

Chapter 31

Safety Concepts for Slope Stability

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Abstract The use of partial safety factors has, with the adaptation of the Eurocodes, become the dominant safety concept, also for slope stability in most European countries. The article discusses the partial safety factor approach based on considerations of uncertainties involved and consequences of failure. Partial safety factors are calculated based on reliability theory, following first the present ideas of the Eurocodes, and second an alternative approach aimed to improve the shortcomings in the code. In the alternative approach, all uncertainties are placed on the material safety factor, which are then calculated for different target reliability index values. Two simple examples are presented, where the differences of the two approaches are outlined.

Keywords Slope stability • Safety • Eurocode • Reliability based design

31.1 Introduction

Slope stability analysis has traditionally been performed using the total factor of safety approach. As it has been used for a long time, there exists a lot of experience based data about the implication of its values. For example, if the total factor of safety is below certain threshold in soft clay, this often implies that there is the potential for larger horizontal movements in the soil, as discussed e.g. by Leroueil et al. (1990). Although the required factor of safety might be case sensitive, it is still often selected rather subjectively based on previous experience or to follow codes and guidelines, while the uncertainties related to loads, material properties, calculation models and construction are usually not considered.

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Reliability based design (RBD) offers a framework to account for all the uncertainties as well as consequences of failure. Although the method is advancing, a full RBD analysis is perhaps still not available for daily engineering purposes.

Many of the present design codes strive between these two options and try to maintain the experience based knowledge of the former and include a better evaluation of risk offered by the later. The partial factor of safety method can be considered as a first step towards RBD. In Europe, the Eurocodes have been adopted in most countries for both structural and geotechnical design. This article reviews Eurocodes way to apply partial safety factors to slope stability analysis, and suggests some improvements.

31.2 Slope Stability Analysis According to EN 1997

31.2.1 Application of Safety in the Eurocodes

The Eurocode consists of several different parts of which EN 1990 defines the principles and requirements for safety, serviceability and durability of structures for all materials. According to EN 1990 the partial safety factors should account for the possible unfavorable deviation of the property from its characteristic value and the uncertainties in the model used in the calculations. Three consequence classes are introduced to account for consequence of failure.

Eurocodes allows the determination of partial safety factors based on reliability theories. In that case, the target reliability index β is set to 3.8 for a 50 years reference period, corresponding to a probability of failure of approximately 1/15,000. A second approach is to calibrate the partial factors of safety to past experience. It is the understanding of the authors that the partial safety factors for material and resistance in geotechnical design is mainly based on calibration to older practice.

31.2.2 Slope Stability According to EN 1997

In EN 1997-1, three alternatives are presented on how safety can be factored in geotechnical design. These design approaches, named DA1, DA2 and DA3 result from the difficulty of finding a single way to combine factors between actions, ground properties and resistances. However, for slope stability most countries have chosen either DA3 or DA1. DA1 consists of checking two combinations, of which combination 2 is relevant for slope stability. As DA3 and DA1 combination 2 are consistent (here after referred as DA3), there seems to be a consensus on how safety should be applied for stability problems.

According to DA3, the safety is placed on material properties (strength) and on actions. The recommended values for partial safety factors on soils resistance are

$\gamma^{\phi'} = \gamma^{c'} = 1.25$ for effective stress analysis and $\gamma_{cu} = 1.4$ for total stress analysis, where the subscripts ϕ' , c' and c_u refer to effective friction angle, cohesion and undrained shear strength, and where γ stands for the partial safety factor. The largest permanent action on slope stability comes often from the soil weight itself. While it is quite difficult to divide soil weight into favorable and unfavorable actions, the permanent loads are left unfactored. Actions from variable loads, like e.g. traffic load, are on the other hand factored. Accordingly, the recommended value in EN-1997 for permanent actions is $\gamma_G = 1.0$ and for variable actions $\gamma_Q = 1.3$, where the subscripts G and Q refer to permanent and variable load.

31.3 Reliability Based Design Partial Safety Factors

31.3.1 Introduction

In the following, partial factors of safety will be calculated based on reliability theory. Firstly, the assumptions made regarding different uncertainties will be presented. Thereafter, the theoretical bases for reliability calculations are outlined. Two different calculation schemes are presented;

- I. Approach I (EN 1997-1): the material partial safety factors are calculated by placing safety on both load and material factors.
- II. Approach II (alternative approach): uncertainty related to loads and material properties are placed on the material factor.

In both of these calculations, a general uncertainty will also be included. The consequence of failure will be introduced into Approach II by setting different target reliability index values.

31.3.2 Assumptions

31.3.2.1 Material Properties

A vital part of reliability based design (RBD), including the partial factor of safety method, is that uncertainties are dealt in a proper way. There exists quite a lot of data about the variation of soil properties, especially on soil strength. The selection of appropriate values to use in RBD is however not obvious. It is not the intention of this article to review data on soil strength variation, but rather to outline some difficulties in addressing the issue.

Traditionally, codes tend to suggest clearly higher partial safety factors for undrained shear strength (s_u) than for effective strength parameters (ϕ' , c'). It is quite correctly thought that there is generally a greater uncertainty in the determination

of s_u from e.g. vane test than in the determination of the friction angle from a triaxial test. But there are many ways to determine undrained shear strength which are rather advanced. The reported coefficient of variation for s_u varies also highly from test to test, see e.g. Phoon and Kulhawy (1999b). It might be argued, that even though the triaxial test gives quite reliable values, the test is not run on a representative sample in the triaxial test, if one is dealing with coarse grained material. In addition if effective strength parameters are used for an undrained analysis of clays, the uncertainty is greater on the pore pressure than on the friction angle. So one might ask should the type of analysis, i.e. drained vs. undrained also be included in the material factor?

A thorough review of the sources of uncertainty can be found in Lacasse and Nadim (1996) and Phoon and Kulhawy (1999a). According to these sources, the uncertainty related to geotechnical properties can be divided into inherent (natural) variability and epistemic uncertainty. The inherent variability represents the natural variation of the property. The epistemic uncertainty is more complex. It includes statistical uncertainty, measurement error and transformation model uncertainty. It is often unclear how these different sources of uncertainty are dealt with. The coefficients of variation might be much higher than the inherent variability due to errors in processing the data. Following the ideas of Phoon and Kulhawy (1999a), a simple example is presented to demonstrate interpretation errors. Let us assume that for a normally consolidated clay layer there are three values for undrained shear strength, i.e. 10, 17.5 and 25 kPa. If the values are treated without further consideration about their origin, one would calculate that the mean as 17.5 kPa and the standard deviation 6.1 kPa, corresponding to a coefficient of variation (CoV) of 35 %. Let us then assume that the values correspond to the depths of 3, 8 and 13 m. Plotting the data versus depth reveals an obvious linear trend in data corresponding to $s_u = \alpha \sigma_v'$ with no fluctuation and that the inherent variability is zero. Phoon and Kulhawy (1999a) pointed out that some statistical analyses reported in the geotechnical literature probably ignore such trends. The resulting coefficient of variation would then be overestimated.

Although the information about the true variation of soil properties in continuously increasing, it is difficult, if not impossible, to fix a general coefficient of variation to a soil property that can be determined in various ways. Therefore the partial material factors in the present paper are calculated for three different CoV, namely 0.1, 0.2 and 0.3. This uncertainty could be included in codes by defining a minimum CoV-value the engineers should use, and adding that if there exist data from which the CoV can be determined with more reliability, the engineer should use the data.

In addition to CoV, one needs to choose the type of distribution for the material strength. Often there are not enough data to get a good statistical fit to a probability density function (PDF). Both normal and lognormal PDF are often considered as valid. For a normal PDF, the data population ranges practically between the mean ± 3 times the standard deviation. A CoV higher than 0.33 indicates thus that negative values are possible. Combining high values of CoV with a normal PDF for strength can lead to erroneous values of s_u in the probabilistic analyses. Lognormal PDFs are often used to characterize properties that have only positive

values. While the authors prefer using lognormal PDFs for soil strength, it is sound to check the possible effect of each PDF assumption. The Approach I calculations are performed for both PDFs.

31.3.2.2 Loads

As mentioned earlier, the permanent load resulting mostly from soil weight is usually left unfactored. The uncertainty can, however, be accounted for by including it in the material factor. Herein a CoV set equal to 0.1 was used for the permanent load.

For the variable load, a CoV_Q of 0.4 might be applicable for structural design as the uncertainty in wind and snow loads is high. For geotechnical design however, the variability should be considered separately. In Approach I the CoV on the variable load was set to 0.4, but for Approach II the calculation was done with CoV-values of 0.4 and 0.25. A normal PDF was used for both the permanent and variable loads and the loads are combined dependently, see Poutanen (2011, 2013).

31.3.2.3 Additional Generic Random Variable

The partial factor of safety method implies indirectly that safety is placed where there is uncertainty. In practice this is not always the case. For example in Europe, the different safety factor approaches, i.e. design approach 1, 2 and 3, are mostly chosen based on previous tradition in each country, not based on the assessment of uncertainty. There is then a danger that all uncertainties are not considered. With a full RBD, it is, at least in theory, possible to directly account for all the uncertainties involved. For a partial safety factor approach, this is not the case. One should thus add a generic variable with an uncertainty to account for factors such as calculation model errors and poor construction.

31.3.2.4 Consequence of Failure

When evaluating a target probability of failure, one should consider not only the uncertainties involved, but also the consequences of a possible failure. It is not economical or logical to require the same safety level for slopes with only a pedestrian way a for a slope in a densely populated residential area. In the Eurocodes, this is addressed by three different consequence classes. A multiplication factor KFI is applied to unfavorable loads and its value depends on the consequence of failure. For slope stability problems, the effect of external actions on the stability varies from nil to rather substantial. It seems thus rather random to apply safety related to the consequence of failure. However, EN 1990 also stated that “Reliability differentiation may also be applied through the partial factors on resistance γ_M ”. In this study, the partial material factors will be calculated for different target reliability index (β) corresponding to the different reliability classes (RC) in the

Eurocode. The target β values were chosen according to EN 1990 as $\beta_{50}=4.3$ (for RC3), $\beta_{50}=3.8$ (for RC2) and $\beta_{50}=3.2$ (for RC1).

31.3.3 Theoretical Bases

The design point was set at unity and the target reliability in the reliability calculation chosen according to EN 1990. The permanent load distribution was assumed as normal, the coefficient of variation equal to 0.1. The cumulative distribution is $FG(x, \mu_G, \sigma_G)$ and the density distribution is $fG(x, \mu_G, \sigma_G)$. For the variable load, a normal distribution was also used. The distributions were $FQ(x, \mu_{QN}, \sigma_{QN})$, $fQ(x, \mu_{QN}, \sigma_{QN})$, and the 0.98 fractile was set at the design point according to the 1-year load. The n-year variable load distribution can be calculated according to:

$$FQ(x, \mu_Q, \sigma_Q)^n \quad (31.1)$$

The material property distribution was assumed as lognormal, the cumulative distribution was $FM(x, \mu_M, \sigma_M)$ and the density distribution $fM(x, \mu_M, \sigma_M)$. The characteristic value was a 5 % fractile value which was set at the design point. With the cumulative distribution of the load as $FL(x, \mu_L, \sigma_L)$, the density distribution of the material property $fM(x, \mu_M, \sigma_M)$, the load safety factor γ_L and the material safety factor γ_M , the failure probability P_f becomes:

$$1 - \int_0^{\infty} FL(x, \mu_L, \sigma_L) \cdot fM(x, \mu_M \cdot \gamma_L \cdot \gamma_M, \sigma_M \cdot \gamma_L \cdot \gamma_M) dx = P_f \quad (31.2)$$

When the two loads $F_1(x, \mu_1, \sigma_1)$, $f_1(x, \mu_1, \sigma_1)$ and $F_2(x, \mu_2, \sigma_2)$, $f_2(x, \mu_2, \sigma_2)$ with the random variable $x_{1,i}$ and $x_{2,i}$ in fractile i are combined dependently in the proportion α and $1-\alpha$ (where α is the proportion of the load 1 in the total load) to obtain random variable $x_{1,2,i}$ of the combination load in fractile i , the variable x_i can be calculated by adding up the partial random variables:

$$x_{1,2,i} = x_{1,i} \cdot \alpha + x_{2,i} \cdot (1 - \alpha) \quad (31.3)$$

The distributions used for the actions and resistances are presented in Fig. 31.1.

31.3.4 Material Factors for Approach I (EN 1997-1)

The Approach I calculations resemble the partial safety factor approach DA3 in the Eurocodes. The loads were combined dependently and the partial safety factors for the loads are $\gamma_G=1$ and $\gamma_Q=1.3$. The load distribution for both the permanent and the variable loads was assumed as normal. The variable load was

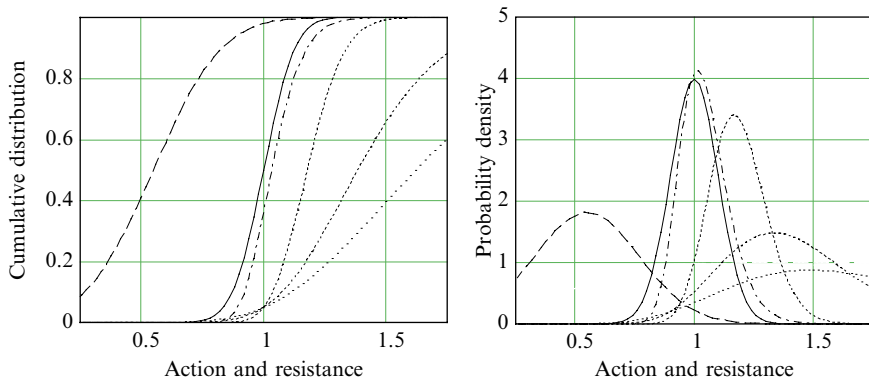


Fig. 31.1 Distributions set at the design point. *Solid line*: permanent load; *dashed line*: variable load 1-year location; *dash-dotted line*: variable load in 50-year location; *dotted lines*: material properties. The uncertainty distribution is equal to the permanent load distribution but it is located at the origin

selected as that with a 50-year return period, and the target reliability corresponds to RC2 of the Eurocodes, i.e. $\beta_{50}=3.8$.

The calculations were done for material CoVs of 0.1, 0.2 and 0.3, using both normal and lognormal PDFs. The generic random variable followed a normal distribution with zero mean and 0.1 CoV. The results of the calculations with Approach I are shown in Fig. 31.2 as a function of the load ratio (the proportion of the variable load to the total load, in %). The case with lognormal PDF for the material factor γ_M is on the left side, the normal PDF on the right side.

Figure 31.2 shows that with the given assumptions, the material factors vary for a given reliability index as a function of load proportion. If one considers a CoV of 0.1 for the effective strength parameters, ignores the generic random variable (as in the Eurocodes), one finds, that the recommended partial material factor γ_{ϕ} in EN 1997-1 is 1.25 for a load proportion of 60 %. To achieve the same probability of failure for a case with no variable load, one would need a material factor of 1.4.

Figure 31.2 also shows that for a normal PDF for the material strength, the partial factors increase significantly with increasing CoV. For a CoV equal to 0.3, the calculation did not converge. This was expected, as a CoV of 0.3 indicates that the strength values can be close to zero.

31.3.5 Material Factors for Approach II, (Alternative)

In Approach II, all uncertainty is placed on the material partial safety factor, i.e. the partial safety factors for both permanent and variable load are set to 1.0. The material factors were calculated for three different reliability index values, corresponding to the three reliability classes in the Eurocodes. The calculations were done for CoVs on the variable load of 0.4 and 0.25. The results are presented in Fig. 31.3, again as

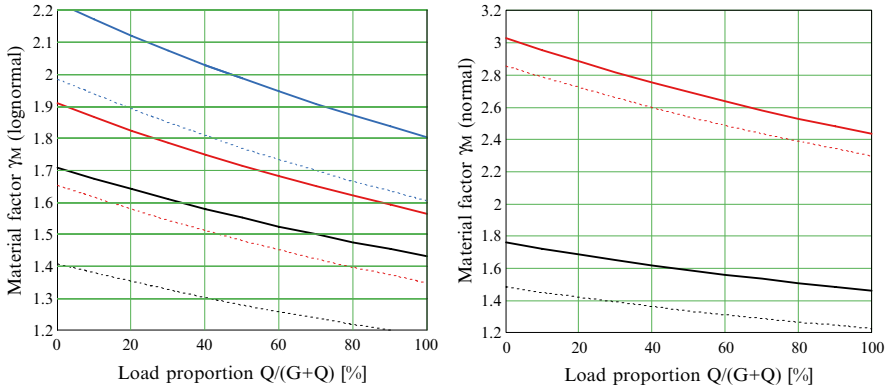


Fig. 31.2 Material factors according to approach I (EN 1997) to $\gamma_G=1$, $\gamma_Q=1.3$ and $\beta_{50}=3.8$ as function of load ratio. *Black lines* denote $CoV_M=0.1$; *red lines* $CoV_M=0.2$ and *blue lines* $CoV_M=0.3$. The *dotted lines* correspond to values without uncertainty, $CoV_U=0$, and the *solid lines* values with uncertainty, $CoV_U=0.1$. *Left side* figure uses lognormal distributions for material and *right side* uses normal distribution

a function of the load ratio. The case with CoV_Q of 0.4 is on the left side, the case with CoV_Q of 0.25 on the right side.

Comparing Figs. 31.2 and 31.3, the material safety factors for the same reliability index were equal when the load proportion was zero. For $CV_Q=0.25$, the material factors were practically not affected by the load proportion. The same probability of failure would be reached with a constant material factor. In addition, the results were comparable, regardless if the safety was set on both the action and resistance or if set on the resistance only. For example, for a load proportion of 60 % and a CoV on material factor 0.1, Fig. 31.2 gives a γ_M of 1.53, resulting in a factor of $[(0.4+0.6*1.3)*1.53]$ or 1.8, which is equal to the value of Fig. 31.3.

31.4 Examples of Application

31.4.1 Case 1, External Load with High Impact

Let us first study a case where the external load has a high impact on the safety. The geometry of the problem is given in Fig. 31.4. A 1-m thick embankment is laid upon a 1-m dry crust layer and a 10-m thick soft clay layer. The unit weight of the embankment material is 20 kN/m^3 and the characteristic friction angle is $\phi' = 38^\circ$. The dry crust layer has a unit weight of 17 kN/m^3 and characteristic undrained shear strength of 30 kPa. The soft clay has unit weight of 16 kN/m^3 and a characteristic s_u of 10 kPa at the top of the layer increasing with 1.4 kPa/m with depth. A 5-m wide load of 40 kPa is placed two meters from the crest of the embankment. According to the recommendations in EN 1997-1, the design (d) values are: ϕ'_d of 32° for the

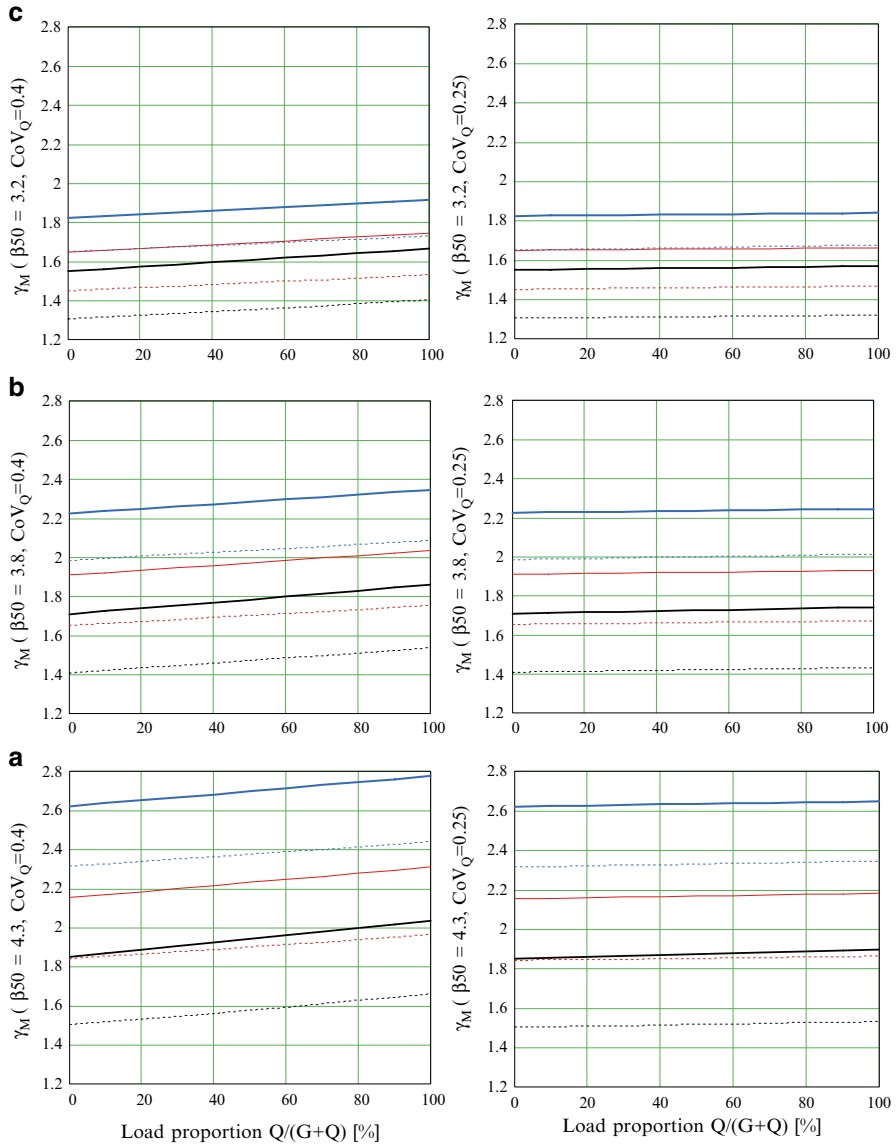


Fig. 31.3 Material factors corresponding to $\gamma_G = \gamma_Q = 1.0$ and (a) $\beta_{50} = 4.3$ (RC3), (b) $\beta_{50} = 3.8$ (RC2) and (c) $\beta_{50} = 3.2$ (RC1) as function of load ratio. *Black lines* denote $CoV_M = 0.1$, *red lines* $CoV_M = 0.2$ and *blue lines* $CoV_M = 0.3$. The *dotted lines* correspond to values without uncertainty, $CoV_U = 0$, and the *solid lines* values with uncertainty, $CoV_U = 0.1$

embankment, s_{ud} of 21.4 kPa for the dry crust and $s_{ud} = 7.1 \text{ kPa} + 1 \text{ kPa/m}$ for the soft clay. For a permanent load G_d is 40 kPa while for a variable load Q_d is 52 kPa.

The total factor of safety calculated with the characteristic values was 1.46 for a circular failure surface applying the Bishop method. According to EN 1997-1 for the permanent load case, the resulting over dimensioning factor (ODF) is 1.04,

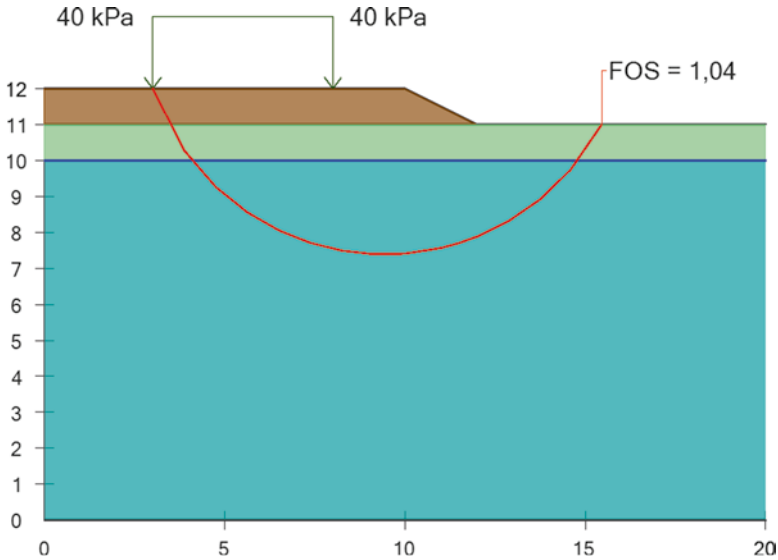


Fig. 31.4 Geometry for Case 1 and calculated ODF according to EN 1997-1

indicating that the situation is safe. For the variable load case, the ODF reduces to 0.88. For this case, to achieve an allowable design, the characteristic initial value of undrained shear strength of the soft clay should increase by about 30 %, up to 13 kPa. This corresponds to a total safety factor of 1.69.

Considering risk and probability of failure, it might be asked if higher soil strength should be required to achieve the same safety level for the variable load case? One might argue that there is more uncertainty in the variable load than in the permanent load. However, if the variable load is due to railway train load, the maximum allowable operation load is set by each track, so the characteristic load is rather a maximum load. It might then be more important to consider the possible consequences of failure. A permanent load might involve a residential building where lives could be lost. On the other hand, if the railway track is in an uninhabited area and the high load comes from a freight train, the risks are merely economical and the owner might consider allowing a higher probability of failure.

The alternative Approach II allows for such considerations. Assuming CoVs of 0.1 for the friction angle and 0.2 for the undrained shear strength, and $CoV_Q=0.25$ for the variable load, Fig. 31.3 gives a partial factor γ^ϕ of 1.3 and γ_{su} of 1.45 for β_{50} of 3.2, and $\gamma^\phi=1.41$ and $\gamma_{su}=1.66$ for $\beta_{50}=3.8$.

For a reliability index β_{50} of 3.2, the design values for approach II would be $\phi_d=31^\circ$ for the embankment, $s_{ud}=20.6$ kPa for the dry crust and $s_{ud}=6.9$ kPa + 0.96 kPa/m for the soft clay. The design load would be 40 kPa for both permanent and variable loads. These values give an ODF = 1.01 indicating that the situation is just acceptable. For β_{50} of 3.8, the design values would be $\phi_d=29^\circ$ for the embankment, $s_{ud}=18$ kPa for the dry crust and $s_{ud}=6.0$ kPa + 0.84 kPa/m for the soft clay and the design load would again be 40 kPa for both permanent and

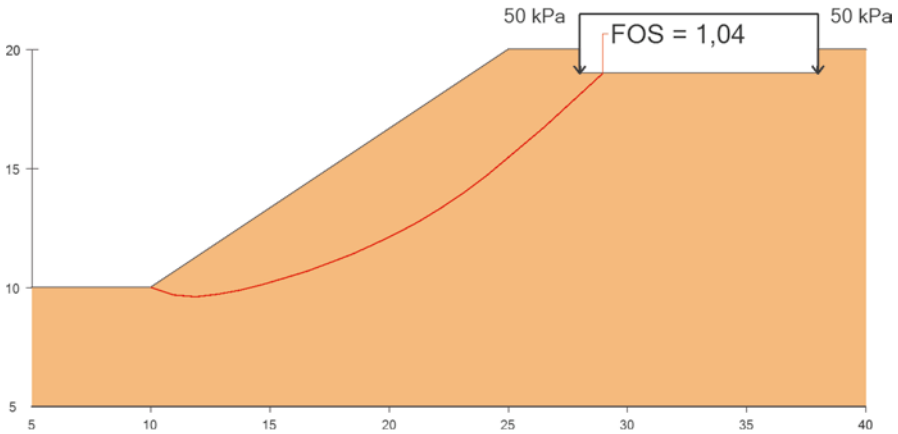


Fig. 31.5 Geometry for Case 2 and calculated ODF according to EN 1997-1

variable loads. These values result in an ODF of 0.88. To have an acceptable safety the initial undrained shear strength of the soft clay would need to increase by 26 % to 12.6 kPa. This corresponds to a total factor of safety requirement of $F = 1.66$.

31.4.2 Case 2, External Load with Minor Impact

Let us then study a case where the external loads have minor impact on the safety. The geometry of the problem is given in Fig. 31.5. The 10-m high slope is in a dry silty soil with a unit weight of 18 kN/m^3 and characteristic effective strength parameters of $\phi'_k = 26^\circ$ and $c_k = 8 \text{ kPa}$. At the top of the slope, there is a 10 m wide load corresponding to a characteristic stress of 50 kPa. According to the recommendations in EN 1997-1, the design values are $\phi'_d = 21.3^\circ$ and $c_d = 6.4 \text{ kPa}$, and $G_d = 50 \text{ kPa}$ if the load is permanent and $Q_d = 65 \text{ kPa}$ if the load is a variable load.

The total factor of safety calculated with the characteristic values was 1.29 for a non-circular failure surface applying the Morgenstern-Price method. In the case of a permanent load, the ODF according to EN 1997-1 was 1.04. In the case of a variable load of 65 kPa, the ODF was 1.03. In this case the load, even if rather high, does not influence significantly the safety factor. If one would then apply a consequence perspective based on loads as in the Eurocodes, it would not improve the reliability. For the alternative approach II, the consequence is on the other hand placed on the material safety factor. Then, for a CoV on the friction angle of 0.1 and a CoV on the variable load of 0.25, one obtains the partial material factors $\gamma^\phi = 1.3$ for $\beta_{50} = 3.2$, $\gamma^\phi = 1.41$ for $\beta_{50} = 3.8$ and $\gamma^\phi = 1.51$ for $\beta_{50} = 4.3$. As there is only one soil layer (only effective strength parameters are used), the material partial factors of safety are in practice also total safety factors.

31.5 Conclusions

The paper reviews the partial safety factor approach adopted in EN 1997-1 for slope stability and presents an alternative approach. The authors conclude that the uncertainties involved in slope stability analysis are not necessarily properly accounted for in EN 1997. Further the method of increasing load factors to account for consequence of failure is not suitable for slope stability. The impact of such factors are rather random in slope stability and do not reflect the risk involved in a failure.

In the proposed approach, all uncertainties are placed on the material factor. The material factors are calculated for different target reliability indices to account for the consequences of failure. A generic random variable is also included to consider model and construction errors. The authors find that such generic uncertainty should be included. However, the value used in the present analyses is probably on the high side.

The coefficient of variation for soil strength varies depending on whether undrained or effective strength parameters are used and which method is used for the determination of the parameters. Codes could account for this by setting a fixed minimum value cases where little or no data are available, but allowing the designer to use lower values if reliable site-specific data on the variability are available.

The authors suggest that, instead of using a normal probability density function for soil strength, a lognormal PDF should be used. Calculated material factors increased exponentially for high coefficient of variation when a normal distribution was used.

If probability theory is used for the determination of partial safety factors in codes, careful consideration is required in the evaluation of the coefficients of variation, PDFs and target reliability index values. For stability analysis, a coefficient of variation of 0.4 for the variable load might be too high and a lower value of 0.25 is suggested.

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