# The *Australian Curriculum: Mathematics*—How Did it Come About? What Challenges Does it Present for Teachers and for the Teaching of Mathematics?

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**Abstract** The Australian Curriculum: Mathematics which incorporates the content descriptions and proficiencies from Foundation Year to Year 10 came into being in December 2010 when all Australian governments—the national government and the governments of the eight States and Territories—gave their approval to the draft which had been in circulation for nearly two years. Prior to that, each State and Territory had responsibility for developing and implementing its own curriculum. In 2008, an Australian Curriculum and Reporting Authority (ACARA) was also established to coordinate and oversee the development of national curricula in all areas of compulsory schooling, and to move towards an agreed upon national curriculum for Years 11 and 12. The formation of ACARA and the adoption of an Australian Curriculum: Mathematics (2010) are interpreted as a result of major transformations of an Australian federalist model over the past twenty years, shaped in large degree by the demands of national assessment and school reporting. This chapter examines how this came about, what has been achieved within Australia's ongoing federalist framework, and also points to some future challenges for teachers in implementing the national curriculum in mathematics.

**Keywords** Mathematics · National curriculum · Australia · Policy

The current Ministerial Council for Education, Early Childhood Development and Youth Affairs (MCEECDYA) has changed its name several times over the years relevant to this chapter. At the time of the Adelaide Declaration (1989), it was known as the Ministerial Council for Education, Employment and Training and Youth Affairs (MCEETYA). These changes in nomenclature reflect changing responsibilities and alignments of portfolio description of Ministers of Education in the Commonwealth and State Governments. For example, Early Childhood Education is now typically included under school education, whereas Training has been shifted to another portfolio. However, the one constant has been school education. My short-hand references to the "Council of Ministers" or simply "the Ministers" are my way of encompassing these various changes over time. These truncated references do not refer to some separate entity.

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## Setting the Australian Context

It is important to understand some key features of the Australian governmental and educational context in which the *Australian Curriculum: Mathematics* was developed in the period from 2008 through 2010, and in which it will continue to be framed. At the most basic level, Australia is a federation of six States and two Territories with an over-arching Commonwealth (Australian) government based in Canberra. These constitutional arrangements clearly distinguish Australia from countries like Singapore, Japan, England and China where the curriculum is framed and promulgated at a national level by an agency established by and responsible to the respective national government. On the other hand, Australia operates quite differently from the USA, where despite a union between Washington and the fifty States, 15,000 local school districts are responsible for the day-to-day administration of school education and contribute an important share of the funding of schools through local property taxes. In Australia, for over 140 years, responsibility for public education including the curriculum has remained with the States, following the passing of various State Education Acts in the 1870s providing for free, compulsory and secular public education. The funding of public education is largely contained with State budgets, even though taxation, principally through income taxes and company taxes, is controlled by the Commonwealth government, and disbursed to the States and Territories under agreed upon funding formulae. Property taxes exist at local level but play no role in the funding of school education.

Until the 1960s, the Australian government played virtually no role in public elementary and high school education—there being no minister responsible for school education and no national education authority. Since that time, the Australian federal model has seen a shift in the balance of powers between the States and the Commonwealth, with the national government taking a more active role in policy development and national accountability for educational expenditure, in partnership with the States and Territories, with the creation and steady growth in importance of a federal Department of Education, Employment and Training (DEET). These changing relationships make Australia different, in my opinion, to Canada which also operates a federal model but where individual Provinces appear to retain a greater degree of independence in the running of schools and in deciding what will be taught.

However, the emergence of an Australian Curriculum, and in particular of an *Australian Curriculum: Mathematics*, does not imply that the States and Territories have been edged out of school education. Each State will be responsible for the implementation of the *Australian Curriculum: Mathematics* in all schools. Several states commenced implementation, in part, during 2011 and others will begin during 2012, with all State and Territories agreeing to implement the Australian Curriculum (Mathematics) by 2013.

Since the *Australian Curriculum: Mathematics* is not intended to occupy all available teaching time, State Education authorities may wish to add additional content, where necessary. This is likely, for example, in providing for different courses in Years 9 and 10 to suit different cohorts of students—some of whom may be planning to specialise in Mathematics in the remaining two years of high school, whereas other students will be expected to continue with mathematics through to the end of school as part of their general education. Moreover, State and Territory curriculum, assessment and certification authorities continue to be responsible for the structure and organization of their senior secondary courses and will determine how they will integrate the Australian Curriculum content and achievement standards into their courses. Australian senior secondary courses, taken in Years 11 and 12, serve a dual purpose: first to certify the successful completion of secondary school, and second to provide the principal—and usually the sole—basis of selection into universities and university courses, as well as entry into courses of continuing technical and vocational education. In contrast to many other countries, Australian universities do not conduct their own entrance examinations.

## Federalist Approaches to the Mathematics Curriculum

Starting in the late 1980s, the various levels of state and national government began working together to create more consistent approaches to the teaching and learning of mathematics. It was argued, for example, that a lack of consistency in the timing in which mathematical content was introduced and taught created problems for those children who moved from one state's jurisdiction to another in the course of their schooling. In addition, the growing status and importance of international assessments such as TIMSS provided a rationale for more coherent approaches across the states and Territories. However, any move to develop a national curriculum was not taken seriously. In December, 1990, for example, *A National Statement on Mathematics for Australian Schools* was completed as a joint project of the States, Territories and the Commonwealth of Australia. This project had been initiated by the Australian Educational Council with the purpose of providing a framework around which all school systems and schools might review and build their own mathematics curricula. Over the next twenty years, various working groups, representative of all levels of governments and of the non-government schools, continued to develop statements of consistency for the mathematics curricula of the States and Territories in the absence of a national curriculum. A major impetus for this continuing work was the growing importance given to State-based assessment and reporting of student achievement in Mathematics and English, and the use of these assessments as a condition of federal funding of education.

From the early 1990s, with eight different forms of State-based achievement testing (initially at Years 3 and 5, and subsequently extending to Years 7 and 9), the then federal Minister decided to develop benchmarks of achievement that could be used to provide a consistent reporting base to the national government as a basis for its funding of education. The debate and policy focus on a national curriculum, student assessments and school reporting was re-invigorated in 2005 when State and Territory education authorities were required by the national government to implement Statements of Learning in subject areas, such as Mathematics, which had been agreed to by the Ministerial Council for Education, Employment, Training and

Youth Affairs (MCEETYA), along with other requirements for student assessment and school reporting (Australian Government 2004).

The Schools Assistance Bill 2008, which provided Australian Government funding for non-government schools contained similar conditions of funding. These conditions were also embedded in National Education Agreements for Australian Government funding for government school systems. What was significant in these new arrangements was the additional requirement for reports about individual school performance, as determined by the Minister. It was no coincidence that this was the year in which the State and Territory governments, along with the Australian Government, agreed to establish ACARA with a mandate to develop an agreed upon national curriculum in all school subjects, with mathematics being in the first group to be so developed. To readers who are not familiar with this new federalist model, it is important to identify some of the key steps along the way.

These federalist approaches over the past twenty five years cannot be viewed as the province of any one Australian political party. From the late 1980s, they were moved forward by a national Labor government, and then from 1996 to 2007 by a Liberal/National Coalition government, which was followed by a returned Labor government, still in office at the time of writing. Over the same twenty-five year period, the political complexions of the various State and Territory governments were also changing.

## Three Declarations on the Way to an Australian Curriculum (1989–2008)

The establishment of ACARA in 2008 was the culmination of a long period of policy debates—in the case of the national curriculum, the debates date back to the 1980s when the then Minister for Employment, Education and Training called for a common curriculum framework that would set out 'the major areas of knowledge and the most appropriate mix of skills and experience for students in all the years of schooling' (Dawkins 1988).

Three important declarations or statements by the Ministers of Education representing the eight States and Territories and the national (Commonwealth) government of Australia the first being the Hobart Declaration in 1989, the next the Adelaide Declaration in 1999, and the last the Melbourne Declaration in 2008, show significant changes in thinking about the meaning of "national curriculum" and "national assessment". The first was the Hobart Declaration on Schooling (MCEECDYA 1989) named after the city in which the Australian Education Council met that year for its sixtieth meeting. In that statement, the ministers agreed for the fist time to improve Australian schooling within a framework of *national collaboration* which embraced:

- Common and agreed upon national goals for schooling in Australia
- (An) Annual National Report on schooling
- National collaboration on curriculum projects

- Establishing the Curriculum Corporation of Australia
- The goal of a common age of entry for Australian schools
- Improving the quality of teaching.

The Curriculum Corporation was intended to be a clearing house for publications to be shared among the States and Territories and to commission new publications. It had no role in curriculum development. Indeed, the very notion of a "national curriculum" is entirely absent from the Hobart Declaration, with its clear commitment to *collaboration* among the States, Territories and the Commonwealth. Among the above goals that directly relate to the teaching and learning of mathematics, one of the stated goals of schooling was "to develop in students skills of numeracy, and other mathematical skills". The term 'numeracy' was not defined, but was generally taken to refer to an ability to use mathematics purposefully in other school subjects, in contexts outside school, and for older students in relation to their future work and life. Indirectly, the Annual National Report on schooling was intended to monitor schools' achievement and their progress towards meeting the agreed upon national goals. It was intended to report on school curriculum (for this, read *what the individual States were doing*), participation and retention rates, student achievements, and the application of financial resources to schools. Reporting on student achievement would rely entirely on whatever measures of achievement were in place at the time in individual State and Territories.

In particular, the Ministers reported that "work has been proceeding through a working party to seek to attain the highest standards of national curriculum, common principles and agreed areas of national collaboration. These will now be defined *for the Mathematics curriculum* taught in Australian schools (i.e. *as taught by the individual States and Territories*). The statement of common principles will identify the knowledge and skills to which all students are entitled, recognize areas of strength and weakness in the mathematics curriculum, and develop recommendations for future collaborative action" (MCEECDYA 1989). The Ministers said that the findings of this process would be presented for public discussion. Significantly, they added: "Their use will *not be compulsory* (my emphasis) but where agreement is reached after full consideration then it is likely that government and non-government systems and schools will use them" (MCEECDYA 1989). It was also agreed that further mapping would continue in the "key curriculum areas of Science, Technology, and English Literacy."

The Hobart Declaration was superseded ten years later by the Adelaide Declaration on National Goals for Schooling in the Twenty-First Century (MCEECDYA 1999). The spirit and letter of federalist collaboration evident in the Hobart Declaration is maintained in this relatively brief declaration. The Ministers' Adelaide Declaration set out eight agreed upon key learning areas through which students were expected to attain high standards of knowledge, skills and understanding in the compulsory years of schooling. These key learning areas were specified as: the Arts, English, Health and Physical Education, Languages other than English, Mathematics, Science, and Studies of Society and Environment. The Ministers' only other direct reference to Mathematics was to say that in terms of the curriculum, "students should have attained the skills of numeracy and English literacy, such that every student should be numerate, able to read, write spell and communicate at an appropriate level" (MCEECDYA 1999). Responsibility for monitoring and reporting on students' attainments *in all these respects* was left to the States and Territories using their own particular forms of assessment and testing.

However, by 2008, the preceding federalist model was transformed and national agreement on *action* was strikingly evident when, in their Melbourne Declaration on Educational Goals for Young Australians (MCEECDYA 2008), the Ministers agreed to take definite steps to "*promoting world class curriculum and assessment*" (my emphasis). Absent from this statement are references to non-binding agreements, as in the Hobart and Adelaide Declarations. In a wide-ranging 20-page document, the Ministers stated that "State, Territory and Commonwealth governments will work together with all school sectors to ensure world class curriculum in Australia" (p. 13). This will require different levels of implementation: a national curriculum, together with curriculum specified at State and Territory, and at local levels. In recognition of the continuing roles of States and Territories, the Ministers said that "schools and school systems are responsible for delivering curriculum programs" (p. 14) that reflect agreed upon learning areas with appropriate flexibility, and the same paragraph singles out English and Mathematics "as being of fundamental importance in all years of schooling and as the primary focus of schooling in the early years" (p. 14).

The Melbourne Declaration makes an explicit reference to national assessment for the first time when it says: "To ensure that student achievement is measured *in meaningful ways* (my emphasis), State, Territory and Commonwealth governments will work with all school sectors to develop and enhance national and school level assessment that focuses on assessment for learning, assessment as learning and assessment of learning to assess student achievement against goals and standards" (p. 14). This statement is followed immediately by a commitment to strengthen accountability and transparency, justified in terms of supporting schools and students, for informing parents and families and the community, and also for governments in order to "analyse how well students are performing, identifying schools with particular needs, to determine where resources are most needed to lift attainment, and to conduct national and international comparisons of approaches and performances" (p. 17). The Ministers reference to measurement "in meaningful ways" could be read in the context of difficulties experienced in the years following the Hobart Declaration (1989) when the eight State and Territory governments tried to compare students' achievement nationally, while still using their own forms of testing. It may also be an implied reference to instances where student achievement appeared to rise over time when smaller States changed from one assessment instrument to another. In 2008, National Assessment of Performance in Literacy and Numeracy (NAPLAN) commenced, replacing all current individual assessment regimes of the States and Territories. It can be argued that. NAPLAN made a national curriculum inescapable.

Prior to the Melbourne Declaration (2008), a National Curriculum Board (NCB) was created by the Australian Government in 2007 to be responsible for carrying forward the initiatives for developing "world class curriculum and assessment". The

NCB was essentially a committee of officials with no statutory power. In October 2008, the Australian Curriculum, Assessment and Reporting Bill (Parliament of Australia 2008) was introduced and enacted creating the Australian Curriculum and Reporting Authority (ACARA) as "an independent statutory authority" which "will manage the creation and implementation of the national curriculum, national student assessment and reporting of school education outcomes" (p. 1). As an independent statutory authority, ACARA derives its plan of work from the Ministerial Council representing all Australian governments and all school sectors. The Ministerial Council approves ACARA's budget with 50 % coming from the Commonwealth and the other 50 % coming from the States and Territories. This ensures that ACARA is independent of any one government. ACARA, which is not an agency of the national (Australian) government acting alone, reflects a new Australian federalist model where responsibility for curriculum and assessment is no longer the exclusive responsibility of the States and Territories. This evolutionary shift gives the national government, acting through ACARA, a greatly enhanced role.

## Key Features of the *Australian Curriculum: Mathematics*

In implementing the *Australian Curriculum: Mathematics*, there is a consensus that it should not be too prescriptive and that there needs to be flexibility in order to cater to local needs (Kelly 2008). The States and Territories, in their proposal for a national curriculum, successfully argued that:

> . . . a national curriculum will benefit if there is flexibility for states and schools to innovate and adapt and to share their experiences of what approaches achieve the best results. A level of autonomy for individual schools and teachers to make professional decisions about curriculum drives the high performance level of a large number of government, Catholic and Independent schools across jurisdictions.
>
> . . . whatever common curriculum standards (that is, what students are expected to achieve in mathematics, science etc.) are adopted by jurisdictions, it is important to allow for flexibility in schools catering for different groups of students to achieve these standards in different ways. This is not an argument for lower standards for some students. On the contrary, it is an argument for flexibility in teaching approach and, in some cases, content in order to reach the standards in different settings. (Council for the Australian Federation 2007).

The federal Minister of the time, Julia Gillard, moved to address these concerns, assuring schools that the national curriculum will 'allow teachers the flexibility to shape their classes around the curriculum in a way that is meaningful and engaging for students' (Gillard 2008). Minister Gillard also recognised particular concerns:

The national curriculum, once agreed upon and completed, will be compulsory. But it will not mean that every school will be required to teach the same subjects, line by line, in the same way (Gillard 2008).

The *Australian Curriculum: Mathematics* does not set out in detail how lessons should be taught each day. This would require a very large and intricate document, which would be difficult to apply across the range and variety of schools and students' backgrounds. State and national curriculum documents serve a variety of purposes. They are intended to express an expectation or broad agreement about what should be taught and in what order, and how some key ideas are to be developed. They are intended to set out what children are expected to know and be able to do' and so provide an agreed upon framework for school and system-wide assessments. They are not recipes for teaching in any day-to-day sense. There is always a gap between what is expressed in curriculum documents and how teachers give shape to their lessons. Occasionally, these documents may recommend or suggest particular ways of teaching particular topics. For example, to support its teachers to implement the *Australian Curriculum: Mathematics*, the State of New South Wales—Australia's most populous State—has produced a five-hundred page syllabus document (Board of Studies 2012). This document can be accessed by teachers in other parts of Australia should they wish to use it. On the other hand, the State of Tasmania—Australia's smallest State—has implemented the *Australian Curriculum: Mathematics* from 2012 using the document in its ACARA format. In yet a further example of federalist approaches to implementation, the State of Victoria has "re-badged" the *Australian Curriculum: Mathematics* for its teachers as *AusVELS* (Victorian Curriculum and Assessment Authority 2012).

## Content Areas and Priories in Australian Curriculum: Mathematics

How does the *Australian Curriculum: Mathematics* differ from its preceding State and Territory documents? A nationally-funded review by Donnelly (2005) and others showed inconsistencies in depth of treatment and the ways in which content was specified across Year levels in the various State and Territory curriculum documents. In curriculum documents prepared by the majority of Australian States and Territories, mathematical content was present in Bands or Levels which were intended to cover two years of schooling. As a result, the level of detail sometimes appeared scanty or too general when compared with the Mathematics curricula of Japan (Japan Society for Mathematical Education 2000) and Singapore (Ministry of Education 2007). By contrast, the *Australian Curriculum: Mathematics* specifies content according to each year level from the Foundation Year to Year 10. Nevertheless, it is important to understand that the *Australian Curriculum: Mathematics* is a consensus document within a continuing federalist model.

One of the key features of the *Australian Curriculum: Mathematics* (2010) is that content is described for each year level, from Foundation Year to Year 10, using three common content categories: Number and Algebra, Measurement and Geometry, and Statistics and Probability. In addition, four Proficiency Strands, Understanding, Fluency, Problem-solving, and Reasoning, are expected to guide teaching and learning at all year levels, across all areas of content.

The following section will compare the content descriptions of the *Australian Curriculum: Mathematics* with related sections from one of the State-based curriculum documents, namely the Victorian Essential Learning Standards (VELS,

**Table 1A** Australian curriculum—foundation year, measurement and geometry

| Content descriptions | Elaborations |
| --- | --- |
| Use direct and indirect comparisons to decide which is larger, heavier or holds more and explain reasoning in everyday language (CMMG006) | Comparing objects directly by pacing one object against another to determine which is larger or longer or by pouring from one container to the other to see which one holds more using suitable language associated with measurement attributes, such as "tall" and "taller", "heavy" and "heavier", "holds more" and "holds less" |

**Table 1B** VELS level 1, measurement, chance and data

| Learning focus | Standards |
| --- | --- |
| (S)tudents learn to compare common objects using terms such as longer, heavier, fuller and hotter | Students compare length, area, capacity and mass of familiar objects using descriptions such as longer, taller, larger, holds more and heavier. They make measurements using informal units such as paces for length, hand spans for area, glasses for capacity and bricks for weight. |

DEECD 2008). As discussed earlier, the VELS is typical of other State-based documents in using Levels, covering a two-year period, instead of year-by-year elaborations. Each level of the VELS is organized according to Learning focus and Standards of performance. The VELS Mathematics domain is organized in five dimensions: Number, Space, Measurement, Chance and Data, Working Mathematically, and Structure. The *Australian Curriculum: Mathematics* uses three content descriptions—Number and Algebra, Measurement and geometry, and Statistics and Probability—to describe the knowledge, skills and processes that teachers are expected to teach and students expected to learn. These descriptions do not prescribe approaches to teaching, but "are intended to ensure that learning is appropriately ordered and that unnecessary repetition is avoided" (ACARA 2010, p. 3). Content elaborations are also given "to illustrate and exemplify content and assist teachers to develop a common understanding of the content descriptions. They are not intended to be comprehensive content points that all students need to be taught" (ACARA 2010, p. 4).

Tables 1A and 1B compare what the two documents prescribe in Measurement for the first year of school. There is a high degree of consistency. Both descriptions are detailed, unambiguous, and measureable (Donnelly 2005). VELS Level 1 is also intended to apply to the first year of school only. Interestingly, VELS includes an informal comparison of areas which is not explicitly included in the *Australian Curriculum: Mathematics* until Year 2.

Tables 2A and 2B compare the two documents in terms of how students in Year 3 are expected to relate knowledge of number facts and relationships for single-digit numbers to mental computation involving larger numbers. This is clearly and unambiguously explained in the *Australian Curriculum: Mathematics*, where the expression "always result in the same answer" is intended to introduce the idea of equiva-

**Table 2A** Australian curriculum—year 3, number and algebra

| Content descriptions | Elaborations |
|---|---|
| Recall addition facts for single digit numbers and related subtraction facts to develop increasingly effective mental strategies for computation (ACMNA055) | Recognise that certain single digit number combinations always result in the same answer for addition and subtraction, and using this knowledge for addition and subtraction of larger numbers using suitable language associated with Combining knowledge of addition and subtraction facts and partitioning to aid computation [for example $57 + 19 = 57 + 20 - 1$] |

**Table 2B** VELS level 3, structure

| Learning focus | Standards |
|---|---|
| (Students) learn to use number properties to support computations (for example, the use the commutative and associative properties for adding or multiplying three numbers in any order or combination | Students understand the meaning of "=" in mathematical statements and technology displays (for example to indicate either the result of a computation or equivalence). They use number properties in combination to facilitate computation [for example, $7 + 10 + 13 = 10 + 7 + 13 = 10 + 20$] |

lence. The elaborations illustrate how this might be applied to developing effective mental strategies. While the VELS expressly refers to equivalence, its references to "number properties" are more general. But the link to computational efficiency is quite clearly shown. This content is expected to be covered by VELS in Years 3 and 4.

In Statistics and Probability, the *Australian Curriculum: Mathematics*, as shown in Table 3A, encourages the use of secondary data from the media and elsewhere to examine how statistics is used to convey messages and how these messages need to be examined carefully with respect to the claims being made and the assumptions made in collecting data, particularly through sampling. On the other hand, VELS Level 4 (covering Years 5 and 6)—shown in Table 3B—treats sampling only indirectly in relation to the collection of primary data through questionnaires and surveys. Incidentally, VELS level 5 Measurement, chance and data has a reference to "Students take samples in order to make inferences and predictions about a population" (DEECD 2008, p. 27). Explicit treatment of the distinction between a sample and its population is given by VELS Level 6, intended for Years 9 and 10 (DEECD 2008, p. 36). These differences could be considered variations in timing and emphasis, but the *Australian Curriculum: Mathematics* places a stronger emphasis on utilising case studies and illustrations from the media and advertising to support the study of Statistics and Probability in the upper primary and early secondary years.

In its treatment of linear and non-linear relations, the *Australian Curriculum: Mathematics* provides more detailed advice than VELS. Tables 4A and 4B set out the content descriptions and elaborations for linear and non-linear relations for Year 9 and Year 10. VELS Level 6 is intended to cover both these year levels. Teachers will see continuities between the content described in Table 4C from VELS Level

**Table 3A** Australian curriculum—year 6, statistics and probability

| Content descriptions | Elaborations |
|---|---|
| Interpret secondary data presented in digital media and elsewhere (ACMSP148) | developing an understanding of sampling and the ability to interpret secondary data in order to critique data-based claims made in the media, advertising and elsewhere investigating data representations in the media and discussing what they illustrate and the messages the people who created them might want to convey considering the need for sampling and recognising when a census of an entire population is not possible or not necessary, and identifying examples of sampling in the media |

**Table 3B** VELS Level 4, Measurement, chance and data

| Learning focus | Standards |
|---|---|
| Students plan and conduct questionnaires to collect data for a specific purpose | Students recognise and give consideration to different data types in forming questionnaires and sampling |

6 and what is recommended for Year 9 and 10 in the *Australian Curriculum: Mathematics*. However, differences between the two documents are more pronounced in this respect: not only does the latter provide greater detail, it also recognizes a need for more challenging content at this Year level for some students. The *Australian Curriculum: Mathematics* achieves this by providing an additional level of content, entitled Year 10A, which while optional "is intended for students who require more content to enrich their mathematical study whilst completing the common Year 10 content" (ACARA 2010, p. 6). Year 10A content descriptions in regard to Linear and non-linear relationships include: Describe, interpret and sketch parabolas, hyperbolas, circles and exponential functions and their transformations (ACMNA267); Solve simple exponential equations (ACMNA270); Apply understanding of polynomials to sketch a range of curves and describe the features of these curves from their equation (ACMNA268); and Factorise monic and non-monic quadratics expressions and solve a wide range of quadratics equations derived from a variety of contexts (ACMNA269). These descriptions recognise that at Year 10 some students require considerably more challenging content than is contained in the "common Year 10" descriptions. This is clearly a limitation of VELS Level 6.

The *Australian Curriculum: Mathematics*, by opting for Year-by-Year descriptions of content, has an advantage over State-based documents which are, in most cases, based on Levels covering two years. Teachers reading the *Australian Curriculum: Mathematics* are intended to see clear continuities between what is currently prescribed in their current State curricula, but they can also expect more detail, less scope for ambiguity, and some definite changes of emphasis. Two areas of changed emphasis are discussed in the following section.

**Table 4A** Australian curriculum—year 9, linear and non-linear relationships

| Content descriptions | Elaborations |
|---|---|
| Find the distance between two points located on a Cartesian plane using a range of strategies, including graphing software (ACMNA214) | investigating graphical and algebraic techniques for finding distance |
| Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software (ACMNA294) | investigating graphical and algebraic techniques for finding midpoint and gradient |
| Sketch linear graphs using the coordinates of two points (ACMNA215) | determining linear rules from suitable diagrams, tables of values and graphs and describing them both using words and algebra |
| Sketch simple non-linear relations with and without the use of technology (ACMNA296) | sketching parabolas, hyperbolas and circles |

## Challenges for Teachers and Teaching

This section will examine two content Strands, (Number and Algebra, and Probability and Statistics) and their implications for different approaches to teaching and learning. (The third Strand of Measurement and Geometry may present fewer challenges for teaching and learning since it largely reiterates the content prescribed in preceding State and Territory curricula.) As mentioned before, the inclusion of these two strands from Foundation Year to Year 10 reflects similar efforts to promote a closer integration between Number and Algebra evident in many other national curriculum documents, and an increased emphasis on the teaching of Statistics and probability which is also present in a number of national curriculum documents—the USA (see NCTM 2006) and China (see, Ministry of Education 2001, 2011) providing just two examples.

### *Number and Algebra*

A more coherent and integrated treatment of Number and Algebra in the elementary and junior high school years raises some challenges for teachers and teaching. Several questions are uppermost in this analysis. How is this expectation interpreted by teachers? Does it imply, as some elementary teachers may think, that there is now less time for teaching Computation? What advantages does a more integrated treatment of Number and Algebra offer students to understand more deeply numbers and number operations, especially in the middle and upper elementary years? And how is this more unified treatment of Number and Algebra intended to assist students to make a smoother transition to a more formal study of algebra in the

**Table 4B** Australian curriculum—year 10, linear and non-linear relationships

| Content descriptions | Elaborations |
|---|---|
| Solve problems involving linear equations including those derived from formulas (ACMNA235) | solving equations that are the result of substitution into common formulas mathematics and elsewhere, including those that involve rearrangement |
| | checking the solution by substitution into the equation |
| Explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technology as appropriate ACMNA239 | identifying, matching and describing algebraic and graphical representations of parabolas, rectangular hyperbolas, exponential functions and circles, including those that have undergone a single transformation sketching the graphical representations of parabolas, exponential functions and circles |
| Solve linear equations involving simple algebraic fractions (ACMNA240) | solving a wide range of linear equations, including those involving one or two simple algebraic fractions, and checking results by substitution representing word problems, including those involving fractions, as equations and solving them to answer the question |
| Solve simple quadratic equations using a range of strategies (ACMNA241) | developing an understanding that many relationships are non-linear and that these can also be represented graphically and algebraically identifying the connection between algebraic and graphical solution of equations (for example understanding that the x-intercepts are the solutions of $f(x) = 0$ exploring the method of completing the square to factorise quadratic expressions and solve quadratic equations |

secondary school? To illustrate these points, let us examine how teachers might approach teaching the following two elements of the Number and Algebra Strand in Year 4 and in Year 5:

> Year 4: Use equivalent number sentences involving addition and subtraction to find unknown quantities (ACMNA083)
>
> Year 5: Use equivalent number sentences involving multiplication and division to find unknown quantities (ACMNA121)

Two contrasting teaching approaches will be discussed. The first might be called a minimalist teaching approach; where the emphasis is focussed on using computation and equivalence to obtain a correct answer to number sentences, involving subtraction such as $39 - 15 = 41 - \square$, for instance, or a sentence involving multiplication such as $5 \times 18 = 6 \times \square$. In this minimalist approach, teachers would encourage students to simplify each number sentence by calculating the value of the known pair of numbers, [24 in the subtraction sentence, and 90 in the case of the multiplication sentence] and then ask what unknown number on the right hand side will be needed to give these results, leading to 17 for the subtraction sentence and

**Table 4C** VELS level 6, structure

| Learning focus | Standards |
|---|---|
| Students work with functions (for example, linear, quadratic, reciprocal, exponential) simple transformation of these functions, their graphs, and related algebraic properties. | Students identify and represent linear quadratic and exponential functions by table, rule and graph (all four quadrants of the Cartesian coordinate system) with consideration of independent variables, domain and range. They distinguish between these types of functions by testing for constant first difference, constant second difference or constant ration between consecutive terms ... They use and interpret the functions in modelling a range of contexts.<br><br>They recognise and explain the roles of the relevant constants in the relationships $f(x) = ax + c$, with reference to gradient and $y$ axis intercept, $f(x) = a(x + b)^2 + c$, and $f(x) = ca^x$ |

15 for the multiplication sentence. Some teachers might think that this is all that is needed to *use equivalent number sentences involving subtraction* (*or multiplication*) to find unknown quantities. However, this minimalist approach omits important opportunities to extend students' understanding of equivalence and its embodiment in different operations.

A mathematically richer approach would be to look more deeply at the structure of these and related equivalent number sentences; noticing especially at how the direction of compensation changes according to the operations involved. In this alternative approach, students are encouraged to refrain from calculating and to look at the numbers either side of the equivalent sign. Some students will express their reasoning verbally, using rich and varied forms of mathematical thinking such as: "*Because 41 is two more than 39, I have to put a number that is two more than 15 in order to keep the same difference*". Other students will express their thinking by using arrows to connect related numbers, 39 to 41 and 15 to the unknown number, concluding that it has to be two more than 15. Other students may write $A_1$ beneath 39 and $A_2$ beneath 41, and place $B_1$ under 15 and $B_2$ under the unknown number, reasoning that "*Since* $A_2$ *is two more than* $A_1$, $B_2$ *has to be two more than* $B_1$". Some students will explicitly use words such as "equivalent" or "to keep both sides equivalent". In all these cases, students know that they are dealing with *equivalent differences*. These students also know that the direction of compensation used the case of subtraction or difference operates in the opposite way to sentences involving addition. Likewise, for the multiplication sentence, students can be encouraged to notice that since 18 is three times the value of 6, the missing number has to be three times 5 in order to maintain equivalence. The fact that the 6 is one more than 5 in the multiplication sentence is not important, whereas the multiplicative relationship between 6 and 18 is all important to reaching a solution.

Unlike the minimalist approach discussed earlier, these approaches focus on important and generalizable features of sentences involving the same number operations. These features are intended to support students' computational fluency, and also to prepare them for algebraic thinking. Research, such as by Carpenter and

Franke (2001) and by Mason et al. (2009), endorse this approach. These possibilities will be quite new to many Australian elementary and junior secondary teachers. Implementation of the *Australian Curriculum: Mathematics* will need to open up teachers' vision to these ideas.

### *Statistics and Probability*

For the Statistics and Probability Strand, different challenges arise for teachers and teaching. The precedence given to Statistics in the title underscores a difference with *Chance and Data* as used in the National Statement on Mathematics for Australian Schools (AEC 1990), and with the various State curricula, where, for example, VELS (DEECD 2008) uses a content heading *Measurement, chance and data*. Many teachers in the upper elementary and junior high school years, who have been accustomed to thinking about probability from a purely theoretical or computational perspective, will need help to develop their understanding of variability and the effects of sampling, which are consequences of the new emphasis on interpreting secondary data presented in digital and printed media. Research by Watson and Nathan (2010) and by Stephens and Zhang (2011), show how teachers can be assisted to think about the effects of sample size on variability of data, and to connect their teaching of probability and statistics to key mathematical ideas such as ratio and proportion. For elementary teachers, in particular, the focus needs to move away from merely collecting and recording data, and to attend more to developing different ways of representing and interpreting data. All teachers will need a clearer appreciation of the key idea of variability and its impact on interpreting data that has already been gathered.

These two illustrations are intended to show that, while the *Australian Curriculum: Mathematics* contains no radical innovations, it has clearly moved beyond existing State and Territory documents; and it can be expected to challenge current levels of practice and mathematical understanding of many teachers. That is its challenge and opportunity for the teaching and learning of mathematics in Australian schools.

## What Does the *Australian Curriculum: Mathematics* Offer to Teachers and Schools?

For the first time in Australian school education, an agreed upon national curriculum sets out clearly for all schools what should be taught and assessed in Mathematics at all levels of schooling from Foundation Year (Kindergarten) to Year 10. Across these eleven years, three continuous content strands—Number and Algebra, Measurement and Geometry, and Statistics and Probability are used. Four Proficiency strands also run across the eleven years with specific elaborations at each year level in Understanding, Fluency, Problem solving, and Reasoning.

The decision to elaborate the curriculum year by year represents a clear departure from the practices of almost all of the States; which had generally used their curriculum documents to describe standards of achievement or outcomes which might be attained by most students over a period of two years. These previous documents, the Victorian Essential Learning Standards (VELS) for instance (DEECD 2008), used six levels to describe the curriculum from Foundation Year (Level 1) to Year 10 (Level 6) using two-year intervals encompassing Years 1 and 2, Years 3 and Year 4, and so on. In England and Wales, the National Curriculum Mathematics (Department of Education 2010) uses only four Key Stages or levels to describe its content for students from the beginning of school to age 16.

While the Content described under Numbers may have been evident in many State-based documents, the joining of Number and Algebra in the primary school years is an important new emphasis. Likewise, the important place given to Statistics and Probability gives a more consistent emphasis to statistical representation and the introduction of probability in the primary years than has been the case in many State-based documents. Some State-based documents included a separate strand on Mathematical thinking. The three Proficiency Strands of the Australian Curriculum are intended to achieve the same purposes.

Smaller States and Territories, which in the past may have experienced difficulty in resourcing the development and updating of their own curriculum, now have access to an agreed upon national curriculum. All States and Territories will also have access to supporting publications and associated teacher development resources. Publishers also can be confident in producing for a national market. In the past, any publisher aiming for a national market had to make significant adjustments in content, timing and terminology to account relatively small differences in curriculum between the States and Territories, which were still significant in terms of teacher acceptance.

Finally, in ACARA there is an independent statutory agency that can undertake systematic evaluations of the current curriculum and initiate revisions in a planned and systematic manner. In the former State-based regimes, reviews and revision were subject to government priorities and changes of government where previously agreed upon priorities might easily be swept aside.

## What Have Been Some Drawbacks of the Current Process?

The *Australian Curriculum: Mathematics*, while an undoubted national achievement, has been the result of a consensus process by representatives of government and non-government schools, who were subject as well to inevitable time constraints. The final statement had to be more or less consistent with what was already contained in the pre-existing State documents; no big departures from current practice could be expected. Moreover, the input of the mathematics education research community which was subject to the same government set timelines was uneven. In Australia the mathematics education research community is not adept at dealing with short response timelines set by bodies such as ACARA.

Is the *Australian Curriculum: Mathematics* the "world class curriculum" that was promised in the 2008 Melbourne Declaration? Given only two years to prepare, it is unreasonable to expect something world class to be prepared in this time using a federalist consensus process. World class Mathematics curricula, such as those of Singapore, China and Japan, are developed over much longer time cycles—up to ten years—with careful input from teachers and schools and usually in the hands of a highly expert group of specialists nominated by respective Ministries of Education. Australia needs to learn from the processes used in these other countries.

## What Lessons Can Be Learned for the Future?

While 2012 has seen the trialling of the *Australian Curriculum: Mathematics* in several of the States, all States and Territories have agreed to fully implement the new curriculum from 2013. Already several publications in the area of assessment, such as *Rich Assessment Tasks in Mathematics: Years 5 to 8*, published by the Catholic Education Office of Melbourne (2011), have aligned student performances with the Content Descriptions of the *Australian Curriculum: Mathematics* and the Standards currently used in the VELS. This has reassured teachers of the high degree of consistency and continuity between current assessment practice based on VELS and what the *Australian Curriculum: Mathematics* expects students to learn.

The fact that Australian Curriculum and Assessment Authority (ACARA) has successfully worked with the States and Territories to develop common and agreed upon courses in Mathematics for the final two years of school does not replace the various State-based assessment and certification procedures. These courses are to be implemented by the States and Territories during 2015–2016. There are no proposals for a national system of certification and assessment. The Results of the final-year high school assessments, across the States and Territories, are currently moderated in a national system which allows students to apply for entry into any Australian university regardless of their state of origin; and those arrangements will continue.

However, the four agreed upon courses in Mathematics for Years 11 and 12 (ACARA 2012), to be implemented in 2015–2016, will be important in reducing current variations in content and in the range of Mathematics courses on offer to senior high school students in the different States. One course, entitled Mathematical Methods, includes algebra, introductory calculus, trigonometry and statistics, and is intended to provide a broad course for the majority of students who wish to undertake university courses in the mathematical sciences, science and economics. A second course, entitled Specialist Mathematics, which must be taken in conjunction with the first is intended to provide more advanced treatment of these topics, especially in calculus, for a subset of students who intend to undertake more specialised mathematical and statistical studies beyond school. A third course, entitled General Mathematics, has a strong foundation in descriptive statistics and in non-calculus applications of mathematics. It is intended to support those students who may wish to pursue courses in business, and the humanities. A fourth course, entitled Essential

Mathematics, is intended to support students who need to apply basic mathematical techniques in their other school subjects and to support future vocationally oriented studies and training. In the past, this latter group of students may have ceased to study any Mathematics after Year 10, and many may have "dropped out" of school altogether. Continuing to engage these students implies that Essential Mathematics be taught in more vocationally oriented contexts and using very different teaching and learning approaches than from what might be expected in the first three courses.

The adoption by ACARA of these four nationally agreed upon courses for Mathematics in the senior high school years (Years 11 and 12) will require some differentiation of content for students in Year 10, and possibly Year 9, to reflect their different academic pathways, and who need to make appropriate choices about the kind of mathematics that is most likely to be relevant to their continuing studies and aspirations beyond school.

The fact that ACARA is funded 50 % by the Commonwealth and 50 % by the States and Territories exemplifies the new federalist model and is intended to ensure that ACARA is robust enough to weather any changes of government at national or state level in the next five years. ACARA will continue to reflect a balance of responsibilities between the States, Territories, and the Australian (Commonwealth) governments, being especially responsive to the needs of schools and students across Australia; and fostering high quality mathematics education in all Australian schools.

## References

Australian Curriculum and Reporting Authority (2010). *Australian curriculum mathematics*. Sydney: ACARA. http://www.australiancurriculum.edu.au/Mathematics/Curriculum/F-10, accessed on 10 January, 2013 [to view one year level per page with elaborations, press the E icon in VIEW on the cover page; Senior Secondary Courses can be accessed through the control bar at the top of the page].

Australian Curriculum and Reporting Authority (2012). *Draft senior secondary Australian Curriculum: Mathematics*, November 2012. Sydney: ACARA. http://www.acara.edu.au/verve/_resources/Consultation_Report_on_the_Draft_Senior_Secondary_Australian_Curriculum__Mathematics.pdf, last accessed 25 September 2013.

Australian Education Council (1990). *A national statement on mathematics for Australian schools*. Melbourne: Australian Education Council.

Australian Government (2004). Schools assistance (learning together—achievement through choice and opportunity) act 2004, paragraphs 14(1)(f) and 14(1)(g) and subsections 31(f) and 31(g). http://www.comlaw.gov.au/ComLaw/Legislation/ActCompilation1.nsf/current/bytitle/B42223A296758099CA257314001D59DD?OpenDocument&mostrecent=1, Accessed on 18 August 2011.

Board of Studies (2012). *NSW syllabuses for the Australian curriculum: mathematics K-10*. Sydney: Board of Studies. http://syllabus.bos.nsw.edu.au/mathematics/mathematics-k10/content/ Accessed on 14 January 2013.

Carpenter, T. P., & Franke, M. L. (2001). Developing algebraic reasoning in the elementary school: generalization and proof. In H. Chick, K. Stacey, J. Vincent, & J. Vincent (Eds.), *Proceedings of the 12th ICMI study conference. The future of the teaching and learning of algebra* (pp. 155–162). Melbourne: University of Melbourne.

Catholic Education Office Melbourne (2011). *Rich assessment tasks in mathematics: years 5 to 8*. Melbourne: Author.

Council for the Australian Federation (2007). *Federalist paper 2: the future of schooling in Australia* (pp. 13–14). Melbourne: Department of Premier and Cabinet. rev. ed. http://dpl/Books/2007/FederalistPaper2_SchoolingInAustraliaRevEd.pdf. Accessed on 31 August, 2011.

Dawkins, J. S. (1988). *Strengthening Australia's schools: a consideration of the focus and content of schooling*. Canberra: Minister for Employment, Education and Training. http://dpl/Books/2008/StrengtheningAustSchools.pdf. Accessed on 8 August 2011.

Department of Education (2010). *National curriculum for mathematics (England)*. London: Department of Education.

Department of Education and Early Childhood Development (DEECD) (2008). *Victorian essential learning standards (mathematics)*. Melbourne: DEECD. http://vels.vcaa.vic.edu.au/downloads/vels_standards/velsrevisedmathematics.pdf, last accessed 6 February, 2012.

Donnelly, K. (2005). *Benchmarking Australian primary school curricula*. Canberra: Department of Education, Science and Training. http://www.dest.gov.au/NR/rdonlyres/B6E3D22B-6E60-4A1F-920A-EFAB3E5C45FA/7940/benchmarking_curricula_report2.pdf, last accessed 6 February, 2012.

Gillard, J. (2008). *Second reading speech: Australian curriculum, assessment and reporting authority bill 2008*. Canberra: Parliament of Australia, Department of Parliamentary Services. http://www.aph.gov.au/library/pubs/bd/2008-09/09bd060.pdf, accessed on 2 September 2011.

Japan Society of Mathematical Education (2000). *Mathematics program in Japan: elementary, lower secondary and upper secondary schools*. Tokyo: JSME.

Kelly, P. (2008). Labor plan is modest progress on education, *Australian*, 26 September 2008, p. 16, http://parlinfo.aph.gov.au/parlInfo/search/display/display.w3p;query=Id%3A%22

Mason, J., Stephens, M., & Watson, A. (2009). Appreciating mathematical structure for all. *Mathematics Education Research Journal*, *21*(2), 10–32.

Ministerial Council for Education, Early Childhood Development and Youth Affairs (MCEECDYA) (1989). The Hobart declaration on national goals for schooling in the twenty-first century. http://www.mceetya.edu.au/mceecdya/hobart_declaration,11577.html, accessed on 2 February, 2012.

Ministerial Council for Education, Early Childhood Development and Youth Affairs (MCEECDYA) (1999). The Adelaide declaration on schooling. http://www.mceetya.edu.au/mceecdya/adelaide_declaration,11576.html, accessed on 2 February, 2012.

Ministerial Council for Education, Early Childhood Development and Youth Affairs (MCEECDYA) (2008). Melbourne declaration on educational goals for young Australians. http://www.mceetya.edu.au/mceecdya/melbourne_declaration,25979.html, accessed on 2 February, 2012.

Ministry of Education of People's Republic of China (2001, 2011). *Mathematics curriculum standards for compulsory education*. Beijing: Beijing Normal University Press.

Ministry of Education of People's Republic of China (2011). *Mathematics curriculum standards for compulsory education (2011 version)*. Beijing: Author.

Ministry of Education, Singapore (2007). *Primary mathematics syllabus*. Singapore: Author.

National Council of Teachers of Mathematics (2006). *Curriculum focal points for prekindergarten through grade 8 mathematics: a quest for coherence*. Reston: Author.

Parliament of Australia: Parliamentary Library (2008). Bills digest no. 60 2008-09: Australian curriculum, assessment and reporting authority bill. http://www.aph.gov.au/library/pubs/bd/2008-09/09bd060.htm, accessed 2 February, 2012.

Stephens, M., & Zhang, Q. (2011). Teacher capacity as a key element of national curriculum reform in mathematics: an exploratory comparative study between Australia and China. In *Mathematics: tradition and [ new] practices, joint AAMT-MERGA conference*, Alice Springs, Northern Territory, July 3–7 (pp. 702–710).

Victorian Curriculum and Assessment Authority (2012). *AusVELS: mathematics*. Melbourne: VCAA. http://ausvels.vcaa.vic.edu.au/Mathematics/Overview/Mathematics-across-Foundation-to-Level-10, accessed on 13 January, 2013.

Watson, J., & Nathan, E. (2010). Biased sampling and PCK: the case of the marijuana problem. In *Shaping the future of mathematics education, 33rd annual conference of the mathematics education research group of Australia*, Fremantle, Western Australia, July 3–7 (pp. 610–617).