Chapter 2
Bargaining Game Based Distributed MPC

F. Valencia, J. D. López, J. A. Patiño and J. J. Espinosa

Abstract Despite of the efforts dedicated to design methods for distributed model predictive control (DMPC), the cooperation among subsystems still remains as an open research problem. In order to overcome this issue, game theory arises as an alternative to formulate and characterize the DMPC problem. Game theory is a branch of applied mathematics used to capture the behavior of the players (agents or subsystems) involved in strategic situations where the outcome of a player is function not only of his choices but also depends on the choices of others. In this chapter a bargaining game based DMPC scheme is proposed; roughly speaking, a bargaining game is a situation where several players jointly decide which strategy is best with respect to their mutual benefit. This allows to deal with the cooperation issues of the DMPC problem. Additionally, the bargaining game framework allows to formulate solutions where the subsystems do not have to solve more than one optimization at each time step. This also reduces the computational burden of the local optimization problems.

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2.1 Introduction

The main goal of the DMPC is to achieve some degree of coordination among subsystems that are solving local MPC problems with locally relevant variables, costs, and constraints, without solving the centralized MPC problem [4, 6, 10]. However, despite of the efforts dedicated to the formulation of the DMPC schemes, the paradigm to formulate DMPC approaches often requires an iterative procedure to compute the control actions to be applied to the controlled system, while stability and controllability properties of such process must be guaranteed. Most approaches also force the subsystems to cooperate without taking into account whether the cooperative behavior gives some operational advantage to the subsystems, and might steer the subsystems to operating points where they do not perceive any benefit. In the DMPC case, the cooperation is not always an advantage because, as it will be shown in the next sections, there exists a compromise between the local and the global behavior. Thus, if the local performance is not good enough, the whole system behavior is not good either. Considering all these issues, game theory arises as an alternative to formulate and characterize the DMPC problem.

Game theory is a branch of applied mathematics attempting to capture behaviors in strategic situations or games, where the outcome of a player (subsystem in the DMPC case) is function not only of his choices but also of the choices of others [8]. Game theory defines a game as formed by the set of all rules used to describe the situation as a mathematical description or model. Every particular instance of the game is called a play, at each play each player performs a move; formally, a move is a choice performed by a player between various alternatives (under conditions precisely prescribed by the mandatory rules of the game that cannot be infringed), following an strategy decided by the player [9].

If the DMPC problem can be viewed as calculated circumstances where the success of each subsystem is based upon the choices of the other subsystems, then it can be viewed as a game. Besides, if this situation can be described, analyzed and solved as a game where the subsystems are able to share information with each other and have a common goal, the DMPC problem can be described as a **bargaining game**. A bargaining situation involves a group of subsystems having the opportunity to collaborate for mutual benefit in more than one way. If an agreement is not achieved, each subsystem selects a course of action according to the available information [9]. In the DMPC case, such course of action is determined by a threshold of maximum lost of performance. This guarantees that the performance of each subsystem is better or at least equal to such threshold. In general terms, all bargaining situations share the following five elements:

1. A group of players involved in the bargaining.
2. A mutual benefit which is the objective of the bargaining, often defined as a profit or cost function.
3. A decision space composed by all the available choices of the subsystems.
4. A disagreement point defined by the minimum expected satisfaction for the bargaining.

5. An utopia point defined by the set of choices where all players involved in the bargaining achieve at the same time their maximum benefit.

In game theory a solution means a determination of the amount of satisfaction each player should expect to get from the situation. In specific games like bargaining ones, the concept of solution is associated with the determination of how much it should be worth to each of these players the right to have a chance for bargaining.

The selection of the bargaining approach is made because its main insight is focusing in others, i.e., "to assess your added value, you have to ask not only what other players can bring to you but what you can bring to other players" [3]. This approach adds flexibility to the DMPC schemes, and taking as a mathematical framework the theory presented by [9] about bargaining games, a bargaining-game-based DMPC scheme can be proposed with the following features:

- An iterative procedure is not required to compute the control actions to be applied to the controlled system.
- The subsystems are not forced to cooperate, i.e., each subsystem is able to decide whether to cooperate or not with other subsystems depending on the benefit received from the cooperative behavior.
- The paradigm to determine the interaction among subsystems is: *Focusing on others*.
- Each subsystem does not have to compute more than one optimization problem at each time step.
- The solution of the game is Pareto optimal.

2.2 Mathematical Formulation of the DMPC as a Game

Mathematically, a game G can be defined as a tuple $G = (\mathcal{N}, \{\Omega_i\}_{i\in\mathcal{N}}, \{\phi_i\}_{i\in\mathcal{N}})$ where:

- $\mathcal{N} = \{1, \ldots, N\}$ is the set of players.
- Ω_i is the decision space of player i determined by the game, i.e., is the set of feasible decisions of player i.
- $\phi_i : \Omega_1 \times \Omega_N \mapsto \mathbb{R}$ is the profit function of the i-th player.

Often, ϕ_i quantifies the preferences of player i (and determines its strategy), and gives to each player some degree of rationality [1]. In this case, it is assumed that the players are able to communicate and "bargain" with each other in order to achieve a common goal. Mathematically, a bargaining game for $\mathcal{N}$ is a pair $(\mathcal{S}, d)$ where [11]:

1. $\mathcal{S}$ denotes the decision space of the game defined as a nonempty closed subset of $\mathbb{R}^N$, i.e., the feasible set of profit functions.
2. $d \in \text{int}(\mathcal{S})$, d being the disagreement point.
3. $\zeta_i(\mathcal{S})$ the maximum profit available in $\mathcal{S}$ for the i-th player, i.e., it is the utopia point of the i-th subsystem: $\zeta_i(\mathcal{S}) := \max\{\phi_i : (\phi_i)_{i\in\mathcal{N}} \in \mathcal{S}\}$ exists for every $i \in \mathcal{N}$.

In general, the outcome of a game $(\mathcal{S}, d)$ is a tuple $\varphi(\mathcal{S}, d) = (\phi_1, \ldots, \phi_N)$ of profits received by the players. If any player does not cooperate, its corresponding position in $\varphi(\mathcal{S}, d)$ is replaced by its disagreement point. Hence, if all subsystems decide not to cooperate: $\varphi(\mathcal{S}, d) = (d_1, \ldots, d_N)$.

Game theory can be applied to the DMPC problem with the following considerations [14]:

- The rules are provided by the physical and operational constraints of the whole system (the DMPC game).
- Each time step k corresponds to an instance at which the optimal control action should be computed (a play of the DMPC game).
- Each time step k is an opportunity for each subsystem to choose a local control action between various alternatives (the choice in a DMPC game). If an iterative procedure is carried out to compute the local control actions, the iterations are the choices in such DMPC game.
- The moves are determined by the procedure to solve the game, often called negotiation model.

Based on these considerations, the DMPC game can be formulated in its strategic form as $G_{\text{DMPC}} = (\mathcal{N}, \{\Omega_i\}_{i\in\mathcal{N}}, \{\phi_i(\widetilde{\mathbf{u}}(k))\}_{i\in\mathcal{N}})$ where $\mathcal{N} = \{1, \ldots, N\}$ is the set of subsystems, Ω_i and $\phi_i(\widetilde{\mathbf{u}}(k))$ are the set of feasible control actions and the cost function of the i-th subsystem respectively, and $\widetilde{\mathbf{u}}(k) = [\mathbf{u}^T(k), \ldots, \mathbf{u}^T(k + N_\text{c}), \ldots, \mathbf{u}^T(k + N_\text{p})]^T$. Note that the unique requirement to formulate the DMPC problem as G_{DMPC} is the dependence of each local cost function in the decisions of the other subsystems.

Moreover, as it was stated before, the game G_{DMPC} is cooperative since it comes from a distributed control situation. Additionally, in G_{DMPC} all subsystems have a common goal. Then, this game satisfies the conditions of a bargaining game $(\mathcal{S}, d)$. However, the game G_{DMPC} has infinite plays and its decision environment has a dynamic evolution. These facts obligate to extend the original bargaining game theory to discrete-time dynamic bargaining games.

2.3 Description of the Approach

In this section the DMPC problem is formulated as a discrete-time dynamic bargaining game (see [14] for details about the definition of discrete-time dynamic bargaining games). With this purpose we recall that (same assumptions were made in [2, 15, 16]):

- $\phi_i(\widetilde{\mathbf{u}}(k); \mathbf{x}(k))$ denotes the cost function for subsystem i, $i = 1, \ldots, N$, where the notation $(\widetilde{\mathbf{u}}(k); \mathbf{x}(k))$ indicates that the function ϕ_i depends on $\widetilde{\mathbf{u}}(k)$, and $\mathbf{x}(k)$ is a time variant parameter.
- It is assumed that the evolution of $\mathbf{x}(k)$ is given by the linear state update equation

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k)$$

where $\mathbf{A}$ and $\mathbf{B}$ are obtained by linearization and discretization of the model describing the behavior of the whole system.
- $\phi_i(\widetilde{\mathbf{u}}(k))$ is a quadratic positive convex function of $\widetilde{\mathbf{u}}(k)$ for $i = 1, \ldots, N$.
- It is assumed that all subsystems are able to "bargain" in order to achieve a common goal: to maintain both the local and the global system performances by driving the system states to their reference values.

Let $\Upsilon(k) := \{\phi_i(\widetilde{\mathbf{u}}(k)) : \widetilde{\mathbf{u}}(k) \in \Omega,\ \forall i \in \mathcal{N}\}$ be a feasible set of cost functions. Since Ω is time-invariant for $i = 1, \ldots, N$, the feasible set $\Upsilon(k)$ is also time-invariant, i.e., $\Upsilon(1) = \Upsilon(2) = \ldots = \Upsilon$. Moreover, since Ω is closed and convex, and by the continuity and convexity of $\phi_i(\widetilde{\mathbf{u}}(k))$ with respect to $\widetilde{\mathbf{u}}(k)$, the set Υ is closed and convex. Note that Υ defines a set of possible values of the cost function of every subsystem given the set Ω. Then, Υ is **the decision space** in the DMPC problem treated in this chapter.

Once defined the decision space in the DMPC problem, it is required to define the disagreement point. The disagreement point is the benefit perceived by the player when an agreement is not possible. Such benefit is associated with an alternative plan carried out by the player in this situation, which is determined by the locally available information [9]. Based on the definition of bargaining game, in [9] the author establishes that the disagreement point should give to the players a strong incentive to increase their demands as much as possible without losing collaboration. Therefore, following these statements let us define **the disagreement point** for the DMPC problem as [14]

$$\eta_i(k+1) = \begin{cases} \eta_i(k) - \alpha(\eta_i(k) - \phi_i(\widetilde{\mathbf{u}}(k))) & \text{if } \eta_i(k) \geq \phi_i(\widetilde{\mathbf{u}}(k)) \\ \phi_i(\widetilde{\mathbf{u}}(k)) & \text{if } \eta_i(k) < \phi_i(\widetilde{\mathbf{u}}(k)) \end{cases} \tag{2.1}$$

$\forall i \in \mathcal{N}$, with $0 < \alpha < 1$.

With this definition of the disagreement point, if the i-th subsystem decides to cooperate it can improve its expected performance by reducing the disagreement point in a factor $\alpha(\eta_i(k) - \phi_i(\widetilde{\mathbf{u}}(k)))$. But if the i-th subsystem decides not to cooperate, its expected performance is increased by a factor $(\phi_i(\widetilde{\mathbf{u}}(k)) - \eta_i(k))$, resulting in a disagreement point equal to $\phi_i(\widetilde{\mathbf{u}}(k))$, allowing the i-th subsystem to cooperate few time steps ahead. For both cooperating and non-cooperating subsystems the disagreement point tends to the optimal expected value of the cost function $\phi(\widetilde{\mathbf{u}}(k))$, given the behavior of the states $\mathbf{x}(k)$ [14].

The definition of the DMPC problem as a discrete-time dynamic bargaining is completed by defining the utopia point. Let $\zeta_i(\Upsilon)$ denote **the utopia point** of the i-th subsystem (in a DMPC problem) defined as $\zeta_i(\Upsilon) := \min\{\phi_i(\widetilde{\mathbf{u}}(k)) : \phi_i(\widetilde{\mathbf{u}}(k)) \in \Upsilon\}$. From such a definition $\zeta_i(\Upsilon)$ exist for every $i \in \mathcal{N}$. Then the DMPC problem can be analyzed as a discrete-time dynamic bargaining game $\{(\Upsilon, \eta(k))\}_{k=0}^{\infty}$, where

1. $\Upsilon := \{\phi_i(\widetilde{\mathbf{u}}(k)) \colon \widetilde{\mathbf{u}}(k) \in \Omega,\ \forall i \in \mathcal{N}\}$ is **the decision space**.
2. $\eta(k) := \{\eta_1(k), \ldots, \eta_N(k)\}$ is **the disagreement point**, whose evolution is determined by (2.1).
3. $\zeta_i(\Upsilon) := \min\{\phi_i(\widetilde{\mathbf{u}}(k)) \colon \phi_i(\widetilde{\mathbf{u}}(k)) \in \Upsilon\}$ is **the utopia point**.
4. The dynamic evolution of the decision environment is given by the dynamic model of the controlled system.

Note that only the disagreement point depends on the time step k in $\{(\Upsilon, \eta(k))\}_{k=0}^{\infty}$.

2.3.1 Symmetric and Non-symmetric Games

It is possible to define (into a mathematical model) the desire of each player of maximizing his own gain in bargaining. However, depending of the complexity of the analyzed situation the resultant model can be intractable. Then, some ideal assumptions must be considered for the players [9]: They are highly rational, can accurately compare their desires for various aspects, are equal in bargaining skill, and have full knowledge of the tastes and preferences of the others.

All the situations satisfying these requirements are known as symmetric bargaining games. If all players have the same characteristics the expected satisfaction after a play should be the same. Note that symmetry conditions imply that all players have the same disagreement point in order to achieve the same utility at the end of the play. However, these symmetry conditions can be heavily restrictive in real applications because often players think different, they do not have the same bargaining skills, or simply the players do not share the same interests. As a consequence of these differences among players the bargaining game becomes non-symmetric [11].

The symmetry conditions of discrete-time dynamic bargaining games establish that the game $\{(\Upsilon, \eta(k))\}_{k=0}^{\infty}$ is symmetric if $\eta_1(k) = \ldots = \eta_N(k)$ for $k = 0, 1, 2, \ldots, \infty$. From the symmetry conditions it is possible to conclude that (naturally) the DMPC problem arising from the distributed control of homogeneous systems is an example of symmetric discrete-time dynamic bargaining games. Here the expression homogeneous systems refers to those systems composed by several subsystems with the same characteristic. Although there exist several real systems satisfying the symmetry conditions of discrete-time dynamic bargaining games, these conditions are heavily restrictive, mainly because real large-scale systems are composed by several different subsystems with different time evolution equations. As a consequence, a DMPC game $\{(\Upsilon, \eta(k))\}_{k=0}^{\infty}$ is non-symmetric in general.

The non-symmetric bargaining solution of a DMPC game $\{(\Upsilon, \eta(k))\}_{k=0}^{\infty}$ at time step k can be computed in a centralized way as a solution of the maximization problem [2, 14, 15]:

Algorithm 2.1 Negotiation model for DMPC games

1: At time step k, each subsystem sends the values of $\mathbf{x}_i(k),\ \eta_i(k)$ to the remaining subsystems.
2: With the information received, each subsystem solves local optimization problem (2.3).
3: Let $\widetilde{\mathbf{u}}_i^*(k)$ denote optimal control actions for subsystem i. If (2.3) is feasible, the subsystem i selects the first control action of $\widetilde{\mathbf{u}}_i^*(k)$. Otherwise, the subsystem i selects the first control action of $\widetilde{\mathbf{u}}_i(k)$, where $\widetilde{\mathbf{u}}_i(k)$ is the initial condition of the subsystem i at time step k.
4: Each subsystem updates its disagreement point. If (2.3) is feasible the update of the disagreement point of the subsystem i is given by $\eta_i(k+1) = \eta_i(k) - \alpha(\eta_i(k) - \phi_i(\widetilde{\mathbf{u}}(k)))$. Otherwise, the update of the disagreement point of the subsystem i is given by $\eta_i(k+1) = \eta_i(k) + (\phi_i(\widetilde{\mathbf{u}}(k)) - \eta_i(k))$ or $\eta_i(k+1) = \phi_i(\widetilde{\mathbf{u}}(k))$.
5: Each subsystem communicates its updated control action and its updated disagreement point to the others.
6: Go to step 1.

$$
\begin{aligned}
&\max_{\widetilde{\mathbf{u}}(k)} \sum_{i=1}^{N} w_i \log(\eta_i(k) - \phi_i(\widetilde{\mathbf{u}}(k))) \\
&\text{Subject to:} \\
&\qquad \eta_i(k) > \phi_i(\widetilde{\mathbf{u}}(k)) \\
&\qquad \widetilde{\mathbf{u}}(k) \in \Omega
\end{aligned}
\tag{2.2}
$$

Note that in the optimization problem of (2.2), $w = \{w_1, \ldots, w_N\}$ requires the selection of the weights for each subsystem. However, there are not guidelines for choosing their values. Often, in control theory field they can be arbitrarily selected as $w_i = \frac{1}{N},\ i = 1, \ldots, N$ (such a selection is made in [5, 16]). However, performing controllability and/or sensitivity analysis or prior knowledge of the system could help to derive guidelines for the selection of the weights. Also note that for $w_i = w_j = \frac{1}{N},\ \forall i, j \in \mathcal{N}$, a symmetric game solution is obtained, i.e., the proposed negotiation model can be used for both symmetric and non-symmetric DMPC games.

Let $\phi_i(\widetilde{\mathbf{u}}(k)) = \sigma_r(\widetilde{\mathbf{u}}_i(k), \widetilde{\mathbf{u}}_{-i}(k))$ with $\widetilde{\mathbf{u}}_{-i}(k)$ denoting the inputs of all subsystems except the i-th one. Then, the maximization problem of (2.2) is still centralized, but it can be solved in a distributed way by locally solving the system-wide control problem [2, 14, 15]

$$
\begin{aligned}
&\max_{\widetilde{\mathbf{u}}_i(k)} \sum_{r=1}^{N} w_r \log(\eta_r(k) - \sigma_r(\widetilde{\mathbf{u}}_i(k), \widetilde{\mathbf{u}}_{-i}(k))) \\
&\text{Subject to:} \\
&\qquad \eta_r(k) > \sigma_r(\widetilde{\mathbf{u}}_i(k), \widetilde{\mathbf{u}}_{-i}(k)),\ r = 1, \ldots, N \\
&\qquad \widetilde{\mathbf{u}}_i(k) \in \Omega_i
\end{aligned}
\tag{2.3}
$$

This formulation, for both symmetric and non-symmetric games, allows each subsystem (see [14] for details):

- To take into account the effect of its decisions in the behavior of the remaining subsystems.
- To include the effect of the local decisions in the profit of the remaining subsystems and in their decision about whether to cooperate or not.
- Combined with the negotiation model, to decide whether or not to cooperate depending on the utility perceived by the cooperative behavior.

These aspects reflect the paradigm underlying the proposed control scheme: *on others*.

2.3.2 Negotiation Model

A negotiation model is a sequence of steps for computing the outcome of a game. The negotiation model proposed to solve DMPC games $\{(\Upsilon, \eta(k))\}_{k=0}^{\infty}$ in a distributed way is based on the model proposed in [9] for two-player games. Such model adapted for solving the DMPC game has the steps as given in Algorithm 1.

The initial condition for solving (2.3) at time step $k+1$ is given by the shifted control input: $\bar{\mathbf{u}}(k+1:k+N_{\mathrm{p}})$, where the bar denotes the selected control input.

As in the case of the negotiation model proposed in [9], the negotiation model to solve the DMPC game in a distributed way represents a two moves game where the decisions are taken in steps 3 and 4. In this negotiation model each subsystem is:

- Fully informed on the structure of the game.
- Fully informed on the utility function of the remaining subsystems.
- Assumed intelligent (intelligence is given by the optimization procedure).
- Assumed rational (rationality is given by the decision procedure).

Additionally, it is assumed that the communication architecture allows each subsystem to communicate with the remaining subsystems in order to transmit their disagreement point and their local measurements of the states and inputs.

2.4 Theoretical Results

Most of the DMPC schemes based on Game theory require to perform two or more optimizations. These optimizations are focused on computing the local control actions of each subsystem and suggesting the other subsystems which control actions to use, and/or on creating a matrix of costs used for each subsystem to select which control action apply (see [7, 13] for examples of these kind of approaches). However, in the proposed negotiation model only one optimization problem should be solved. This allows to reduce the computational burden of the DMPC scheme associated with the communications among subsystems.

In addition, only local functions that depend from decisions of the other subsystems are required. This makes more flexible the bargaining approach to the DMPC problem than almost all the DMPC schemes presented in the literature. This statement is validated in [12], where a nonlinear DMPC is formulated for traffic control based on the approach described in this chapter. Moreover, in [15] the proposed control scheme was also used to formulate a DMPC scheme for the chain of reactors followed by a flash separator proposed in [16]. Besides, a comparison of several DMPC schemes (including the control scheme described in this chapter) using a quadruple tank process as a testbed is presented in [2].

The stability of the proposed DMPC method depends on the decision of each subsystem about cooperation. In [14] the case when some subsystems initially do not cooperate, but few steps ahead they start cooperating has been considered in order to demonstrate the stability of the closed-loop system.

2.5 Application Results

In this section the results of the application of the bargaining game based DMPC to an HPV are presented. Here, two scenarios were considered: a power tracking scenario and a price based operation scenario. The system description and model can be found in [14]. The HPV considered in this Chapter is presented in Fig. 2.1 and the full mathematical deployment of both examples is shown in [14].

As in almost all large-scale systems, the most common control scheme employed in practice for controlling the HPV comprises a PI controller with disturbance feed-

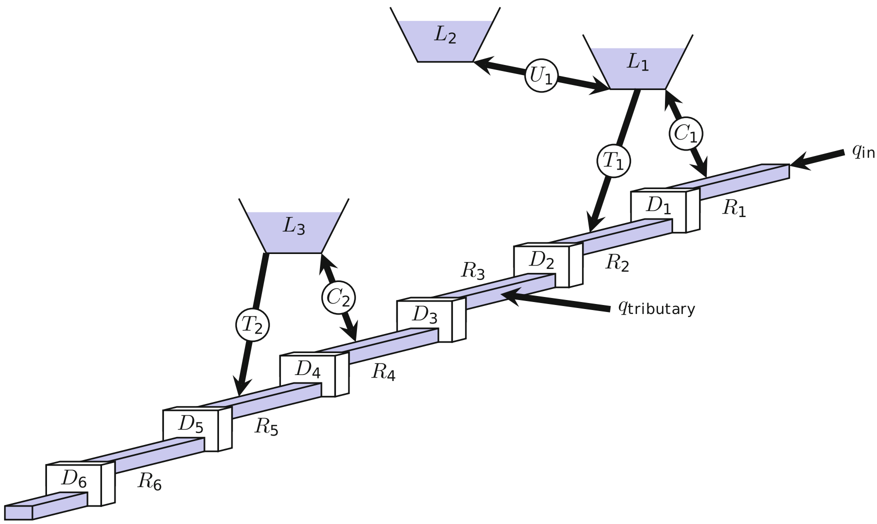

Fig. 2.1 Hydro-power valley used as case of study [14]

forward installed on each individual power plant. However, the use of local PI controllers does not guarantee an efficient use of the stored water, and in presence of disturbances the performance of the entire system could be compromised. For tackling these issues, multivariable control structures have been proposed for controlling HPV systems. Often, these are centralized optimal control schemes. But, since an HPV is a large scale system, a centralized MPC is inflexible and unsuitable. Therefore distributed and/or hierarchical MPC controllers are required (several references regarding the implementation of centralized optimal control schemes in HPV systems can be found in [14]).

2.5.1 The Power Tracking Scenario

In this scenario the power output of the system should follow a given reference while keeping the water levels in the lakes and at the dams as constant as possible. Such power reference is determined by the expected daily demand in the zone fed by the HPV. So, the global cost function considered for the DMPC is composed by two terms [14]:

- The first term penalizes the 1-norm of the power tracking error.
- The second term penalizes the 2-norm of the deviations of the levels in the lakes and dams from their steady state values.

The power reference to be followed by the entire system is known 24 h in advance and the inputs of the system can be changed every 30 min.

From [14], it is possible to divide the HPV of Fig. 2.1 into eight subsystems:

- Subsystem 1: lakes L_1 and L_2, turbine T_1, and turbine-pump C_1
- Subsystem 2: lake L_3, turbine T_2, and turbine-pump C_2
- Subsystems 3–8: reaches R_1 to R_6 respectively

Based on this system decomposition, the local cost function for each of the $\mathcal{N} = \{1, \ldots, 8\}$ subsystems can be obtained. For each subsystem, there exists a decision space Ω_i determined by the state and input constraints, and a performance index $\sigma_i(\widetilde{\mathbf{u}}_i(k), \widetilde{\mathbf{u}}_{-i}(k))$ indicating the preferences of each subsystem. Hence, the DMPC for the HPV can be viewed as a strategic game

$$G_{\text{HPV}} = \{\mathcal{N}, \{\sigma_i(\widetilde{\mathbf{u}}_i(k), \widetilde{\mathbf{u}}_{-i}(k))\}_{i \in \mathcal{N}}, \{\Omega_i\}_{i \in \mathcal{N}}\}.$$

Since all subsystems have the same goal: to minimize the power tracking error while keeping the levels in the lakes and dams as close as possible to their steady state values; the game G_{HPV} can be analyzed and solved as a discrete-time dynamic bargaining game $\{(\Upsilon, \eta(k))\}_{k=0}^{\infty}$. In addition, due to the diversity of the physical phenomena involved in an HPV, it is expected that the game G_{HPV} belongs to the class of non-symmetric bargaining games.

Since the power produced by the HPV at time step k is equal to the sum of the powers generated by all subsystems, and assuming that each one communicates the value of its states and inputs to the remaining subsystems, it is possible to locally compute the power produced by the others. Hence, a reference value for each subsystem is not required. This also reduces the computational and communication burden of the proposed DMPC scheme.

2.5.1.1 Simulation Results

A closed-loop simulation of the HPV was performed along 24 h (simulation time). In this simulation the sampling time was $T_s = 1{,}800$ s (30 min), $N_\mathrm{p} = 48$ (corresponding to a day), $N_\mathrm{c} = 32$, $w_{1,2} = \frac{0.4}{2}$, $w_{3-8} = \frac{0.6}{6}$ (the weights of subsystems 1 to 8), $\eta_i(0) = 1 \times 10^5$ (the initial disagreement point of subsystems 1–8), $\gamma = 50$, and $\mathbf{Q} = \mathbf{I}$ ($\mathbf{I}$ being the identity matrix). The values of the parameters as well as the lower and upper limits of the inputs and the states were taken as proposed in [14].

Figure 2.2 shows the comparison between the power produced by the HPV and the power reference when the proposed DMPC scheme computes the inputs of each subsystem. This Fig. 2.2 shows how the power produced by the HPV followed the power reference, satisfying one of the objectives proposed for the control scheme. However, there was an oscillation at the beginning of the experiment due to the transient generated by the change of power from 175 MW (equilibrium power) to the initial required power 150 MW. Recall that in the proposed control scheme there is not a power reference for each subsystem; hence, the initial change requires a negotiation among subsystems in order to decide the amount of power delivered by each power plant to supply the demanded power. The evolution of the disagreement points is presented in Fig. 2.3. In this Figure, the disagreement started at the same point but as they were evolving each subsystem had its own value, indicating the non-symmetry of the game G_HPV. Figure 2.3 also shows a zoom between 4×10^4 s and 7.5×10^4 s, note that all disagreement points decreased with low frequency oscillations; such oscillations were associated to the decision process of each subsystem.

2.5.2 The Economic Scenario

The economic scenario concerns the operation of the HPV based on the electricity price. Here, the objective is to compute the optimal control actions such that the profit of the HPV is maximized. Following the economic scenario proposed by [14], the profit maximization relates two important elements:

- The amount of money perceived by the production of electricity in the HPV.
- The opportunity cost determined by the water remaining in the system at the end of the prediction horizon.

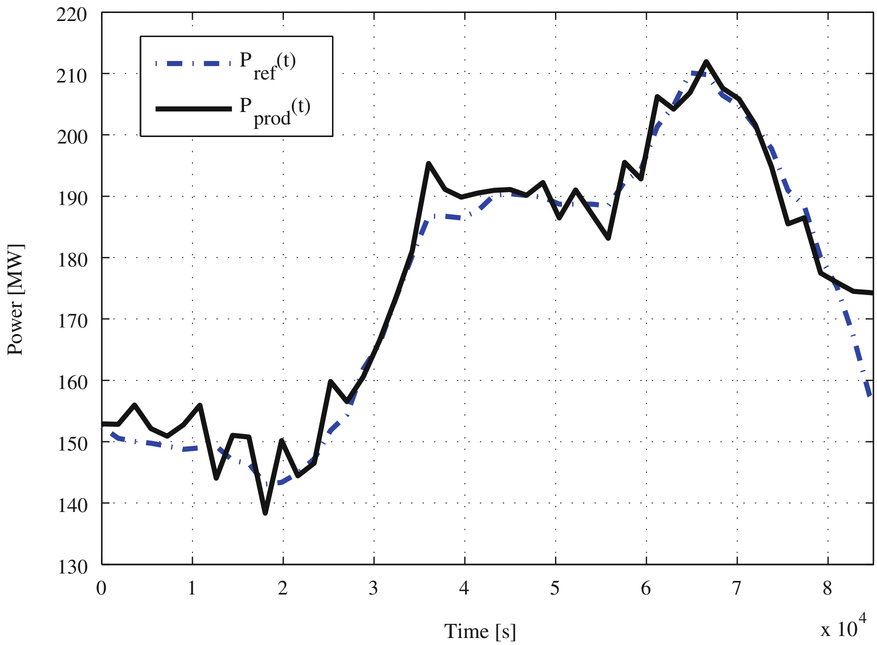

Fig. 2.2 Comparison between the power produced by the HPV and the power reference, when the proposed game-theory-based DMPC is used for computing the inputs of the subsystems

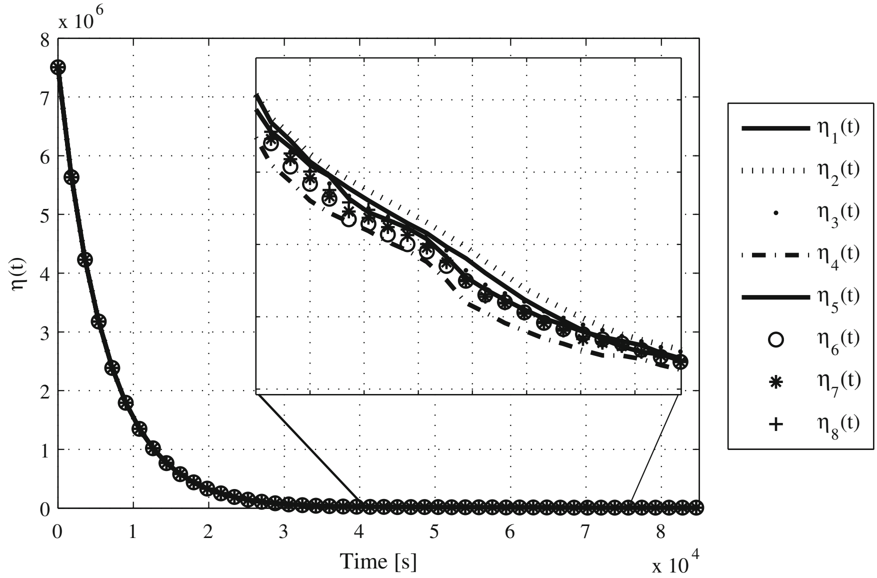

Fig. 2.3 Behavior of the disagreement points. This figure shows an overall evolution and presents a detailed view that allows to evidence the non-symmetry of the game

In order to solve this problem it is assumed that the electricity prices are known 24 h in advance and change every hour, they have the same behavior during the next day, and the prices of the remaining water in the system are constant.

As in the power tracking scenario, there exists a set of subsystems $\mathcal{N} = \{1, \ldots, 8\}$ in the price based operation of the HPV, each one with a decision space Ω_i determined by the input and state constraints, and with a profit function $\sigma_i(\widetilde{\mathbf{u}}_i(k), \widetilde{\mathbf{u}}_{-i}(k))$ indicating their preferences. Hence the DMPC for the price based operation of the HPV can be viewed as an strategic game

$$G_{\text{ec}} = \{\mathcal{N}, \{\sigma_i(\widetilde{\mathbf{u}}_i(k), \widetilde{\mathbf{u}}_{-i}(k))\}_{i\in\mathcal{N}}, \{\Omega_i\}_{i\in\mathcal{N}}\}.$$

Furthermore, since the value of $\sigma_i(\widetilde{\mathbf{u}}_i(k), \widetilde{\mathbf{u}}_{-i}(k))$ depends of the decisions of the other subsystems, all subsystems are able to communicate with each other in order to decide the local control action to be applied. All of them have the same goal: to maximize the profit perceived by the electricity production in the HPV. The game G_{ec} can be analyzed and solved as a discrete-time dynamic bargaining game $\{(\Upsilon, \eta(k))\}_{k=0}^{\infty}$ (due to the diversity of the physical phenomena involved in the HPV, it is expected that the game G_{ec} belongs to the class of non-symmetric bargaining games).

There must be noted that in the power tracking scenario the objective is to minimize a cost function, while the objective in the economic scenario is to maximize a profit function. As a consequence the value of the disagreement point $\eta(k)$ in the economic scenario indicates the minimum profit expected by each subsystem for the production of electricity in the HPV.

2.5.2.1 Simulation Results

In order to test the performance of the proposed control scheme in the economic scenario, a closed-loop simulation of the HPV was carried out along 24 h (simulation time). The sampling time, prediction horizon, control horizon, and weights of the subsystems were the same of the power tracking scenario. However, in the economic scenario the disagreement point of all subsystems was started in zero. The values of c_l, $c_{f,i}$, and the lower and upper limits of the inputs and the states were taken from [14].

Figure 2.4 shows the behavior of the prices and the power produced by the HPV along the simulation. In this Fig. 2.4, despite of the changes of the prices of the electricity, the power produced by the HPV remained almost constant along the simulation. Such behavior was determined by the prices of changing the levels of the reaches. If the use of the stored water to produce energy provided a higher profit than keeping the water at the same levels while producing the same amount of power, then the water levels were changed; otherwise, the levels at the lakes and reaches were kept at a constant value. Recall that the profit of each subsystem is given by the difference between its disagreement point $\eta_i(k)$ and its local profit function $\sigma_i(\cdot)$.

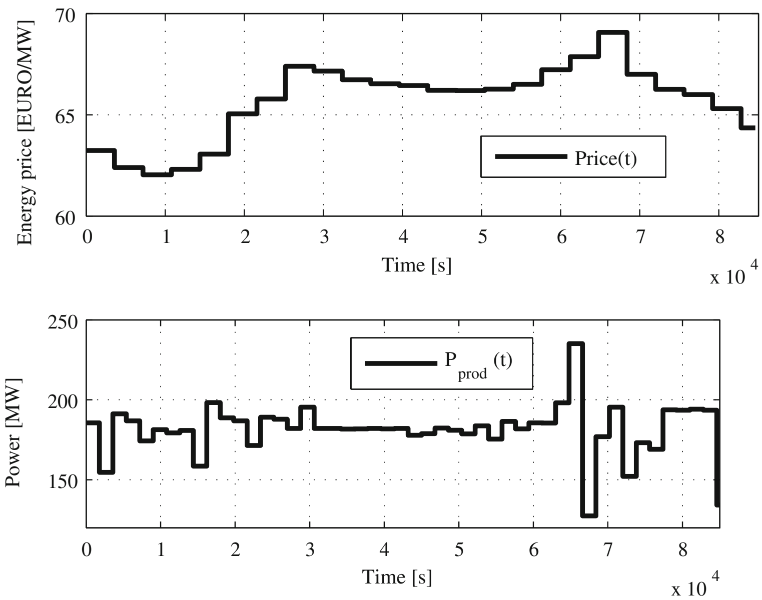

Fig. 2.4 Economic operation of the HPV. The *top panel* shows the behavior of the electricity prices along the simulation. The *bottom panel* shows the power produced by the HPV. Despite of the changes in the electricity prices the power produced by the HPV remains constant along the simulation

From [14], the water of the lake 1 provides more profit than the water in lakes 2 and 3, being the cheapest water the water stored in lake 2. Furthermore, the water stored at the dam of the reach 3 provides more profit than the water stored at the dams in the remaining reaches. Since the price of the water is higher (in all the cases) than the price of the electricity, the results shown in Fig. 2.4 were the expected: a constant production of electricity in order to maximize the water stored in the lakes and reaches.

Figure 2.5 presents the evolution of the disagreement points. The cooperation among subsystems was evident, all disagreement points started at the same value but their increase was cooperative (with different rates, which means that the game G_{ec} is non-symmetric) until 6×10^4 s where a non-cooperative behavior appeared. At that moment, some subsystems did not perceived any benefit by the cooperation, i.e., there were no feasible control actions such that the minimum expected profit was achieved. As a consequence, a decrease of the disagreement point of these subsystems was done in order to promote the cooperation of these subsystems several time steps ahead (note that the cooperation started again at the next time step).

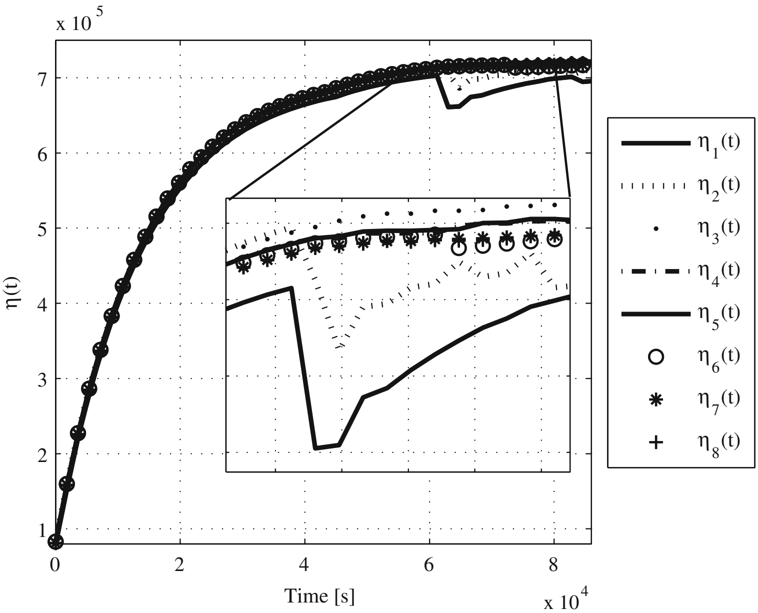

Fig. 2.5 Evolution of the disagreement points. Here, the cooperative behavior of the subsystems remained during 6×10^4 s where the non-cooperative behavior emerged. This is reflected in the decrease of the value of the disagreement point of some subsystems

2.6 Conclusions

In this chapter, the formulation of the distributed model predictive control as a bargaining game was presented. With this purpose, the concept of discrete-time dynamic bargaining game was considered. Moreover, a negotiation model to solve the distributed model predictive control game was proposed. Properties like closed-loop stability of the system when the control actions are computed by the proposed control scheme were also discussed. The application of the DMPC scheme presented in this chapter to an HPV was also included. In this application two scenarios were considered: a power tracking scenario and a price based operation scenario. In both cases the original control problem was formulated and solved as a bargaining game.

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