Chapter 6
The Node Element

Abstract The modelling for nodes in contact with the sea bed is described. The forces as well as the stiffness are given in case of contact as well as of friction.

Keywords Forces contact with the sea bed · Drag on the sea bed.

6.1 Principle

The contact of a marine structure with the sea bed has to be taken into account. It is of great importance for structures such as chains lying on the sea-bed or bottom trawls.

In the following sections a few forces related to this contact are described.

6.2 Contact on Bottom

In this model, the main hypothesis for these contact forces is that the bottom is elastic. That means that if a node is in contact with the bottom, the force reaction (N) is vertical and equal to the product of the node depth (m) in the soil by the soil stiffness (N/m).

6.2.1 Force Vector

The vertical force on a node due to its potential contact with the bottom is

$$if z < Z_b \;\; F_z = B_k(Z_b - z) \tag{6.1}$$

$$if z \geq Z_b \;\; F_z = 0 \tag{6.2}$$

With:
F_z: the vertical force on the node (N),
B_k: the bottom stiffness (N/m),
Z_b: the vertical position of the bottom (m),
z: the vertical position of the node (m).

6.2.2 Stiffness Matrix

$$if z < Z_b \quad -\frac{\partial F_z}{\partial z} = B_k \tag{6.3}$$

$$if z \geq Z_b \quad -\frac{\partial F_z}{\partial z} = 0 \tag{6.4}$$

6.3 Drag on Bottom

Contact of a node with the bottom could lead to a wearing force. This force is taken into account when there is a movement of the structure on the bottom. This force is horizontal and opposite to the motion. This wearing depends on the depth on which the node digs the bottom, on the bottom stiffness, and on the node speed displacement on the bottom.

6.3.1 Force Vector

As mentioned earlier (Sect. 6.2, p. 87), the vertical force on a node due to its contact ($z < Z_b$) to the bottom is:

$$F_c = B_k(Z_b - z) \tag{6.5}$$

With:
F_c: the vertical force on the node due to the contact to the bottom (N),
B_k: the bottom stiffness (N/m),
Z_b: the vertical position of the bottom (m),
z: the vertical position of the node (m).

The drag force on the bottom has been modelled as a function of the displacement speed of the node on the bottom. Figure 6.1 shows this relation.

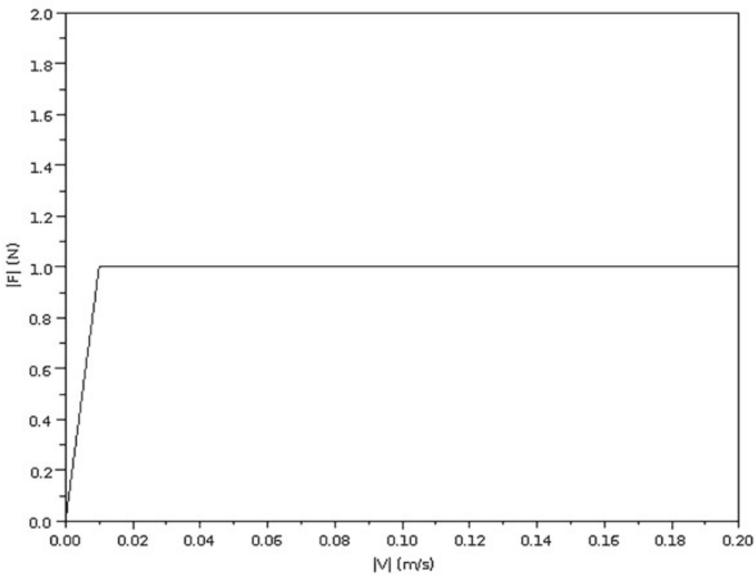

Fig. 6.1 Example of amplitude of wearing force $|\mathbf{F}|$ depending on the node displacement speed on the bottom $|\mathbf{V}|$

$$if|\mathbf{V}| < V_l \ |\mathbf{F}| = F_c B_f \frac{|\mathbf{V}|}{V_l} \tag{6.6}$$

$$if|\mathbf{V}| \geq V_l \ |\mathbf{F}| = F_c B_f \tag{6.7}$$

With:

$$\mathbf{V} = \begin{vmatrix} V_x \\ V_y \\ V_z \end{vmatrix} \tag{6.8}$$

The components of speed are calculated as follows:

$$V_x = \frac{x - x_p}{\Delta t} \tag{6.9}$$

$$V_y = \frac{y - y_p}{\Delta t} \tag{6.10}$$

$$V_z = \frac{z - z_p}{\Delta t} \tag{6.11}$$

V_x (V_y, V_z): component of the speed of the node along the x (y, z) axis (m/s),
x (y, z): coordinate of the node along the x (y, z) axis (m) calculated at time t,

x_p (y_p, z_p): previous coordinate of the node along the x (y, z) axis (m) calculated at time $t - \Delta t$.

Two cases are defined: a high-speed case ($|\mathbf{V}| \geq V_l$) and a low-speed case ($|\mathbf{V}| < V_l$). The wearing force is calculated in the two cases such as there is continuity between the two cases (at $|\mathbf{V}| = V_l$).

6.3.1.1 High-Speed

In this case, $|\mathbf{V}| \geq V_l$.

That means that the components of this force are the following:

$$F_x = -F_c B_f \frac{V_x}{|\mathbf{V}|} \tag{6.12}$$

$$F_y = -F_c B_f \frac{V_y}{|\mathbf{V}|} \tag{6.13}$$

$$F_z = -F_c B_f \frac{V_z}{|\mathbf{V}|} \tag{6.14}$$

6.3.1.2 Low-Speed

In this case, $|\mathbf{V}| < V_l$.

That means that the components of this force are the following:

$$F_x = -F_c B_f \frac{V_x}{V_l} \tag{6.15}$$

$$F_y = -F_c B_f \frac{V_y}{V_l} \tag{6.16}$$

$$F_z = -F_c B_f \frac{V_z}{V_l} \tag{6.17}$$

6.3.2 Stiffness Matrix

6.3.2.1 High-Speed

$$\frac{\partial F_x}{\partial x} = -\frac{F_c B_f}{|\mathbf{V}|^2} \frac{\partial V_x}{\partial x} \left[|\mathbf{V}| - \frac{V_x^2}{|\mathbf{V}|} \right] \tag{6.18}$$

$$\frac{\partial F_x}{\partial y} = -\frac{F_c B_f}{|\mathbf{V}|^2} \frac{\partial V_y}{\partial y} \left[-\frac{V_x V_y}{|\mathbf{V}|} \right] \tag{6.19}$$

$$\frac{\partial F_x}{\partial z} = B_k B_f \frac{V_x}{|\mathbf{V}|} - \frac{F_c B_f}{|\mathbf{V}|^2}\left[-\frac{V_x V_z}{|\mathbf{V}|}\frac{\partial V_z}{\partial z}\right] \tag{6.20}$$

$$\frac{\partial F_y}{\partial x} = \frac{F_c B_f}{|\mathbf{V}|^2}\left[\frac{V_x V_y}{|\mathbf{V}|}\frac{\partial V_x}{\partial x}\right] \tag{6.21}$$

$$\frac{\partial F_y}{\partial y} = -\frac{F_c B_f}{|\mathbf{V}|^2}\frac{\partial V_y}{\partial y}\left[|\mathbf{V}| - \frac{V_y^2}{|\mathbf{V}|}\right] \tag{6.22}$$

$$\frac{\partial F_y}{\partial z} = B_k B_f \frac{V_y}{|\mathbf{V}|} - \frac{F_c B_f}{|\mathbf{V}|^2}\left[-\frac{V_x V_z}{|\mathbf{V}|}\frac{\partial V_z}{\partial z}\right] \tag{6.23}$$

$$\frac{\partial F_z}{\partial x} = \frac{F_c B_f}{|\mathbf{V}|^2}\left[\frac{V_x V_z}{|\mathbf{V}|}\frac{\partial V_x}{\partial x}\right] \tag{6.24}$$

$$\frac{\partial F_z}{\partial y} = \frac{F_c B_f}{|\mathbf{V}|^2}\left[\frac{V_y V_z}{|\mathbf{V}|}\frac{\partial V_y}{\partial y}\right] \tag{6.25}$$

$$\frac{\partial F_z}{\partial z} = B_k B_f \frac{V_z}{|\mathbf{V}|} - \frac{F_c B_f}{|\mathbf{V}|^2}\left[\frac{\partial V_z}{\partial z}|\mathbf{V}| - \frac{V_z^2}{|\mathbf{V}|}\frac{\partial V_z}{\partial z}\right] \tag{6.26}$$

With:

$$\frac{\partial V_x}{\partial x} = \frac{1}{\Delta t} \tag{6.27}$$

$$\frac{\partial V_y}{\partial y} = \frac{1}{\Delta t} \tag{6.28}$$

$$\frac{\partial V_z}{\partial z} = \frac{1}{\Delta t} \tag{6.29}$$

The stiffness matrix becomes:

$$K = -\frac{B_f F_c}{|\mathbf{V}|^2 \Delta t}\begin{pmatrix} \frac{V_x^2}{|\mathbf{V}|} - |\mathbf{V}| & \frac{V_x V_y}{|\mathbf{V}|} & \frac{V_x V_z}{|\mathbf{V}|} \\ \frac{V_x V_y}{|\mathbf{V}|} & \frac{V_y^2}{|\mathbf{V}|} - |\mathbf{V}| & \frac{V_y V_z}{|\mathbf{V}|} \\ \frac{V_x V_z}{|\mathbf{V}|} & \frac{V_y V_z}{|\mathbf{V}|} & \frac{V_z^2}{|\mathbf{V}|} - |\mathbf{V}| \end{pmatrix} - \frac{B_f B_k}{|\mathbf{V}|}\begin{pmatrix} 0 & 0 & V_x \\ 0 & 0 & V_y \\ 0 & 0 & V_z \end{pmatrix} \tag{6.30}$$

6.3.2.2 Low-Speed

$$\frac{\partial F_x}{\partial x} = -\frac{F_c B_f}{V_l}\frac{\partial V_x}{\partial x} \tag{6.31}$$

$$\frac{\partial F_x}{\partial y} = 0 \tag{6.32}$$

$$\frac{\partial F_x}{\partial z} = B_k B_f \frac{V_x}{V_l} \tag{6.33}$$

$$\frac{\partial F_y}{\partial x} = 0 \tag{6.34}$$

$$\frac{\partial F_y}{\partial y} = -\frac{F_c B_f}{V_l} \frac{\partial V_y}{\partial y} \tag{6.35}$$

$$\frac{\partial F_y}{\partial z} = B_k B_f \frac{V_y}{V_l} \tag{6.36}$$

$$\frac{\partial F_z}{\partial x} = 0 \tag{6.37}$$

$$\frac{\partial F_z}{\partial y} = 0 \tag{6.38}$$

$$\frac{\partial F_z}{\partial z} = B_k B_f \frac{V_z}{V_l} - \frac{F_c B_f}{V_l} \frac{\partial V_z}{\partial z} \tag{6.39}$$

The stiffness matrix becomes:

$$K = -\frac{B_f}{V_l} \begin{pmatrix} \frac{F_c}{\Delta t} & 0 & -B_k V_x \\ 0 & \frac{F_c}{\Delta t} & -B_k V_y \\ 0 & 0 & \frac{F_c}{\Delta t} - B_k V_z \end{pmatrix} \tag{6.40}$$