Kinetostatic Benchmark of Rear Suspension Systems for Motorcycle

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Abstract. This paper provides a benchmark for motorcycle rear suspension systems. The main goal is to determine whether any of the suspension systems provides clear advantages over the others when seeking for a previously defined progressive wheel rate. A kinetostatic formulation of the mechanism is therefore presented. In this formulation, kinematics is based on groups of elements, while statics is based on the principle of virtual work. This formulation has been proved to be efficient and robust. It allows for building objective functions which are especially suitable for evolutionary algorithm optimization. Results show that there are no significant differences between the four types of analysed suspensions.

Keywords: motorcycle, rear suspension, dimensional synthesis, progressiveness, groups of elements.

1 Introduction

The main function of the suspension of a vehicle is to maximize comfort by reducing vertical acceleration of the passenger seat. Furthermore, the suspension system should maximize tyre-road contact to ensure traction and braking. To achieve these goals and, in addition, offer an improved driving feeling, the rear suspension of the motorcycle should have a progressive wheel rate. Therefore, roughness of the road can be adequately filtered while, simultaneously, large vertical displacements under heavy braking or traction are avoided.

A non-progressive behaviour shall be obtained by connecting a spring (with K stiffness constant) and a viscous damper (with a C damping constant) directly to the swing-arm. If a non-linear behaviour is desired, it shall be obtained using tie rods, with K and C values remaining constant. In motorcycles, tie rod systems are usually planar mechanisms (four-bar linkage) which are placed between the rear wheel swing-arm and the rear spring-damper to achieve a non-linear wheel rate.

Different configurations of tie rod systems have been introduced by the major motorcycle manufacturers during the past 30 years.

Almost all of these systems are variations of the four-bar linkage. The most common types can be classified according to the different possibilities of joints for the two ends of the spring-damper, as the lower one can be jointed to the tie rod or

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to the rocker, and the upper one can be hinged to the frame or to the swing-arm. Fig. 1 shows schemes of the tie rod systems analysed in this paper which are among the most common types.

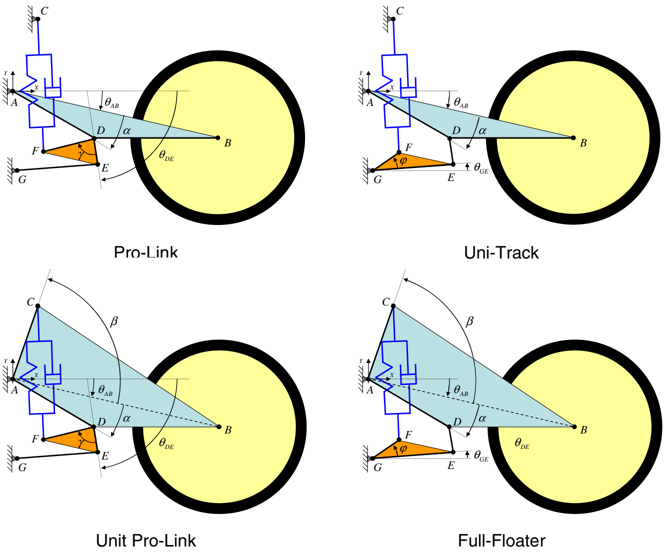

Fig. 1 Kinematic schemes for the tie rod systems

Foale [3] has indicated that all these systems are very similar to each other, in spite of the affirmations of the manufacturers. This work will explore whether this assertion is truthful through comparison of the ability of each system for achieving pre-defined wheel rate.

2 Kinetostatic Dimensional Synthesis

In order to obtain a proper comparison among the four types of suspensions included in this study, a group of fixed geometric parameters shall be defined. This group includes:

- Values of X and Y coordinates for point A: $x_A = 0$ mm, $y_A = 0$ mm
  This point indicates the position where the swing-arm is jointed to the frame, and it has been used as the origin of the coordinate system.
- Value of X coordinate for point G: $x_G = 0$ mm
  This point indicates the position where the rocker is jointed to the frame.
- The length of the swing-arm: $L_{AB} = 500$ mm
- Minimum length of the spring-damper: $L_{sd\_\min} = 210$ mm
- Maximum length of the spring-damper: $L_{sd\_\max} = 260$ mm

- Limits for the displacement along Y axis for the rear wheel: $y_B \in [-100 \quad 20]$

The remaining lengths, angles, stiffness and preloads defined for each suspension system evaluated in this work have been considered as variables in the dimensional synthesis problems. These parameters are depending on the characteristics of each single type of suspension.

In order to simplify the problem and eliminate the influence of the damper, movement of the suspension has been considered quasi-static. Furthermore, neither the mass nor the inertia of moving parts have been considered in the dimensional synthesis problems.

Taking these considerations into account, modelling of each suspension system has the objective of determining length variation of the spring-damper for a given Y coordinate variation of the rear wheel axle, under certain values of the geometric and load parameters.

Castillo et al. [1] had proposed the use of closed loop equations for modelling the Pro-Link suspension kinematic. Nevertheless, it is not clear how they deal with those cases where the mechanism cannot be disassembled. Fernández de Bustos et al. [2] had modelled the kinematics by means of finite elements, so that mentioned problem is avoided.

Problem modelling must be robust and contemplate the most appropriate way of dealing with situations where the mechanism cannot be assembled. This way on, it will be possible to obtain surjective and monotonous functions for the synthesis problems. In addition, modelling has to provide an unambiguous definition for the configuration of the mechanism, which will be maintained throughout the movement. For this purpose, a kinematic modelling based on groups of elements has been used in this work. This model has been previously described in [4], and additionally used for modelling suspensions in synthesis problems [5]. The geometry of each single link can be defined through lengths and/or angles. Therefore, bounds can be easily established for these geometric parameters in the dimensional synthesis problem. Moreover, to obtain a full definition, the assembly mode for certain groups must be introduced as constants of the mechanism. The nomenclature that has been used in this work is as follows:

L_{ij} : Distance between points i and j

θ_{ij} : Angle between the *ij* vector and the X positive axis. Counter-clockwise is assumed to be positive

$\alpha, \beta, \gamma, ...$: Angle between two vectors belonging to the same rigid body. Counter-clockwise is assumed to be positive

Desired progressiveness curve is defined in Eq. (1) based on practical knowledge and previous experiences.

$$F_{y_B-desired} = 0.1025 \cdot y_B^2 + 27.3457 \cdot y_B + 2412,1 \quad (1)$$

2.1 Parametric Formulation of Kinematics

Formulation of kinematics for considered suspension types has been defined following the sequential procedure described in [5]. The steps of this procedure are:

1. Calculate the θ_{AB} angle from y_B and L_{AB} using the arcsine function.
2. Calculate the θ_{AD} angle from θ_{AB} and α. Additionally, when modelling the Unit Pro-Link or the Full-Floater suspensions, calculate the θ_{AC} angle from θ_{AB} and β. In both cases the consideration of the swing-arm as a rigid body has been applied.
3. Calculate the coordinates for the D point. Additionally, calculate the coordinates for the C point when modelling the Unit Pro-Link or the Full-Floater suspensions.
4. Calculate the coordinates for the E point as a function of the positions of points D and G, by solving the RRR group according to [2] and indicating the assembly mode. It has to be remarked that this group suitability for assembling depends on actual positions and dimensions. Mentioned solving procedure takes into account all possible options and provides a value for the *assembly error* parameter. If assembly error equals 0, the group can be assembled. Otherwise, if assembly error is above 0, then the RRR group cannot be assembled.
5. Once the coordinates for the D, E and G points are already known, and using the arctangent function, calculate the θ_{DE} angle for the Pro-link and the Unit Pro-Link suspensions, or the θ_{GE} angle for the Uni-Track and the Full-Floater ones.
6. Calculate the coordinates of the F point using the coordinates of point D (cases of the Pro-link and the Unit Pro-Link suspensions) or the coordinates of point G (when modelling the Uni-Track or the Full-Floater)
7. Once the coordinates for the C and F points are known, calculate the L_{sd} length of the spring-damper.

Each suspension system has its own geometric parameters. These parameters have been grouped in the $\mathbf{p}$ vector. According to this, the Pro-Link suspension geometric parameters are:

$$\mathbf{p} = [x_C \quad y_C \quad y_G \quad \alpha \quad L_{AD} \quad L_{DE} \quad L_{EG} \quad \gamma \quad L_{DF}]$$

While the Uni-Track ones are:

$$\mathbf{p} = [x_C \quad y_C \quad y_G \quad \alpha \quad L_{AD} \quad L_{DE} \quad L_{EG} \quad \varphi \quad L_{FG}]$$

And in the case of the Unit Pro-Link:

$$\mathbf{p} = [y_G \quad \beta \quad L_{AC} \quad \alpha \quad L_{AD} \quad L_{DE} \quad L_{EG} \quad \gamma \quad L_{DF}]$$

Finally, the parameters of the Full-Floater suspension are:

$$\mathbf{p} = [y_G \quad \beta \quad L_{AC} \quad \alpha \quad L_{AD} \quad L_{DE} \quad L_{EG} \quad \varphi \quad L_{FG}]$$

2.2 Parametric Formulation of Statics

Formulation of kinematics from section 2.1 allows for determining the value of L_{sd}. The error related to the assembly of the DEG dyad can be also determined using the y_B value and the geometric parameters in $\mathbf{p}$ for each single type of suspension.

$\mathbf{y}_B$ and $\mathbf{L}_{sd}$ have been defined as the n-component vectors constructed from y_B and L_{sd} values for each j-th evenly spaced position in the $[-100 \quad 20]$ range, given $j = 1,...,n$. Eq. (2) shows the expression that has been obtained for F_{y_B-mech} applying the principle of virtual work for $j = 2,...,n$ and neglecting the force at the damper.

$$\mathbf{F}_{y_B-mech}(j) = \frac{\left(K \cdot (\mathbf{L}_{sd}(j) - \mathbf{L}_{sd}(1)) - F_{preload}\right) \cdot (\mathbf{L}_{sd}(j) - \mathbf{L}_{sd}(j-1))}{\mathbf{y}_B(j) - \mathbf{y}_B(j-1)} \tag{2}$$

2.3 Formal Approach for the Optimization Problem

The variables that have been considered as unknown for the optimization problem have been grouped in vector $\mathbf{v}$, defined as follows:

$$\mathbf{v} = \begin{bmatrix} \mathbf{p} & K & F_{preload} \end{bmatrix}$$

Therefore, this problem has been formulated as:

"Determining the values for the components in $\mathbf{v}$, that make the value for the vertical force in the wheel (F_{y_B-mech}) as close as possible to a desired value ($F_{y_B-desired}$)". Two additional constraints have been imposed for this problem: firstly, vertical displacement in the wheel must fulfil the working range of the spring-damper; secondly, the mechanism does not get locked in an intermediate position.

The so defined optimization problem can be formulated as Eq. (3).

$$\begin{aligned}
&\min \sum_{j=2}^{n} \left(\mathbf{F}_{y_B-desired}(\mathbf{y}_B(j)) - \mathbf{F}_{y_B-mech}(\mathbf{y}_B(j), \mathbf{v})\right)^2 \\
&\text{with} \\
&\qquad L_{sd_\max} = \mathbf{L}_{sd}(1) \\
&\qquad L_{sd_\min} = \mathbf{L}_{sd}(n) \\
&\qquad \sum_{j=1}^{n} error_j = 0 \\
&\text{being} \qquad \mathbf{y}_B(j) = \frac{-100 \cdot (n-j) + 20 \cdot (j-1)}{n-1}
\end{aligned} \tag{3}$$

The $error_j$ value represents assembly error for the DEG dyad in the position j. It must be remarked that this error is not calculated for the first point in the wheel rate curve, as F_{y_B-mech} is calculated through increments.

2.4 Solving the Optimization Problem

Four synthesis problems have been therefore posed independently, one for each type of suspension. All of them share a series of characteristics:

- They are minimization problems with equality constraint.
- They have eleven continuous type variables
- The derivatives of the objective functions with respect to the variables are not available
- They have low computational costs
- Neither the number of optima nor their possible values are known.
- Though an approximation to the optimum solution is not available, it is relatively simple to set bounds based on practical considerations

Once these characteristics have been analysed, the evolution strategy DDM-ES has been finally selected for the optimization problem. This election is based on the ability of the DDM-ES strategy for natively working with bounded and continuous variables, an also for its good performance when working with multimodal functions [6]. After this, solutions are refined with the SQP method.

Nevertheless, this algorithm does not admit constraints. Therefore, the original problem has been transformed into a new unconstrained optimization problem. The formulation of this new problem is reflected on Eq. (4).

$$\begin{aligned} \min \ & \sum_{j=2}^{n}\left(\mathbf{F}_{y_B-desired}\left(\mathbf{y}_B\left(j\right)\right)-\mathbf{F}_{y_B-mech}\left(\mathbf{y}_B\left(j\right),\mathbf{v}\right)\right)^2+\ldots \\ & \ldots+\omega_1\cdot\left(L_{sd\_\max}-\mathbf{L}_{sd}\left(1\right)\right)^2+\omega_2\cdot\left(L_{sd\_\min}-\mathbf{L}_{sd}\left(n\right)\right)^2+\omega_2\cdot\sum_{j=1}^{n}error_j^2 \end{aligned} \tag{4}$$

In this equation ω_1 and ω_2 are used for weighting possible failures on the extreme lengths of the damper. ω_3, on the other hand, is used for weighting the relative effect of error assembly.

3 Results

Formulations of the synthesis problems for the suspension systems have been programmed using MATLAB. These problems have later been solved, considering variable limits as shown in Table 1.

Table 1 Bounds for the variables

Variable	*Min*	*Max*	*Variable*	*Min*	*Max*
$x_C\,(mm)$	-30	100	$y_C\,(mm)$	30	100
$y_G\,(mm)$	-150	-50	$\beta\,(rad)$	$\pi/10$	$\pi/2$
$L_{AC}\,(mm)$	50	200	$\alpha\,(rad)$	$-\pi/10$	0
$L_{AD}\,(mm)$	50	200	$L_{DE}\,(mm)$	50	200
$L_{EG}\,(mm)$	50	200	$\gamma\,(rad)$	$-\pi/3$	0
$L_{DF}\,(mm)$	50	200	$\varphi\,(rad)$	$-\pi/10$	$\pi/10$
$L_{FG}\,(mm)$	50	200	$K\,(N/mm)$	10	200
$F_{preload}\,(N)$	0	1000			

Weights have been fixed as: $\omega_1 = \omega_2 = \omega_3 = 10^6$. Population size for the DDM-ES has been fixed to a 500 value, while the number of generations has been fixed to a 40 value. Fig. 2 shows the desired and optima wheel rate curves for each single suspension system.

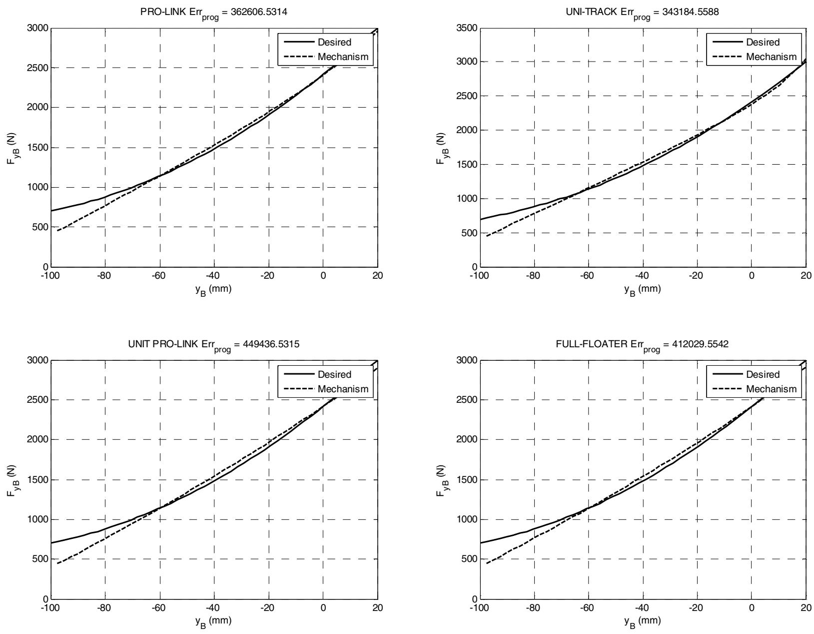

Fig. 2 Desired and optima wheel rate curves for each single suspension system

For the evaluated suspension systems, optimal solutions show a zero assembly error in all positions. Likewise, in the four cases, error related to the achievement of maximum and minimum damper lengths shows values below 0.2 mm.

Computational time for solving each optimization problem has been as follows: Pro-Link suspension: 27.43 s, Uni-Track suspension: 27.79 s, Unit Pro-Link suspension: 28.66 s, Full-Floater suspension: 28.41 s.

4 Conclusions

Firstly, a brand new kinetostatic formulation has been proposed. This formulation, based on groups of elements and the principle of virtual work, allows for defining a robust and efficient objective function. Secondly, a comparative among different types of motorcycle rear suspensions systems is presented. This comparative provides a promising approach to an integral synthesis of mechanisms (structural + dimensional). Thirdly, results reveal that none of the suspension systems studied is qualitatively superior to the others under a kinetostatic criterion. This statement agrees with Foale conclusions. Finally, it can be established that all the analysed mechanism are only capable for accurately fulfilling the final 2/3 of the desired wheel rate curve.

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