Study of the Stress Intensity Factor of an Unbalanced Rotating Cracked Shaft

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Abstract. The components of machines very often present defects that can grow to fatigue cracks when they are submitted to the working solicitations. In the case of shaft,this is particularly important because the catastrophic failures can lead to personal injuries or economic problems. When a cracked shaft rotates, the breathing mechanism appears. The crack opens and closes passing from the open state to the close state with a transition that produces a partial opening. The shafts present additionally misalignments or/and unbalances that alter their normal function. In this paper, we present a Finite Element Method (FEM) study of the influence of the eccentricity in the breathing mechanism of a rotating cracked shaft. The classical Jeffcott rotor model has been chosen for this study. To simulate the rotation of the shaft, different angular positions have been considered. The Stress Intensity Factor (SIF) along the crack front during the rotation has been studied considering different angles of eccentricity. The work allows to know the influence of the unbalance of rotating shafts in the crack breathing mechanism, in the values of the Stress Intensity Factor and in the propagation of cracks.

Keywords: Rotating shaft, crack, unbalance, Stress Intensity Factor.

1 Introduction

The increasing importance of safety and costs derived from failure in machinery has pushed the researchers in the field of damage detection to analyze the behavior of mechanical components with defects. The failures of machines are produced quite often by the presence and propagation of fatigue cracks due to the loads and solicitations they carry. Those failures sometimes are catastrophic and produce personal injuries or economic problems.

Among the machine parts, shafts are one of the main components of machines. They work in rotation, usually carrying bending and torsion efforts. All of them together, can produce the shaft failure by generation and propagation of fatigue cracks.

V. Petuya et al. (eds.), *New Advances in Mechanisms, Transmissions and Applications*, Mechanisms and Machine Science 17,
DOI: 10.1007/978-94-007-7485-8_49,

When a cracked shaft rotates, the crack can open and close once per revolution depending on the loads, the stiffness of the shaft at the cracked section. The opening and closing of the crack has been modeled in different ways. The simplest one, very much used because of that, is the switch model. It considers that the crack is open or closed, so that the crack is half the rotation in the open state and the other half in the closed one. However, the breathing behavior is the most feasible behavior of the crack, although it is more complex. In this case, the crack passes from the closed state to the open state gradually in a rotation. As in the switch model, the crack is closed when it is situated in the compression zone of the shaft and it is open when situated in the tensile zone. The transition between both situations produces partial opening (or closing) of the crack when the static deflection dominates the behavior of the rotating shaft. The partial opening/closing of the crack has been studied, numerically or analytically, by different authors [1, 2, 3] always considering an aligned and balanced shaft. The opening and closing of the crack is very much connected with the the values taken by the the Stress Intensity Factor (SIF) at the crack front as the shaft rotates. The SIF is a parameter of fracture mechanics that allows to predict the stress state in the proximity of the crack front due to remote loads. The crack is open while the SIF remains positive, otherwise the crack will be closed.

Unfortunately, in the real work of large shafts, it is very usual that the shafts present unbalances or misalignments that modify the normal behavior of the component. The presence of eccentric masses [4, 2, 5, 6, 12, 7] can be considered as a very common unbalance case in rotors. The unbalance, as mention before, modifies the dynamic behavior of the rotating shafts and may hide the presence of the cracks or, on the other hand, can increase their effects.

In this paper we present the numerical study of the influence of the eccentricity in a rotating cracked shaft using a finite element model of a cracked Jeffcott rotor. The analysis has been made using the commercial finite element code ABAQUS. We have analyse the SIF along the crack front, for each angle of rotation and for different positions of the eccentricity. We can identify, through the study of the SIF, when and where the opening takes place.

2 The Cracked Shaft Model

As mentioned before, the model chosen for this study is the classical Jeffcott rotor widely used in rotordynamics [11, 8, 9, 10, 5, 6, 7]. This simple but useful model consists in a massless shaft simply supported at the ends, with a concentrated mass (a disc). The crack is situated at the mid span of the shaft having a straight front, for sake of simplicity, oriented on a plane normal to the axis of the shaft. The eccentric mass has been placed on the disc of the Jeffcott rotor as an additional mass as can be seen in Figure 1.The round bar total length is equal to 900mm, whereas the diameter is 20mm. The material of the shaft is aluminium with the following mechanical properties: Youngs Modulus E=72GPa, Poisson ratio μ=0.33 and density ρ=2800Kg/m^3.

The rotation of the shaft has been simulated considering eight different angular positions, one for every eighth of a rotation, called angle of rotation ϕ, see Figure 2. At each angular position given, we analyze the static behavior of the shaft (considering the gravity effect), variables such as displacements, open portion of open crack and SIF, among others. On the other hand, the influence of the mass eccentricity on the opening of the crack has been studied considering different positions of the eccentric mass on the disc, given by different angles measured from the position of the crack, called angle of eccentricity θ, as shown in Figure 3. The effect of the eccentric mass, has been included as an inertial force, F_e calculated as a mass m located at a distance e from the center of the Jeffcott rotor rotating with the angular rotating velocity Ω, see Figure 3. Values taken for this analysis are m=0.2 Kg, e=80mm and Ω= 1000 rpm.

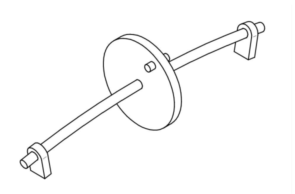

Fig. 1 Jeffcott rotor with an eccentric mass

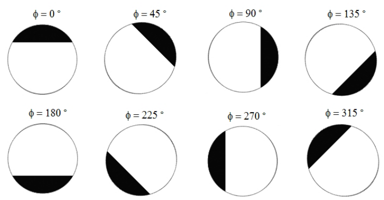

Fig. 2 Angular positions of the crack during one rotation

In order to know how much other variables are involved in the problem, like the size of the crack, three crack lengths, $\alpha = a/D$=0.1; 0.25 and 0.5, have been considered.

The numerical simulation of the problem has been carried out using the Finite Element commercial code ABAQUS. As there is no symmetry in the problem, the analysis has been developed for a complete 3D model of the shaft. The mesh of the three dimensional model is made employing 8 node linear brick elements. To avoid the interpenetration between the crack faces when the crack is in the compression zone (closed), a surface-to-surface contact interaction has been defined.

The analyzed cases has been done accordingly with the following:

- crack length α=0.1; 0.25; 0.5
- eccentricity angle θ= 0^o; 45^o; 90^o; 135^o; 180^o
- rotation angle ϕ= 0^o; 45^o; 90^o; 135^o; 180^o; 225^o; 270^o; 315^o; 360^o

To simulate the balanced shaft, the cases corresponding to the same three crack lengths and eight rotation angles have been also modeled in order to compare with the corresponding unbalanced cases.

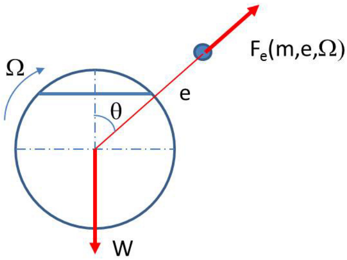

Fig. 3 Resume of the applied forces on the shaft

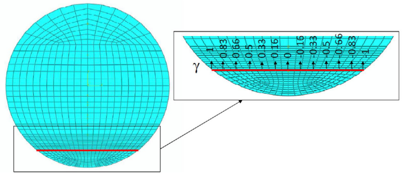

Fig. 4 Positions on the crack front, γ

Although the behavior of a rotating cracked shaft is a nonlinear problem due to the vibrations produced and also to the breathing mechanism of the crack, we have evaluated the SIF treating the problem as a succession of static ones in which the nonlinearity is reduced to the contacts in the crack. The commercial code ABAQUS linearizes internally this problem, so that our study can be considered lineal.

3 Stress Intensity Factor at the Crack Front

The model developed using commercial code ABAQUS, allows to obtain the SIF along the crack front in a total of 13 locations, given by variable γ, as indicated in Figure 4.

The non-dimensional values of the SIF versus non-dimensional location on the front, γ, have been plotted in Figure 5 cases (a,b,c,d,e). In this case, we have chosen the crack length α=0.25. Each figure shows, for a given eccentricity (θ) the SIF values for five angles of rotation (ϕ=0^0, 45^0, 90^0, 135^0, 180^0) (only half the rotation has been considered in the plots). Looking to those figures we can see that when the crack and the eccentricity are at the same location, (θ=0^0), Figure 5 (a), the SIF is always positive which means that the crack is always open. It is true for the whole front and the whole rotation. On the other hand, when the crack is just opposite to the eccentric mass (θ=180^0) Figure 5 (e), the values of the SIF are always null which means that the crack is completely closed (at any moment of the rotation and at any location in the front). The cases corresponding to the other three (b,c,d), represent intermediate cases where the crack is partially open with positive and negative values of the SIF along the front. The figures show clearly the influence of the eccentricity and the angle of rotation on the breathing mechanism of the crack.

For the sake of clarity, the results in terms of stress and open area of the crack corresponding to the case $\alpha = 0.25$ and $\theta = 90^0$ for the whole rotation, have been plotted in Figures 6 and 7. In Figure 6 the tensile stress at the

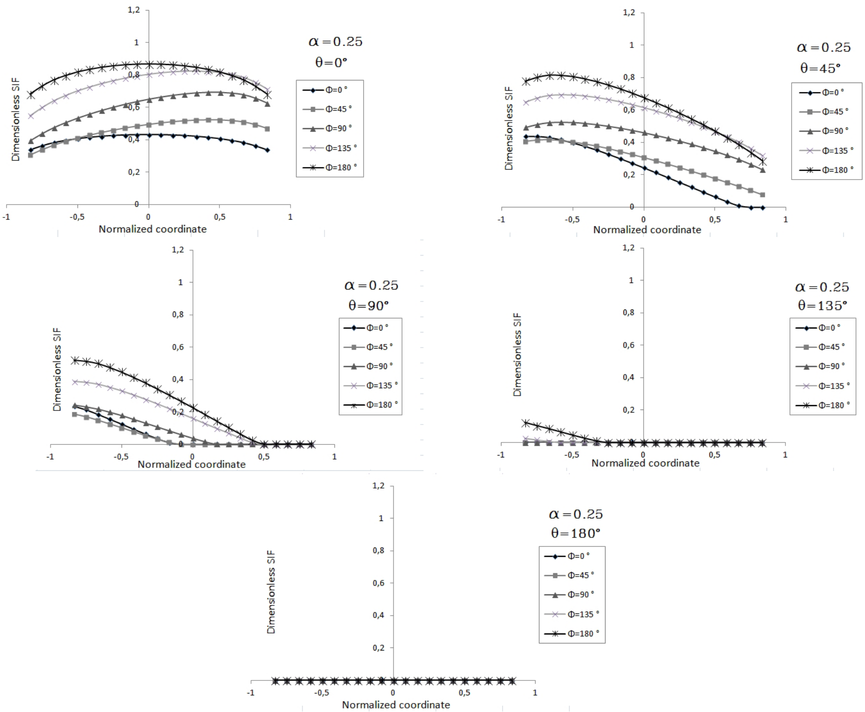

Fig. 5 Non-dimensional SIF vs location in the front, γ. Case α=0.25.

front of the crack is plotted. The white colour corresponds to the open part of the crack (tensile stress). Note the agreement with the plotted results of SIF in Figure 5 c). More clear is the plot of the open part of the crack for the same case that can be observed in Figure 7.

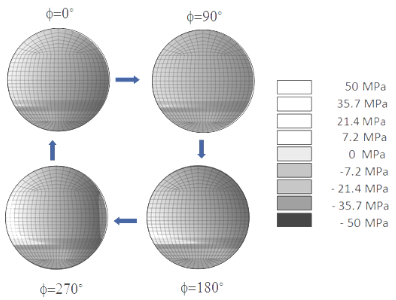

Fig. 6 Stress map at the cracked section. Case $\alpha = 0.25$ and $\theta = 90^0$

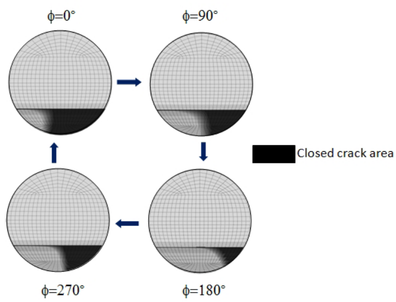

Fig. 7 Opening map at the cracked section. Case $\alpha = 0.25$ and $\theta = 90^0$

In order to know the influence of the crack length, the values of the SIF have been plotted versus the crack front. In this case, we have selected a specific eccentricity $\theta = 45^0$, so we have plotted a figure for each crack length for different angles of rotation. In Figure 8 we can observe that as the crack is larger the values of the SIF are greater. On the other hand, the behavior of the front in terms of opening and closing is nearly the same for the different values of length, being more similar the cases of medium and large cracks.

4 Analysis of the Breathing Mechanism Using the SIF

To have a better interpretation of the data obtained for the SIF, we have calculated for each location on the front, a new variable, ΔK, given by the following expression:

$$\Delta K(\gamma) = K_{max}(\gamma) - K_{min}(\gamma) \tag{1}$$

where $K_{max}(\gamma)$ and $K_{min}(\gamma)$ corresponds to the maximum value of the SIF and the minimum value of the SIF, respectively, both in a rotation for each location γ on the front of the crack.

ΔK is a good indicator of the opening of the crack. If a point of the crack front is open at least once in a rotation, ΔK for this point will be positive. If ΔK takes value 0, that means that the point will be always closed during the rotation. That indicator shows the feasibility of the crack to grow. In Figure 9, we have plotted the value of ΔK in each location of the front, for each angle of rotation, θ. The figures corresponds to the three values of α considered

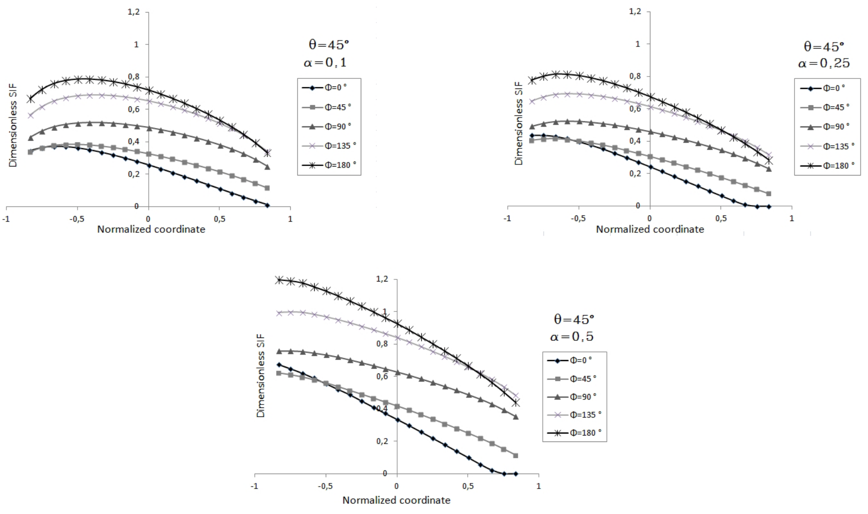

Fig. 8 Non-dimensional SIF vs location in the front, γ. Case $\theta = 45^0$.

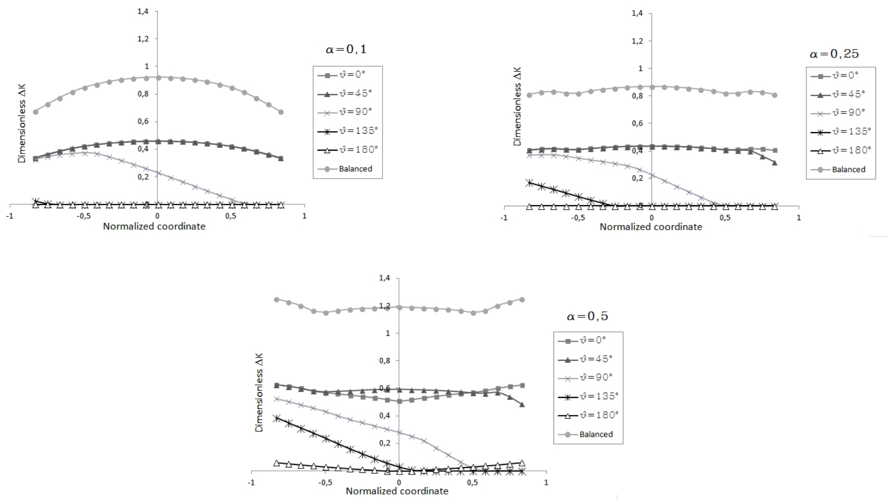

Fig. 9 ΔK vs location in the front. Case $\theta = 45^0$.

in this study. The figures include, just for comparison, the behavior of the balanced cracked shaft.

5 Conclusions

In this paper a numerical study of the evolution of the Stress Intensity Factor of an unbalanced cracked shaft during a rotation has been developed. The influence of some variables involved in the problem have also been analyzed. The conclusions of the paper can be summarized in the following:

- Regarding the angle of eccentricity, data show that the position of the eccentric mass with respect to the crack (angle of eccentricity) influences very much the opening and closing of the crack. When both crack and eccentric mass are at the same position, the crack is always open along the rotation. However, if the eccentric mass is opposite to the crack, then the crack never opens. With other relative positions of eccentric mass and crack, data show the partial opening of the crack during the rotation.
- With respect to the crack length, data show that the SIF grows with the length of the crack. The pattern is the same independently of the crack length.
- For a better interpretation of the data, a new variable is proposed, ΔK, calculated at each location on the front. This variable allows to know if the crack would probably grow or not depending on its value. For a positive value, the crack very probably will grow.

Acknowledgements. The authors gratefully acknowledge the financial support given by the Spanish Ministerio de Economía y Competitividad (Project DPI2009-13264).

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