately compose a table. Thus there is still the sort of existing, the degrees of intensity like light, and that is just as strange – but perhaps not very strange – as there being only a vague object indeterminately composed of entities that don't strictly compose anything.

Unlike Morreau, I don't find it hard to imagine some things vaguely existing without being constituted by a determinately existing constituter.[8,9] I suggest that the reader just imagine smaller parts coming together and not being tightly enough tied to each other to provide the requisite causal connections and processes

[8]I will qualify this claim in the conclusion. It is difficult to make sense of a vaguely existing *thinking* being. It is hard to imagine oneself sort of existing but in great pain. The obstacle is that thought seems to be all or nothing, not fluctuating as life or existence does with the degree of underlying vital physiological processes. See the below discussion of Chalmers's notion of reductive explanation.

[9]Similar judgments are rendered by Rosen and Smith (2004), Smith (2005), and Hawley (2002).

constitutive of a determinate existence. Before that they merely composed a vaguely existing object. For example, imagine molecules coming increasingly closer together and exerting more and more causal influences on each other, the eventual outcome being they form the first living cell. Or picture a collection of logs gradually becoming a log cabin as more and more of them become firmly interlocked. Or think of a glue hardening and things that were loosely connected becoming increasingly more so. Or envision a liquid in a mold could congeal (like Jell-O) and come to constitute a new entity.[10] And one could with any of the above, conceive the composite entity as composed only of simples rather than parts of composites that were themselves composites. So there wasn't a liquid or organic molecules but just simples arranged liquid-wise or molecule-wise. Therefore, I don't think being *somewhat present* is spooky or unintelligible, especially where the kind-bestowing properties or processes supervene, that is, they are nothing over and above other properties or processes.[11] What I have in mind here is akin to what Chalmers speaks of as "reductive explanation" when discussing learning, reproduction, and life (1996, 44, 108). This can be captured by the phrase that all w consists of is x, y, and z. So to be alive just consists of such and such facts that can be functionally described.[12] Imagine an organism dying and the processes constitutive of life are there just in degree. One doesn't even have to imagine any *composites* constituting or composing the cell.[13] One can work within a sparse van Inwagen-style metaphysics of only simples and organisms (cells and multicellular organisms). It isn't hard to imagine some but not all of the life processes continuing. Or one can imagine some much lower than typical amount of cellular activity, some substandard entropy resistance, some subnormal maintenance of temperature, and similarly with other metabolic and homeostatic functions that render the life a borderline case. What I mean by the vague existence of object O is just that its candidate parts aren't related to the appropriate degree so it can be said that they either determinately compose or determinately don't compose O and instantiate its essential properties, in this case being caught up in a life in the case of an organism.

Lynne Baker, like Morreau, appeals to constitution to avoid certain puzzles of vagueness. She too combines constitution with the view that composition is

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[10]The Jell-O Museum in Le Roy New York sells molds shaped so as to produce brains made of Jell-O, arguably an object distinct from the liquid.

[11]The vague existence of *simples* is spooky and I share Morreau's skepticism.

[12]Parfit might have meant something along similar lines with his reductionist account of personhood that involved no further fact in his *Reasons and Persons* (1984, 240). Perhaps *no further fact* is to be taken as meaning no separately existing fact, that is, one couldn't have one fact without the other, even though they are not the same fact.

[13]Hawley (2002) maintains that there is a *modest vague existence* which is coherent and her van Inwagen-inspired conception of that is just like my response to Morreau. The *immodest vague existence* that she rejects would involve existence as a first-order property of a Meinongian-like object "which somehow straddles two domains, the existent and non-existent" (2002, 135). She denies that there is an object that sort of instantiates a first-order property of existence.

unrestricted. Such precise aggregates would not be vague, though it could be vague which aggregate constituted something. But this vagueness would not be vague identity for the vagueness is in the constitution relation not the identity relation. However, Baker more clearly embraces the coherence of vague existence than Morreau. Besides there being vagueness in which of many microphysical aggregate constitutes some object, Baker believes it could also be vague whether there exists a constituted object. She defends this view with what she calls an *Argument from Natural Processes* (2007, 126). According to this view, there are natural processes that occur independently of our concepts and they do not have precise beginnings. She says that if we take biology and astronomy at face value, we'll recognize that there is no precise moment at which the solar system or later an organism came into existence.

Baker's view of vague existence differs from that of other well-known proponents of the position in that she claims vagueness is parasitical on determinacy. Something coming into existence could be a vaguely existing X only if there is a later determinate X. If the intended X never gets finished, there was no vaguely existing X (128).[14] So objects will exist vaguely at a time only if they will also later exist determinately. There won't be an aggregate that constitutes an indeterminately existing house if there is never a finished house.[15] And that indeterminately existing house at t is identical to the determinately existing house at t1. She writes, "So there is no vague identity . . . Thus we do not need indeterminate identity statements that, as Gareth Evans has shown, lead to contradiction when coupled with the thesis that there are vague objects" (131).

Evans (1978, 208) seems to think the problem is that if the world is itself vague and the vagueness is not due to deficiency in our describing it, then combining that with the belief that identity statements may lack determinate truth values, will mean that the world will "contain certain objects about which it is a fact that they have fuzzy boundaries" (208). He then asks, "Is this idea coherent?" and precedes to show that *de re* indeterminate identity will lead to a contradiction. So he takes the answer to the question that is the title of his article "Can there be Vague Objects?" to be no. The impossibility of *de re* vague identity reveals the impossibility of *de re* vague objects. Morreau says, "the main problem with this argument from definite identities is just that there is no reason to think that things with fuzzy boundaries must have indefinite identities. Strangely, Evans did not even try to show that they must . . . However this might be, the omission hides a crucial difficulty with his argument which comes to light as soon as we try to complete it" (338).

[14] Anticipating the objection that something must have been partially there, she compares building something that never gets built to hunting unicorns. They are both intentional activities "subject to the phenomenon of intentional nonexistence" (2007, 131).

[15] So "there may be indeterminacy about the number of things that exist at a time, there is no indeterminacy in the number of things that ever exist, or exist simpliciter" (Baker, 135 note 31).

Morreau speculates that it might not even have occurred to Evans that having a fuzzy boundary and having an indefinite identity might be different things.[16]

A related but different failure to make the distinctions might be thought to occur if any worldly vagueness of an object is taken to mean that the object in question must exist vaguely. It just might be that there are objects that indeterminately have some parts while determinately possessing a sufficient number of other parts to determinately exist. So there is a kind of vague object that should be understood in a way that doesn't involve them vaguely existing at any time they vaguely possess parts. Michael Tye understands a concrete "object o to be vague (as Everest is) if and only if (a) o has borderline parts and (b) there is no determinate fact of the matter about whether there are objects that are neither parts, borderline parts or non-parts of o" (1990, 535–36). So on such a construal, there might be no fact of the matter whether a thing has certain parts without that involving any vague existence.

Tye thought worldly vagueness is coherent and real, and he accepted vague existence. Some people may sort of exist. Tye (2003, 154–63) draws upon Lewis's depiction of Methuselah (1983, 66) in which there is one person embedded within the entity called Methuselah[17] ceasing to exist where another is beginning to exist but each distinct from the other. Tye claims that in a Methuselah-like case that it is vague when Anthony (who is embedded within Methuselah) ceases to exist and Tony (who is also embedded within Methuselah but only partially overlaps Anthony) begins to exist. But there is no vague identity between them; in fact, they are determinately not identical since Tony and Anthony have properties the other lacks.

Tye is not opposed to vague existence and insists that vague existence is not the same thing as vague identity nor does it imply it. Tye insists (1990, 538) worldly vagueness isn't indeterminate (vague) identity. O′ could be Mount Everest and a vague object while o could be a determinate object. Assuming for the sake of argument that vague identity is possible, it could be that o′ and o are vaguely identical but the vagueness resides in o′ (Everest). So the claim that o′ is vague is not a result of o′ being indeterminately identical to o. Secondly, Tye notes that

[16]Perhaps the fuzzy boundaries that Evans envisioned were those of overlapping entities that raise the problem of distinguishing them and therefore the impossibility of indeterminate identity would mean the impossibility of such fuzzy bounded overlapping objects. Maybe he thought that *all* vague objects were already overlapping other objects in an Unger-style problem of the many (1980). Or maybe he conceived that it was possible for any fuzzy bounded entity to become an overlapping entity and thus had in mind arguments like those I give below. One can't tell from his cryptic paper. But a charitable read of a brilliant philosopher is that he wasn't blind to some rather obvious distinctions, just assumed without argument that one would bring the others.

[17]Methuselah is not really a person on Lewis's treatment. Although there is psychological *continuity* (an overlap of memories and other mental states) for over nine centuries, there isn't a single person where Methuselah's organism is. The reason is that there aren't any of the same psychological *connections* (the same memories, desires, intentions, etc.) persisting across the 969 years of Methuselah's life and they are important to the persistence of the same person. So Lewis stipulates that a person persists for roughly every 137 years so Methuselah contains more than one embedded and overlapping person.

there is an argument given by Evans and separately by Salmon that if the singular terms are rigid, then there will never be indeterminate identity claims. "But nothing in this argument undermines the *intuitive* claim that Everest, for example, is a vague object" (1990, 538).[18]

Evans's argument against vague identity (1978) is usually mentioned along with Nathan Salmon's (1981).[19] Salmon, unlike Evans but like Tye, thinks there could be vague existence. But Salmon differs from Tye in that he insists that there couldn't be a vague object. Existence is a property, and worldly vagueness is always just indeterminately having a property. So even the vaguely existing thing is a determinate object. He writes, "Objects are not vague or indeterminate. Of course, some objects – ordinary physical objects, for example, – have indeterminate boundaries. It might also be indeterminate whether a given object exists, e.g. as it is fading away into nonexistence. Both of these phenomena are cases of indeterminacy among an object's properties, not indeterminacy of the object itself. And object itself is just the thing it is, and as Bishop Butler observed, not another thing . . . " (2010, 20–1). An object is distinct from its properties; it is not a bundle of properties, but that which has the qualities. At times it almost appears that Salmon construes objects to be like propertyless substrata, writing, "The object is not it-with-such-and-such-properties. It is the very object itself, without even the clothes on its back" (2010, 20). I would think my response to Morreau's qualms about vague existence can be extended to Salmon's dismissal of vague objects. There are vague objects when smaller objects are insufficiently causally connected to determinately compose the larger object. But it won't matter for my purposes in the next section since Salmon allows vague existence and determinately existing things with fuzzy boundaries. I will show that he can only do so if he allows vague identity. Since he rules out the latter (1981, 243), he should exclude the former. Although Salmon's argument against vague identity is similar to Evans's, Evans was right to assume that a compelling disproof of vague identity meant the claim that there were objects with fuzzy boundaries was equally suspect.

It is worth noting that there are other philosophers who believe that the advocates of the possibility of worldly vagueness should avoid positing the vagueness in the object. Their reasoning is different from Salmon's. Their concern is that there is no more reason to posit the vagueness in the object than in the property (Williamson, 706; Williams, 768). Likewise for ontic indeterminacy stated in terms of parthood or boundaries (Hawley, 106–09; Williamson, 707). If the claim is that it is indeterminate that Everest includes some rock as a part, there is no more reason

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[18]Noonan reaches a similar conclusion: "Everyone knows that Evans's argument against vague identity in-the-world doesn't show that there aren't vague objects. Even if the argument succeeds all it proves is that every vague object is determinately distinct from every precise object and every other vague object" (2004, 131).

[19]Salmon writes that the main idea underlying their proofs is disarmingly simple: "What would *y* have to be like in order for there to be no fact of the matter whether it just is *x*? One thing is clear: it would not be exactly like x in every respect. But in that case it must be something else, so that there is a fact of the matter after all" (2002, 239).

to place the vagueness in Everest than in the rock or the relation of parthood. If the rock had been differently located, then the statement that Everest has a rock as a part would have been determinate. Likewise for the parthood relation, if parthood had a more determinate extension, then the statement would have been more determinate. Williams (2008, 768) compares the mistake of thinking that the "blame" for metaphysical vagueness must belong to either the object or the property to assigning the responsibility of the metaphysical contingency of his sitting to either an object (himself) or a property (sitting down). Williamson likewise compares the mistake to attributing falsehood at the level of subsentential expressions. One should not ask whether the falsehood of "cats bark" is due to the falsity in cats or falsity in barks. Williamson (2003, 700) suggests we abandon the query of "whether there are vague objects?" for the question "Is reality vague?" Williamson argues that the ontological correlate of a sentence is a state of affairs and thus we should interpret the question of vagueness in reality to arise with states of affairs rather than objects or properties and relations (2003, 699). Abandoning such talk won't affect my thesis; everything can be rephrased in terms of vague states of affairs.

The thesis defended in the next section is that one can't accept vague existence without admitting vague identity. So Morreau, Baker, Tye, Salmon, and many others are mistaken to believe they can endorse the former but not the latter. In fact, they shouldn't even accept something determinately existing but vaguely possessing a part, if they don't allow vague identity. I don't mean that something couldn't vaguely have parts at T without vaguely existing at that time. Nor do I mean that something couldn't vaguely exist at T without at that time overlapping another object to which it is vaguely identical. Rather, my claim is that the same relation that brings vague parthood could occur to such an extent that vague existence is the result.[20] While the vague object at one time may have enough determinate parts so its existence is not vague, it could become vague or perhaps have earlier existed vaguely during the process of its origination. Moreover, the same relation that allows for vague existence will lay the groundwork for vague identity. Now I admit that if something could vaguely exist, then it could do so without at that time being vaguely identical to anything. Its identity with itself is determinate, it just sort of exists. It is a determinately self-identical but vaguely existing object. But in the end I will be open to the possibility, even sympathetic to the idea, that there can be no such things (or states of affairs) because the commitment to vague existence brings a commitment to the possibility of vague identity. The latter may doom the former even though the former would not have involved every vaguely existing object always being vaguely identical to anything. We shall see that if the problem is that vague identity is susceptible to Salmon/Evans style *reductio* or otherwise shown to necessarily never occur, then that makes vague existence unacceptable and renders both impossible. What is perhaps even more surprising is that the same type of reasoning would make it unacceptable that there was no fact of the matter whether objects possessed certain parts

[20]This can occur by taking away too much matter of various kinds or if there is an essential part, say an organ like the brain in persons or nucleus in cells.

even though that would not have involved a commitment to everything with vague parts always existing vaguely or being indeterminately identical to something else.

So vague existence is in the same boat as vague identity. If one sinks, so does the other. Now I don't believe *indeterminate identity* can be defined because I don't believe *identity* can be defined. But I think I can give a mereological construal of what would likely be held by those who believed that there could be indeterminately identical composite objects – at least if they shared my hostility to there being spatially coincident objects with the same proper parts. (I provide a slight variant for those theorists who don't share my dislike of coincident objects.) Since an axiom of classical mereology is that A and B are identical if they have all the same parts, I will characterize vague identity as follows: A and B are indeterminately identical iff A and B have all of their parts in common, and there is at least one part that A or B determinately has and the other indeterminately possesses.[21] So entities partially overlapping would not have enough part sharing to constitute a case of vague identity. A and B would have to possess every part in common, it just being that either A or B had some parts indeterminately possessed that are determinately possessed by the other. That seems to be the indeterminate extension of the mereological claim that identical entities have the same parts. But it rules out A being a quantity or aggregate having very precise parts and being indeterminately identical to B which vaguely has some parts that A lacks. So contrary to much of the literature, I don't think there is any danger of there being an indeterminate identity between a mountain and the various overlapping precise aggregates of mountainous matter. The aggregates would each lack parts that the mountain indeterminately possesses. Likewise, a precise cat-like aggregate (Lewis 1993 called them P-cats) would not, as Parsons and Woodruff propose (1997), be indeterminately identical to the vague cat in the area that had indeterminate boundaries and thus indeterminately have some parts that were not possessed at all by the precise cat-like aggregate. I am assuming if A indeterminately has parts that B lacks having in any manner, that isn't vague identity for one would possess some parts the other doesn't.[22] But maybe it is best to relax my account and just rule out A and B as a case of vague identity where either A or B determinately has a part that the other neither determinately nor indeterminately possesses.[23]

[21]My account would not render Baker-like constitution in cases where one vague object constituted another vague object into an instance of indeterminate identity since any determinately possessed parts by the constituted are likewise determinately possessed by the constituted and the same goes for their indeterminately possessing parts. The difference between constituter and constituted is that some may have parts derivatively that the other has and nonderivatively. See Hershenov (2008) for an explanation of that difference.

[22]Perhaps I am wrong to hold or find too significant the claim that there is a greater "ontological distance" between A and B when either completely lacks a part that the other has than there is between C and D when either determinately has a part that the other indeterminately possesses.

[23]So in that case P-cats which are precise aggregates and equally good candidates to compose Tibbles would each be indeterminately identical to Tibbles even though Tibbles indeterminately possessed some parts that they didn't possess in any manner.

A worry is that virtually anyone who believes in coincident objects standing in a *constitution* relationship (which includes four of my targets, Baker, Tye, Salmon, and Morreau) would probably not accept the mereological account of A and B being identical if they have the same parts. For example, the person and the animal have the same parts but aren't identical.[24] Because many theorists hold the coinciding objects in a constitution relation can have the same parts, my extending the mereological characterization of identity that they reject to then characterize indeterminate identity might seem to them to be instead described as just "indeterminate coincidence."[25] While I don't accept there are any coincident objects, in part for the familiar considerations about the grounding of their sortal and modal differences, my constitution opponents are not so bothered. (Baker, for instance, just considers essential properties distinguishing coincident objects to be primitive and not in need of further grounding.) So if constitution theorists don't accept the classical mereological construal of identity, then they won't accept my indeterminacy variation of it.

However, most defenders of coincidence will accept that there can't be two objects of the *same* kind in the same place. So there can't be two distinct spatially coincident persons composed of the same parts and there can't be two distinct spatially coincident tables composed of the same parts, even though there can be an animal distinct from but colocated with the person and a mass of wood distinct from but colocated with the table. Thus I suspect that defenders of coincidence would probably accept something like a mereological characterization of identity, appropriately qualified, for objects of the *same* kind. I don't think any have so formulated it. And since the four constitution theorists that I mention don't believe there is indeterminate identity, they won't advocate any characterization of indeterminate identity. But my contention is that they should be able to agree that my mereological characterization does justice to what defenders of indeterminate identity would have in mind about the mereological makeup of indeterminately identical entities and what they themselves can avoid by following my epistemicist recommendations in my large part replacement scenarios in the next section.

14.2 No Vague Existence Without Vague Identity

The following table parts replacement scenario seems to me to show that that there can't be *de re* vague existence without *de re* vague identity. Let's assume a commonsense ontology in which you can only remove so much of a table's

[24]Some constitution theorists like Lowe would accept that spatially coincident objects don't have the same parts, that is, the statue has hands and head while its constitution lump does not. Baker, the only one of the above four who has laid out her mereological claims in any detail, would allow that the lump derivatively has the statue's hands and head as parts.

[25]This characterization is due to Ken Akiba.

top and legs before it becomes vague whether the table still exists. Too large a removal, the original table is gone. Too small a removal, the original table remains though reduced in size. Removing pieces of sizes in between will bring vague existence.[26]

But imagine "immaculate replacements" – God instantaneously replaces the parts that he removes. When a small part is removed and replaced, the original table still exists. When a very large part is removed and replaced, the original table is destroyed and a new table takes its place. When God removes and replaces an "in-between" size piece of the table with a qualitatively similar duplicate piece, it is indeterminate whether the original table still exists and if the replacement parts are possessed by the original table. But there is always a table and a determinate one at that. There is never a moment where there isn't clearly a table after the initial table is made. So our replacement case means that along with the indeterminately existing table A, there is a determinately existing table B that consists of all the parts determinately possessed by table A and the latter's newer indeterminately possessed parts. Given the earlier characterization of vague identity, then A is indeterminately identical to B for they have all the same parts, one just having some parts determinately that the other has indeterminately.

A judgment of indeterminate identity avoids there being two tables and persons, an increase of colocated objects of the same kind that is anathema to most defenders of coincident objects. So the indeterminate identity of A and B would enable the defender of the vague existence of A to account for its relationship to B after the part replacement. Those theorists hostile to colocated objects of the *same* kind as well as indeterminate identity – myself included – will instead be compelled to describe the part replacement case as not involving A or B indeterminately sharing all their parts, some determinately and some indeterminately, but rather as either A being still there and there not existing B that is indeterminately sharing some parts with A, or B having replaced A and there being no other candidate table or person that it shares parts with. But if theorists resist indeterminate identity in the thought experiment and accept a sharp boundary where A goes out of existence, then they should abandon their claim that there would be vague existence when the same parts are removed but not replaced.

If one doesn't like the divine replacement example, perhaps because one doesn't accept such instantaneous simultaneous removals and replacements, then consider a similar scenario involving mortals removing and replacing parts over a very brief period. It is safe to assume that if you destroy too large of a chunk of table A in your living room that the table ceases to exist. Remove and destroy a small part of the table and it continues to exist. Removing and destroying pieces of wood somewhere in between renders it vague whether the table continues to exist.

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[26]The semantic vagueness alternative is that there are countless overlapping tables and no fact of the matter whether the term applies to any one rather than all the others. See Smith (2005) and Hershenov (2001) for why de re vagueness is the commonsensical notion.

Now consider a twist on the three previous cases. In the first case where the loss was so great that the table went out of existence, we replace the extremely large missing part with a numerically distinct but qualitatively similar chunk of wood. This doesn't bring the original living room table A back into existence. If one thinks it does, it might help to imagine that the large replacement chunk was taken from another qualitatively similar table D in one's study, leaving just a few splinters behind there. The better interpretation is that we have moved table D from the study to the living room, adding a little wood to it that remained in the study after table A was destroyed.

Our second case involves our first removing a small chunk of wood from the living room table A and then replacing it with a new small chunk of wood. The original table that had become smaller when it lost a part is restored back to its original size when it gets a part added.

In the third case where the loss of an intermediate size chunk of wood had left a vaguely existing table, we replace that missing wood with an intermediate chunk. It is still indeterminate whether the original table A continues to exist because of the size of the replacement part. However, if the total amount of replacement wood had been added gradually in very small portions, then there would be reason to think the indeterminately existing table A had come back into determinate existence. The reason why is that it is very plausible that objects can undergo full part replacement if this gradually occurs. So it is as plausible, or nearly as plausible, to think that a vaguely existing object could come back into existence if it had the missing parts slowly replaced bit by bit by qualitatively similar duplicates. The ontologically significant difference between the gradual small replacements and a single very large replacement is evident in that in the latter it makes sense to think that we have just moved a different table D to the spot where table A was rather than just provided the original living room table A with some new parts. But in the case described at the beginning of this paragraph of the intermediate size replacement, it is indeterminate whether the original table A continues to exist because the replacement part was of the size that it would have been indeterminate whether or not we had just moved another table into the living room were A was located.

So as not to complicate matters with the possibility that we have created or moved an indeterminate existing table into the living room to overlap indeterminately existing table A, let's make the intermediate size replacement matter consist of two separate pieces of wood rather than an intact larger piece. These two pieces are combined at the same time with each other that they are attached to the rest of the wood of the vaguely existing table. Since we are replacing the missing wood in the living room table with two new chunks that were each too small to themselves be even vaguely existing tables, nor had ever been parts of a table located elsewhere, then there is no additional complication of our moving into or reassembling in the living room an indeterminately existing table. Moreover, the fact that the replacement wood consists of two fairly large pieces shouldn't give us any more reason to claim that the vaguely existing table has been restored to determinate existence than we had in the first case where the replacement wood had come from another table D located in your study. Since the two replacement parts in our thought experiment are combined with each at the same time that they are being attached to the rest of the vaguely existing living room table, there is no reason to think that table A is just getting first a smaller part then subsequently another and so on.

Thus our tweaked version of the third scenario again means that along with the indeterminately existing table A, there is a determinately existing table B that consists of all the parts determinately possessed by table A and the latter's newer indeterminately possessed parts. Given the earlier characterization of vague identity, then A is indeterminately identical to B for they have all the same parts, one just having some parts determinately that the other has indeterminately. I think that even believers in constitution should accept something similar to my account as the mereological construal of the view of indeterminate identity that they will likely deny because they hold that identity is determinate and so as a result are committed to there being a precise moment of substantial change in my part replacement thought experiment. So if readers are convinced by Salmon/Evans style arguments against vague identity and thus believes that there is a difference between A and B that makes them distinct, then it seems that there had to be a last splinter in which its removal or replacement made it the case that table B replaced table A.

Such readers will not admit that there is indeterminate identity, but should understand the mereological conception of the indeterminate identity view that they are rejecting as amounting to roughly what I sketched for the mereological relationship of things of the *same* kind. They won't accept that all things with the same parts are identical as does the classical mereologist nor that things having all the same parts but differing in whether they determinately possess all of those same parts are indeterminately identical. But they should accept that there could not be two things of the *same* kind that have all their parts in common. That leaves them no recourse but to favor the second of two options. The first was to accept that A and B are indeterminately identical, sharing many of the same parts, one indeterminately sharing some parts the other has determinately. But since most constitution theorists (certainly those I discuss in this chapter) will deny that there is a table or person indeterminately identical to another table or person, they will advocate the second option which is that if too much matter is replaced, then one table or person will suddenly replace the other, though they won't claim to know which was the decisive splinter or cell. Right up to that point, there is just one table or person that isn't indeterminately identical to another. So it appears that the options are that one accepts vague identities because vague existence implies vague identity, or modus tollens, given that vague identity is impossible, then there isn't any vague existence.

Thus there will be indeterminate identity unless one takes a page from the epistemicist in such a scenario and judges there to be a (unknowable) decisive splinter of wood that determines the coming into and going out of existence of tables. But if one takes an epistemicist approach here in order to avoid vague identity, then there is no need to posit *de re* vague existence when the table has a large part or many parts removed that aren't replaced. It would be arbitrary to claim that the possession of one last splinter of wood was a principled demarcation in the case in which vague identity threatened but there wasn't an equally significant splinter in the case where just vague existence lurks. It shouldn't matter that the table's parts were being removed and replaced rather than just removed.[27] In cases where too much matter is removed for A to still exist, we don't reach a different judgment about A's fate if the same amount of removed matter was immaculately replaced. So I don't think removals that leave it indeterminate whether A still exists should elicit different judgments if the matter is suddenly replaced.[28] If there must be a chunk of wood (CW) that is just big enough that if removed and replaced would determine that table B has replaced table A, then there is a precise chunk of wood (CW-1) with one less splinter that if removed and replaced would mean table A still exists and table B doesn't yet exist. The removal of the latter chunk (CW-1) should also dictate that we have reached the smallest size that table A can be reduced to and still exist in the case where its parts are removed and not replaced. So the removal of CW which has one more splinter than CW-1 would doom table A in the case of a removal without replacement.

Likewise, it seems arbitrary to admit a fact of the matter that a splinter of wood connected to other pieces of wood in a certain manner can determine a table's origins or endings, but there is no fact of the matter whether a splinter connected in the same manner to other pieces of wood in the determinately existing table is or is not a part of that table. This is queer because there is the same degree and manner of part separation that will manifest itself in non-vaguely existing cases as in cases where there is a threat of vague existence or vague identity. The only difference is that in the first case the distance or causal ties between the part of wood and the rest of the wood will be between a determinately existing table and a wooden part that doesn't have existential import for the table, while in the other two cases, it will be the same distance or causal ties between the one part of wood and the rest of the wood that will determine the existence of one table rather than another (where vague identity threatens) or the existence of just a single table (where vague existence

[27] So if one believes as Salmon does that there can be a single splinter that is decisive in avoiding indeterminate identity of tables (2005, 343–44), then one shouldn't be so hostile to the epistemicist claim about the sorites as Salmon is. He states that "it is excessively implausible that removing a single grain from a heap of sand can make for a non-heap . . . " (2010, 22 nt 1). Or at least one shouldn't say what Salmon does if heaps are going out of existence rather than a persisting structure undergoing a phase change from heap to non-heap. Given that a final splinter is decisive in avoiding vague identity, it should also be decisive in determining the passage from existence to nonexistence in non-replacement cases. So Salmon should not write that "The vagueness-in-the-world approach offers a simple, straightforward, and I believe obviously correct diagnosis of sorites arguments . . . the inductive premise [for every n: If $F(n)$, then $F(n + 1)$] (e.g., 'the result of removing a single grain from a heap of sand is still a heap') is not false. Although the vast majority of its instances are true, not all are. Specifically, each of the conditionals whose antecedent or consequent is about a borderline case is neither true nor false. The inductive claim itself is also therefore neither true nor false" (2010, 26 note 28). The existentially significant cases show us that the sorites arguments are unsound for the inductive premise is false rather than neither true nor false. There is a decisive splinter. Perhaps only sorites arguments that involve parts or properties that are *never* existentially significant will avoid falling prey to my extension of arguments against vague identity to vague existence.

[28] See note 29 for further support of this point.

lurks). In the second case (where vague identity threatens), the piece of wood in question will become the existentially crucial part that will determine which table it and others bits of wood compose, while in the third case (where vague existence lurks), that piece of wood will determine whether it and the other pieces compose any table at all. If the Salmon/Evans reductio shows that there will have to be a decisive number of splinters *and* a manner or degree in which they are connected in the vague identity case, then the same precise manner or degree of connection should be expected to govern parthood in cases where vague identity is not a concern.[29]

A similar argument can be run with vaguely existing human organisms.[30] I am assuming that we human persons are human organisms. In my view, it is better to identify them to avoid the problem of too many thinkers if persons and animals are distinct.[31] We can run the same thought experiment with the organism/person A undergoing too much sudden part replacement to still determinately exist but not enough removal so it determinately no longer exists. There still seems to be a determinately existing organism B that includes both old and new parts. Therefore, indeterminately existing organism A and determinately existing organism B would each not have any parts that the other lacks. They will either be indeterminately identical or there would be final atom or cell which if removed means B has replaced A.[32]

[29]I can still maintain my thesis even if one believes that objects which vaguely exist when they lose sufficient material would still determinately exist if that same material had been immaculately replaced. I will just have to make my argument that there isn't any de re vague existence in a more indirect manner. There will still be a need for an epistemicist cutoff in cases where too much replacement matter means table B has replaced table A. There will be a last appropriately attached splinter for A to survive. Since there must be an epistemicist solution to the parthood relationship to avoid the vague identity of A and B, that precise parthood relationship can also be relied upon to prevent vague existence. De re vague existence doesn't just occur just when too much matter is taken away but also when too much matter moves from being determinately attached to indeterminately attached. But the requirement of an appropriately attached last splinter to avoid vague identity will render it arbitrary to claim that there is no such precise part relationship preventing vague existence due to indeterminate part attachment.

[30]So appealing to a sparse ontology that lacks artifacts won't evade the problem. And we shall see in the conclusion that there is an additional benefit to use thinking human organisms to illustrate the puzzle for it provides a reason to resolve the puzzle one way rather than the other.

[31]See Olson (2007) for reasons why coincidence should be avoided.

[32]One might object that this case isn't analogous to that of the table. The reason would be that organism B hasn't assimilated the new and old parts since they aren't caught up in the same life processes but remain briefly "frozen" in an indeterminate status. But think of a new organism just composed *ex nihilo* or given life and existence by an electrical shock *a la* Dr. Frankenstein. Does the creature really need time to assimilate those parts? No biological assimilation is needed at its origins. It seems to exist immediately when there are life processes. Each part is beginning to play its role in metabolism or homeostasis which is different from whether something is not yet assimilated and not yet playing a role in the process. Beginning to play a role is determinate in a way that *sort of* playing a role is not. To see this, contrast your beginning to digest something with some entity only able to *sort of* digest something because it is missing too much physical structure for bona fide (determinate) digestion to occur. Also, it might help to think that the Biblical

One other possibility besides an epistemicist-like precise boundary between A and B or treating A and B as indeterminately identical in my part replacement scenario is open to those who are not opposed to objects of the *same* kind being in the same place at the same time. This third alternative has two variants. The first involves there being an indeterminately existing table (or person) and a colocated distinct determinately existing table (or person) after the replacement of the removed matter. So the original entity that indeterminately possesses the new parts would continue to exist indeterminately because it didn't determinately acquire the new parts. That would mean that there were two colocated things of the *same* kind, though only one indeterminately existing and the other determinately existing. On the second variation, one could insist that both A and B determinately existed and it was just that A indeterminately possessed the new parts while B determinately possessed both the new parts and A's earlier parts that were never removed. Either variant could absurdly lead to hundreds of such tables (or persons) because the process of removing and replacing a large chunk could be repeated over and over, so, to use just the first variant, what before had been a determinately existing table would then become an indeterminately existing table, and it would be the second indeterminately existing table there. So theorists accepting this interpretation of part replacement have to suffer the absurdity of hundreds of *thinking* persons in the same place. That might be as unattractive as vague identity – both are very unattractive. If so, then my epistemicist-friendly argument about a last splinter or cell becomes more attractive.

14.3 Conclusion: Is Vague Identity Incoherent?

I have defended the thesis that vague existence can't be accepted without bringing vague identity along in its wake. If there are vaguely existing objects, then there will be vaguely identical objects. And avoiding vague identity involves positing a last wooden splinter or organic atom rendering replacement tables or human persons distinct from their predecessors. But that last splinter or atom will also be existentially decisive when there isn't replacement but just loss of parts. And the precision governing whether that last splinter or atom is still a part in the existentially significant cases will be no different when possession of that part has no bearing on the table or person's continued existence. So avoiding *de re* vague identity involves avoiding either of the two other kinds of worldly vagueness and makes doing so in the latter pair seem less arbitrary and implausible.

Adam with normal biological dispositions would not have had to digest and metabolize before his existence would be determinate.

But is vague identity so bad? There is much literature about the logic of vague identity.[33] Evans famously claimed that vague identity was incoherent. But he didn't finish his proof and left matters with some dubious appeals to determinacy and indeterminacy operators being duals and obeying a logic at least as strong as S5. Other critics challenged whether there aren't properties expressed by the predicates introduced by Evans or whether the inferences would go through if objects were indeterminately identical, perhaps the worldly indeterminacy bringing referential indeterminacy (Williams, 778–79). Maybe the most common objection is that Evans can't help himself to the contrapositive of Leibniz's law if there is the possibility of truth value gaps and so it being false that A and B are identical doesn't make it true that they are not indeterminately identical.[34] However, the logical and semantic issues are adjudicated[35]; I want to end by suggesting a different tack. I think it is generally helpful in metaphysics to see how any account of material constitution works with people. For example, positing the spatial coincidence of objects may not seem so bad if one is thinking of statues and lumps. But it will seem a lot less attractive when imagining oneself (a person or an organism) coinciding with another being (i.e., a person or an organism) and a *thinking* one at that. Eric Olson's strategy is to reveal how his rivals' accounts of material objects like ourselves lead to a problem of too many thinkers (2007).[36] So let's try to imagine being vaguely identical to another thinking entity, that is, there being no fact of the matter that I am identical or distinct from the other thinker. If identity can be indeterminate, then there would be experiences that were just partly one's and partly someone else's. I tend to agree with Madell who has used this tactic when criticizing Parfit's claims that reductionist accounts of personal identity allowed vague identity and who earlier wrote:

> What I fear is that the future pain will be mine; the fact that it may or may not be accompanied by a particular set of memory impressions and personality traits seems quite irrelevant . . . what I fear about the future pain is . . . simply its being felt by me. It is equally clear that our ordinary attitude toward future pain leaves no room for the notion that whether or not some future pain is mine could be a matter of degree. We, rightly, find unintelligible that there could be a pain in the future which is part mine and part not. (1989, 31–32)

So it seems that we can't make sense of a pain being partly mine and partly not. It is no easier to do this from the third-person perspective than trying to conceive of it from our first-person perspective. Pains seem to need an owner and only one owner.

Thus I sympathize, if not align myself, with those who consider vague identity to be incoherent and perhaps when the indeterminate identity involves thinking beings to be not even fully intelligible. Since vague parthood and vague existence brings vague identity, they must be abandoned as well. And as my replacement

[33]See Parsons (2001) and Williams (2008) for extensive bibliographies.

[34]Evans can't respond, but Salmon can. See his "Identity Facts" (2002).

[35]For what it is worth, I am very sympathetic to Salmon's arguments in his 2002 paper.

[36]Unger (2004) does the same with overlap in a version of the problem of the thinking many and moves towards dualism as a result.

table scenario revealed, appealing to a decisive splinter is not more arbitrary in cases of vague parthood and vague existence than it is to avoid vague identity. So the epistemicist-friendly arguments against vague identity show far more than many have realized. On the other hand, if *de re* vague parthood and vague existence are accurate descriptions of the world, then worldly indeterminate identity is also the case.[37]

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Chapter 15
Castles Built on Clouds: Vague Identity and Vague Objects

Benjamin L. Curtis and Harold W. Noonan

15.1 Evans's Argument Stated

Evans's 1978 article is entitled 'Can There Be Vague Objects?' and opens with the following passage:

> It is sometimes said that the world might itself *be* vague. Rather than vagueness being a deficiency in our mode of describing the world, it would then be a necessary feature of any true description of it. It is also said that amongst the statements which may not have a determinate truth value as a result of their vagueness are identity statements. Combining these two views we would arrive at the idea that the world might contain certain objects about which it is a *fact* that they have fuzzy boundaries. But is this idea coherent? (Evans 1978, p. 208)

Evans then presents the following argument:

(1) $\nabla(a = b)$
(2) $\lambda x\ [\nabla(x = a)]\ b$
(3) $\sim\nabla(a = a)$
(4) $\sim\lambda x\ [\nabla(x = a)]\ a$
(5) $\sim(a = b)$

'∇' is a sentential operator that is to be read 'it is indeterminate whether'. (1) is an assumption for *reductio*. (3) is supposed to be self-evident. (2) is supposed to follow from (1), and (4) from (3), by property abstraction. And (5) is supposed to follow from (2) and (4) by an application of (the contrapositive of) Leibniz's Law. It is clear from the title of Evans's paper and the passage quoted above that Evans himself takes this argument to establish both that there cannot be identity statements that are indeterminate for ontic reasons and that there cannot be vague objects. As we will

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see, the argument does not establish either of these conclusions. But it does establish another important conclusion, namely, that identity *itself* cannot be vague. And we take this to be the main lesson that Evans wishes to impart.

Identity itself can be vague iff identity statements can be indeterminate *in virtue of* the vagueness of identity. To see what this means, first consider that one can hold that an identity statement is indeterminate for a number of reasons. One can hold that it is indeterminate because either (or both) of the terms flanking the identity sign is referentially indeterminate. To do so is to hold that it is indeterminate *in virtue of* the referential indeterminacy of (at least one of) its singular terms. One can also hold that an identity statement is indeterminate because we are irremediably ignorant about its truth-value. To do so is to hold that it is indeterminate *in virtue of* our epistemic limitations. But suppose that we have an identity statement that is indeterminate, but for neither of the reasons above. Then, it seems, the only place left to locate the source of its indeterminacy is in the identity that it expresses. Thus, the view that identity itself can be vague is a negative thesis; it is the view that identity statements can be indeterminate, but not in virtue of referential indeterminacy or our epistemic limitations. Evans's argument is directed at those who believe that identity itself can be vague. (1) is supposed to be a representative expression of this view. So '∇' is to be taken as a non-epistemic operator and 'a' and 'b' to be referentially determinate.

15.2 Resisting Evans's Argument

How might the defender of the view that identity itself can be vague resist Evans's argument? Note once more that (1) is an assumption for *reductio*, (2) is supposed to follow from (1) by abstraction, (4) is supposed to follow from (3) by abstraction, and (5) is supposed to follow from (2) and (4) by (the contrapositive of) Leibniz's Law. So, there are only four possible ways to resist the argument:

(i) By denying premise (3)
(ii) By rejecting one or both abstraction steps
(iii) By rejecting (the contrapositive of) Leibniz's Law
(iv) By denying that the argument is a genuine case of *reductio*

Of course, it is possible to consistently take any of these ways and so resist Evans's argument. But this should not be surprising. *Any* argument can be resisted somehow. One could take way (iv), for example, by adopting dialetheism and claiming that the assumption and conclusion of the argument constitute an example of a true contradiction. But because dialetheism is far more controversial than the thesis that identity itself cannot be vague, unless one has independent reasons for endorsing dialetheism, to resist Evans's argument in this way would be inapposite. The conclusion of Evans's argument is not an unpalatable one, so one cannot reasonably reject otherwise plausible assumptions purely in order to resist it.

Williams (2008) makes just this point and identifies the following conditions (amongst others) that any way of resisting Evans's argument must meet if it is to be acceptable:

A. Classical logic should be preserved.
B. 'Properties' should be taken in a thin or 'merely abundant' sense; so even if there is no Armstrongian Universal corresponding with 'being identical with a', there is still (in standard cases) a property accurately so described.
C. Leibniz's Law (the principle of the indiscernibility of identicals) [and its converse] should be recognised as holding.
D. The logic of 'indeterminately' is to be S5; a consequence of this is that if something is indeterminate, it is determinate that it is indeterminate (Williams 2008, p. 136).

Williams goes on to justify these conditions (see pp. 136–138). We do not repeat those justifications here but merely signal our agreement with Williams about these conditions and note that they immediately rule out most of the possible ways of resisting Evans's argument (and with them nearly all of the published responses to Evans's argument).[1] Way (iii) of resisting the argument is obviously immediately ruled out by C. The most obvious way of developing way (ii) (namely, by claiming that $\lambda x\ [\nabla(x = a)]$ is not a genuine property) is immediately ruled out by B. Way (iv) is ruled out by D. One might deny that (1) contradicts (5) as each stands, but in the presence of the characteristic S5 axiom mentioned by Williams above, the argument can be strengthened so that $\sim\nabla(a = b)$ can be derived from $\nabla(a = b)$.[2] D (along with A) also rules out way (i), for it is an axiom of S5 that all theorems of classical logic are determinately true. (Indeed, this is also an axiom of S4, and even the much weaker T.) All instances of the schema $(\alpha = \alpha)$ are theorems of classical logic, whence it follows from D that $\Delta(a = a)$ is true, and this of course is equivalent to $\sim\nabla(a = a)$. In fact, quite independently of D, we consider the truth of premise (3) to be unimpeachable. The very concept of an individual is tied up with the notion of self-identity. No individual can fail to be self-identical. So (3) cannot fail to be true. Thus, even if one adopts a non-standard modal logic in which the axiom mentioned above is rejected, we do not think one can reject premise (3).[3]

Given the above conditions, the only remaining possible way of resisting Evans's argument (to defend the view that identity itself is vague) is to develop way (ii)

[1] To give just a few examples, the responses to Evans's argument given in each of the following rely, in one way or another, on the rejection of one of the conditions A to D and are thus immediately ruled out as admissible responses on our view: Broome (1984), Cook (1986), Johnsen (1989), Garrett (1991), Parsons and Woodruff (1995), Copeland (1997), Parsons (1987, 2000), Lowe (2005), French and Krause (1995, 2003, 2006), van Inwagen (1988, 1990, 2009), and Cowles and White (1991) (the response given in this last paper is explicitly directed at Pelletier (1989) but is also applicable to Evans's argument). (See also fn. 7 for additional comments on French and Krause (1995, 2003, 2006).)

[2] See Heck (1998, pp. 282–283) for precisely how to derive $\sim\nabla(a = b)$ from $\nabla(a = b)$.

[3] See also fn. 7 below for more on this.

somehow. There are, so far as we know, only two places in the published literature where anyone has attempted this.[4] The first is Lowe (1994, 1997, 1998), and the second Barnes (2009). So (given conditions A–D above), in order to defend Evans's argument, all that remains is to reject Lowe's and Barnes's arguments, to which we now turn.

15.3 Lowe's Denial of Abstraction

Lowe's original argument for his denial of the abstraction steps in Evans's argument is in his 1994 'Vague Identity and Quantum Indeterminacy'. He asks us to consider the property $\lambda x\,[\nabla(x = b)]$. This property, he argues, is symmetrical to the property ascribed to b in (2) (i.e. $\lambda x\,[\nabla(x = a)]$) because it differs from it only by permutation of 'a' and 'b'. By parity of reasoning, if b possesses $\lambda x\,[\nabla(x = a)]$, a must possess $\lambda x\,[\nabla(x = b)]$. But given the premise that it is indeterminate whether $a = b$, he argues, it must be indeterminate whether these two properties are identical. So it cannot be right to say, as (4) does, that a does *not* possess $\lambda x\,[\nabla(x = a)]$. Thus, Lowe argues, the step from (3) to (4) must be an illegitimate one.

Two responses can be given to this argument. The first is that even if the argument succeeds, it does so only for cases that involve only identity-involving properties. But concrete examples of supposed vague identity invariably involve features that allow us to apply Evans-style arguments without making use of identity-involving properties. We first illustrate this using Lowe's own example.

Example: Lowe's Electrons

Electron a is free at time t_0 and whizzing around an ionisation chamber. At time t_1 it becomes trapped by a helium atom (whereupon the atom becomes a negative helium ion) and enters into a superposed state with the two other electrons within it. At a later time t_2, electron b is emitted by the atom (whereupon the atom returns to a neutral state).

Here 'a' and 'b' are supposed to refer to single electrons, but it is also supposed to be indeterminate whether $a = b$. There are a number of ways in which an Evans-style argument that makes no reference to identity-involving properties can be utilised here. Here is one way:

(1) ∇ (a is emitted at t_2)
(2) λx [∇ (x is emitted at t_2)] a

[4] Some of the papers cited in fn. 1 above do also reject one or both of the abstraction steps in Evans's argument. But where they do so, they do so not for independent reasons, but on the basis of one of the other rejected ways of rejecting Evans's argument. For example, Parsons (2000, ch. 4) argues that the abstraction step from (1) to (2) fails *because* there is no property of 'being indeterminately identical with a' (see pp. 50–52). Lowe and Barnes, by contrast, offer independent reasons for rejecting the abstraction steps that (at least *prima facie*) do not rely upon a prior commitment to one of the other rejected ways of rejecting Evans's argument.

(3) $\sim\nabla$ (b is emitted at t_2)
(4) $\sim\lambda x$ [∇ (x is emitted at t_2)] b
(5) $\sim(a = b)$

Here no property can be constructed that is symmetrical to the one ascribed to a in (2), so Lowe's argument does not apply.

Lowe's example is quantum mechanical. But nothing hangs on this. A second illustration of this style of argument is Hawley's (2001: pp. 118ff.) application of it to van Inwagen's fiendish Cabinet thought experiment (van Inwagen 1990, Ch. 18):

Example: van Inwagen's Cabinet

A person, Alpha, steps into a fiendish Cabinet at t_1, which then disrupts those features, whatever they are, that are relevant to personal identity. Later at t_2 someone, Omega, steps out.

Here it is supposed to be indeterminate whether Alpha is Omega. Hawley gives the following Evans-style argument for their distinctness:

(1) It is indeterminate whether Alpha steps out of the Cabinet.
(2) Alpha is such that it is indeterminate whether she steps out of the Cabinet.
(3) It is not indeterminate whether Omega steps out of the Cabinet.
(4) Omega is not such that it is indeterminate whether she steps out of the Cabinet.
(5) So, Alpha is distinct from Omega.

Again, Lowe's argument does not apply.

Lowe replies to this response in his 1997 'Reply to Noonan on Vague Identity'. He argues that *at* t_2 it is indeterminate whether *a has been emitted* (in the electron example) but that *at* t_1 it is indeterminate whether *b is going to be emitted*. He then goes on to say:

> We have to remember that the names '*a*' and '*b*' have been introduced with an implicit time reference built into them: *a* has been introduced as the *captured* electron and *b* as the *emitted* electron ... Granted that there is a property that is assignable to *a* in virtue of the fact that at t_2 it was indeterminate whether *a* had been emitted from the atom, we can nonetheless see that there is no reason to suppose that this property is determinately distinct from the property that is assignable to *b* in virtue of the fact that at t_1 it was indeterminate whether *b* was going to be emitted from the atom. Consequently, *a*'s possession of that 'first' property can provide no reason for thinking that *a* determinately differs in at least one of its properties from *b*. (Lowe 1997, p. 90)[5]

The difficulty with this reply is that it is just false to say at t_1 that it is indeterminate whether b will be emitted (mutatis mutandis, in Hawley's case that Omega will step out). If the reference of the name 'b' is fixed by the description 'the emitted electron' (or the reference of 'Omega' by 'the person who steps out'), then b is the emitted electron (Omega is the person who steps out), but it is not correct

[5] Again, nothing hangs on the fact that the example is quantum mechanical. Lowe does not think this reply appropriate only with respect to employment of the non-identity-involving property style of argument in quantum mechanical examples. He would give the same reply to Hawley.

to say at t_1 that it is indeterminate whether the emitted electron (the person who steps out) is going to be emitted (is going to step out). So it is equally not correct to say that it is indeterminate whether b (Omega) is going to be emitted (step out). Lowe says in the passage quoted, 'we have to remember that the names "a" and "b" have been introduced with an implicit time reference built into them'. It is this, we think, that has misled him. All that is true is that the names 'a' and 'b' (and 'Alpha' and 'Omega') have had their references fixed by descriptions which identify their denotata by properties which they (the denotata) only possess at certain times. But this makes them no different from any other name whose reference is fixed by such a description, and insofar as the references of names are fixed by descriptions, it will typically be by appeal to such descriptions that their references are fixed. (Maybe the reference of 'Einstein' can be fixed by the description 'the creator of the Special Theory of Relativity' – no need to say when he created it; but most people are no Einsteins, with such unique and time-independent achievements; rather they are known by, and can only be identified by, properties which involve reference to time. Who is John Major? The man who led the Conservative Party to electoral disaster? No, that has been done before. He is the man who led the Conservative Party to electoral disaster *in 1997*. The case is entirely typical.)

Lowe returns to the topic in his book (1998, pp. 67–69) and replies slightly differently. But the modified reply still exhibits a confusion about historical properties. A historical property, like being a mother, may be had at some time and not at another (earlier) time. A woman becomes a mother when she gives birth. So suppose Mary gave birth in 1950. Then in 1960 it was correct to say 'Mary gave birth in 1950'. But in 1940 it was not correct to say 'Mary gave birth in 1950' (but only, 'Mary will give birth in 1950'). However, the form of words 'In 1960, Mary gave birth in 1950' is meaningless. It is to just such a meaningless form of words that Lowe appeals in his modified reply. The argument he is opposing is that at t_2 one can correctly assert that it is indeterminate whether b was captured at t_1 (Omega stepped in at t_1) and correctly deny that it is indeterminate whether a was captured at t_1 (Alpha stepped in at t_1). Lowe objects that no mention of the time at which this property is allegedly had by b has been made. But there is no need to mention an additional time. Lowe suggests that what should be said is 'at t_2, b has the property of being such that it is indeterminate whether it was captured by the helium atom at t_1'. But this is comparable to the meaningless 'In 1960, Mary gave birth in 1950'.[6]

[6]Lowe again returns to the topic of vague identity in his 2005 'Identity, Vagueness, and Modality', developing earlier material (from his 1982), though he does not specifically discuss the variant of Evans's argument which appeals only to non-identity-involving properties. His main claim in this paper is that the Evans argument, like the Barcan-Kripke argument for the necessity of identity, involves a transition (in the Evans argument in the move from (3) to (4)) from an ascription to an object a of the property of being determinately/necessarily self-identical to an ascription to a of the property of being necessarily identical to a, but that this transition is illegitimate since these are different properties (everything has the first, only a has the second). The flaw in Evans's argument (and mutatis mutandis in the Barcan-Kripke argument) which he thinks this reveals is that (3) and (in the Barcan-Kripke argument) '$\Box(a = a)$' are ambiguous between two modally

The first response to Lowe's (original) argument might save the spirit of Evans's argument, but it does not save the letter of it. Whether this matters depends on whether concrete cases of vague identity that involve only identity-involving properties can be described. We are not convinced that they can be, but offer no argument for this here because the second response to Lowe's argument does save the letter of Evans's argument. (If this is so, why do we bother giving the first response? Because its main point will be important later.) Lowe claims that he is rejecting the abstraction step from (3) to (4). But he must also reject the abstraction step from (1) to (2) and, ultimately, premise (3) itself. To see why note that in giving his argument Lowe first assumes that the abstraction step from (1) to (2) is legitimate. It is only because he makes this assumption that he is able to maintain on the basis of (1) that b possesses $\lambda x\, [\nabla(x = a)]$. He then claims that $\lambda x\, [\nabla(x = a)]$ and $\lambda x\, [\nabla(x = b)]$ are indeterminately identical and so claims that it is not right to say that a does *not* possess $\lambda x\, [\nabla(x = a)]$, which renders the abstraction step from (3) to (4) illegitimate. But suppose we reason in the opposite direction. Suppose we assume that the abstraction step from (3) to (4) is legitimate, and so assume that a does not possess $\lambda x\, [\nabla(x = a)]$. Then we can conclude that b does not possess $\lambda x\, [\nabla(x = b)]$. So, from the claim that $\lambda x\, [\nabla(x = a)]$ and $\lambda x\, [\nabla(x = b)]$ are indeterminately identical, we can conclude that it is not right to say that b possesses $\lambda x\, [\nabla(x = a)]$, which renders the abstraction step from (1) to (2) illegitimate instead. If Lowe's argument is sound, so is this one, and so the abstraction steps cannot both be true. But neither of these arguments, in the presence of the other, can give us reason to deny one of the abstraction steps rather than the other. So, both must be rejected.

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non-equivalent readings, one ascribing to a the property of being determinately/necessarily self-identical and the other ascribing to a the property of being determinately/necessarily identical to a. (As he notes, he has to make this ambiguity claim since he must deny the validity of the 'stripped down' Evans argument which omits steps (2) and (4) and the original Kripkean formulation of the Barcan-Kripke argument (2005, p. 305).) Despite the obvious non-identity of the properties, the supposed non-equivalence seems difficult to defend. Suppose a possesses the property of being determinately/necessarily self-identical. Then it is a determinate/necessary truth that a possesses the property of being self-identical. But if a has the property of being self-identical, then a has the property of being identical to a. That is also a determinate/necessary truth. Whence we can conclude that it is a necessary truth that a has the property of being identical to a (not just the distinct property of being self-identical, which everything has). So a has the property of being necessarily identical to a. Whatever may be said about this argument, the response to Evans in Lowe's 2005 paper is different from the one presently under discussion in this section and, in fact, is a version of way (i) of responding to Evans listed above (i.e. not accepting the determinate truth of (3)) which is why we list Lowe's 2005 paper in fn. 1 above.

The other development in Lowe's 2005 paper is that he now denies that the names 'a' and 'b' in the electron example make determinately identifying reference. But he insists that these terms could not be made determinate by precisification. If he is right, then this is another example of what we emphasise the possibility in Sect. 15.5 below – singular terms which are referentially indeterminate but not on account of semantic indecision. But as Williams (2008) explains, the possibility of such cases (of 'ontically induced') referential indeterminacy is no objection to Evans.

Now, Lowe does not deny that $\lambda x\ [\nabla(x = a)]$ is a genuine property. Rather, he denies that it is determinately distinct from $\lambda x\ [\nabla(x = b)]$. But if it is a genuine property, there should be some fact of the matter regarding b's possession of it. According to the reasoning above, we cannot conclude on the basis of $\nabla(a = b)$ that b *possesses* $\lambda x\ [\nabla(x = a)]$. But given that $\nabla(a = b)$, surely it cannot be that b *fails* to possess it. So why *can't* we conclude that b possesses it on the basis of $\nabla(a = b)$? The answer must be that, on Lowe's view, the most that we can conclude on the basis of (1) is that it is indeterminate whether b possesses $\lambda x\ [\nabla(x = a)]$. And so, by parity of reasoning, it must be indeterminate whether a possesses $\lambda x\ [\nabla(x = b)]$ too. But given that the properties are not determinately distinct, it must therefore also be indeterminate whether a possesses $\lambda x\ [\nabla(x = a)]$. But if it is indeterminate whether a possesses $\lambda x\ [\nabla(x = a)]$, then it cannot be determinately true that $\sim\nabla(a = a)$. So, ultimately, Lowe must also reject premise (3). As we have already argued that premise (3) is beyond reproach, we reject Lowe's argument on this basis. This does not, of course, offer a diagnosis of what is wrong with Lowe's argument. But we think the matter is clear enough. Despite Lowe's claim to the contrary, '$\lambda x\ [\nabla(x = a)]$' and '$\lambda x\ [\nabla(x = b)]$' must pick out properties that are determinately distinct.[7]

[7]French and Krause have, in various places (e.g. 1995, 2003, 2006), defended the view that in quantum mechanical cases such as the one that Lowe describes, it is indeterminate whether the particles involved are identical. Unlike Lowe, however, they explicitly endorse the view that such quantum particles are non-identical with themselves (they refer to particles with such a characteristic as 'non-individuals') (1995, p. 24; 2003, p. 109; 2006, p. 143). This does allow them to respond to Evans's argument, but in a way that we have explicitly rejected (i.e. way (i) – by rejecting premise (3)). Above we said that it is dialectically inappropriate to deny premise (3) purely in order to reject Evans's argument. But French and Krause take themselves to have independent reasons for rejecting premise (3) with regard to quantum particles – that is, they think that considerations from quantum mechanics itself strongly support the view that quantum particles are non-individuals. To consider French and Krause's arguments for this claim in any detail would take us beyond the scope of the current essay. But briefly, we do not think French and Krause are right about this. French and Krause's main argument for the conclusion that quantum particles are non-individuals, and so indeterminately identical with each other and non-identical with themselves, is in effect that such particles are absolutely indistinguishable (i.e. that they share all of their non-identity-involving properties) (2003, p. 99; 2006, ch. 4). But we simply do not see why we are supposed to conclude that two entities are indeterminately identical, and so non-identical with themselves, from the fact that they are indistinguishable. And even prescinding from this, we have further worries about French and Krause's view. For one, it is difficult to see how it is possible to secure determinate reference to a non-individual in the first place. And secondly, if it is true that for some quantum particle a, that it is not the case that $a = a$, then how can it be indeterminate, for some quantum particle b, whether $a = b$ (for surely, if it is not the case that $a = a$, then it should also be that it is not the case that $a = b$). But whether or not these latter worries amount to anything, we do reject the view that anything can fail to be self-identical, and can see no reason to revise our opinion based on quantum mechanical considerations. So we reject French and Krause's response to Evans's argument along with all other responses that deny premise (3).

15.4 Barnes's Denial of Abstraction

Barnes, in giving her argument for rejecting the abstraction steps in Evans's argument, first gives an interpretation of the determinacy operators wherein they are considered to be quantifiers that range over the possible worlds that are admissible precisifications of the actual world (Barnes 2009, pp. 83–85). Her thought is that if identity itself is vague, then (because this is a species of ontic vagueness) the world itself is vague, so there will be many precise ways that the world could have been, and it will be indeterminate whether the world itself is one of those ways. Thus, ΔP iff P is true in all possible worlds that are admissible precisifications of the actual world or false in all possible worlds that are admissible precisifications of the actual world, and ∇P iff P is true in some, but not all, possible worlds that are admissible precisifications of the actual world. The result is that if there is vagueness in the actual world with regard to any P, then there are (at least) two possible worlds in the space of (admissible) precisifications, one of which represents the world as being P and one of which represents the world as being not-P.

In Evans's argument, it is crucial that the predicate '$\lambda x\ [\nabla(x = a)]$' picks out the same property when used in (2) as it does when used in (4). But Barnes proposes that we read *de re* property ascriptions that are prefixed with the determinacy/indeterminacy operator in a counterpart-theoretic manner. So, because in counterpart theory *de re* modal predication is inconstant, if we do so, then the predicate '$\lambda x\ [\nabla(x = a)]$' picks out a different property in (2) from the property it picks out in (4) (2009, pp. 89–93). Taking counterparts of actual individuals to be individuals from worlds that are admissible precisifications of the actual world, when prefixed to 'b', Barnes argues, '$\lambda x\ [\nabla(x = a)]$' refers to the property *having some b-counterparts that are a-counterparts and some b-counterparts that are not a-counterparts*, but when prefixed to 'a' it refers to the property *having some a-counterparts that are a-counterparts and some a-counterparts that are not a-counterparts*. So, premise (2) of Evans's argument now says that b has the former property and (4) says that a lacks the latter property. Evans's argument would still go through if a and b differed over whether they have *these* properties – but they do not. Just as b has the former property, so does a, and just as a lacks the latter property, so does b.

All of the above should sound quite familiar to those who know the literature on contingent identity.[8] The response isn't just analogous to the response given by the defender of contingent identity – it *is* that response with the modal quantifiers restricted to precisifications. Barnes, of course, is fully aware of this and in fact presents her account as standing or falling with that of the contingent identity theorists:

> Indeterminate identity simply is contingent identity, where 'contingency' here is the contingency defined on the restricted necessity that is determinacy. Thus, any vindication

[8] For those who do not, see Gibbard (1975) for a classic introduction.

> of the coherence of (absolutely) contingent identity will automatically yield a vindication of the coherence of indeterminate identity. The coherence of both stands or falls together. (Barnes 2009, p. 93)

However, we think that Barnes obscures the real issue here. It is true that if either of the accounts above is *consistent*, then so is the other, and if either is *inconsistent*, then so is the other. But the real question is whether the account Barnes develops deserves the name she bestows upon it. It is one thing to develop a consistent account that diagnoses a fallacy in the Evans argument, and quite another to develop a consistent account that diagnoses a fallacy in the Evans argument *and* deserves to be called an account of vague identity. We are in no doubt that Barnes achieves the first of these things, but we do not think that she achieves the second.

To see this, first consider contingent identity again. The thought that an object a is contingently identical to an object b is the thought that a and b *are* identical, but that they might not have been identical (and, in general, the thought that an object a is contingently F is the thought that a is F, but might not have been). Anyone wishing to defend contingent identity has to respect this thought. If they fail to do so, no matter whether their account is consistent or not, it will not deserve to be called an account of contingent identity. To illustrate this idea further, first note that by replacing the delta operator '∇' in Evans' argument against indeterminate identity with an operator 'C' that is read 'it is contingent whether', we can obtain an exactly parallel argument against contingent identity:

(1) $\mathrm{C}\,(a = b)$
(2) $\lambda x\,[\mathrm{C}\,(x = a)]\,b$
(3) $\sim\mathrm{C}\,(a = a)$
(4) $\sim\lambda x\,[\mathrm{C}\,(x = a)]\,a$
(5) $\sim(a = b)$[9]

Suppose one responds to this argument by diagnosing a subtle fallacy, but that in order to make the diagnosis one ends up being committed to the claim that all instances of contingent identity are cases in which a and b actually designate *distinct* objects. In making such a reply, one wouldn't necessarily have made any logical error – but one would have changed the subject. One's account will not deserve to be called an account of contingent identity. Of course, by adopting a counterpart-theoretic account of modal predication, contingent identity theorists can identify a fallacy in the argument without having to change the subject in this way. According to their account, when an object a and an object b are contingently identical, they actually share all properties (including modal ones) and so turn out to be *actually* identical.

But now consider Barnes's so-called account of vague identity. Precisely because it is modelled on the contingent identity account, Barnes is committed to the view

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[9]That this is so is by no means a new insight. Noonan (1991) makes just this point. Incidentally, in that paper it is also argued that one cannot respond to Evans's argument in a counterpart-theoretic manner (see p. 191).

that whenever we have a case such that it is indeterminate whether an object a and an object b are identical, then it might be that a and b are *actually* identical. That is, the truth of 'it is indeterminate whether $a = b$' is consistent with the truth of '$a = b$'. But this, we think, *does* change the subject. The claim that an identity statement '$a = b$' is indeterminate due to the vagueness of identity itself precludes the actual identity of a and b (and in general the claim that 'a is F' is indeterminate due to ontic vagueness precludes the actual truth of 'a is F'). If identity itself is vague, then it must be that a and b are *not* actually identical, *nor* actually non-identical. So, we think that even though Barnes produces a consistent account that Evans's argument does not refute, we do not think that it deserves to be called an account of vague identity.

That completes our defence of Evans's argument. We thus endorse the conclusion that identity itself cannot be vague.

15.5 Ontic Indeterminacy of Identity Survives

Evans's argument establishes that identity cannot be vague, and so that if a statement of identity is indeterminate in truth-value one of the terms flanking the identity sign must be referentially indeterminate. Evans's article is, of course, in fact entitled 'Are there vague objects?' It clearly never occurred to him that someone might take on board its lesson but endorse the possibility of vague objects, but to many subsequent authors it has seemed evident that there is no inconsistency in this position. A vague object is not an object with indeterminate identity. It is an object, such as the eponymous cloud, with indeterminate boundaries and it seems obvious that Evans's argument cannot show that vague objects, so defined, are impossible (e.g. Edgington 2000; Tye 2000). Even if the argument succeeds, it seems, all it can establish is that every vague object is determinately distinct from every precise object and every other vague object. So it is consistent to hold both that there are vague objects and that the identity relation is precise – in the sense that in any identity statement of indeterminate truth-value, one of the terms flanking the sign of identity must be indeterminate in reference between several, possibly vague, candidate referents.

Furthermore, Evans's argument does not, at least by itself, establish that either of the following two theses is false:

1. Identity statements can be indeterminate for ontic reasons.
2. Identity statements can be indeterminate *in virtue of* the existence of vague objects.

One might think that theses 1 and 2 amount to the same thing. But they do not. In this section, we first show, by drawing upon Williams (2008), how the existence of vague objects can give rise to indeterminate identity statements in a way that is consistent with the soundness of Evans's argument. We then show that there can be ontic indeterminacy that involves only *precise* objects and that gives rise to indeterminate identity statements in a way that is consistent with the soundness of Evans's argument. In the sections that follow, we offer supplementary arguments to show that vague objects cannot, after all, exist (i.e. that thesis 2 is false). But the supplementary arguments will not show that there cannot be ontic indeterminacy that involves only precise objects, and so will leave thesis 1 standing.

Above we said that the terms 'a' and 'b' are to be taken as being referentially determinate in Evans's argument. If instead we take either (or both) terms to be referentially indeterminate, it is easy to show that the abstraction steps fail and so that the argument is invalid. If 'b' is referentially indeterminate, for example, it does not follow from $\nabla(a=b)$ that there is some object that satisfies the predicate 'λx $[\nabla(x=a)]$', and a fortiori it does not follow that b does. It is perhaps easiest to see that this is so by considering a concrete example (the example is due originally to Shoemaker (Shoemaker and Swinburne 1984: 146)):

Example: Alpha Hall and Beta Hall

> Jones is lecturing in Alpha Hall. Alpha Hall is linked by a flimsy walkway to Beta Hall in such a way that is unclear whether Alpha Hall and Beta Hall count as two distinct buildings or merely as two parts of one and the same building.

It is clear that here the identity statement 'The building in which Jones is lecturing is identical to Alpha Hall' is indeterminate in truth-value. But it also seems perfectly clear why this is so. The term 'building' is such that it is indeterminate whether it applies to the whole structure or just to the two halls. So, it is indeterminate what 'the building in which Jones is lecturing' denotes. There are two perfectly precise candidates for the denotation of the term, namely, Alpha Hall itself, or the structure consisting of Alpha Hall and Beta Hall. But neither of these perfectly precise things is such that it has the property of being indeterminately identical to Alpha Hall. The first is determinately identical to it, and the second determinately non-identical. So the abstraction step fails.

According to the most well-known account of the phenomenon, referential indeterminacy is a matter of semantic indecision. The world contains only precise objects, but we have not fully fixed the application conditions of our terms, so that in certain cases it is indeterminate which precise objects they apply to. So, the reason why the term 'the building in which Brown is lecturing' is referentially indeterminate is that we have just not decided how to apply the general term 'building' in this case. The application conditions of the general term are such that in borderline cases like this they supply no determinate answer to questions regarding their application. On this view, however, if we liked, we could precisify our terms by laying down more precise application conditions that do supply answers in all cases.

The above account has dominated the literature on semantic indeterminacy, and many seem to assume that it is the *only* account of referential indeterminacy available. But it is not. This is what Williams (2008) makes clear. If we suppose that there can be vague objects, namely, objects that are such that it is indeterminate where their boundaries are, then referential indeterminacy can arise without semantic indecision. Williams gives an example involving identity over time to illustrate the point in his 2008 'Multiple Actualities and Ontically Vague Identity', but that it involves identity over time is not an essential feature of the example and in fact adds extra complications. In a later paper, Barnes and Williams (2009, p. 181) give a synchronic example to illustrate the point that serves just as well, so here we use that example.

Example: Table and Front

Table is a vague object and it is ontically indeterminate whether it is located between you and Wardrobe or in another room. 'Front' is a name introduced to refer to whatever object is in front of you. 'Front' is thus referentially indeterminate between Table and Wardrobe, and thus the identity statement 'Table = Front' is indeterminate in truth-value.

Here, then, we have an example of an identity statement that is indeterminate in virtue of the existence of a vague object that is consistent with the soundness of Evans's argument.[10]

The above kinds of case arise because, as Lewis (1984) has argued, reference is both a matter of what we do to fix the meanings of the terms we use and the way the world is. Sometimes, as the defender of semantic indecision maintains, referential indeterminacy arises because we have not done enough: 'our reference-fixing procedures fail to isolate one amongst a range of suitable candidates' (Williams 2008, p. 147). But it is also possible that we have done all that we can, but that we still fail to refer determinately because the world does not play its part. Such cases will be cases of ontic indeterminacy of reference. Williams seems to think that ontic indeterminacy of reference can only arise in the kind of way he describes, that is, in virtue of the existence of vague objects. But this, we think, is false. Ontic indeterminacy of reference can arise even in the absence of vague objects. And when it does it can also give rise to identity statements that are indeterminate in a way that is consistent with the soundness of Evans's argument.

Consider the following suggestive passage from Quine (drawn to our attention by Greenough (2008)):

> Where to draw the line between heaps and non-heaps... or between the bald and the thatched, is not determined by the distribution of microphysical states, known or unknown it remains an open option... On this score the demarcation of the table surface is on a par with the cases of heaps and baldness. But it differs in those cases in not lending itself to any stipulation, however arbitrary, that we can formulate; so it can scarcely be called conventional. It is neither a matter of convention nor a matter of inscrutable but objective fact. (Quine 1981, p. 94)

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[10]Vague objects, if such there be, must be weird. But Table is very weird indeed. An example due originally to Hawley (2002) provides us with a (comparatively) less weird example:

Example: Hawley's Mouse

Algernon and Socrates are two mice in a cage. Whilst Socrates is a perfectly precise mouse, Algernon is a vague object. Algernon's vagueness consists in indeterminacy in whether his tail, which is hanging by a thread, is a part of him. It may then be that 'the largest mouse in the cage' is indeterminate in reference between Algernon and his more fortunate companion Socrates with an intact, but shorter, tail. So, the identity statement 'the largest mouse in the cage = Socrates' is indeterminate in truth-value.

This passage is drawn from a discussion of bivalence, and Quine goes on to say that if we commit ourselves to bivalence, then we are committed to 'treating the table as one and not another of this multitude of imperceptibly divergent physical objects' (ibid.). But if, as we should, we allow statements to be indeterminate in truth-value and so reject bivalence, what becomes of the point Quine is making? In short, that sometimes there may be nothing that we can do, even in principle, to secure reference to one thing rather than another. To successfully refer requires not only that we engage in reference-fixing activities but also that there be eligible referents. But even once we have done everything that we can do, there may still be ties in eligibility, and if there is, there will be referential indeterminacy. And such referential indeterminacy will be properly classified as *ontic* rather than semantic, for it will be the world, and not us, that isn't playing its part.

That there is ontic referential indeterminacy of this kind is, we think, entirely plausible. Indeed, it is plausible that it is widespread. If there are no vague objects in the vicinity of the table, or the cloud, or the mountain, or indeed any macroscopic object, but just a host of overlapping precise ones, it is overwhelmingly plausible that we do not secure reference to any one of them rather than another using those general terms of our language that we use to construct singular referring terms. The defender of semantic indecision supposes that there are precisifications, that is, ways in which we could, in principle, revise our general terms in order to secure a determinate reference. But it is plausible that this is mere fantasy. We are not suggesting here that it is a fantasy to suppose we can always find a way to refer to any of the precise objects that stand in the vicinity of ordinary macroscopic objects. What we are suggesting is a fantasy is that we can always do so using terms that count as precisifications of our extant general terms. At any rate, if what we are suggesting is right in even a single possible case, then there will be the possibility of identity statements that are ontically indeterminate in the absence of vague objects in a way that is consistent with the soundness of Evans's argument. Suppose, for example, that we single out a precise mountain-like object in the vicinity of some mountain using the singular term 'M'. And suppose that 'mountain' is one of those general terms that cannot be precisified to secure a determinate reference. Now consider the identity statement 'the mountain on the horizon is identical with M'. This may be indeterminate in truth-value due to the referential indeterminacy of the term 'the mountain on the horizon'. But the referential indeterminacy will be ontic rather than being a matter of semantic indecision.

15.6 Ontic Indeterminacy in Boundaries Entails Vague Identity

So far we have argued that Evans's argument against vague identity can be defended. Our conclusion is that it establishes that if a statement of identity is indeterminate in truth-value, one of the terms flanking the sign of identity must be referentially indeterminate – an imprecise designator. We have acknowledged, however, that the soundness of Evans's argument is consistent with the existence of vague objects and that it is even consistent with existence of identity statements that are indeterminate in virtue of the existence of vague objects. But are there vague objects and can Evans's argument be used to in fact demonstrate that they are impossible?

The question we are concerned with can be expressed as follows. Does ontic indeterminacy in boundaries entail ontic indeterminacy in identity? One understanding of this question can be expressed in this way. If a is an object with an indeterminate boundary, does it follow that there must be an indeterminate statement of identity '$a = b$', in which 'b' as well as 'a' is determinate in reference? For example, if Kilimanjaro (henceforth, K) has a vague boundary because a particle, Sparky (henceforth S), is neither indeterminately part of it nor determinately not a part of it, does it follow that there must be some indeterminate statement of identity 'K $= c$', in which 'c' as well as 'K' is determinate in reference (see Weatherson 2003, p. 222)? The sense in which it is correct to say that Edgington and Tye and the view they represent are right is that if the question whether ontic indeterminacy in boundaries entails ontic indeterminacy in identity is understood in this way, its answer is negative.

It will be useful to look at an argument for an affirmative answer (given in Weatherson 2003) and how it must be answered, to see to what the defender of ontic indeterminacy in boundaries is committed if he endorses Evans's argument. Suppose it is indeterminate whether K contains S, 'K' and 'S' being precise designators and 'contains' determinately having as its extension the set of pairs $<x, y>$ such that y is part of x. Now let us dub the fusion of K and S 'K+' and the fusion of the parts of K not overlapping S, 'K−'. It is definitely the case that K is K+ if and only if S is part of K. And it is definitely the case that K is K− if and only if S is not part of K. But it is indefinite whether K has S as a part. So it is indeterminate whether K is K+ and it is indeterminate whether K is K−. Generalising, if a is any object with indeterminate boundaries, there must be an indeterminate statement of identity '$a = b$', in which 'b' as well as 'a' has a determinate reference ('b' standing to 'a' either as 'K+' stands to 'K' or as 'K−' stands to 'K').

The believer in ontic indeterminacy in boundaries who endorses Evans's reasoning can resist this argument in only one way. He must refuse to assent to the proposition that it is definitely the case that K is K+ if and only if S is part of K. Mutatis mutandis, he must refuse to assent to the proposition that it is determinately true that K is K− if and only if S is not part of K. He must say that in each case the left-hand side of the biconditional of which the proposition is a definitisation is indeterminate in truth-value (since it is an identity statement containing no imprecise terms). So since the right-hand side is indeterminate, the biconditional is not true and its definitisation is false. Hence, he must say that it is not definitely the case that objects with the same parts are identical.

He can say this without inconsistency and still endorse the inference from 'a and b share all their parts' to '$a = b$', just as a supervaluationist who equates truth with truth under all precisifications (supertruth) can endorse the inference from 'p' to 'it is (definitely) true that p' and not accept the conditional 'if p then it is (definitely) true that p'.

Hence, just as the supervaluationist identifier of truth with supertruth must say that 'p' and 'it is definitely true that p' differ in ingredient sense, though they are identical in assertoric content (Dummett 1991, p. 47), the defender of ontic indeterminacy in boundaries who does not wish to dispute Evans's reasoning must distinguish the ingredient senses of 'a and b have the same parts' and '$a = b$', but may maintain the identity of their assertoric contents.[11]

That this is the *only* way the proponent of this position can respond to Weatherson's argument from ontic indeterminacy in boundaries to ontic indeterminacy in identity has been disputed, in effect, by Barnes and Williams in their (2009) paper mentioned earlier, who argue that the believer in vague objects can accept that it is definitely true that things with the same parts are identical and so accept that it is definitely true that K is K+ (K−) if and only if S is (not) part of K, whilst denying that it can be inferred from these propositions and the indeterminacy of 'S is part of K' that there is vague identity in the world in the sense Evans denies. He can do so, they say, because he can reject the assumption that 'K+' and 'K−' are referentially determinate. In fact, they say, the believer in vague objects can say that these terms are another example of referential indeterminacy in the absence of semantic indecision. He can say that the situation with Kilimanjaro is the following. There are two distinct (and hence, consistently with Evans's reasoning, determinately distinct) objects, Kilimanjaro and dual-Kilimanjaro (henceforth K and dual-K). Of each of these, it holds that it is indeterminate whether S is part of it. For each x apart from S, it is determinate that x is part of K if and only if it is part of dual-K. But determinately, S is part of K just in case it is not part of dual-K. So K and dual-K are determinately distinct. Each of 'K+' and 'K−', understood as introduced above, is referentially indeterminate between K and dual-K. Hence, even though 'K is K+' and 'K is K−' are both indeterminate in truth-value, this does not mean that there is ontic indeterminacy in the sense intended to be ruled out by Evans's argument.

The problem with this proposal is straightforward. 'K+' and 'K−' relate to K and dual-K as 'the smartest child' relates to Mary and Jane (if these are the two competing candidates for the title). Just as 'the smartest child has brown hair' is determinately true if and only if each of the candidates has brown hair, 'K+ contains Sparky' is determinately true if and only if each of its candidate referents contains Sparky – and the same is true of 'K− contains Sparky'. So neither of these is determinately true, and neither is determinately false. However, both 'K+ is such that it indeterminately contains S' and 'K− is such that it indeterminately contains S' are true since each of the candidate referents is such that it indeterminately contains S. In fact, if 'K+' and 'K−' are referentially indeterminate terms whose contribution to the truth-conditions of predications of which they are the subject is fixed in the supervaluational manner indicated, they mean the same – misleading spelling aside. So if 'K is K+ if and only if S is part of K' is determinately true and 'K is K− if and only S is not part of K' is determinately true, each of 'K is K+'

[11]This is assuming, of course, that such a defender does not wish to reject the classical mereological inference from identity of parts to identity.

and 'K is K−' is determinately false. (Another comparison may be helpful. K and dual-K may be compared to *i* (the positive square root of minus one) and –*i*. The proposal that 'K+' and 'K−' may be introduced as terms with distinct meanings each referentially indeterminate between K and dual-K is like the proposal that two referentially indeterminate terms may be introduced for each of which the two equally eligible candidate referents are *i* and –*i*.)

The conclusion must be that the believer in ontic indeterminacy in boundaries who does not wish to dispute Evans's argument must say that 'K is K+' is false and deny that it is definitely true that K is K+ if and only if K has S as a part. But this is an option he can take, and he can point to the precedent set by the supervaluationist identifier of truth and supertruth.

Nevertheless, we shall now argue, Evans's argument is not one a believer in vague objects can ultimately endorse, for there are additional kinds of vague object to be considered, as well as those, like Kilimanjaro, that can, prima facie, be pictured as having fuzzy boundaries, and the existence of vague objects of these kinds requires ontic indeterminacy in identity. But it is plausible that if any case of ontic indeterminacy in boundaries is possible – for example, the Kilimanjaro case – then some (other) case that involves ontic indeterminacy in identity is possible. Hence, the possibility of *any* vague objects is incompatible with the soundness of Evans's argument.

We have already seen two examples of vague objects whose vagueness is prima facie different from that of Kilimanjaro, namely, Table and Hawley's mouse. Here's another (gleaned from Lewis 1993, pp. 35–36) for good measure:

Example: Fred's House

> Fred's house is a vague object of which a newly constructed garage is a questionable part. So the reference of 'the largest house in the street' may be indeterminate between it and, say, No. 27.

Fred's house, considered as a vague object, either contains a garage or does not. Its size is either 1,200 m^2 of floor space or 1,000 m^2 of floor space. It is not correctly pictured as, prima facie, Kilimanjaro, or still more obviously a cloud, as having a fuzzy boundary.[12] Of course, someone who wishes to deny the existence of vague objects can redescribe this case as one in which there are two objects – the home, as we may call it, and the mereological sum of the home and the garage – and say that 'Fred's house' is referentially indeterminate, no doubt as a result of semantic indecision. But someone who wishes to say that K is a vague object determinately designated by 'K' can also say the same, without any of evident absurdity, of Fred and 'Fred's house'.

An example of another type of vague object can be provided by again considering Shoemaker's structure mentioned earlier. There we supposed that the example was

[12] Whether it is right so to picture mountains and clouds, of course, depends on whether they have minimal extended parts – a mountain-sized heap of footballs does not have a fuzzy boundary in the sense in use here.

one in which 'the building in which Jones is lecturing' is referentially indeterminate. But suppose now we reconstrue it as an example of a vague object. We said Jones is lecturing in Alpha Hall. Suppose also that Smith is lecturing in Beta Hall. It is definitely true that Jones is located in just one building – even those who think that Alpha Hall and Beta Hall are two buildings agree – so we can speak of '*the* building in which Jones is located'. Mutatis mutandis, we can speak of '*the* building in which Smith is located'. If we think of these descriptions as precise designators of vague objects, we again cannot picture these objects as having fuzzy boundaries. Rather, they must be thought of, like Fred's house or Hawley's mouse, as having either a minimum or maximum spatial extent, but as overlapping:

____________...................... THE BUILDING IN WHICH JONES IS LOCATED
.... ____________ THE BUILDING IN WHICH SMITH IS LOCATED

The continuous line represents the determinate region of each vague object and the line of dots the indeterminate region.[13]

It is tempting to describe cases such as this in a way that is consistent with the non-existence of vague objects by regarding the relevant definite descriptions as referentially indeterminate, but there is no obvious reason why a believer in vague objects cannot regard the descriptions instead as precise designators of vague objects.

These cases, of course, have temporal analogues which it is much more tempting to describe as involving vague objects.

van Inwagen describes a case (mentioned earlier) in which a man, Alpha, goes into a fiendish Cabinet which disrupts those identity-relevant elements of his existence to just such an extent that Omega, the man who emerges, is neither definitely Alpha nor definitely not Alpha. It is tempting to think that the descriptions 'the man who entered' and 'the man who emerged' are neither indeterminate in reference nor empty. But if so Alpha and Omega have to be regarded as vague objects, and they must also be regarded as having vague temporal extents: it is indeterminate whether Alpha, the man who enters, exists later and indeterminate whether Omega, the man who emerges, existed earlier. Another example of the same form is Shoemaker's Brown/Brownson case (in which Brownson post-transplant has the brain and psychology of the pre-transplant Brown and the body of the pre-transplant brain donor Robinson) if we suppose (contrary to Shoemaker's own view) that the case is one of indeterminacy in temporal boundaries (because our criteria for personal identity give weight both to psychological continuity and bodily continuity or because, say, Brown's psychology is not perfectly replicated in Brownson).

Now in cases of this type, it can be argued that ontic indeterminacy in boundaries entails ontic indeterminacy in identity. Assume that Brown is alone in Room 100

[13]Other examples of the same type are Edgington's landmass which is either two mountains divided by a valley or one twin-peaked mountain and van Inwagen's example of two places connected by a narrow and frequently inundated isthmus that is not definitely land or sea (Edgington 2000, p. 40; van Inwagen 1990, p. 243).

before the transplant and Browson is alone in Room 101 afterwards. Then (1) It is determinately the case that there is just one person in Room 100 before the transplant, (2) it is determinately the case that there is just one person in Room 101 after the transplant, (3) someone who is determinately in Room 100 before the transplant is such that it is indeterminate whether he is in Room 101 afterwards, and (4) someone who is determinately in Room 101 after the transplant is such that it is indeterminate whether he is in Room 100 before the transplant.

Now the person in Room 100 before the transplant is not determinately identical with the person in Room 101 after the transplant. Are they determinately *non*-identical? This is inconsistent with the description given of the case as one in which there is just one person in Room 100 before the transplant (for Brown is determinately there, and Brownson is indeterminately there, so if Brown and Brownson are determinately distinct, it cannot be that it is determinately true that there is *just one* person there). But it cannot be denied even by the vague object theorist that there is just one person in Room 100 before the transplant since even those who think that the man who receives the new body dies and Brownson is someone else (Robinson or someone new), so that the situation involves two determinately distinct people, agree that before the transplant just one man is in Room 100, and, of course, those who think of the transplant as merely providing Brown with a new body think there is just one person all along and hence just one person in Room 100 before the transplant. So the vague object theorist who thinks of the case as involving ontic indeterminacy in temporal boundaries must accept that the person in Room 100 before the transplant is indeterminately identical with the person in Room 101 after the transplant.

The same reasoning applies in the structurally analogous case of putative indeterminacy in spatial boundaries. Given that (1) it is determinately the case that there is just one building in which Jones is located, (2) it is determinately the case that there is just one building in which Smith is located, (3) some building in which Jones is determinately located is such that it is indeterminate whether Smith is located in it, and (4) some building in which Smith is determinately located is such that it is indeterminate whether Jones is located in it, it follows that the building in which Jones is located is indeterminately identical with the building in which Smith is located if each of the descriptions 'the building in which Jones is located' and 'the building in which Smith is located' is a referentially determinate designator of a vague object. For it cannot be denied that there is just one building in which Jones (mutatis mutandis, Smith) is located since even those who think that Alpha Hall and Beta Hall are two buildings agree with that.

Thus, some cases of ontic indeterminacy in boundaries do involve ontic indeterminacy in identity, even if others, like the Kilimanjaro case, do not.

But, as said previously, it is plausible that if there are (in logical space) any cases of ontic indeterminacy in boundaries at all there must be (in logical space) cases of the type just described. So in this sense it is true that the possibility of cases of ontic indeterminacy in boundaries entails the possibility of ontic indeterminacy in identity.

15.7 Vague Identity Entails Ontic Indeterminacy in Boundaries

However, as we shall now argue, even if the possibility of ontic indeterminacy in identity is acknowledged, it cannot by itself account for all vagueness in reality. Not only does ontic indeterminacy in boundaries entail ontic indeterminacy in identity, but also the converse holds.

The crucial point here is that whenever a singular statement of identity is indeterminate, there will also be a statement not involving the concept of identity which is indeterminate.[14] It is indeterminate whether van Inwagen's Alpha, the person who enters the fiendish Cabinet, is Omega, the person who emerges. So it is indeterminate whether someone both enters and emerges. It is indeterminate whether Brown is Brownson. So it is indeterminate whether someone is both in Room 100 pre-transplant and in Room 101 post-transplant. Other statements not involving identity will similarly be indeterminate in truth-value. Thus, if Brown is thin and Robinson, the body donor, is fat, it will be indeterminate whether someone is both thin at the earlier time and fat at the later time. The point, of course, holds generally, not just for persons. If it is indeterminate whether the new church is the old church, it will be indeterminate, say, whether some church which was made wholly of granite is now partly made of brick. And in Shoemaker's case of Alpha Hall and Beta Hall, it is indeterminate whether there is a building in which both Jones and Smith are located.

Of course, all these statements are logically equivalent to ones in which the concept of identity figures (so is any statement). But their indeterminacy cannot be explained just by reference to indeterminacy in identity – just as we cannot explain the (possibly multiply determined) indeterminacy in 'the smartest child is tall' just by reference to indeterminacy in identity. If a statement is indeterminate in truth-value, some expression occurring in *it* (or some grammatical feature of *it*) must be the location of the indeterminacy; it can be neither necessary nor sufficient that some expression not occurring in it is characterised by indeterminacy.

So consider again the case of Brown and Brownson. It is indeterminate whether Brown is Browson. So it is indeterminate whether the man in Room 100 before the transplant is the man in Room 101 afterwards. So it is indeterminate whether there is a man who is both in Room 100 before and in Room 101 afterwards.[15] To make sense of this reference to indeterminate identity is insufficient. We need, if we are thinking of the indeterminacy as ontic, to appeal to the notion of a vague object. Moreover, because of the temporal symmetry of the situation, comparable to the spatial symmetry in the Alpha Hall/Beta Hall case, *one* vague object cannot suffice. It is definitely true that some person exists at the earlier time and definitely true that some person exists at the later time. So to postulate a single vague object which is

[14]If only because if '$a = b$' is true '$\exists x(Ax\&Bx)$' is true in which the predicates 'A' and 'B' relate to 'a' and 'b' as 'Socratizes' relates to 'Socrates'.

[15]Though it is definitely true that there is a man, just one, in Room 100 before and definitely true that there is a man – just one – in Room 101 afterwards.

indeterminately a person is inconsistent with the description of the situation as one in which it is definitely true that a *person* exists at the earlier time and definitely true that a *person* exists at the later time. And if there is a single vague object which is determinately a person but is such that it is indeterminate whether it exists at the earlier time and indeterminate whether it exists at the later time, it is false that the situation is one in which it is *definitely* true that a person exists at the earlier time and *definitely* true that a person exists at the later time. So to accept the case of Brown and Brownson as a case of ontic indeterminacy in identity, one must accept that there are at least two vague objects sometime present, one of which determinately exists at the earlier time and is such that it is indeterminate whether it exists at the later time and the other of which determinately exists at the later time and is such that it is indeterminate whether it exists at the earlier time. If the Brown/Brownson case is one of ontic indeterminacy in identity, therefore, it is also one of ontic indeterminacy in boundaries.

15.8 Evans's Conclusion by a Safer Route

But, of course, we can now argue that, contrary to the description given of the case as one in which it is definitely true that there is just one person present earlier and definitely true that there is just one person present later, these two vague objects are determinately distinct, since one of them has the property: *is such that it determinately exists at the earlier time (and is thin then)* and the other lacks this property. The reasoning here parallels that in Evans's argument but appeals only to the identity-free property: *is such that it determinately exists at the earlier time (and is thin then)*, the possibility of which was drawn attention to earlier in the discussion of Lowe.

It may be that there are vague objects which can only be distinguished by identity-involving properties. The reasoning just given cannot refute the possibility of their ontic indeterminacy in identity. For that we need Evans's argument. Setting these aside, we can conclude that any case of ontic indeterminacy in identity must involve ontic indeterminacy in boundaries and hence, by the reasoning just given, will after all be one in which the putatively ontically indeterminately identical objects are distinct after all. If, as is plausible, the possibility of cases of ontic indeterminacy in the identity of vague objects only distinguished by identity-involving properties entails the possibility of cases of ontic indeterminacy in the identity of vague objects which can be distinguished by identity-free properties, we can conclude that vague identity is impossible *simpliciter*.

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Chapter 16
Evans Tolerated

Elia Zardini

16.1 Borderline Identity In Re

Consideration of *vagueness* and *soritical series* strongly suggests that there have to be cases in which it is *borderline* whether an object x is *identical* with an object y. For example, it is a very plausible metaphysical assumption that we can go from a human being to a dumpling through a very long but finite stepwise process, where at each step we simply move one single atom in the whole universe merely within a nanometre's distance. Consider then Greg's seamless transformation into a dumpling, dividing it into a very long finite sequence of subsequent times $t_0, t_1, t_2 \ldots, t_k$ and, for every i such that $0 \leq i \leq k$,[1] calling 'Greg$_i$' the Greg-related substance (Greg, the dumpling, sort of Greg and what have you) wholly present at t_i. Given the other very plausible, mildly essentialist metaphysical assumption that nothing that is human at a time can be a dumpling at the same or other time, Greg$_0$ (i.e. Greg) is distinct from Greg$_k$ (i.e. the dumpling). However, since the transformation of the former into the latter is vague, it cannot be the case that, for every i, either it is definite[2] that Greg$_0$ is identical with Greg$_i$ or it is definite that

[1]Henceforth, this or similar qualifications on the values of 'i' will be implicitly understood.

[2]Throughout, I follow the established practice in the literature of using 'It is definite that P' and its like in such way that 'It is borderline whether P' is *strongly equivalent* with 'It is neither definite that P nor definite that it is not the case that P' (e.g. so strongly as to guarantee that they are *fully intersubstitutable* even in logics—which will be prominent in this paper—in which mere logical equivalence does not guarantee that).

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Greg_0 is distinct from Greg_i; for whence could the vagueness of the transformation then arise? Thus, it has to be the case that, for some i, it is borderline whether Greg_0 is identical with Greg_i (let such i be l).[3] And that is really a question about whether Greg_0 is identical with the *object itself* that is Greg_l, rather than really a question about the *referent* of 'Greg_l': for that referent has in effect been introduced as "the Greg-related substance wholly present at t_l", and where are the two or more objects that are equally good candidates for being such a substance?[4] An analogous consideration holds of course for the referent of 'Greg_0',[5] with the upshot that the

[3]The text has in effect just moved from a claim of the form 'It is not the case that, for every x, P' to a claim of the form 'For some x, it is not the case that P'. That move is notoriously *intuitionistically* invalid and admittedly less than self-evident in contexts in which reference is made to "*unsurveyable*" totalities like soritical series. However, for the purposes of this paper, the move will be taken as granted (see Zardini 2013c for some discussion).

[4]As the phrase 'wholly present' indicates, I'm assuming a plausible *three-dimensionalist* view of *persistence* of substances across time (according to which, roughly, substances are typically present at different times and may be so present without different parts of them existing at different times). But, contrary to what some authors seem to think (see e.g. Noonan 1982), borderline identity in re (henceforth, simply 'borderline identity') cannot be avoided by simply switching to a *four-dimensionalist* view (e.g. saying that, although it is definite that 'Greg_l' refers to a substance having as part the temporal slice present at t_l, it is borderline whether it refers to a substance having also as part the temporal slice present at t_0 or to a substance having also as part the temporal slice present at t_k). For an analogous example could be given in *modal* rather than *temporal* terms, and few of us would be tempted by the idea—discussed e.g. by Weatherson (2013)—that a typical substance is not wholly present in all the worlds in which it is present (nor would many of us be tempted by the idea—defended e.g. by Lewis 1986—that, in a fundamental sense, a typical substance is present in only one world). In fact, given the plausible assumptions that there could be extended simples (as argued e.g. by Markosian 1998) and that these could have fuzzy boundaries, an analogous example could be given in *spatial* rather than *modal* or *temporal* terms (contrary to the suggestion of Williamson 1994, pp. 255–256, to the effect that fuzzy spatial boundaries—which are certainly one of the prominent ways in which objects themselves can be thought to be vague—do not imply borderline identity). Notice that the thrust of these points consists in foreclosing the existence of (temporal, modal, spatial) *parts* that would give rise to several equally good candidates for being the referent of the relevant expression. However, the foe of borderline identity might try to conjure up such candidates in other ways. For example, in the case of Greg_0, she might claim that there are at least two three-dimensional substances, Greg′ and Greg″, such that, while Greg′ has ceased to exist at t_l, Greg″ still exists at t_l. She might then hold that it is borderline whether Greg is Greg′ or Greg″, so that there is nothing 'Greg' definitely refers to. Unfortunately for the foe of borderline identity, the metaphysics required by this non-mereological move—implying that there are indefinitely many human beings, with slightly different persistence conditions, all of which are spatially co-located at t_0 and collectively undergo a dramatic mass death as Greg's transformation unfolds—is still in these respects wildly implausible (notice that some of the problematic features just alluded to are shared by the mereological move, which can however at least provide a ready explanation of the different persistence conditions and of the spatial co-location at a time). I'm grateful to Aurélien Darbellay, Dan López de Sa and Giovanni Merlo for urging clarifications of an earlier version of this footnote.

[5]For which we could also adduce the additional consideration that, extremely plausibly, there is something (i.e. Greg) that is definitely Greg, from which it follows that there is something that 'Greg' definitely refers to (since it is definite that being Greg implies being referred to by 'Greg'), and so that there is something that 'Greg_0' definitely refers to (since it is definite that being referred to by 'Greg' implies being referred to by 'Greg_0').

question facing us is really whether an object x (i.e. Greg$_0$) is identical with an object y (i.e. Greg$_l$).

Once we grant ourselves a liberal enough ontology of properties (according to which, roughly, for every ordinary predicate 'F', there is such an object as the property of being F, and according to which properties are wholly present in each of the objects that exemplify them), cases of borderline identity multiply (as sketched by Priest 1991, p. 294, who attributes the point to Jean Paul van Bendegem). Consider a standard soritical series for baldness, with a seamless transition from bald people to non-bald people, dividing it into a very long finite sequence of people $b_0, b_1, b_2 \ldots, b_k$ (with, for every i, b_i having exactly i hairs on his scalp) and, for every i, calling 'baldness$_i$' the baldness-related property (baldness, non-baldness, sort of baldness and what have you) exemplified by, and wholly present in, b_i. Given that b_0 is bald while b_k is not, baldness$_0$ (i.e. baldness) is distinct from baldness$_k$ (i.e. non-baldness). However, since the transformation of the former into the latter is vague, it cannot be the case that, for every i, either it is definite that baldness$_0$ is identical with baldness$_i$ or it is definite that baldness$_0$ is distinct from baldness$_i$, for whence could the vagueness of the transformation then arise? Thus, it has to be the case that, for some i, it is borderline whether baldness$_0$ is identical with baldness$_i$ (let such i be l). And that is really a question about whether baldness$_0$ is identical with the *object itself* that is baldness$_l$, rather than really a question about the *referent* of 'baldness$_l$': for that referent has in effect been introduced as "the baldness-related property exemplified by, and wholly present in, b_l", and where are the two or more objects that are equally good candidates for being such a property?[6] An analogous consideration holds of course for the referent of 'baldness$_0$',[7] with

[6]It is common to think that such candidates are easily available. For example, in the case of 'baldness$_0$', it is common to think that such candidates could be: the property of having at most l hairs on one's scalp, the property of having at most $l + 1$ hairs on one's scalp, the property of having at most $l + 2$ hairs on one's scalp etc. Unfortunately, this common thought forgets that the stuff about hairs on one's scalp is just a useful oversimplification of what baldness consists in: it is definite that a man with l hairs on his scalp that are however much thicker than normal and uniformly distributed so as to cover the whole of his scalp with a bushy mane is not bald, and so, after all, the simple property of having at most l hairs on one's scalp is not a good candidate for being the referent of 'baldness$_0$'. And, as in so many other cases of conceptual analysis, the prospects of coming up with a series of more complex precise properties strong enough as to rule out all definite cases of non-baldness and at the same time weak enough as to rule in all definite cases of baldness are bleak. Even setting this aside, points similar to those raised at the end of Footnote 4 apply here: it is metaphysically wildly implausible to think that there are indefinitely many trichological disorders, with slightly different exemplification conditions, all of which conspire to affect poor b_0.

[7]For which we could also adduce the additional consideration that, extremely plausibly, there is something (i.e. baldness) that is definitely baldness, from which it follows that there is something that 'baldness' definitely refers to (since it is definite that being baldness implies being referred to by 'baldness'), and so that there is something that 'baldness$_0$' definitely refers to (since it is definite that being referred to by 'baldness' implies being referred to by 'baldness$_0$').

the upshot that the question facing us is really whether an object x (i.e. baldness$_0$) is identical with an object y (i.e. baldness$_l$). Thus, once we grant ourselves a liberal enough ontology of properties, every case of a soritical series can be turned into a case of borderline identity. Although *predication* is distinct from *identification*, there is arguably enough of a connection between the two to turn cases of borderline predication into cases of borderline identification.

16.2 Evans' Argument and Extensions Thereof

The considerations in Sect. 16.1 notwithstanding, Evans (1978) heroically argued that borderline identity is impossible (and went even so far as to suggest that this would show that vague objects are impossible).[8] Evans' argument in favour of the *definiteness* of identity is really just a rewrite of the argument in favour of the *necessity* of identity popularised by Kripke (see e.g. Kripke 1971, p. 136) and, freely adapted, goes as follows (letting $\mathcal{D}\varphi$ mean 'It is definite that φ'):

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\varnothing \vdash x=x}\,\text{reflexivity}}{\varnothing \vdash \mathcal{D}(x=x)}\,\text{simple definitisation} \qquad \dfrac{}{x=y, \mathcal{D}(x=x) \vdash \mathcal{D}(x=y)}\,\text{indiscernibility of identicals}}{x=y \vdash \mathcal{D}(x=y)}\,\text{transitivity}}{x=y, \neg\mathcal{D}(x=y) \vdash \varnothing}\,\neg\text{-L}}{\neg\mathcal{D}(x=y) \vdash x \neq y}\,\neg\text{-R}$$

(let's call this argument 'argument E').[9]

[8]A related argument is offered in Salmon (2005), pp. 243–246 (whose first edition is from 1982), which however also requires some basic principles from the theory of ordered pairs. *Mutatis mutandis*, all the discussion to follow applies equally well to Salmon's argument.

[9]Contrary to other discussions of the same topic, ours is conducted having in view as paradigmatic candidate cases of borderline identity mainly cases of *cross-temporal* and *cross-modal* identity (see Sect. 16.1 and in particular Footnote 4). But, as is well-known, in such cases indiscernibility of identicals has to be handled with some care: for example, we don't want to conclude from the fact that in a world Greg is German and in another world Greg is not German that Greg (in the former world) is distinct from Greg (in the latter world). The properties discernibility across which entails cross-temporal and cross-modal distinctness are naturally thought of as properties exemplified *in an atemporal and amodal way*, like the property of being human, the property of being concrete and the property of being identical with Greg. We do want to conclude from the fact that Greg is human and a dumpling is not human that Greg (at least in any world in which he exists) is distinct from the dumpling (at least in any world in which it exists), no matter precisely in which worlds they happen to exemplify these properties (e.g. Greg may not be human in worlds in which he does not exist). Since the property of being definitely identical with Greg$_0$ is arguably one of the properties that are exemplifiable in an atemporal and amodal way, the application of indiscernibility of identicals required by argument E in cases of borderline cross-temporal or cross-modal identity is legitimate. I'm grateful to Aurélien Darbellay for raising this issue.

Without further justification, Evans claims that $x \neq y$ is inconsistent with $\mathcal{B}(x{=}y)$ (letting $\mathcal{B}\varphi$ mean 'It is borderline whether φ'), presumably on the grounds of the general principle of *inconsistency*:

(INC) φ is inconsistent with $\neg\mathcal{D}\varphi$,

and, I suppose, implicitly assumes that, if $x \neq y$ is inconsistent with $\neg\mathcal{D}(x \neq y)$, since argument E shows that $\neg\mathcal{D}(x = y)$ entails $x \neq y$ it follows that $\mathcal{B}(x = y)$ is inconsistent, presumably on the grounds of the general principle of *transitivity of adjunction of inconsistents*:

($\mathrm{TR}^{\mathrm{ADJ}^{\mathrm{INC}}}$) If φ entails ψ, φ entails χ and ψ is inconsistent with χ, then φ is inconsistent.

Even without (INC) in its full generality, if the logic of definiteness is as strong as **KB**, rewriting the standard argument for the necessity of *distinctness* we could indeed show that $x \neq y, \neg\mathcal{D}(x \neq y) \vdash \varnothing$ holds. In any event, even without assuming that the logic of definiteness is as strong as **KB** (see Field 2000, p. 19 for an objection against the **B**-axiom for $\mathcal{D}$), and, more generally, even without going for a ($\mathrm{TR}^{\mathrm{ADJ}^{\mathrm{INC}}}$)-route, the conclusion of argument E would still seem to suffice for committing one thinking that $\neg\mathcal{D}(x = y)$ holds to think that $x \neq y$ holds, presumably on the grounds of the general principle of *closure of commitment to thinking under logical consequence*:

(CCTLC) If φ entails ψ and one thinks that φ holds, then one is committed to thinking that ψ holds.

And that may seem to be rebarbative, since, if one thinks that $\mathcal{B}(x = y)$ holds, it may seem that one would precisely like to avoid commitment to thinking that $x \neq y$ holds.

Instead of making the simple point made at the end of the previous paragraph, Evans puzzlingly suggests instead that the logic of definiteness may be as strong as **S5**. If that were the case, given the conclusion of argument E, since in **S5** $\neg\mathcal{D}(x{=}y) \vdash \mathcal{D}\neg\mathcal{D}(x = y)$ holds it would follow, by *single-premise closure of definiteness under logical consequence*:

(M) If $\varphi \vdash \psi$ holds, then $\mathcal{D}\varphi \vdash \mathcal{D}\psi$ holds

and transitivity of logical consequence, that $\neg\mathcal{D}(x = y) \vdash \mathcal{D}(x \neq y)$ holds, and so it would follow, by $\neg$-L, that $\mathcal{B}(x = y)$ is indeed inconsistent. Unfortunately for Evans' suggestion, because of *higher-order vagueness* it is widely assumed that the logic of definiteness cannot be as strong as **S5**. However, given simply (M), from the conclusion of argument E it follows that $\mathcal{D}\neg\mathcal{D}(x = y) \vdash \mathcal{D}(x \neq y)$ holds, from which one would expect it to follow that $\mathcal{D}\mathcal{B}(x = y)$ is indeed inconsistent.

Thus, the conclusion of argument E provides the materials for a variety of concerns about the possibility of borderline identity (let's use 'Evans' argument' to refer to argument E plus whatever is one's favoured way of arguing that the conclusion of argument E creates problems for the possibility of borderline identity). In the remainder of this paper, I'll show that there is at least one independently motivated logic of vagueness and definiteness in which argument E fails, and at least one independently motivated logic of vagueness and definiteness in which the conclusion of argument E is harmless. Either way, Evans' argument will fail.

Before seeing that, however, it'll be helpful to reach a more comprehensive perspective on Evans' argument, in particular on argument E and on the style of reasoning it exemplifies. To understand the real punch of argument E, it is useful to ask first why an argument using an equivalence relation weaker than identity like *material equivalence* would not go through. Letting $\equiv$ and B express material equivalence and baldness respectively, since Bb_0 holds, Bb_l is tantamount to $Bb_0 \equiv Bb_l$; therefore, since $\neg\mathcal{D}Bb_l$ holds, so should $\neg\mathcal{D}(Bb_0 \equiv Bb_l)$. But one could then have thought that one can create problems for borderline cases in general *via* an argument analogous to argument E, where the formulas $x = x$ and $x = y$ are replaced by $Bb_0 \equiv Bb_0$ and $Bb_0 \equiv Bb_l$ respectively and the principles of reflexivity of identity and indiscernibility of identicals are replaced by *reflexivity of material equivalence* and *intersubstitutability of materially equivalents* respectively. However, of course, on minimal assumptions about definiteness, the last principle fails in those cases in which the envisaged substitution is within a $\mathcal{D}$-context, at least if the ensuing entailment is then used as an input for $\neg$-L and $\neg$-R (in other words, the last principle fails at least in its contrapositive form of *material non-equivalence of non-intersubstitutables*). The real punch of argument E is then that, contrary to material equivalence, identity does seem to be a strong enough relation as to validate intersubstitutability of its terms also within a $\mathcal{D}$-context, even if the ensuing entailment is then used as an input for $\neg$-L and $\neg$-R (in other words, indiscernibility of identicals does not seem to fail even in its contrapositive form of *distinctness of discernibles*, see also Footnote 24).

But, if that is the real punch of argument E, one should indeed be able to extend it so as to create problems for borderline cases in general, by using not of course material equivalence but a stronger equivalence relation such as *semantic equivalence* (in the sense of identity of semantic value). Letting $\cong$ express semantic equivalence, since Bb_0 holds, Bb_l is tantamount to $Bb_0 \cong Bb_l$ (for it is tantamount to its semantic value being the result of applying the semantic value of 'bald' to a bald man); therefore, since $\neg\mathcal{D}Bb_l$ holds, so should $\neg\mathcal{D}(Bb_0 \cong Bb_l)$. But one can indeed then create problems for borderline cases in general *via* an argument analogous to argument E, where the formulas $x = x$ and $x = y$ are replaced by $Bb_0 \cong Bb_0$ and $Bb_0 \cong Bb_l$ respectively and the principles of reflexivity of identity and indiscernibility of identicals are replaced by *reflexivity of semantic equivalence* and *intersubstitutability of semantically equivalents* respectively. Notice in particular that intersubstitutability of semantically equivalents is, if anything, even more compelling than indiscernibility of identicals, since it is a straightforward consequence of *compositionality of semantic value*; for that reason, semantic equivalence has an even better claim than identity to be a strong enough relation as to validate intersubstitutability of its terms also within a $\mathcal{D}$-context, even if the ensuing entailment is then used as an input for $\neg$-L and $\neg$-R (in other words, intersubstitutability of semantically equivalents has an even better claim than identity not to fail even in its contrapositive form of *semantic non-equivalence of non-intersubstitutables*).

Relatedly, even without talk of semantic equivalence, one should be able to extend argument E to cover at least borderline cases in re in general, whose existence would seem undeniable on the strength of the considerations developed in Sect. 16.1 (especially in Footnotes 6 and 7). As a warm-up argument, focus first on borderline *parthood* in re (henceforth, simply ‘borderline parthood’). Obviously, having something as a part does not imply having all its properties *of every kind whatsoever*; yet, it does seem to imply having all its properties of *definite positive partial location*:[10] how can a place be a place of definite location for a part without being a place of definite location for every larger whole of which it is a part?[11] We thus seem to have the principle of *monotonicity of definite positive location over parthood*:

(MDPLP) If x is part of y and it is definite that x is at p, then it is definite that y is at p.

But, letting $\trianglelefteq$ and L and express parthood and location respectively, one can indeed then create problems for borderline parthood *via* an argument similar to argument E:

$$\cfrac{\cfrac{\cfrac{\cfrac{}{L(x,p) \vdash \mathcal{D}L(x,p)}\,\text{fact entails definite fact} \qquad \cfrac{}{x \trianglelefteq y, \mathcal{D}L(x,p) \vdash \mathcal{D}L(y,p)}\,\text{(MDPLP)}}{x \trianglelefteq y, L(x,p) \vdash \mathcal{D}L(y,p)}\,\text{transitivity}}{x \trianglelefteq y, L(x,p), \neg\mathcal{D}L(y,p) \vdash \varnothing}\,\neg\text{-L}}{L(x,p), \neg\mathcal{D}L(y,p) \vdash x \ntrianglelefteq y}\,\neg\text{-R}$$

(let’s call this argument ‘argument F’). Now, in the cases of interest, while $L(x, p)$ is uncontroversial $x \trianglelefteq y$ is tantamount to $L(y, p)$, and so $\neg\mathcal{D}(x \trianglelefteq y)$ is tantamount to $\neg\mathcal{D}L(y, p)$. Therefore, the conclusion of argument F provides the materials for a variety of concerns about the possibility of borderline parthood analogous to the materials for a variety of concerns about the possibility of borderline identity provided by argument E.[12]

Given this warm-up argument, it’s easy to see how, appealing to the liberal ontology of properties introduced in Sect. 16.1 and letting E express exemplification, one

[10] Henceforth, ‘location’ and its relatives will be understood as ‘partial location’ and its relatives.

[11] To elaborate a bit, if x is part of y, y is identical with $y + x$. But, if it is definite that x is at p, surely it is definite that $y + x$ is at p. That is not only intuitively compelling; if the logic of definiteness is as strong as **K**, it follows from the uncontroversial fact that it is definite that, if x is at p, $y + x$ is at p. An application of indiscernibility of identicals that is as legitimate as the one made in argument E (since it only involves expressions—‘y’ and ‘$y + x$’—such that there is something they definitely refer to) yields then the desired conclusion.

[12] It’s worth mentioning that there is a relative of argument F which even more closely resembles argument E and which is thereby more general (in that it does away with talk of location). That relative can be got from argument E by replacing the formulas $x = x$ and $x = y$ with $x \trianglelefteq x$ and $x \trianglelefteq y$ respectively and by replacing the principles of reflexivity of identity and indiscernibility of identicals with *reflexivity of parthood* and *monotonicity of definite parthood over parthood* (if x is part of y and it is definite that z is part of x, then it is definite that z is part of y) respectively. Notice that the latter principle, although it may initially come across as slightly less plausible than (MDPLP), can be supported in a way analogous to how (MDPLP) has been supported in Footnote 11.

can create problems for borderline cases in re in general *via* an argument analogous to argument F, where the formula $x \trianglelefteq y$ is replaced by $E(x, y)$ and (MDPLP) is replaced by the equally compelling principle of *monotonicity of definite positive location over exemplification*:

(MDPLE) If x exemplifies y and it is definite that x is at p, then it is definite that y is at p.[13,14]

To sum up, Evans' argument can be extended in various ways so as to cover borderline cases in re in general and indeed borderline cases in general (whether in re or not). Given the extensions to borderline cases in re in general, and *contra* the approach exemplified for instance by Salmon (2005), pp. 240–252, *any theorist admitting such cases must find fault with Evans-style reasoning* (rather than accepting Evans' argument as a sound demonstration that borderline identity, contrary to other kinds of borderline cases in re, is impossible). And given the extension to borderline cases in general (whether in re or not), and *contra* the

[13](MDPLE) can be supported in a way analogous to how (MDPLP) has been supported in Footnote 11. Letting pla(x) be the sum of the places at which x is located (and assuming that there is something 'pla' definitely refers to), if x exemplifies y, pla(y) is identical with pla(y) + pla(x). But, if it is definite that p is part of pla(x) (which is tantamount to its being definite that x is at p), surely it is definite that p is part of pla(y) + pla(x). That is not only intuitively compelling; if the logic of definiteness is as strong as **K**, it follows from the uncontroversial fact that it is definite that, if p is part of pla(x), p is part of pla(y) + pla(x). An application of indiscernibility of identicals that is as legitimate as the one made in argument E (since it only involves expressions—'pla(y)' and 'pla(y) + pla(x)'—such that there is something they definitely refer to) yields then that it is definite that p is part of pla(y) (which is tantamount to the desired conclusion).

[14]It's worth mentioning that there is a relative of the argument considered which even more closely resembles argument E and which is thereby more general (in that it does away with talk of location). That relative employs plural talk and assumes that there are some things 'the bald people' definitely refers to. (Notice that this is a fair assumption given that, at this point in the text, we're working under the hypothesis that there are borderline cases in re: if there is something 'baldness' definitely refers to, the things exemplifying it surely are the things 'the bald people' definitely refers to. Notice also that, independently of these issues, the arguments to follow can be reworked as arguments targeting at least borderline being-some-of in re.) Letting $\sqsubseteq$ and bb express being-some-of and the bald people respectively, the argument in question can be got from argument E by replacing the formulas $x = x$ and $x = y$ with $xx \sqsubseteq xx$ and $xx \sqsubseteq bb$ respectively and by replacing the principles of reflexivity of identity and indiscernibility of identicals with *reflexivity of being-some-of* and *monotonicity of definite being-some-of over being-some-of* (if the xx are some of the yy and it is definite that the zz are some of the xx, then it is definite that the zz are some of the yy) respectively. Notice that the latter principle, although it may initially come across as slightly less plausible than (MDPLP) (or (MDPLE)), can be supported in a way analogous to how (MDPLP) has been supported in Footnote 11. Indeed, letting $\doteqdot$ and ll express plural identity and the plurality formed by the bald people together with b_l respectively, taking argument E, replacing the formulas $x = x$ and $x = y$ with $ll \doteqdot ll$ and $ll \doteqdot bb$ respectively and replacing the principles of reflexivity of identity and indiscernibility of identicals with *reflexivity of plural identity* and *indiscernibility of identical pluralities* respectively, we obtain $\neg\mathcal{D}(ll \doteqdot bb) \vdash \neg(ll \doteqdot bb)$, which is tantamount to $\neg\mathcal{D}Bb_l \vdash \neg Bb_l$ (since $ll \doteqdot bb$ is tantamount to Bb_l).

approach exemplified for instance by Lewis (1988), *any theorist whatsoever must find fault with Evans-style reasoning and must do so at some step other than the one at which equivalent expressions are substituted*.

16.3 The Naive Theory of Vagueness and Tolerant Logics

In earlier work (Zardini 2006a,b, 2008a,b, 2009, 2013a,b,c,d), I've developed and defended a *naive theory of vagueness*, according to which, very roughly, the vagueness of an expression consists in its *tolerance* (see Wright 1975)—that is, in its *not drawing a sharp boundary between positive and negative cases of application*. Thus, for example, going back to the second soritical series mentioned in Sect. 16.1 the vagueness of 'bald' (in that situation) consists in the fact that, for every i, if b_i is bald, so is b_{i+1}. The theory applies straightforwardly also to soritical series involving identity. Thus, for example, going back to the first soritical series mentioned in Sect. 16.1 the vagueness of 'identical with Greg$_0$' (in that situation) consists in the fact that, for every i, if Greg$_i$ is identical with Greg$_0$, so is Greg$_{i+1}$.

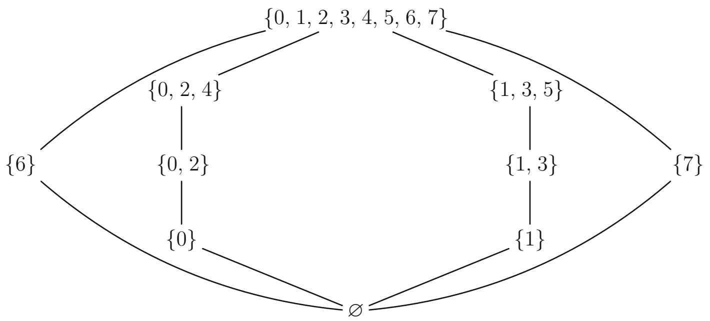
The naive theory of vagueness arguably enjoys a number of important advantages over its rivals (see Zardini 2008b, pp. 15–16, 21–71; Sweeney and Zardini 2011; Zardini 2013d). It however faces a significant problem that has usually been taken to be fatal: as shown by *standard soritical reasoning*, it is *inconsistent in almost any logic of vagueness*. The naive theory thus requires a novel logic, and I do provide a family of logics—*tolerant logics*—that are hospitable to the theory in Zardini (2008a, b, pp. 93–173, 2009, 2013b, c). The main feature of tolerant logics is that they place restrictions on *transitivity* of logical consequence, a restriction which proves crucial in blocking standard soritical reasoning and, more generally, in restoring the consistency of the naive theory.

Instead of offering the full framework in which tolerant logics can be developed, in this paper I'll simply introduce my favoured broad kind of tolerant logic, with the main aim of using it to offer an analysis of where and why Evans' argument fails. I'll offer two versions of that kind of logic, basically differing on the treatment of definiteness. In this regard, I should stress that the focus of this paper is on using those two tolerant logics to explore the issues raised by Evans' argument rather than on offering two philosophically and formally complete theories of definiteness. As a consequence, I'll only give as much philosophical and formal detail about those two theories as is necessary for such an exploration, leaving it to another occasion to offer a philosophically and formally adequate presentation and comparison of them (notice that, among other things, I'll mostly ignore the considerable philosophical and formal complications required by higher-order vagueness). Also, although, in both cases, the analysis of where and why Evans' argument fails will straightforwardly cover also the extensions of Evans' argument developed in Sect. 16.2, I'll save the reader the details of this.

16.4 The Tolerant Logic $\mathbf{V_0}$

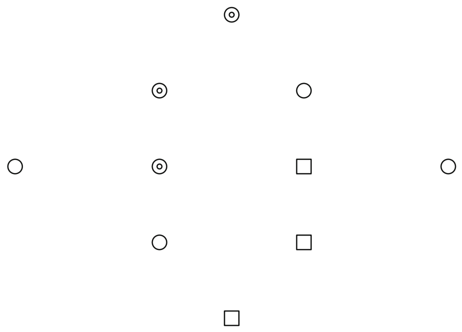
Let's start then with the logic $\mathbf{V_0}$, which treats definiteness as obeying a relatively standard modal logic. For our purposes, $\mathbf{V_0}$ and its like can be introduced *model-theoretically*. Informally, we consider a certain class of *lattice-theoretic* models rich enough as to include operations sufficient for interpreting a standard first-order modal language. Such models include a set of *designated* values and also a set of *tolerated* values which is a *proper superset* of the former set. Rather than defining, as usual, logical consequence in terms of *preservation* in every model of designated (or tolerated) value from the premises to the conclusions, we define it instead as *connection* in every model of designated value in the premises with tolerated value in the conclusions. It is this style of definition of logical consequence which will give rise to failures of transitivity for $\mathbf{V_0}$ and its like.

Definition 1. A $\mathbf{V_0}$-*model* $\mathfrak{M}$ is a 9ple $\langle U_{\mathfrak{M}}, V_{\mathfrak{M}}, \preceq_{\mathfrak{M}}, D_{\mathfrak{M}}, T_{\mathfrak{M}}, \mathrm{neg}_{\mathfrak{M}}, \mathrm{def}_{\mathfrak{M}}, \mathrm{id}_{\mathfrak{M}}, \mathrm{int}_{\mathfrak{M}}\rangle$. $U_{\mathfrak{M}}$ is a *domain* of objects. $V_{\mathfrak{M}}$ is a set of *values* representable as: $\{X : X \in \mathrm{pow}(\{i : 0 \leq i \leq 7\})$ and, if $X \neq \{i : 0 \leq i \leq 7\}$, either, [[for every $i \in X$, i is even][15] and, [for every i and j, if $i \in X$ and ≤ 4 and j is even and $< i$, $j \in X$] and, [for every i and $j \in X$, $|i - j| < 6$]] or, [[for every $i \in X$, i is odd] and, [for every i and j, if $i \in X$ and ≤ 5 and j is odd and $< i$, $j \in X$] and, [for every i and $j \in X$, $|i - j| < 6$]]$\}$. $\preceq_{\mathfrak{M}}$ is a *partial order* on $V_{\mathfrak{M}}$ representable as: $\{\langle X, Y\rangle : X \subseteq Y\}$. Thus, $V_{\mathfrak{M}}$ and $\preceq_{\mathfrak{M}}$ jointly constitute the lattice depicted by the following Hasse diagram:

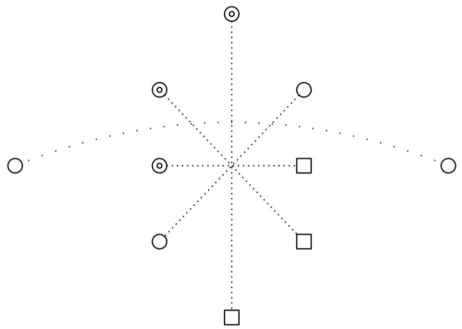


$D_{\mathfrak{M}}$ is a set of *designated* values while $T_{\mathfrak{M}}$ is a set of *tolerated* values, with $D_{\mathfrak{M}} \subset T_{\mathfrak{M}}$. Indicating designated values with doubly circular nodes, tolerated but not designated values with simply circular nodes and neither designated nor tolerated values with square nodes, they can be depicted as:

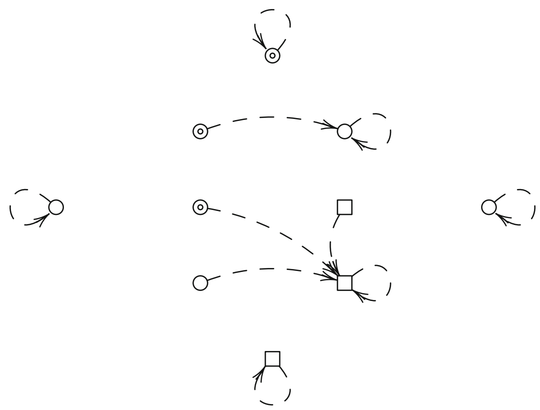
[15]Throughout, I use square brackets to disambiguate constituent structure.



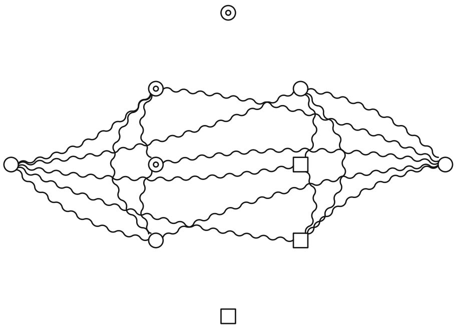
$\mathrm{neg}_{\mathfrak{M}}$ is a *negation* operation on $V_{\mathfrak{M}}$. Indicating it with pointed edges, it can be depicted as:



$\mathrm{def}_{\mathfrak{M}}$ is a *definitisation* operation on $V_{\mathfrak{M}}$. Indicating it with dashed arrows, it can be depicted as:



$\mathrm{id}_{\mathfrak{M}}$ is an *identification* function from the Cartesian product of $U_{\mathfrak{M}}$ with itself to $V_{\mathfrak{M}}$. In addition to being commutative in its arguments, it can be represented as being such that, for every $u \in U_{\mathfrak{M}}$, $\{0, 2, 4\} \subseteq \mathrm{id}_{\mathfrak{M}}(u, u)$ and as being such that, if $\mathrm{id}_{\mathfrak{M}}(u, v) \in D_{\mathfrak{M}}$, for every atomic $i$ary predicate Φ^i (including $=$) $\mathrm{val}_{\mathfrak{M},\mathrm{ass}}(\Phi^i)(u_0, u_1, u_2 \ldots, v \ldots, u_{i-1})$[16] either is identical with $\mathrm{val}_{\mathfrak{M},\mathrm{ass}}(\Phi^i)(u_0, u_1, u_2 \ldots, u \ldots, u_{i-1})$ or at most deviates from it in the way depicted by the squiggle dashes:



$\mathrm{int}_{\mathfrak{M}}$ is an *interpretation* function from the union of the Cartesian products of the sets of $i$ary atomic predicates (minus the identity predicate) with the sets of $i$tuples of members of $U_{\mathfrak{M}}$ to $V_{\mathfrak{M}}$.

$\mathrm{int}_{\mathfrak{M}}$ can be extended to a full *valuation* function $\mathrm{val}_{\mathfrak{M}}$ (relative to assignments) in the usual way (using glb in $\preceq_{\mathfrak{M}}$ for interpreting conjunction and universal quantification, $\mathrm{neg}_{\mathfrak{M}}$ for interpreting negation, $\mathrm{def}_{\mathfrak{M}}$ for interpreting definiteness and $\mathrm{id}_{\mathfrak{M}}$ for interpreting identity).[17] With such function in place, the advertised style of definition of logical consequence becomes available:

Definition 2. $\Gamma \vdash_{\mathbf{V_0}} \Delta$ holds iff, for every $\mathbf{V_0}$-model $\mathfrak{M}$ and assignment ass, if, for every $\varphi \in \Gamma$, $\mathrm{val}_{\mathfrak{M},\mathrm{ass}}(\varphi) \in D_{\mathfrak{M}}$, then, for some $\psi \in \Delta$, $\mathrm{val}_{\mathfrak{M},\mathrm{ass}}(\psi) \in T_{\mathfrak{M}}$.

In Zardini (2008a, b, pp. 93–173, 2009, 2013c), I explore in great detail the properties of what is essentially the $\mathcal{D}$-free fragment of $\mathbf{V_0}$. Here, I only pause to record the main properties of $\mathcal{D}$ and the other properties that are relevant for our purposes:

Theorem 1. *In* $\mathbf{V_0}$*, definiteness obeys a modal logic* at least as strong as $\mathbf{CN}^{\neg,\mathcal{D}}\mathbf{DT}^{\subset}\mathbf{B45}$*, in the sense that, in* $\mathbf{V_0}$*, the principles:*

(C) $\mathcal{D}\varphi, \mathcal{D}\psi \vdash \mathcal{D}(\varphi \wedge \psi)$ *holds;*

($\mathrm{N}^{\neg,\mathcal{D}}$) *If* φ *does not contain either* $\wedge$ *or* $\forall$ *and* $\varnothing \vdash \varphi$ *holds, then* $\varnothing \vdash \mathcal{D}\varphi$ *holds;*[18]

[16] $\mathrm{val}_{\mathfrak{M},\mathrm{ass}}$ is the model- and assignment-relative valuation function that will be defined below in the text in terms of $\mathfrak{M}$ (and in particular in terms of $\mathrm{id}_{\mathfrak{M}}$). The circularity is not vicious as we're here only constraining rather than explicitly defining $\mathrm{id}_{\mathfrak{M}}$. It is of course not guaranteed that such constraint is satisfiable. Models in which it is satisfied will be sketched in Theorems 5 and 6.

[17] In turn, we can assume that disjunction, implication and particular quantification are defined in the usual way.

[18] ($\mathrm{N}^{\neg,\mathcal{D}}$) was more informally labelled 'simple definitisation' in Sect. 16.2.

(D) *If* $\varphi \vdash \varnothing$ *holds, then* $\mathcal{D}\varphi \vdash \varnothing$ *holds;*
($T^{\subset}$) $\varnothing \vdash \mathcal{D}\varphi \supset \varphi$ *holds;*
(B) $\varnothing \vdash \varphi \supset \mathcal{D}\neg\mathcal{D}\neg\varphi$ *holds;*
(4) $\varnothing \vdash \mathcal{D}\varphi \supset \mathcal{D}\mathcal{D}\varphi$ *holds;*
(5) $\varnothing \vdash \neg\mathcal{D}\varphi \supset \mathcal{D}\neg\mathcal{D}\varphi$ *holds*

hold.[19]

Theorem 2. *In* $\mathbf{V_0}$*, negation is* exclusive *and* exhaustive*, in the sense that, in* $\mathbf{V_0}$*, the principles:*

(EXC) *If* $\Gamma \vdash \Delta, \varphi$ *holds, then* $\Gamma, \neg\varphi \vdash \Delta$ *holds;*
(EXH) *If* $\Gamma, \varphi \vdash \Delta$ *holds, then* $\Gamma \vdash \Delta, \neg\varphi$ *holds*[20]

hold.

Theorem 3. *In* $\mathbf{V_0}$*, identity is* reflexive *and entails* indiscernibility across non-composite properties,[21] *in the sense that, in* $\mathbf{V_0}$*, the principles:*

(R) $\varnothing \vdash \tau = \tau$ *holds;*
(II) *If* φ *does not contain either* $\wedge$ *or* $\forall$*, then* $\tau = \sigma, \varphi \vdash \varphi[\tau/\sigma]$[22] *holds*[23]

hold.

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[19]Notice that, with *classical* logic as *background* logic for the *modal* logic, the list of modal principles given in the text is multiply redundant. But this need no longer be so if a *non-classical* logic is used as background logic for the modal logic, in particular if the non-classical logic in question is in the relevant respects so weak as $\mathbf{V_0}$ is. Notice also that, with the exception of ($N^{\neg,\mathcal{D}}$), the listed principles will not be directly concerned by our discussion. However, the validity of principles (C) and ($N^{\neg,\mathcal{D}}$) serves to mark the extent to which definiteness is still *closed under logical consequence* in $\mathbf{V_0}$ and its like (see the discussion in Sect. 16.6), while the validity of principles (D)–(5) serves to show that Evans' argument fails in $\mathbf{V_0}$ and its like even under *very strong assumptions* about definiteness (setting aside that, as I've noted in Sect. 16.2, at least (5) is widely rejected because of higher-order vagueness).

[20](EXC) and (EXH) were more neutrally labelled '$\neg$-L' and '$\neg$-R' respectively in Sect. 16.2.

[21]A property is *non-composite* iff it is neither conjunctive nor disjunctive. Arguably, a composite property is exemplified in virtue of some non-composite properties being exemplified and of logical facts about conjunction and disjunction. That in turn plausibly suggests that, if identicals are indiscernible across composite properties, they are such in virtue of their being indiscernible across non-composite properties and of logical facts about conjunction and disjunction. But, for reasons I'll partially adumbrate in Sect. 16.6, the required logical facts about conjunction and disjunction do not obtain in $\mathbf{V_0}$ and its like and in fact cannot possibly obtain in any naive theory of vagueness adopting as tolerant logic $\mathbf{V_0}$ or one of its like. Therefore, (II) is arguably the proper formulation of the principle of indiscernibility of identicals in $\mathbf{V_0}$ and its like. Even more strongly, for reasons I cannot go into in this paper, identicals cannot possibly be indiscernible across composite properties in any naive theory adopting as tolerant logic $\mathbf{V_0}$ or one of its like, and, on a natural way of making metaphysical sense of any such theory, this is so because indiscernibility across composite properties may concern the same object in different circumstances (and it is uncontroversial that the same object may exemplify different properties in different circumstances, see Footnote 9).

[22]With the usual proviso that τ be free for σ in φ.

[23](R) and (II) were more informally labelled 'reflexivity' and 'indiscernibility of identicals' respectively in Sect. 16.2.

Corollary 1. *In* $\mathbf{V_0}$*, identity is* transitive *and distinctness is entailed by* discernibility across non-composite properties*, in the sense that, in* $\mathbf{V_0}$*, the principles:*

(TR$^{=}$) $\tau = \sigma, \sigma = \upsilon \vdash \tau = \upsilon$ *holds;*
(DD) *If* φ *does not contain either* $\wedge$ *or* $\forall$*, then* $\varphi, \neg\varphi[\tau/\sigma] \vdash \tau \neq \sigma$ *holds*[24]

hold.

Theorem 4. $\mathbf{V_0}$ *is* non-transitive*, in the sense that, in* $\mathbf{V_0}$*, the principle:*

(TR$^{\vdash}$) *If, [for every* $\varphi \in \Theta$*,* $\Gamma \vdash \Delta, \varphi$ *holds] and* $\Lambda, \Theta \vdash \Xi$ *holds, then* $\Lambda, \Gamma \vdash \Delta, \Xi$ *holds*[25]

does not hold.[26]

16.5 The Failure of Argument E in $\mathbf{V_0}$

Argument E employs (TR$^{\vdash}$), and so it fails in $\mathbf{V_0}$ (indeed, since, in $\mathbf{V_0}$, all the other principles employed by argument E hold, in a good sense in $\mathbf{V_0}$ argument E fails *exactly* at (TR$^{\vdash}$)). However, the fact that argument E in its specificity fails in $\mathbf{V_0}$ is no great reassurance that its conclusion does not hold in $\mathbf{V_0}$, since, for all that has been shown, there could be more ingenious arguments that establish that conclusion and that are valid in $\mathbf{V_0}$. What needs to be shown is that the conclusion does not hold in $\mathbf{V_0}$, which is to say that there is a $\mathbf{V_0}$-model $\mathfrak{M}$ such that $\text{val}_{\mathfrak{M},\text{ass}}(\neg\mathcal{D}(x = y)) \in D_{\mathfrak{M}}$ and $\text{val}_{\mathfrak{M},\text{ass}}(x \neq y) \notin T_{\mathfrak{M}}$. Indeed, to make sure that the borderline identity between x and y does not have problematic consequences for the naive theory of vagueness, and that, even together with it, it remains silent on whether x

[24]Many theories of borderline identity block Evans' argument by rejecting (DD) while preserving (II), claiming that (DD) is only a degenerated principle resulting from the compelling (II) plus questionable assumptions about negation like (EXC) and (EXH) (see e.g. Parsons 1987, pp. 9–11). However, contrary to such assessment, (DD) strikes me as equally compelling as (II): if x and y are discernible with respect to a property, how could they fail to be distinct? Moreover, the move in question does not apply at least to the semantic-equivalence extension of Evans' argument developed in Sect. 16.2.

[25](TR$^{\vdash}$) was more informally labelled 'transitivity' in Sect. 16.2.

[26](TR$^{\vdash}$) is a version of transitivity for logical consequence with a somewhat *intermediate* strength. It is *strong* in that, for example, it allows one to apply transitivity in the presence of *side premises and conclusions* (contrast with the principle saying that, if $\varphi \vdash \psi$ and $\psi \vdash \chi$ hold, then $\varphi \vdash \chi$ holds). It is *weak* in that, for example, it does not allow one to apply transitivity in order to dispense with intermediate premises taken *distributively* rather than *collectively* (contrast with the principle saying that, if $\Xi \vdash \Lambda, \Theta$ holds and, for every $\varphi \in \Theta$, $\Delta, \varphi \vdash \Gamma$ holds, then $\Delta, \Xi \vdash \Lambda, \Gamma$ holds). In fact, the naive theory of vagueness requires failures of transitivity *even in the absence of side premises and conclusions* and, unsurprisingly, *even that weak version of transitivity fails in* $\mathbf{V_0}$ *and its like* (see Weir 2005 for a different family of non-transitive logics in which such weak version of transitivity is preserved). However, argument E requires transitivity *in the presence of side premises*, and so we focus on (TR$^{\vdash}$).

is identical with y, we should show that, in $\mathbf{V_0}$, $\mathcal{B}(x = y)$ together with the relevant fragment of the naive theory entails neither $x \neq y$ nor $x = y$, which is to say that there is a $\mathbf{V_0}$-model $\mathfrak{M}$ of the relevant fragment of the naive theory such that $\mathrm{val}_{\mathfrak{M},\mathrm{ass}}(\mathcal{B}(x = y)) \in D_{\mathfrak{M}}$ and $\mathrm{val}_{\mathfrak{M},\mathrm{ass}}(x \neq y) \notin T_{\mathfrak{M}}$ and a $\mathbf{V_0}$-model $\mathfrak{M}'$ of the relevant fragment of the naive theory such that $\mathrm{val}_{\mathfrak{M}',\mathrm{ass}}(\mathcal{B}(x = y)) \in D_{\mathfrak{M}'}$ and $\mathrm{val}_{\mathfrak{M}',\mathrm{ass}}(x = y) \notin T_{\mathfrak{M}'}$.

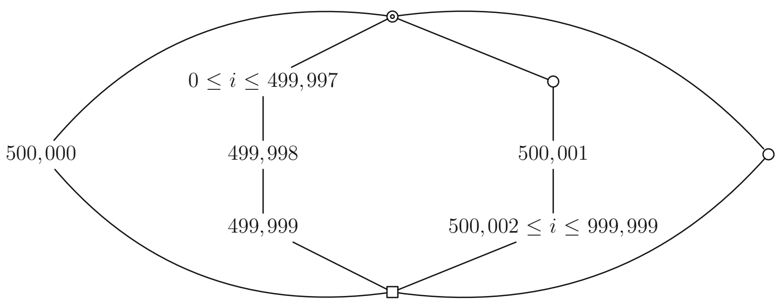
So let's consider the fragment of the naive theory of vagueness concerning Greg's transformation as described in Sect. 16.1. Let's assume that $H\tau$ translates 'τ is human', g_i translates 'Greg$_i$' and τ' translates 'the substance canonically present at the time that is the successor of the time at which τ is canonically present' (with the understanding that, for every i, Greg$_i$ is the substance canonically present at t_i). Setting, for merely illustrative purposes, $k = 999{,}999$, the relevant fragment of the naive theory is then:

(H^p) Hg_0;
(H^n) $\neg Hg_{999{,}999}$;
(H^t) $\neg\exists x(Hx \wedge \neg Hx')$;
($=^n$) $g_0 \neq g_{999{,}999}$;
($=^t$) $\forall x(x = x')$,

and we can set the borderline identity to be $\mathcal{B}(g_0 = g_{499{,}998})$.

Theorem 5. *There is a* $\mathbf{V_0}$*-model* $\mathfrak{C}_0$ *such that* $\mathrm{val}_{\mathfrak{C}_0,\mathrm{ass}}((\mathrm{H}^p))$, $\mathrm{val}_{\mathfrak{C}_0,\mathrm{ass}}((\mathrm{H}^n))$, $\mathrm{val}_{\mathfrak{C}_0,\mathrm{ass}}((\mathrm{H}^t))$, $\mathrm{val}_{\mathfrak{C}_0,\mathrm{ass}}((=^n))$, $\mathrm{val}_{\mathfrak{C}_0,\mathrm{ass}}((=^t))$ *and* $\mathrm{val}_{\mathfrak{C}_0,\mathrm{ass}}(\mathcal{B}(g_0 = g_{499{,}998})) \in D_{\mathfrak{C}_0}$ *while* $\mathrm{val}_{\mathfrak{M},\mathrm{ass}}(g_0 \neq g_{499{,}998}) \notin T_{\mathfrak{C}_0}$, *and there is a* $\mathbf{V_0}$*-model* $\mathfrak{C}_0'$ *such that* $\mathrm{val}_{\mathfrak{C}_0',\mathrm{ass}}((\mathrm{H}^p))$, $\mathrm{val}_{\mathfrak{C}_0',\mathrm{ass}}((\mathrm{H}^n))$, $\mathrm{val}_{\mathfrak{C}_0',\mathrm{ass}}((\mathrm{H}^t))$, $\mathrm{val}_{\mathfrak{C}_0',\mathrm{ass}}((=^n))$, $\mathrm{val}_{\mathfrak{C}_0',\mathrm{ass}}((=^t))$ *and* $\mathrm{val}_{\mathfrak{C}_0',\mathrm{ass}}(\mathcal{B}(g_0 = g_{499{,}998})) \in D_{\mathfrak{C}_0'}$ *while* $\mathrm{val}_{\mathfrak{C}_0',\mathrm{ass}}(g_0 = g_{499{,}998}) \notin T_{\mathfrak{C}_0'}$.

Proof. Sketch for $\mathfrak{C}_0$. Take $\mathfrak{C}_0$ to be a $\mathbf{V_0}$-model such that, for every i, $\mathrm{val}_{\mathfrak{C}_0,\mathrm{ass}}(g_i = g_{i+1}) = \{0, 2, 4\}$ and, letting a number i occupying the place of a value X mean that $\mathrm{val}_{\mathfrak{C}_0,\mathrm{ass}}(Hg_i) = \mathrm{val}_{\mathfrak{C}_0,\mathrm{ass}}(g_0 = g_i) = X$, such that:



□

Thus, at a *formal* level, one can adopt $\mathbf{V_0}$ and thereby be committed to accepting the claim that it is definite that Greg$_0$ is identical with Greg$_0$, accept the (H^p)–($=^t$)-fragment of the naive theory of vagueness and the claim that it is borderline whether Greg_0 is identical with $\text{Greg}_{499,998}$, accept that that claim together with the previous claim that it is definite that Greg_0 is identical with Greg_0 entails that Greg_0 is distinct from $\text{Greg}_{499,998}$ and yet not be committed to accepting that Greg_0 is distinct from $\text{Greg}_{499,998}$ (or that Greg_0 is identical with $\text{Greg}_{499,998}$).

But how can we make sense of this position at a *philosophical* level? A natural way to do so starts by distinguishing *two grades* in which a sentence ψ can be (strictly or non-strictly) *weaker* than a sentence φ (see Zardini 2013b for more details). Very roughly, the first grade of being weaker (which I'll denote with single starring) is present as soon as φ *could not hold without* ψ *holding*; the second grade of being weaker (which I'll denote with double starring) is present when ψ *could not be less (epistemically) likely than* φ. *Like* many other logics, tolerant logics are constructed with the aim of forcing the conclusion of a valid argument to be *weaker** than the premise, but, *unlike* many other logics, tolerant logics achieve that without forcing the conclusion to be *weaker*** than the premise—in tolerant logics, φ may entail ψ (and so it may be impossible that φ holds without ψ holding) even if ψ is less likely than φ. For example, in $\mathbf{V_0}$ $(\mathrm{H}^t) \wedge Hg_{499,998} \vdash Hg_{499,999}$ holds (it is impossible that $(\mathrm{H}^t) \wedge Hg_{499,998}$ holds without $Hg_{499,999}$ holding) even if $Hg_{499,999}$ may be less likely than $(\mathrm{H}^t) \wedge Hg_{499,998}$ (for it is natural to think that the former may make a "weightier" claim than the latter, in that, contrary to it, it straightforwardly decides that $\text{Greg}_{499,999}$ is bald). Tolerant logics thus have the best of both the *non-deductive* and the *deductive* world: they allow for valid arguments in which the conclusion goes beyond the premise while ensuring that nevertheless the truth of the premise guarantees the truth of the conclusion. The non-transitivity of tolerant logics can then be seen to arise from this *mismatch* between the relation of being weaker** on the one hand and the relations of being weaker* and of logical consequence on the other hand, for, under these circumstances, chaining together enough valid arguments may eventually lead from a very likely (indeed, certainly true) initial premise to a very unlikely (indeed, certainly false) final conclusion.

Now, $\mathbf{V_0}$ encodes the idea that *definitely being is a mode of being that is not weaker than merely being*. For sentences about borderline cases like '$\text{Greg}_{499,998}$ is human', this feature is manifested as a relatively straightforward failure of definitely being to be weaker* than merely being: by the law of excluded middle,[27] either $\text{Greg}_{499,998}$ is human or $\text{Greg}_{499,998}$ is not human, but he is nevertheless neither a definite human nor a definite not human. A borderline case of humanity may be human (non-human), but if it is human (non-human), it is not so much of a human (non-human) as to be a definite human (non-human). For logical truths like 'Greg_0 is identical with Greg_0', the feature is manifested not as any failure of definitely being

[27]Notice that the law of excluded middle does hold in $\mathbf{V_0}$ and its like, being a straightforward consequence of (EXH).

to be weaker* than merely being (for every i, it is definite^{i}[28] that Greg_0 is identical with Greg_0), but still it is manifested as a failure of definitely being to be weaker** than merely being. Given that it is a logical truth that Greg_0 is identical with Greg_0, by ($\text{N}^{\neg,\mathcal{D}}$) it is definite that Greg_0 is identical with Greg_0, but, although that could not fail to be so, it is nevertheless to some extent less likely than Greg_0's being identical with Greg_0. Similarly, under the assumption that $\text{Greg}_{499,998}$ is identical with Greg_0, by (II) it would follow from its being definite that Greg_0 is identical with Greg_0 that it is definite that $\text{Greg}_{499,998}$ is identical with Greg_0, but, although that could not fail to be so, it would nevertheless be to some extent less likely than its being definite that Greg_0 is identical with Greg_0. And, since it is in fact already less than certain that it is definite that Greg_0 is identical with Greg_0, the further uncertainty generated by (II) determines that, under the assumption that $\text{Greg}_{499,998}$ is identical with Greg_0, it could in fact be so unlikely that it is definite that $\text{Greg}_{499,998}$ is identical with Greg_0 that 'It is definite that $\text{Greg}_{499,998}$ is identical with Greg_0' cannot be taken to follow simply from '$\text{Greg}_{499,998}$ is identical with Greg_0'.

Relatedly, since, on this position, in general φ could hold without $\mathcal{D}\varphi$ holding, the argument from '$\text{Greg}_{499,998}$ is identical with Greg_0' to 'It is definite that $\text{Greg}_{499,998}$ is identical with Greg_0' would indeed be an argument whose conclusion is not only not weaker** but also not weaker* than its premise and so an invalid argument—unless it is rescued by a suitable finer-grained analysis of its logical form. What is supposed to come to the rescue is the observation that the premise is an identity, and the assumption that 'It is definite that Greg_0 is identical with Greg_0' as a further premise turning the argument into an instance of (II) can be *suppressed* as it is a logical truth. But, while all logical truths are indeed likely enough as to guarantee that they must hold, as we've seen in the previous paragraph some of them are not so likely that they can be suppressed in a valid argument.[29]

[28]Henceforth, 'definitely^{i}' or its like is the result of concatenating the empty string with i occurrences of 'definitely' or its like.

[29]It is not unusual to admit that the analogue of this can happen in a *non-deductive* case. Consider, for example, a non-deductive consequence relation $\Vdash$ relativised to a biographer's evidence (where $\Gamma \Vdash \varphi$ holds iff, given the biographer's evidence, the state of information represented by Γ lends credibility to φ), with the biographer's evidence being good enough as to lend credibility both to 'On 25/12/1915, Kafka had ham for dinner' and to 'On 26/12/1915, Kafka had sausages for dinner'. Then, while we have that both $\varnothing \Vdash$ 'On 25/12/1915, Kafka had ham for dinner' and $\varnothing \Vdash$ 'On 26/12/1915, Kafka had sausages for dinner' hold, and presumably also that 'On 25/12/1915, Kafka had ham for dinner', 'On 26/12/1915, Kafka had sausages for dinner' $\Vdash$ 'On 25/12/1915, Kafka had ham for dinner and, on 26/12/1915, Kafka had sausages for dinner' holds, it is a familiar point that 'On 25/12/1915, Kafka had ham for dinner' and 'On 26/12/1915, Kafka had sausages for dinner' cannot be suppressed in the last argument on pain of giving rise to the *preface paradox* (see Makinson 1965). What is distinctive of the position under discussion is to maintain that the analogue of this can happen in the *deductive* case (I explore further the illuminating similarities between tolerant logics and certain non-deductive consequence relations in Zardini 2013b).

16.6 The Conclusion of Argument E and the Properties of Definiteness

As I've already implied, a crucial feature of the construction in Sect. 16.5 is that a relative of the converse of ($T^{\subset}$):

($T^{\vdash}$) $\varphi \vdash \mathcal{D}\varphi$ holds[30]

does not hold. ($T^{\vdash}$), as well as its strengthening:

($T^{\vdash^{\omega}}$) For every i, $\varphi \vdash \mathcal{D}^{i}\varphi$ holds,

is arguably necessary for an appealing conception of definiteness according to which *everything is definitely what it is* (although the argument lies beyond the scope of this paper), and, together with (EXC), it does imply (INC). Yet, it *has* to fail in the construction of Sect. 16.5: for, by Theorem 2, (EXC) and (EXH) are both crucial properties of negation in $\mathbf{V_0}$ and its like,[31] and they jointly imply that, if $\varphi \vdash \mathcal{D}\varphi$ holds, so does $\neg\mathcal{D}\varphi \vdash \neg\varphi$, so that ($T^{\vdash}$) would lead straight away to the conclusion of argument E. But the construction of Sect. 16.5 is predicated precisely on the assumption that argument E should fail, and, more generally, that its conclusion should not hold.

Recall the reasons in Sect. 16.2 for that assumption. One reason (the one Evans himself seems to have had in mind) appealed to ($TR^{ADJ^{INC}}$) (and (INC), which does hold if ($T^{\vdash}$) and (EXC) do). But, just as with ($TR^{\vdash}$), ($TR^{ADJ^{INC}}$) does not hold in $\mathbf{V_0}$ and its like, and in fact *cannot possibly hold in any naive theory of vagueness adopting as tolerant logic* $\mathbf{V_0}$ *or one of its like*. In its essence, the reason is this (Zardini 2013c gives more details). Given that, in $\mathbf{V_0}$ and its like, (H^{p}) $\wedge$ (H^{n}) $\wedge$

[30] ($T^{\vdash}$) was more informally labelled 'fact entails definite fact' in Sect. 16.2. The proper converse of ($T^{\subset}$) is of course:

($T^{\supset}$) $\varnothing \vdash \varphi \supset \mathcal{D}\varphi$ holds,

which is typically rejected even by logics of definiteness that accept ($T^{\vdash}$) (as, by contraposition on material implication and a version of *modus ponens*, it yields the contrapositive of ($T^{\vdash}$)). Given the usual definition of implication, in $\mathbf{V_0}$ and its like ($T^{\supset}$) follows from ($T^{\vdash}$) by (EXH) and the properties of disjunction in $\mathbf{V_0}$ and its like. I think that this is as it should it be, as ($T^{\supset}$) no less than ($T^{\vdash}$) is arguably integral to the appealing conception of definiteness that I'm about to introduce in the text. As for the alleged bad consequence of ($T^{\supset}$) ($\neg\mathcal{D}\varphi \vdash \neg\varphi$), I'm going to argue in this section that, in a naive theory of vagueness adopting a tolerant logic, it is not such.

[31] I'm well aware that (EXH) is rejected by many non-classical logics of vagueness (and that even (EXC) is rejected by some of them). But, setting aside the question of the meaning and logic of the negative constructions ordinarily used in natural language, I think that we clearly do have a notion of a sentence φ *failing to hold* that, on the face of it, is exclusive and exhaustive with respect to φ. $\mathbf{V_0}$ is a theory, among other things, of that arguably theoretically fundamental notion and of the borderline cases that arise with respect to it.

$(\mathrm{H}^t) \vdash Hg_1$ and $(\mathrm{H}^p) \wedge (\mathrm{H}^n) \wedge (\mathrm{H}^t) \wedge \neg Hg_2 \vdash \neg Hg_1$ hold (and that, by (EXC), $Hg_1, \neg Hg_1 \vdash \varnothing$ holds), by the properties of conjunction in $\mathbf{V_0}$ and its like and $(\mathrm{TR}^{\mathrm{ADJ}^{\mathrm{INC}}})$ it would follow that, in $\mathbf{V_0}$ and its like, $(\mathrm{H}^p) \wedge (\mathrm{H}^n) \wedge (\mathrm{H}^t) \wedge \neg Hg_2 \vdash \varnothing$ holds, and so, by the properties of conjunction and negation in $\mathbf{V_0}$ and its like, it would follow that, in $\mathbf{V_0}$ and its like, $(\mathrm{H}^p) \wedge (\mathrm{H}^n) \wedge (\mathrm{H}^t) \vdash Hg_2$ holds. Applying analogous arguments another 999,997 times, we would reach the tragic conclusion that, in $\mathbf{V_0}$ and its like, $(\mathrm{H}^p) \wedge (\mathrm{H}^n) \wedge (\mathrm{H}^t) \vdash Hg_{999,999}$ holds (and so, since, in $\mathbf{V_0}$ and its like, $(\mathrm{H}^p) \wedge (\mathrm{H}^n) \wedge (\mathrm{H}^t) \vdash \neg Hg_{999,999}$ holds, by $(\mathrm{TR}^{\mathrm{ADJ}^{\mathrm{INC}}})$ we would reach the even more tragic conclusion that, in $\mathbf{V_0}$ and its like, $(\mathrm{H}^p) \wedge (\mathrm{H}^n) \wedge (\mathrm{H}^t) \vdash \varnothing$ holds).

Another reason for the assumption that the conclusion of argument E should not hold appealed to (CCTLC). But, just as with $(\mathrm{TR}^{\vdash})$, (CCTLC) cannot possibly hold in any naive theory of vagueness adopting a tolerant logic. For, clearly, it is crucial for such a theory sometimes *to accept* φ*, accept that* φ *entails* ψ *but not to accept* ψ (otherwise, since the naive theory accepts $(\mathrm{H}^p) \wedge (\mathrm{H}^t)$, as well as the relevant logical principles like *modus ponens*, it will be committed to accepting $\neg(\mathrm{H}^n)$). Generally, a naive theory of vagueness adopting a tolerant logic will want to draw a distinction between, very roughly, acceptance for *non-inferential* reasons and acceptance for *inferential* reasons and will want to hold that, if φ entails ψ, if one accepts φ one is committed [to accepting ψ] only if one accepts φ for non-inferential reasons, while that may not be so if one merely accepts φ for inferential reasons. Similar points apply for attitudes other than acceptance, like conditional acceptance, supposition, desire etc. (see Footnote 32; Zardini 2013b offers an extended discussion of the normative import of non-transitive logical consequence).

So, in a naive theory of vagueness adopting a tolerant logic, the two main reasons against the conclusion of argument E no longer have force, this being so for the second reason at least if one does not have non-inferential reasons for accepting $\mathcal{B}(x = y)$. And it does seem to be the case that, *quite generally, one does not have non-inferential reasons for accepting* $\mathcal{B}\varphi$. For such reasons are typically supposed to consist in the fact that, roughly, there is something to be said in favour of φ and also something to be said against φ; but, at least if $(\mathrm{T}^{\vdash})$ holds, if there is something to be said in favour of φ (against φ), there is something to be said in favour of $\mathcal{D}\varphi$ ($\mathcal{D}\neg\varphi$), and so something to be said against $\mathcal{B}\varphi$, which plausibly defeats the putative reason in favour of $\mathcal{B}\varphi$. This claim about borderline cases also dovetails nicely with the way in which the notion has been introduced in Sect. 16.1. For, on the approach of Sect. 16.1, our immediate reason in favour of borderline cases is a *general* one concerning overall features of soritical series: on the way from Greg_0 to Greg_k, it cannot be the case that, for no i, it is borderline whether Greg_0 is identical with Greg_i (for whence could the vagueness of the transformation then arise?), and so it has to be the case that, *for some* i, it is borderline whether Greg_0 is identical with Greg_i. Thus, on that approach, our immediate reason in favour of borderline cases is not a *particular* one concerning a specific i to the effect that it is borderline whether Greg_0 is identical with Greg_i, and so, since any non-inferential reason would count as an immediate reason, it follows that, on that approach, for no i does one have non-inferential reasons for accepting that it is borderline whether Greg$_0$ is identical with Greg$_i$.[32]

It still remains the case that, as I've noted in Sect. 16.2, the conclusion of argument E would seem to imply that *nothing is a definite borderline case of identity*. However, as I've also noted in Sect. 16.2, that implication only holds if (M) holds. But, quite obviously, on a view accepting the contrapositive of (T$^{\vdash}$) just as there are reasons *independent of argument* E for accepting its conclusion—reasons that rely rather on (T$^{\vdash}$), (EXC) and (EXH)—so there may be reasons *independent of argument* E for rejecting (M) in its full generality—reasons that would basically rely rather on (T$^{\vdash}$), (EXC), (EXH) and the very plausible claim that there can be definite borderline cases of some kind or other. For, by (T$^{\vdash}$), (EXC) and (EXH), $\neg\mathcal{D}\varphi \vdash \neg\varphi$ holds, and so, by (M), $\mathcal{D}\neg\mathcal{D}\varphi \vdash \mathcal{D}\neg\varphi$ holds, from which one would expect it to follow, even in a tolerant logic, that $\mathcal{D}\neg\mathcal{D}\varphi \vdash \neg\mathcal{D}\mathcal{B}\varphi$ holds. But one would also expect, even in a tolerant logic, that $\mathcal{D}\mathcal{B}\varphi \vdash \mathcal{D}\neg\mathcal{D}\varphi$ holds and that it does so in such a way as to combine with the previous valid argument to yield $\mathcal{D}\mathcal{B}\varphi \vdash \neg\mathcal{D}\mathcal{B}\varphi$. Therefore, by (EXC) (and intersubstitutability of a sentence with its double negation), $\mathcal{D}\mathcal{B}\varphi \vdash \varnothing$ holds, and so, by (EXH), $\varnothing \vdash \neg\mathcal{D}\mathcal{B}\varphi$ holds. Thus,

[32] I've briefly argued in the text that, for no i, one has non-inferential reasons for accepting $\mathcal{B}(g_0 = g_i)$. If one does not have inferential reasons either for accepting it, that is obviously sufficient for undercutting the *letter* of the second reason against the conclusion of argument E, quite independently of all the previous stuff in the text about (CCTLC). But, even assuming that one does not have inferential reasons either for accepting $\mathcal{B}(g_0 = g_i)$, the richness of articulation afforded by the normative theory which accompanies a naive theory of vagueness adopting a tolerant logic and which leads to the rejection of (CCTLC) is needed in order to undercut subtler renderings of the *spirit* of the second reason against the conclusion of argument E. For example, given that (CCTLC) has deliberately been formulated in terms of the broader notion of *thinking* (which encompasses also attitudes other than acceptance), the foe of the conclusion of argument E could observe that, in $\mathbf{V_0}$ and its like, given the obvious additional constraints on the domain $\exists x\mathcal{B}(g_0 = x) \vdash \mathcal{B}(g_0 = g_0), \mathcal{B}(g_0 = g_1), \mathcal{B}(g_0 = g_2) \ldots, \mathcal{B}(g_0 = g_{999,999})$ holds and that there should be no objection in a naive theory adopting a tolerant logic to the normative principle concerning multiple-conclusion arguments saying that, if one has non-inferential reasons to accept all the premises of a valid argument, one has inferential reasons for *conditionally accepting each of its conclusions* (i.e. *conditionally on the rejection for non-inferential reasons of all the other conclusions*). Letting conditional acceptance be the relevant mode of thinking, the foe of the conclusion of argument E could then use the conclusion of argument E and (CCTLC) to infer that, for every i, one is committed to accepting conditionally $g_0 \neq g_i$ (i.e., conditionally, for every other j, on the rejection for non-inferential reasons of $\mathcal{B}(g_0 = g_j)$). These are already in themselves rebarbative commitments. And, if the logic of definiteness also yields $x \neq y, \neg\mathcal{D}(x \neq y) \vdash \varnothing$ (see Sect. 16.2), the foe of the conclusion of argument E could turn those commitments into downright unacceptable ones by inferring that, for every i, one is committed to accepting conditionally the jointly inconsistent $g_0 \neq g_i$ and $\neg\mathcal{D}(g_0 \neq g_i)$ (again, conditionally, for every other j, on the rejection for non-inferential reasons of $\mathcal{B}(g_0 = g_j)$), from which it follows that, unacceptably, all options open to one are unacceptable options on which one is committed to accepting two jointly inconsistent sentences. This more sophisticated argument is effectively blocked by rejecting (CCTLC) on the grounds adduced in the text.

on the view under consideration, for reasons independent of argument E (M) implies that *nothing is a definite borderline case of any kind whatsoever*. On the view under consideration then, if there are reasons for rejecting the claim that nothing is a definite borderline case of identity on the grounds that, in general, there can be definite borderline cases, these should be taken to be reasons for rejecting (M) rather than reasons for rejecting the conclusion of argument E.

An even more general reason for rejecting (M) that does not rely on ($\mathrm{T}^{\vdash}$) (nor on (EXC) or (EXH)) emerges if we consider the specific situation of a soritical series. Let's focus again on Greg's transformation. Because of higher-order vagueness, it is extremely plausible to think that, for every open formula φ containing only H, $\neg$ and $\mathcal{D}$, φ is vague in that situation. Now, here is, somewhat roughly stated, a plausible requirement of *closure under no definite sharp boundaries* on our theory Γ of Greg's transformation:

(CNDSB) For every i, if φ is a vague open formula (with respect to ξ) and $\Gamma \vdash \mathcal{D}\varphi[g_i/\xi]$ holds, then $\Gamma \vdash \neg\mathcal{D}\neg\varphi[g_{i+1}/\xi]$ holds.

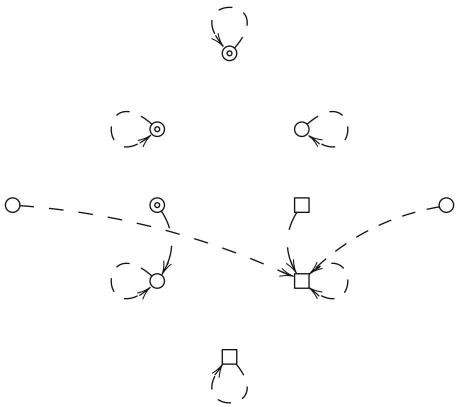
In other words, for everything that our theory entails to be a candidate for being a point at which the transition from a vague property to its negation is not vague, our theory also entails that it is after all not such a point. However, in virtually every logic of vagueness (including tolerant logics), (CNDSB) is inconsistent with (M), at least given the two extremely plausible assumptions that, for a large "enough" i, $\mathcal{D}^i Hg_0$ as well as $\mathcal{D}^i \neg Hg_{999{,}999}$ hold and that there is "enough" higher-order vagueness (see in particular Zardini 2013a for the precise sense of 'enough', for the details of the demonstration and for a discussion of its assumptions).[33]

16.7 The Tolerant Logic $\mathbf{V_1}$

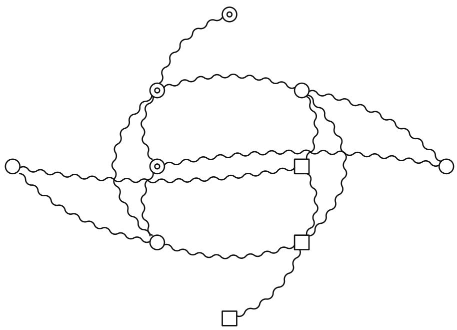
In Sect. 16.6, I've argued in effect that all of ($\mathrm{T}^{\vdash}$), ($\mathrm{T}^{\vdash^{\omega}}$) and ($\mathrm{T}^{\supset}$) may after all still be acceptable in a naive theory of vagueness adopting a tolerant logic. Let's see then how they can be added to $\mathbf{V_0}$ (thus giving rise to the logic $\mathbf{V_1}$).

Definition 3. A $\mathbf{V_1}$-model $\mathfrak{M}$ is a 9ple $\langle U_{\mathfrak{M}}, V_{\mathfrak{M}}, \preceq_{\mathfrak{M}}, D_{\mathfrak{M}}, T_{\mathfrak{M}}, \mathrm{neg}_{\mathfrak{M}}, \mathrm{def}_{\mathfrak{M}}, \mathrm{id}_{\mathfrak{M}}, \mathrm{int}_{\mathfrak{M}}\rangle$. $U_{\mathfrak{M}}$, $V_{\mathfrak{M}}$, $\preceq_{\mathfrak{M}}$, $D_{\mathfrak{M}}$, $T_{\mathfrak{M}}$, $\mathrm{neg}_{\mathfrak{M}}$ and $\mathrm{int}_{\mathfrak{M}}$ are as *per* Definition 1. $\mathrm{def}_{\mathfrak{M}}$ is an operation of the same kind as *per* Definition 1 and can be depicted as:

[33] (M) may be rescued from the problems discussed in the text if one were to go tolerant *in reasoning about* $\vdash$ *itself*. But, on a natural way of implementing it, this move would equally block as tolerantly invalid the reasoning based on the conclusion of argument E and (M) to the effect that nothing is a definite borderline case of identity.



$\mathrm{id}_{\mathfrak{M}}$ is an operation of the same kind as *per* Definition 1, with the modification that, for every $u \in U_{\mathfrak{M}}$, $\{0, 1, 2, 3, 4, 5, 6, 7\} = \mathrm{id}_{\mathfrak{M}}(u, u)$ and with the relevant deviations being:



Again, $\mathrm{int}_{\mathfrak{M}}$ can be extended to a full valuation function $\mathrm{val}_{\mathfrak{M}}$ (relative to assignments) in the usual way. With such function in place, the same style of definition of logical consequence yields:

Definition 4. $\Gamma \vdash_{\mathbf{V_1}} \Delta$ holds iff, for every $\mathbf{V_1}$-model $\mathfrak{M}$ and assignment ass, if, for every $\varphi \in \Gamma$, $\mathrm{val}_{\mathfrak{M},\mathrm{ass}}(\varphi) \in D_{\mathfrak{M}}$, then, for some $\psi \in \Delta$, $\mathrm{val}_{\mathfrak{M},\mathrm{ass}}(\psi) \in T_{\mathfrak{M}}$.

$\mathbf{V_1}$ enjoys all the properties recorded for $\mathbf{V_0}$ in Sect. 16.4 (and we can now explicitly note that (M) fails both in $\mathbf{V_0}$ and in $\mathbf{V_1}$); moreover, contrary to what is the case for $\mathbf{V_0}$, in $\mathbf{V_1}$ ($\mathrm{T}^{\vdash}$), ($\mathrm{T}^{\vdash^{\omega}}$) and ($\mathrm{T}^{\supset}$) hold and the argument $\varnothing \vdash \mathcal{D}(\tau = \tau)$ can no longer give rise to failures of ($\mathrm{TR}^{\vdash}$) (and so $\mathcal{D}(\tau = \tau)$ can be suppressed).[34]

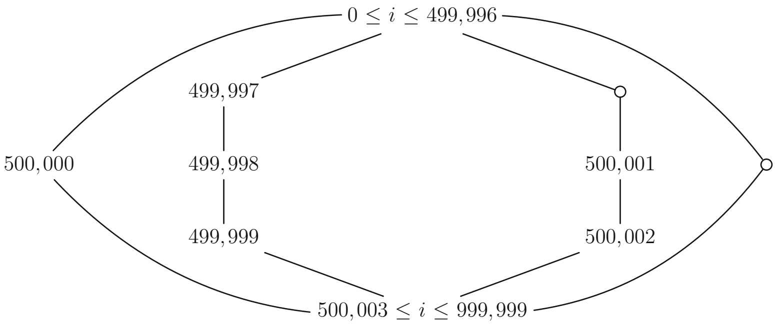
[34] All this implies that not only does the conclusion of argument E hold in $\mathbf{V_1}$ but also argument E itself is valid.

16.8 The Harmlessness of Argument E in $\mathbf{V_1}$

The upshot of Sect. 16.6 was that, in a naive theory of vagueness adopting a tolerant logic, the two main reasons against the conclusion of argument E no longer have force and that, very plausibly, in every theory accepting ($\mathrm{T}^{\vdash}$), (EXC) and (EXH) (and, plausibly, in virtually every theory whatsoever), that conclusion does not imply that nothing is a definite borderline case of identity. Focussing now on a naive theory adopting $\mathbf{V_1}$, what needs to be shown is, in line with one of the main contentions of Sect. 16.6, the consistency in $\mathbf{V_1}$ of the claim that, for some object or other, it is definite that it is borderline whether Greg_0 is identical with it, which is to say that there is a $\mathbf{V_1}$-model $\mathfrak{M}$ such that $\mathrm{val}_{\mathfrak{M},\mathrm{ass}}(\exists x\mathcal{DB}(g_0 = x)) \in D_{\mathfrak{M}}$. Indeed, and again in line with one of the main contentions of Sect. 16.6, to make sure that the existence of a definite borderline identity does not have problematic consequences for the naive theory and that, even together with it, it remains silent on whether any specific identity is borderline, we should show that, in $\mathbf{V_1}$, $\exists x\mathcal{DB}(g_0 = x)$ together with the (H^p)–($=^t$)-fragment of the naive theory does not entail $\mathcal{B}(g_0 = g_i)$ for any i, which is to say that, for every i, there is a $\mathbf{V_1}$-model $\mathfrak{M}$ of the (H^p)–($=^t$)-fragment of the naive theory such that $\mathrm{val}_{\mathfrak{M},\mathrm{ass}}(\exists x\mathcal{DB}(g_0 = x)) \in D_{\mathfrak{M}}$ and $\mathrm{val}_{\mathfrak{M},\mathrm{ass}}(\mathcal{B}(g_0 = g_i)) \notin T_{\mathfrak{M}}$.

Theorem 6. *For every i, there is a $\mathbf{V_1}$-model $\mathfrak{C}_1$ such that* $\mathrm{val}_{\mathfrak{C}_1,\mathrm{ass}}((\mathrm{H}^p))$, $\mathrm{val}_{\mathfrak{C}_1,\mathrm{ass}}((\mathrm{H}^n))$, $\mathrm{val}_{\mathfrak{C}_1,\mathrm{ass}}((\mathrm{H}^t))$, $\mathrm{val}_{\mathfrak{C}_1,\mathrm{ass}}((=^n))$, $\mathrm{val}_{\mathfrak{C}_1,\mathrm{ass}}((=^t))$ *and* $\mathrm{val}_{\mathfrak{C}_1,\mathrm{ass}}(\exists x\mathcal{DB}(g_0 = x)) \in D_{\mathfrak{C}_1}$ *while* $\mathrm{val}_{\mathfrak{C}_1,\mathrm{ass}}(\mathcal{B}(g_0 = g_i)) \notin T_{\mathfrak{C}_1}$.

Proof. Sketch for $i \neq 500{,}000$. Take $\mathfrak{C}_1$ to be a $\mathbf{V_1}$-model such that, for every j, $\mathrm{val}_{\mathfrak{C}_1,\mathrm{ass}}(g_j = g_{j+1}) = \{0, 1, 2, 3, 4, 5, 6, 7\}$ and, letting a number j occupying the place of a value X mean that $\mathrm{val}_{\mathfrak{C}_1,\mathrm{ass}}(Hg_j) = \mathrm{val}_{\mathfrak{C}_1,\mathrm{ass}}(g_0 = g_j) = X$, such that:



□

Thus, at a *formal* level, one can adopt $\mathbf{V_1}$ and thereby be committed to accepting the claim that it is definite that Greg_0 is identical with Greg_0, accept the (H^p)–($=^t$)-fragment of the naive theory of vagueness and the claim that, for some i, it is definite that it is borderline whether Greg_0 is identical with Greg_i, accept that, for every i, the claim that it is borderline whether Greg_0 is identical with Greg_i entails that Greg_0 is distinct from Greg_i (and entails that Greg_0 is identical with Greg_i) and yet, for no i, be committed to accepting that Greg_0 is distinct from Greg_i (or that Greg_0 is identical with Greg_i).

But how can we make sense of this position at a *philosophical* level? The position is somehow the opposite of the position discussed in Sect. 16.5, as it encodes the idea that, by ($\text{T}^{\vdash}$) and ($\text{T}^{\supset}$), definitely being is a mode of being that is weaker* than merely being (and that, by ($\text{T}^{\subset}$), merely being is a mode of being that is weaker* than definitely being). On this position, for example, nothing could be human without being a definite human—there is no limbo of degenerated substances that are human but not so much as to be definite humans. On a natural way to support this intuitively appealing claim, that is so because, to be human, one must be marked as *different enough* from what is not human, and so be a definite human. Every human—no matter how deformed—is a definite human. More generally, and as already suggested in Sect. 16.6, everything is definitely what it is.

The contrapositive of ($\text{T}^{\vdash}$) follows just as naturally from these considerations: for example, if one is not a definite human, one is not marked as different enough from what is not human, and so one is not human after all. And that can in fact most clearly be seen as an application of what it is for 'not human' to be *tolerant*: if x is not human and y is not very different from x, y is also not human. Being thus arguably grounded in tolerance, ($\text{T}^{\vdash}$) exhibits the same feature exhibited by other valid arguments that are so grounded: although its conclusion is weaker* than its premise, it is not weaker**, and so ($\text{T}^{\vdash}$) can give rise to failures of ($\text{TR}^{\vdash}$) (as it should in order for the defence offered in Sect. 16.6 to be ultimately viable).[35]

All this may sound alright in itself, but can this conception of definiteness still square with the claim with an instance of which this paper began and which passed through the sieve of the criticisms in Sect. 16.6—that is, with the claim that there are borderline cases? It can. Firstly, the conception of definiteness in question still supports the justification for that claim: if, for every i, either Greg_i is marked as different enough from what is not human or Greg_i is marked as different enough from what is human, whence could the vagueness of human Greg_0's transformation into not human Greg_k arise? Secondly, as was already implicit in our discussion of ($\text{TR}^{\text{ADJ}^{\text{INC}}}$) in Sect. 16.6, the conception of definiteness in question only implies [that, under the assumption that it is borderline whether Greg_i is human, it follows that Greg_i is not human] and [that, under the same assumption, it follows that

[35]Thus, generally, both $\mathbf{V_0}$ and $\mathbf{V_1}$ think that the problem with Evans' argument consists in the fact that $\mathcal{D}\varphi$ is not weaker** than φ. The particular instance on which $\mathbf{V_0}$ focusses is $\mathcal{D}(x = x)$ not being weaker** than $x = x$ while, contrary to what $\mathbf{V_0}$ thinks happens in other instances, being weaker* than it (with the consequence that the former cannot be suppressed in a valid argument although it is a logical truth). On the contrary, $\mathbf{V_1}$ thinks that $\mathcal{D}(x = x)$ is not only weaker* but also weaker** than $x = x$ (with the consequence that the former can be suppressed in a valid argument) and focusses instead on $\mathcal{D}(x = y)$ not being weaker** than $x = y$ while being weaker* than it. Thanks to Krzysztof Posłajko for questions that led to this observation.

Greg_i is human]. It does not imply that, under the same assumption, it follows that [Greg_i is not human and Greg_i is human]. Because it does not imply that (nor any other relevant inconsistency claim), the conception of definiteness in question does not imply that the assumption that it is borderline whether Greg_i is human is inconsistent (which, by (EXH), would in turn imply that it is not borderline whether Greg_i is human). And so it does not imply a sort of "n-inconsistency" in which, although it is asserted that something among a finite number n of things is thus and so, it is also asserted of each of these things that it is not thus and so (a pattern that, under minimal assumptions, would lead to a straightforward inconsistency).[36] Our concept of definiteness pulls in two different directions: on the one hand, everything that is some way is definitely that way; on the other hand, vague transitions require that there be things that are neither definitely one way nor definitely the other way. And while these two different directions are indeed contradictory in virtually every other logic of vagueness, they are not so in tolerant logics, and can in fact be upheld in a tolerant logic like $\mathbf{V_1}$.

To sum up, on this position 'It is borderline whether Greg_0 is identical with Greg_i' does entail 'Greg_0 is distinct from Greg_i'. Evans was right about that. He was even right for correct reasons: argument E is valid. But, although correct, those reasons are fatally misleading insofar as they suggest that the entailment is due to peculiar features of identity: rather, it is a completely general fact that 'It is borderline whether P' entails 'It is not the case that P' (and that it also entails 'P'). What Evans with his argument was definitely wrong about was to assume, implicitly appealing to principles untenable in a naive theory of vagueness adopting a tolerant logic, that these results show that borderline identity is impossible.

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[36]Nor does it imply a sort of "supervaluationist thingy" in which, although it is asserted that it is definite that something among a finite number n of things is thus and so, it is also asserted of each of these things that it is not definite that it is thus and so (a pattern that, under minimal assumptions, ($\mathrm{T}^{\vdash}$), (EXC) and (EXH), would lead to a straightforward inconsistency). For example, as we've seen, the position under discussion asserts not only $\mathcal{D}\exists x \mathcal{B} H x$ but also $\mathcal{D}\exists x \mathcal{D}\mathcal{B} H x$, and, for many is, it does not assert $\neg\mathcal{D}\mathcal{B} H g_i$ (although for no i does it assert $\mathcal{D}\mathcal{B} H g_i$).

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(Note: Only selected page numbers are provided for oft-used terms such as “vague” and “indeterminate.”)

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