Tapered Beams Made of Cross Laminated Timber

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Abstract. A method to determine strength reduction factors for tapered beams made of cross laminated timber (CLT) is presented. The method is based on EC5 equations for the calculation of strength reduction factors for tapered glulam beams. For CLT-beams, however, the required strength properties, i.e. the shear strength and the tensile or compressive strength perpendicular to the longitudinal direction, are determined considering the beam layup and the different failure modes both affecting the characteristics of CLT-beams. The analytical approach was substantiated by the good agreement that was found between the strength reduction factors obtained from the analytical approach and the results of tests performed with double tapered CLT-beams.

Keywords: cross laminated timber, tapered beams, strength reduction factors, failure modes, beam lay-up.

1 Introduction

Pitched members with variable height are commonly used in timber structures for both, aesthetic and economic reasons. Typical examples are tapered or double tapered beams and columns in three hinged frames. However, since glulam, the most established material in engineered timber structures, has poor strength properties perpendicular to the grain and also a relatively small shear strength compared to its strength properties in grain direction, transverse and shear stresses arising at the pitched edges of tapered beams cause a rapid decrease of bending strength of tapered glulam beams with increasing taper angle. In contrast, beams made of cross laminated timber (CLT), a material consisting of several orthogonally bonded layers of board lamellae, provide similar strength properties in two directions: the main beam axis, which is the direction of longitudinal layers, and the direction of transversal layers. Consequently, the tensile and the compressive strength perpendicular to the longitudinal direction and also the shear strength of CLT-beams are considerably higher than the corresponding values for glulam. These comparably high strength

DOI: 10.1007/978-94-007-7811-5_60,

properties make CLT particularly suitable for the production of tapered beams where both stresses perpendicular to the longitudinal direction and shear stresses occur simultaneously at the pitched edges.

2 Analytical Approach

In most CLT products neighboured lamellae within individual layers are not bonded to each other directly, at their facing edge sides, but indirectly via the crossing areas with lamellae of neighbouring orthogonally arranged layers. Yet the bonding between the individual longitudinal lamellae is strong enough to ensure that the layers act as solid units. It can therefore be assumed that the normal stress distribution in tapered CLT-beams is similar to the distribution in tapered glulam beams and that the reduced bending strength of tapered CLT-beams can be calculated as a function of bending strength, shear strength and tensile or compressive strength perpendicular to the grain, like the reduced bending strength of glulam.

For tapered glulam beams Eurocode 5 [1] provides the strength reduction factors given in Eq. (1) with values of $k_s = 0,75$ and $k_s = 1,5$ for beams with tapered tension and compression zone, respectively. The factors take into account the interaction of shear stresses and stresses perpendicular to the grain direction which in glulam is beneficial for compression and adversely for tension.

$$k_\alpha = \frac{1}{\sqrt{1 + \left(\frac{f_m}{k_s \cdot f_v} \cdot \tan\alpha\right)^2 + \left(\frac{f_m}{f_{t(c),90}} \cdot \tan^2\alpha\right)^2}} \tag{1}$$

where:

f_m is the bending strength,
f_v is the shear strength,
$f_{t(c),90}$ is the tensile or compressive strength perpendicular to the grain

The application of Eq. (1) to CLT-beams requires the knowledge of the strength properties used in the equation. For any approved CLT-product the in plane bending strength, which mainly depends on the quality of the lamellae within longitudinal layers but is also influenced by the number of longitudinal layers, is provided by the respective technical approval. Since the transversal layers do not contribute to the in-plane bending resistance of CLT-beams, the bending strength is usually related to the net cross section of longitudinal layers. With regard to the strength reduction factors discussed here it is useful to relate the shear strength and strength properties in direction of the transversal layers to the same cross section.

In CLT-beams both the shear strength and the strength in direction of transversal layers do not only depend on the respective strength properties of the lamellae, but also on the strength of the crossing areas between orthogonally arranged longitudinal and transversal layers. Therefore the number and the arrangement of layers as well as the width of lamellae have great influence on these strength properties of CLT-beams, too. Since shear stresses and stresses in direction of transversal layers cause stresses within both the lamellae and the crossing areas different failure modes have to be considered when determining the strength properties.

Shear Strength

Three different failure modes can be distinguished in CLT-beams subjected to shear stresses as shown in Fig. 1. The first failure mode takes into account a simultaneous shear failure in all longitudinal and transversal layers within a beam. In the second failure mode the possibility of shear failure in sections that coincide with joints between the unglued edges of lamellae is considered. In these joints only the cross section of orthogonally arranged layers is available for the transfer of shear stresses. The third failure mode takes account of shear stresses in the crossing areas of orthogonally bonded layers.

Shear stresses in the gross cross-section in failure mode 1 can be calculated according to the Euler-Bernoulli theory taking into account the gross cross section of a beam (see Eq. (2)).

Shear stresses in the net cross-section in failure mode 2 can be calculated according to the beam theory taking into account the net cross section of a beam, which is the smaller value of longitudinal and transversal layers (see Eq. (3)).

Shear stresses in the crossing areas of orthogonally bonded longitudinal and transversal layers can be derived from the model of a composite beam as shown in Fig. 2. From the transfer of differential normal forces dN_i between longitudinal lamellae two shear stress components result: Shear stresses τ_{yx} (see Eq. (4)) in longitudinal direction and torsional shear stresses τ_{tor} (see Eq. (5)).

Shear stresses within the lamellae and the crossing areas of CLT-beams can be calculated from the equations given below. Detailed information on the derivation of Eq. (4) and Eq. (5) can be found in [2].

$$\tau_{xz,gross} = \frac{V_z \cdot S_{y,gross}}{I_{y,gross} \cdot t_{gross}} \quad (2)$$

$$\tau_{xz,net} = \frac{V_z \cdot S_{y,net}}{I_{y,net} \cdot t_{net}} \quad (3)$$

$$\tau_{yx} = \frac{6 \cdot V_z}{b^2 \cdot n_{CA}} \cdot \left(\frac{1}{m^2} - \frac{1}{m^3}\right) \quad (4)$$

$$\tau_{tor} = \frac{3 \cdot V_z}{b^2 \cdot n_{CA}} \cdot \left(\frac{1}{m} - \frac{1}{m^3}\right) \quad (5)$$

In Eq. (2) through (5) the following symbols apply.

V_z	Shear force,
S_y	First moment of area about the weak axis of the beam,
I_y	Second moment of area about the weak axis of the beam,
t	Thickness of the considered cross section,
b	Width of lamellae in longitudinal and transversal layers,
n_{CA}	Number of crossing areas within the thickness of the beam,
m	Number of lamellae within a longitudinal layer

When determining the effective shear strength of CLT-beams the interaction of simultaneously acting shear stress components within the crossing areas needs to be considered. The failure criterion given in Eq. (6) has been derived by evaluating test results performed on different types of CLT-beams [3].

$$\frac{\tau_{tor}}{f_{v,tor}} + \frac{\tau_{yx}}{f_R} \leq 1 \tag{6}$$

The effective shear strength of rectangular CLT-beams can generally be defined as the minimum strength resulting from the three failure modes described. However, when calculating strength reduction factors for tapered CLT-beams the second failure mode can be neglected, since shear stresses in transversal layers are small near the tapered edges. Taking into account the first and the third failure mode only, the effective shear strength of tapered CLT-beams related to the net cross-section of longitudinal layers can be calculated according to Eq. (7).

$$f_{v,CLT} = min \begin{cases} f_{v,lam} \cdot \dfrac{t_{gross}}{t_{net,long}} \\ \dfrac{n_{CA} \cdot b}{2 \cdot t_{net,long}} \cdot \dfrac{1}{\dfrac{1}{f_{v,tor}} \cdot \left(1 - \dfrac{1}{m_x^2}\right) + \dfrac{2}{f_R} \cdot \left(\dfrac{1}{m_x} - \dfrac{1}{m_x^2}\right)} \end{cases} \tag{7}$$

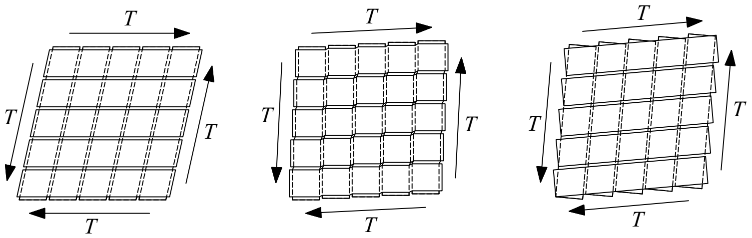

Fig. 1 Failure modes in CLT-beams subjected to transversal forces acting in plane direction: shear failure in the gross cross-section (*left*), shear failure in the net cross-section (*middle*), shear failure in crossing areas of orthogonally bonded layers (*rigth*)

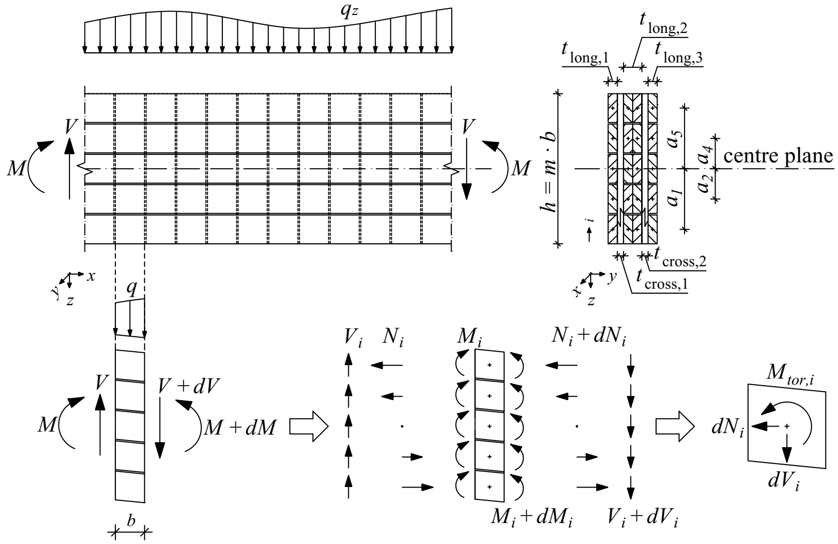

Fig. 2 Side view and cross-section of a six-layered CLT-beam loaded in plane (*top*) and internal forces in the beam, in the lamellae and in the crossing areas (*bottom, from left to right*)

In Eq. (6) and (7) the following symbols apply.

$f_{v,tor}$	Torsional shear strength of crossing areas,
f_R	Rolling shear strength of crossing areas,
$f_{v,lam}$	Shear strength of lamellae,
t_{gross}	Total thickness of the beam,
$t_{net,long}$	Summarised thickness of longitudinal layers,
m	Number of lamellae within a longitudinal layer at position x

For European softwood the shear strength $f_{v,lam}$ of lamellae can be taken from EN 338. In CLT, due to the crosswise arrangement of layers, the influence of cracks on the shear strength of lamellae is low and a crack reduction factor of k_{cr} = 1,0 may be assumed. The torsional shear strength and the rolling shear strength, determined by tests during the assessment procedure of CLT-products, can be found in technical approvals. For spruce (*Picea abies*) and fir (*abies alba*) the characteristic rolling shear strength usually ranges between 0,9 and 1,2 N/mm^2 whereas the characteristic torsional shear strength varies between 2,0 and 2,7 N/mm^2.

Strength in Direction of Transversal Layers

In CLT-beams stresses perpendicular to the longitudinal direction which arise in longitudinal lamellae at tapered edges can be transferred by transversal layers. The transfer of these stresses into transversal layers also causes shear stresses within the crossing areas. The tensile or compressive strength in direction of transversal layers can therefore be defined as the minimum strength resulting from shear failure in the crossing areas and failure in transversal layers due to tensile or compressive stresses. Assuming uniformly distributed shear stresses within the crossing areas the effective strength direction of transversal layers, again related to the cross section of longitudinal layers, can be calculated according to Eq. (8).

$$f_{90,CLT} = min \begin{cases} f_{c,0,lam} \cdot \dfrac{t_{net,cross}}{t_{net,long}} \quad or \quad f_{t,0,lam} \cdot \dfrac{t_{net,cross}}{t_{net,long}} \\ f_R \cdot \dfrac{n_{CA} \cdot b}{2 \cdot t_{net,long}} \end{cases} \tag{8}$$

In Eq. (8) the following symbols apply.

$f_{c(t),0,lam}$	Compressive (tensile) strength of transversal lamellae,
$t_{net,cross}$	Summarised thickness of transversal layers,
n_{CA}	Number of crossing areas within the element thickness
b	Width of lamellae in longitudinal and transversal layers

Strength Reduction Factors for Tapered CLT-Beams

In CLT, unlike in glulam, stresses acting in transversal direction do not adversely affect the shear strength. In general a factor $k_s = 1,0$ Eq. (1) can be used when calculating the strength reduction factors for CLT-beams according to Eq. (1). However, in CLT-beams with small gaps between the lamellae (less than 1 mm), compressive stresses will activate friction forces between the edges of lamellae which increase the shear strength. Therefore in beams with small or no gaps a factor $k_s = 1,5$ may be assumed. As an example the strength reduction factors for CLT-beams with a total thickness of 150 mm and lamellae of strength class C24 are given in Fig. 3 and Fig. 4. In the chosen example small gaps between the lamellae and an effective bending strength of $f_{m,CLT} = 24\, N/mm^2$ have been assumed. The rolling shear strength and the torsional shear strength of crossing areas were assumed as f_R = 1,0 N/mm^2 and $f_{v,tor}$ = 2,5 N/mm^2.

3 Experimental Investigations

Twenty double tapered CLT-beams with two different taper angles were tested. Half of the beams of each type were tested with compressive stresses (series C) and tensile stresses (series T) at the tapered edge, respectively. All beams had the same layup consisting of four longitudinal and two transversal layers. The dimensions of the tested beams are given in Tab. 1. All specimens were produced from lamellae of strength class C24 with a mean density of 438 kg/m^3 at an average moisture content of 10,6%. To determine the bending strength at the tapered edges two single loads were applied in the third points of the span (Fig. 5).

In all specimens of series C6 failure was caused by compressive stresses at the tapered edge (Fig. 6, *left*). In series C10 only in two specimens the compressive strength was reached whereas in three specimens failure occurred in the crossing areas between longitudinal and transversal layers (Fig. 6, *right*). In series T6 and T10 failure occurred exclusively in the crossing areas. In all specimens a subsequent splitting-off of the tapered ends of longitudinal lamellae was observed (Fig. 7).

Test results are summarised in Tab. 2. The given bending stresses $\sigma_{m,\alpha,net}$ at the tapered edge are related to the net cross section of longitudinal layers. The values were calculated according to Eq. 9 taking into account the non-linear distribution of bending stresses [4]. The given 5%-percentiles of bending strength were calculated according to EN 14358.

$$\sigma_{m,\alpha,net} = \frac{6 \cdot F_{max} \cdot x}{t_{net,long} \cdot h(x)^2} \cdot (1 - 3,7 \cdot \tan^2 \alpha) \tag{9}$$

In Eq. (9) the following symbols apply.

F_{max}	Maximum load determined in the test,
$t_{net,long}$	Summarized thickness of longitudinal layers,
x	Position of maximum bending stress,
$h(x)$	Height of the beam at position x

For the tested double tapered beams the position x, where the ratio of $M(x)/W(x)$ becomes maximum, was x = 1933 *mm* and x = 1697 *mm* for series C6/T6 and C10/T10, respectively. The height of the beams at the respective positions was 500 mm for series C6/T6 and 600 mm for series C10/T10.

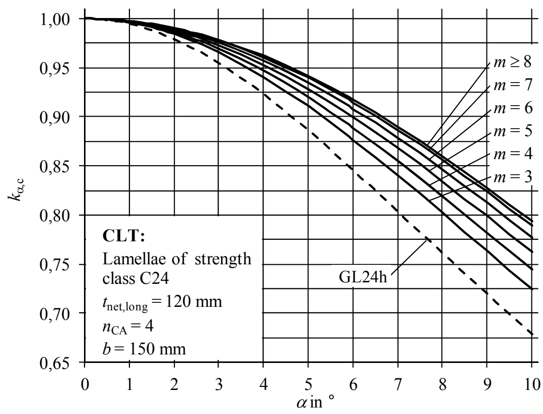

Fig. 3 Strength reduction factors $k_{\alpha,c}$ for CLT beams and glulam beams with compressive stresses at the tapered edge

Table 1 Dimensions and layup of the tested double-tapered CLT-beams

Series	Length in mm	Span in mm	Angle of taper in °	Height at apex in mm	Total thickness in mm	Layer thickness in mm	Number of layers
C6 / T6	6000	5800	6	600	150	30 / 15	4 / 2
C10 / T10	7000	6800	10	900	150	30 / 15	4 / 2

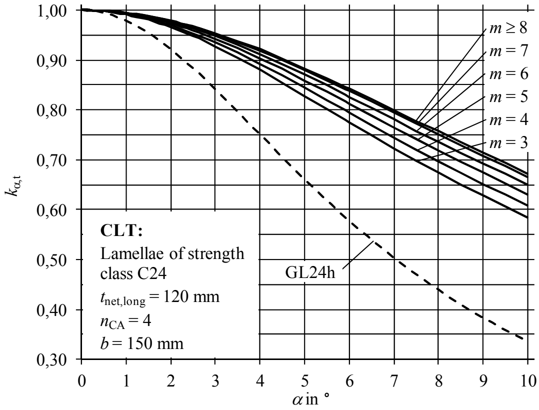

Fig. 4 Strength reduction factors $k_{\alpha,t}$ for CLT beams and glulam beams with tensile stresses at the tapered edge

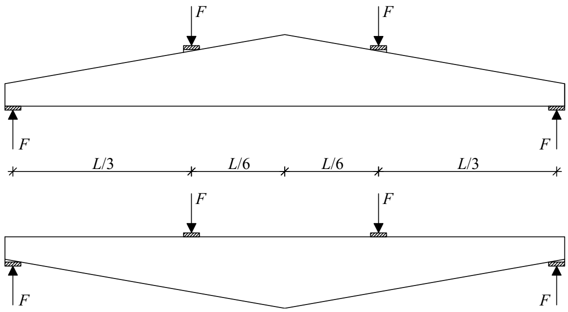

Fig. 5 Test setup for double-tapered beams *Top*: Beams with compressive stresses at the tapered edge (series C) *Bottom*: Beams with tensile stresses at the tapered edge (series T)

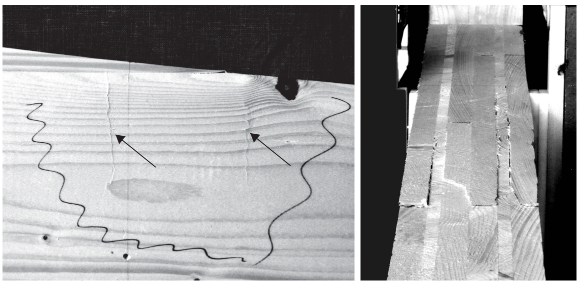

Fig. 6 Failure in specimens with tapered compression zone *Left*: Failure caused by compressive stresses in longitudinal lamellae *Right*: Failure caused by shear stresses in the crossing areas

Both mean and characteristic values of the bending strength determined by tests were used to calculate strength reduction factors k_α. Since the bending strength parallel to the grain of the sample was unknown, the strength reduction factor for series C6 was calculated from the analytical approach as described above and set as reference level. The strength reduction factors of the remaining series were then calculated from the ratios of the experimentally obtained bending strengths. In Tab. 3 the strength reduction factors k_α evaluated from the test results and the respective values obtained from the analytical approach, calculated with characteristic values of $f_{R,k} = 1,0\,N/mm^2$ and $f_{v,tor,k} = 2,5\,N/mm^2$, are summarised and compared.

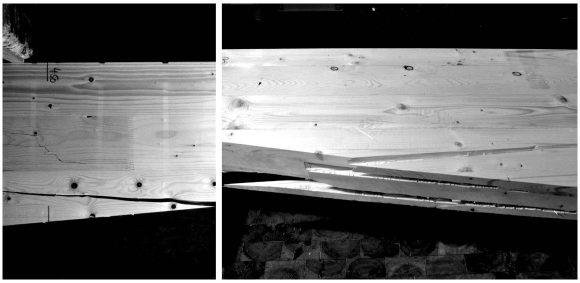

Fig. 7 Failure in specimens with tapered tension zone *Left* and *right*: Failure caused by shear stresses in the crossing areas and split-off longitudinal layers

Table 2 Test results

Series	No.	F_{max} in kN	$\sigma_{m,\alpha,net}$ in N/mm^2	$\rho_{mean,long}$ in kg/m^3	Series	No.	F_{max} in kN	$\sigma_{m,\alpha,net}$ in N/mm^2	$\rho_{mean,long}$ in kg/m^3
C6	1	106	39,4	424	T6	1	97,6	36,2	417
	2	111	41,1	427		2	99,2	36,7	445
	3	86,5	32,0	430		3	75,2	27,8	464
	4	100	36,9	440		4	102	37,7	438
	5	90,7	33,6	441		5	79,6	29,5	435
Mean value			36,6	432	Mean value			33,6	440
5%-Quantile			28,1		5%-Quantile			23,6	
C10	1	155	32,4	429	T10	1	116	24,2	431
	2	165	34,5	467		2	128	26,8	455
	3	154	32,2	441		3	121	25,3	450
	4	130	27,2	430		4	110	23,0	437
	5	135	28,2	439		5	101	21,1	424
Mean value			30,9	441	Mean value			24,1	439
5%-Quantile			24,0		5%-Quantile			19,1	

Table 3 Comparison of experimentally and analytically obtained strength reduction factors k_α for tapered CLT-beams

Series	$f_{m,\alpha,mean}$ in N/mm^2	$k_{\alpha,exp}$	$k_{\alpha,anlyt}$	Ratio	$f_{m,\alpha,k}$ in N/mm^2	$k_{\alpha,exp}$	$k_{\alpha,anlyt}$	Ratio
C6	36,6	0,880		1,0	28,1	0,880		1,0
C10	30,9	0,743	0,744	1,002	24,0	0,752	0,744	0,990
T6	33,6	0,808	0,780	0,966	26,3	0,824	0,780	0,947
C10	24,1	0,579	0,608	1,050	19,1	0,598	0,608	1,017

4 Summary and Conclusions

A method for the calculation of strength reduction factors for tapered CLT-beams is presented. The method is based on EC5 equations for the calculation of strength reduction factors for tapered glulam beams. For CLT-beams the required strength properties are determined considering possible failure modes and the beam layup.

Considering the variability of the material involved, good agreement has been found between the strength reduction factors evaluated from tests and analytically obtained values. Therefore it can be stated that EC5 equations for the calculation of strength reduction factors are also suitable for CLT-beams, if the required strength properties are determined with respect to the materials peculiarities.

The higher strength properties perpendicular to the longitudinal direction of CLT-beams result in significantly smaller strength reductions than for glulam, making CLT a viable alternative to the established material.

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