Forging New Opportunities for Problem Solving in Australian Mathematics Classrooms through the First National Mathematics Curriculum

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Abstract Although the Federal government in Australia has tried on previous occasions to exert a greater influence on curriculum development, curriculum development was the responsibility of each of the eight states and territories until quite recently. The new Labour Government in 2007 has employed increased central control and accountability measures, with national testing in grades 3, 5, 7 and 9 from 2008, publication of school results on a MySchool website, and the development of the first national curriculum in English, Mathematics, Science and History. States are still responsible for implementation, but the new funding model means they must comply with national curriculum implementation up to grade 10. Developing the first national curriculum for mathematics has been a challenge, but a plan of mathematics learning for each grade level organised into three content strands has now been developed. In addition, four proficiency (or process) strands describe the actions associated with doing mathematics. Since problem solving has been a key component of previous curriculum documents and there is evidence of limited use of complex problem solving in some Australian mathematics classrooms, the representation of problem solving in curriculum documents is examined in this chapter to explore whether the new national curriculum for Australia forges new opportunities for teachers and students.

Keywords National curriculum · Historical perspectives · Problem solving · Proficiencies · Teacher interpretation · Authentic problems

How difficult can it be to develop a national mathematics curriculum in a country with fewer than 23 million people? The Australian experience over the past 50 years exposes a rocky road to success. However, this has not deterred the Federal government who, in 2008, began yet another attempt to develop the first national curriculum in English, Mathematics, Science and History—designed to improve quality, equity and accessibility (McGaw 2010). After much debate and consternation, the

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first national curriculum for mathematics was endorsed by state and territory Ministers of Education in December 2010. The development of this curriculum required navigating the obstacles of divided responsibilities for education between the state and Federal governments, as well as negotiating many stakeholder concerns.

This chapter presents a brief historical account of the development of the first national mathematics curriculum in Australia, and outlines the challenges presented at various phases of the curriculum development process. For clarification, I use the term 'curriculum' to represent the official policies or plans of mathematics content to be taught in schools—also referred to as "the intended curriculum" (Robitaille et al. 1996) or the "specific set of instructional materials that order content" (Clements 2007, p. 36).

Because of its importance in mathematics teaching and learning (Schoenfeld 2007) and because there is evidence of limited use of complex problem solving in Australian classrooms (Hollingsworth et al. 2003), this chapter also examines the ways problem solving has been described and presented to teachers in previous curriculum documents and reports research identifying teachers' interpretation of the curriculum advice about teaching problem solving. Finally, the chapter considers whether the first national curriculum for mathematics provides new problem-solving opportunities for Australian students and teachers, particularly since "problem solving is one of the most fundamental goals of teaching mathematics, but also one of the most elusive" (Stacey 2005, p. 341).

The Australian Context

Before the national curriculum was developed, each of the eight Australian states and territories used state-developed curriculum documents. Some were broad frameworks allowing for school-based curriculum development (e.g., South Australia) while others were more detailed and highly prescriptive (e.g., New South Wales [NSW]). Usually separate curriculum documents were developed for the elementary grades (the first six or seven years of schooling), the secondary grades (in most states from grades 7 to 10), and the senior secondary grades (11 and 12). There is no middle school structure in Australia, although some independent schools that cater to students from the first years of schooling to grade 12 have used state-based curriculum to design alternative experiences for students in the middle years (typically grades 5 to 8).

Usually state-based curriculum documents in Australia present the school mathematics curriculum as lists of topics or 'content' and a set of 'processes'. Content includes the fundamental ideas of mathematics, historically grouped into such topics as number, algebra, measurement, geometry, and chance and data. While processes include the actions associated with using and applying mathematics to solve a range of problem types including applications of mathematics in authentic contexts and other non-routine problems.

Problem solving is recognised as an important life skill involving a range of processes including analysing, interpreting, reasoning, predicting, evaluating, and reflecting. It is either an overarching goal or a fundamental component of the school mathematics curriculum in many countries (Stacey 2005). One of the challenges in curriculum development is to present the mathematics curriculum in a way which encourages teachers to embrace reforms or new approaches to teaching and learning. One new approach was the introduction of problem solving into curriculum documents in Australia in the late Eighties but there has been limited evidence of complex problem-solving opportunities in elementary classrooms (e.g., Anderson et al. 2004) or in secondary classrooms (e.g., Hollingsworth et al. 2003). Because of the diversity of curriculum documents in the Australian context, in this chapter I examine the evolution of problem solving in one state (New South Wales [NSW]) context and compare this with the new Australian curriculum approach.

Historical Perspective of Australian Curriculum Development

I present a brief historical perspective to set the context for the development of the first national Australian curriculum since as Kennedy (2005, p. 1) notes, "the school curriculum is tightly bounded by the social, political and economic contexts in which it is located". This overview is necessarily brief and seeks to identify key drivers of curriculum change, particularly those impacting mathematics.[1]

In Australia, the constitutional responsibility for curriculum resides with the state and territory governments who have "jealously guarded their curriculum sovereignty, overtly or passively resisting attempts to engineer national approaches" (Reid 2005, p. 39). However, curriculum has become a "state and Commonwealth [Federal] political football" (Yates et al. 2011b, p. 4) with the Federal government making several unsuccessful attempts at implementing a national curriculum. Reid (2005) argues the lack of success goes beyond the political agenda to the lack of an adequate rationale for a national curriculum, a failure to develop a rigorous theoretical base for the curriculum, and a failure to consider key aspects of managing curriculum change. Others have argued there has also been a lack of consultation with key stakeholders (e.g., Ellerton and Clements 1994). Reid (2005) outlines four phases in the move towards a national curriculum, particularly after 1963 when the Federal government in Australia began to fund aspects of school education.

In the first phase (1968–1988) the Federal government sought to influence the state-based curriculum using 'indirect' approaches by funding projects for the production of resources for teachers and students. During this phase, the *Mathematics Curriculum and Teaching Program* [MCTP] (Lovitt and Clarke 1988) was developed to address concerns about the teaching of mathematics in Australia and in particular, to address issues about students' attitudes to mathematics

[1]For a more detailed historical account of curriculum development in Australia, I recommend Yates et al. (2011a) and Marsh (2010). Both volumes describe case studies of curriculum change in particular states and territories as well as the prevailing political agendas leading to the rejection of earlier attempts at national curriculum development.

and the diversity of students' needs, as well as shallow teaching and narrow assessment practices (Lovitt and Clarke 2011). To support teacher professional development, the program identified and captured good practice in a collection of exemplary lessons—these resources have been sold internationally and are now available online through the Maths300 website (http://www.maths300.esa.edu.au/). While widely recognised as an outstanding resource, it is debatable how much influence this resource has had on addressing the concerns and issues mentioned above, particularly given these same issues continue to be raised (see for example the *AAMT Position on National Curriculum in Mathematics* at http://www.aamt.edu.au/Publications-and-statements/Position-statements/National-Curriculum).

The second phase of national curriculum development (1988–1993) saw the design of *Statements* and *Profiles* in each of eight key learning areas including mathematics. A detailed historical account of the failure of the Australian Education Council [AEC] to develop and then endorse the national curriculum is contained in *The National Curriculum Debacle* (Ellerton and Clements 1994). Through reference to meeting minutes, letters, and personal accounts of events over this period, Ellerton and Clements describe the key issues associated with the failure of this enterprise as:

- the lack of a strong and agreed upon theoretical base, in particular the use of an outcomes-based education approach which the authors align with behaviourist principles suggesting this was "totally at odds with the directions and findings of mathematics education research over the past two decades" (p. 7);
- a lack of consultation with key stakeholders in the curriculum development process, in particular lack of involvement with mathematicians and mathematics educators; and
- being guided by the national curriculum approach in the United Kingdom, which was reported in 1994 as 'disastrous'.

While Ellerton and Clements argue that the approach was strongly influenced by developments in the United Kingdom [UK] at that time, there was a significant difference between the two countries regarding curriculum control. The UK Government had "the constitutional authority to impose a national curriculum" whereas the Australian Government did not and had to "negotiate and persuade" (Piper 1989, p. 22). According to Ellerton and Clements, if the AEC had more closely considered the national *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM] 1989) developed in the United States of America, the approach would have been far more acceptable to those who were so strongly opposed to the enterprise.

The first stage of development of the *Statement* and *Profiles* began with mathematics and involved a mapping of state and territory curriculum documents to identify similarities and differences. At the time there were

> ... large differences in the ways in which school mathematics was organised in the different states and territories. For example, four systems (Victoria, South Australia, Tasmania, and the Australian Capitol Territory) did not mandate any aspect of the mathematics curriculum, and in these systems, centrally issued guidelines served as the basis for school-based

> curriculum development for primary school mathematics. In New South Wales, Queensland and Western Australia, however, the aims and content of primary school mathematics were centrally specified, mandatory, and were backed up by centrally specified notes and suggestions. (Ellerton and Clements 1994, p. 52)

While this attempt at developing a national curriculum failed, the mathematics *Statements* and *Profiles* were used as a framework to guide curriculum development in some states. Identified legacies from these documents include an increased emphasis on mental computation, an increased focus on probability and statistics, and the articulation of a separate strand of 'processes' in most state and territory curriculum documents (Morony 2011). These processes tended to be included in a 'working mathematically' strand (e.g., Board of Studies NSW 2003), which also included reference to problem solving. Further elaboration of the approaches taken to embed problem solving into curriculum documents in NSW is examined later in this chapter.

The third phase (1993–2003) witnessed a return to indirect Federal government involvement where significant funds were devolved to schools to support professional development in the move towards a national curriculum, similar to the first phase. Through the *Australian Government Quality Teaching Program* [AGQTP], projects were funded to focus on literacy, numeracy, mathematics, science and/or technology. This phase led to many school- and system-based projects focused on numeracy and/or mathematics throughout Australia (Vincent 2004).

The fourth phase began in 2003 with the Federal Minister for Education suggesting the need for a common school starting age, common assessments for grade 12, and the need for a common curriculum in all states and territories. Another mapping exercise was undertaken to identify overlap and difference between state and territory curriculum documents, leading to the development of four *Statements of Learning* in English, Mathematics, Science and Civics. These 'statements' introduced the notion of 'national curriculum consistency' guiding national testing in literacy and numeracy from 2008 in grades 3, 5, 7 and 9.

The rationale for one curriculum for *all* Australian students was to improve quality, equity and accessibility. The rhetoric suggested "a national curriculum would play a key role in delivering quality education" and that it would be "world class" (ACARA 2010a). Further, one curriculum would mean:

- a united focus on how student learning can be improved to achieve national goals;
- greater attention devoted to equipping students with skills, knowledge and capabilities necessary to enable them to effectively engage with and prosper in society;
- more efficient development of high quality resources; and
- greater consistency for mobile student and teacher populations. (ACARA 2010a)

The curriculum development process began in 2008 with four academics writing framing papers in English, Mathematics, Science and History—Professor Peter Sullivan from Monash University was the author of the early papers for mathematics and a lead writer for the first Australian mathematics curriculum. A brief account of the development of the first national curriculum for mathematics is presented in the next section with the outcome of the process of development in this phase achieving more than in any earlier attempt.

The Development of the First Mathematics Curriculum in Australia

The process of development of the first national curriculum for mathematics by the National Curriculum Board [NCB] began with a *Framing Paper for Mathematics* (NCB 2008). Based on stakeholder feedback, the *Shape of the Australian Curriculum: Mathematics* (NCB 2009) guided the writing of the curriculum with content for each year of schooling, and achievement standards presenting a continuum of typical growth. The *Shape* paper outlined the goals, key terms and structure of the new curriculum. The structure included three content strands—Number and algebra, Measurement and geometry, and Statistics and probability—as well as four proficiency (or process) strands—understanding, fluency, problem solving, and reasoning (adapted from Kilpatrick et al. 2001).[2]

Three key issues were to be addressed in the development of the first Australian mathematics curriculum. First, improve quality and address concerns about the 'syndrome of shallow teaching' (Hollingsworth et al. 2003) by engaging more learners with complex problem solving. Second, improve equity and address differential mathematics achievement among particular groups of students. For example, from PISA 2009 data, differences in performance were related to socio-economic status, geographical location and cultural background (particularly between non-Indigenous and Indigenous students) (see Thomson et al. 2010). Third, increase accessibility with "a commitment to ensuring that all students experience the full mathematics curriculum until the end of Year 10" (NCB 2009, p. 10). This third effect challenges the common practice of 'streaming' or 'tracking' which typically leads to offering a limited mathematics curriculum for groups of students considered not able to learn more challenging mathematics content. For example in NSW, earlier curriculum documents differentiated the mathematics curriculum in grades 9 and 10.

The drafted mathematics curriculum for grades up to 10 was released for consultation in May 2009. Also during this period, the National Curriculum Board became a statutory body, the Australian Curriculum, Assessment and Reporting Authority [ACARA], responsible for curriculum and associated accountability processes including national testing. Feedback to ACARA on the draft curriculum was extensive with many recommendations supported by evidence from research. For example, Siemon (2011) indicated the 'number' content did not clearly identify and articulate the 'big ideas', including numeration. She suggested there were inconsistencies in content sequencing and language, particularly because different people wrote different sections of the draft document. Sadly, as can often occur with curriculum development, Siemon suggested the task "became one of managing competing interests rather than making hard, futuristic decisions based on research and practical experience" (p. 68).

[2]For a more detailed account of the development of the first national mathematics curriculum document, see Anderson et al. (2012), and for a detailed critique of sections of the curriculum document, see Atweh et al. (2012a, 2012b).

Table 1 The definitions for each of the proficiencies (ACARA 2010b, p. 3)

Understanding	Students build a robust knowledge of adaptable and transferable mathematical concepts. They make connections between related concepts and progressively apply the familiar to develop new ideas. They develop an understanding of the relationship between the 'why' and the 'how' of mathematics . . .
Fluency	Students develop skills in choosing appropriate procedures, carrying out procedures flexibly, accurately, efficiently and appropriately, and recalling factual knowledge and concepts readily. Students are fluent when they calculate answers efficiently, when they recognise robust ways of answering questions, when they choose appropriate methods . . .
Problem Solving	Students develop the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively. Students formulate and solve problems when they use mathematics to represent unfamiliar or meaningful situations . . .
Reasoning	Students develop an increasingly sophisticated capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying and generalising. Students are reasoning mathematically when they explain their thinking, when they deduce and justify strategies used and conclusions reached . . .

Additional concerns included the inadequate representation of the proficiency or process strands in the content descriptions, the need for further reduction of content to provide time for more problem solving and modelling, the poor sequencing of some content, and the need to further consider current research (AAMT 2010; Mathematics Education Research Group of Australasia 2010; Siemon 2011). In addition, the consultation process was limited to feedback from teachers at large on only one draft of the curriculum, suggesting a lack of transparency (Morony 2011). However, it should be noted here that ACARA regularly consulted with small numbers of teachers who represented a broad range of professional associations and systems.

After revisions, the *Australian Curriculum: Mathematics* was released online in December 2010 with opportunities for schools to trial some aspects of the curriculum and provide feedback to the writers so that revisions could be made during 2011 (ACARA 2010b). The final document presents mathematics for Foundation (the first year of schooling) to Year 10. There is evidence that some issues raised during the consultation have been addressed with a review of the sequencing of concepts within the three content strands, and organisation of content into sub-strands. In addition, the embedding of the proficiency strands was revised with the use of more 'actions' at the beginning of content statements. The definitions of each of the proficiencies (see Table 1) highlight the types of verbs used to represent the actions recommended.

Atweh et al. (2012a, 2012b) argue that while the proficiencies are described as 'actions', their descriptions as presented in Table 1 suggest a different interpretation. They state, "these articulations imply that the proficiencies describe dimensions of student performance within mathematics rather than a type of experience they have in its study" (p. 7). This may be because the proficiencies were informed by the work of Kilpatrick et al. (2001) who explained, "proficiency" was used to describe what "it means for anyone to learn mathematics successfully" (p. 5). The language used in the proficiency descriptions describes the outcomes of successful learning rather than the potential actions or experiences. Clearly it is still up to teachers to determine how this might occur and what experiences will be necessary to support the development of these proficiencies.

While the initial *Framing Paper* and the subsequent *Shape* paper articulated a vision for mathematics curriculum few would disagree with, many now feel "there is little to distinguish it from the content of 20 years ago" (Morony 2011, p. 64). Coupled with this, Thornton (2011) argues that in the curriculum documents for mathematics,

> ... the rationale and aims do little to convey a sense of what the practice of mathematics is really like and continue to promote an absolutist view of mathematics as a body of knowledge that needs to be taught and has little or no room for questioning. (p. 75)

Atweh and Goos (2011), and Siemon (2011) question whether the curriculum is 'futures oriented' and prepares young Australians for a 21st Century world. Irrespective of these criticisms, Australia now has a national mathematics curriculum for the first eleven years of schooling.

It is unclear whether this first national curriculum has addressed the challenges of improving quality, equity, and accessibility. As noted above, the final product does present content that is similar to the documents used previously by some states and territories so the question remains as to whether the quality has improved. Producing equitable learning outcomes and improving accessibility will depend very much on how the curriculum is implemented by teachers at the local school level (Atweh and Singh 2011)—clearly teachers will need more support if any real change is to ensue and differential outcomes are to be addressed. Sullivan (2012) argues that

> ... the challenge of equity can be addressed by focusing on depth of learning rather than breadth, by specifically supporting the learning of those students who need it and by extending more advanced students within the content for that level rather than isolating such students into different classes. (p. 175)

However, the embedding of the proficiencies (which include problem solving) into the content statements offers some hope. This may assist teachers in overcoming the 'syndrome of shallow teaching' if they follow the recommendations and provide students with increased opportunities to engage in complex problem solving. The *Melbourne Declaration on Educational Goals for Young Australians* (MCEETYA 2008, p. 8) also informed the development of the national curriculum. Goals for 21st Century learners suggested they "are creative, innovative and resourceful, and are able to solve problems". One way to engage and motivate students in mathematics is through problem solving and investigations (Schoenfeld 2007).

The following sections of this chapter examine the evolution of problem solving in curriculum documents from one Australian state, NSW, and consider whether the national curriculum approach to problem solving forges new opportunities for students and teachers in mathematics classrooms. I draw on Lester's (1994) reflections of 25 years of problem-solving research in reviewing this evolution.

The Evolution of Problem Solving in NSW Curriculum Development

Curriculum documents typically promote reform-oriented approaches and recognise the importance of engaging students in worthwhile mathematics through a range of actions or processes. For example, the *Principles and Standards for School Mathematics* (NCTM 2000) includes standards related to five processes—problem solving, reasoning and proof, communication, connections, and representations. Similar processes have been included in NSW curriculum documents with the most recent including a range of processes under the umbrella term "Working Mathematically". This section documents the evolution of problem solving in the curriculum documents in NSW, and examines some of the research into teachers' knowledge and understanding of the curriculum approach to problem solving in that state.

In NSW, problem solving was made explicit for the first time in the mathematics curriculum documents or syllabuses developed and introduced into elementary (NSW Department of Education 1989) and junior secondary (Board of Secondary Education [BSE] NSW 1989) classrooms in the Eighties. In the introduction to the elementary syllabus, problem solving and applications were described as important components of mathematics teaching and learning and a *problem* was described as having three characteristics:

- there is a goal to be reached
- an obstacle prevents ready solution
- the solver is motivated to reach a solution. (NSW Department of Education 1989, p. 22)

In both the elementary and the lower secondary curriculum documents, advice was provided on how problem solving could be implemented including the possibilities associated with teaching *for* problem solving, teaching *about* problem solving and teaching *through* problem solving (Siemon and Booker 1990). The lower secondary document included problem solving as one of six strands and included examples of problem types. It also advised that problem solving should involve interpretation, use of a range of heuristics, and evaluation of solutions. To support teachers, textbook writers developed chapters devoted to practising particular problem-solving strategies (e.g., Barry et al. 1988) or they presented problems associated with particular content at the end of each chapter.

Alongside the development of the curriculum, teaching support documents were produced to assist the implementation of problem solving in classrooms. However, implementation was limited (Anderson 1996, 1997)—a popular approach in lower secondary contexts was to timetable one lesson a week on 'problem solving' with sets of problem-solving tasks set up in a special classroom or 'laboratory'. Students referred to their lessons as either 'mathematics' or 'problem solving' so that problem solving was viewed as an add-on to the curriculum and not integrated into regular mathematics lessons as a way of learning and a way of doing mathematics. Perhaps this situation was mirrored in the USA as reflected in Lester's (1994) comments.

> To date, no mathematics program has been developed that adequately addresses the issue of making problem solving the central focus of the curriculum. Instead of being given coherent

> programs with clear direction, teachers have had to be satisfied with a well-intentioned melange of story problems, lists of strategies to be taught, and suggestions for classroom activities. (p. 661)

A revised curriculum for grades 9 and 10 was released in NSW in 1996 with three differentiated courses (advanced, intermediate, and general). Curriculum documents provided advice about problem-solving processes and heuristics with the introductory pages referring to problem solving as "a major aspect of mathematics" accompanied by the recommendation for teachers to consider "four important elements of solving problems" (BOS NSW 1996, p. 14), mirroring Polya's (1945) phases. Instead of a problem solving strand, the curriculum writers adopted the term *mathematical investigations* with the advice students should undertake an investigation associated with *Chance and data* as well as "one other, longer investigation which might take up to five hours" (p. 173).

The term 'working mathematically' was also introduced into NSW curriculum at this time. This introduction appeared to be informed by the document *Mathematics—A Curriculum Profile for Australian Schools* (Curriculum Corporation 1994) where *working mathematically* was described as comprising six processes—investigating, conjecturing, using problem-solving strategies, applying and verifying, using mathematical language, and working in context, each with their own outcomes and presented as a developmental continuum across all of the years of schooling. However, the term 'working mathematically' in the NSW grade 9 and 10 syllabus was presented as an objective with no clear description of the associated knowledge, skills or understandings.

The content was presented in each strand as detailed statements with 'applications, suggested activities and sample questions'. These sample questions represented activities not typically found in available textbooks, thus providing opportunities for students to engage in higher-level thinking tasks and investigations of mathematical ideas (see the example presented in Fig. 1). As Anderson (2002) notes, this appeared to be an attempt to align problem solving with the content in a more explicit manner.

In revisions to NSW curriculum in the late Nineties when an outcomes-based approach was adopted (BOSNSW 1998, 1999), *working mathematically* was also introduced into the elementary grades curriculum. More clearly aligned to the *Profile* document (1994), *working mathematically* was described as encompassing the processes of questioning, problem solving, communicating, verifying, reflecting and using technology (BOSNSW 1998). There was no similar list in mathematics curriculum documents for the secondary grades. It is evident from Table 2 that, because of the development of each of these curriculum documents at different times, the approach to mathematics curriculum design for the first 11 years of schooling in NSW was inconsistent, sending mixed messages to teachers of mathematics, particularly in relation to the implementation of problem solving.

At this stage, I pause to include a reflection from Lester (1994) on what should occur in mathematics classrooms to address the lack of engagement with problem solving. Based on his review of the research into problem solving, Lester suggested there were five clear messages about improving this situation for teachers and students.

N2: Consumer Arithmetic Content	N2: Consumer Arithmetic Applications, suggested activities and sample questions
iv) Consumer Problems Learning experiences should provide students with opportunity to: • identify best buys • compare the cost of loans using flat and reducible interest for a small number of repayment periods • find the value of an item after certain time period of depreciation or appreciation • ...	**iv) Consumer Problems** Students should: • devise and compare strategies to determine best buys in a realistic context • compare the cost of the same item in different sizes: does the ratio of cost to size remain constant as the size of the item increases? • Use a spreadsheet and graph to investigate the effect of different repayment schedules on the cost of a housing loan • ...

Fig. 1 Content and applications for Consumer Arithmetic from the Number strand of the Advanced Years 9 and 10 Syllabus (BOS NSW 1996, pp. 80–81)

Table 2 Names of the strands for the elementary and secondary school curriculum documents in NSW

Grades K to 6 (BOSNSW 1989)	Grades 7 and 8 (Board of Secondary Education NSW 1989)	Grades 9 and 10 (BOSNSW 1996)
Working Mathematically	Problem solving	Working Mathematically
Number	Number	Number
	Algebra	Algebra
	Statistics	Chance and data
Space	Geometry	Geometry
Measurement	Measurement	Measurement (including trigonometry)

1. Students must solve many problems in order to improve their problem-solving ability.
2. Problem-solving ability develops slowly over a prolonged period of time.
3. In order for students to benefit from instruction, they must believe that their teacher thinks problem solving is important.
4. Most students benefit greatly from systematically planned problem-solving instruction.
5. Teaching students about problem-solving strategies and heuristics and phases of problem solving does little to improve student's ability to solve mathematics problems in general. (p. 666)

To send clear messages to teachers, Lester's list suggests problem solving needs to be embedded in curriculum documents from the early years of schooling, with recommendations for regular, well-planned learning experiences for students. While there was considerable advice in the NSW curriculum documents of the late Eighties and the Nineties, problem solving was still presented as a separate strand or included as a process of *working mathematically*, and was usually represented within

Table 3 Working mathematically processes (BOSNSW 2003, p. 16)

Process	Description of the Process
Questioning	Students ask questions in relation to mathematical situations and their mathematical experiences
Applying Strategies	Students develop, select and use a range of strategies, including the selection and use of appropriate technology, to explore and solve problems
Communicating	Students develop and use appropriate language and representations to formulate and express mathematical ideas
Reasoning	Students develop and use processes for exploring relationships, checking solutions and giving reasons to support their conclusions
Reflecting	Students reflect on their experiences and critical understanding to make connections with, and generalisations about, existing knowledge and understanding

content strands as examples of activities or 'good questions'. Depending on one's view of what problem solving is, this was not necessarily visible to teachers (Anderson 2005). The curriculum documents in NSW listed problem-solving strategies and heuristics and emphasised the phases of problem solving—none of these approaches were supported by the research into improving students' problem-solving competence according to Lester (1994).

Beginning in 2000, all of the mathematics curriculum documents from Kindergarten to grade 10 were revised together to ensure "consistency, continuity and coherence" (Anderson 2002, p. 14). Led by the author, the curriculum development process began by mapping content from the three existing sets of curriculum documents, removing overlap and repetition, and realigning content based on research into developmental continua of learning (e.g., Harel and Confrey 1994). Mathematical ideas for all curriculum documents up to grade 10 were grouped into the content strands—number, algebra, data, geometry, and measurement. One process strand, *working mathematically*, was used to describe the mathematical actions or processes associated with doing mathematics. The overarching description of *working mathematically* included reference to problem solving:

> Students will develop knowledge, skills and understanding through inquiry, application of problem-solving strategies, including the selection and use of appropriate technology, communication, reasoning and reflection. (BOSNSW 2003, p. 12)

The *working mathematically* processes included questioning, applying strategies, communicating, reasoning and reflecting (see Table 3 for a description of each process). Anderson and Bobis (2005) argued that when teachers use rich tasks in mathematics lessons so that students are engaging with all of these processes, the students are likely to be experiencing more complex problem-solving situations.

While much of the mathematical content remained the same as in previous curriculum documents, the new curriculum approach required teachers to review their practice by:

Area MS3.2 Selects and uses the appropriate unit to calculate area, including the area of squares, rectangles and triangles	Key Ideas Select and use the appropriate unit to calculate area Recognise the need for square kilometres and hectares Develop formulae in words for finding area of squares, rectangles and triangles
Knowledge and skills	**Working Mathematically**
• recognising the need for a unit larger than the square metre • identifying situations where square kilometres are used for measuring area eg a suburb • recognising and explaining the need for a more convenient unit than the square kilometre • measuring an area in hectares eg the local park • using the abbreviations for square kilometre (km^2) and hectare (ha) • recognising that one hectare is equal to 10 000 square metres • selecting the appropriate unit to calculate area	• apply measurement skills to everyday situations eg determining the area of the basketball court (*Applying Strategies*) • use the terms 'length', 'breadth', 'width' and 'depth' appropriately (*Communicating, Reflecting*) • extend mathematical tasks by asking questions eg 'If I change the dimensions of a rectangle but keep the perimeter the same, will the area change?' (*Questioning*) • interpret measurements on simple plans (*Communicating*) • investigate the areas of rectangles that have the same perimeter (*Applying Strategies*)

Fig. 2 Content for the Measurement strand, Area sub-strand for grades 5 and 6 (BOSNSW 2003, p. 123)

1. assessing students' current knowledge and planning learning experiences informed by the developmental continuum regardless of the grade they were in at school;
2. designing programs that enabled students to be extended in their learning rather than stopping at some predetermined endpoint which occurred in the previous curriculum with its differentiated three course structure for grades 9 and 10;
3. designing lessons that integrated the process strand, *working mathematically*, with the content so that problem solving became a central focus of learning; and
4. using a range of assessment strategies that included assessment *for* learning as well as assessment *of* learning (BOSNSW 2003).

To encourage teachers to integrate the *working mathematically* processes into everyday learning experiences for students, examples were listed beside the appropriate content in the curriculum document—see Fig. 2 for an example for the Measurement strand, Area substrand. Each example was labelled with one or more processes to assist teachers in their understanding of each term.

Beginning in 2004 the revised curriculum was implemented in NSW classrooms. Extensive professional development was provided by school system personnel, professional associations, and private providers including the University of Sydney (Anderson and Moore 2005). Professional learning experiences focused on the new curriculum approach, particularly how embedding *working mathematically* into mathematics lessons would provide students with increased problem-solving opportunities.

To investigate if teachers understood this approach and whether it assisted them in the integration of *working mathematically* into classroom practice, Cavanagh (2006) interviewed 39 secondary mathematics teachers of grades 7 to 10 from a range of school contexts across NSW. While a small number of teachers had embraced the *working mathematically* approach, most had limited understanding and reported few changes to their practice. In the elementary school context, Anderson and Bobis (2005) surveyed 40 teachers of Kindergarten to grade 6 to evaluate their understanding of *working mathematically*. Based on their responses to open-ended questions about the *working mathematically* processes, only two teachers appeared to have a comprehensive understanding of all five processes with another five teachers revealing a good understanding of most. Eight teachers who reported planning *working mathematically* experiences for their students in most lessons were also interviewed to confirm their knowledge and understanding of the curriculum. While these studies explored the curriculum knowledge of a small number of teachers, they revealed the majority of teachers had a limited understanding of *working mathematically* and problem solving as they were represented in the NSW curriculum documents.

To summarise, in the NSW context, problem solving has been described in curriculum documents since 1989. Problem solving was first represented as a separate strand with accompanying advice about teaching problem-solving processes, heuristics and the phases of problem solving. Examples of problems were frequently presented. During the Nineties, problem solving was included in a *working mathematically* strand and typically described as a set of processes. To assist the integration of problem solving with content, curriculum documents presented lists of 'good questions' or 'activities'. Given this evolution of representations, there is still limited evidence of implementation in mathematics classrooms in NSW. Similar changes have occurred in the curriculum documents in other states and territories in Australia with mixed success (for further information see Clarke et al. 2007; Stacey 2005). Therefore, a valid challenge in developing the first national curriculum was to determine how problem solving should be represented to assist teachers and increase the level of implementation in classrooms. Handal and Herrington (2003) argue

> Successful curriculum change is more likely to occur when the curricular reform goals relating to teachers' practice take account of teachers' beliefs. (p. 65)

While I acknowledge teachers' beliefs filter curriculum advice, I also agree with Kennedy (2009) who states curriculum should articulate the "valued knowledge, skills and beliefs that will benefit young people in the future" (p. 278). Most teachers believe problem solving is an important life skill and that it should be included in the school curriculum (Anderson 2003, 2005; Anderson and Bobis 2005; Cavanagh 2006). Our challenge is to find effective ways to represent problem solving in curriculum documents so that teachers feel better equipped to respond positively to the advice (Stacey 2005; Sullivan 2012). The challenge for curriculum developers

Table 4 The proficiencies in the Australian national mathematics curriculum matched to the names used by Kilpatrick et al. (2001)

Australian Curriculum Proficiency Strands	Mathematical Proficiencies (Kilpatrick et al. 2001)
Understanding	Conceptual understanding
Fluency	Procedural fluency
Problem solving	Strategic competence
Reasoning	Adaptive reasoning
	Productive disposition

is to clearly articulate the expected standards for both content and problem solving at each grade level and to assist teachers by integrating the content with problem solving so that problem-solving approaches to teaching and learning mathematics are explicit and easily understood.

The Approach to Problem Solving in the First Australian Mathematics Curriculum

As reported earlier in this chapter, the Australian curriculum for mathematics has three content strands (Number and algebra, Measurement and geometry, Statistics and probability) and four process strands which are based on four of the five proficiencies described by Kilpatrick et al. (2001) (see Table 4). In this section, the approach taken to embed problem solving into the new national curriculum will be reviewed to determine whether it provides new opportunities for teachers and students.

As noted in the *Shape of the Australian Curriculum: Mathematics* (NCB 2009, p. 5) document, the term 'working mathematically' was not considered to adequately represent the full range of actions so the new proficiencies have been adapted from the mathematical proficiencies proposed by Kilpatrick et al. (2001). Sullivan (2012) argues

> … the four proficiencies … provide a clearer framework for mathematical processes than "working mathematically" and are more likely to encourage teachers and others who assess student learning to move beyond a focus on fluency, however, there will need to be support for teachers if they are to incorporate them into the curriculum. (p. 175)

While there needs to be a balance of the proficiencies in mathematics classrooms (Sullivan 2011), if problem solving and reasoning are to be promoted as important components of the curriculum it is necessary to reconsider the advice from Lester in 1994 and the types of problems used by teachers in mathematics lessons must be carefully considered (Clarke 2009; Sullivan 2011). Curriculum developers recognise that providing problem-solving experiences is critical if students are to be able to use and apply mathematical knowledge in meaningful ways. It is through problem solving that students develop deeper understanding of mathematical ideas, become more engaged and enthused in lessons, and appreciate the relevance and usefulness of mathematics.

In the new *Australian Curriculum: Mathematics* (ACARA 2012) problem solving is described as follows.

> Students develop the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively. Students formulate and solve problems when they use mathematics to represent unfamiliar or meaningful situations, when they design investigations and plan their approaches, when they apply their existing strategies to seek solutions, and when they verify that their answers are reasonable. (p. 6)

For students to become the successful problem solvers that this description suggests, they will need to actively engage with a range of important processes during mathematics lessons. For this to occur teachers will need to select tasks, which allow for student choice about the mathematics they might use and the problem-solving strategies they select to model and investigate mathematical situations. Importantly, they also need to be able to effectively communicate their solutions. According to the NCTM *Standards* (2000, p. 52) "problem solving means engaging in a task for which the solution method is not known in advance". So problem solving frequently involves investigating new and somewhat challenging situations that require time and effort. Problem solving needs to be more than just doing questions that are applications of the mathematics students are learning right now.

To aid teacher understanding of the proficiencies in the Australian curriculum, the following statement is presented at each grade level:

> The proficiency strands Understanding, Fluency, Problem Solving and Reasoning are an integral part of mathematics content across the three content strands Number and Algebra, Measurement and Geometry, and Statistics and Probability. The proficiencies reinforce the significance of working mathematically within the content and describe how the content is explored and developed. They provide the language to build in the developmental aspects of the learning of mathematics.

Under this statement, a brief description is presented for each of the proficiencies, which is appropriate to the grade level. While there is a statement for every grade level, Table 5 presents the description for problem solving for some levels.

These statements include actions associated with learning mathematics and combine types of problem-solving tasks with the content relevant for the particular grade. Several of these statements mention "authentic problems" or "authentic situations", neither of these terms is defined for teachers so while the statements generally suggest engagement with problem-solving experiences, they may not be necessarily clear. Atweh and Goos (2011) offer the suggestion that "authentic activities" would involve "using examples from the real world of the student". At the grade 10 level, problem solving refers to applying formulae and procedures. Depending on student understanding, these could be routine applications and have limited opportunity for problem solving as defined in the curriculum document.

In addition to these problem-solving statements, the content descriptions also include some reference to solving problems. This was the strategy used by the curriculum writers to embed problem solving into the content and to address the concern that teachers believe problem solving is an added extra. Table 6 presents some examples for the Number and Algebra strand at different grade levels.

Table 5 Problem-solving statements at the beginning of several grades (ACARA 2012)

Grade	Problem Solving statement
Foundation	Problem solving includes using materials to model **authentic problems**, sorting objects, using familiar counting sequences to solve unfamiliar problems, and discussing the reasonableness of the answer
2	Problem solving includes formulating problems from **authentic situations**, making models and using number sentences that represent problem situations, planning routes on maps, and matching transformations with their original shape
4	Problem solving includes formulating, modelling and recording **authentic situations** involving operations, comparing large numbers and time durations, and using properties of numbers to continue patterns
6	Problem solving includes formulating and solving **authentic problems** using numbers and measurements creating similar shapes through enlargements, representing secondary data and calculating angles
8	Problem solving includes formulating and modelling, with comparisons of ratios, profit and loss, authentic situations involving areas and perimeters of common shapes and analysing and interpreting data using two-way tables
10	Problem solving includes calculating the surface area and volume of a diverse range of prisms, finding unknown lengths and angles using applications of trigonometry, using algebraic and graphical techniques to find solutions to simultaneous equations and inequalities, and investigating independence of events and their probabilities

Table 6 Examples of embedding problem solving into content descriptions in Number and Algebra at several grade levels (ACARA 2012)

Grade	Content descriptions in Number and Algebra which refer to problem solving
1	Represent and solve simple addition and subtraction problems using a range of strategies including counting on, partitioning and rearranging parts.
3	Apply place value to partition, rearrange and regroup numbers to at least 10 000 to assist calculations and solve problems
5	Solve problems involving multiplication of large numbers by one- or two-digit numbers using efficient mental, written strategies and appropriate digital technologies
7	Recognise and solve problems involving simple ratios
9	Solve problems involving simple interest
10	Solve problems involving linear equations, including those derived from formulas

Table 6 reveals that there has been an attempt to embed problem solving into content, but it is possible teachers may interpret these statements as 'simple word problems'. Anderson (2005) found many teachers believed they were implementing problem solving as required in the curriculum by presenting students with a range of word problems—many of the examples teachers provided were lower order applications requiring little mathematical thinking for students who were able to read and interpret the language in the problem.

It should be noted here that a review of the content descriptions and their elaborations across all grade levels reveals "a heavy focus on" the first two proficiencies of understanding and fluency and "to a lower level on reasoning and problem solving" (Atweh and Goos 2011, p. 221). Atweh et al. (2012a, 2012b) analysed all of the content elaborations for grade 8 and found that while 56 % related to fluency, only 12 % related to problem solving and 7 % to reasoning. Further, they suggest that the problem solving elaborations are limited in their scope and "may not inspire teachers to appreciate the importance of these proficiencies and to think of valuable and exciting ways in which they can be used or developed in the classroom" (p. 9).

The Australian national curriculum does provide advice about problem solving that is different to previous documents, particularly when compared to the NSW context. There is an overarching definition of problem solving, there are statements about problem solving at each grade level, and problem solving has been embedded into several content descriptions. This may provide new opportunities for teachers to engage their students with more problem solving in mathematics lessons. At this early stage of implementation of the new Australian curriculum, no research has been published into teachers' use of the new curriculum documents. It will be critical to examine the impact of this approach to determine whether it assists teachers and improves the level of engagement with problem solving in Australian mathematics classrooms.

It is certainly true that Australia does have its first national curriculum for mathematics and it was implemented in some schools in some jurisdictions in 2012. However, how it is being implemented in each state and territory differs. Several states (e.g., NSW and Victoria) are using the new national curriculum as a framework to develop their own curriculum documents. Implementation in these locations will follow in 2013 or 2014. Others are providing teachers with extensive professional development to use the national curriculum as a planning document for school-based curriculum (e.g., Australian Capital Territory). Given that the responsibility for curriculum implementation rests with the state and territory governments, it is not surprising the approaches to curriculum delivery and teacher support varies. It is historically difficult to change deeply held beliefs and practices, so the implementation of the national curriculum varies depending on which state you visit.

While there appear to be new opportunities for Australian teachers and students to engage in more complex problem solving in the new national curriculum, the fundamental issue of clarity on the meanings of 'problem' and 'problem solving' appears to remain—although research is needed to ascertain whether this is the case. From her review of problem solving in the mathematics curriculum documents from several countries as well as some Australian states in 2005, Stacey recommended:

> Research could examine whether and how these curriculum structures from different countries influence teachers' understanding of the goals of teaching mathematics, and whether these different understandings make a real difference in the attention that teachers give to mathematical problem solving beyond the routine. (p. 345)

It is a shame this recommendation was not heeded—it would have assisted the curriculum developers of the first national curriculum in Australia. But it is not too late to further explore the ways problem solving is represented in other countries and whether alternative approaches may better support teachers' understanding.

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