Section IV Commentary: The Perspective of Mathematics Education

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The ten chapters selected for this section were chosen to reflect the perspective of Mathematics Education. The first task of this commentary is hence to reflect on this perspective in comparison or contrast with the perspectives of the other three sections of the book: Mathematics and Philosophy, Psychology, Stochastics. The next consideration is how the topics of the ten chapters fit within the perspective of Mathematics Education and the contributions they make to our understanding of probability within Mathematics Education. This leads to further suggestions for extension of the projects and issues covered in the chapters. Finally, some comments are made about other past, current, and potential contributions from researchers in probability to the field of Mathematics Education.

1 Why Mathematics Education?

In line with the title of the series of which this volume is a part, "*Advances in Mathematics Education*," on the one hand, it seems appropriate to label the final section, Mathematics Education. On the other hand, the question then arises in relation to how the contributions in this section are different from those in the other three sections if the entire series is about Mathematics Education.

To think about this question, it is necessary to explore the scope of Mathematics Education itself. Unfortunately, in this series it appears to be taken as an undefined term. For the purposes of this commentary, Mathematics Education is assumed to encompass broadly the "teaching and learning of mathematics." Although teaching usually comes first in such a phrase, it is learning that is the goal of Mathematics Education. Students and teachers are hence the focus of studies in Mathematics Education. The environments in which they interact, however, influence the outcomes of

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the education process. These environments include the local culture, language and learning issues, and economic conditions, as well as the all-important curriculum set down by education authorities.

As well as the environments within which the learning and teaching take place, a significant factor is the method by which the mathematics, in this case probability, becomes a part of the learner's repertoire. Over time suggested methods have evolved, reflecting to some extent the impact of research, for example, into the detail of students' development of conceptual understanding and into the requirements of the pedagogy of teachers that will assist that development. These broad interpretations of Mathematics Education allow the suggestion that the initial section of this book, on Mathematics and Philosophy, contributes to Mathematics Education in relation to curriculum content and perhaps pedagogy. The section on Psychology definitely contributes to the teacher's appreciation of the learner's development of understanding, at times supplemented by suggestions for the curriculum. The section on Stochastics, in developing links with statistics, not only provides further implications for the curriculum, but also offers advice on teaching, including the use of technology.

If these other sections contribute to the position of probability within Mathematics Education, how does the final section contribute something extra? It appears that the chapters in this section were mainly chosen because they focus more specifically on some feature of learning probability that is classroom based and the product of the author/s' research. Except for the chapter on frameworks by Mooney, Langrall and Hertel, all of the chapters in this section present data of some type related to interventions with groups of learners ranging from age $5\frac{1}{2}$ years to practicing secondary teachers. Four chapters deal with elementary school learners, three report on high school students, one discusses longitudinal development from Grade 4 to graduate school, and one considers the thinking of secondary teachers. The topics range from basic concepts such as sample spaces and equally likely outcomes to the law of large numbers, with problems related to binomial and conditional probabilities considered at the high school level.

2 Contributions to This Volume

In grouping the chapters in the section on Mathematics Education, four perspectives from the probability curriculum are addressed: attitudes and beliefs, with the potential for a cultural influence on them; the necessity to understand and appreciate the sample space as underlying the calculation of probabilities; the law of large numbers; and two advanced secondary topics, binomial and conditional problems. In various ways these chapters, as well as the theoretical chapter on frameworks, are a representative sample of the classroom research carried out in the past decade or two. Only the chapter by Mamolo and Zazkis begins to move beyond this focus into considering the educational imperative to look beyond students and content to the realms of teachers and the pedagogical content knowledge required to address the needs of learners as they are developing the understanding exemplified in the other chapters.

What do we learn and what further questions should be asked at the conclusion of the studies reported here? In terms of attitudes and beliefs towards probability and how they are influenced by several chance games or activities, Nisbet and Williams provide much detail but it would be very useful in assessing outcomes if further information were also made available, for example, about what actually happened in the intervention in terms of instructions given or worksheets provided to students, why the exact same lesson was presented twice (by the researcher and then the classroom teacher) and how that affected the interest level of the students, and what probability content was expected to be learned by students. Although "test" questions were asked, there is no indication of their content and their relationship to the games played. The issue of playing games and gaining insight into probabilistic understanding has been considered for a long time (e.g., Bailey 1981; Bright et al. 1981) and perhaps a longer intervention that could document transfer, for example, like that carried out by Prediger and Schnell, would lead to even more positive outcomes for attitudes and beliefs.

Sharma presents a different perspective on beliefs about probability in looking at their direct influence on responses to probability questions rather than their influence on general views of chance. Beliefs about events derived from Fijian culture are used to explain students' interview responses to questions about equally likely outcomes and independence. Because studies from other cultures have found similar results (e.g., Watson et al. 1997), the question arises about the relative frequency of the types of responses in different cultural settings around the world. Although suggestions are made for teacher interventions, the possibility of knowing where the students' cultural beliefs begin may not be all that helpful in moving them meaningfully into the abstract mathematical sphere. There is the danger of students learning that there is one set of rules in the world of mathematics and another set when they go home to discuss chance events with their parents. This is reminiscent of the pre-service teacher known to the author who explained completely correctly the theory behind tossing a coin repeatedly but added, "But I will always call 'tails' because it is lucky for me." Throughout this volume pleas are made for a curriculum that specifically addresses three perspectives on probability—theoretical, frequency, and subjective. Listed in this order there is implicit support for a hierarchical order of importance with theoretical at the top and subjective at the bottom. Sharma and other authors in this section raise the importance of this triad and lament the lack of research interest, Sharma in particular, in the subjective sphere. In thinking from the students' perspectives of beginning as subjective learners, rather than from the historical perspective of probability as pure mathematics to be embedded in students' minds, perhaps the teachers' knowledge of "student as learner" can contribute to an improved pedagogical content knowledge in transitioning from subjective, to frequentist, to theoretical probability understanding.

Once moving past the attitudes and beliefs that can influence outcomes related to determining probabilities, what is the foundation concept without which Mathematics Education cannot hope to build meaningful student understanding of probability?

Excluding the chapters on technical problems (binomial and conditional) it is possible to imagine a fierce debate between the authors of the three chapters focusing on sample spaces and the authors of the two chapters on the law of large numbers. It is not possible to calculate probabilities without valid sample spaces but the point of practical probability is lost without the law of large numbers.

The importance of sample space to mathematics education as seen through the eyes of researchers is highlighted in the varied approaches in the three chapters on the topic in this section. Nilsson "begins at the very beginning," describing the authentic environment of a Grade 6 classroom where students are asked to determine the sample space, and hence the chance of picking a yellow piece of candy from a bag, by repeated replacement sampling. Hence the students start within a frequency/experimental context to predict the theoretical context, which can ultimately be determined by opening the bag to count the contents. For Nilsson the sample space is the link between sample and population and although increasing sample size is a factor, the law of large numbers is not an explicit feature. The proportion of yellow pieces within the sample size used (number of single draws with replacement) becomes the salient issue for the Grade 6 students. Although the conceptual expectation is sophisticated, the context is basic and does not change throughout the teaching. A follow-up research question could usefully address the transfer of learning to another context.

In contrast to the Nilsson approach, Maher and Ahluwalia present a longitudinal cross-sectional study that focuses much more specifically on determining the specific elements of a sample space only from a theoretical perspective based on counting techniques and combinatorial reasoning. The increased sophistication of arguments is mathematical rather than statistical in nature; hence, model building rather than sampling is the mechanism for reaching results. Starting when the students are in Grade 4, they are allowed to explore, interact, and create models, which are then rejected, accepted, or extended by their fellow students over the years until the last description is of one of the participants as a graduate student. In contrast to Nilsson who uses only one context for the consideration of the sample space, Maher and Ahluwalia use several settings to follow increasingly complex configurations to model the sample spaces. In beginning and ending in the world of theoretical probability, the issue of the law of large numbers does not arise.

Although still based on a sample space, the chapter by Mamolo and Zazkis provides a very different perspective in setting up an infinite sample space of all real numbers between 0 and 10. Teachers are then given the task to adjudicate an imagined argument between two students about the probably of one of them identifying the secret number chosen by the other. Although any mathematical resolution is highly theoretical and based in understanding the infinite cardinality of the real numbers, the discussion contextualizes the responses in terms of the teachers' conceptualizations of randomness, the meaning of "real" in real numbers, the concreteness or otherwise of real numbers, and the relationship of a "zero" chance and impossibility. In considering the various potential beliefs of students about the context presented, the message becomes similar to that of Sharma in relation to the Fijian culture that also influences students' decisions on answers to probability problems. In the case of Mamolo and Zazkis, the culture rests in the level of understanding of the mathematics of infinity and infinite sets, and it must be assumed that appreciating sets of measure zero is likely to be beyond the reach of school students and most of their teachers. As a topic such as this is not in any school curriculum, the practicalities of dealing with it might be questioned within the usual realms of Mathematics Education. The apparent paradox of dealing with actual numbers such as 3 or 6.79 and saying they have a probability of zero of being guessed by another person, is similar to the paradox of the expected number of children in families if they continue to have children until the family has an equal number of boys and girls. Any family that is imagined will eventually be of finite size but the expected (average) number of children is infinite (Straub 2010). The culture of social understanding in Fiji and the culture of pure mathematical understanding in the real number system are very different but both can be seen as significant influences on decision making in probabilistic situations.

The approaches to explore children's understanding of the law of large numbers by Pararistodemou and by Prediger and Schnell both are based on computer game simulations, which facilitate completing a large number of trials more quickly than doing them by hand. More details of the goal and operation of the space game and how situated abstractions are developed by the children would have been helpful in interpreting the outcomes related to students' expressed robust intuitions for the law of large numbers, as presented by Pararistodemou. The students' responses in the Prediger and Schnell study are analyzed with a detailed scheme that follows the development of their appreciation of the importance of a large number of trials through their explanation of bets on which of four "animals" will win a race. The distinction between the internal number of trials within a game and the total number of games played is useful in tracking development because each can be separated (large and small) to create a two-dimensional grid. The question might be asked of the ultimate importance of the large number of trials over a large number of games but the analysis suggests the students who were the focus of the investigation realized this. It would be interesting to apply this model in an analysis of the Pararistodemou data as one suspects it would support the conclusions claimed there. The ultimate question for both of these studies is: What is the evidence of transfer of understanding of the law of large numbers to other settings? This question should provide a starting point for future research.

The chapters of Sanchez and Landin and of Huerta focus on particular, more advanced topics in the high school probability curriculum, namely binomial problems and conditional probability problems. Sanchez and Landin focus on a structural approach to analyzing students' responses to the problems in a manner similar to that suggested by Mooney et al. at the beginning of the section. Although structures are designed to display increasingly complex understanding, if the highest level of response is the "correct" answer that may be shown in a strictly procedural fashion without any accompanying description of the associated understanding, there may be a question of whether the solution should genuinely be regarded as the most complex structurally. Although the potential confounding by the alternate statement of the question is noted in explaining the results, this is not clarified to the extent of distinguishing between two possibilities in detail. A further discussion of the implications of the results, either in terms of teaching or in terms of desirable problem statements, would be useful to teachers.

In a similar fashion, Huerta presents an even more detailed account than Sanchez and Landin, of the solving of conditional probability problems. Using a trinomial graph to map the relationships among the components of a ternary problem, he presents an exceedingly complex model. Again the model is used to classify student solutions to conditional problems and particular areas of error are highlighted. Although interesting from a theoretical point of view, Huerta gives no suggested teaching intervention or trajectory to avoid the difficulties observed and claims the problems arise from the "usual teaching" based on techniques. This is unfortunate given that the claim is also made that "the most efficient [approach] was the arithmetical one of using a 2×2 table." Both Huerta and Sanchez and Landin are concerned with obtaining the correct answers to conventionally stated problems, and although this is obviously important, a display of a correspondingly high level of descriptive understanding is also necessary.

As noted in relation to several of the chapters, when employing concrete contexts such as games (either hands on or with technology) to introduce probability concepts, it is necessary to conduct follow-up research to find out if the understanding is meaningful and transferrable. The dilemmas are sometimes explored within the original contexts (e.g., Prediger and Schnell) and Maher and Ahluwalia provide both this kind of evidence and evidence of transfer across contexts in their longitudinal study. Transfer is a continuing issue with research into both students' learning and teachers' pedagogical content knowledge.

It is also possible to ask whether some of the chapters from other sections of this volume can be claimed to contribute to Mathematics Education in the sense of improving learning for students. The chapter by Jolafee, Zazkis, and Sinclair, for example, reports on learners' descriptions of randomness and the chapter by Martignon suggests the introduction of tools for dealing with risk and decision-making under uncertainty. Careful reading of other chapters is also likely to reveal hints for improved classroom practice.

Mooney, Langrall, and Hertel provide a transition of thinking, from the section focusing on Psychology to this one on Mathematics Education. They claim that in his observations of children Piaget was not interested in the influence of education on the development of the thinking he observed and note the influence of Fischbein in moving to the need to acknowledge the interaction of developmental thinking with the context of education. Biggs and Collis (1982), for example, extended Piaget's idea to develop a "structure of observed learning outcomes" (SOLO) without necessarily going behind the observations to make assumptions about their origin. Such a developmental framework, if understood by teachers can be used to observe a current level of development and devise learning activities to assist students in moving to higher levels. From a Mathematics Education perspective, this is the usefulness of such frameworks that have their basis in Psychology. In using such a framework to describe solutions to binomial problems, Sanchez and Landin could provide a useful extension to their research by suggesting detailed specific classroom interventions to assist in moving responses to higher levels. Further, the research of Huerta on conditional probably could be more useful for teachers in planning lessons if he included a similar developmental framework with details of students' increasingly appropriate steps toward solutions. It may be that other authors, such as Martignon and Chilsi and Primi, who write in the Psychology section, provide contributions in this area.

At the end of their chapter, Mooney et al. discuss the influence of probability frameworks on the recently released Common Core State Standards for Mathematics (CCSSM) (Common Core State Standards Initiative 2010) in the United States. It would be useful to consider similar initiatives in other countries. In New Zealand, for example, the title of the country's Mathematics curriculum was changed in 2007 to Mathematics and Statistics, reflecting the importance of statistics for students entering the world of the twenty-first century (Ministry of Education 2007). At every level of the curriculum, one to eight, Probability is given a similar heading to Statistics under the overall Statistics section. Although not specifically based on a particular framework, the content reflects research into probabilistic understanding over the last few decades. Similarly *The Australian Curriculum: Mathematics* (Australian Curriculum, Assessment and Reporting Authority 2013) for Years Foundation to 10 contains one of three major sections on Statistics and Probability, with Chance as a subheading for content at every year level. Although not as research-based as the New Zealand document, it recognizes the development of intuitive ideas in the early years, which is not acknowledged in the CCSSM content. The approach of the CCSSM to Probability appears more mathematical (i.e., theoretical) than would seem appropriate from the research findings of recent years, particularly those that illustrate the development of understanding of variation and expectation in quite young children (e.g., Watson 2005).

In a recent review of children's understanding of probability that could also contribute to curriculum development and classroom planning, Bryant and Nunes (2012) cover the four cognitive demands they believe are made on children learning about probability: randomness, the sample space, comparing and quantifying probabilities, and correlations. Although somewhat limited in scope and sources, the review highlights some of the issues that are noted elsewhere in this commentary and require further research. There is a need for more detailed analysis of classroom interventions (e.g., Nilsson; Prediger and Schnell) and of longitudinal studies (e.g., Maher and Ahluwalia). Again this reflects the call of Mooney et al. for considering the practical side of applying the psychological frameworks they introduce. This approach is indeed the Mathematics Education perspective on probability that contributes to this volume in the "*Advances in Mathematics Education*" series.

3 Other Aspects and Future Directions

What are some of the other aspects of Probability education within the scope of Mathematics Education that deserve attention of researchers and educators? Although throughout most of the twentieth century, probability was treated as pure mathematics in school curricula, the necessity for probability in making decisions about the confidence one has in carrying out statistical tests, has meant that these ideas, often treated informally, have filtered down to the school curriculum. Informal probability has had to be applied to decision making. This intuitive conceptual use of probability in judging uncertainty is not related to counting elements in sample spaces or calculating basic probabilities but is equally important. It may involve frequency representations or even subjective decision making with limited evidence but it provides a foundation for a critical use of probability in formal inference later. Makar and Rubin (2009) provide an excellent framework for informal inference to get students started: *generalizations* for populations are made based on *evidence* collected from samples, acknowledging the *uncertainty* in the claim. Studies around students' informal decision making and uncertainty are beginning to appear on the research scene (e.g., Ben-Zvi et al. 2012).

Again in looking behind the formal calculation of probabilities of events, there is the issue of the relationship of expectation and variation as students are developing their conceptions. As noted by Prediger and Schnell, students experience variation in their small sample trials, which at times confounds their predictions of outcomes. Given that the traditional exposure in the school curriculum has been to expectation as probability (or averages) first and to variation as standard deviation much later, the research of Watson (2005) with students from ages 6 to 15 years suggests, perhaps surprisingly, that students develop an intuitive appreciation of variation before an intuitive appreciation of expectation. The complex development of interaction between the two concepts has been shown based on in depth interviews with students employing protocols in chance contexts, as well other contexts relevant in statistics (Watson et al. 2007). This fundamental conflict of variation and expectation underlies decision making and is as important a topic to explore as is the law of large numbers, with which there are connections.

Except for the chapter by Mamolo and Zazkis, which looks at the content knowledge of secondary teachers, the chapters in this section focus on school student learning, its development, associated success, attitudes and beliefs. Since the seminal work of Green (1983), students' understandings and misunderstandings of chance, probability and randomness have been a focus of research. This and the suggestion of developmental models (e.g., Mooney et al.) have contributed much to curricula around the world and to suggestions for teaching, for example, from the National Council of Teachers of Mathematics in the US (e.g., Burrill 2006). In other volumes (e.g., Jones 2005), suggestions are made explicitly as to how teachers can assist learners in developing their probabilistic understanding. Pratt (2005), for example, provides ample justification for the use of technology in terms of building models, testing conjectures, completing large-scale experiments, and considering various contexts linked to basic models. Other researchers make specific suggestions for building concepts based on observed developmental frameworks (such as those reviewed by Mooney et al.) to cover topics such as compound events, conditional probability and independence (Polaki 2005; Tarr and Lannin 2005; Watson and Kelly 2007, 2009). Although a step in the right direction, in contributing to teachers' knowledge of students as learners and the difficulties they are likely to meet, there is still the dilemma of the actual teacher encounter with the confused or blank look of a student who is stuck or has no idea where to start. Prediger and Schnell begin to offer ideas in this regard with their detailed case study of two learners with a betting game.

Compared to student understanding of probability, much less is known about teachers' development of understanding for teaching, especially recognizing how broad it must be in the light of the work of Shulman (1987) on the seven types of knowledge required for teaching and the adaptations by Ball and colleagues for "Mathematical knowledge for teaching" (e.g., Hill et al. 2004). Many variations on these beginnings have been suggested and in some instances the pedagogical content knowledge (PCK) of Shulman has been expanded to encompass not only "pedagogical" knowledge and "content" knowledge and their interaction, but also knowledge of students as learners (Callingham and Watson 2011). Given the previous decades of research on students' errors and development of understanding, knowledge of students as learners must be a critical component of the knowledge teachers bring to the classroom. One approach to exploring such knowledge in probability is based on either asking teachers to suggest both appropriate and inappropriate responses to particular questions or asking teachers to suggest remedial action for authentic inappropriate student responses to questions (Watson and Nathan 2010). This approach may help teachers become aware of the potential difficulties students experience but there are also the issues of planning for teaching episodes and responding on-the-fly to students' unusual classroom contributions. These aspects cannot be explored in teacher interviews or surveys, but must be observed in real time in classrooms. Brousseau et al. (2002) presented an early detailed account describing interactions when teaching probability and statistics in Grade 4, without specific reference to PCK but with insights into what students needed in order to take on the desired understanding. Similarly, observing two Grade 5 classrooms for two lessons on probability, Chick and Baker (2005) found the teachers differed in their content knowledge as well as their PCK and, although there were interactions with students noted, these exchanges were not documented in detail with the knowledge of students as learners being made explicit. It is the ability to react meaningfully on-the-spot in response to a student's answer or question that combines content knowledge and pedagogical knowledge with knowledge of students as learners. Being well-read on the research into students' potential errors and developmental progressions is a great help to teachers but much experience is needed in starting "where the student is at" and not making assumptions based on a formulaic description from a text book.

More research is needed on all aspects of teachers' PCK with respect to probability including documented experiences with students in actual classrooms. This is likely to be time-consuming with much non-relevant material included in recordings. Perhaps a starting point could be based on earlier research related to cognitive conflict (Watson 2002, 2007), where students are presented with genuine conflicting responses from other students and asked to respond. Such genuine student responses could be used as starting points with pre-service or in-service teachers in workshops where they debate the best way to respond. A model for this is provided by Chernoff and Zazkis (2011) with pre-service teachers. When given an inappropriate response to a sample space problem the pre-service teachers could only offer didactic suggestions that did not appreciate the starting point for the student response. It was not until the pre-service teachers were themselves given a sample space problem they could not solve and were dissatisfied with a similar response from the lecturer that they saw the point of "starting where the learner is at." It seems likely that the chapter by Jolafee, Zazkis, and Sinclair earlier in this volume may provide further examples in relation to children's ways of talking about randomness.

Mathematics Education is about teaching and learning. It would appear that finding evidence of facets of student learning has been the major focus of research based around classrooms. Deep thinking needs to occur into the place of teaching in mathematics education and how it can be enhanced to provide improved support for the learning of probability.

4 Final Comment

The existence of a volume of this size solely devoted to Probability in a series on *Advances in Mathematics Education* is a measure of the increased interest in research on the understanding, learning, and teaching of probability in the last 30 years. This growth reflects a growing academic field of Mathematics Education, as well as an intrinsic interest of researchers in the complex topic of probability. The appearance of probability, with one interpretation or another, in the curricula of many countries for over 25 years (e.g., NCTM 1989), is evidence of a synergistic relationship of these two perspectives of Mathematics Education: the outcomes of research have influenced curriculum writers of the importance of probability for school students and its presence in the curriculum has encouraged more research on the topic. It is critical that this continued research focus on teachers as well as students with the aim of producing a generation of citizens who understand the chances and risks involved in the decisions they make.

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