Reflections on Curricular Change

Alan H. Schoenfeld

Abstract Within any national perspective, curricular change may be viewed as evolutionary, with curricula evolving in ways responsive to the surrounding political and intellectual environments. There is, however, less global coherence than any intra-national perspective might suggest. Historical and political contexts matter, just as ecological niches do in evolutionary biology. This chapter begins with a meta-level discussion describing the consequential nature of (typically national) values, goals, and cultural context and traditions as shapers of curricula. It then proceeds with a discussion of curricular trends in the United States over the past decades, and thumbnail descriptions of changes in the Netherlands, Great Britain, Germany, France, China, and Japan. A concluding discussion reflects on the diversity of curricular directions worldwide, and suggests some ways in which we can profit from it.

Keywords Curriculum · Curriculum change · High stakes assessment · International trends

I begin with two meta-level issues.

Issue 1: Values and goals—typically, at the national level—are consequential.

If "rich and powerful mathematical understanding" were a straightforward goal, then curricula worldwide would presumably be aiming for it. But, the fact that a nation's rankings on TIMSS and PISA can differ significantly suggests that some national curricula emphasize skills more than mathematical modeling and problem solving (the rough foci of TIMSS and PISA, respectively) and vice-versa. Thus, for example, Russia scored above the United States (and above average) on the TIMSS 8th grade 2007 test, while it scored below the U.S. (and below average) on the PISA 2009 mathematics exams (Mullis et al. 2008; OECD 2010). The differences are not huge (scores on any two mathematics tests will correlate to a significant degree), but they reflect non-trivial differences in curricular emphases in the two nations.

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A nation such as the Netherlands, which emphasizes modeling and applications, would expect to do relatively well on PISA. Nations that do not have such emphases (the U.S. among them) would not expect to do as well.

This should not be surprising. Stigler and Hiebert (1999) indicated that there is much greater pedagogical variability between nations than within nations. The same is the case with regard to curricula, especially in nations where there is essentially one curriculum, specified by a national agency such as a ministry of education. Different nations aim in different directions.

This is consequential with regard to both pedagogy and curricula. In China and Korea, for example, curricular specifications and standards have focused largely if not exclusively on content, and the primary pedagogical "ideal" is a beautifully constructed lecture, which makes the mathematics involved absolutely clear to the students (Li and Huang 2013; Park and Leung 2006). The teacher may check in with students to determine their understanding, but it is the teacher's responsibility to lay out the curriculum content, and the student's responsibility to master what has been presented. In contrast, a significant trend in the U.S. has been to break away from this direction—to provide students with opportunities to engage with mathematical ideas and to develop some of the core mathematics (under the teacher's careful guidance) for themselves. Some American curricula, then, have been evolving to provide such support structures. This differs from curricular practice in some (but not all: cf. Japan) Asian nations. Indeed, the contrast between Japanese and Chinese curricula provides an indication of significantly different trends in those two nations. Many Japanese lessons depend very heavily on the orchestration of student responses to carefully chosen problems, while comparably rich Chinese lessons emphasize the unfolding of the mathematics from the teacher.

In sum, different national premises about the nature of thinking and learning lead to different premises about the most effective forms of instruction, which lead to different forms of curricula. Here is an illustrative anecdote. In my problem solving courses I have students work together in small groups for a significant part of class, before I discuss the work that has emerged and move things forward. One of my students, who comes from Korea, expressed bafflement at the class organization. "Why am I listening to other students," she said, "when you know so much more than we do? Shouldn't you be telling us about the mathematics?"

Issue 2: Cultural context and traditions are consequential.[1]

Curricula are tools, to be used in the hands of teachers. Like any tools, their effectiveness depends on the preparation of those who will be using them. Those who construct curricula make assumptions about the people who will be using them, and construct the curricula accordingly. Thus, a particular curriculum may work well in certain contexts and be problematic in others.

[1]What follows contains broad generalities. I am describing trends, in the way that (for example) Stigler and Hiebert (1999) describe trends.

Consider Singaporean curricula. There is no questioning the effectiveness of Singaporean teaching: Singaporean students have consistently scored near the top on both TIMSS and PISA. Singaporean textbooks are models of clarity, and some school districts in the U.S. have tried to adopt them—with mixed success. Why? To put it bluntly, Singaporean texts are designed for Singaporean teachers. Those teachers know the curriculum, and (broadly speaking) they are well versed in the relevant mathematics. Thus, having a "lean" textbook is not an issue: teachers can be trusted to flesh out the examples and make connections between the examples and the underlying mathematical concepts by themselves. In contrast, teachers in the U.S. tend not to have the kinds of backgrounds that would enable them to make good use of the Singaporean texts. Textbooks in the U.S. tend to be "fat" because textbook publishers make the assumption that American teachers require support in implementing curricula. If the publisher expects something to happen in the classroom, then explicit guidance for that event is likely to be presented in the text.

National pedagogical style also makes a huge difference—see, for example, the contrast between the TIMSS videotapes of American and Japanese mathematics classrooms. For many years in the US, "traditional" instruction followed what Lappan and Phillips (2009) called the "show and practice" model of instruction, in which a teacher demonstrates and explains a particular procedure, and students are then given extensive practice at working similar examples. Traditional textbooks in the U.S. were designed to support this approach. Many texts offered a "two page spread" per lesson. The student edition contained worked examples on two facing pages demonstrating the procedures to be learned, and a series of exercises for the students to complete. The teacher's edition contained the student edition as an inset, surrounded by a pre-made lesson plan for the teacher that contained descriptions of the sequence of activities, timing for the lesson, things to highlight, possible assignments, and answers and/or worked-out solutions to the exercises in the inset. Thus, teachers could teach a lesson by opening the text to the day's two-page spread and following the script it embodied. Given the factory model of much mathematics instruction in the U.S.—a typical secondary teacher might be responsible for teaching five or six classes with 30 students each, with three different course preparations—such easy-to-pick-up-and-use texts were in essence a survival tool for many teachers.

This teacher-plus-textbook picture must be seen against the backdrop of teacher preparation and the opportunities for professional development in the US. Typically, a candidate for a teaching license in the U.S. can either obtain certification as an education major during a 4-year baccalaureate career or in a 1-year post-baccalaureate teacher certification program. During the teacher preparation program, the candidate will typically observe and discuss instruction by a "master teacher," and then "take over" the master teacher's classroom instruction for some weeks. This is *the entirety of* the candidate's pedagogical apprenticeship. Once hired, the teacher will tend to have classroom autonomy—but little opportunity to interact with colleagues, either during the workweek or during (rare) "professional development days." (Lortie 1975 likened the insularity of teaching to that of an egg crate, with each separate cell its own private province, insulated from the others.) Within this context, "ready-to-use" curricula make it possible for the teacher to get through the day. Given that extended opportunities to learn on or outside the job are rare in the US, more ambitious curricular goals have to be supported by more ambitious curricula that provide extensive support.

This picture stands in stark contrast to the national norms in Japan. To oversimplify somewhat, the assumption underlying the professional development of teachers in Japan is that even the most talented beginning teachers will require a decade of supported professional growth before they become truly expert teachers. The work setting for teachers in Japan is radically different from the work setting for teachers in the U.S. Working with colleagues is defined as part of one's responsibilities, and the workweek is arranged so that some percentage of one's official work time is spent collaborating with one's colleagues.

The Japanese practice of lesson study (see, e.g., Fernandez and Yoshida 2004) exemplifies the difference. Obvious contrasts between the social contexts represented by lesson study and typical U.S. practice are that (1) teachers in Japan are given time to collaborate on lesson design as part of their defined work, in contrast to the isolation experienced by most U.S. teachers; and (2) the lesson as designed is taught by one member of the lesson study team and observed and refined by the entire team. In particular, that means that *Japanese classrooms are open to knowledgeable visitors as a matter of practice*. This openness provides opportunities for beginning teachers to learn by observation, discussion, and practice—opportunities that are typically not available for teachers in the U.S.

There is more. Typically, lessons in the U.S. are focused on the mathematics, or the mathematical activities, that students are to experience. In the U.S. a good lesson is typically considered to be one that contains an engaging activity or series of activities that highlight the important mathematical content and practices to be learned. At least as represented in the literature (Fernandez and Yoshida 2004; Takahashi 2004), a major focus of lesson study lessons concerns not just the mathematics itself, but *student thinking about the mathematics*. Questions that shape lesson design include, what understandings do students bring to the topic? what choice of examples will best reveal student understandings? how can one build on what is solid, and lead students to see the limitations or errors in what they understand? how can a lesson be sequenced to help students see connections across ideas, and to build deeper understandings? Building and using such lessons calls for mathematical knowledge, every bit as much as the content-oriented lessons in the U.S. But, content knowledge is not enough to make such lessons succeed. Thus, teachers steeped in lesson study approach the lessons with a different mindset than teachers who have not benefitted from that cultural surround. This is one reason that attempts to use lesson study in the U.S. have generally not succeeded. Unless the introduction to lesson study provides mechanisms for U.S. teachers to become comfortable with the "student thinking focus" of lesson study lessons, and the cultural surround provides opportunities for shared think time and development, the skills sets of U.S. teachers are likely to put them in a position where they are not prepared to implement lesson study in the ways Japanese teachers do.

To be clear, I am talking about structural and cultural issues here. The cultural and administrative contexts of education in the U.S. result in most teachers having limited opportunities to develop certain kinds of skills. In some other nations teachers have much greater opportunity to develop those skills. The differences have nothing to do with the inherent capacity of teachers, or how hard they work. A secondary teacher in the U.S. may meet with 6 classes of 30 students during the workday. In addition to planning for the next day's lessons, that teacher will, if he or she spends just one minute per student looking over homework, spend three hours during the evening doing so. Part of the challenge in the U.S. context is that the workday is defined in ways that deny teachers the opportunities to grow professionally that are available to teachers in other nations.

A final contextual factor that must be considered as a component of teacher professionalism is teacher autonomy. The starkest current contrast may be between the U.S. and Finland. In Finland, trust in adequately prepared teachers, school systems willing to take responsibility for educational outcomes, and adequate resources are the key:

> Experience from Finland … suggests that it is not enough to establish world-class teacher education programs or pay teachers well. Finland has built world-class teacher education programs. And Finland pays its teachers well. But the true Finnish difference may be that teachers in Finland may exercise their professional knowledge and judgment both widely and freely. They control curriculum, student assessment, school improvement, and community involvement. (Sahlberg 2012, p. 4)

Trends in those directions since the 1990s, says Sahlberg, are the main reasons for the stellar performance of the Finnish educational system. By contrast, the U.S. has seen strong trends in precisely the opposite direction. As elaborated below, the "standards" championed by the National Council of Teachers of Mathematics (1989) were intended in the following sense:

> A standard is a statement that can be used to judge the quality of a mathematics curriculum or methods of evaluation. Thus, standards are statements about what is valued. (NCTM 1989, p. 2)

However, the "standards movement," epitomized by the federal "No Child Left Behind" law (U.S. Department of Education; see http://www2.ed.gov/nclb/landing.jhtml) came instead to focus on "accountability"—the idea that students, schools, districts and states *must meet certain standards* or suffer the consequences. Standards became targets for performance, with rewards for meeting or exceeding them, and penalties for failing to do so. Federally enforced policy moved in reverse of the direction taken by Finland: in districts that failed to meet statewide standards (as determined by statewide examinations), teachers and schools were given less and less autonomy. In many schools "teaching to the test" became the norm, essentially negating the very idea of teacher autonomy.

In summary, values, goals and context matter. Different nations (if they operate at the national level; some devolve significant authority to states, provinces, or other such entities) emphasize different aspects of mathematical proficiency in their standards or curricula. The teaching forces in various nations have significantly different levels of preparation before they enter the classroom, comparably different opportunities for professional development or growth, and radically different levels of autonomy in structuring what takes place in their classrooms. A curriculum well suited to one context will be poorly suited for another. There is no "one size fits all" and no one curriculum direction, given the diversity of contexts in which teachers do their work. Attempts to move educational systems in any particular directions will have to be suited to the local educational ecologies—or (see the concluding discussion) conscious efforts will need to be made to alter those local ecologies.

Curricular Stories

Prologue: There Is More than Can Be Summarized in a Chapter

The forces that shape curricular evolution, and the tangled histories that result, are far more complex than can be dealt with in a chapter of this length (Just one example: As (some of) the U.S. was becoming enthralled with the kinds of lessons exemplified in the TIMSS videos of Japanese classrooms, the Japanese ministry of education was backing off from some of the underpinnings of those lessons.) Fine-grained detail is impossible here, but it exists. For a general overview of curriculum change in 15 nations spanning the globe, see Törner et al. (2007). That volume focuses on problem solving, but in that context, national curricular histories are given. In particular, my article (Schoenfeld 2007) provides a more extended discussion of trends in the U.S. up to 2007 than I can give here. I summarize those telegraphically, and then give more detail about events over the past half dozen years.

The United States

I think it is fair to say that for the most part, mathematics education received little attention during the bulk of the 20th century save for times when the nation was in crisis.[2] After World War II broke out, for example, the U. S. Office of Education and the National Council of Teachers of Mathematics (NCTM) jointly characterized the level of mathematical competency that schools needed to provide prior to students' entry into the military (see NCTM 1943). Likewise, the cold war had a significant impact on mathematics education. Following the launch of Sputnik in 1957, science and mathematics education were seen as major national security issues; in response to the Soviet threat, alliances of scientists, mathematicians, and educators produced novel curricula in the sciences and mathematics. In mathematics, the School Mathematics Study Group or SMSG curriculum (see http://www.lib.utexas.edu/taro/utcah/00284/cah-00284.html) exemplified what

[2]It was this perception that led to the formation of the Mathematical Sciences Education Board (MSEB) at the National Research Council. MSEB "was established in 1985 to provide "a continuing national overview and assessment capability for mathematics education." (National Research Council 1989, p. ii) The idea was to keep mathematics education from being put back on the "back burner" after the flurry of attention it was getting in the wake of the Japanese "economic miracle" of the 1970s.

came to be known as the "New Math"—which was perceived to have "failed" over the course of the 1960s and was replaced by a decade of "back to basics" instruction in the 1970s.[3]

The next crisis was economic rather than military. The "Japanese economic miracle" of the 1970s threatened to unseat the U.S. as the world's dominant economic power. The iconic response was the production of the report *A Nation at Risk* (National Commission on Excellence in Education 1983), which described the crisis as follows:

> Our Nation is at risk. Our once unchallenged preeminence in commerce, industry, science, and technological innovation is being overtaken by competitors throughout the world ... If an unfriendly foreign power had attempted to impose on America the mediocre educational performance that exists today, we might well have viewed it as an act of war. We have, in effect, been committing an act of unthinking, unilateral educational disarmament. (p. 1)

In addition to the economic crisis, there was evidence of American students' poor mathematical performance on the Second International Mathematics Study (McKnight et al. 1985a, 1985b, 1987). These could be seen as a mandate for change.

Change came. For political reasons (see Schoenfeld 2004, 2007) this change was stimulated not by the government, but from the National Council of Teachers of Mathematics, which undertook the task of creating a nationwide statement of high quality expectations—"standards"—for mathematics curriculum and evaluation. The NCTM's (1989) *Curriculum and evaluation standards for school mathematics*, which was grounded in the research of previous decades, opened up significant new territory. Previous discussions of curriculum desiderata focused on the content—the body of mathematics students should learn. The *Standards* broke new ground, focusing on mathematical processes as well as content. As in previous documents, the *Standards* listed (by grade band) the essentials of number, patterns, measurement, geometry, algebra, and pre-calculus (for college-intending students) that students should learn. But for every grade band, the first four standards concerned essential processes: mathematics as problem solving; as communication; as reasoning, and mathematical connections. Being able to *think mathematically*, as well as knowing certain bodies of mathematics, became part of the goal of a mathematics education. This was revolutionary.[4]

Aware of the fact that commercial publishers would not produce "standards-based" curricula on their own, the National Science Foundation supported the development of curricula aligned with the standards. Fast forward twenty-plus years, and standards-based curricula—which, although varied in style, are all demonstrably different from the "traditional" curricula that predominated in 1989[5]—hold a

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[3]This is a gross over-simplification. Some of the ideas behind the creation of the New Math, such as attention to mathematical structure and the idea of "hands on" mathematics (parallel to "hands on" activities introduced in all of the alphabet curricula) live on to this day.

[4]And, it was controversial, giving rise to the "math wars." I will not discuss those here; see Schoenfeld (2004, 2007).

[5]Roughly speaking, the "traditional" curricula placed significant attention on conceptual and procedural knowledge, focusing on the bodies of skill that students were intended to master, and their

significant portion of the curriculum market. (Some estimates are that 25 % of the texts sold nationwide are standards-based, but publishers are notoriously secretive about sales figures and various hybrids exist, so it is hard to know what the actual figure might be.) What is not in doubt, however, is the main finding of the research literature: students who studied curricula that offered a balance of concepts, procedures, and problem solving did as well on tests of skills as students who studied from skills-oriented curricula, and far better on tests that called for using concepts and doing problem solving (Senk and Thompson 2003; Schoenfeld 2007). Here we shall simply stipulate that finding, and address two questions:

1. What are some of the main changes that characterize the standards-based curricula?
2. What trends have shaped the evolution of curricula over the past two decades, and what changes may they produce over the decade to come?

1. What are some of the main changes that characterize the standards-based curricula?

The 1989 NCTM *Standards* provided curriculum standards by grade bands (grades K-4, grades 5–8, grades 9–12), rather than by grade. They were also pedagogically agnostic, in that a wide range of pedagogical strategies were consistent with the intentions of the *Standards*. Thus they provided tremendous latitude in interpretation: As long as they were consistent with the broad outlines of the content standards in the *Standards* and paid specific attention to the process standards, curricula could claim to be standards-based. The NSF-supported standards-based curricula reflected a broad range of approaches and foci.[6] These included experiential, hands-on curricula, curricula that focused on applications, and curricula that used large thematic units in order to provide rich "surrounds" for the mathematical content they offered. Once again, I am painting with a very broad brush when I make statements about trends in those curricula. But, there were some clear trends. In what follows I draw very heavily on Lappan and Phillips (2009). Lappan and Phillips describe the history and development of the *Connected Mathematics Project* (*CMP*) curriculum, a widely distributed middle school standards-based curriculum. I also draw on exchanges with Zalman Usiskin and Diane Resek, who, respectively, played pivotal roles in the University of Chicago School Mathematics Project (2009) (which produced the preK-6 series *Everyday Mathematics* and UCSMP texts for middle and high school) and the *Interactive Mathematics Project* curriculum.

Lappan and Phillips describe their goals as follows:

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conceptual underpinnings. Standards-based curricula placed a greater emphasis on the process standards discussed above: problem solving, communication, reasoning, connections.

[6]This was done in part because it makes good sense to have a range of models when trying something new, and in part in order to avoid putting NSF in the position of advancing a "national curriculum," which would have been extremely dangerous politically (see Schoenfeld 2004, 2007).

> All students should be able to reason and communicate proficiently in mathematics. They should have knowledge of and skill in the use of the vocabulary, forms of representation, materials, tools, techniques, and intellectual methods of the discipline of mathematics. This knowledge should include the ability to define and solve problems with reason, insight, inventiveness, and technical proficiency. (Lappan and Phillips 2009, p. 4)

For many educators today, these goals—which are, I believe, shared by all of the standards-based curricula—may seem unexceptional. Of course one wants students to be powerful mathematical thinkers! But from the vantage point of the 1970s and to some degree the 1980s, they represent a major paradigm shift. No longer is the sole focus of the curriculum the specifics of the content that students should learn. That content is part of a student's "mathematical tool kit," with which the student solves problems,[7] reasons, and communicates.

This stance has not only curricular but also pedagogical implications. Standard 1 in NCTM's (1991) *Professional standards for teaching school mathematics*, entitled "Worthwhile Mathematical Tasks," says:

> The teacher of mathematics should pose tasks that are based on—
>
> - sound and significant mathematics;
> - knowledge of students' understandings, interests, and experiences;
> - knowledge of the range of ways that diverse students learn mathematics;
>
> and that
>
> - engage students' intellect;
> - develop students' mathematical understandings and skills;
> - stimulate students to make connections and develop a coherent framework for mathematical ideas;
> - call for problem formulation, problem solving, and mathematical reasoning;
> - promote communication about mathematics;
> - represent mathematics as an ongoing human activity;
> - display sensitivity to, and draw on, students' diverse background experiences and dispositions;
> - promote the development of all students' dispositions to do mathematics. (NCTM 1991, p. 25)

Although this is presented as a curricular challenge it is also a fundamental professional development challenge. Having students grapple successfully with "worthwhile mathematical tasks" of the type described above requires a significant shift in classroom activity structures. Crafting environments in which students feel comfortable grappling with tasks that they do not necessarily know how to solve, often in small groups (remember the communication goals!), and providing enough support so that students do not flail but are not simply told "how to do it," is an enormous pedagogical challenge. This challenge is faced by teachers working with any of the

[7] All curricula have students solve problems, of course. But for pre-standards curricula, those problems were typically exercises similar to the examples students had been shown how to solve. In the new curricula, "problem solving" came to mean working on problems for which the precise solution methods had not been demonstrated.

standards-based curricula. Consider, for example, what it takes to orchestrate a productive conversation about problem 1.1 from the *Connected Mathematics* text given in Fig. 1.

Lappan and Phillips (2009) describe the challenges of designing tasks, such as Fig. 1, that present enough of the real world context so that the challenges can be meaningful, but do not, in asking students to perform various tasks, provide non-mathematical distractions. Resek (January 10, 2012) notes the degree of support required for differing classrooms—with "warm-ups" being useful for some classrooms to remind students of relevant skills and understandings, while they may be superfluous in others. The challenges of tailoring curricula to student needs should not be underestimated. Nor should the challenges of teacher knowledge. It is one thing to show students how to implement a particular procedure and monitor their execution of it, in "show and practice" mode. It is quite something else to be prepared to respond in the moment to the things students say (some seemingly sensible, some not) in ways that build on and shape their current understandings.

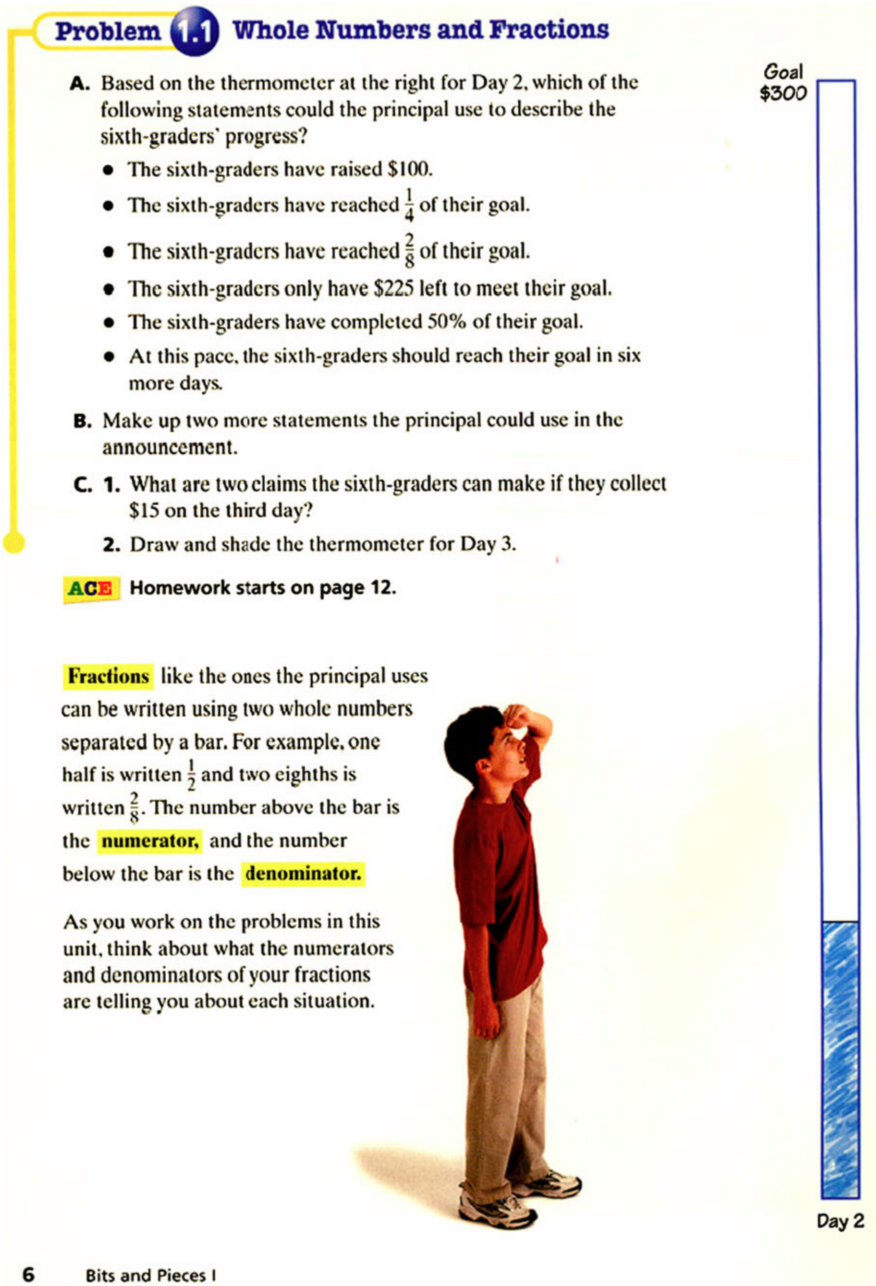
To take a somewhat different perspective on changes in curricula, let me move from goals to beliefs. Volume 40 of the *UCSMP Newsletter*, which introduced the third edition UCSMP materials, provided a recap of "some beliefs underlying the UCSMP Pre-K-12 curriculum." Here I cite three:

3. The scope of school mathematics should expand at all levels, including number and operation, algebra and functions, geometry and measurement, probability and statistics, and discrete mathematics.
4. The classroom should reflect the real world both in the choices of activities and problems and the choices of methods (paper and pencil, calculator, computer).
5. Students learn best when they are actively involved in their learning, and usually need practice and review over time in order to achieve mastery. (*UCSMP Newsletter, 40*, p. 1.)

These, too, reflect general trends in standards-based curricula, and are broadly consistent with the trends in the NCTM Standards documents (1989, 1991, 1995, 2000).

Finally, in terms of curricular criteria (in the U.S.), I note that the NCTM documents (particularly the 1991 NCTM *Professional standards for teaching school mathematics*) placed much greater emphasis on supporting classroom discourse around the relevant mathematics. This too makes significant demands on teachers. But it also makes significant demands on curriculum designers, in that curricula should provide the affordances for such classroom discourse. Thus, for example, one sees the following "criteria for a mathematics task" in the Connected Mathematics Project:

> In our work a good task is one that supports some or all of the following:
>
> - The problem has important, useful mathematics embedded in it.
> - Investigating the problem should contribute to the conceptual development of important mathematical ideas.
> - Work on the problem promotes the skillful use of mathematics.
> - The problem has various solutions paths or allows different decisions or positions to be taken and defended.



Problem 1.1 Whole Numbers and Fractions

A. Based on the thermometer at the right for Day 2, which of the following statements could the principal use to describe the sixth-graders' progress?

- The sixth-graders have raised $100.
- The sixth-graders have reached $\frac{1}{4}$ of their goal.
- The sixth-graders have reached $\frac{2}{8}$ of their goal.
- The sixth-graders only have $225 left to meet their goal.
- The sixth-graders have completed 50% of their goal.
- At this pace, the sixth-graders should reach their goal in six more days.

B. Make up two more statements the principal could use in the announcement.

C. **1.** What are two claims the sixth-graders can make if they collect $15 on the third day?

2. Draw and shade the thermometer for Day 3.

ACE **Homework starts on page 12.**

Fractions like the ones the principal uses can be written using two whole numbers separated by a bar. For example, one half is written $\frac{1}{2}$ and two eighths is written $\frac{2}{8}$. The number above the bar is the **numerator,** and the number below the bar is the **denominator.**

As you work on the problems in this unit, think about what the numerators and denominators of your fractions are telling you about each situation.

Goal $300

Day 2

6 Bits and Pieces I

Fig. 1 Problem 1.1 from Bits and Pieces II, CMP 2

- The mathematical content of the problem should build on and connect to other important mathematical ideas.
- The problem requires higher-level thinking, reasoning, and problem solving.
- The problem should engage students and encourage classroom discourse.
- The problem creates an opportunity for the teacher to assess what his or her students are learning and where they are experiencing difficulty. (Lappan and Phillips 2009, p. 8)

In sum, standards-based curricula—a significant chunk of the textbook market in the U.S., but not a majority—have moved in very clear directions, consistent with the directions outlined in the various NCTM *Standards* documents. But what of the rest, and what is to come? There we must discuss politics.

2. What trends have shaped the evolution of curricula over the past two decades, and what changes may they produce over the decade to come?

I introduce the issue of politics with a quote from a piece I wrote at the turn of the century.

> In the political arena, "standards" may be evolving from a progressive to a conservative force. The move toward standards catalyzed by the National Council of Teachers of Mathematics was designed to focus on mathematical understanding. However, in the very recent past "standards" have been adopted as rhetorical banners for programs of testing and accountability. Many states have instituted strict testing regimens... These accountability tests tend to focus on the mastery of facts and procedures, since that is what can be tested cheaply and easily... Since the accountability measures are "high stakes," teachers feel compelled to focus on them, with a corresponding de-emphasis on the aspects of mathematics learning (reasoning, representation, problem solving, communication, making connections) that are not tested. (Schoenfeld 2001, p. 274)

This turned out, alas, to be prophetic. In 2001 the U.S. congress passed PL 107-110, the *No Child Left Behind Act*, known as NCLB. Paying homage to a political tradition of states' rights, NCLB said that each State was entitled to establish its own set of standards, and its own tests of those standards. But, to receive federal funding, the state had to produce a plan that would result in 100 % of its students being proficient (as measured by the state test) by 2014. Because of this, Hugh Burkhardt's coinage, WYTIWYG ("What You Test Is What You Get") became increasingly true nationwide. When test scores determine whether a student passes to the next grade, whether a teacher gets a raise or gets fired, whether a principal keeps his or her job, whether a school or school district has its management replaced, then testing drives the curriculum. If the tests do not represent high quality mathematics, then the quality of instruction suffers. It takes a brave teacher to teach for complex problem solving skills when the high stakes assessments focus on more mundane skills.

Consider, for example, the California State Tests. The entire test is multiple choice. Figures 2 and 3 offer two typical items from the Algebra I test.

These problems are trivial—and representative. For the past decade, tasks like this have been shaping instruction in California. That is about to change, and not just in California.

The largest change in the American educational landscape of the past decade is the emergence of the Common Core State Standards in Mathematics (CCSS-M;

Fig. 2 Released item from the CST

23 **What is the *y*-intercept of the graph of $4x + 2y = 12$?**

A –4

B –2

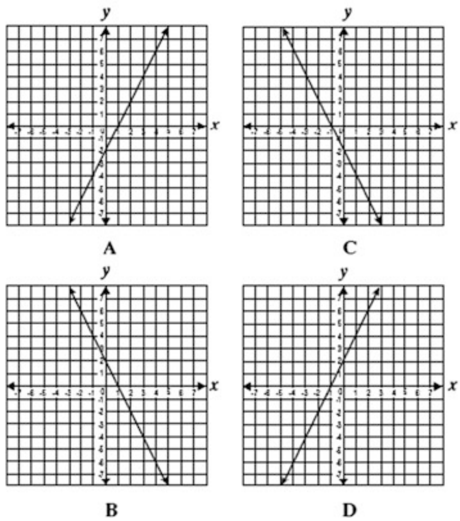
C 6

D 12

CSA00239

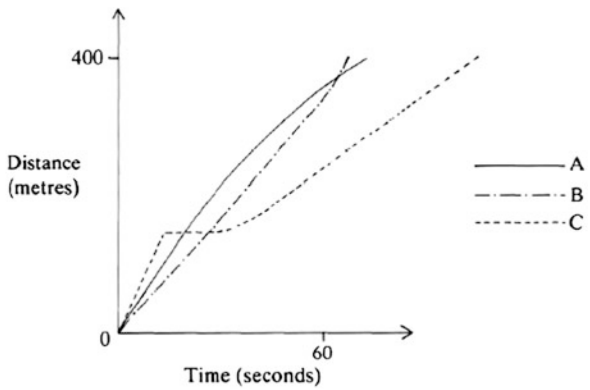
Fig. 3 Released item from the CST

25 Which *best* represents the graph of $y = 2x - 2$?



see www.corestandards.org/the-standards/mathematics). Because of a strong tradition of states' rights, suggestions of national standards, curricula and/or testing in the U.S. would provoke extremely strong resistance. Thus the successor to NCLB, called the "Race to the Top," set no mandate—but, it offered funding to consortia of states that produced collections of "high standards" and plans to meet them. The National Governors' Association and the Council of Chief State School Officers obtained funding to create a set of "voluntary" standards that were offered to all the states. States had the option to adopt the CCSS-M standards (which were likely to be "pre-approved" by the government), or they could join a consortium to craft their own. Ultimately all but five states (Alaska, Minnesota, Nebraska, Texas, and Virginia) adopted the CCSS-M, making them a de facto set of national standards.

The CCSS-M specify content progressions at the grade level. This may require some rearrangement on the part of curriculum developers who want to be "standards compliant." At least as important, however, is the fact that CCSS-M maintain the previous commitment to classroom activities that engage students *doing* mathematics. In the language of CCSS-M, the desired activities are called *practices*, but the



The rough sketch graph shown above describes what happens when 3 athletes A, B, and C enter a 400 metres high hurdles race.

Imagine that you are the race commentator. Describe what is happening as carefully as you can. You do not need to measure anything accurately.

Fig. 4 Hurdles Race

CCSS-M commitment to NCTM's framing of problem solving, reasoning and proof, communication, representation, and connections, as well as the NRC's (2001) concepts of adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition, are made clear.

In short, there is great continuity with the process-oriented view that was a key component of the NCTM Standards volumes and the standards-based curricula. The question is, how much will the emphasis on *practices* matter in classrooms? The answer, assuming WYTIWIG, is that what happens in classrooms will be a function of the assessments that students and teachers face. Here too there is a new landscape.

As part of the Race to the Top, Federal funding supported the development of two assessment consortia, the Smarter Balanced Assessment Consortium (SBAC; see www.smarterbalanced.org/) and the Partnership for Assessment of Readiness for College and Careers (PARCC; see www.parcconline.org/). Roughly half of the states that have adopted the Common Core State Standards have signed up with each of the two assessment consortia, and will be using their assessments. Thus, rather than there being a patchwork of assessments across the nation, there will be (save for the five non-CCSS states) just two.

Both consortia have issued sets of specifications describing their intended assessments. As noted below, there is significant potential for things to change, so nothing that follows should be taken as carved in stone. But if things stay as they are now, things will be very different than they have been—at least in some states.

Consider, for example, the task "Hurdles Race," given in Fig. 4.

Content-wise, Hurdles Race demands a good deal more than the tasks in Figs. 2 and 3. To deal with it successfully, students must interpret distance-time graphs in a real-world context. This includes realizing that "to the left" is faster in the context of a distance-time graph (the racer whose graph crosses the $d = 400$ line to the left has reached that mark in less time). It means interpreting the point where two runners' graphs cross as meaning that, at that point in the race, the two runners are tied. It means recognizing that runner C was ahead at the beginning of the race, and that the horizontal line segment in C's graph indicates that C has stopped moving forward (presumably having tripped over a hurdle).

Moreover, the student must put all of this information together in an explanation that respects the chronology of the graph. This calls for perseverance. It calls for producing a coherent narrative that does justice to a real-world event. This is the kind of task that is likely to appear on the SBAC assessments: see SBAC (2012).[8]

The key to understanding the character of the SBAC summative assessments lies in their reporting structure. Typically, a mathematics test reports one score for a student. In contrast, the SBAC assessments are intended to report four scores, along the following dimensions:

Dimension 1, concepts and procedures: "Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency."

Dimension 2, problem solving: "Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies."

Dimension 3, reasoning: "Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others."

Dimension 4, modeling: "Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems."

Given that the tests will be "high stakes," classroom activities are likely to fall more in line with these dimensions of mathematical activity. Note that this is entirely consistent with the goals for curricula quoted above from Lappan and Phillips (2009), and, more generally with the two volumes of NCTM standards, the CCSS-M (by design), and, in broad-brush terms, the standards-based curricula.

There are similarities and differences for the PARCC draft specifications. Somewhat in parallel, the PARCC summative assessments have three classes of tasks:

Type I: Tasks assessing concepts, skills and procedures;
Type II: Tasks assessing expressing mathematical reasoning;
Type III: Tasks assessing modeling / applications.

However, it is not clear whether PARCC will assign separate scores to student proficiency on tasks of types I, II, and III. PARCC appears to have committed itself to computer-based tests: sample tasks may be found, for example, through the links at http://www.parcconline.org/samples/mathematics/high-school-mathematics. Some of the questions (e.g., the golf balls" modeling task), given the technological medium, look very different from an "essay test" with the same questions and pose different demands.

[8] Hugh Burkhardt and I were the lead authors of the SBAC content specifications.

Because of the power of high stakes tests to drive curricula, it is hard to know where instruction in the U.S.—not just curricula—will be heading over the next decade. As I write, SBAC has not fully adopted the specs discussed above or released sample tests. (Sample items have been released.) Ultimately, what does get adopted will in large measure be a political decision. To date, PARCC has released even less information. Moreover, it is not yet clear how well the SBAC and PARCC assessments are aligned with each other. How they will drive curricula, and in what directions, remains to be seen.

Snapshots of other Nations

As noted above, curricular trends vary substantially across the globe, as do the traditions underlying them and the support structures for implementing them. Here I offer either summaries or quotes in response to my requests for information about trends in various nations. See the ZDM special issue *Problem solving around the world—summing up the state of the art* (Törner et al. 2007) for additional detail.

The Netherlands

Paul Drijvers (personal communication, May 21, 2012) reported that trends in Dutch mathematics education are diverse.

> On the one hand, we still have investigation tasks in secondary school (e.g., see http://www.fisme.science.uu.nl/alympiade/en/welcome.html and http://www.fisme.science.uu.nl/wisbdag/, the latter only in Dutch). Also, the new 2015 curriculum explicitly mentions problem solving and modeling skills, in the frame of overall "thinking activities." ... Most textbooks for secondary school still make use of many contexts, even if these problem situations are in many cases far from realistic and their use is very limited, and may even have a tendency to just 'dress up' the mathematics.
>
> On the other hand, there is an important back-to-the-basics movement, with a strong focus on arithmetic and algebraic by-hand skills, knowledge on how to carry out rules for simplification, solving equations, and differentiation. From this perspective, the realistic approach is questioned and we see textbook series and national examinations following this trend. The role of technology is criticized as well. The ministry is introducing examinations in arithmetic skills in addition to the mathematics national examinations.

Great Britain

There is a significant amount of detail in Hugh Burkhardt's chapter in this volume, "Curriculum Design and Systemic Change." Burkhardt's jaundiced summary of problem solving (personal communication, May 21, 2012) is that curricular work on problem solving "goes back to 1870, peaks in 1945, with a false dawn in the

1980s, [and was] choked off by the National Curriculum.” This is not unlike the history of mathematical modeling, in which there have been a number of exemplary projects (existence proofs) but nothing that has had lasting impact and is deeply embedded in current curricula. More generally, there appears to be a certain level of entropy: the National Curriculum has had various revisions since 1989, with the most recent promised revisions under the Tory government delayed.

Germany

Germany appears to be somewhat in a state of flux, according to information from Guenter Toerner and Kristina Reiss. In some ways, the situation appears to parallel the situation in the U.S. (though with significant allowances for cultural context, teacher support, etc.):

> The state of the education system and its prospects for development have become the subject of increasing debate in Germany since publication of the TIMSS results … In recent months, the Forum Bildung, a working party set up by the German federation and states to elaborate recommendations for educational reform, and the tremendous response to the PISA study … have given this debate an intensity and a range not seen in years. (German Federal Ministry of Education and Research 2004, p. 7)

With sixteen Bundesländer and perhaps 50 different curricula at primary, lower secondary, and upper secondary school (gymnasium), the quest for national standards and coherence has been a challenge; it is difficult to paint a simple picture of current events. Yet, there are trends. Modeling and applications are getting more attention than before. And, perhaps most interestingly, there is a change in language: German standards now emphasize *competencies* rather than *knowledge*. Competencies are defined as “cognitive abilities and skills possessed by or able to be learned by individuals that enable them to solve particular problems, as well as the motivational, volitional and social readiness and capacity to utilise the solutions successfully and responsibly in variable situations” (Weinert 2001, p. 27).

One sees, then, moves toward coherence grounded in a set of standards that emphasize a broader set of competencies than descriptions of content knowledge.

France

The French, as Artigue and Houdement (2007) indicate, have a curricular and didactic history that is at some remove from the traditions of the other nations discussed in this chapter. Problem solving *per se* has never been a major theme in French curricula, although problems themselves play a central role. Artigue and Houdement indicate that the dominant theoretical frames in French didactic research are Brousseau’s theory of didactic situations (Brousseau 1997) and Chevallard’s anthropological theory of didactics (Chevallard 1992). Both of these theoretical orientations bring unique perspectives to classroom activities in France: Brousseau’s by virtue of the core idea that classroom activities can be structured in ways that are inherently mathematical, and that reveal student conceptions to the teacher (see, e.g., Schoenfeld 2012), Chevallard's by way of situating classroom actions within the larger context of school, schooling, and society. These, of course, are developments over the past quarter century; and they have not fully permeated the French school system. In another paper, for example, Artigue (2011) traces the history of the high school teaching of calculus in France as an example of curricular change. That paper documents seven distinct periods of instruction over the course of the 20th century.

China

Chinese mathematics education has its own very strong traditions, which tend to separate curricular development in China from that in other nations. As Cai and Nie (2007) indicate, "research in China has been much more content and experience-based than cognitive and empirical-based." (p. 459) That said, it should be noted that the tradition of honoring and sharing beautifully designed lessons—with journals devoted to such activities—flourishes in China, as a robust complement to the kind of research done in the West.

But, curricula in China are changing rapidly, partly as a function of deliberate study of some practices in the West. Liu and Li (2010) report that, prior to 2001, "mathematics education in China held an important but simplified objective: acquisition of knowledge and skills." (p. 11) From 1996 to 1998 the Chinese undertook the systematic study of curricula in Western nations, and curriculum reform and curriculum standards for all disciplines were released in 2001. There were major changes, these being the first three:

> 1. Curriculum objectives: moving away from over-emphasizing knowledge acquisition to emphasizing the formation of students' positive attitudes toward learning, so that students can learn how to learn and develop positive attitudes in the process of learning basic knowledge and skills.
> 2. Curriculum structure: moving away from over-emphasizing content-based subjects, having too many school subjects and lack of integration. School curriculum needs to be structured with balance, comprehensiveness, and selectivity.
> 3. Curriculum content … needs to emphasize its connections with students' life and knowledge development in science and technology, pay close attention to students' interests and their experiences, and carefully select those basic knowledge and skills needed for students' life-long learning." (Liu and Li 2010, p. 14)

One can be sure that, although these goals echo those from the west, their instantiation in curricular practice will take root in ways that are unique to Chinese context and history.

Japan

For my final example we return to the somewhat more familiar territory. For many years American mathematics educators have been inspired by aspects both of Japanese curriculum and professional development, e.g., the ideas that one rich problem could be the source of a whole lesson's activities, and that (at least at the elementary level, through lesson study) teacher communities could serve as the ongoing homes for the professional development of the teaching force. As Hino (2007) portrays it, Japanese mathematics educators were strongly influenced by the NCTM's recommendations that "problem solving should be the focus of school instruction," from the 1980s on. Some would argue that the Japanese, with help from their Ministry of Education (which revises the Course of Study every 10 years) were more successful than the U.S. in doing so.

Here too, it would be a mistake to assume stability or unidirectionality. As Koyama (2010) notes in his chronological survey of Japanese curricular trends, there have been some significant changes, even after the adoption of "problem solving" as a major theme. Here is Koyama's survey of the two decades from 1988 to 2007.

- *Integration of Cognitive and Affective Aspects (1988–1997)*
  In 1989 the Courses of Study were revised to integrate cognitive and affective aspects. For example: ... "To help students develop their abilities to consider daily life problems insightfully and logically, and thereby foster their attitudes to appreciate the way of thinking mathematically, and to willingly make use of the above mentioned qualities and abilities in their lives."

- *Latitude through Intensive Selection of Teaching Contents (1998–2007)*
  During these ten years, such problems as 'un-schooling' and 'classroom in crisis' have became quite notable and they were attributed to the excessively stressed life of students. Therefore the Courses of Study were revised and the teaching and learning contents were slimmed down intensively. About 30 % of mathematical content was removed from elementary school and lower secondary school levels. (Koyama 2010, p. 62)

Discussion

I trust that the snapshots above have made one point clear: there is no single worldwide trend in curricula. Curricula are, in deep ways, a function of a nation's history and culture, its governance structures, and the kinds of support given teachers in their professional lives. Moreover, within any one nation, one sees radical shifts over the course of time—often in contradictory directions. This raises two questions: (1) is there such a thing as progress? And (2) what can one do, profitably, with the great diversity of curricular trends that one sees worldwide?

Is there Such a Thing as Progress?

I believe so. When I entered the field as a researcher, the dominant paradigm for classroom research was the process-product paradigm, which was essentially correlational; curricula were similarly evaluated by controlled experiments; we had little or no idea of how to understand a student's 20-minute attempt at problem solving, much less the blooming complexity of the classroom; and, curricula focused on content, with little or no explicit attention to the concepts of mathematical thinking processes or practices. Over the decades since the 1970s, our understanding of learning and teaching has grown dramatically—and, there has been a lovely dialectic between what we understand and the evolution of curricula over that time period. The 1989 NCTM *Curriculum and evaluation standards* were inspired by research, and set the stage for the first wave of standards-based curricula. There is clear evidence (e.g., Senk and Thompson 2003) that they make a difference. The evolution of standards in the U.S., from the 1989 *Standards* to the 2000 volume of *Principles and Standards* to the Common Core State standards, represents increasing sophistication as well as a better understanding of the political surround that envelops curricular practices.

One can hardly be sanguine about the role (or power) of politics; it can be a powerful force both for the benefit of students and to their detriment. But, looking at curricula now in place in the U.S. as opposed to the one I learned from, one sees significant progress. There are many ups and downs, and retrogressions—but on average, curricula are more mathematically rich, as well as being tailored to be accessible to a far larger proportion of the school population. That is in the U.S., but I am reasonably confident that comparable statements (regarding the long view) could be made across much of the globe.

What can one do, profitably, with the great diversity of curricular trends that one sees worldwide?

As I have noted, it is a mistake to think one can simply import good curricula or effective pedagogical practices from one country into another. Singaporean textbooks "work" because Singaporean teachers are well prepared to teach from them, for example. So, what can one do?

Cross-national comparisons are tremendously valuable in helping one to understand one's strengths and one's weaknesses, and also to realize that things don't necessarily have to be the way they are. I grew up, for example, assuming that the "ninth grade algebra, tenth grade geometry, eleventh grade advanced algebra" curriculum was the only way to do things. It was a surprise to discover that the U.S. was a singular point in that regard, and that most of the world offered integrated curricula. And, when I began studying problem solving, I was astounded by the challenges offered by the Russian and Hungarian problem books. It would have been unthinkable at the time to imagine that such problems could be offered to students in the U.S. But, once one realizes that such things can be done, doors are opened. The same goes for structural supports for the professional development for teachers, and organizing the contexts of work in ways that become learning communities for teachers (cf. Japan and Finland).

There is, of course, a substantial amount of cross-national comparative research. But, I think it could be expanded or focused in ways that would be beneficial to all concerned. For one thing, it would be good to be much more explicit about the goals and underpinnings of the exams that are used for cross-national comparisons (TIMSS and PISA). Better yet, it would be good to construct exams that explicitly aim at assessing the wide range of understandings that represent the *union* of mathematical goals for students, and that report out different dimensions of mathematical proficiency. The dimensions of concepts and procedures, problem solving, producing and critiquing reasoning, and mathematical modeling would seem to be a good start in this direction.

Alongside this, the systematic study of how different nations are organized systemically with regard to mathematics education could help all of us learn from one another. Some of this information exists, but not in a way that supports a meaningful and potentially productive compare-and-contrast across nations, or the meaningful adaptation of ideas and artifacts from one culture to another. One could imagine a project in which teams of scholars from nations across the globe put together the following kinds of information about each nation:

- What is the history of curricular change?
- How does curriculum change take place? Who makes decisions, and how are support structures put in place? What are the impediments to change?
- What is the role of research in the process?
- What are the systemic levers (e.g., high stakes testing) for supporting change and how powerful a (positive or negative) role do they play?
- The nature and processes of change: are they stable and evolutionary (as Japan was for many years), unpredictable (as the U.S. has been), or something in between?
- Systemic affordances and constraints—what resources are available; what kinds of changes are plausible, given systemic organization and resources; what kinds of changes would be difficult because of various limiting factors?
- As one major example, consider teaching as a profession:
  - How well are teachers regarded, how well paid are they?
  - What opportunities for learning and professional development do teachers have?
  - How knowledgeable are teachers? How well prepared might they be for implementing problem solving, supporting robust mathematical conversations among students, etc.?
  - What is the career trajectory of teachers? What are the demographics of the teaching force? (For example, in the U.S., 50 % of new teachers leave the profession within 5 years—a higher percentage in urban districts.)
- Case studies of curriculum change: what succeeded, for what reasons (in terms of organization, supports, etc.); what did not succeed, for what reasons?

With such information it might be possible to understand, for example, how Japanese lesson study functions as a form of professional development and what cultural and intellectual support structures are necessary for it to be productive; what kinds of preparation and what kinds of structural supports are necessary for teachers to make effective use of Singaporean curricula; what kinds of teacher preparation, societal incentives, and institutional "surrounds" are needed in order for the kinds of teacher autonomy heralded in Finland to be productive; and so on. Understanding the culturally embedded nature of curricular change in other nations may enable people to think more productively about how to foster effective curriculum change within their own cultural and educational ecologies.

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