Dialogues and Monologues in Logic

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Abstract The dialogical framework is an approach to meaning that provides a pragmatist alternative to both the model-theoretical and the proof-theoretical semantics. However, since dialogic had and still has a bias towards antirealism, it has been quite often seen as a version of the proof-theoretical approach. The main claim of this chapter is that the proof-theoretical approach as displayed by a tableaux system of sequent calculus is, from the dialogical point of view, a monological approach and cannot provide a purely dialogical theory of meaning. Indeed, in general, validity is monological, in the sense that a winning strategy is defined independently of the moves of the Opponent. In the dialogical framework, validity should be based bottom up on a dialogical semantics.

The dialogical approach to logic is not but a semantic rule-based framework where different logics could be developed, combined or compared. But are there any constraints? Can we introduce rules ad libitum to define whatever logical constant? The answer is no, for logical constants must be governed by player-independent dialogical rules. The approach of the present chapter has been influenced by Marcelo Dascal's reflections on meaning, pragmatics and dialogues. In fact, on my view the dialogical approach to logic offers a framework for developing logic as closest as possible to his own theory of meaning and soft rationality.

Keywords Dialogical logic • Strategies • Proof theory • Pragmatics

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1 Introduction

The dialogical framework is an approach to meaning that provides a pragmatist alternative to both the model-theoretical and the proof-theoretical semantics. However, since dialogic had and still has a bias towards antirealism, it has been quite often seen as a version of the proof-theoretical approach. The main claim of the chapter is that the proof-theoretical approach as displayed by a tableaux system of sequent calculus is, from the dialogical point of view, a monological approach and cannot provide a purely dialogical theory of meaning. Indeed, in general, validity is monological, in the sense that a winning strategy is defined independently of the moves of the Opponent. In the dialogical framework, validity should be based bottom up on a dialogical semantics.

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2 Dialogical Logic

2.1 The Dialogical Framework

Dialogical logic, developed by Paul Lorenzen and Kuno Lorenz, was the result of a solution to some of the problems that arouse in Lorenzen's *Operative Logik* (1955, cf. Lorenz 2001). We cannot discuss here thoroughly the passage from the operative to the dialogical approach, though as pointed out by Peter Schroeder-Heister, the insights of operative logic had lasting consequences in the literature on proof theory and still deserve attention nowadays (see Schröder-Heister 2008). Moreover, the notion of *harmony* formulated by the antirealists and particularly by Dag Prawitz has been influenced by Lorenzen's notions of *admissibility*, *eliminability* and *inversion*. However, the dialogical tradition is rather a rupture than a continuation of the operative project, and it might be confusing to start by linking conceptually both projects.

Dialogical logic was suggested at the end of the 1950s by Paul Lorenzen and then worked out by Kuno Lorenz.[1] Inspired by Wittgenstein's *meaning as use,* the

[1]The main original papers are collected in Lorenzen and Lorenz (1978). A detailed account of recent developments can be found in Felscher (1985), Keiff (2004a, b, 2007, 2009), Rahman (2009), Rahman and Keiff (2004), Rahman et al. (2009), Fiutek et al. (2010), Rahman and Tulenheimo (2009), and Rückert (2001, 2007). For text book presentations see Fontaine and Redmond (2008) and Redmond and Fontaine (2011).

basic idea of the dialogical approach to logic is that the meaning of the logical constants is given by the norms or rules for their use. This feature of its underlying semantics quite often motivated the dialogical approach to be understood as a *pragmatic* approach to meaning. I concede that the terminology might be misleading and induce one to think that the theory of meaning involved in dialogic is not semantics at all. Helge Rückert proposes the formulation *pragmatistische Semantik* (*pragmatist semantics*) that might be more appropriate.

Anyway, the point is that those rules that fix meaning may be of more than one type, and that they determine the kind of reconstruction of an argumentative and/or linguistic practice that a certain kind of language games called dialogues provide. As mentioned above the dialogical approach to logic is not a logic but a semantic rule-based framework where different logics could be developed, combined or compared. However, for the sake of simplicity and exemplification, I will introduce only the dialogical version of classical and intuitionist logics.

In a dialogue two parties argue about a thesis respecting certain fixed rules. The player that states the thesis is called Proponent (**P**); his rival, who puts into question the thesis, is called Opponent (**O**). In its original form, dialogues were designed in such a way that each of the plays ends after a finite number of moves with one player winning, while the other loses. Actions or moves in a dialogue are often understood as *utterances* (cf. Rahman and Rückert 2001: 111; Rückert 2001, Chapter 1.2.) or as speech acts (cf. Keiff 2007). The point is that the rules of the dialogue do not operate on expressions or sentences isolated from the act of uttering them (Tulenheimo 2009). The rules are divided into particle rules or rules for logical constants (*Partikelregeln*) and structural rules (*Rahmenregeln*). The structural rules determine the general course of a dialogue game, whereas the particle rules regulate those moves (or utterances) that are requests (to the moves of a rival) and those moves that are answers (to the requests).

Crucial for the dialogical approach and that distinguishes it from all other approaches are the following points (that will motivate some discussion further on):

- The distinction between local (rules for logical constants) and global meaning (included in the structural rules).
- The symmetry of local meaning. This feature amounts to player independence. Let me call it the *purely dialogical approach to meaning*.
- The distinction between the play level (local winning or winning of a play) and the strategic level (global winning or existence of a winning strategy).

2.2 Local Meaning and Global Meaning

2.2.1 Particle Rules

In dialogical logic, the particle rules are said to state the *local semantics*: what is at stake is only the request and the answer corresponding to the utterance of a given logical constant, rather than the whole context where the logical constant is embedded.

- The standard terminology makes use of the terms *challenge* or *attack* and *defence*.

Tero Tulenheimo pointed out that this might lead the reader to think that already at the local level, there are strategic features and that this contravenes a crucial feature of the dialogical framework. Indeed, it is better to think at this level of meaning of the allowed moves as *requests* and *answer* (see Keiff 2007; Rahman et al. 2009). However, the dialogical vocabulary has been established with the former choice, and it would be perhaps confusing to change it once more. Thus, after some hesitation I will continue to use the terminology of challenges, etc. but insist once more that at the local level (the level of the particle rules), these words should be devoid of strategic underpinning.

The following table displays the particle rules, where X and Y stand for any of the players **O** or **P**:

$\vee, \wedge, \rightarrow, \neg, \forall, \exists$	Challenge	Defence
X: $A \vee B$	**Y**: ?-$\vee$	**X**: A *or* **X**: B (**X** chooses)
X: $A \wedge B$	**Y**: ?$\wedge$1 or **Y**: ?$\wedge$2 (**Y** chooses)	**X**: A *respectively* **X**: B
X: $A \rightarrow B$	**Y**: A (**Y** challenges by uttering *A* and requesting *B*)	**X**: B
X: $\neg A$	**Y**: A	— (no defence available)
X: $\forall xA$	**Y**: ?-$\forall x/k$ (**Y** chooses)	**X**: $A[x/k]$
X: $\exists xA$	**Y** ? $\exists$	**X**: $A[x/k]$ (**X** chooses)

In the table, A[*x*/*k*] stands for the result of substituting the constant *k* for every occurrence of the variable *x* in the formula A.

Let us briefly mention a crucial issue to which we will come back later on.

- *Dialogical meaning*: The particle rules are symmetric in the sense that they are player independent – that is why they are formulated with the help of variables for players. Compare with the rules of tableaux or sequent calculus that are asymmetric: one set of rules for the *true*(left)-side, other set of rules for the *false*(right)-side. The symmetry of the particle rules provides, as we will see below, the means to get rid of tonk-like-operators.

2.2.2 Structural Rules

(SR 0) (starting rule):
The initial formula is uttered by **P** (if possible). It provides the topic of the argumentation. Moves are alternately uttered by **P** and **O**. Each move that follows the initial formula is either a request or an answer.

Comment The proviso *if possible* relates to the utterance of atomic formulae. See formal rule *(SR 2)* below.

(SR 1) (no delaying tactics rule):
Both **P** and **O** may only make moves that change the situation.

Comments This rule should assure that plays are finite (though there might be an infinite number of them). There are several formulations of it with different advantages and disadvantages. The original formulation of Lorenz made use of ranks; other devices introduced explicit restrictions on repetitions. Ranks seem to be more compatible with the general aim of the dialogical approach of distinguishing between the play level and the strategic level. Other non-repetition rules seem to presuppose the strategic level. One disadvantage of the use of ranks is that they make metalogical proofs quite complicated. Let us describe here the rule that implements the use of ranks.

- After the move that sets the thesis players **O** and **P,** each chooses a natural number n and m, respectively (termed their repetition ranks). Thereafter the players move alternately, each move being a request or an answer.
- In the course of the dialogue, **O** (**P**) may attack or defend any single (token of an) utterance at most n (or m) times.

(SR 2) (formal rule):
P may not utter an atomic formula unless **O** uttered it first. Atomic formulae cannot be challenged.

Comments One way to see this rule is to assume that the atomic formulae encode a certain kind of justification or grounding and the proponent can, in Andreas Blass' words (Blass 1992), copycat it.

Indeed, assume that there is no such formal rule, that atomic formulae can be challenged, and that **P** uttered an atomic formula that **O** uttered before. Assume further that the atomic formula encodes a certain kind of justification. If **O** challenges a given atomic formula, **P** can also challenge the same atomic formula and then copycat the justification that **O** provides when he responds to **P**'s challenge. The formulation of the formal rule above abbreviates this process.

The formal rule introduces an asymmetry in relation to the commitments of **O** and **P,** particularly so in the case of the utterance of the conditional. Indeed, if **O** utters a conditional, then **P** commits him to a series of moves that must at the end be based on atomic moves of **O**. If it is **O** that challenges a conditional, no such commitment will be triggered. But it would be a mistake to draw from this fact the conclusion that the meaning of the conditional is not symmetric. The very point of symmetry is that it is a property of the meaning of the logical particles rather than of the dialogue as a whole where **P** is committed to the validity of the thesis. More precisely the asymmetry of the winning strategy is triggered by the semantic asymmetry of the formal rule. It is the possibility to isolate meaning (local and global) from validity commitments that allows dialogicians to speak of the symmetry of the logical constants, and this prevents tonk-like operators from being introduced in the dialogical framework (see 3 below).

Another way suggested by Helge Rückert is to see the formal rule as establishing a kind of game where the Proponent must play without knowing what the Opponent's justifications of the atomic formulae are.[2]

However, the dialogical framework is flexible enough to define the so-called material dialogues that assume that atomic formulae have a fixed truth-value:

*(SR *2) (rule for material dialogues):*

Only atomic formulae standing for true propositions may be uttered. Atomic formulae standing for false propositions cannot be uttered.

(SR 3) (winning rule):

X wins iff it is Y's turn but he cannot move (either challenge or defend).

Global Meaning of the Logical Constants

These rules complete the local meaning of the logical constants by establishing rights and obligations of a player in relation to the moves of the rival.

(SR 4i) (intuitionist rule):

In any move, each player may challenge a (complex) formula uttered by his partner or he may defend himself against the last challenge that has not yet been defended.

or

(SR 4c) (classical rule):

In any move, each player may challenge a (complex) formula uttered by his partner or he may defend himself against any challenge (including those challenges that have already been defended once).

- Notice that the dialogical framework offers a fine-grained answer to the question: Are intuitionist and classical negation the same negations? Namely, the particle rules are the same but it is the global meaning that changes.

In the dialogical approach, validity is defined via the notion of *winning strategy*, where winning strategy for X means that for any choice of moves by Y, X has at least one possible move at his disposal such that he (X) wins.

[2]Personal communication, Nancy April (2010).

Validity (definition):

A formula is valid in a certain dialogical system iff **P** has a formal winning strategy for this formula.

Thus,

- *A* is classically valid if there is a winning strategy for **P** in the formal dialogue Dc(A).
- *A* is intuitionistically valid if there is a winning strategy for **P** in the formal dialogue Dint (A).

Examples See Appendix.

2.2.3 Addenda

Lorenz (2001: 259) adds a condition (rather than a rule) that he calls the *crucial-dialogical* condition:

> Neither player is forced to defend himself against a challenge (where a formula has been uttered) unless the formula uttered in such a challenge has been defended upon finitely many counterattacks.

As we will discuss below, this rule understands the switch of "utterance-sides" triggered by a conditional, such as the core of dialogicity in logic.

3 Dialogues and Monologues: Play Level, Strategic Level and Tonk

3.1 Monologues: Strategic Level and Tableaux

As mentioned above in the dialogical approach, validity is defined via the notion of *winning strategy.*

A systematic description of the winning strategies available for **P** in the context of the possible choices of **O** can be obtained from the following considerations[3]:

If **P** is to win against any choice of **O**, we will have to consider two main different situations, namely:

- The dialogical situations in which **O** has uttered a complex formula
- Those in which **P** has uttered a complex formula

[3]Lorenzen (1978: 217–220). The relation with natural deduction has been recently worked out in Rahman et al. (2009: 301–336).

We call these main situations the **O**-cases and the **P**-cases, respectively. In both of these situations, another distinction has to be examined:

1. **P** wins by *choosing* between two possible challenges in the **O**-cases or between two possible defences in the **P**-cases, iff he can win with *at least one* of his choices.
2. When **O** can *choose* between two possible defences in the **O**-cases or between two possible challenges in the **P**-cases, **P** wins iff he can win *irrespective* of **O's** choices.

The description of the available strategies will yield a version of the semantic tableaux of Beth that became popular after the landmark work on semantic trees by Raymond Smullyan (1968), where **O** stands for **T** (left side) and **P** for **F** (right side) and where situations of type **ii** (and not of type **i)** will lead to a branching rule.

However, tableaux are not dialogues. The main point is that dialogues are built up bottom up, from local semantics to global semantics and from global semantics to validity. This establishes the priority of the play level over the winning-strategy level. The levels are to be thought as defining an order. From the dialogical point of view, to set the meaning of the logical constants via validity is like trying to define the (meaning) moves of the king in the game of chess by the strategic rules of how to win a play. Neither semantic tableaux nor sequent calculus gives priority to the play level. The point is not really that sequent calculus or tableaux do not have a play level, if with this we mean that one could not find the steps leading to the proof though there is one. What distinguishes the dialogical approach from other approaches is that in the other approaches – if there is something like a play level – the play level is ignored: the logical constants are defined via the rules that define validity.[4] The dialogical approach takes the play level as the level where meaning is set and on the basis of which validity rules should result. Within the dialogical approach, the more basic step of meaning at the play level is the setting of player-independent particle rules (i.e. symmetric rules): the difference between **O (T**)-rules and the **P** (**F**)-rules is a result of the strategic level and the asymmetry introduced by the formal rule. These considerations lead us to tonk. One can build tableaux-rules for tonk and tonk-like operators but, from the dialogical point of view, they have no semantic underpinning.

3.2 Tunk and Tonk

Let us discuss this point with the example of tableaux-rules for a tonk-like operator that we call *tunk*. Assume that we take tableaux-rules (or sequent-calculus) for **T**(left)-side and **F**(right)-side to set the meaning of logical constants. Under this assumption the following rules set the meaning of tunk:

[4]The point that other systems have also a play level has been stressed by Luca Tranchini in the workshop Workshop Amsterdam/Lille: *Dialogues and Games: Historical Roots and Contemporary Models*, 8–9 February 2010, Lille.

(**O**) [(**T**)] *A*tunk*B*	(**P**) [or (**F**)] *A*tunk*B*
————————-	————————-
(**O**) [(**T**)] *A*	(**P**) [(**F**)] *A*
(**O**) [(**T**)] *B*	(**P**) [(**F**)] *B*

Such a constant, when added to the standard tableaux-rules of, say, classical logic, renders proofs for

$$A\text{tunk}\neg A \text{ and for} \neg\,(A\text{tunk}\neg A)$$

Moreover, if we apply the cut-rule based on the formula *A*tunk*B,* it is possible to obtain a closed tableau for **T***A*, **F***B* for any *A* and *B.* The point is that in dialogues tonk-like operators are rejected because there is no symmetric particle rule that justifies the tableaux-rules designed for these operators. Indeed, let us attempt to define a particle rule for tunk. Let us thus assume that for a given player **X** that uttered *A*tunk*B,* the challenge (if it should somehow meet the tableaux-rules) must be one of the following:

1. (**Y**) show me the left side and (**Y**) show me the right side. Here it is the challenger who has the choice.
2. (**Y**) show me at least one of the both sides. Here it is the defender who has the choice.

Now whatever the options are, one of them will clash with one of the tableaux-rules described above:

- If we take option one, **O** has the choice and this should yield a branching on the **P**-rule (the **P**-rule is of the type of situations **ii** mentioned above).
- If we take option 2, **O** has the choice rule and this should produce a branching on the **O**-rule (the **O**-rule is of the type of situation **ii** mentioned above).

Prior's original tonk takes half of the rule that delivers the grounds for the assertion of a disjunction (half of the introduction rule) and half of the inference rule for the conjunction (half of the elimination rule). This renders the following tableaux version:

(**O**) [(**T**)] *A*tonk*B*	(**P**) [or (**F**)] *A*tonk*B*
————————-	————————-
(**O**) [(**T**)] *B*	(**P**)[(**F**)] *A*

From the dialogical point of view, the rejection tonk is simpler than the case of tunk: the defence must yield a different formula, namely, the tail of tonk if the defender is **O** and the head of tonk if the defender is **P**. This means, once more, that the attempted particle rule for tonk is player dependent, and this should not be the case.

The point is that the tableaux-rules for tunk and tonk are not based on particle rules that are player independent and are thus not apt to render a *purely dialogical* *local meaning*. Moreover, the only rules tableaux have are player dependent, they are so to say *monological*. That is why, according to the dialogical analysis, external criteria such as harmony have to be introduced in order to reject tonk-like operators. The dialogical analysis sketched above seems to suggest a generalization that should capture some of the effects of Lorenzen's inversion rule and that will take the form of kind of local soundness and completeness for the dialogical framework. In fact the argument sketched above shows that the tableaux-rules find no correspondence in the dialogical framework. The tableaux-tunk rules allow proving formulae that correspond to no winning strategy of **P**.

4 Conclusions

Is there then any limit to the dialogical framework for the introduction of logical constants? As well known, since the work of Dag Prawitz,[5] the natural deduction framework provides some criteria for the introduction of logical constants, which, as mentioned above, are rooted in Lorenzen's inversion principle and are known as *harmony*. In the natural deduction framework, there are only two sets of rules, and thus one might be thought as setting the meaning and the other as setting inferences that vehicle this meaning. In such a framework, *local soundness* or reducibility says that any derivation containing, say, an introduction of a logical constant followed immediately by its elimination can be turned into an equivalent derivation without this detour. It is a check on the *strength* of elimination rules: they must not be so strong that they include knowledge not already contained in its premises. Dually, *local completeness* says that the elimination rules are strong enough to decompose a connective into the forms suitable for its introduction rule. It is still an open question whether harmony should or not be based on the introduction rules as setting meaning rather than in the elimination rules.

The nice point is that in the dialogical framework, we have in fact several different sets of rules. I will separate two of them, those that set the meaning (particle + structural rules) and those responsible for the inferences setting validity, that is, the winning strategies described by an adequate tableaux or sequent calculus. Thus, the version of harmony appropriate to dialogical logic is the local soundness and completeness of the calculus purported to describe the winning strategies of a given system. In the particular case of tunk mentioned above, the point is that the tableaux-rules are unsound in relation to the semantics established at the play level by the joint collaboration of the particle and structural rules. The basis of the latter set of rules is the purely *dialogical feature* of the particle rules, that is, their player independence. The tunk-operator is rejected since the tableaux-rules with the help of which it has been described are too strong, i.e. the tableaux-rules prove more than the dialogical semantics allow. What we must do then in order to test if an operator

[5] Prawitz (1979, Ch. IV). See too Sundholm (2000, 2001, 1983a, b), Read (2008, 2010).

is or not a tonk-like operator is to prove soundness and completeness in relation to the dialogical system described above. Winning strategies involving the notion of validity (that are in fact essentially *monological*) should be based on a purely dialogical semantics.

According to this argument, such metalogical proofs are crucial for the general means of the dialogical framework as a framework. In fact, Lorenzen and Lorenz started this path, which must now be worked out for the dialogical systems recently developed.

But can we not produce tonk-like operators only at the play level? For example, let us take a particle rule for a tonk-like operator that gives as an answer to the challenger, say, the head of tonk. Such a rule is possible, but the result is not very harmful, as it amounts to the introduction of an operator that is equivalent to any formula. Thus, it is fully redundant. A similar variation of tonk seems hard to find. Indeed, if we fix the meaning of tunk, say, by establishing that the defender has the choice, then the particle rule will be exactly the one for disjunction. Another open question is about the limits on the structural rules. Can we freely combine a structural rule with the introduction of an arbitrary particle? The results coming from linear logic and substructural logics seem to indicate that there are many delicate interrelations to be taken into consideration. Deeper research is still due; however, let us fix some points towards *dialogical harmony*:

Dialogical Harmony

1. Particle rules must be player independent.
   This should also be understood, as pointed out by Keiff, that the particle rules should be defined independently of who is the player that is restricted by the formal rule.[6]
2. Global meaning of the logical constants must be player independent.
   This assumes that within the structural rules, a global meaning for the logical constants can be distinguished. This also assumes that the global meaning does not "undo" the player independence of the particle rules.
3. The particle rule of a logical constant must be given independently of the inner structure of the formula in which this logical constant occurs as a main operator.
4. Appropriate tableaux systems must be build up bottom up.
   In other words, those tableaux systems (or sequent calculi) that render a proof theory for a given dialogical semantics must be sound and complete in relation to the latter.

[6]Personal discussion with Keiff. Keiff has in mind a kind of negation introduced by Rahman and Rückert (2001).

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Appendix

Examples

In the following examples, the outer columns indicate the numerical label of the move and the inner columns state the number of a move targeted by an attack. Expressions are not listed following the order of the moves, but writing the defence on the same line as the corresponding attack, thus showing when a round is closed. Recall, from the particle rules, that the sign "—" signalizes that there is no defence against the attack on a negation.

For the sake of a simpler notation, we will not record in the dialogue the rank choices but assume the uniform rank: **O**: n = 1 **P**: m = 2.

In the following dialogue played with classical structural rules **P**' move 4 answers **O**'s challenge in move 1, since **P**, according to the classical rule, is allowed to defend (once more) himself from the challenge in move 1. **P** states his defence in move 4 though, actually **O** did not repeat his challenge – we signalize this fact by inscribing the not repeated challenge between square brackets.

O			P	
			$p \vee \neg p$	0
1	$?_\vee$	0	$\neg p$	2
3	p	2	—	
[1]	$[?_\vee]$	[0]	p	4

Classical rules. **P** wins

In the dialogue displayed below about the same thesis as before, **O** wins according to the intuitionistic structural rules because, after the challenger's last attack in move 3, **t**he intuitionist structural rule forbids **P** to defend himself (once more) from the challenge in move 1.

O			P	
			$p \vee \neg p$	0
1	$?_\vee$	0	$\neg p$	2
3	p	2	—	

Intuitionist rules. **O** wins

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