Aspects of the Difference between the Local and Global Modulus of Elasticity of Structural (hardwood) Timber

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Abstract. The modulus of elasticity of structural timber (MOE) may be determined by 2 methods according to the European standard EN 408 [1]. The ratio between the so-called local and global MOE that is found from tests according to these two methods, cannot be explained by the ratio between the MOE and shear modulus G that is assumed in the strength class tables of EN 338 [2]. The relationship between MOE_{local} and MOE_{global} from EN 384 [3] is not consistent with the shear values given in [2]. In this study the shear modulus for samples of the tropical hardwood species massaranduba and of softwood species spruce was determined. The shear modulus G was found to be not related to the MOE and was shown to be constant at around 550 N/mm^2 for massaranduba and 190 N/mm^2 for spruce. With these values, the ratios between MOE-local and MOE-global that were found in the test series could be explained. The found values for the shear moduli differ from previous research. The study concludes that it is unclear which parameters determine the magnitude of the shear modulus of a single piece of timber and that this needs to be investigated.

Keywords: structural (hardwood) timber, ratio of local and global Modulus of Elasticity, shear modulus, weak zones.

1 Introduction

The Modulus of Elasticity of structural timber can be determined by 2 methods according to the European standard EN 408. The first method is based on measurements of the deformation within the zone with a constant bending moment in a 4-point bending test arrangement. The relation between the applied force and the measured deformation is used to calculate the so-called MOE_{local} of the beam. The second method uses the same four point bending test, but now the mid-span deflection relative to the supports of the test piece is measured. In this case, the total deformation is caused by both bending and shear deformations in the beam. In the 2010+A1:2012 version of EN 408 [1] the expression for this so-called MOE_{global} includes the shear modulus G. However it is stated that the shear

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modulus G should be taken as infinite when the MOE_{global} is used to determine the MOE_{local} according to the relationship in EN 384, which should be used for strength class assignments. Therefore in this paper MOE_{global} is defined as the property calculated out of deflection measurements where the shear modulus G taken as infinite in the calculation.

EN 384 [3] gives the following equation to calculate MOE_{local} out of the $MOE_{global:}$

$$MOE_{local} = 1{,}3 * MOE_{global} - 2690$$

This equation is derived from a dataset with results for softwood specimens, consequently with a limited range of MOE values. For low values and for high values of $MOE_{global,G=\infty}$ the resulting values for MOE_{local} are doubtful.

In the strength class tables of EN 338, the shear modulus G is given as MOE_{local} divided by 16. When this ratio is used to calculate G for individual test pieces, the ratio between MOE_{global} and MOE_{local} that is found in literature cannot be explained adequately with the equation for MOE_{glob} in EN 384. Under the assumption that the MOE does not vary along the beam, the expected ratio between MOE_{local} and MOE_{global} is approximately 1.04 for a ratio $MOE_{local}/G = 16$. However, Ravenshorst and Van de Kuilen[4] report tests on softwood, temperate hardwoods and tropical hardwoods in which a ratio MOE_{local}/MOE_{global} of around 1.15 is found for all species. Brandner et al. [5] report from literature that MOE_{local}/G values for softwoods can vary between 12 to 36, and that the shear modulus G for structural softwood timber could be more or less constant around $G=600\ N/mm^2$, slightly increasing for higher MOE.

The aim of this study is to explain the differences between tested MOE_{local} and MOE_{global} for 2 samples of a tropical hardwood species and 1 softwood sample, to test the quality of the EN 384-equation and to evaluate the assumed dependency of the shear modulus with MOE_{local}.

2 Theoretical Considerations

The difference in the local and global Modulus of Elasticity of structural timber are influenced by the following two phenomena:

- The shear deformation in the side parts of the test set-up.
- The influence of zones with low bending stiffness.

2.1 The Shear Deformation in the Side Parts of the Test Set-Up

As mentioned in the introduction, there is no consensus in literature for the magnitude of the shear modulus G in relation to the modulus of elasticity for softwoods. For tropical hardwoods there is almost no data available for the shear modulus of structural timber. The effect of the shear modulus is therefore studied on both softwood and tropical hardwood.

2.2 The Influence of Weak Zones with Low Bending Stiffness

In the standard EN 384 an equation for the MOE_{local} is given when the MOE_{global} determined. According to this equation MOE_{local} can become lower than MOE_{global}. Denzler et al. [6] confirmed this phenomenon. The relevant studies are mostly done on softwood, so including pieces with low MOE, with a lower limit of approximately 5000 N/mm^2. That MOE_{local} becomes lower than the MOE_{global} can only be explained by the occurrence of a zone with a low bending stiffness within the MOE_{local} area, which has more influence than the deflection due to shear in the outer parts. This seems logical as it is prescribed that the weakest part should be placed in the center of the span. However, for most tropical hardwoods, the only stiffness reducing characteristic is the grain angle deviation. Since there is no clear visible weak part, it can be assumed that the MOE is more constant over the beam length than for softwoods with knots, which cause low local bending stiffness. This is another argument to test and compare a tropical hardwoods species and a softwood species. In this paper the zones with low bending stiffness will be referred to as weak zones.

3 Materials and Methods

3.1 Materials

In table 1 the materials that are used are listed. The tropical hardwood species massaranduba has no visible weak zones due to defects like knots, so a constant value of the MOE and G can be assumed over the length of the beam. Two samples of massaranduba with different dimensions and different moisture contents are studied. In the softwood sample of spruce knots, causing weak zones are be present.

Table 1 Description of the hardwood and softwood samples used in the study

Sample code	n	Wood species	Botanical name	source	Dimensions t (mm) x h (mm)	Mean moisture content (%)	Mean density (kg/m^3)
M1	26	Massaranduba	Manilkara spp.	Brazil	60 x 150	20.2	1088
M2	40	massaranduba	Manilkara spp	Brazil	50 x 100	11.9	979
S1	31	spruce	Picae abies	Austria	40 x200	9.0	453

3.2 Methods

3.2.1 Test Set-Up

For all measurements a 4-point bending test arrangement was used, similar to the setup described in EN408. In the test arrangement special care was taken to minimize local restrain of deformations. The detailing of both supports and the devices used to apply the loads ensure this (see figure 1).

MOE_{local} and MOE_{global} were determined in accordance with EN408, where for the determination of MOE_{global} the relation between the applied force and the displacement of the section of the beam where the load is applied (points P in figure 1) was used instead of the displacement of the middle section of the beam. The displacement of point P was derived from the displacement of the actuator that applied the force F. Effectively corrections were made for the stiffness of the test rig elements as well as for local indentations at both supports and loading point locations.

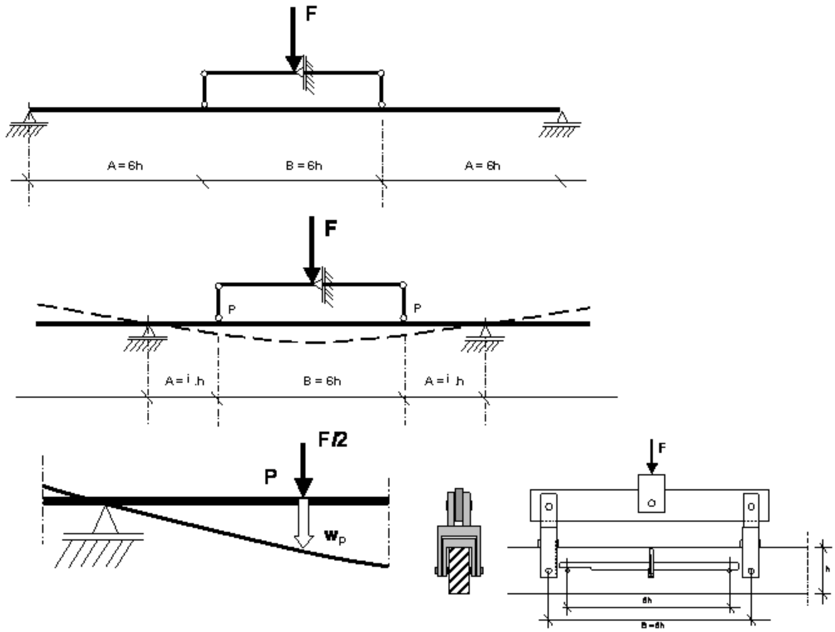

Fig. 1 Test set-up

To investigate the influence of the shear deformation, the distance A from the support to the location where the load is applied (points P in figure 1) was varied. For all specimens the relation between the load F and the deflection w_p was measured for A = i.h, where i=2,3,4,5,6. These 5 measurements were used to derive an effective shear modulus G and an effective MOE for each specimen.

Using Timoshenko beam theory the deflection w_p of point P caused by load F in the test setup in figure 1 is equal to:

$$w_p = \underbrace{\frac{F \cdot A^2 \cdot B}{4EI} + \frac{F \cdot A^3}{6EI}}_{bending} + \underbrace{\frac{F \cdot k_s \cdot A}{2 \cdot G \cdot b \cdot h}}_{shear}$$ The measurements for A = i.h result each in a value for $\left(\frac{dw_p}{dF}\right)_i$

$$\frac{w_p}{F} = \underbrace{\frac{A^2 \cdot B}{4EI} + \frac{A^3}{6EI}}_{bending} + \underbrace{\frac{k_s \cdot A}{2 \cdot G \cdot b \cdot h}}_{shear}$$

$$\left(\frac{w_p}{F}\right)_i = \frac{1}{E}\left(\frac{(i \cdot h)^2 \cdot 6h}{4 \cdot \frac{1}{12} bh^3} + \frac{(i \cdot h)^3}{6 \cdot \frac{1}{12} bh^3}\right) + \frac{k_s \cdot i \cdot h}{2 \cdot G \cdot b \cdot h}$$

$$\left(\frac{w_p}{F}\right)_i = \frac{1}{E}\left(\frac{18 \cdot i^2 + 2 \cdot i^3}{b}\right) + \frac{k_s \cdot i}{2 \cdot G \cdot b}$$

$$\underbrace{\frac{\left(\frac{dw_p}{dF}\right)_i}{\left(\frac{18 \cdot i^2 + 2 \cdot i^3}{b}\right)}}_{MR_i} = \frac{1}{E} + \underbrace{\frac{1}{\left(18 \cdot i + 2 \cdot i^2\right)} \frac{k_s}{2}}_{F_i} \cdot \frac{1}{G} \quad \Rightarrow MR_i = \frac{1}{E} + F_i \cdot \frac{1}{G}$$

For a rectangular section $k_s = 1.2$. This yields the f values for F_i according to table 2.

The slope of the linear trend line through the points (F_i, MR_i) now yields the reciprocal value of the effective shear modulus G for the specimen. The cutting point of the trend line with the vertical axis is the reciprocal value of the MOE for pure bending of the beam. This value will be tagged MOE_{beam}.

Table 2 Values for F_i

i	F_i
2	0.0136
3	0.0083
4	0.0058
5	0.0043
6	0.0033

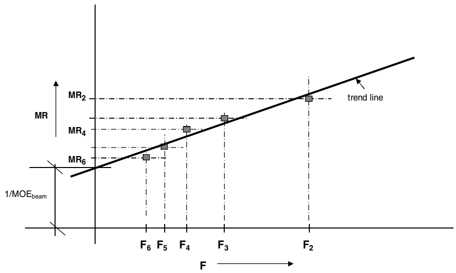

Fig. 2 Principle for deriving of MOE_{beam} and G out of MOE_{global} measurements

4 Results

In table 3 the values for MOE_{local} and MOE_{global} are given and the ratio between them. The values found are in line with the ratios found in Ravenshorst and van de Kuilen [4]. Figure 3 shows a scatter plot for the 3 samples with regression lines forced through the origin. What can be observed is that the 2 massaranduba samples almost follow the same regression line but that sample M2 is shifted giving higher values for both MOE_{local} and MOE_{global}. Thisis is caused by the difference in moisture content between the 2 samples.

Table 4 gives the values for MOE_{beam} and G determined with the variable span method, the ratios between MOE_{local} and G and the ratio between MOE_{bend} and MOE_{local}. Table 4 shows that ratios between MOE_{local} and G are much higher than the value of 16 which is used in the strength class tables of EN 338. Remarkable are the low values for G. Particular for the spruce sample very low values for G are found. (Remark: to verify these values with an alternative method the dynamic torsional stiffness G_{dyn} was determined for samples M2 and S1. Values found with this method gave similar results as the static G).

Table 3 Test results for MOE_{local} and MOE_{global}

Sample code	MOE_{local}		MOE_{global}		MOE_{local}/MOE_{global}
	Mean (N/mm^2)	cov	Mean(N/mm^2)	cov	mean
M1	22580	0.17	19500	0.15	1.16
M2	26100	0.12	22580	0.11	1.15
S1	13190	0.21	11680	0.21	1.14

Table 4 Calculated values for MOE_{beam} and G

Sample code	MOE_{beam}		G		MOE_{local}/G	MOE_{local}/MOE_{beam}
	Mean (N/mm^2)	cov	Mean(N/mm^2)	cov	mean	mean
M1	23200	0.17	500	0.12	45.1	0.97
M2	25220	0.12	590	0.22	44.3	1.04
S1	14790	0.27	190	0.26	69.4	0.89

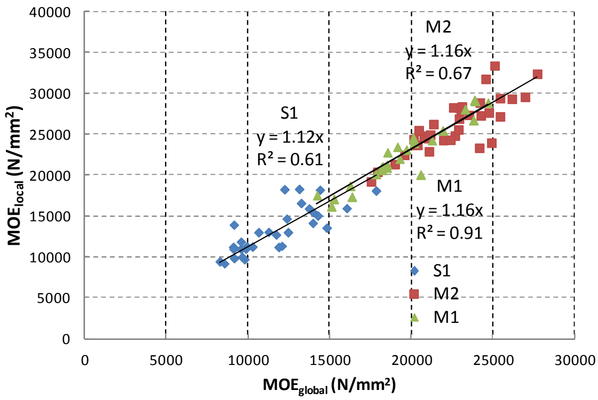

Fig. 3 Scatter plots for MOE_{local} against MOE_{global} for the tested samples

5 Analysis

5.1 *Prediction of MOE$_{global}$*

To study the influence of G on the ratio between MOE$_{local}$ and MOE$_{global,}$ the MOE$_{global}$ is predicted out of the found values of MOE$_{beam}$ and G, both determined with the variable span method for every test piece. Out of the share of deflection of both properties the MOE$_{global;pred}$ can then be calculated, assuming an infinite value for the shear modulus, as if the beam was tested according to EN 408.

The found MOE$_{bend}$ and G-values for a piece are average values over the entire beam (MOE$_{beam}$) and the end parts (G). This is a simplification of reality, because the weak zones might have a local influence, especially for softwoods. This can explain that the ratio between MOE$_{local}$ and MOE$_{beam}$ deviates more from 1 for softwoods than for hardwoods. Since the weak zone is placed in the MOE$_{local}$ zone it can be expected that the MOE$_{local}$ for softwood is lower than the MOE$_{beam}$, which is confirmed with the value of 0,89 between MOE$_{local}$ and MOE$_{beam}$ for softwood.

In table 5 the mean values for the calculated MOE$_{global,pred}$ are given, together with the ratio between MOE$_{local}$ and MOE$_{glob,pred}$. In figure 4 the MOE$_{global,}$ is plotted against the MOE$_{global,pred}$. Figure 4 shows that the MOE$_{global;pred}$ gives a good prediction of MOE$_{global}$. This gives evidence that the shear modulus G is the main cause for the difference between MOE$_{local}$ and MOE$_{global.}$

Table 5 Calculated values for MOE$_{global;pred}$

Sample code	MOE$_{global;pred}$		MOE$_{local}$/ MOE$_{glob;pred}$
	Mean (N/mm^2)	cov	Mean
M1	20400	0.15	1.11
M2	22480	0.11	1.16
S1	11830	0.20	1.12

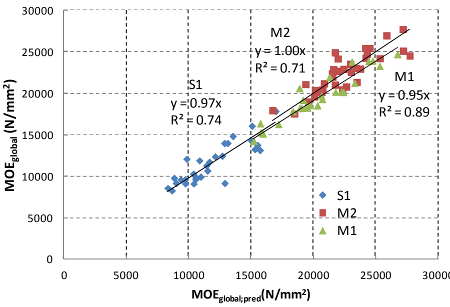

Fig. 4 Scatter plots for MOE$_{global}$ against the calculated MOE$_{global;pred}$ or the tested samples

5.2 *Dependency of G from MOE$_{local}$*

In figure 5 the G is plotted against MOE$_{local}$. Figure 5 shows that within the samples there seems to be no clear relation between the MOE$_{local}$ and G. Between the samples there seems to be a relation between the MOE$_{local}$ and G, suggesting a higher MOE$_{local}$ leads to a higher G$_{stat}$. However, in previous studies mean values for samples of sawn beams of sitka spruce of 500 N/mm^2 and Norway spruce of 650 N/mm^2 (Kokhar [7]) and spruce glued laminated timber between 600 N/mm^2 and 800 N/mm^2 (Brandner et al. [5]) were found, with comparable values for

MOE_{local} as in this study. This suggests that the G is not only dependent of MOE_{local} but also on other factors that could not be identified in this study. Options are the density, the dimensions or the quality. Regarding this last point, in Nocetti et al. [8] was found that for increasing knot ratio the ratio MOE_{loc} /MOE_{global} decreased, which might be caused by the influence of knots on the G. The results found in Nocetti et al. [8] also supports also the influence of weak zones on the ratio MOE_{loc} /MOE_{global}.

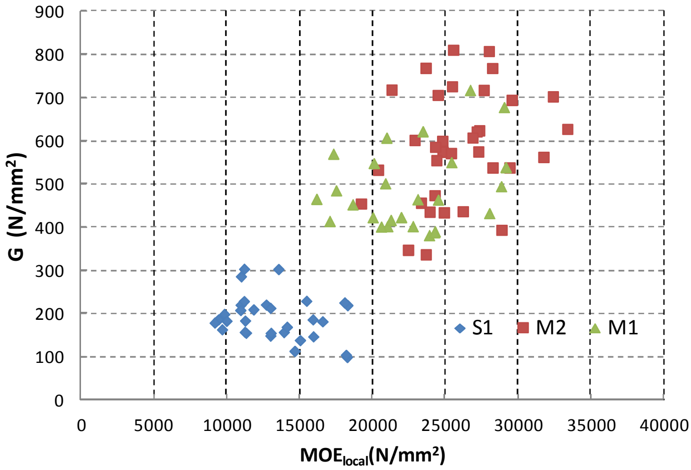

Fig. 5 Scatterplot for MOE_{local} against G for the tested samples

5.3 The Influence of Weak Zones on the Ratio between MOE_{local} and MOE_{global}

The influence of weak zones as a function of its position in the beam on the MOE_{local} and the MOE_{global} can be calculated, and as a result the effect on the ratios between them. In figure 6 the influence of the position of a weak zone on the ratio between MOE_{local} and the MOE_{global} is shown, also depending on the value of MOE_{local} and G. There are many possibilities , but the figures assume that the weak zone has a length of 0,5 the height of the beam and one third of the bending stiffness of a clear part. For softwood the shear modulus G is kept a constant value of 190 N/mm^2 which was found in this study. The figure gives the effect on the ratio between MOE_{loc} and MOE_{glob} when the position of the weak zone changes from the center to the outside of the beam. The weak zone is assumed to be asymmetric (so a weak zone only on one side, the other side is clear). For tropical hardwood no weak zone is assumed and a constant shear value of 550 N/mm^2 was used. The ratio between MOE_{loc} and MOE_{glob} is calculated for different values for the MOE_{local}.

Figure 6 shows that for a low MOE_{local} the ratio MOE_{local}/MOE_{global} can become lower than 1. Based on figure 6 the expected values and ratios for MOE_{local} and MOE_{global} can be calculated for the shear values found in this research. In the range of MOE_{global} for softwoods in this research (between 10.000 N/mm^2 and 18.000 N/mm^2) the influence of a weak zone leads to a minimum and maximum value for every MOE_{global} depending on the position of the weak zone. In the range of MOE_{global} (15.000 N/mm^2 to 30.000 N/mm^2) for hardwoods no weak zone was assumed, giving one value.

In figure 7 the ratio MOE_{local} / MOE_{global} is plotted against the MOE_{local} for the 3 samples. For softwood for three values of the MOE_{local} the ratio MOE_{local} / MOE_{global} is plotted for the minimum and maximum value found in figure 6 for a combination of E and G. The influence of the weak zones will lead to a ratio MOE_{local} / MOE_{global} somewhere between these limits. For hardwoods also for three values of the MOE_{local} the ratio MOE_{local} / MOE_{global} is plotted. In this case there is only one value, because no weak zones are assumed. It must be stated that these are average expected ratios, where scatter due to normal variation of timber can be expected. Figure 7 shows that the hardwood test data are around the predicted theoretical line, and that for softwoods the test data is between the maximum and minimum theoretical line. That means that for softwoods the ratio MOE_{local} / MOE_{global} is a combined effect of low G and weak zones. However, the weak zone effect can only be evaluated properly when also timber with low MOE is tested.

In figure 8 the predicted theoretical values from figure 7 are plotted for MOE_{local} against MOE_{global}. In figure 8 also the EN 384 equation is shown. Figure 8 shows that the different regression lines that can be found depending on the presence of weak zones and the value of the shear modulus. When one single regression line is plotted through all theoretical data points which is then forced through the origin a slope of 1,15 is found, similar to that for all tested data. The En 384 equation does not follow exactly this slope, but the error between the 2 lines is relatively small.

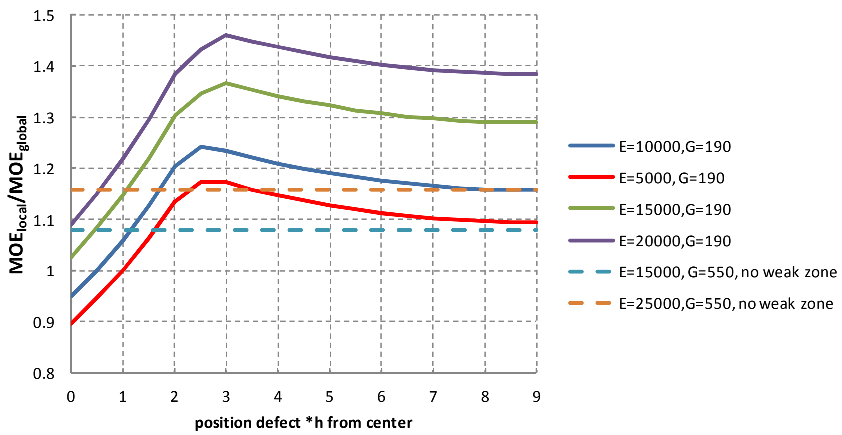

Fig. 6 Influence of position of weak zone, MOE and G on the ratio MOE_{local}/MOE_{global}

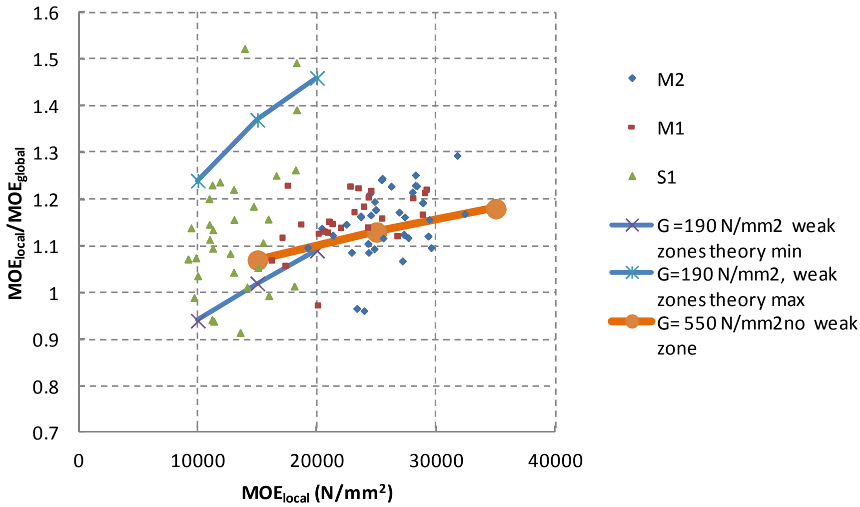

Fig. 7 Ratio MOE_{local}/MOE_{global} plotted against MOE_{local} with theoretical expected values

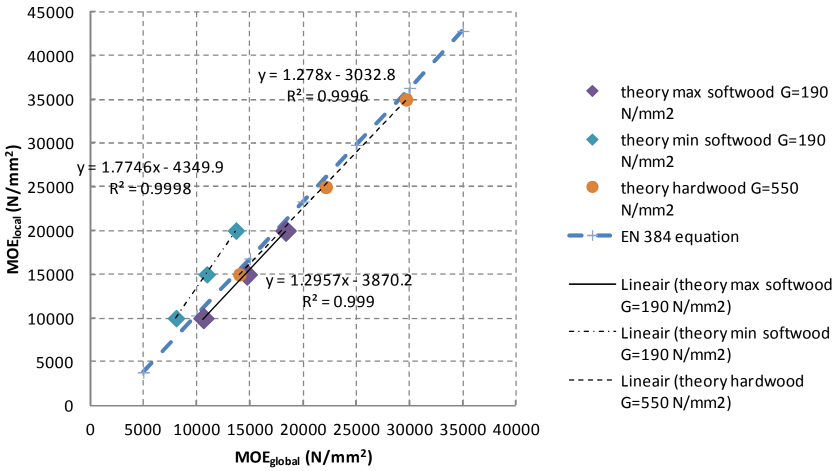

Fig. 8 Theoretical values from figure 8 for MOE_{local} plotted against MOE_{global} together with the EN 384 equation

6 Conclusions

- The ratio for MOE_{local}/MOE_{global} for the studied dataset can be explained by the found low values for the shear modulus for the samples determined with the variable span method for the softwood and hardwood samples. For the softwood sample the presence of weak zones is also expected to have an effect, but this aspect could not be identified for individual softwood pieces, because of the lack of low MOE_{local} values in the tested sample.

- The found G-values are not in line with current standardized values. The current assumed dependency from the MOE_{local} is questioned. No final statement from which parameters the shear modulus G is dependent can be made.
- The EN 384 equation for MOE_{local} out of MOE_{global} is not similar with the equation found in this study, but the differences are relatively small.

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