Preface to Perspective I: Mathematics and Philosophy

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> The theory of probability has a mathematical aspect and a foundational or philosophical aspect. There is a remarkable contrast between the two. While an almost complete consensus and agreement exists about the mathematics, there is a wide divergence of opinions about the philosophy. (Gillies 2000, p. 1)

Within the wide divergence of opinions about the philosophy of probability, there is one significant bifurcation that has been recurrently acknowledged since the emergence of probability around 1600 (Hacking 1975). Hacking describes this duality of probability as the "Janus-faced nature" (p. 12) of probability, explaining "on the one side it is statistical, concerning itself with stochastic laws of chance processes" (ibid.); and "on the other side it is epistemological, dedicated to assessing reasonable degrees of belief in propositions quite devoid of statistical background" (ibid.). Although the phrase "Janus-faced" continues to be used throughout probability (related) literature, the terms used to describe the two different faces (i.e., the different theories or interpretations) of probability have not been similarly adopted.

In fact, the nomenclature associated with the duality of probability is (for various reasons) inconsistent. For example, Hacking (1975) used the terms "aleatory" and "epistemic" for the two faces. Gillies (2000), while adopting Hacking's epistemic, found the term aleatory "unsatisfactory" (p. 20) preferring the term "objective" instead. Although different terms (all with their own specific issues) have been used by different individuals to describe the duality of probability, the terms that (perhaps) most familiarly represent the Janus-faced nature of probability are "Bayesian" and "frequentist" or, respectively, "subjective" and "objective." (See McGrayne 2011 for a detailed account of the history of the Bayesians and the frequentists.)

Arguably, the theory of probability, currently, has four (not two) primary interpretations. Whereas "logical" and "subjective" denote edges of the division within subjective or Bayesian (or "belief-type" Hacking 2001) theory, "frequency" and "propensity" denote the extremes of the division within the objective or frequentist

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(or "frequency-type" Hacking 2001) theory. Worthy of note, issues surrounding the terms used for these four main interpretations persist. For example, Gillies (2000) notes that "subjective" is used, concurrently, as a general classifier and as a specific theory. Resolutions to this issue include Hacking's (2001) use of the term "personal" instead of "subjective" when discussing specific theory. Regardless of the terminology used, however, the four primary interpretations in probability theory are not likewise found in the field of mathematics education.

In the field of mathematics education, only three (currently) dominant philosophical interpretations of probability are present: classical, frequentist and subjective. Akin to probability theory, nomenclatural issues (e.g., inconsistent use of multiple terms) also exist. The classical interpretation of probability in mathematics education is known alternatively as "a priori" or "theoretical;" whereas the frequentist interpretation of probability is differently known as: "a posteriori," "experimental," "empirical" or "objective." Through informal consensus, despite the multitude of terms, when an individual uses the term "classical probability" or "frequentist probability," there is little confusion. Unfortunately, the same cannot be said for the "subjective probability."

Chernoff (2008), in an examination of the state of probability theory specific to the field of mathematics education, concluded that the state of the term "subjective probability" is *subjective*. There is, once again, inconsistent use of multiple terms (e.g., "subjective," "Bayesian," "intuitive," "personal," "individual," "epistemic," "belief-type," "epistemological" and others). Moreover, Chernoff also determined that the term subjective probability in mathematics education has dual meaning—"subjective probability" is concurrently used as a general classifier and as a specific theory. In an attempt to rectify the dual usage of the term, Chernoff suggested subjective probability remain the general classifier and a further distinction be made, more aligned with the distinction found in probability theory, for the specific theory. In further examining parallels between mathematics education and probability theory, Chernoff contended that subjective probability (the specific theory not the general classifier) in the field of mathematics education trended toward the "personal" (or "subjective") interpretation found in probability theory rather than the "logical" interpretation. As a result he suggested that "subjective probability" be used as a general classifier while "personal probability" (or other terms) could be used for the specific theory.

In a recent, comprehensive synthesis of probability research in mathematics education, Jones et al. (2007), declared that "it is timely for researchers in mathematics education to examine subjective probability and the way that students conceptualize it" (p. 947). Their declaration—(perhaps) made in part because the authors (i) "were not able to locate cognitive research on the subjective approach to probability" (p. 925) and (ii) found that subjective probability "is not widely represented in mathematics curricula" (p. 947)—is well received. However, it must be noted, the nomenclatural issues inherent to subjective probability, documented above, remain unresolved as the subjective interpretation makes its way into curricula, classrooms and mathematics education research around the globe.

Mathematics education and probability theory are similar, in that the issues associated with differing philosophical interpretations are found in both domains; however, the domains differ with respect to the issues surrounding the infamous feud between the Bayesians and the frequentists. A close read of important pieces of literature in mathematics education reveals individuals' support for particular interpretations of probability (e.g., Hawkins and Kapadia 1984; Shaughnessy 1992). Despite these declarations of affinity for one interpretation over another, the feud between Bayesians and frequentists does not (appear) to exist in the field of mathematics education. The most likely reason that this feud is not found in mathematics education is because the very same research that advocates one interpretation over another also champions an approach to the teaching and learning of probability that "utilize[s] subjective approaches in addition to the traditional 'a priori' and frequentist notions" (Hawkins and Kapadia 1984) or, alternatively stated, "involves modeling several connections of probability" (Shaughnessy 1992, p. 469). In more general terms, research in mathematics education continues to advocate for "a more unified development of the classical, frequentist, and subjective approaches to probability" (Jones et al. 2007, p. 949).

As you will encounter, the chapters in this perspective all contribute to a more unified development of the three different approaches to probability in mathematics education, albeit in different ways. Through historical accounts of mathematics and philosophy, discussion of the importance of puzzles and paradoxes as a bridge between philosophy and mathematics and research focused on modeling and technology, these chapters may ultimately help align the interpretations of probability in mathematics education with those found in probability theory.

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