School Mathematics Textbook Design and Development Practices in China

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Abstract In this chapter, we present and discuss school mathematics textbook design and development in China, with a special focus on high school mathematics textbooks. Textbook development in China has its own history. This chapter highlights several design guidelines and common development practices used in selecting, presenting, and organizing content in mathematics textbooks over time. With the recent curriculum reform in China, we also discuss some new developments in designing high school mathematics textbooks. The implication of these Chinese practices in textbook development are then discussed in a broad context.

Keywords China · Curriculum reform · Mathematics textbook · School mathematics · Textbook design · Textbook development

Introduction

It is generally recognized that developing and using textbooks can serve as an important channel for promoting changes in teaching and learning mathematics (e.g., Ball and Cohen 1996; Beagle 1973; Hirsch 2007; Weiss et al. 2002). Although textbooks' effectiveness in improving classroom teaching and learning relates to many factors, including the teachers who use them (e.g., Kilpatrick 2003; National Research Council [NRC] 2004), the textbook quality itself is often an important concern to many teachers and educators (e.g., American Association for the Advancement of Science [AAAS] 2000; Kulm 1999; Trafton et al. 2001). Efforts to develop new, high-quality textbooks have received ever-increasing attention and sup-

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port over time (e.g., Senk and Thompson 2003). In contrast, there are a very limited number of studies available that examine and discuss textbook design and the process of textbook development. In fact, during the 10th International Congress of Mathematics Education (ICME-10) held in Denmark in 2004 (Fan et al. 2008), the development and research of mathematics textbooks was the topic of a Discussion Group for the first time in the history of the International Commission on Mathematical Instruction (ICMI). As textbook development is a process that integrates many different considerations in content and instruction, a better understanding of textbook design and development practices helps enhance the quality of the textbooks being developed. Because many issues related to textbook design and development (e.g., "the relationship between mathematics curriculum standards/syllabi and textbooks" and "possibly good practices and approaches in presenting and organizing content in textbooks") are not restricted to specific regions, in this chapter we focus on the guidelines and practices used in developing mathematics textbooks in China.[1]

China's case becomes especially interesting since textbooks play a very important role in developing effective classroom instruction and teachers' knowledge in Chinese education overall, and especially in mathematics (e.g., Ding et al. 2012; Li and Li 2009; Ma 1999; Wang and Paine 2003). Ma found that Chinese teachers even consider their use and study of textbooks the most important of the factors contributing to their professional growth over the years. Ma's findings suggest that Chinese textbooks have been well accepted and are adhered to by teachers and students for day-to-day classroom instruction, and have been very successful in promoting mathematics teaching and learning in China. The important role of textbooks in mathematics teaching and learning has undoubtedly led to the emphasis placed on developing high-quality textbooks for Chinese education.

Although some studies are now available that reveal various features of mathematics textbooks from China (e.g., Fan et al. 2004; Li 2000, 2007), much remains unknown to outsiders about the design considerations and development practices of mathematics textbooks. Mathematics textbook development in China should provide useful information for curriculum developers and mathematics educators in many other education systems to reflect on their own textbook design and development practices.

The following sections are organized into four parts. In the first part (Section "School Mathematics Textbook Development: Issues and Focuses"), we outline issues and focuses on school mathematics textbook development. Descriptions of some issues in mathematics textbook development in the international context are provided in this part to help outline further discussions of mathematics textbook development in China. The second part (Section "General Characteristics of Mathematics Textbook Development in China Before 2000") provides detailed descriptions of various characteristics of mathematics textbook development in China before the dramatic curriculum reform taking place in 2000. Pre-

[1] If not specified otherwise, China refers to the Chinese mainland in this chapter.

vious practices and approaches commonly used in selecting, presenting and organizing content in mathematics textbooks are addressed and discussed. With recent curriculum reform taking place in China, the third part (Section "Developing Secondary Mathematics Textbooks in the Context of the Mathematics Reform After 2000") focuses on the new development in designing and authoring mathematics textbooks, with an emphasis on secondary mathematics textbooks. In the last part (Section "Concluding Remarks"), we conclude the article with discussions on textbook design and development practices in China, as well as possible directions for research.

School Mathematics Textbook Development: Issues and Focuses

Similar to many other education systems, curriculum is a key component in Chinese education. Because of its centralized education system, China uses nationwide unified curriculum standards (previously called the "teaching and learning syllabus") which provide guidelines for all teaching and learning activities at different grade levels and serve as a direct channel for major education reforms. Mathematics textbooks have been developed in alignment with the unified curriculum standards, and play an important role in guiding day-to-day teaching and learning activities in classrooms across the system. Given the large impact that textbooks wield on daily classroom teaching and learning, textbook quality is predictably important in the eyes of education policy makers, schoolteachers, and students.

A recent effort to examine textbook development in the United States has placed a focus on textbook design principles and development processes (Hirsch 2007). In particular, those textbook development teams, funded by the US National Science Foundation (NSF), were asked to contribute and share explicitly how they designed and developed their textbooks. The book edited by Hirsch (2007) includes contributions from four textbook development teams at the primary school level (Kindergarten through Grade 5), four at the middle school level (Grades 6–8), and seven at the high school level (Grades 9–12). The book's emphasis on design principles and development processes for textbooks at all grade levels in school mathematics is consistent with the focuses of Discussion Group on mathematics textbooks (DG14) during the ICME-10 (Fan et al. 2008). Similar focuses were also taken by Pang (2008) when she discussed mathematics textbook design and implementation in South Korea. Thus, in this chapter, we focus on similar issues in the case of China. The following three aspects are used as a guideline to frame our discussion in this chapter.

(1) The typical process of textbook development.
(2) General considerations in textbook structural design and content selection.
(3) General guidelines for authoring textbooks in terms of content organization and presentation.

General Characteristics of Mathematics Textbook Development in China Before 2000

The Typical Process of Textbook Development

Back in the 1950s, the Chinese Ministry of Education (MOE) founded the People's Education Press (PEP): a specialized organization dedicated to studying, authoring, and publishing textbooks for primary and secondary school education as well as a contact for designing and revising the "National Mathematics Teaching and Learning Syllabus". Since then, the PEP, under the direct administration of the MOE, has taken a leading role in curriculum development in China.

Because the PEP played a major role in textbook development before 2000, some common practices were formed along the way. For instance, authoring textbooks had always followed the process of "research, authoring—review, experimental use—revise, publish—experimental use again, review—revise again, ..." strictly. During this process, it was emphasized that great efforts are needed to prepare the text, review and experimentally use it several times.

One major feature in developing textbooks was to learn from other education systems and consider different opinions. For instance, since 1961 researchers and authors have begun to reflect on previous textbook development experiences and, more recently, conducted international comparative studies on teaching requirements, content, and method and content organization (Wei 1996). In the 1980s a national investigation was also conducted to examine the need for mathematics essential knowledge and techniques based on the economic and social development in China. The investigation involved sixteen vocations including engineering, water and electricity, aerospace, and agriculture. Questionnaires on mathematics knowledge requirements were collected from 692 engineers, technicians, or administrators and more than 300 experts from over 60 higher education institutes. Furthermore, 76 journals were randomly selected from 21 categories; and the mathematics knowledge used in the articles of those journals was categorized. Findings indicated that knowledge other than existing mathematics in curriculum materials should be added; including probability, statistics, calculus, optimization, linear programming, vectors, and analytic geometry.

Structural Design of Textbooks—Combination or Separation

There has always been heated discussion on the structure of mathematics textbooks in China, which mainly involves two questions: (1) Which approach is better; a separated, subject linear organization or an integrated, gradual organization? (2) What may be the overall design guideline, in consideration of these two different approaches?

Traditionally, separated content branches such as "algebra", "plane geometry", "solid geometry" and "analytic geometry" have been adopted in Chinese textbooks.

The guidelines for arranging mathematical content in different branches are as follows:

For algebra: content is organized as developing from number → expression → equation → function, constant to variable, concrete to abstract, simple to complex. "Function" is the core concept throughout the textbook.

For plane geometry: content is arranged as going from relationships between lines, equal or unequal relation → circle → similar triangles → solving triangles → measurement (e.g., circumference and area).

For solid geometry: content is organized as developing from lines and planes → polyhedron, evolving shape. If we take relative positions as a theme, contents are then organized from 'line and line' → 'line and plane' → 'plane and plane'.

To connect plane geometry and solid geometry, it is emphasized to extend the main properties of geometry from plane to solid, and from simple to complex.

For plane analytic geometry: content is arranged as emphasizing its transitional function from elementary mathematics to advanced mathematics by focusing on the integration of number and shape and its comprehensive application. It can be placed in a later part of the high school mathematics curriculum. Content organization can be arranged from Cartesian coordinate → polar coordinate; line → conic section; standard equation → general equation.

Since the late 1990s, all the above content divisions have been integrated into one subject—mathematics. The guidelines for the course structure include: reducing the number of mathematical curriculum branches; modernizing the course content; helping students learn the "Two Basics";[2] and facilitating students' application of mathematics knowledge. Principles for content arrangement include: (a) integrating all mathematical branches into one course; (b) proceeding step by step from simple to complex, shallow to deep, based on students' learning curves and abilities; (c) strengthening the systematic characteristic of textbook content; (d) separating the contents into two parts with different emphases for junior and senior high schools respectively; (e) integrating with related science subjects.

Textbook Content Selection

Changes to mathematics curriculum and textbooks have been made frequently. There are various reasons for the changes in Chinese mathematics curriculum and textbooks. One lasting concern is to reduce students' learning burden and improve teaching quality.

General Guidelines of Textbook Content Selection

Standards of textbook content selection that are used in China include the following considerations:

[2] "Two Basics" refer to basic knowledge and skills in school mathematics that are often identified and emphasized in Chinese mathematics curriculum (e.g., Zhang 2006).

First, is it the basic knowledge of algebra, geometry, statistics and probability?
Second, is it widely used in everyday life, production and technology activities?
Third, is it prerequisite knowledge for students' future study?
Fourth, is it feasible for students to learn?

The above considerations are generally accepted in many other education systems as well. However, the application of the standards can vary due to different natures of specific content topics and different understanding of the standards. The following sub-section will provide several examples of textbook content changes over the years.

Selection and Arrangement of Examples and Exercise Problems

An important principle in selecting and arranging examples and exercise problems is matching the exercise problems with the provided example problems. Memorizing and understanding basic concepts is highlighted with the use of similar problems and therefore the difficulty of study is reduced.

At present, the textbook differentiates exercise problems by difficulty to meet various students' needs. Problems are divided into group A and group B: A is fundamental and B is focused on applying the "two basics", improving students' ability and meeting high-achieving students' extra needs. Chapter review problems (also in A and B groups) and "self tests" are designed to help students study and examine their progress.

Besides textbooks, there are other additional curriculum materials, such as exercise problem books. In those books, challenging worked-out examples and exercise problems are provided for capable students.

General Guidelines and Highlights for Organizing and Presenting Content in Mathematics Textbooks

Logical Order of Mathematics Knowledge

This is a textbook compilation method: textbook content is arranged according to content logic connections in quantity and spatial forms. For example, in junior high algebra textbooks, rational number, real number, algebraic expression, linear equation and quadratic equation are introduced to prepare students for learning algebra transformations and solving equations. Function is then introduced. In senior high school, knowledge of algebraic expressions, equations, and inequalities are developed first and then the following concepts are introduced: exponential functions, logarithmic functions, trigonometric functions, sequences, induction, permutation and combination, probability and statistics, and basic calculus.

The logical order of mathematical concepts is also emphasized. In textbook development, there is an unwritten rule that states: a mathematical concept without a strict definition cannot be used. Every concept must be defined with previously introduced terms. For instance, concepts such as square roots and real numbers are defined before introducing the Pythagorean Theorem.

Introducing Concepts Clearly

Chinese textbook authors take the understanding of mathematical concepts as a necessary condition to learning mathematics well. Without understanding, students cannot apply concepts to solve problems. Therefore, Chinese textbook authors consider "introducing concepts clearly" the most important feature of textbooks. Various approaches have been developed and used, such as starting from examples of previously learned concepts, explaining concepts with real-world examples, or introducing concepts through comparison with related concepts. For instance, the concept of "irrational numbers" is introduced in a Chinese textbook as follows (Secondary School Mathematics Section 2001a, pp. 154–155):

The textbook begins with the review of "rational numbers can be written in forms of finite decimals or circulating decimals" and provides examples.

Second, $\sqrt{2}$, $\sqrt{3}$, $\sqrt[3]{2}$, $-\sqrt{7}$, and π are used to illustrate the existence of infinite, non-repeating decimals.

Third, infinite, non-repeating decimals are defined as irrational numbers.

Fourth, students are asked to think about the following questions; "Are $\sqrt{4}$, $-\sqrt[3]{27}$ irrational numbers? Are all numbers with radical signs irrational numbers?"

Finally, irrational numbers are classified.

In the above process, the definition of an irrational number is introduced through a comparison of irrational numbers with rational numbers by examples. π is used deliberately, as an example to explain that irrational numbers are not always numbers with radical signs. Questions are also raised to help students understand that "numbers with radical signs are not necessarily irrational numbers". This sequence of steps presents competently the idea of introducing concepts clearly.

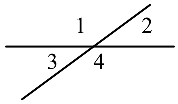
Important Points, Hinge Points, and Difficult Points

In textbook development important points, hinge points, and difficult points of content are highlighted.

Important points refer to mathematical knowledge that plays an important role in the current content level and further studies. It is necessary to devote more time and effort to introducing such knowledge. For example, in middle school algebra, important points include rational number computation, solving linear equations, quadratic equations and systems of equations. In plane geometry, the properties of plane figures are important points.

A hinge point indicates significant mathematics content, which influences students' understanding of other relevant mathematics content knowledge to a great extent. Such content points should receive great instructional attention in order to

Fig. 1 Two intersecting lines



foster students' conceptual understanding in depth. For example, identical transmutation of algebra expressions is a hinge point in middle school algebra. The textbook thus places much emphasis on the inclusion of various types of worked-out examples and a large number of exercise problems with various levels of cognitive demands.

Difficult points refer to mathematical knowledge that is hard for students to understand. Ways to deal with such contents include the use of more real-life examples, emphasis on visualization, relevant knowledge preparation in advance, and step-by-step exercises. For example, proof in plane geometry is a difficult point, which is demonstrated in textbooks as containing the following several steps across several content sections (Secondary School Mathematics Section 2001b):

First, in the section of "intersecting lines and opposite vertical angles", "opposite vertical angles are equal" is derived first from informal reasoning and then through a formal proof.

As shown in Fig. 1, $\angle 2$ and $\angle 1$ are complementary; and $\angle 3$ and $\angle 1$ are complementary. Because both $\angle 2$ and $\angle 3$ are complementary angles of $\angle 1$, therefore $\angle 2$ is equal to $\angle 3$ since complementary angles of the same angle are the same. Similarly, $\angle 1$ is equal to $\angle 4$.

Then a formal proof is introduced:

$\because \angle 2$ and $\angle 1$ are complementary; $\angle 3$ and $\angle 1$ are complementary (Definition of adjacent complementary angles).

$\therefore \angle 2 = \angle 3$ (Complementary angles of the same angle are equal).

By repeatedly demonstrating the above process from reasoning to proof in the content of parallel lines, a norm of proving is developed.

Second, in the section of "proposition, theorem, and proof", concepts such as propositions, true or false propositions, axioms, theorems, and proofs of axioms and theorems are introduced together with examples of writing formal proof process (i.e., Known $\rightarrow$ To be proved $\rightarrow$ Analyze $\rightarrow$ Prove).

Third, in the "triangle" chapter, topics such as "theorem of the sum of a triangle's internal angles" and "congruent triangles" are included and discussed to strengthen geometrical proving.

Basic Mathematical Skills

Emphasis on basic skills is presented with the inclusion of worked-out examples and exercise problems. Problem analysis is emphasized in the worked-out examples in textbooks to show how to analyze and solve them and why certain methods are used. Problem analysis in worked-out examples is followed by problem solution procedures, which show students a problem's solution step by step. Corresponding basic skill exercises are provided after worked-out examples so as to consolidate concepts and methods. For instance, in the section of "Using the method of substitution to solve a binary linear system of equations" (Secondary School Mathematics Section 2001a), worked-out examples and exercise problems are arranged as below:

Example 1 Solve the system of equations

$$\begin{cases} y = 1 - x, & (1) \\ 3x + 2y = 5. & (2) \end{cases}$$

Problem analysis: y is equal to $1 - x$ in Eq. (1); therefore, y in Eq. (2) can be substituted by $1 - x$, which transforms Eq. (2) into a linear equation.

Example 2 Solve the system of equations

$$\begin{cases} 2x + 5y = -12, & (1) \\ x + 3y = 8. & (2) \end{cases}$$

Problem analysis: consider substituting one unknown with an algebraic expression containing the other unknown. In Eq. (2), the coefficient of x is 1, therefore, Eq. (2) can be transformed by denoting x with an algebraic expression of y. Then such a substitution can be put into the Eq. (1).

The above examples introduced the method of substitution. The underlying basic thinking of this method is by means of "substitution" to eliminate one variable. The following procedures are usually used,

(1) Choose one equation containing one unknown with a simpler coefficient from the system of equations and substitute the unknown, for instance, y, with an algebraic expression containing x, that is, the form of $y = ax + b$;
(2) Substitute y for $y = ax + b$ in the other equation and get a linear equation of x;
(3) Solve this linear equation and get the value of x;
(4) Substitute x in $y = ax + b$ with the value of x and then get the value of y to obtain the solution of the system of equations.

Example 3, solving the system of equations with more complicated coefficients, is then provided.

Relevant exercise problems are provided and they can be divided into the following four types:

Type 1, transform $2x + y = 3$, $5x - 2y + 12 = 0$ into the form of denoting y with an algebraic expression that contains x;

Type 2, solve the system of equations;

Type 3, solve word problems that can be transformed into systems of equations;

Type 4, solve non-traditional problems. For example, the solutions to $\begin{cases} ax+by=8, \\ ax-by=2 \end{cases}$ are $x = 5$, $y = 3$. What are the values of a and b?

In summary, worked-out examples are first provided with different coefficients that are arranged from easy to difficult. Exercise problems are provided in a large quantity and corresponding to the worked-out examples.

Emphasis on Improving Students' Mathematical Ability

Chinese textbook authors pay close attention to improving students' mathematical abilities, and focus on computation, logic and reasoning, and spatial visualization. Textbook authors believe that students' abilities are developed together with "Two Basics" training and problem solving. In the process of problem solving, students' abilities of observation, comparison, analysis, synthesis, abstraction and generalization and their thinking habits of induction, deduction and analogy are developed through analyzing quantitative relationships in problems and reasoning on various solution methods. In addition, connections between different concepts, for example, solving geometric problems with algebraic methods, or investigating function graphs with geometric knowledge can develop students' mathematical abilities as well. Thus, exercise problems are carefully selected and arranged in textbooks.

System-unified Common Requirements with Elective Contents

With the use of the national teaching and learning syllabus and university entrance examinations in China, textbooks mainly focus on system-unified common requirements, though they are flexible to some extent. Less flexibility and more common requirements are presented in textbooks for middle school than those for high school. The elective contents in middle school textbooks include:

(1) "Reading" for broadening students' knowledge, such as "decomposing quadric trinomial with the method of making square", "complex fraction" and "why $\sqrt{2}$ is not a rational number?"

(2) "Thinking about it" to develop students' thinking and improve their abilities to analyze and solve problems, through enhancing knowledge connections.

(3) "Trying it" to improve students' abilities to handle practical tasks and interests in geometric learning. For example, on a piece of paper, if we draw a line l, and randomly take a point P on it and a point Q not on it. Through folding the paper, we get a perpendicular line l_1 of line l so that line l_1 should (a) pass through point P; or (b) pass through point Q. How many folding lines can we get?

Furthermore, the second group of exercise problems (i.e., group B) is included to strengthen the applications of "Two Basics", improve students' abilities, and meet the needs of high-achieving students.

Summary of Textbook Design and Development Practices Before 2000

Chinese primary and secondary curriculum has adopted a national unified model. Chinese school textbooks were also nationally unified for a long period of time. As a major resource for teachers and students, textbooks played a significant role in Chinese primary and secondary teaching and learning. Consequently, the Chinese government emphasized textbook development through organizing the specialized educational press and convening experts to develop textbooks. Much experience has been accumulated, especially in the following aspects:

(1) It is emphasized that textbooks need to embody mathematics education objectives. That is, "Two Basics", intellectual and ability, and ideological education should all be treated as significant and important, and they should supplement each other.

(2) When developing and selecting instructional objectives and textbook contents, social needs (after graduation, students have to seek jobs or pursue further education) and the intellectual development of students should be the two main considerations. The selected contents must be fundamental, useful for production, science, and technology, and connected with higher education. The content requirements should have a certain flexibility to accommodate individual differences among the students.

(3) Instructional objectives and textbook contents should be updated to align with social and knowledge development, but priority ought to be given to strengthening "Two Basics". Reform can not be completed in one day; instead, textbooks should be relatively consistent during a certain period. It would benefit teachers to master textbooks and use them to improve teaching quality. It is better to "revise" or "supplement" textbooks rather than completely change everything in the textbooks.

(4) The organization of textbook content should consider the connections and relations between mathematics knowledge (e.g., connection and difference between numbers and shapes), and also take into account cognitive and intellectual development of students. With regard to the organization of textbook content, linear curriculum and spiral curriculum content organizations have their own advantages and disadvantages. Linear organization of textbook content avoids repetition, but some content knowledge are not easily obtained by students; spiral organization of textbook content fits students' cognitive development, but may cause unnecessary repetitions. Consequently, linear organization should be taken as the main approach with some difficult contents being arranged spirally.

(5) Textbook authors should pay close attention to the relationship between learning and the application of mathematics knowledge, as well as the implementation of the principle of learning for application.

Developing Secondary Mathematics Curriculum Textbooks in the Context of the Mathematics Reform After 2000

The Need for Developing New Textbooks to Meet Mathematics Curriculum Reform Requirements

At the beginning of the 21st century, China embarked on a new round of basic education curriculum reform. In 1999, the Ministry of Education began to design the new basic education curriculum for the 21st century. School mathematics in China has experienced dramatic changes, especially since 2001. In 2001, the new Curriculum Standards for the nine-year compulsory education were formally established for experimentation. The ongoing "standards-based education reform" specifies the mathematics knowledge and ability requirements for students, advocates the advancement of students' conceptual understanding, basic skills, problem solving simultaneously and the improvement of teaching quality.

Because the previous mathematics curriculum focused on knowledge acquisition, textbooks failed to meet the needs of society's development. Some contents in previous textbooks were "complicated, difficult, insignificant and outdated" with few options available for students. The curriculum implementation lacked flexibility and opportunities for self-directed learning. Because too much attention was paid to the drilling of "Two Basics", students were restricted in the development of creativity and critical thinking. It is advocated that new mathematics textbooks should be fundamental, diversified and selective. The textbooks should help arouse students' interest in learning mathematics, help students to study mathematics actively, develop students' potential in creativity through learning basic knowledge, improve students' mathematical thinking when trying to understand mathematics, and raise students' awareness of applying mathematics knowledge in everyday lives. The new expectations placed on textbooks have inevitably called for changes in textbooks.

New Textbook Development

Purposes

The primary aims of the reform include changes to curriculum systems, structure, and content in order to construct a new basic education curriculum system for teaching and learning development (Liu and Li 2010). There are specific tasks textbook development should include. First, textbook reforms should be oriented towards the exploration of knowledge and student development, based on their prior knowledge and creative teaching. Second, textbook content should meet the requirements of the curriculum standards, embody physical and cognitive development of primary and secondary school students, and reflect social, political, economic and technological needs. Third, textbook contents should be presented in various ways to facilitate students' active engagement in activities such as observation, experimentation, survey, and communication. Fourth, the adoption and use of diversified curriculum materials for primary and secondary schools should be advocated. Education institutions and publishing companies are encouraged to develop textbooks that are aligned with the national curriculum standards.

Diversification of curriculum materials has been achieved by developing new reform-oriented school mathematics textbooks. In particular, nine series of middle school mathematics textbooks and six series of high school mathematics textbooks have been developed with approval from the education administration in China. With the brief history of the development of new textbooks in the context of mathematics curriculum reform, it is difficult to summarize and discuss specific design principles and practices that may be common to different textbook series. To illustrate the diversity of new textbooks, we describe some features of selected reform-oriented high school mathematics textbooks in the following sub-sections.

Features of Selected Reform-Oriented High School Mathematics Textbooks

People's Education Press's High School Mathematics Textbooks (Version A)

A team effort to develop a series of high school mathematics textbooks was carried out by the People's Education Press. The textbook development in this project basically follows the process of "literature review–theoretical framework construction–textbook development–experimental use of textbooks–results and reflection–textbook revision". Based on literature review and relevant researches in this project, the team expects to explore the design and development of mathematics textbooks that can help students learn mathematics in an active way, integrating teacher-guided and student-centered teaching and learning. Specific instructional strategies and methods are designed and integrated in mathematics textbooks to help actively engage students in the classroom. The team develops high school mathematics textbooks, and examines their scientific accuracy, rationality and effectiveness with experimental use. Through analyzing the data collected from the experiments and summarizing feedbacks from teachers and students, the results are used as guidelines for textbook revisions.

Several aspects have been emphasized in developing new textbooks. They include the following four aspects. Some specific examples are also provided to illustrate the changes.

(1) Placing more emphases on context, mathematical thinking, and application.

Introducing context, mathematical thinking and application are intended to integrate mathematical logic and students' thinking. Students' thinking and cognitive development are emphasized as well as the development logic of mathematical knowledge. In this way, mathematics content is not only organized by algorithms, but also by students' cognitive development.

Example: changes in introducing the concept of function.

In previous textbooks, the concept of "function" was introduced through the sequence of "set and corresponding $\to$ mapping $\to$ function". Mathematical logic was highlighted such as "mapping is a special case of corresponding", and "function is a special case of mapping". Consequently, students felt that the concept of function was abstract and disconnected from their lives.

In new textbooks, "function" is introduced through "real-life examples" with an emphasis on "function is a mathematical model descriptive of the rules of change in the real world". By going through the process from "analyzing features of real-life examples" to "summarizing common features to understand the concept of function", students learn to apply the concept of function into analyzing and solving problems.

(2) Problematizing mathematics.

Problematizing mathematics in textbooks is intended to improve students' creative spirit and ability in knowledge applications. In the new mathematics textbooks, content introduction is changed from the simple form of 'given' to include some new sections, such as "observation", "think about it", and "exploration" that are set up to pose questions in order to facilitate students' thinking and exploration. During that process, students experience the joy of discovery and creativity in mathematics.

(3) Emphasizing the guidance for developing mathematical reasoning.

The new mathematics textbook authors tried to change the approach from "what is it?" without providing explanation of "why is it?" in the previous textbooks to "guide students' mathematical reasoning".

Example: changes in introducing "the basic properties of inequality".

In previous textbooks, the content was introduced through the following sequence: "stating eight basic properties of inequality $\rightarrow$ proving the properties of inequality $\rightarrow$ working through examples $\rightarrow$ doing exercises".

In new textbooks, an introduction is presented as an advanced organizer: two real numbers, a and b, can have one of the following relationships: $a < b$, $a = b$ or $a > b$. These relationships are represented as: $a < b \Leftrightarrow a - b < 0$; $a = b \Leftrightarrow a - b = 0$; $a > b \Leftrightarrow a - b > 0$. These equivalent forms are called basic facts about the magnitude of real numbers.

Inequality and equality are basic quantity relationships. Equality is described with an equal sign and an unequal sign for inequality. To solve problems involving equality, we need to use the basic properties of equality. Similarly, we need to use the properties of inequality to solve problems involving inequality. Because both "equal to" and "unequal to" refer to relationships of numbers and expressions, we can explore inequality properties with an approach similar to those used in discussing equality properties.

Question #1: can you recall the basic properties of equality?

Question #2: what is the approach used in discussing equality properties? (Hint: "invariance" in operations.)

Question #3: based on the discussion about the basic properties of equality, can you guess the basic properties of inequality?

Question #4: the proof of the basic properties of equality is mainly based on "basic facts" (Note: this permeates axiom reasoning). Can you give the proof based on the "basic facts"?

(4) Emphasizing connections among different content knowledge.

With more emphasis on connections among different content knowledge, students are exposed to various approaches to obtaining conceptual understanding.

Example: changes in presenting and organizing the content of vector.

In previous textbooks, it was directly stated that mathematical operations such as addition and subtraction can be performed with vectors. The addition of vectors satisfies the communicative law and the associative law, etc.

In new textbooks, introduction is first provided as numbers can be operated. In light of the physical features of vector and number operations, the concept of vector operation is introduced.

Then, derived from the concept of displacement in physics and the experiment of the composition of forces, the "addition of vectors" is introduced. A follow-up question is raised to encourage students to think more about it: "Number operations and operation laws are closely connected since operation laws simplify operations and addition satisfies the commutative and associative laws. Are there similar operation laws for vector addition?" Students can then be guided to derive their conclusions.

Hunan Education Press Version of High School Mathematics Textbooks

Features of the Hunan Education Press version of high school mathematics textbooks include: focusing on inter-relationship among different mathematics contents and stressing underlying thinking and methods of various mathematics contents. For instance, other textbooks often lack a common theme for connecting contents such as trigonometry, analytic geometry, and complex numbers. This textbook series treats vectors as a unifying theme. Focusing on the concept of vector, the textbook introduces various contents such as trigonometry, analytic geometry, and complex numbers coherently.

This textbook series also gives prominence to mathematical methods rather than simply deleting contents to reduce students' burden. Newly added contents such as vectors are helpful for students to learn other contents and solve problems. Learning vectors well is useful for plan geometry, analytic geometry, solid geometry, trigonometry, complex numbers and even high school physics. In this way, although students learn new contents, their burden of learning other contents is actually reduced. Sufficient exercise problems are also included in the textbook to further facilitate students' learning of the mathematical methods.

The textbook series stresses the importance of geometry and the inter-relationship between numbers and geometric shapes. Specifically, the textbook emphasizes the importance of geometrical visualization in the introduction and explanation of mathematical reasoning and algebraic operations. For instance, complex numbers are generally introduced as $i^2 = -1$ from the algebraic perspective in mathematics textbooks. However, the Hunan Education Press version textbook introduces complex numbers through geometric transformations. Therefore, students come to know that explorations of geometric transformations can lead to complex numbers. Such an introduction relates numbers and geometry, embodies mathematical thinking and makes the content easier for students to understand.

The textbook series introduces mathematical knowledge through questioning students. In attempting to solve the problems, students learn concepts and algorithms and develop theoretical awareness. The mathematics experiments included in the textbooks involve students in activities. While thinking about and doing mathematics, students realize the needs of mathematical concepts, experience failure or success in exploration and discovery, and trace the development of mathematical knowledge. In this way, students explore more mathematical knowledge by themselves than in being led by teachers.

This textbook series emphasizes the expression of mathematical concepts. Based on practice and research in mathematics education, this series presents rigorous mathematical concepts with easy access. For example, it provides the elementary expression of fundamental theorems of calculus and relatively rigorous proofs. In this way, students can understand the essence of the significant outcome in the history of mathematics, even if they do not have the opportunity to learn more calculus. It is also helpful for students who are going to learn more calculus in their further studies. This series does not decrease the difficulty for easiness nor abandon rigor of mathematics. It does not increase the difficulty for the sake of rigor of mathematics, either.

The textbook series also gives prominence to in-depth thinking on mathematical problems and discovering the fun in mathematics. For example, it raises the possibility of an operation in which 0 is a divisor by introducing a new number. Such a question may promote critical thinking among students about the rules of introducing new numbers in the system of numbers, and thus elevate their mathematical habit of mind.

Last, the textbook series includes and arranges mathematical experiments for high school students. Besides mathematics experiments in the text, the textbook authors also include experiments in the exercises following the text. In this way, students are provided with opportunities to understand the text more deeply through observation, experimentation and induction. This also encourages students to seek evidence to verify the new mathematics knowledge in the text. For example, after introducing the concept of definite integrals, the textbook arranges an experiment to calculate the area of a circle using a computer. Other experiments include extended exercises integrating mathematics and physics. For instance, students are required to draw the reflection of parallel rays through spherical mirrors to observe the light gathering power of spherical mirrors. Another instance is to observe sine curves using spring vibration, based on Hooke's law.

Beijing Normal University Press's Version of High School Mathematics Textbooks

There are several important features for this version of textbooks. They are highlighted as follows.

First, function is taken and used as the core concept connecting almost all content areas in high school mathematics, including function and equations, inequalities, linear programming, algorithms, derivatives and their applications, and random variables in probability and statistics. It is important to understand these content areas from the perspective of functions. At the same time, learning these contents deepen students' understanding of functions.

Second, geometry is used to develop students' abilities of observation and reasoning. In this series, geometry integrates logical thinking and visual imagination using texts, pictures, and graphs. Two parts of geometry content are designed as follows: (1) the content of geometric shapes is designed and arranged as to developing students' visual imagination. (2) logical thinking is integrated into geometric content.

Third, operations are fundamental to mathematics. In this series, operations and methods of operation are significant and indispensable. The textbook introduces two components of operations: first, objects of operation; second, rules of operation. Objects of operation include numbers, variables or algebraic expressions, exponents, logarithms, trigonometric functions, and vectors. Rules of operation include associative law, commutative law, distributive law and "$a + (-a) = 0$". The textbook focuses on operations of various contents, including exponential operation, algorithmic operation, trigonometric function operation, vector operation (including plane and space vectors), complex number operation and derivative vectors.

Fourth, in terms of statistics and probability, the textbook series focuses on the following content areas with students' active involvement: the ability to deal with data, the process of gathering and analyzing data, case studies in statistics, inductive thinking and random sampling in statistics.

Summary of New Textbook Development After 2000

Grounded in Chinese mathematics education traditions, the reform-oriented high school mathematics textbooks discussed above show distinctive features. These newly developed textbook series contain not only specific considerations in content selection and mathematical treatment, but also aspects of students' learning and cognitive development. In a way, these new textbooks embody expectations for teaching and learning that are advocated in the mathematics curriculum reform. In contrast to previous textbook development in China, the involvement of many educators and researchers promotes the development of multiple textbooks available for students and teachers. However, there are no specific studies that have been reported to empirically document possible advantages and disadvantages of these new textbooks.

The most widely adopted and used high school mathematics textbook series is developed by People's Education Press (Version A). Based on the marketing data collected by PEP, more than 60 % of high school students in China are using mathematics textbooks published by PEP (Version A). Yet, this particular series lacks solid empirical evidence to document its effectiveness, though some data collection has been conducted in recent years.

In fact, the development of many new textbooks in China was a bit rushed (Li 2008). With the formal release of the new curriculum standards in 2001, some textbook developers took less than one year's time to develop and publish their new textbooks. The quality of the new textbooks has become a concern. The 'rushed' development of textbooks further suggests the importance of quality control, which can and should be supplemented with empirical data collected from textbooks' experimental uses.

The Integration and Use of Technology in Mathematics Textbooks and Instruction

The above discussion does not provide specific information about the use of technology in mathematics textbooks and instruction. The integration and use of technology did not receive much attention before 2000, but the situation in China changed dramatically after 2000. The integration and use of technology in mathematics instruction in China can be outlined as going through three different stages over the years.

The first stage can be characterized with the use of electric equipment in classroom instruction, such as slide projectors, projectors, and TV sets. The use of such electric equipment helps the presentation of mathematics content, but not necessarily changes the way of how students learn. There are also very limited impacts on students' learning content and their thinking.

The second stage can be characterized as computer-assisted teaching and learning. With the advancement of technology, more and more teachers have changed their views about the use of technology in classroom instruction. Teachers also become used to computers, which provides the base for the follow-up stage of using information technology.

The third stage can be designated as the integration of information technology and mathematics instruction. In addition to scientific calculators and computers, many other technology equipment (e.g., graphing calculators, internet) is also used. Many more software and applications become readily available for teaching and learning almost all contents in high school mathematics. For example, multi-media teaching courseware is made available for selected chapters of People's Education Press version textbook. Teachers are able to use the courseware to teach high school mathematics in interactive ways. In addition, the software of "Scilab" developed collaboratively by China and France is also used as a platform for mathematics teaching and learning. As free software, "Scilab" can be downloaded by teachers and schools from the official website. The publisher plans to develop many more textbook-related courseware that will be free to all teachers and students to download and use. Figure 2 shows the textbook introduction of information technology use for collecting water temperatures and building a function model.

Concluding Remarks

The above discussions and summaries outline a number of guidelines and practices used in developing mathematics textbooks in China. Because textbook development

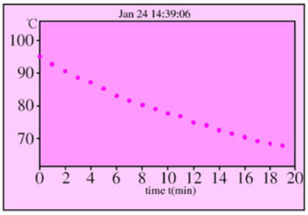
收集数据并建立函数模型

我们周围的绝大多数变化现象，是难以根据已知理论直接建立函数模型的．但只要能收集到变化过程中变量的数据，利用信息技术就可以建立起能大致反映变化规律的函数模型.

下面就向大家介绍如何用计算机、数据采集器、温度传感器等信息技术工具收集水温变化数据，并建立温度与时间的函数模型.

（1）连接计算机、数据采集器、温度传感器，并在数据采集器上，将要采集的温度个数和每两个温度的间隔时间设置好，然后将温度传感器放入热水杯中.

（2）将计算机和数据采集器中的运行功能打开．这时，计算机和数据采集器上就会同时显示出温度随时间的变化情况（图 1(1)，(2)).



(1)

	Time[s]	Temp[-25 – 110]
1	0.000	94.7
2	10.000	94.2
3	20.000	94.5
4	30.000	93.7
5	40.000	93.4
6	50.000	92.6
7	60.000	92.1
8	70.000	91.8
9	80.000	91.3
10	90.000	91.1

(2)

图 1

（3）通过对整个温度变化过程的观察，根据图 1(1)，在计算机中选择一个能大致反映其变化规律的函数模型，如 $y=ae^{bt}+c$，计算机便立即画出这个函数的图象并求出其解析式（图 2).

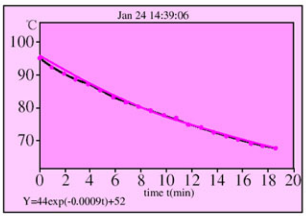


图 2

以上建立函数模型的过程简单、方便，形象直观，是传统手段难以比拟的．只要我们掌握好所学的函数模型，利用信息技术，就可以探索复杂现象的变化规律.

Fig. 2 Collecting water temperatures and building a function model (People's Education Press textbook)

was viewed more of a profession before 2000, it is clear that Chinese textbook developers have accumulated many practical experiences in textbook development. These accumulated experiences in textbook development should provide a rich source of information for textbook writers and mathematics educators in other education systems to reflect on their own practices.

At the same time, textbook development itself has undergone many changes in China. Although various changes to textbooks happen all the time, the recent curriculum reform in China has brought the biggest change in textbook development. Before 2000, textbooks were mainly focused on mathematical knowledge itself. Specifically, presentation and organization of mathematics content in textbooks focused on knowledge accuracy and connections. Because textbooks have an important role in guiding day-to-day classroom instruction in China (e.g., Li et al. 2009a), students' high achievement and teachers' textbook use with fidelity suggest that Chinese textbooks were effectively implemented. Although textbooks were developed by a group of professionals housed in a large education press, textbook development was mainly an experience-based practice. It becomes important to elevate accumulated experience in Chinese textbook development for theoretical awareness.

After 2000, however, more attention has been given to students and their needs, which is reflected in textbook development with both content adjustment and considerations for teaching and learning in classrooms. Efforts are needed to focus on meeting the needs of students with diverse socioeconomic background, especially those coming from rural areas.

In contrast to practices before 2000, textbook development and publication is now open to all publishers. It is not only a few full-time professionals housed in a large education press, but experienced teachers and university professors, that become the main working force of textbook development in China. The situation therefore becomes more and more similar to other education systems, such as the United States (Hirsch 2007). As diversity can certainly promote creativity and competition in textbook development, changes in textbook developers would eventually benefit the improvement of textbook quality and thus mathematics teaching and learning in classrooms. Yet, textbook development still remains largely an experience-based practice in China. The successful changes in curriculum reforms in China may require specific strategies and considerations (e.g., Huang 2004). With recent growing interest in the quality and process of textbook development, making textbook development a scientific endeavor becomes a significant challenge to educators and researchers in many education systems, including China.

Knowing and understanding curriculum and textbook development cannot be separated from the social and political contexts of a system (Apple 2004). The situation in China is no exception. Although we intended to focus and frame our discussions on several aspects of textbook development more from an academic perspective (see Section "School Mathematics Textbook Development: Issues and Focuses"), the history of textbook development in China suggests that changes in mathematics textbooks and textbook development are closely related to social and political changes in the system over the years (Li et al. 2009b). Our inclusion and discussion of textbook development in China over the years helps highlight the importance of attending to the contextual changes in the educational system. By doing so, we encourage mathematics educators and curriculum developers in other education systems to reflect on their own practices.

Finally, it is important to point out that textbook development itself is not a result by itself, but a process that aims to produce high-quality textbooks. Textbook development closely relates to textbook studies that examine the quality of textbooks and their impact on teaching and learning mathematics. Efforts to improve textbook development can be informed and facilitated by the ever-growing research interests in examining and documenting teachers' use of textbooks and use of textbooks in students' learning (e.g., Ding et al. 2012; Li et al. 2009a; Stein et al. 2007). Further research is needed on textbook development, the connection of textbook development and textbook use, and the impact of textbook development on teaching and learning mathematics.

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