Towards Replacing Lyapunov's "Direct" Method in Adaptive Control of Nonlinear Systems

József K. Tar

Abstract In adaptive nonlinear control Lyapunov's 2nd or "Direct" method became a fundamental tool in control design due to the typical practical difficulties viz. a) most of the control problems do not have closed analytical solutions; b) from numerical calculations "well behaving within a finite period" the stability cannot be taken for granted. According to Lyapunov, guaranteeing negative time-derivative of the Lyapunov function by relatively simple estimations the stability of the solution can theoretically be guaranteed. However, finding an appropriate Lyapunov function to a given problem is rather an "art" that cannot algorithmically be automated. Adaptivity normally requires slow tuning of numerous model parameters. This process is sensitive to unknown external disturbances, and the tuning rule is determined by numerous other, more or less arbitrary "adaptive control parameters". Furthermore, making the necessary estimations is a laborious, tedious work that normally results in "very strange conditions" to be met for guaranteeing stability of the solution. In the present paper the application of "Robust Fixed Point Transformations" is proposed instead of the Lyapunov technique. It can find the proper solution without any parameter tuning and depends on the setting only of three "adaptive control parameters". As application example direct control of a "Single Input—Single Output (SISO)" system, and a novel version of the "Model Reference Adaptive Control (MRAC)" of a "Multiple Input—Multiple Output (MIMO)" system is presented. Since this method cannot automatically guarantee global stability, as a novelty, a possible adaptive tuning of one of the adaptive control parameters is proposed for SISO systems to keep the control within the local basin of attraction of the proper convergence. Its operation is presented via simulations at first time in this paper.

Keywords Robust fixed point transformation-based adaptive control · Model reference adaptive control · Control parameter tuning · Local stability · Lyapunov's direct method

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J. K. Tar (✉)
Institute of Applied Mathematics, Óbuda University, Budapest, Hungary
e-mail: tar.jozsef@nik.uni-obuda.hu

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1 Introduction

Lyapunov's 2nd Method is a widely used technique in the analysis of the stability of the motion of the *non-autonomous dynamic systems* of equation of motion as $\dot{x} = f(x, t)$. The typical stability proofs provided by Lyapunov's original method published in 1892 [5] (and later on in e.g. [6]) have the great advantage that they do not require to analytically solve the equations of motion. Instead of that the *uniformly continuous nature* and non-positive time-derivative of a positive definite Lyapunov-function V constructed of *quadratic terms* of the tracking and modeling errors of the system's parameters are assumed in the $t \in [0, \infty)$ domain. From that the convergence $\dot{V} \to 0$ can be concluded according to Barbalat's lemma [4] utilizing the uniform continuity of $\dot{V}$. It used to be guaranteed by showing that $\ddot{V}$ is bounded. Due to the positive definite nature of V from that it normally follows that the tracking errors have to remain bounded, or in certain special cases, has to converge to 0. To illustrate the difficulties related to the "orthodox use of Lyapunov's direct method", on the basis of [4, 11, 14], and [13] a brief summary will be given in the next subsection.

1.1 Example for Orthodox Use of Lyapunov Functions

The most "historical" adaptive controllers used in *robotics* are the methods of "Adaptive Inverse Dynamics" and the "Adaptive Slotine–Li" controllers [4]. Since similar observations can be done for both of them, in the present considerations we recapitulate only the latter one. It utilizes subtle details of the Euler–Lagrange equations of motion, viz. that the terms quadratic in the generalized velocity components can specially be symmetrized. In this approach the exerted generalized torque/force components are constructed by the use of the *actual model marked by the symbol* ^ and causes $\ddot{q}$ according to the exact model values:

$$
\begin{aligned}
Q &= \hat{H}(q)\dot{v} + \hat{C}(q,\dot{q})v + \hat{g} + K_D r = H(q)\ddot{q} + C(q,\dot{q})\dot{q} + g \\
e &:= q^N - q, v := \dot{q}^N + \Lambda e, \quad r := \dot{e} + \Lambda e, \tilde{p} := \hat{p} - p \\
C_{ij} &= \frac{1}{2}\sum_z \dot{q}_z \left(-\frac{\partial \hat{H}_{zj}}{\partial q_i} + \frac{\partial \hat{H}_{ij}}{\partial q_z} + \frac{\partial \hat{H}_{iz}}{\partial q_j}\right), \quad Q = Y(q,\dot{q},v,\dot{v})\hat{p} + K_D r
\end{aligned}
\tag{1}
$$

in which q^N and q denote the generalized co–ordinates of the *nominal* and the *actual* motion, K_D and Λ are symmetric positive definite matrices, matrices H, C, and g denote the system's inertia matrix, the Coriolis, and the gravitational terms. The possession of the exact form of the dynamical model makes it possible to linearly separate the system's dynamic parameters p in the expression of the physically interpreted *generalized forces* Q by the use of matrix Y that exclusively consists of known kinematical data. The Lyapunov function of this method is $V = r^T H r + \tilde{p}^T \Gamma \tilde{p}$, with positive definite symmetric matrix Γ. For guaranteeing negative derivative of the Lyapunov function the *skew symmetry* of the C_{ij} matrix and the parameter tuning rule $\dot{\hat{p}} = \Gamma^{-1} Y^T r$ are utilized. The above results well exemplify the difficulties with the application of the Lyapunov function: (a) no unknown external perturbations can be present; (b) for a complex Classical Mechanical System $\hat{p}$ may have many (say m) independent components; besides the elements of the positive definite matrices Λ, K_D we have further $m + (m^2 - m)/2$ independent elements in the positive definite matrix Γ (the main diagonals plus the parameters of the arbitrary *orthogonal matrix* O that can transform a *positive definite diagonal matrix* D into a more general non-diagonal form $\Gamma = O^T D O$); (c) the tuning process is too slow, since it happens according to the matrix Γ in spite of the fact that more explicit information can be obtained for the parameter errors if we subtract $\hat{H}\ddot{q}$, $\hat{C}\dot{q}$, and $\hat{g}$ from both sides of (1) [13]: $\hat{H}\dot{r} + \hat{C}r + K_D r = (H - \hat{H})\ddot{q} + (C - \hat{C})\dot{q} + g - \hat{g} = \Upsilon(q, \dot{q}, \ddot{q})(p - \hat{p})$. Since both the LHS of this equation and Υ are known, an SVD-based generalized inverse of Υ can provide direct information for optimal parameter tuning. Regarding the variation of the "error metrics" from both sides of the 1st line of (1) $H\dot{v}$, $C\dot{v}$, $K_D r$, and g can be subtracted so again some information can be obtained on the modeling errors: $Y\tilde{p} = -K_D r - H\dot{r} - Cr$. The fragment of the Lyapunov function $r^T H r$ itself can serve as a metrics for r. It has the time-derivative $d(r^T H r)/dt = 2r^T H\dot{r} + r^T \dot{H} r = r^T(\dot{H} - 2\,C)r - 2r^T K_D r - 2r^T Y\tilde{p} = -2r^T K_D r - 2r^T Y\tilde{p}$. That is this metrics is kept at bay during the new tuning process by the negative quadratic term and it is perturbed only by a linear one with a coefficient $\tilde{p}$ that converges to zero as the tuning proceeds. That is asymptotic stability can be also maintained without using the original Lyapunov function.

1.2 Adaptive Control Based on Robust Fixed Point Transformations

Certain control tasks can be formulated by using the concept of the appropriate *"excitation"* U of the controlled system to which it is expected to respond by some *"desired response"* r^d. The appropriate excitation can be computed by the use of the *inverse dynamic model of the system* as $U = \varphi(r^d)$. Since normally this inverse model is neither complete nor exact, the actual response determined by the system's dynamics, ψ, results in a *realized response* r^r that differs from the desired one: $r^r \equiv \psi(\varphi(r^d)) \equiv f(r^d) \neq r^d$. The controller normally can manipulate or "deform" the input value from r^d to $r^d_\star$ so that $r^d \equiv \psi(r^d_\star)$. Such a situation can be maintained by the use of some local deformation that can properly "drag" the system's state in time while it meanders along some trajectory. To realize this idea a fixed point transformation was introduced in [12] that is quite "robust" as far as the dependence of the resulting function on the behavior of $f(\bullet)$ is concerned. This robustness can approximately be investigated by the use of an affine approximation of $f(x)$ in the vicinity of $r^d_\star$ and it is the consequence of the strong nonlinear saturation of the sigmoid function tanh (x):

$$\begin{aligned}G(r|r^d) &:= (r+K)\left[1+B\tanh\left(A[f(r)-r^d]\right)\right]-K\\ G(r^d_\star|x^d) &= r^d_\star,\ \text{if } f(r^d_\star)=r^d \text{ then } G(-K|r^d)=-K,\\ G(r|r^d)' &= \tfrac{(r+K)ABf'(r)}{\cosh\left(A[f(r)-r^d]\right)^2}+1+B\tanh\left(A[f(r)-r^d]\right),\\ G(r^d_\star|r^d)' &= (r^d_\star+K)ABf'(r^d_\star)+1.\end{aligned} \tag{2}$$

It is evident that the transformation defined in (2) has a proper ($r^d_\star$) and a false ($-K$) fixed point, but by properly manipulating the control parameters A, B, and K the good fixed point can be located within the basin of attraction of the iteration obtained by the repetitive use of function $r_{n+1} := G(r_n|r^d)$ if the requirement of $|G'(r|r^d)| < 1$ can be guaranteed in the vicinity of $r^d_\star$: if $|G'| \le H$ $[0 \le H < 1]$ can be maintained then a Cauchy sequence is obtained via the iteration that is convergent in the real numbers and it converges to the solution of the *Fixed Point Problem* $r_n \to r^d_\star = G(r^d_\star)$ [12]. Instead of the function tanh any sigmoid function, i.e. any bounded, monotone increasing, smooth function $\sigma(x)$ with the property of $\sigma(0) = 0$ can naturally be used (e.g. $\sigma(x) := x/(1+|x|)$), too. A possibility for applying the same idea outlined in (2) of adaptivity is the application of a sigmoid function projected to the direction of the response-error defined in the n^{th} control cycle as $\vec{h} := \vec{f}(\vec{r}_n) - \vec{r}^d$, $\vec{e} := \vec{h}/||\vec{h}||$, $\tilde{B} = B\sigma(A||\vec{h}||)$, so that $\vec{r}_{n+1} = (1+\tilde{B})\vec{r}_n + \tilde{B}K\vec{e}$. If $||\vec{h}||$ is very small, instead of normalizing with it the approximation $\vec{r}_{n+1} = \vec{r}_n$ can be applied since then the system already is in the very close vicinity of the fixed point.

This idea can be used in the following manner for SISO systems: on the basis of the available rough system model a simple PID controller can be simulated that reveals the order of magnitude of the occurring responses. Parameter K can be so chosen for which the $r+K$ values are considerable negative numbers. Depending on $sign(f')$ let $B \pm 1$ and let $A > 0$ be a small number for which $|\partial G(r|r^d)/\partial r| \approx 1-\varepsilon$ for a small $\varepsilon > 0$. For r^d varying in time the following estimation can be done in the vicinity of the fixed point when $|r_n - r_{n-1}|$ is small: $r_{n+1} - r_n = G(r_n|r^d_n) - G(r_{n-1}|r^d_{n-1}) \approx \frac{\partial G(r_{n-1}|r^d_{n-1})}{\partial r}(r_n - r_{n-1}) + \frac{\partial G(r_{n-1}|r^d_{n-1})}{\partial r^d}(r^d_n - r^d_{n-1})$. Since from the analytical form of $\sigma(x)$ $\frac{\partial G(r_{n-1}|r^d_{n-1})}{\partial r^d}$ is known, and the past "desired" inputs as well as the arguments of function G are also known, this equation can be used for realtime estimation of $\frac{\partial G(r_{n-1}|r^d_{n-1})}{\partial r}$. ε can be tried to be fixed around -0.25 by a slow tuning of parameter A as $\dot{A} = \alpha(\varepsilon_{est} + 0.25)A$ ($\alpha > 0$) to keep the system in the local basin of attraction. The simulations revealed that increasing A resulted in smooth control, decreasing A caused small fluctuations. To avoid the occurrence of such fluctuations instead of a single α different values were chosen for "slow increase" (α_{incr}) and "very fast decrease" was prescribed by $\alpha_{decr} = 20\alpha_{incr}$. In the sequel a simple possible application is outlined for a strongly nonlinear system, the Electrostatic Microactuator (EμA). In connection with this problem in [10] the possibility of tuning A was not considered.

2 A Potential Example for Tuning the Adaptive Control Parameter

Paper [10] was inspired by the work by Vagia, Nikolakopoulos and Tzes who suggested the application of a robust switching PID controller coupled to a feed-forward compensator for controlling an electrostatic micro-actuator (EμA) in [15]. In their approach the precise non-linear model of a given EμA was linearized in certain set-points as typical operating points and the LMI technique was used in the design phase to stabilize separate PID controllers that were determined in the vicinity of these set points. Such kinds of controllers have to switch at the boundaries within which static PID parameters are set. The design typically was made by minimization of quadratic cost functions. The EμA corresponds to a micro-capacitor whose one plate is attached to the ground while its other moving plate is floating in the air. In the present paper the model considered was taken from [15]. Accordingly, the equation of motion of the system is given as follows

$$\ddot{q} = \frac{-b\dot{q} - kq + \varepsilon A_{Pl} U^2/(2(\eta_{max} - q)^2) + Q_d}{m} \tag{3}$$

in which $b = 1.4 \times 10^{-5}$ kg $\cdots$ is the viscous damping of the motion of the EμA in air, $k = 0.816$ N/m is a spring constant, $A_{Pl} = (400 \times 10^{-6}\ \mathrm{m})^2$ denotes the area of the plate, $m = 7.096 \times 10^{-10}$ kg is its mass, $\eta_{max} = 4 \times 10^{-6}$ m is the distance between the plates when the spring is relaxed, q is the displacement of the plates from the relaxed position, $\varepsilon = 9 \times 10^{-12}\ \mathrm{C}^2/(\mathrm{N} \cdot \mathrm{m}^2)$ is the dielectric constant, Q_d denotes the external disturbance forces, and U denotes the control voltage e.g. the physical agent by the help of which the plate's displacement can be controlled. It can be seen that (3) is singular near $q = \eta_{max}$, therefore for controllability allowable displacements of the micro-capacitor's plate in the vertical axis were $q \in [0.1, 1.3] \times 10^{-6}$ m that was deemed necessary in order to guarantee the stability of the linearized open-loop system in [15]. In that paper only responses to step-like inputs were considered.

In the present simulations continuous variation of the nominal motion was prescribed by 3rd order spline functions in which the 2nd derivatives linearly vary with the time within neighboring intervals. A the boundaries of these intervals the accelerations are continuous functions. To study the effect of the modeling errors in the simulations the controller assumed the *approximate model parameters* as follows: $\hat{A}_{Pl} = 0.8\mathrm{A}_{\mathrm{Pl}}$, $\hat{m} = 1.2$ m, $\hat{b} = 1.2\mathrm{b}$, $\hat{k} = 1.2\mathrm{k}$, $\hat{\eta}_{max} = 0.8\eta_{\mathrm{max}}$, and $\hat{\varepsilon} = 0.8\varepsilon$. Their effects can well be traced in the first row of Fig. 1 that reveals erroneous trajectory and acceleration (response) tracking in the case of a common PID controller defining the prescribed relaxation as $\ddot{q}^{Des} = \ddot{q}^N + 3\Lambda^2(q^N - q) + 3\Lambda(\dot{q}^N - \dot{q}) + \Lambda^3 \int_{t_0}^{t} (q^N(\xi) - q(\xi))d\xi$ with $\Lambda = 8500$/s in which $q^N(t)$ denotes the nominal trajectory. The *adaptive controller* used the following parameters: $K = -500\mathrm{m/s}^2$, $B = 1$, and as an initial estimation $A_{ini} = 3 \times 10^{-3}\ \mathrm{s}^2/\mathrm{m}$, and $\alpha_{incr} = 10^3$/s. As it can be seen from the 2nd and 3rd rows of Fig. 1, the tracking errors quickly relaxed, and in the non-transient phase of the motion fine tracking and acceleration tracking were realized. It is also clear that the initial A_{ini} parameter was "overestimated", it

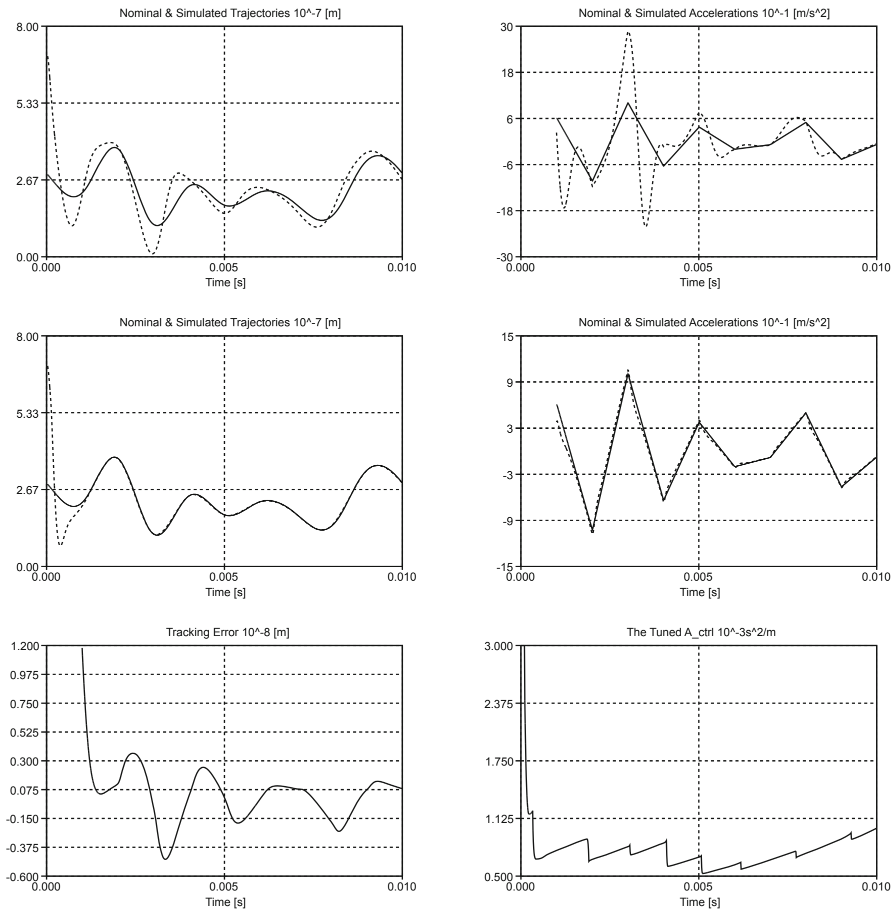


Fig. 1 The operation of the *non–adaptive* controller (1st row), and that of the *adaptive one*: the trajectory and acceleration tracking (2nd row), the tracking error and the tuned adaptive control parameter A (3rd row)

was quickly decreased in the initial phase of the motion and later was finely tuned to keep the control near the center of the local basin of attraction by decreasing and increasing sessions. *The simulations well exemplify the expected behavior of the simple adaptive controller.*

3 The Traditional and the Novel MRAC Approaches

The *MRAC* technique is a popular and efficient approach in the adaptive control of nonlinear systems e.g. in robotics. A great manifold of appropriate papers can be found for the application of MRAC from the early nineties (e.g. [4]) to our days (e.g. [2]). One of its early applications was a breakthrough in adaptive control. In [7] C. Nguyen presented the implementation of a joint-space adaptive control scheme that was used for the control of a non-compliant motion of a Stewart platform-based manipulator that was used in the *Hardware Real-Time Emulator* developed at Goddard Space Flight Center to emulate space operations. The mainstream of the adaptive control literature at that time used some parametric models and applied Lyapunov's "direct method" for parameter tuning (e.g. [4, 3]). The essence of the idea of the *MRAC* is the transformation of the actual system under control into a well behaving reference system (reference model) for which simple controllers can be designed. In the practice the *reference model* used to be stable linear system of constant coefficients. To achieve this simple behavior normally special adaptive loops have to be developed.

In our particular case the reference model can be *the nonlinear analytical model of the system built up of its nominal parameters*. Assume that on purely kinematical basis we prescribe a trajectory tracking policy that needs a desired acceleration of the mechanical system as $\ddot{q}^D$. From the behavior of the reference model for that acceleration we can calculate the physical agent that could result in the response $\ddot{q}^D$ for the reference model (in our case the generalized force components U^D). The direct application of this U^D for the actual system could result in different response since its physical behavior differs from that of the reference model. Therefore it can be "deformed" into a "*required*" U^{Req} value that directly can be applied to the *actual system*. Via substituting the realized response of the actual system $\ddot{q}$ into the reference model the "realized control action" U^R can be obtained instead of the "desired one" U^D. Our aim is to find the proper deformation by the application of which U^R well approaches U^D, that is at which the controlled system seems to behave as the reference system. The proper deformation may be found by the application of an iteration as follows. Consider the iteration generated by some function G as $U_{n+1}^{Req} = G(U_n^{Req}, U_n^R, U_{n+1}^D)$ in which n is the index of the control cycle. For slowly varying desired value U^D can be considered to be constant. In this case the iteration is reduced to $U_{n+1}^{Req} = G(U_n^{Req}, U_n^R | U^D)$ that must be made convergent to $U_\star^{Req}$. It is evident that the same function G and the same considerations can be applied in this case as that detailed in Sect. 1.2. In the sequel an possible application is outlined via simulation.

3.1 Novel MRAC Control of a 3 DOF System

The sketch of the system considered is given in Fig. 2. In the dynamical model it was assumed that the hamper is assembled to the end of the beam at its mass center point. The Euler-Lagrange equations of motion also given in Fig. 2 are valid in this case.

The dynamic parameters of the *actual system* were assumed to be $M = 30$ kg (the mass of the cart moving in the horizontal direction), $m = 10$ kg (the mass of the hamper), $L = 2$ m (the length of the beam), $\Theta = 20\ \mathrm{kg} \cdot \mathrm{m}^2$ (the inertia momentum

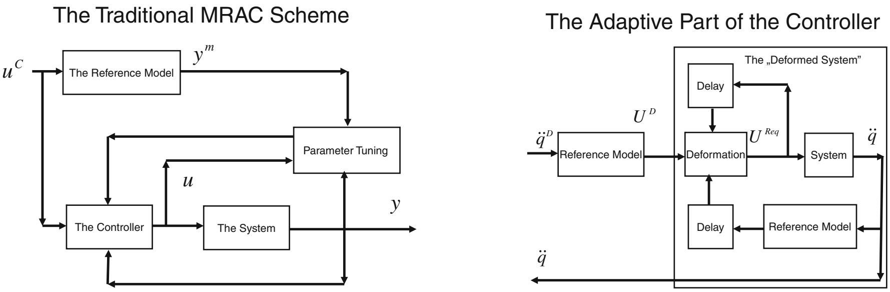
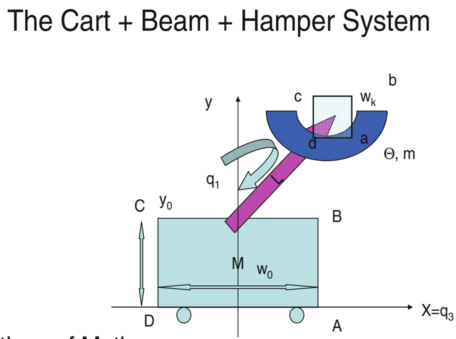


Fig. 2 The "traditional" MRAC scheme (LHS) operated by some Lyapunov function based parameter tuning, and the novel one (RHS) based on "Robust Fixed Point Transformations"



Equations of Motion:

$$\begin{bmatrix} (mL^2+\Theta) & \Theta & mL\cos q_1 \\ \Theta & \Theta & 0 \\ mL\cos q_1 & 0 & (m+M) \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} + \begin{bmatrix} -mgL\sin q_1 \\ 0 \\ -mL\sin q_1 \dot{q}_1^2 \end{bmatrix} = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix}$$

Fig. 3 The paradigm considered: the cart + beam + hamper system

of the hamper), and $g = 10\ \mathrm{m/s^2}$ the gravitational acceleration. For the simplicity the mass and the inertial momentum of the beam was neglected. The appropriate "nominal parameters of the reference model" quite considerably differed from the actual one as follows: $\hat{M} = 60$ kg, $\hat{m} = 20$ kg, $\hat{L} = 2.5$ m (only within the *dynamic model*, the *kinematic model* that is responsible for trajectory tracking used the exact value), $\hat{\Theta} = 50\,\mathrm{kg \cdot m^2}$, and $\hat{g} = 8\,\mathrm{m/s^2}$. The appropriate results for the *non-adaptive* and *adaptive* approaches for the PID type prescribed tracking error relaxation as $\ddot{q}^D = \ddot{q}^N + 3\Lambda^2(q^N - q) + 3\Lambda(\dot{q}^N - \dot{q}) + \Lambda^3 \int_{t_0}^{t} (q^N(\xi) - q(\xi))d\xi$ with a small $\Lambda = 1/s$ resulting in lose tracking [$q^N(t)$ denotes the nominal trajectory that was generated by 3rd order spline functions to produce linearly varying nominal acceleration within well defined intervals with continuous connection at their boundaries]. The fixed *adaptive parameters* were: $K = 7000$, $B = -1$, and $A = 10^{-5}$. Representative results are given in Fig. 3. The improvement due to the adaptivity is quite illustrative:

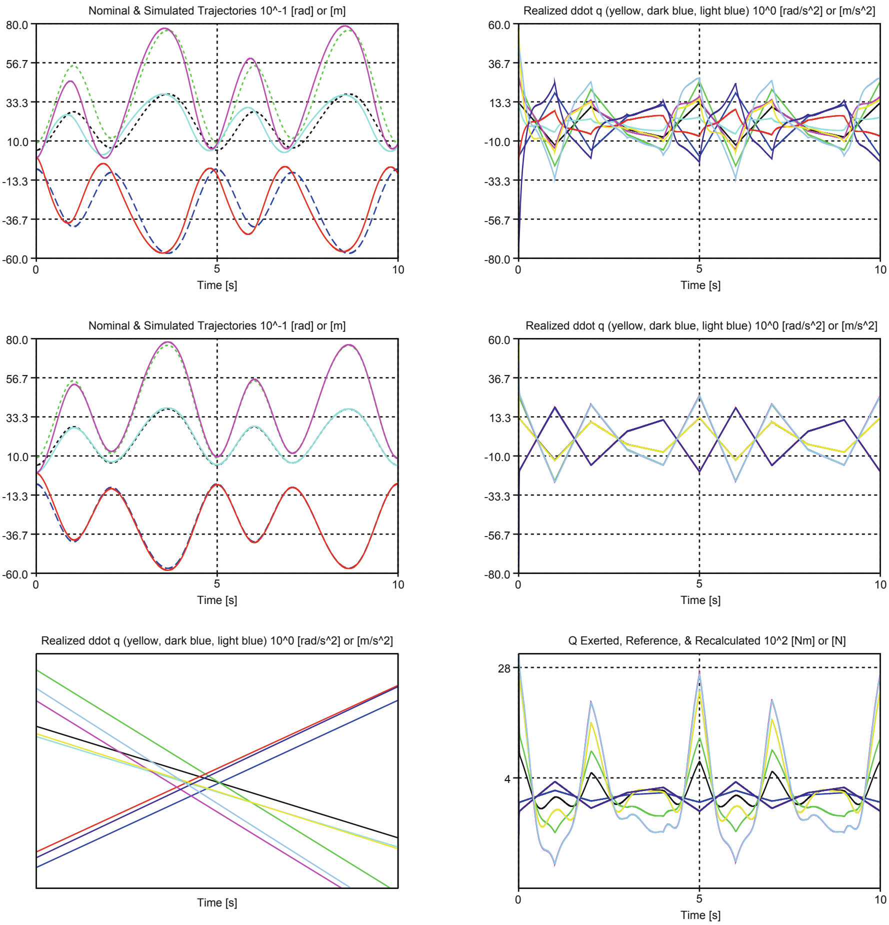


Fig. 4 The operation of the *non–adaptive* controller (1st row), and that of the *adaptive one* (2nd and 3rd rows) (color coding for the trajectories: $q_1^N = black$, $q_2^N = blue$, $q_3^N = green$, $q_1 = bright \quad blue$, $q_2 = red$, $q_3 = magenta$); for the accelerations: $\ddot{q}_1^N = black$, $\ddot{q}_2^N = blue$, $\ddot{q}_3^N = green$ for the nominal values, $\ddot{q}_1^D = bright \quad blue$, $\ddot{q}_2^D = red$, $\ddot{q}_3^D = magenta$ for the kinematically corrected *"desired"* values, and $\ddot{q}_1 = yellow$, $\ddot{q}_2 = dark \quad blue$, $\ddot{q}_3 = light \quad blue$ for the realized values)]; generalized forces: exerted: $Q_1 = black$, $Q_2 = blue$, $Q_3 = green$; calculated from the reference model: $Q_1 = bright \quad blue$, $Q_2 = red$, $Q_3 = magenta$, recalculated from the realized acceleration and the parameters of the reference model: $Q_1 = yellow$, $Q_2 = dark \quad blue$, $Q_3 = light \quad blue$)

the *nominal trajectories* are well approximated by the *"realized"* (simulated) ones while the Q^D forces calculated from the *reference model* are well approximated by the recalculated Q^R values and both considerably differ from the really exerted forces Q^{Req} that is needed for properly manipulating the actual physical system under control. *This altogether proves that the MRAC controller works, and the "deformed part" in the RHS of Fig. 4 really behaves like the reference model.*

4 Concluding Remarks

In this paper possible substitution of Lyapunov's "Direct Method" by the application of "Robust Fixed Point Transformations (RFPT)" in the adaptive control of nonlinear dynamic systems was suggested. For this purpose two typical frameworks, the "Model Based Computed Force Control" using approximate model and the "Model Reference Adaptive Controllers" were considered for a SISO and a MIMO system to be controlled. It was shown that this latter method is far less complicated and works with far less "arbitrary" parameters than the Lyapunov function based tuning approaches. Illustrative examples obtained by simulation have shown that in spite of the fact that this latter method cannot guarantee global asymptotic stability, it can work for a wide set of physical systems to be controlled. For compensating this deficiency for the case of SISO systems additional tuning of one of the altogether three control parameters proposed to keep the control near the center of the local basin of attraction of the RFPT transformation. Its operation was illustrated via simulations. In the next phase of the research this tuning is expected to be extended for the adaptive control of MIMO systems.

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