# Improving the Alignment Between Values, Principles and Classroom Realities

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**Abstract** The curricular reforms described in this book are wide-ranging and are driven by many external factors and value systems. They usually begin with a vision of 'how things should be', but as we have seen, their implementation is often a travesty of their aims. In this chapter I begin with a synthesis of the values exhibited in curricula across the world, then go on to analyse the kinds of classroom activity that are implied when these are taken seriously. This process will be illustrated through a specific case—a national consultation in England that attempted to elicit, prioritise and exemplify apparently competing values held by mathematics educators. I argue that the misalignment of the intended and enacted curriculum is at least partly due to the almost universal lack of vivid exemplification in curriculum specifications and consequent reductive interpretations of them by their users. An argument is thus made for a serious systematic design-research effort into the production of beautiful examples that illustrate and effectively communicate our core values to the key educational stakeholders.

**Keywords** Alignment · Classroom reality · Curriculum reform · Principle · Value

## Introduction

Across the world politicians are demanding that more citizens should study mathematics to a higher level than ever before. The reforms described in this book appear primarily to arise from a desire for change in:

- *Economic competitiveness*. Most nations view the quality of mathematics education in schools as an indicator of their economic prospects in the 21st century. International comparisons of standards are rife, and the uses and abuses made of TIMSS and PISA studies in arguing for reform are reported in many chapters.
- *Student participation and dispositions towards mathematics*. There is a great concern in many countries that students cease to study mathematics at the earliest

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possible opportunity and that even those who are most successful have such negative dispositions towards the subject that they avoid scientific careers. The negative correlation between attitude and performance in TIMSS is striking. This is attributed in many cases to the way the subject is taught and is evidenced by attempts to reduce the amount of content (such as in the 2011 changes in Korea) and increase the emphasis on inquiry-based learning, such as those currently being promoted across the EU (Rocard 2007).
- *Control and coherence*. In countries with a history of state autonomy, such as the USA and Australia, new curricula have been introduced in an attempt to centralise and regain control of the curriculum. This is usually seen as a necessary precursor to further major reform (e.g. Reys, Anderson, this volume).

For these reasons, curriculum documents are created to specify those aspects of Mathematics that are to be valued and taught. There are, almost universally, major mismatches between the intended curriculum described in policy documents, the tested curriculum embodied in examinations, and the implemented curriculum taught in most classrooms. Burkhardt (this volume) notes the main causes: "underestimating the challenge; misalignment and mixed messages; unrealistic pace of change; pressure with inadequate support; inadequate evaluation in depth; and inadequate design and 'engineering' ". A recent statement by a Dutch politician on the release of a new curriculum specification illustrates the problem rather vividly: "The hard work has been done now all you have to do is implement it" (van den Akker 2012). Other authors in this volume describe how centralised, 'top down' reforms have mostly resulted in only superficial implementations (Brown, Cavanah), whereas the more successful cases have been mostly local, and underpinned with sustained professional development and aligned assessment and curriculum materials (e.g. Hoe, Brown, Ma).

In this chapter, I focus on the challenges that influencing and implementing policy reform offers to curriculum and assessment designers. I look again at the values that are commonly emphasised in policy documents and consider the implications that these pose for the design of classroom activities. For me, the greatest research needs lay at the interface of policy and implementation, in particular the almost universal lack of quality exemplification in policy documents. Such documents begin by conveying 'worthy values', with which most agree, including processes (or 'practices') that students should learn to perform, and the hierarchical content domain that students should 'master' ('scope and sequence'). Unfortunately, it is usually only the latter that is assessed in most high stakes examinations and little attention is paid to the design challenges of drawing connections between values, principles, practices and content in curriculum implementation. In addition, for reforms to have impact, there should be some succinct attempt to articulate the research-based principles that underlie effective teaching of the various curriculum elements.

## The Nature of Values

Values may be characterized as those preferences, principles, and convictions that act to guide our actions and the standards by which we judge particular actions to be desirable. (Halstead and Taylor 2000, p. 2). They are what we consider 'ought to be the case', and as such have an almost moral dimension. They may be held to different degrees, from simple preferences, reflecting tastes or sentiments, to more complex organised states of commitment and prioritization (Atweh 2008; Krathwohl et al. 1964). Attitudes become values as they are thoughtfully chosen, prized, cherished, affirmed and acted on repeatedly (Raths et al. 1987, p. 199). This is not a straightforward process, particularly when values conflict. A teacher may simultaneously value opportunities to develop a deep understanding of mathematics, to broaden students' awareness of its applications, of its cultural and historical evolution, of the need to cover content and to develop the fluency and speed needed for examination success. Prioritizing these, particularly in a results-oriented culture can lead to painful and difficult decisions and inconsistencies between values and actions (Bishop et al. 2003).

Values may be both individually and culturally based. Education systems are often determined by tacit cultural values that cannot be ignored when, for example, making international comparisons. In a recent review of mathematics teaching in higher attaining countries, it was argued that high attainment was more closely linked to cultural values than to specific mathematics teaching practices (Askew et al. 2010).

While the pedagogical practices clearly vary considerably between nations (Schoenfeld, this volume), the aspirations exhibited in Mathematics curricula reform documents are often strikingly similar (Askew et al. 2010; Stigler et al. 1999; Stigler and Hiebert 1999). They typically emphasise the societal, personal and intrinsic value of studying mathematics.

In the current English national curriculum: Mathematics is deemed essential for 'national prosperity', 'public decision-making' and 'participation in the knowledge economy'; It equips pupils with 'uniquely powerful ways to describe, analyse and change the world' and can 'stimulate moments of pleasure and wonder'; and it provides an 'international language' that transcends cultural boundaries and is therefore worth studying 'as a means for solving problems' and 'for its own sake' (QCA 2007). Mathematics is even seen to offer opportunities for spiritual, moral, social, and cultural development (DfEE/QCA 1999). The current national documents then go on to describe the importance of developing key 'concepts' (competence, creativity, applications and implications, critical understanding) and 'processes' (representing, analyzing, interpreting and evaluating, communicating and reflecting), before listing the content to be covered. This list is extensive and, currently, the only one taken seriously in assessment. As I write this, however, the national curriculum is being rewritten under political direction that it is to be focused on only 'core knowledge', with a stronger emphasis on 'fluency in arithmetic' (DfE 2013). 'Key concepts' and 'key processes' are being replaced with more general statements requiring reasoning and problem solving.

In the US, The NCTM Standards have aspirations similar to the current English national curriculum. It emphasises that mathematics is important for 'one's personal life,' as part of our 'cultural heritage', for 'the workplace', and for 'the scientific and technical community' and then details the processes of problem solving, communicating, reasoning, making connections, concepts, procedures and dispositions (NCTM 1989, 2000). The recent Common Core State Standards for Mathematics (NGA and CCSSO 2010) emphasises the importance of both technical procedures and understanding, along with the development of eight 'mathematical practices' that include making sense, reasoning, constructing arguments, modelling, choosing and using appropriate tools, attending to precision, making use of structure and regularity in repeated reasoning.

The high performing countries along the Pacific Rim have values that resonate with these. In Singapore, for example, the current curriculum has mathematical problem solving at its heart, and is summarized by the five inter-related components of concepts, skills, processes, attitudes, and metacognition (Soh 2008). The concepts and skills aspects are subdivided into mathematical content areas (e.g. numerical, algebraic); the processes into reasoning, communication, connections, applications, modelling; meta-cognition into monitoring of one's own thinking and self-regulation; and attitudes into beliefs, interest, appreciation, confidence and perseverance. In reaction to the transmission styles of the past, the Chinese national curriculum reform stresses the importance of students becoming active and creative students. "'Exploration', 'co-operation', 'interaction', and 'participation' are central leitmotifs of its theory of student learning (Halpin 2010, p. 259). Citing the general secretary (2004), Guan and Meng (2007, p. 595) state that:

> The form of instructions should no longer follow the "teacher-talk, student-listen" model, rather, there should be dynamic interactivity, an engaged cooperation between teachers and students. Instructions should focus on a student's comprehensive development instead of exam-oriented education.

Lew (2008) summarises the 'ultimate goal' of the Korean curriculum as to cultivate students with creative and autonomous minds by achieving three aims: (i) to understand basic mathematical concepts and principles through concrete and everyday experiences; (ii) to foster mathematical modelling abilities through the solving of various problems posed with and without mathematics, and (iii) to keep a positive attitude about mathematics and mathematics learning by emphasizing a connection between mathematics and the real world.

## A Synthesis of Values

From these and other documents, it is possible to synthesise five distinct aspects of learning mathematics: (i) developing fluency when recalling facts and performing skills; (ii) interpreting concepts and representations; (iii) developing strategies for investigation and problem solving; (iv) awareness of the nature and values of the educational system and (v) an appreciation of the power of mathematics in society.

The table below (Swan 2006; Swan and Lacey 2008b) expands and develops these categories in order to explore appropriate types of classroom activity that might result. Mathematics teaching will look very different depending on the relative value that is ascribed to these purposes.

The rows in this table resonate with complementary theories/metaphors of learning. The first row is the focus of 'behaviourists', who emphasise the value of terminology and fluency in the performance of 'skills'. This trend is evident in learning activities that break 'mathematics' up into 'subskills' and 'key facts' that are taught until fluency is attained. Complex skills are then built by learning sequences of subskills. The process of learning is generally conducted by clear exposition, followed by consolidation and practice. The second and third rows reflect the focus of 'constructivists' who recognise the value of encouraging students to construct concepts and strategies through exploration or creativity and discussion. Also reflected is the emphasis on metacognitive aspects in monitoring decisions in the course of problem solving. The fourth and final rows reflect the current focus of 'social constructivists' who emphasise that students should appreciate the way mathematics has evolved historically, how it is used by the world, and how they may use their mathematics to gain power over their own environment. This also includes students reflecting on their own role as a student in an educational environment and combines elements of metacognition, in which a student develops an awareness of effective personal strategies for learning, with an awareness of the social values and discourses of education. The intention is also that students become aware of the nature of the assessment system and how they may portray their own abilities to their best advantage when presenting themselves to the world. On the right of Table 1, I have begun to list a few of the activities implied by these outcomes. It is immediately clear that most textbooks (at least in England) do not embody the full range of activity types.

The inclusion of learning objectives is usually non contentious in general curriculum descriptions. We want students to be able to perform in all of these aspects. As they become elaborated and incorporated into implemented curricula, however, the time and emphasis each is given becomes an issue. There are also potential tensions and incompatibilities in the teaching methods that need to be employed.

## What Types of Classroom Activity Are Implied by These Values?

Teaching methods for developing factual knowledge and procedural fluency and for developing conceptual understanding are quite different.[1] By facts we mean items of information that are unconnected or arbitrary, including notational conventions (Cockcroft 1982). By fluency we mean the ability to carry out a mathematical procedure quickly and efficiently without effortful thought. In both cases, individual

[1]In England, the current draft national curriculum states that 'varied and frequent' practice for fluency *will lead* to improved conceptual understanding. This paragraph explains why this may not be the case.

**Table 1** Values in learning mathematics and implications for classroom activities

| Outcomes | Examples of types of mathematical learning activity implied |
|---|---|
| Fluency in recalling facts and performing skills | Memorising names and notations |
| | Practising algorithms and procedures for fluency and 'mastery' |
| Conceptual understanding and interpretations for representations | Discriminating between examples and non-examples of concepts |
| | Generating representations of concepts |
| | Constructing networks of relationships between concepts |
| | Interpreting and translating between representations of concepts |
| Strategies for investigation, problem solving and modelling | Formulating situations and problems for investigation |
| | Constructing, sharing, refining, and comparing strategies for exploration and solution |
| | Monitoring one's own progress during problem solving and investigation |
| | Interpreting, evaluating solutions and communicating results |
| Awareness of the nature and values of the educational system | Recognising different purposes of learning mathematics |
| | Developing appropriate strategies for learning/reviewing mathematics |
| | Appreciating aspects of performance valued by the examination system |
| Appreciation of the power of mathematics in society | Appreciating mathematics as human creativity (+ historical aspects) |
| | Creating and critiquing 'mathematical models' of situations |
| | Appreciating uses/abuses of mathematics in social contexts |
| | Using mathematics to gain power over problems in one's own life |

work on exercises in which the facts and procedures are used repeatedly with immediate feedback are undoubtedly helpful, though one might argue that all such practice should be set within the context of meaningful, substantial problems. The development of conceptual structures, (which of course should underpin procedural knowledge) requires the careful negotiation of meaning in which objects are compared and classified, definitions are built, and representations are created, shared, interpreted and compared. These are essentially social, collaborative activities. There is considerable research evidence to show, for example, the superiority of conflict discussion over guided discovery methods for concept development. (Bell 1993; Swan 2006). The creation of a *network* of connections between concepts requires non-linear exploratory work—difficult to design and embody in hierarchical curricula specifications.

The fundamental differences between teaching for concept development and for problem solving strategies are less well understood. In a current project for which we are developing formative assessment lessons to support the Common Core State Standards in the US, we are discriminating carefully between these two types of lessons (Swan et al. 2012). A concept-focused lesson concerns interpreting and representing a predetermined ‘big idea’, such as place value or proportion. Where applications or ‘word problems’ are used in such a lesson, they are purely illustrative. In a problem-solving lesson, however, students are offered a substantial problem to tackle for which no solution method is obviously apparent. The purpose of the lesson is for students to develop the ability to *select*, *apply* and *compare* appropriate mathematical methods. In a true problem-solving lesson the teacher therefore cannot predict which methods the students will choose. We do know, however, that students are unlikely to choose methods that they have only just acquired. There is often a several year gap between being introduced to a method and being able to select and use it autonomously. We also know that students usually prefer more ‘tangible’ numerical or graphical approaches to algebraic ones. This presents the teacher with a dilemma—how does one reveal the power of an algebraic approach without ‘forcing’ students to use it, in which case the lesson is no longer a true problem-solving lesson, but a mere exercise in algebra? One possible solution is to follow up students’ own attempts to solve a problem with a critiquing activity. We offer students a range of pre-prepared alternative attempts at solving the problem, all of which are imperfect, and invite students to try and improve and complete these. As different approaches are then contrasted and compared in whole class discussions (akin to the Japanese practice of ‘neriage’), ideas are combined and refined into collaborative solutions.

Currently, we are also elaborating a limited number of different task *genres* that seem essential for concept development. All involve collaborative work in which students create a shared product, for example, posters describing their ideas. Research is needed to elicit the design principles for their effective construction. Examples are:

- *Classifying and defining*. Students are presented with a collection of mathematical objects (numbers, expressions, graphs etc), and are asked to create /or apply classifications devised by others. They discriminate, recognise properties and develop mathematical language and definitions.
- *Interpreting and translating between multiple representations*. Students are given a collection of cards that show different representations of mathematical objects—words, diagrams, algebraic symbols, tables, graphs. They share interpretations, compare and group the cards in ways that made connections between underlying concepts. They show how one may be transformed into another by linking cards. The discussion of common ’misconceptions’ is encouraged by the inclusion of distracters.
- *Creating and solving variants of mathematical problems*. Students devise new problems or variants of existing problems, prepare solutions then challenge other students to solve them. They offer support when the solver becomes stuck. This promotes awareness of the structures underlying problems, and focuses attention on the doing and undoing processes in mathematics.

- *Analyzing and challenging generalizations*. Students are given statements or assertions that typically embody general principles or common 'misconceptions'. (Such as "The shape with the greater area has a greater perimeter", "the more digits in the number the greater is its value"). Their task is to challenge these and define domains for their validity.

Teaching for 'awareness' is a further distinctive curriculum goal. This includes cultivating students' awareness of how mathematics fits together as a discipline, how best students may learn something new and how they may best communicate their ideas to others, for example in a high stakes examination. It also includes those 'metacognitive aspects of learning, such as 'monitoring one's own thinking' while solving a problem (as referred to in, for example, the Singaporean National Curriculum). It is widely recognised that when students remain unaware of the purpose of an activity, they often pay undue attention to unimportant or superficial aspects of it. They may, for example, focus more on the appearance of their work or the coverage of material rather than the quality and depth of reasoning employed. Twenty years ago, we conducted a curriculum research project to develop a range of reflective experiences in real classroom settings through which students might acquire such awareness (Bell et al. 1993). These usually required students to change their classroom roles, from consumers of learning to task designers, assessors, textbook authors, and so on. Examples of effective curriculum activities included:

- Preparing summary materials from which other, younger, students could learn.
- Conducting student-student interviews on what has been learned.
- Construct tests of other students' understanding (and mark schemes).
- Planning and teaching a topic to students from another class.
- Planning an outline for a new textbook; deciding which concepts are important and describing how these link together.
- Observing other students working and decide how their problem solving approaches might be improved.
- Conducting 'mini debates' on general learning issues such as: "Do we learn more from working on a few hard problems or from working on many short exercises?"
- Assessing their own progress against given criteria.

Finally, few would dispute that developing an appreciation of the evolution, importance and power of mathematics in society is a laudable goal for the mathematics curriculum. Across the world, however, this only occupies a small part of a teacher's normal agenda. In Science teaching in the UK, there has been a lively debate about the relative emphases that should be on students appreciating the significance and impact of scientific ideas (such as pollution and climate change) and students doing their own science. A similar debate has not been evident in Mathematics. Notably, there is almost no teaching of the cultural history of Mathematics in English schools. Over the years, however, there have been a number of projects across the world to introduce real world modelling and simulation into the curriculum. Recently, for example, we designed a lesson sequence in which students are invited to role play a town planning situation in which their task is to spend a given budget on reducing the number of road accidents in the town (Swan and Pead 2008). They are supplied with a computer database showing the locations and police records of the accidents and their task is to present a convincing case to the town council by analysing these. The activity, which is designed to take about 5 hours, models how planners have actually used mathematics to reduce road accidents by 25 % in one English city. It is interesting to note that this particular activity is now being taken up and used in Japan to plug a perceived gap in the curriculum.

## The Perceived Mismatch Between Ideal and Implemented Values in England

In England, there is a clear mismatch between the values and principles held by the educational community and those implemented in classrooms. A few years ago, I chaired a national consultation, commissioned by the National Centre for Excellence in Teaching Mathematics (NCETM), to review and describe the values and practices considered to be most important and effective by the community (Swan and Lacey 2008a).[2] This consultation involved 150 mathematics educators, with representation drawn mostly from secondary teachers, adult education teachers and teacher educators. An initial conference was held to stimulate debate by: (i) identifying, confirming and agreeing values and principles that underpin the effective teaching and learning of mathematics; (ii) illustrating, through examples, how practice may reflect and interpret these values and principles, and (iii) exploring the factors that inhibit or modify their implementation. This was followed by a series of six one-day regional colloquia that were designed to test levels of agreement with the values and principles articulated at the initial conference, and to amend and refine them as appropriate, as well as to begin to build a collection of lesson accounts that illustrate what the values and principles may look like in practice. These days began with the participants writing descriptions of the most inspirational mathematics lesson they had ever experienced. Later these were discussed in relation to the values they revealed.

In both the initial conference and subsequent colloquia, there was broad agreement as to the type of learning outcomes valued and the different types of classroom activity that these outcomes might imply, as summarized in Table 1 above. Participants were asked to compare their "vision for an ideal mathematics curriculum" with the values that are implied by the "curriculum that is currently implemented in most schools and other settings". In Table 2, I have separated out the teachers' responses from those obtained from university educators and other participants. The results show a remarkable consistency: both sets of participants consider that fluency in recalling facts and performing skills currently dominates the curriculum while in fact it should be the least valued of the five outcomes. Both sets of participants also agree that conceptual understanding and strategies for investigation and problem solving should take up the most curriculum time.

[2] Although the report of the project was 2008, the analysis presented here is new and previously unpublished.

**Table 2** The values of teachers compared with the values of university and other educators. Mean ratings showing how frequently mathematics respondents felt that lessons should *ideally* include each learning outcome and also how frequently mathematics lessons, *actually do* reflect each learning outcome (1 = hardly ever, 4 = almost every lesson). Standard deviations are in brackets. The final column shows the proportion of lesson accounts that participants allocated to each category

| Purposes | Mean ratings (S.D.) Teachers in schools and colleges ($n = 45$) | | Mean ratings (S.D.) University and other educators ($n = 89$) | | % of lesson descriptions in each category |
|---|---|---|---|---|---|
| | Ideal | Actual | Ideal | Actual | |
| A. Fluency in recalling facts and performing skills | 2.58 (0.75) | 3.61 (0.54) | 2.60 (0.78) | 3.81 (0.5) | 33 % |
| B. Conceptual understanding and interpretations for representations | 3.56 (0.59) | 2.29 (0.59) | 3.49 (0.55) | 1.99 (0.66) | 60 % |
| C. Strategies for investigation and problem solving | 3.69 (0.63) | 2.18 (0.76) | 3.71 (0.48) | 2.00 (0.70) | 61 % |
| D. Awareness of the nature and values of the educational system | 3.13 (0.76) | 1.70 (0.85) | 3.02 (0.71) | 1.59 (0.86) | 11 % |
| E. Appreciation of the power of mathematics in society | 3.13 (0.79) | 1.34 (0.68) | 3.05 (0.71) | 1.28 (0.57) | 23 % |

Each colloquium day started with an invitation to each participant to write an account of a memorable, inspirational mathematics lesson, either taught or observed. Over seventy rich lesson descriptions emerged. These offer an alternative perspective on participants' values. As may be seen from the final column in Table 2, most of the lessons were related to conceptual understanding and strategies for investigation and problem solving. Each lesson description was coded and analyzed. In almost all of the lessons reported, students were clearly actively engaged in constructing their own mathematical meanings and methods using the types of activities reported earlier. Below I briefly describe the categories and offer one or two examples of each. (Numbers in brackets refer to the number of examples of each type generated).

- *Students creating definitions* (5). E.g. Students were asked to bring a selection of reading books to school and they then discussed different ways of defining and measuring 'readability'.
- *Students comparing representations and solution methods* (15). E.g. Students sorted cards that contained different representations such as travel graphs and written descriptions of journeys.
- *Students generating their own examples and problems* (10). E.g. Students devising their own financial problems, equations, probability tasks, geometry questions and 'magic tricks'. Other students then had to try to solve or explain these. When

solvers became stuck or were unable to understand the problems, they asked the originators for help or clarification.

- *Students justifying and proving conjectures* (15). E.g. Students set out to find the number of factors of $n!$ (factorial $n$). After an initial conjecture that the answer was $2n - 1$ (this works when $n = 1, 2, 3, 4, 5$), students found that this failed for $n = 6$. This was resolved in discussion by relating the number of factors to the prime factorization.
- *Students tackling ill-defined problems* (2). E.g. Students were given incomplete problems and were asked what additional information they needed to know. They estimated the missing data and then attempted to solve them.
- *Students learning through practical work* (23). Examples included the use of measuring and weighing devices to check estimations; plastic strips to explore properties of triangles; plastic cubes for constructing geometric solids; paper folding for exploring properties of polygons; and even a 'washing line' to help order statements written on cards. Five participants also emphasised the use of students' own bodies to represent mathematical objects and/or as sources for data. One example involved students standing outside on a grid to represent data points on a graph. Students positioned themselves according to their shoe sizes and hand spans. The resulting human scatterplot was filmed from above and played back afterwards for analysis.
- *Students working with electronic resources* (12). E.g. Students began by imagining and mentally manipulating sets of parallel lines, and then subsequently constructed their own geometric computer animations. This was linked to the Hungarian mathematician, Bolyai's excitement at his discovery of hyperbolic geometry.

In reporting this brief summary of the lesson descriptions, I hope to have captured some of the richness and excitement that was conveyed by participants. Throughout, the overriding theme that emerged was one of students' active involvement and enthusiasm in constructing their own mathematics. What seemed to be missing from the lessons reported by participants, yet was clearly valued, was the power of mathematics in society. Perhaps, as noted earlier, this was simply due to the fact that this aspect is almost entirely missing from mathematics classrooms in England.

It should also be noted that the disconnection between the values endorsed by participants through these lesson accounts and the reality in most classrooms was universally recognised. Participants identified four related obstacles to change: the narrow set of values implied by the nature and content of national tests and examinations; the poor quality of textbooks and other resources (many produced by examiners that work for awarding bodies); the social acceptance that it is 'OK' to be mathematically incompetent; and teachers' own lack of confidence in their subject knowledge and fear of stepping 'out of line' with local interpretations of national inspection criteria.

## Principles for Teaching and Learning

Unlike the values listed above, where arguments over relative worth are subjective, principles for learning have been established on the basis of more solid research. These are not normally included in curriculum documents as it is declared that the specified curriculum should only specify *what* is taught, not *how* it should be taught. This argument, however, dodges our responsibility to help teachers apply the wisdom of research to daily practice. Without such principles, we find external pressures (such as those from senior managers in schools) compel teachers to aim for short term, superficial goals, such as 'curriculum coverage' rather than deeper learning.

In the national consultation, participants were also asked to develop a set of research-based generic principles that they believed would improve the quality of lessons in mathematics. In preparation for this, the list of principles in Table 3, drawn from our previous research (Swan 2006), were offered as a starting point. Participants were asked to critique this list and add their own modifications. 64 % (46/72) of participants totally agreed with the initial version of the principles presented, 32 % mainly agreed, expressing reservations whilst 4 % expressed particular concerns. Of those who expressed concern, 35 % (9/26) related to the use of technology and 23 % cited concerns about the confusion that may be caused when exposing and discussing common misconceptions. The principles were subsequently revised, based on suggestions from participants. These revisions are also shown in the table.

Space does not permit me to describe the many research foundations for this list here, but they are considerable (for example Askew 2001). The choices of principles, however, were made deliberately in order to challenge common practices that undermine effective practices. For example the final statement is an attempt to counteract the common request from senior school managers for teachers to list the objectives on the board in front of the class at the start of each lesson. As such it is a political tool to assist teachers in counteracting such pressures. In a mischievous mood, we asked participants to list the most *unhelpful* principles that they have heard articulated. These usually contain just enough validity to undermine our best efforts to reform school practices. Here is their list (without comment):

- Learn how to do it first—understanding can always come later.
- Practice makes perfect, mnemonics and short cuts are helpful.
- Reinforcement/consolidation tasks improve understanding.
- There is a correct way to teach, an optimal sequence to learn.
- Learning must be preceded by instruction.
- Share lesson objectives with students beforehand. Lessons should be in 3-parts.
- Cover the syllabus (at all costs).
- Presentation and neatness are very important.
- There is a right way to solve problems.
- Knowing the answer is important.
- Keep learners busy. Learners go off-task if they talk.
- Don't confuse learners by showing them incorrect methods.
- Use technology wherever possible.

**Table 3** Principles for the effective teaching of mathematics. Final changes or additions made after consultation with participants are shown in italics

| Teaching is more effective when it… | |
|---|---|
| builds on the knowledge learners already have | This means developing formative assessment techniques and adapting our teaching to accommodate individual learning needs. |
| exposes and discusses common misconceptions *and other surprising phenomena* | Learning activities should expose current thinking, create 'tensions' by confronting learners with inconsistencies and surprises, and allow opportunities for resolution through discussion. |
| uses higher-order questions | Questioning is more effective when it promotes explanation, application and synthesis rather than mere recall. |
| makes appropriate use of *whole class interactive teaching, individual work and* cooperative small group work | *Collaborative group work is more effective after learners have been given an opportunity for individual reflection.* Activities are more effective when they encourage critical, constructive discussion, rather than argumentation or uncritical acceptance. Shared goals and group accountability are important. |
| creates connections between topics *both within and beyond mathematics and with the real world* | Learners often find it difficult to generalise and transfer their learning to other topics and contexts. Related concepts (such as division, fraction and ratio) remain unconnected. Effective teachers build bridges between ideas. |
| encourages reasoning rather than 'answer getting' | Often, learners are more concerned with what they have 'done' than with what they have learned. It is better to aim for depth than for superficial 'coverage'. |
| uses rich, collaborative tasks | The tasks we use should be accessible, extendable, encourage decision-making, promote discussion, encourage creativity, encourage 'what if' and 'what if not?' questions. |
| uses *resources, including* technology, in *creative and* appropriate ways | *ICT offers new ways to engage with mathematics. At its best it is dynamic and visual: relationships become more tangible. ICT can provide feedback on actions and enhance interactivity and learner autonomy. Through its connectivity, ICT offers the means to access and share resources and—even more powerfully—the means by which learners can share their ideas within and across classrooms.* |
| *confronts difficulties rather than seeks to avoid or pre-empt them* | *Effective teaching challenges learners and has high expectations of them. It does not seek to 'smooth the path' but creates realistic obstacles to be overcome. Confidence, persistence and learning are not attained through repeating successes, but by struggling with difficulties.* |
| *develops mathematical language through communicative activities* | *Mathematics is a language that enables us to describe and model situations, think logically, frame and sustain arguments and communicate ideas with precision. Learners do not know mathematics until they can 'speak' it. Effective teaching therefore focuses on the communicative aspects of mathematics by developing oral and written mathematical language.* |
| *recognises both what has been learned and also how it has been learned* | *What is to be learned cannot always be stated prior to the learning experience. After a learning event, however, it is important to reflect on the learning that has taken place, making this as explicit and memorable as possible. Effective teachers will also reflect on the ways in which learning has taken place, so that learners develop their own capacity to learn.* |

## Implications for Research

The values, principles and sample lesson activities articulated within the consultation seem to go to the heart of what it means to be mathematical. Most curricula specifications are sterile artefacts that, whatever the aspirations of the 'worthy words' in their introductions, continue to be interpreted by politicians, assessors and teachers in conservative, reductive ways. The descriptive language we use changes, but the reality in classrooms does not.

We need to develop a clearer vision of how the values, principles and content relate and the direct implications this has for the tasks we offer to students. Detailed exemplification is essential and this must be designed in a careful, systematic, research-based way. Here is not the place here to review research methodologies, but it seems clear to me that more serious effort needs to be devoted to *Design Research* approaches to curriculum development. Design research seeks the *transformation* of educational practices in typical classrooms, reducing the credibility gap between educational research and classroom practice through interventionist, iterative, theory-driven studies of designs in action (Burkhardt and Schoenfeld 2003; Kelly 2003; van den Akker et al. 2006). The main research question (in education) is 'How is this design (curriculum specification) interpreted and enacted by its intended audience (typical teachers and students), and how can it be redesigned and supported in ways that more fully realise our values?' Currently we are undertaking such an exercise in order to support the implementation of the Common Core State Standards in the US by the careful design of exemplary lessons (Swan et al. 2012). This is a slow process requiring much more time and funding than educational publishers are usually willing to provide. In engaging in this process, however, we are slowly developing and sharing a *professional vision* (Schoenfeld 2009) for designing learning experiences (not just 'tasks') that are not only engaging, but also take account of the teachers' role in facilitating learning.

In this chapter I have attempted to illustrate the importance of explicitly reconciling our theories, values, principles and curricular aspirations with the design of lessons for real children, and the importance of exemplification. We need exemplary design, not only for teachers and classrooms, but also to communicate our core values to politicians, examination bodies and other key educational stakeholders.

Although I cannot pretend to have done justice to the wonderful range of contributions that others have made within this book, I hope to have drawn out some common themes and provided a provocation for future research.

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