Chapter 4
The Triangular Finite Element for Netting

Abstract The modellings for netting are fully described. The usual modellings based on numerical twines or globalization of twines are partly explained with their limitations. These limitations have drove to the creation of the triangular finite element for netting. This triangular element for netting is fully described. The forces required for the equilibrium calculation are fully described, as well as the stiffness in case of—twines elasticity,—hydrodynamic forces,—twine flexion,—mesh opening stiffness,—fish catch pressure,—inertia,—buoyancy and weight.

Keywords Triangular finite element for netting · Twines tension in netting · Hydrodynamic forces on netting · Twine flexion in netting · Mesh opening stiffness of netting · Fish catch pressure in cod-end

4.1 State-of-the-Art of Numerical Modelling for Nets

4.1.1 Constitutive Law for Nets

There is little or no published work on the constitutive law for nets. Only Rivlin [23], to our knowledge, begins to express the stresses in a net surface, but only under conditions of symmetrical deformation twine. If such constitutive law could be defined, usual finite element softwares could be adapted for nettings.

4.1.2 Twine Numerical Method

The twine numerical method includes almost all the work on numerical modelling of the net [2, 6, 9, 10, 11, 24]. The initial idea is simple: the twines of the net are modelled by bars (called here numerical twines). Then a few adjustments are required.

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The twines could be modelled by two bars to account for the shortening, which appears as an angle between the bars. The twines could be modelled with a single bar, but Young's modulus in compression is almost zero to account for the shortening. Given the large number of twines in some structures (up to one million), a numerical bar refers to several true twines (Fig. 4.1). This is called globalization.

The major difficulty with this method of globalization lies in the description of the net by numerical twines. Indeed, a structure is very often the assembly of several panels of nets. Therefore, the creation of numerical twines in a panel will generate nodes on its contour. These nodes are the basis for the creation of numerical twines of the adjacent panel (Figs. 4.2 and 4.3).

Figure 4.2a shows four panels (50 by 50 meshes) whose numerical twines connect perfectly (Fig. 4.2b): the nodes on the edges are perfectly aligned with the nodes of the adjacent panels.

Figure 4.3a shows the same example, except that panel 1 is only 45 meshes horizontally. In this case the nodes on the borders do not connect perfectly between panels 4 and 1 (Fig. 4.3b), whereas the connections are perfect on the other three seams. This approach requires facilities such modification of the design of the netting panels. These facilities are not well described in the literature dedicated to this method.

4.2 The Finite Element for Netting

Triangular elements have been developed to model the net (Fig. 4.4). A number of approximations are made in these triangular elements, with the aim of calculating the forces at the vertices of these elements. These are calculated based on the positions

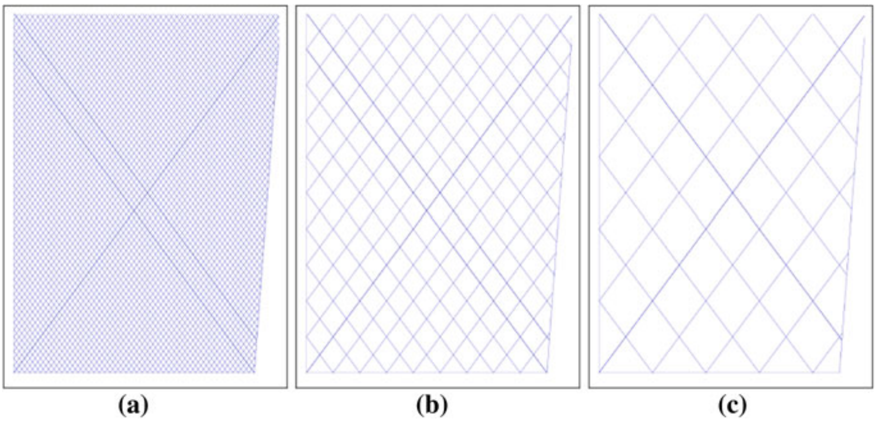
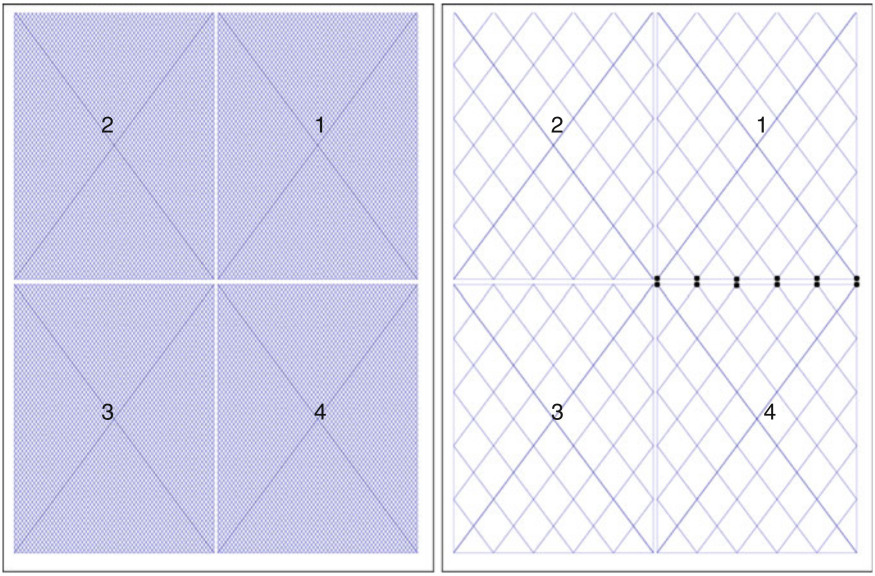


Fig. 4.1 Control net 50 meshes high by 50 and 45 wide (**a**), with a ratio of globalization of 5 (**b**) and 10 (**c**)



(a) (b)

Fig. 4.2 Structure of four panels of 50 by 50 meshes (**a**) discretized in numerical twines (**b**; globalization ratio of 10): the connection between numerical nodes on the borders of panels is perfect (*black dots* for the border between panels)

of the vertices. The basic assumption in modelling nets by triangular elements is that the twines remain parallel. Under these conditions the twines of the same direction have the same deformation. The second assumption is that the twines are modelled as elastic rods.

One difficulty with the method of numerical globalized twines (or numerical twines) was described earlier: nodes on the edges of the panels do not always coincide perfectly (Fig. 4.3b). This difficulty disappears with triangular elements, since the discretization of a netting panel is independent of the discretization of adjacent panels, except on the border. The same panels of Fig. 4.3 are discretized in Fig. 4.5 with triangular elements. Panel 2 in (Fig. 4.5a) is discretized with large triangular elements and in (Fig. 4.5b) with smaller elements. It is clear that triangular element discretization is done very easily, unlike the numerical twines technique. This flexibility in the creation of triangular elements overcomes the cumbersome tool for creating globalized twines. This burden results from many different cases to be processed and consequently adjustments that sometimes make it impossible to fully describe the structure to be studied with the method of numerical twines.

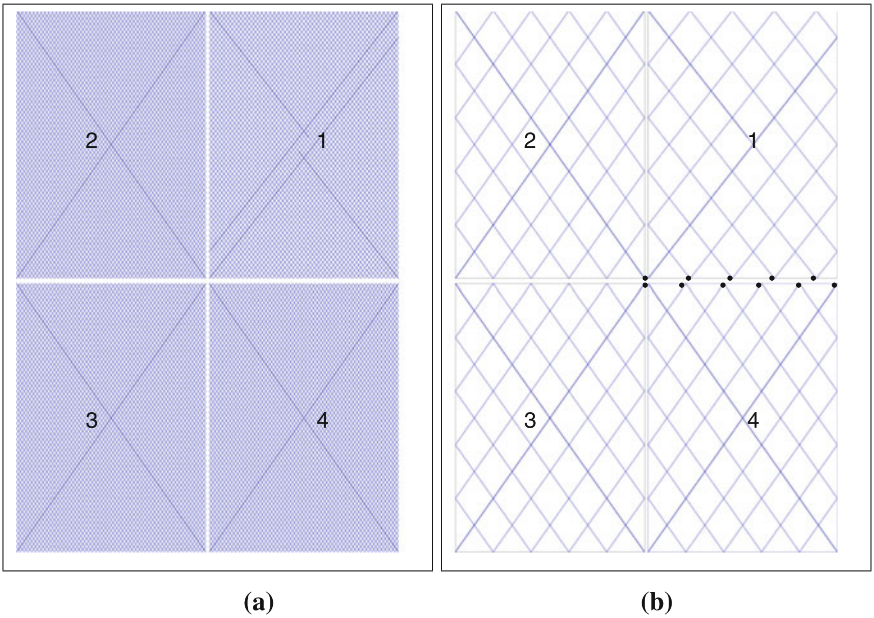


Fig. 4.3 **a** Four netting panels 50 by 50 meshes except for panel 1, which has only 45 meshes horizontally. **b** The globalization of 10 leads the nodes on the common border of panels 1 and 4 to not connect perfectly: panel 1 has five nodes on its bottom border, while the top border of panel 4 has six nodes (*black dots*)

4.2.1 The Basic Method: Direct Formulation

The triangular finite element dedicated to diamond mesh nets is described here.

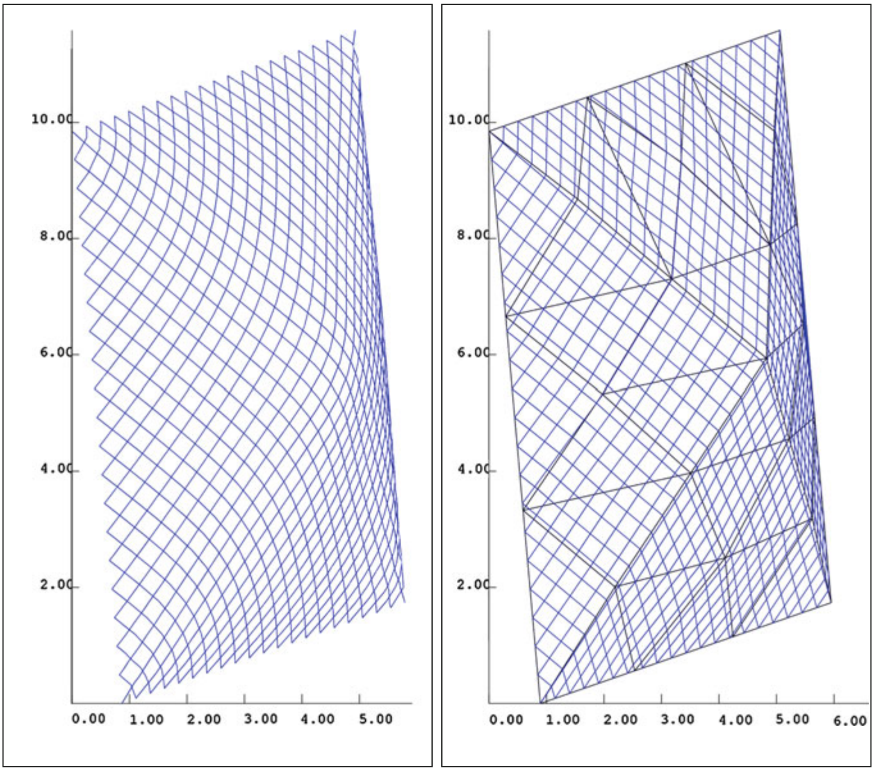
The triangular element is defined by its three vertices, which are connected to the net. The coordinates of the vertices in number of twine vectors are then constant, whatever the deformation of the triangle. Figure 4.6 shows an example. In this example the coordinates in twine number of node 1 are 1.5 along the **U** twine and −3.5 along the **V** twine. It is clear that if the origin of coordinates in twine number changes, the twine coordinates of nodes will change but will not affect the equilibrium position of the net.

These twines are parallel inside the triangular element, which means that the sides of the triangle (**12**, **23**, **31**) are linear combinations of twine vectors (**U** and **V**, cf. Fig. 4.6). This point is the main foundation of the model. These combinations are as follows:

$$\mathbf{12} = (U_2 - U_1)\mathbf{U} + (V_2 - V_1)\mathbf{V} \tag{4.1}$$

$$\mathbf{13} = (U_3 - U_1)\mathbf{U} + (V_3 - V_1)\mathbf{V} \tag{4.2}$$

12 (**13**): vector from vertex 1 (1) to vertex 2 (3).



(a) **(b)**

Fig. 4.4 The diamond mesh (**a**) is decomposed into triangular elements (**b**). The approximation in each triangle is that twines are parallel and therefore have the same deformation, and that the twines are elastic

The two previous equations with two unknowns (**U** and **V**) then give the following:

$$\mathbf{U} = \frac{V_3 - V_1}{d}\mathbf{12} - \frac{V_2 - V_1}{d}\mathbf{13} \tag{4.3}$$

$$\mathbf{V} = \frac{U_2 - U_1}{d}\mathbf{13} - \frac{U_3 - U_1}{d}\mathbf{12} \tag{4.4}$$

With side vectors:

$$\mathbf{12} = \begin{vmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{vmatrix} \tag{4.5}$$

$$\mathbf{13} = \begin{vmatrix} x_3 - x_1 \\ y_3 - y_1 \\ z_3 - z_1 \end{vmatrix} \tag{4.6}$$

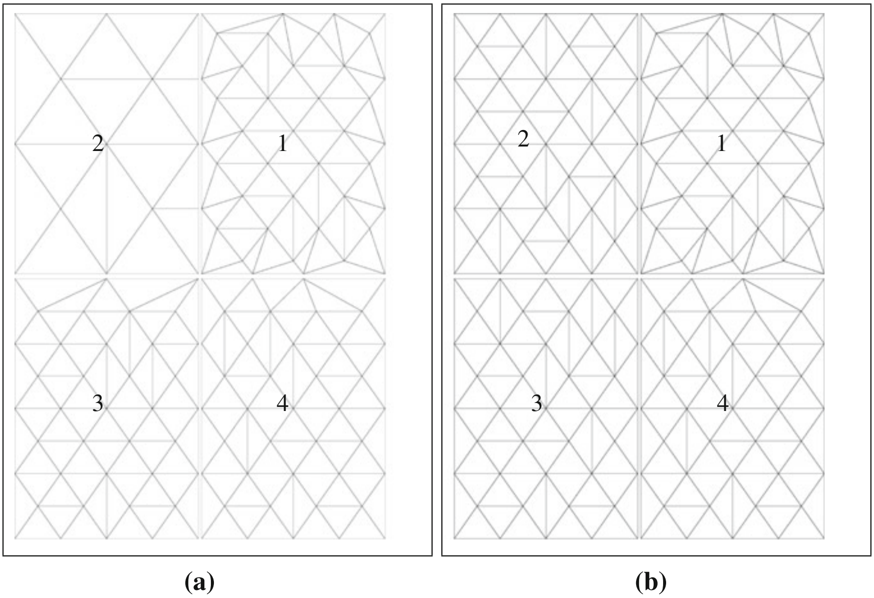
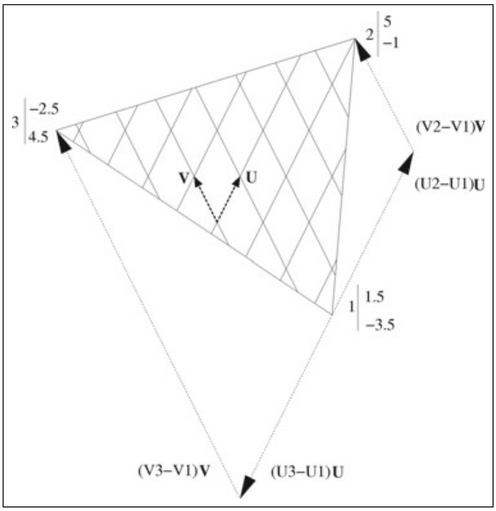


Fig. 4.5 Case identical to Fig. 4.3. Although the netting in panel 1 has only 45 meshes horizontally, the triangular element discretization is easy. The step size of panel 2 is larger in (**a**) than in (**b**)

Fig. 4.6 A triangular element: the sides of the triangle are linear combinations of twine vectors (**U** and **V**). The coordinates in twine number are noted. The origin of theses coordinates is the intersection of **U** and **V**



and

$$d = (U_2 - U_1)(V_3 - V_1) - (U_3 - U_1)(V_2 - V_1) \tag{4.7}$$

x_i, y_i, z_i: Cartesian coordinates of vertex i,
U_i, V_i: coordinates of vertex i in number of twines (twine coordinates).

The twine vectors (**U**, **V**) are calculated from the Cartesian coordinates (x_i, y_i, z_i) of the vertices of the triangular element.

It appears that nothing implies that the number of twine coordinates of the vertices of the triangle consists of integers. Therefore, these coordinates can be real. This implies that the vertices of the triangle are not necessarily located on knots of the net (Fig. 4.4). Similarly, nothing prevents the triangle from being smaller than a mesh. It appears that while the triangle does not contain any piece of twine of the net, d is not null, and therefore the triangle contains twines and consequently a deformation energy. In other words, the triangular finite element is a homogenization of the mechanical properties of the net.

It also appears that every point of the twines belongs to only one triangular element and still the same, regardless of the deformation of the net. Points on the contour of a triangular element also belong to the neighbours.

4.2.2 Metric of the Triangular Element

The objective of the finite element method is to calculate the Cartesian coordinates of the numerical nodes. These nodes are, for the netting, the vertices of the triangular elements (Figs. 4.7 and 4.8a).

The nodes are fixed relative to the netting, which means that the coordinates of the nodes in twines or meshes remain constant regardless of the netting deformation.

Figure 4.8b and c show an example of coordinates of a triangular element. Generally speaking, the mesh coordinates are used by the netting maker.

There are relations between the mesh coordinates and the twine coordinates, the bases of which are noted in Fig. 4.8b and c.

The relations between the bases are the following:

$$\mathbf{u} = \mathbf{U} - \mathbf{V} \tag{4.8}$$

$$\mathbf{v} = \mathbf{U} + \mathbf{V} \tag{4.9}$$

This leads to:

$$\mathbf{U} = \frac{\mathbf{u} + \mathbf{v}}{2} \tag{4.10}$$

$$\mathbf{V} = \frac{\mathbf{v} - \mathbf{u}}{2} \tag{4.11}$$

u, **v**: mesh coordinates base,
U, **V**: twine coordinates base.

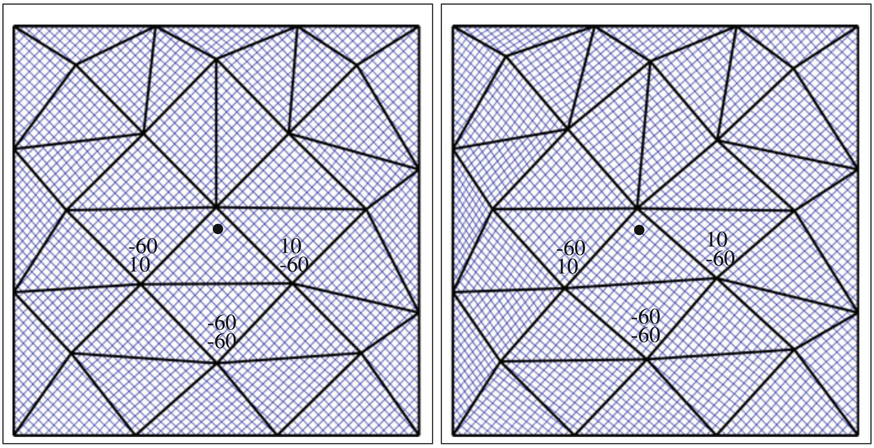
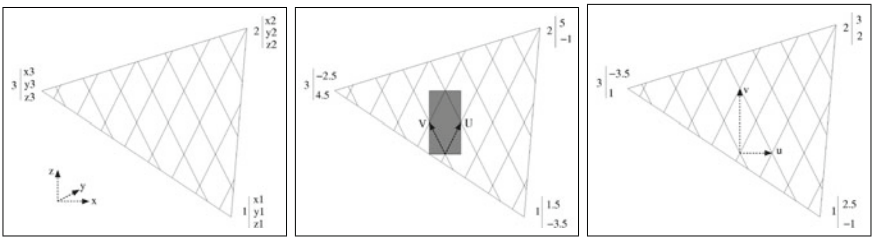


Fig. 4.7 Two deformations of the same structure. The twines coordinates of vertices remain constant. The twines coordinates of three vertices are noted. The *dot* is the origin of twines numbering. Only 1 twine on 5 is drawn



(a) **(b)** **(c)**

Fig. 4.8 Triangular element: Cartesian coordinates (**a**), twines coordinates (**b**), and mesh coordinates (**c**). The *grey* surface is a mesh surface (b)

This means that the relations between the twine coordinates and the mesh coordinates of the node P are the following:

$$U_P = u_P + v_P \tag{4.12}$$

$$V_P = v_P - u_P \tag{4.13}$$

and

$$u_P = \frac{U_P - V_P}{2} \tag{4.14}$$

$$v_P = \frac{U_P + V_P}{2} \tag{4.15}$$

Here, U_P and V_P are the twine coordinates, and u_P and v_P are the mesh coordinates of the same node P. In these conditions the vector from origin to node P could be written as follows:

$$\mathbf{OP} = U_P\mathbf{U} + V_P\mathbf{V} \tag{4.16}$$

$$\mathbf{OP} = u_P\mathbf{u} + v_P\mathbf{v} \tag{4.17}$$

Because the amplitude of a cross product of vectors is twice the surface of the triangle made of these two vectors, the Cartesian surface of the triangular element (in m^2) is half the amplitude of the cross product of the side vectors of the triangular element:

$$S = \frac{1}{2}|\mathbf{12} \wedge \mathbf{13}| \tag{4.18}$$

The side vectors in Cartesian coordinates are as follows:

$$\mathbf{12} = \begin{vmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{vmatrix} \tag{4.19}$$

$$\mathbf{13} = \begin{vmatrix} x_3 - x_1 \\ y_3 - y_1 \\ z_3 - z_1 \end{vmatrix} \tag{4.20}$$

By the same way, the number of meshes, as defined in Fig. 4.8b, is

$$nb_m = \frac{1}{4}|\mathbf{12} \wedge \mathbf{13}| \tag{4.21}$$

with side vectors in twine coordinates:

$$\mathbf{12} = \begin{vmatrix} U_2 - U_1 \\ V_2 - V_1 \\ 0 \end{vmatrix} \tag{4.22}$$

$$\mathbf{13} = \begin{vmatrix} U_3 - U_1 \\ V_3 - V_1 \\ 0 \end{vmatrix} \tag{4.23}$$

The number of meshes in a triangular element is

$$nb_m = \frac{1}{4}\left[(U_2 - U_1)(V_3 - V_1) - (U_3 - U_1)(V_2 - V_1)\right] = \frac{d}{4} \tag{4.24}$$

Because there are two twines U and two twines V per mesh, the number of twines U and V is calculated as follows:

$$nb_U = \frac{d}{2} \quad (4.25)$$

$$nb_V = \frac{d}{2} \quad (4.26)$$

Because there are also two knots per mesh, the number of knots in a triangular element is

$$nb_k = \frac{d}{2} \quad (4.27)$$

The surface (m^2) of one mesh is calculated through the cross product of twines vectors (**U** and **V**):

$$Ms = 2|\mathbf{U} \wedge \mathbf{V}| \quad (4.28)$$

which is also the surface of the triangular element divided by the number of meshes in the element:

$$Ms = \frac{S}{nb_m} \quad (4.29)$$

In the case of Figs. 4.6 and 4.8, $d = 38$, the number of meshes is 9.5, the number of **U** twines is 18, the number of **V** twines is 18, and the number of knots is 18.

4.3 The Forces on the Netting

4.3.1 Twine Tension in Diamond Mesh

The tensions in the twines are required to estimate the forces on the vertices due to these tensions. In the hypothesis of linear elasticity, these tensions are deduced from **U** and **V**, which have been previously calculated. In these conditions the twine tensions are as follows:

$$T_u = EA\frac{|\mathbf{U}| - l_0}{l_0} \quad (4.30)$$

$$T_v = EA\frac{|\mathbf{V}| - l_0}{l_0} \quad (4.31)$$

E : Young's modulus of the material (N/m^2),
A : mechanical section of the twines U and V (m^2),
l_o : unstretched length of twine vectors (m).

The principle of virtual work is used here to calculate the forces on the vertices due to the tension in the twines.

The force component along X on vertex 1 of a triangular element is estimated by considering a virtual displacement ($\partial x1$) along the axis x of vertex 1. This displacement leads to an external work:

$$W_e = F_{x1}\partial x1 \tag{4.32}$$

This displacement also induces a change in the length of mesh bars ($\partial|\mathbf{U}|$ and $\partial|\mathbf{V}|$), an internal work per twine $\partial|\mathbf{U}|T_u$ and $\partial|\mathbf{V}|T_v$ and therefore an internal work for the triangular element:

$$W_i = (\partial|\mathbf{U}|T_u + \partial|\mathbf{V}|T_v)\frac{d}{2} \tag{4.33}$$

The principle of virtual work implies that the external work equals the internal work, since the forces represent the tension in the twines. That gives for each component of force on the three vertices:

$$F_{x1} = \left(T_u\frac{\partial|\mathbf{U}|}{\partial x1} + T_v\frac{\partial|\mathbf{V}|}{\partial x1}\right)\frac{d}{2} \tag{4.34}$$

$$F_{y1} = \left(T_u\frac{\partial|\mathbf{U}|}{\partial y1} + T_v\frac{\partial|\mathbf{V}|}{\partial y1}\right)\frac{d}{2} \tag{4.35}$$

$$F_{z1} = \left(T_u\frac{\partial|\mathbf{U}|}{\partial z1} + T_v\frac{\partial|\mathbf{V}|}{\partial z1}\right)\frac{d}{2} \tag{4.36}$$

$$F_{x2} = \left(T_u\frac{\partial|\mathbf{U}|}{\partial x2} + T_v\frac{\partial|\mathbf{V}|}{\partial x2}\right)\frac{d}{2} \tag{4.37}$$

$$F_{y2} = \left(T_u\frac{\partial|\mathbf{U}|}{\partial y2} + T_v\frac{\partial|\mathbf{V}|}{\partial y2}\right)\frac{d}{2} \tag{4.38}$$

$$F_{z2} = \left(T_u\frac{\partial|\mathbf{U}|}{\partial z2} + T_v\frac{\partial|\mathbf{V}|}{\partial z2}\right)\frac{d}{2} \tag{4.39}$$

$$F_{x3} = \left(T_u\frac{\partial|\mathbf{U}|}{\partial x3} + T_v\frac{\partial|\mathbf{V}|}{\partial x3}\right)\frac{d}{2} \tag{4.40}$$

$$F_{y3} = \left(T_u\frac{\partial|\mathbf{U}|}{\partial y3} + T_v\frac{\partial|\mathbf{V}|}{\partial y3}\right)\frac{d}{2} \tag{4.41}$$

$$F_{z3} = \left(T_u\frac{\partial|\mathbf{U}|}{\partial z3} + T_v\frac{\partial|\mathbf{V}|}{\partial z3}\right)\frac{d}{2} \tag{4.42}$$

The derivatives $\frac{\partial|U|}{\partial x1}$ ... $\frac{\partial|V|}{\partial z3}$ can be calculated, as the equations relating to U, V and X_i, Y_i, Z_i have already been described. This gives the following vectors force for the three vertices:

$$\mathbf{F_1} = (V_3 - V_2)T_u\frac{\mathbf{U}}{2|\mathbf{U}|} + (U_2 - U_3)T_v\frac{\mathbf{V}}{2|\mathbf{V}|} \tag{4.43}$$

$$\mathbf{F_2} = (V_1 - V_3)T_u\frac{\mathbf{U}}{2|\mathbf{U}|} + (U_3 - U_1)T_v\frac{\mathbf{V}}{2|\mathbf{V}|} \tag{4.44}$$

$$\mathbf{F_3} = (V_2 - V_1)T_u\frac{\mathbf{U}}{2|\mathbf{U}|} + (U_1 - U_2)T_v\frac{\mathbf{V}}{2|\mathbf{V}|} \tag{4.45}$$

The Newton-Raphson method, described earlier, requires the calculation of the stiffness matrix, which is calculated from the derivatives of effort with respect to the positions of the vertices of the triangular element. The 81 derivatives, that is to say, by 9 by 9 component coordinates, are then the following:

The stiffness matrix:

$$K = \begin{pmatrix} -\frac{\partial F_{x1}}{\partial x_1} & -\frac{\partial F_{x1}}{\partial y_1} & \cdots & -\frac{\partial F_{x1}}{\partial z_3} \\ -\frac{\partial F_{y1}}{\partial x_1} & -\frac{\partial F_{y1}}{\partial y_1} & \cdots & -\frac{\partial F_{y1}}{\partial z_3} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ -\frac{\partial F_{z3}}{\partial x_1} & -\frac{\partial F_{z3}}{\partial y_1} & \cdots & -\frac{\partial F_{z3}}{\partial z_3} \end{pmatrix} \tag{4.46}$$

The components are calculated as follows:

$$\begin{aligned}\frac{\partial F_{w1}}{\partial t} =& \frac{EA_u(V_3 - V_2)}{2}\left[\frac{\partial U_w}{\partial t}\left(\frac{1}{n_0} - \frac{1}{|\mathbf{U}|}\right) + \frac{\partial|\mathbf{U}|}{\partial t}\frac{U_w}{|\mathbf{U}|^2}\right] \\ &+ \frac{EA_v(U_2 - U_3)}{2}\left[\frac{\partial V_w}{\partial t}\left(\frac{1}{n_0} - \frac{1}{|\mathbf{V}|}\right) + \frac{\partial|\mathbf{V}|}{\partial t}\frac{V_w}{|\mathbf{V}|^2}\right]\end{aligned} \tag{4.47}$$

$$\begin{aligned}\frac{\partial F_{w2}}{\partial t} =& \frac{EA_u(V_1 - V_3)}{2}\left[\frac{\partial U_w}{\partial t}\left(\frac{1}{n_0} - \frac{1}{|\mathbf{U}|}\right) + \frac{\partial|\mathbf{U}|}{\partial t}\frac{U_w}{|\mathbf{U}|^2}\right] \\ &+ \frac{EA_v(U_3 - U_1)}{2}\left[\frac{\partial V_w}{\partial t}\left(\frac{1}{n_0} - \frac{1}{|\mathbf{V}|}\right) + \frac{\partial|\mathbf{V}|}{\partial t}\frac{V_w}{|\mathbf{V}|^2}\right]\end{aligned} \tag{4.48}$$

$$\begin{aligned}\frac{\partial F_{w3}}{\partial t} =& \frac{EA_u(V_2 - V_1)}{2}\left[\frac{\partial U_w}{\partial t}\left(\frac{1}{n_0} - \frac{1}{|\mathbf{U}|}\right) + \frac{\partial|\mathbf{U}|}{\partial t}\frac{U_w}{|\mathbf{U}|^2}\right] \\ &+ \frac{EA_v(U_1 - U_2)}{2}\left[\frac{\partial V_w}{\partial t}\left(\frac{1}{n_0} - \frac{1}{|\mathbf{V}|}\right) + \frac{\partial|\mathbf{V}|}{\partial t}\frac{V_w}{|\mathbf{V}|^2}\right]\end{aligned} \tag{4.49}$$

With:
$w = x, y, z,$
$t = x1, y1, z1, x2, y2, z2, x3, y3, z3.$

The following derivatives are also required.

The derivatives of the components of **U** are as follows:

$$\frac{\partial U_x}{\partial x1} = \frac{\partial U_y}{\partial y1} = \frac{\partial U_z}{\partial z1} = \frac{V_2 - V_3}{d} \tag{4.50}$$
$$\frac{\partial U_x}{\partial x2} = \frac{\partial U_y}{\partial y2} = \frac{\partial U_z}{\partial z2} = \frac{V_3 - V_1}{d} \tag{4.51}$$
$$\frac{\partial U_x}{\partial x3} = \frac{\partial U_y}{\partial y3} = \frac{\partial U_z}{\partial z3} = \frac{V_1 - V_2}{d} \tag{4.52}$$
$$\frac{\partial U_x}{\partial yi} = \frac{\partial U_x}{\partial zi} = \frac{\partial U_y}{\partial zi} = \frac{\partial U_y}{\partial xi} = \frac{\partial U_z}{\partial xi} = \frac{\partial U_z}{\partial yi} = 0 \tag{4.53}$$

The derivatives of the components of **V** are the following:

$$\frac{\partial V_x}{\partial x1} = \frac{\partial V_y}{\partial y1} = \frac{\partial V_z}{\partial z1} = \frac{U_3 - U_2}{d} \tag{4.54}$$
$$\frac{\partial V_x}{\partial x2} = \frac{\partial V_y}{\partial y2} = \frac{\partial V_z}{\partial z2} = \frac{U_1 - U_3}{d} \tag{4.55}$$
$$\frac{\partial V_x}{\partial x3} = \frac{\partial V_y}{\partial y3} = \frac{\partial V_z}{\partial z3} = \frac{U_2 - U_1}{d} \tag{4.56}$$
$$\frac{\partial V_x}{\partial yi} = \frac{\partial V_x}{\partial zi} = \frac{\partial V_y}{\partial zi} = \frac{\partial V_y}{\partial xi} = \frac{\partial V_z}{\partial xi} = \frac{\partial V_z}{\partial yi} = 0 \tag{4.57}$$

The derivatives of $|\mathbf{U}|$ follow:

$$\frac{\partial |\mathbf{U}|}{\partial x1} = \frac{V_2 - V_3}{d^2}\left[(x_2 - x_1)(V_3 - V_1) - (x_3 - x_1)(V_2 - V_1)\right] \tag{4.58}$$
$$\frac{\partial |\mathbf{U}|}{\partial x2} = \frac{V_3 - V_1}{d^2}\left[(x_2 - x_1)(V_3 - V_1) - (x_3 - x_1)(V_2 - V_1)\right] \tag{4.59}$$
$$\frac{\partial |\mathbf{U}|}{\partial x3} = \frac{V_1 - V_2}{d^2}\left[(x_2 - x_1)(V_3 - V_1) - (x_3 - x_1)(V_2 - V_1)\right] \tag{4.60}$$
$$\frac{\partial |\mathbf{U}|}{\partial y1} = \frac{V_2 - V_3}{d^2}\left[(y_2 - y_1)(V_3 - V_1) - (y_3 - y_1)(V_2 - V_1)\right] \tag{4.61}$$
$$\frac{\partial |\mathbf{U}|}{\partial y2} = \frac{V_3 - V_1}{d^2}\left[(y_2 - y_1)(V_3 - V_1) - (y_3 - y_1)(V_2 - V_1)\right] \tag{4.62}$$
$$\frac{\partial |\mathbf{U}|}{\partial y3} = \frac{V_1 - V_2}{d^2}\left[(y_2 - y_1)(V_3 - V_1) - (y_3 - y_1)(V_2 - V_1)\right] \tag{4.63}$$
$$\frac{\partial |\mathbf{U}|}{\partial z1} = \frac{V_2 - V_3}{d^2}\left[(z_2 - z_1)(V_3 - V_1) - (z_3 - z_1)(V_2 - V_1)\right] \tag{4.64}$$
$$\frac{\partial |\mathbf{U}|}{\partial z2} = \frac{V_3 - V_1}{d^2}\left[(z_2 - z_1)(V_3 - V_1) - (z_3 - z_1)(V_2 - V_1)\right] \tag{4.65}$$
$$\frac{\partial |\mathbf{U}|}{\partial z3} = \frac{V_1 - V_2}{d^2}\left[(z_2 - z_1)(V_3 - V_1) - (z_3 - z_1)(V_2 - V_1)\right] \tag{4.66}$$

The derivatives of $|\mathbf{V}|$ are shown below:

$$\frac{\partial|\mathbf{V}|}{\partial x1} = \frac{U_2 - U_3}{d^2}\left[(x_2 - x_1)(U_3 - U_1) - (x_3 - x_1)(U_2 - U_1)\right] \tag{4.67}$$

$$\frac{\partial|\mathbf{V}|}{\partial x2} = \frac{U_3 - U_1}{d^2}\left[(x_2 - x_1)(U_3 - U_1) - (x_3 - x_1)(U_2 - U_1)\right] \tag{4.68}$$

$$\frac{\partial|\mathbf{V}|}{\partial x3} = \frac{U_1 - U_2}{d^2}\left[(x_2 - x_1)(U_3 - U_1) - (x_3 - x_1)(U_2 - U_1)\right] \tag{4.69}$$

$$\frac{\partial|\mathbf{V}|}{\partial y1} = \frac{U_2 - U_3}{d^2}\left[(y_2 - y_1)(U_3 - U_1) - (y_3 - y_1)(U_2 - U_1)\right] \tag{4.70}$$

$$\frac{\partial|\mathbf{V}|}{\partial y2} = \frac{U_3 - U_1}{d^2}\left[(y_2 - y_1)(U_3 - U_1) - (y_3 - y_1)(U_2 - U_1)\right] \tag{4.71}$$

$$\frac{\partial|\mathbf{V}|}{\partial y3} = \frac{U_1 - U_2}{d^2}\left[(y_2 - y_1)(U_3 - U_1) - (y_3 - y_1)(U_2 - U_1)\right] \tag{4.72}$$

$$\frac{\partial|\mathbf{V}|}{\partial z1} = \frac{U_2 - U_3}{d^2}\left[(z_2 - z_1)(U_3 - U_1) - (z_3 - z_1)(U_2 - U_1)\right] \tag{4.73}$$

$$\frac{\partial|\mathbf{V}|}{\partial z2} = \frac{U_3 - U_1}{d^2}\left[(z_2 - z_1)(U_3 - U_1) - (z_3 - z_1)(U_2 - U_1)\right] \tag{4.74}$$

$$\frac{\partial|\mathbf{V}|}{\partial z3} = \frac{U_1 - U_2}{d^2}\left[(z_2 - z_1)(U_3 - U_1) - (z_3 - z_1)(U_2 - U_1)\right] \tag{4.75}$$

4.3.2 Twine Tension in Hexagonal Mesh

The same technique for the diamond mesh netting is used for hexagonal ones. The triangular element dedicated to the hexagonal mesh netting has the same assumption as previously adopted: the three families of twines inside the element are parallel, i.e., **l**, **m**, and **n** twine vectors, are parallel (Fig. 4.9).

The mesh base (shaded area in Fig. 4.9) is first defined. This base mesh is defined as a parallelogram; its corners coincide with knots, and it includes two **l** twine vectors, two **m** twine vectors, and two **n** twine vectors. This base mesh is also used to quantify the number of meshes inside the triangular element. The vertices of the triangular element then have coordinates in base meshes (U_1, U_2, U_3, V_1, V_2, V_3; Fig. 4.9).

Vectors **U** and **V** are the sides of the mesh base. There are linear relations between these two vectors and the sides of the triangular element (arrows on Fig. 4.9):

$$\mathbf{12} = (U_2 - U_1)\mathbf{U} + (V_2 - V_1)\mathbf{V} \tag{4.76}$$

$$\mathbf{13} = (U_3 - U_1)\mathbf{U} + (V_3 - V_1)\mathbf{V} \tag{4.77}$$

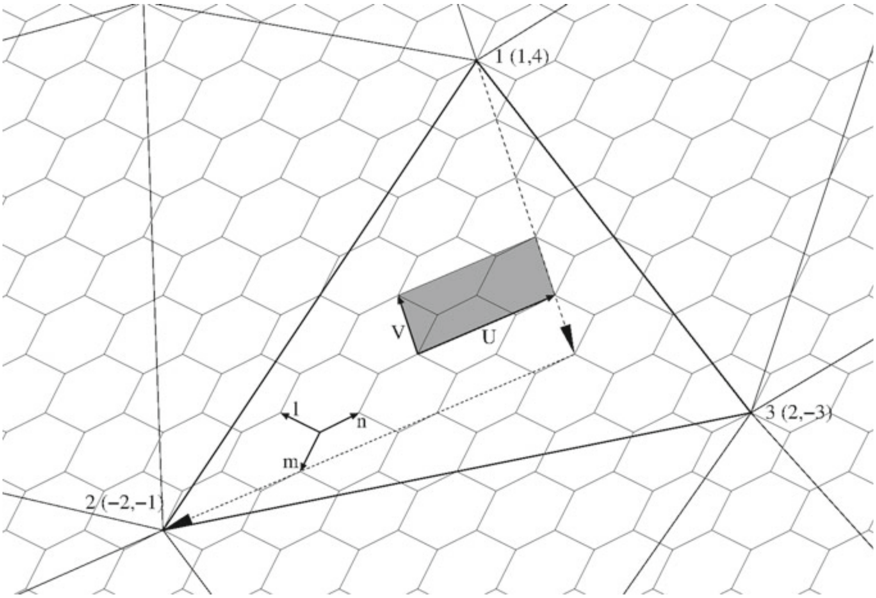


Fig. 4.9 Triangular element dedicated to the hexagonal mesh nets. The twine vectors are **l**, **m**, and **n**. The number of meshes are noted for each vertex. The mesh base is in *grey* and is defined by vectors **U** and **V**

The two previous equations give the following as in the case of diamond mesh (see Sect. 4.2.1, page 30), namely:

$$\mathbf{U} = \frac{V_3 - V_1}{d}\mathbf{12} - \frac{V_2 - V_1}{d}\mathbf{13} \tag{4.78}$$

$$\mathbf{V} = \frac{U_3 - U_1}{d}\mathbf{12} - \frac{U_2 - U_1}{d}\mathbf{13} \tag{4.79}$$

With vectors of the sides of the mesh base:

$$\mathbf{12} = \begin{vmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{vmatrix} \tag{4.80}$$

$$\mathbf{13} = \begin{vmatrix} x_3 - x_1 \\ y_3 - y_1 \\ z_3 - z_1 \end{vmatrix} \tag{4.81}$$

and

$$d = (U_2 - U_1)(V_3 - V_1) - (U_3 - U_1)(V_2 - V_1) \tag{4.82}$$

x_i, y_i, z_i: Cartesian coordinates of vertex i.

The number of base meshes in a triangular element is equal to $d/2$, the total number twine vectors is $3d$, the number of twine vectors **l**, **m**, or **n** is d, and the number of nodes is $2d$.

Tensions in twine vectors **l**, **m**, and **n** are now calculated. This is done by solving the force balance of the twines. This is solved by writing the following equations:

(1) The base mesh definition leads to (Fig. 4.9) :

$$\mathbf{U} = -\mathbf{m} + 2\mathbf{n} - \mathbf{l} \tag{4.83}$$

$$\mathbf{V} = -\mathbf{m} + \mathbf{l} \tag{4.84}$$

(2) The amplitude of tension in the twines gives:

$$|\mathbf{T}_l| = EA_l \frac{|\mathbf{l}| - l_0}{l_0} \tag{4.85}$$

$$|\mathbf{T}_m| = EA_m \frac{|\mathbf{m}| - m_0}{m_0} \tag{4.86}$$

$$|\mathbf{T}_n| = EA_n \frac{|\mathbf{n}| - n_0}{n_0} \tag{4.87}$$

(3) The balance of tensions leads to:

$$\mathbf{T}_l + \mathbf{T}_m + \mathbf{T}_n = \mathbf{0} \tag{4.88}$$

This gives six equations with six unknowns ($\mathbf{l}$, $\mathbf{m}$, $\mathbf{n}$, $\mathbf{T}_l$, $\mathbf{T}_m$, $\mathbf{T}_n$).

4.3.2.1 Equilibrium of the Joint Knot

The six previous equations can be reduced to the two that follow with two unknowns (m_x and m_y components of **m**), since the triangular element has been turned in the plane XOY [17, 19]:

$$\frac{m_x + V_x}{\sqrt{(m_x + V_x)^2 + (m_y + V_y)^2}} \frac{E_l A_l}{l_o} \left[\sqrt{(m_x + V_x)^2 + (m_y + V_y)^2} - l_o\right]$$
$$+ \frac{m_x}{\sqrt{m_x^2 + m_y^2}} \frac{E_m A_m}{m_o} \left[\sqrt{m_x^2 + m_y^2} - m_o\right]$$
$$+ \frac{m_x + \frac{U_x + V_x}{2}}{\sqrt{\left(m_x + \frac{U_x+V_x}{2}\right)^2 + \left(m_y + \frac{U_y+V_y}{2}\right)^2}} \frac{E_n A_n}{n_o}$$

$$\times \left[\sqrt{\left(m_x + \frac{U_x + V_x}{2}\right)^2 + \left(m_y + \frac{U_y + V_y}{2}\right)^2} - n_o \right]$$
$$= 0 \tag{4.89}$$

$$\frac{m_y + V_y}{\sqrt{(m_x + V_x)^2 + (m_y + V_y)^2}} \frac{E_l A_l}{l_o} \left[\sqrt{(m_y + V_y)^2 + (m_y + V_y)^2} - l_o \right]$$
$$+ \frac{m_y}{\sqrt{m_x^2 + m_y^2}} \frac{E_m A_m}{m_o} \left[\sqrt{m_y^2 + m_y^2} - m_o \right]$$
$$+ \frac{m_y + \frac{U_y + V_y}{2}}{\sqrt{\left(m_x + \frac{U_x + V_x}{2}\right)^2 + \left(m_y + \frac{U_y + V_y}{2}\right)^2}} \frac{E_n A_n}{n_o}$$
$$\times \left[\sqrt{\left(m_y + \frac{U_y + V_y}{2}\right)^2 + \left(m_y + \frac{U_y + V_y}{2}\right)^2} - n_o \right]$$
$$= 0 \tag{4.90}$$

m_x, m_y: components of m twine (m),
l_o, m_o, n_o: unstretched length of twines l, m, and n (m),
U_x, U_y, V_x, V_y: components of the sides of the mesh base (m; see Fig. 4.9),
E_l, E_m, E_n: Young modulus of twines l, m, and n (Pa),
A_l, A_m, A_n: section of twines l, m, and n (m^2).

These two equations describe the equilibrium of the joint knot of three twines in a triangle, the sides of which are $\frac{\mathbf{U}+\mathbf{V}}{2}$ and $\mathbf{V}$ (Fig. 4.10). These equations are in newtons.

4.3.2.2 Approximation of the Equilibrium of the Joint

The analytical solution of the two previous equations has not been found. Therefore, the following approximation has been made to simplify the equations. This approximation is acceptable because the stretched lengths of the twines are close to the unstretched length.

$$\frac{m_x}{|\mathbf{m}|} \approx \frac{m_x}{m_o} \tag{4.91}$$

$$\frac{m_y}{|\mathbf{m}|} \approx \frac{m_y}{m_o} \tag{4.92}$$

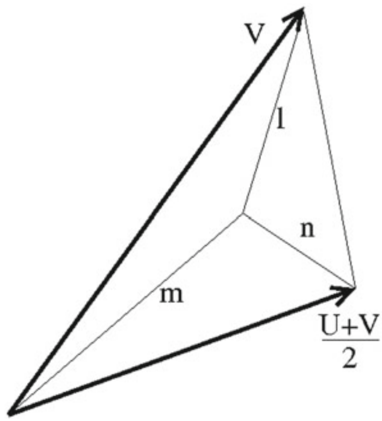


Fig. 4.10 The three twines are in the triangle defined by $\frac{\mathbf{U}+\mathbf{V}}{2}$ and **V** (cf. Fig. 4.9)

With this approximation the two previous equilibrium equations are reduced to the following:

$$(m_x + V_x)\frac{E_l A_l}{l_o^2}\left(\sqrt{(m_x + V_x)^2 + (m_y + V_y)^2} - l_o\right) + m_x \frac{E_m A_m}{m_o^2}\left(\sqrt{m_x^2 + m_y^2} - m_o\right) + \left(m_x + \frac{U_x + V_x}{2}\right)\frac{E_n A_n}{n_o^2}\left(\sqrt{\left(m_x + \frac{U_x + V_x}{2}\right)^2 + \left(m_y + \frac{U_y + V_y}{2}\right)^2} - n_o\right) = 0 \tag{4.93}$$

$$(m_y + V_y)\frac{E_l A_l}{l_o^2}\left(\sqrt{(m_x + V_x)^2 + (m_y + V_y)^2} - l_o\right) + m_y \frac{E_m A_m}{m_o^2}\left(\sqrt{m_x^2 + m_y^2} - m_o\right) + \left(m_y + \frac{U_y + V_y}{2}\right)\frac{E_n A_n}{n_o^2}\left(\sqrt{\left(m_x + \frac{U_x + V_x}{2}\right)^2 + \left(m_y + \frac{U_y + V_y}{2}\right)^2} - n_o\right) = 0 \tag{4.94}$$

They are the complete form of the following:

$$l_x \frac{E_l A_l}{l_o^2}(|\mathbf{l}| - l_o) + m_x \frac{E_m A_m}{m_o^2}(|\mathbf{m}| - m_o) + n_x \frac{E_n A_n}{n_o^2}(|\mathbf{n}| - n_o) = 0 \tag{4.95}$$

$$l_y \frac{E_l A_l}{l_o^2}(|\mathbf{l}| - l_o) + m_y \frac{E_m A_m}{m_o^2}(|\mathbf{m}| - m_o) + n_y \frac{E_n A_n}{n_o^2}(|\mathbf{n}| - n_o) = 0 \tag{4.96}$$

4.3.2.3 Newton-Raphson Method

The previous approximation has not been sufficient to reach the analytical solution. The Newton-Raphson method is used to find a numerical solution [4].

For each iteration the displacement h is searched to find the equilibrium:

$$h_k = \frac{F(x_k)}{-F'(x_k)} \tag{4.97}$$

$$x_{k+1} = x_k + h_k \tag{4.98}$$

k: iteration number,
F: force on nodes,
x: position of nodes.
Here:

$$\mathbf{F} = \begin{cases} l_x \frac{E_l A_l}{l_o^2}(|\mathbf{l}| - l_o) + m_x \frac{E_m A_m}{m_o^2}(|\mathbf{m}| - m_o) + n_x \frac{E_n A_n}{n_o^2}(|\mathbf{n}| - n_o) = F_1 \\ l_y \frac{E_l A_l}{l_o^2}(|\mathbf{l}| - l_o) + m_y \frac{E_m A_m}{m_o^2}(|\mathbf{m}| - m_o) + n_y \frac{E_n A_n}{n_o^2}(|\mathbf{n}| - n_o) = F_2 \end{cases} \tag{4.99}$$

$$\mathbf{x} = \begin{cases} m_x \\ m_y \end{cases} \tag{4.100}$$

The derivative is:

$$F' = \begin{vmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{vmatrix}. \tag{4.101}$$

With:

$$D_{11} = -\left[\frac{EA_l}{l_o^2}\left(\mathbf{l} - l_o + \frac{l_x^2}{\mathbf{l}}\right) + \frac{EA_m}{m_o^2}\left(\mathbf{m} - m_o + \frac{m_x^2}{\mathbf{m}}\right) + \frac{EA_n}{n_o^2}\left(\mathbf{n} - n_o + \frac{n_x^2}{\mathbf{n}}\right)\right] \tag{4.102}$$

$$D_{12} = D_{21} = -\left[\frac{EA_l}{l_o^2}\frac{l_x l_y}{\mathbf{l}} + \frac{EA_m}{m_o^2}\frac{m_x m_y}{\mathbf{m}} + \frac{EA_n}{n_o^2}\frac{n_x n_y}{\mathbf{n}}\right] \tag{4.103}$$

$$D_{22} = -\left[\frac{EA_l}{l_o^2}\left(\mathbf{l} - l_o + \frac{l_y^2}{\mathbf{l}}\right) + \frac{EA_m}{m_o^2}\left(\mathbf{m} - m_o + \frac{m_y^2}{\mathbf{m}}\right) + \frac{EA_n}{n_o^2}\left(\mathbf{n} - n_o + \frac{n_y^2}{\mathbf{n}}\right)\right] \tag{4.104}$$

With the previous conditions the displacement (**h**) can be calculated:

$$\mathbf{h} = \begin{cases} \frac{D_{22}F_1 - D_{12}F_2}{D_{22}D_{11} - D_{12}D_{21}} \\ \frac{D_{22}F_2 - D_{21}F_1}{D_{22}D_{11} - D_{12}D_{21}} \end{cases} \tag{4.105}$$

4.3.2.4 Forces on Nodes

The forces on the sides of the triangular element are calculated from the twine tension. These forces are related to the number of twines through the sides of the triangle.

This number of twines through each side can be calculated based on the number of base mesh of each vertex.

The effort on the side along **U** of the base mesh (Fig. 4.9) is

$$\mathbf{F}_U = \mathbf{T}_l - \mathbf{T}_m \tag{4.106}$$

The effort along **V** is

$$\mathbf{F}_V = -\mathbf{T}_n \tag{4.107}$$

Under these conditions, the effort on each side of the triangle can be deduced:

$$\mathbf{T}_{12} = (U_2 - U_1)(\mathbf{T}_l - \mathbf{T}_m) + (V_2 - V_1)(-\mathbf{T}_n) \tag{4.108}$$

$$\mathbf{T}_{23} = (U_3 - U_2)(\mathbf{T}_l - \mathbf{T}_m) + (V_3 - V_2)(-\mathbf{T}_n) \tag{4.109}$$

$$\mathbf{T}_{31} = (U_1 - U_3)(\mathbf{T}_l - \mathbf{T}_m) + (V_1 - V_3)(-\mathbf{T}_n) \tag{4.110}$$

Here, $\mathbf{T}_{ij}$ is the effort on the side ij of the triangular element.

Each side effort is distributed on each end of this side as the twines are evenly distributed along the sides of the triangle:

$$\mathbf{F}_1 = \frac{\mathbf{T}_{12} + \mathbf{T}_{31}}{2} \tag{4.111}$$

$$\mathbf{F}_2 = \frac{\mathbf{T}_{23} + \mathbf{T}_{12}}{2} \tag{4.112}$$

$$\mathbf{F}_3 = \frac{\mathbf{T}_{31} + \mathbf{T}_{23}}{2} \tag{4.113}$$

$\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ are the forces on the three vertices of the triangular element due to the tension in the twines.

The contribution of the stiffness matrix is not described here.

4.3.3 Hydrodynamic Drag

4.3.3.1 Introduction

The drag force on the netting is calculated in this model as the sum of the drag force on each twine (**U** and **V**). This assumption is probably questionable, because the drag on a twine alone is surely not exactly the same as the drag on this twine among other twines as it is the case in a netting. Anyway, this assumption leads to the calculation of the drag of each triangular element because for each the twines vectors are known, as described earlier. The formulation for the twine vector drag is based on the assumptions of Morrison adapted by Landweber and Richtmeyer [8, 22].

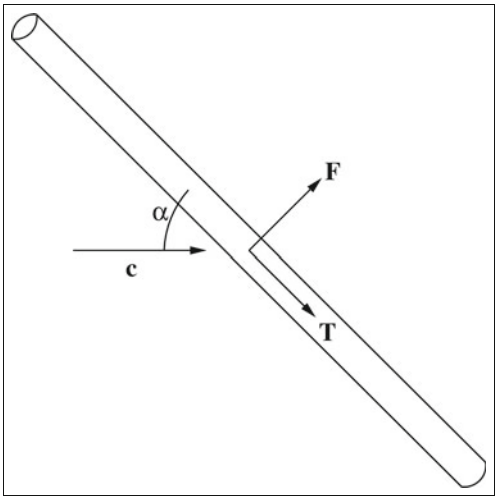


Fig. 4.11 Normal (**F**) and tangential (**T**) forces on a twine due to the relative velocity of water (**c**)

The drag amplitudes on the U twines used in the model (Fig. 4.11) are:

$$|\mathbf{F}| = \frac{1}{2}\rho C_d D l_0 \left[|\mathbf{c}| sin(\alpha)\right]^2 \frac{d}{2} \tag{4.114}$$

$$|\mathbf{T}| = f\frac{1}{2}\rho C_d D l_0 \left[|\mathbf{c}| cos(\alpha)\right]^2 \frac{d}{2} \tag{4.115}$$

The directions of the drag on the **U** twine vectors are:

$$\frac{\mathbf{F}}{|\mathbf{F}|} = \frac{\mathbf{U} \wedge (\mathbf{c} \wedge \mathbf{U})}{|\mathbf{U} \wedge (\mathbf{c} \wedge \mathbf{U})|} \tag{4.116}$$

$$\frac{\mathbf{T}}{|\mathbf{T}|} = \frac{\mathbf{F} \wedge (\mathbf{c} \wedge \mathbf{F})}{|\mathbf{F} \wedge (\mathbf{c} \wedge \mathbf{F})|} \tag{4.117}$$

F: normal drag (N) on the U twines, following the assumptions of Landweber,
T: tangential drag (N) on the U twines, Richtmeyer hypothesis,
ρ: density of water (kg/m^3),
C_d: normal drag coefficient,
f: tangential drag coefficient,
D: diameter of twine (m),
l_0: length of twine vector (m),
c: water velocity relative to the twine (m/s),
α: angle between the U twine and the water velocity (radians),
$d/2$: number of U twine vectors in the triangular element.

In the equations of drag amplitude, the expressions $|\mathbf{c}|sin(\alpha)$ and $|\mathbf{c}|cos(\alpha)$ are the normal and tangential projections on **c** along the U twine vector.

The drag on V twines for a triangular element are similar: **U** is replaced by **V** and α by β.

The length of twine vectors used in the formulation of drag amplitude can be assessed by $|\mathbf{U}|$ for the U twines and by $|\mathbf{V}|$ for the V twines. That would mean it takes into account the twine elongation. Generally speaking, a twine elongation is associated with a diameter D reduction by the Poisson coefficient. Because this Poisson coefficient is not taken into account in the present modelling, the twine surface is approximated by Dl_0, where D is the diameter of the twines and l_0 is the unstretched length of the twine vectors.

All parameters, including the angles α and β, are constant and known for each triangular element. Therefore, the drag can be calculated for each triangular element. The drag force for a triangular element is spread over the three vertices of the element at $1/3$ per vertex.

4.3.3.2 Definitions of the Variables

The Cartesian coordinates of the three nodes (1, 2, 3) of the triangular element (cf. Fig. 4.12) follow:

$$\mathbf{1} = \begin{vmatrix} x_1 \\ y_1 \\ z_1 \end{vmatrix} \tag{4.118}$$

$$\mathbf{2} = \begin{vmatrix} x_2 \\ y_2 \\ z_2 \end{vmatrix} \tag{4.119}$$

$$\mathbf{3} = \begin{vmatrix} x_3 \\ y_3 \\ z_3 \end{vmatrix} \tag{4.120}$$

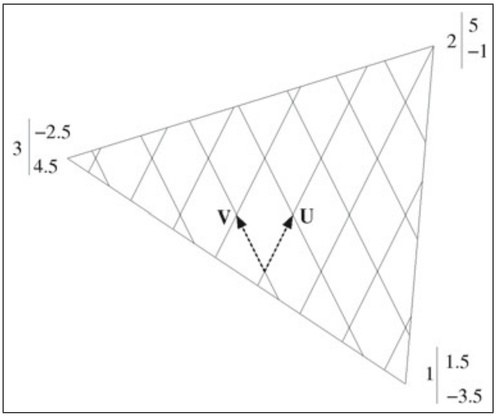


Fig. 4.12 Example of triangular element. The drag forces are calculated for U twines and for V twines. The twine coordinates are noted in this example

The twine coordinates of the three nodes (1, 2, 3) of the triangular element are as follows:

$$\mathbf{1} = \begin{vmatrix} U_1 \\ V_1 \end{vmatrix} \tag{4.121}$$

$$\mathbf{2} = \begin{vmatrix} U_2 \\ V_2 \end{vmatrix} \tag{4.122}$$

$$\mathbf{3} = \begin{vmatrix} U_3 \\ V_3 \end{vmatrix} \tag{4.123}$$

The vector current is

$$\mathbf{c} = \begin{vmatrix} c_x \\ c_y \\ c_z \end{vmatrix} \tag{4.124}$$

Generally speaking, c_z is null.
It has been seen previously:

$$\mathbf{U} = \frac{V_3 - V_1}{d}\mathbf{12} - \frac{V_2 - V_1}{d}\mathbf{13} \tag{4.125}$$

$$\mathbf{V} = \frac{U_2 - U_1}{d}\mathbf{13} - \frac{U_3 - U_1}{d}\mathbf{12} \tag{4.126}$$

with sides vectors:

$$\mathbf{12} = \begin{vmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{vmatrix} \tag{4.127}$$

$$\mathbf{13} = \begin{vmatrix} x_3 - x_1 \\ y_3 - y_1 \\ z_3 - z_1 \end{vmatrix} \tag{4.128}$$

and

$$d = (U_2 - U_1)(V_3 - V_1) - (U_3 - U_1)(V_2 - V_1) \tag{4.129}$$

The components of U twine vectors are as follows:

$$\mathbf{U} = \begin{vmatrix} U_x \\ U_y \\ U_z \end{vmatrix} \tag{4.130}$$

$$\mathbf{U} = \begin{vmatrix} \frac{1}{d}\left[(V_3 - V_1)(x_2 - x_1) - (V_2 - V_1)(x_3 - x_1)\right] \\ \frac{1}{d}\left[(V_3 - V_1)(y_2 - y_1) - (V_2 - V_1)(y_3 - y_1)\right] \\ \frac{1}{d}\left[(V_3 - V_1)(z_2 - z_1) - (V_2 - V_1)(z_3 - z_1)\right] \end{vmatrix} \tag{4.131}$$

The angle between current and U is

$$cos(\alpha) = \frac{\mathbf{c}.\mathbf{U}}{|\mathbf{c}||\mathbf{U}|} \tag{4.132}$$

The components of V twine vectors are as follows:

$$\mathbf{V} = \begin{vmatrix} V_x \\ V_y \\ V_z \end{vmatrix} \tag{4.133}$$

$$\mathbf{V} = \begin{vmatrix} \frac{1}{d}\left[(U_2 - U_1)(x_3 - x_1) - (U_3 - U_1)(x_2 - x_1)\right] \\ \frac{1}{d}\left[(U_2 - U_1)(y_3 - y_1) - (U_3 - U_1)(y_2 - y_1)\right] \\ \frac{1}{d}\left[(U_2 - U_1)(z_3 - z_1) - (U_3 - U_1)(z_2 - z_1)\right] \end{vmatrix} \tag{4.134}$$

The angle between current and V is

$$cos(\beta) = \frac{\mathbf{c}.\mathbf{V}}{|\mathbf{c}||\mathbf{V}|} \tag{4.135}$$

4.3.3.3 Stiffness of the Normal Force on the U Twines

The normal force on U twines is

$$\mathbf{F} = |\mathbf{F}| \frac{\mathbf{U} \wedge (\mathbf{c} \wedge \mathbf{U})}{|\mathbf{U} \wedge (\mathbf{c} \wedge \mathbf{U})|} \tag{4.136}$$

That means that the x y and z components are as follows:

$$\mathbf{F}_x = |\mathbf{F}| \frac{\mathbf{E}_x}{|\mathbf{E}|} \tag{4.137}$$

$$\mathbf{F}_y = |\mathbf{F}| \frac{\mathbf{E}_y}{|\mathbf{E}|} \tag{4.138}$$

$$\mathbf{F}_z = |\mathbf{F}| \frac{\mathbf{E}_z}{|\mathbf{E}|} \tag{4.139}$$

With:

$$\mathbf{E} = \mathbf{U} \wedge (\mathbf{c} \wedge \mathbf{U}) \tag{4.140}$$

and

$$\mathbf{E} = \begin{vmatrix} E_x \\ E_y \\ E_z \end{vmatrix} \tag{4.141}$$

The x component of the derivative is

$$\mathbf{F}'_x = |\mathbf{F}|' \frac{\mathbf{E}_x}{|\mathbf{E}|} + |\mathbf{F}| \frac{\mathbf{E}'_x |\mathbf{E}| - \mathbf{E}_x |\mathbf{E}|'}{|\mathbf{E}|^2} \tag{4.142}$$

Which gives for the x y and z components:

$$\mathbf{F}'_x = |\mathbf{F}|' \frac{\mathbf{E}_x}{|\mathbf{E}|} + \frac{|\mathbf{F}|}{|\mathbf{E}|^2} \left\{ \mathbf{E}'_x |\mathbf{E}| - \frac{\mathbf{E}_x}{|\mathbf{E}|} (\mathbf{E}_x \mathbf{E}'_x + \mathbf{E}_y \mathbf{E}'_y + \mathbf{E}_z \mathbf{E}'_z) \right\} \tag{4.143}$$

$$\mathbf{F}'_y = |\mathbf{F}|' \frac{\mathbf{E}_y}{|\mathbf{E}|} + \frac{|\mathbf{F}|}{|\mathbf{E}|^2} \left\{ \mathbf{E}'_y |\mathbf{E}| - \frac{\mathbf{E}_y}{|\mathbf{E}|} (\mathbf{E}_x \mathbf{E}'_x + \mathbf{E}_y \mathbf{E}'_y + \mathbf{E}_z \mathbf{E}'_z) \right\} \tag{4.144}$$

$$\mathbf{F}'_z = |\mathbf{F}|' \frac{\mathbf{E}_z}{|\mathbf{E}|} + \frac{|\mathbf{F}|}{|\mathbf{E}|^2} \left\{ \mathbf{E}'_z |\mathbf{E}| - \frac{\mathbf{E}_z}{|\mathbf{E}|} (\mathbf{E}_x \mathbf{E}'_x + \mathbf{E}_y \mathbf{E}'_y + \mathbf{E}_z \mathbf{E}'_z) \right\} \tag{4.145}$$

For this assessment the derivative of **E** is required:

$$\mathbf{E}' = \mathbf{U}' \wedge (\mathbf{c} \wedge \mathbf{U}) + \mathbf{U} \wedge (\mathbf{c} \wedge \mathbf{U}') \tag{4.146}$$

This leads to:

$$\mathbf{E}' = 2(\mathbf{U}'.\mathbf{U})\mathbf{c} - (\mathbf{U}'.\mathbf{c})\mathbf{U} - (\mathbf{U}.\mathbf{c})\mathbf{U}' \tag{4.147}$$

Which is:

$$\mathbf{E}'_x = 2(\mathbf{U}'.\mathbf{U})\mathbf{c}_x - (\mathbf{U}'.\mathbf{c})\mathbf{U}_x - (\mathbf{U}.\mathbf{c})\mathbf{U}'_x \tag{4.148}$$

$$\mathbf{E}'_y = 2(\mathbf{U}'.\mathbf{U})\mathbf{c}_y - (\mathbf{U}'.\mathbf{c})\mathbf{U}_y - (\mathbf{U}.\mathbf{c})\mathbf{U}'_y \tag{4.149}$$

$$\mathbf{E}'_z = 2(\mathbf{U}'.\mathbf{U})\mathbf{c}_z - (\mathbf{U}'.\mathbf{c})\mathbf{U}_z - (\mathbf{U}.\mathbf{c})\mathbf{U}'_z \tag{4.150}$$

With:

$$\mathbf{U}'.\mathbf{U} = \mathbf{U}_x \mathbf{U}'_x + \mathbf{U}_y \mathbf{U}'_y + \mathbf{U}_z \mathbf{U}'_z \tag{4.151}$$

$$\mathbf{U}'.\mathbf{c} = \mathbf{c}_x \mathbf{U}'_x + \mathbf{c}_y \mathbf{U}'_y + \mathbf{c}_z \mathbf{U}'_z \tag{4.152}$$

$$\mathbf{U}.\mathbf{c} = \mathbf{U}_x \mathbf{c}_x + \mathbf{U}_y \mathbf{c}_y + \mathbf{U}_z \mathbf{c}_z \tag{4.153}$$

The derivative of the amplitude of the normal force is

$$|\mathbf{F}|' = \frac{1}{2} \rho C_d D l_0 |\mathbf{c}|^2 \left([sin(\alpha)]^2 \right)' \frac{d}{2} \tag{4.154}$$

Which is

$$|\mathbf{F}|' = \frac{d}{2} \rho C_d D l_0 |\mathbf{c}|^2 cos(\alpha) sin(\alpha) \alpha' \tag{4.155}$$

The derivative of α is

$$\alpha' = \frac{-1}{\sqrt{1-\left(\frac{\mathbf{c}.\mathbf{U}}{|\mathbf{c}||\mathbf{U}|}\right)^2}}\left[\frac{\mathbf{c}.\mathbf{U}}{|\mathbf{c}||\mathbf{U}|}\right]' \tag{4.156}$$

That gives

$$\alpha' = \frac{-1}{\sqrt{1-\left(\frac{\mathbf{c}.\mathbf{U}}{|\mathbf{c}||\mathbf{U}|}\right)^2}}\left[\frac{\mathbf{c}}{|\mathbf{c}|}.\left(\frac{\mathbf{U}}{|\mathbf{U}|}\right)'\right] \tag{4.157}$$

The derivative of the U twine direction is

$$\left(\frac{\mathbf{U}}{|\mathbf{U}|}\right)' = \frac{\mathbf{U}'|\mathbf{U}| - \mathbf{U}|\mathbf{U}|'}{|\mathbf{U}|^2} \tag{4.158}$$

That means that the derivative of α is

$$\alpha' = \frac{-1}{\sqrt{1-\left(\frac{\mathbf{c}.\mathbf{U}}{|\mathbf{c}||\mathbf{U}|}\right)^2}}\left(\frac{\mathbf{c}}{|\mathbf{c}|}\right).\left(\frac{\mathbf{U}'|\mathbf{U}| - \mathbf{U}|\mathbf{U}|'}{|\mathbf{U}|^2}\right) \tag{4.159}$$

or

$$\alpha' = \frac{-1}{|\mathbf{U}|^2|\mathbf{c}|\sin\alpha}\left\{|\mathbf{U}|\left[c_x\mathbf{U}'_x + c_y\mathbf{U}'_y + c_z\mathbf{U}'_z\right] - (\mathbf{c}.\mathbf{U})|\mathbf{U}|'\right\} \tag{4.160}$$

In this case $\mathbf{U}'_x$ is the component along x of $\mathbf{U}'$.
The derivative of vector $\mathbf{U}$ is

$$\mathbf{U}' = \begin{vmatrix} \mathbf{U}'_x \\ \mathbf{U}'_y \\ \mathbf{U}'_z \end{vmatrix} \tag{4.161}$$

Which is

$$\frac{\partial U_x}{\partial x_1} = \frac{\partial U_y}{\partial y_1} = \frac{\partial U_z}{\partial z_1} = \frac{1}{d}(V_2 - V_3) \tag{4.162}$$

$$\frac{\partial U_x}{\partial x_2} = \frac{\partial U_y}{\partial y_2} = \frac{\partial U_z}{\partial z_2} = \frac{1}{d}(V_3 - V_1) \tag{4.163}$$

$$\frac{\partial U_x}{\partial x_3} = \frac{\partial U_y}{\partial y_3} = \frac{\partial U_z}{\partial z_3} = \frac{1}{d}(V_1 - V_2) \tag{4.164}$$

$$\frac{\partial U_x}{\partial y_1} = \frac{\partial U_x}{\partial y_2} = \frac{\partial U_x}{\partial y_3} = \frac{\partial U_x}{\partial z_1} = \frac{\partial U_x}{\partial z_2} = \frac{\partial U_x}{\partial z_3} = 0 \tag{4.165}$$

$$\frac{\partial U_y}{\partial z_1} = \frac{\partial U_y}{\partial z_2} = \frac{\partial U_y}{\partial z_3} = \frac{\partial U_y}{\partial x_1} = \frac{\partial U_y}{\partial x_2} = \frac{\partial U_y}{\partial x_3} = 0 \tag{4.166}$$

$$\frac{\partial U_z}{\partial x_1} = \frac{\partial U_z}{\partial x_2} = \frac{\partial U_z}{\partial x_3} = \frac{\partial U_z}{\partial y_1} = \frac{\partial U_z}{\partial y_2} = \frac{\partial U_z}{\partial y_3} = 0 \tag{4.167}$$

On vector form and for the nine coordinates of the triangular element it is:

$$\frac{\partial \mathbf{U}}{\partial x_1} = \left| \begin{array}{c} \frac{V_2 - V_3}{d} \\ 0 \\ 0 \end{array} \right. \tag{4.168}$$

$$\frac{\partial \mathbf{U}}{\partial y_1} = \left| \begin{array}{c} 0 \\ \frac{V_2 - V_3}{d} \\ 0 \end{array} \right. \tag{4.169}$$

$$\frac{\partial \mathbf{U}}{\partial z_1} = \left| \begin{array}{c} 0 \\ 0 \\ \frac{V_2 - V_3}{d} \end{array} \right. \tag{4.170}$$

$$\frac{\partial \mathbf{U}}{\partial x_2} = \left| \begin{array}{c} \frac{V_3 - V_1}{d} \\ 0 \\ 0 \end{array} \right. \tag{4.171}$$

$$\frac{\partial \mathbf{U}}{\partial y_2} = \left| \begin{array}{c} 0 \\ \frac{V_3 - V_1}{d} \\ 0 \end{array} \right. \tag{4.172}$$

$$\frac{\partial \mathbf{U}}{\partial z_2} = \left| \begin{array}{c} 0 \\ 0 \\ \frac{V_3 - V_1}{d} \end{array} \right. \tag{4.173}$$

$$\frac{\partial \mathbf{U}}{\partial x_3} = \left| \begin{array}{c} \frac{V_1 - V_2}{d} \\ 0 \\ 0 \end{array} \right. \tag{4.174}$$

$$\frac{\partial \mathbf{U}}{\partial y_3} = \left| \begin{array}{c} 0 \\ \frac{V_1 - V_2}{d} \\ 0 \end{array} \right. \tag{4.175}$$

$$\frac{\partial \mathbf{U}}{\partial z_3} = \left| \begin{array}{c} 0 \\ 0 \\ \frac{V_1 - V_2}{d} \end{array} \right. \tag{4.176}$$

The derivative of the norm of vector **U** is

$$|\mathbf{U}|' = \frac{U_x U_x' + U_y U_y' + U_z U_z'}{|\mathbf{U}|} \tag{4.177}$$

This gives for the nine coordinates of the triangular element:

$$\frac{\partial|\mathbf{U}|}{\partial x_1} = \frac{U_x(V_2 - V_3)}{d|\mathbf{U}|} \tag{4.178}$$

$$\frac{\partial|\mathbf{U}|}{\partial y_1} = \frac{U_y(V_2 - V_3)}{d|\mathbf{U}|} \tag{4.179}$$

$$\frac{\partial|\mathbf{U}|}{\partial z_1} = \frac{U_z(V_2 - V_3)}{d|\mathbf{U}|} \tag{4.180}$$

$$\frac{\partial|\mathbf{U}|}{\partial x_2} = \frac{U_x(V_3 - V_1)}{d|\mathbf{U}|} \tag{4.181}$$

$$\frac{\partial|\mathbf{U}|}{\partial y_2} = \frac{U_y(V_3 - V_1)}{d|\mathbf{U}|} \tag{4.182}$$

$$\frac{\partial|\mathbf{U}|}{\partial z_2} = \frac{U_z(V_3 - V_1)}{d|\mathbf{U}|} \tag{4.183}$$

$$\frac{\partial|\mathbf{U}|}{\partial x_3} = \frac{U_x(V_1 - V_2)}{d|\mathbf{U}|} \tag{4.184}$$

$$\frac{\partial|\mathbf{U}|}{\partial y_3} = \frac{U_y(V_1 - V_2)}{d|\mathbf{U}|} \tag{4.185}$$

$$\frac{\partial|\mathbf{U}|}{\partial z_3} = \frac{U_z(V_1 - V_2)}{d|\mathbf{U}|} \tag{4.186}$$

This leads to the derivatives of α (angle between c and U):

$$\frac{\partial\alpha}{\partial x_1} = \frac{V_3 - V_2}{d|\mathbf{U}|^2|\mathbf{c}|\sqrt{1 - \left(\frac{\mathbf{c}.\mathbf{U}}{|\mathbf{c}||\mathbf{U}|}\right)^2}}\left[c_x|\mathbf{U}| - \frac{U_x}{|\mathbf{U}|}\mathbf{c}.\mathbf{U}\right] \tag{4.187}$$

$$\frac{\partial\alpha}{\partial y_1} = \frac{V_3 - V_2}{d|\mathbf{U}|^2|\mathbf{c}|\sqrt{1 - \left(\frac{\mathbf{c}.\mathbf{U}}{|\mathbf{c}||\mathbf{U}|}\right)^2}}\left[c_y|\mathbf{U}| - \frac{U_y}{|\mathbf{U}|}\mathbf{c}.\mathbf{U}\right] \tag{4.188}$$

$$\frac{\partial\alpha}{\partial z_1} = \frac{V_3 - V_2}{d|\mathbf{U}|^2|\mathbf{c}|\sqrt{1 - \left(\frac{\mathbf{c}.\mathbf{U}}{|\mathbf{c}||\mathbf{U}|}\right)^2}}\left[c_z|\mathbf{U}| - \frac{U_z}{|\mathbf{U}|}\mathbf{c}.\mathbf{U}\right] \tag{4.189}$$

$$\frac{\partial\alpha}{\partial x_2} = \frac{V_1 - V_3}{d|\mathbf{U}|^2|\mathbf{c}|\sqrt{1 - \left(\frac{\mathbf{c}.\mathbf{U}}{|\mathbf{c}||\mathbf{U}|}\right)^2}}\left[c_x|\mathbf{U}| - \frac{U_x}{|\mathbf{U}|}\mathbf{c}.\mathbf{U}\right] \tag{4.190}$$

$$\frac{\partial\alpha}{\partial y_2} = \frac{V_1 - V_3}{d|\mathbf{U}|^2|\mathbf{c}|\sqrt{1 - \left(\frac{\mathbf{c}.\mathbf{U}}{|\mathbf{c}||\mathbf{U}|}\right)^2}}\left[c_y|\mathbf{U}| - \frac{U_y}{|\mathbf{U}|}\mathbf{c}.\mathbf{U}\right] \tag{4.191}$$

$$\frac{\partial \alpha}{\partial z_2} = \frac{V_1 - V_3}{d|\mathbf{U}|^2|\mathbf{c}|\sqrt{1-\left(\frac{\mathbf{c}.\mathbf{U}}{|\mathbf{c}||\mathbf{U}|}\right)^2}}\left[c_z|\mathbf{U}| - \frac{U_z}{|\mathbf{U}|}\mathbf{c}.\mathbf{U}\right] \quad (4.192)$$

$$\frac{\partial \alpha}{\partial x_3} = \frac{V_2 - V_1}{d|\mathbf{U}|^2|\mathbf{c}|\sqrt{1-\left(\frac{\mathbf{c}.\mathbf{U}}{|\mathbf{c}||\mathbf{U}|}\right)^2}}\left[c_x|\mathbf{U}| - \frac{U_x}{|\mathbf{U}|}\mathbf{c}.\mathbf{U}\right] \quad (4.193)$$

$$\frac{\partial \alpha}{\partial y_3} = \frac{V_2 - V_1}{d|\mathbf{U}|^2|\mathbf{c}|\sqrt{1-\left(\frac{\mathbf{c}.\mathbf{U}}{|\mathbf{c}||\mathbf{U}|}\right)^2}}\left[c_y|\mathbf{U}| - \frac{U_y}{|\mathbf{U}|}\mathbf{c}.\mathbf{U}\right] \quad (4.194)$$

$$\frac{\partial \alpha}{\partial z_3} = \frac{V_2 - V_1}{d|\mathbf{U}|^2|\mathbf{c}|\sqrt{1-\left(\frac{\mathbf{c}.\mathbf{U}}{|\mathbf{c}||\mathbf{U}|}\right)^2}}\left[c_z|\mathbf{U}| - \frac{U_z}{|\mathbf{U}|}\mathbf{c}.\mathbf{U}\right] \quad (4.195)$$

4.3.3.4 Stiffness of the Tangential Force on the *U* Twines

The tangential force on U twines is

$$\mathbf{T} = |\mathbf{T}|\frac{\mathbf{F} \wedge (\mathbf{c} \wedge \mathbf{F})}{|\mathbf{F} \wedge (\mathbf{c} \wedge \mathbf{F})|} \quad (4.196)$$

Following the definition of $\mathbf{F}_1$:

$$\mathbf{T} = |\mathbf{T}|\frac{[\mathbf{U} \wedge (\mathbf{c} \wedge \mathbf{U})] \wedge \{\mathbf{c} \wedge [\mathbf{U} \wedge (\mathbf{c} \wedge \mathbf{U})]\}}{|\,[\mathbf{U} \wedge (\mathbf{c} \wedge \mathbf{U})] \wedge \{\mathbf{c} \wedge [\mathbf{U} \wedge (\mathbf{c} \wedge \mathbf{U})]\}\,|} \quad (4.197)$$

It follows that

$$\mathbf{T} = |\mathbf{T}|\frac{[(\mathbf{U}.\mathbf{U})(\mathbf{c}.\mathbf{c}) - (\mathbf{U}.\mathbf{c})^2](\mathbf{U}.\mathbf{c})\mathbf{U}}{|[(\mathbf{U}.\mathbf{U})(\mathbf{c}.\mathbf{c}) - (\mathbf{U}.\mathbf{c})^2](\mathbf{U}.\mathbf{c})\mathbf{U}|} \quad (4.198)$$

or

$$\mathbf{T} = |\mathbf{T}|\frac{[|\mathbf{U}|^2|\mathbf{c}|^2 - (|\mathbf{U}||\mathbf{c}|cos\alpha)^2]|\mathbf{U}||\mathbf{c}|cos\alpha\mathbf{U}}{|[|\mathbf{U}|^2|\mathbf{c}|^2 - (|\mathbf{U}||\mathbf{c}|cos\alpha)^2]|\mathbf{U}||\mathbf{c}|cos\alpha\mathbf{U}|} \quad (4.199)$$

and

$$\mathbf{T} = |\mathbf{T}|\frac{\cos\alpha\mathbf{U}}{|\cos\alpha||\mathbf{U}|} \quad (4.200)$$

The x y and z components are as follows:

$$\mathbf{T}_x = |\mathbf{T}|\frac{\cos\alpha\mathbf{U}_x}{|\cos\alpha||\mathbf{U}|} \quad (4.201)$$

$$\mathbf{T}_y = |\mathbf{T}|\frac{\cos\alpha\mathbf{U}_y}{|\cos\alpha||\mathbf{U}|} \quad (4.202)$$

$$\mathbf{T}_z = |\mathbf{T}| \frac{\cos\alpha \mathbf{U}_z}{|\cos\alpha||\mathbf{U}|} \tag{4.203}$$

The derivative of $\mathbf{T}_x$ is:

$$\mathbf{T}'_x = |\mathbf{T}|' \frac{\cos\alpha \mathbf{U}_x}{|\cos\alpha||\mathbf{U}|} + |\mathbf{T}| \frac{(\cos\alpha \mathbf{U}_x)'|\cos\alpha||\mathbf{U}| - \cos\alpha \mathbf{U}_x (|\cos\alpha||\mathbf{U}|)'}{(|\cos\alpha||\mathbf{U}|)^2} \tag{4.204}$$

$$\begin{aligned}\mathbf{T}'_x = {} & |\mathbf{T}|' \frac{\cos\alpha \mathbf{U}_x}{|\cos\alpha||\mathbf{U}|} \\ & + \frac{|\mathbf{T}|}{|\cos\alpha||\mathbf{U}|}(\cos\alpha \mathbf{U}'_x - \sin\alpha\alpha' \mathbf{U}_x) \\ & - \frac{|\mathbf{T}|\cos\alpha \mathbf{U}_x}{(|\cos\alpha||\mathbf{U}|)^2}\left[|\cos\alpha| \frac{\mathbf{U}_x\mathbf{U}'_x + \mathbf{U}_y\mathbf{U}'_y + \mathbf{U}_z\mathbf{U}'_z}{|\mathbf{U}|} - \frac{\cos\alpha}{|\cos\alpha|}\sin\alpha\alpha'|\mathbf{U}|\right]\end{aligned} \tag{4.205}$$

$$\begin{aligned}\mathbf{T}'_x = {} & |\mathbf{T}|' \frac{\mathbf{T}_x}{|\mathbf{T}|} + \frac{|\mathbf{T}|}{|\cos\alpha||\mathbf{U}|}(\cos\alpha \mathbf{U}'_x - \sin\alpha\alpha' \mathbf{U}_x) \\ & - \frac{\mathbf{T}_x}{|\cos\alpha||\mathbf{U}|}\left[|\cos\alpha| \frac{\mathbf{U}_x\mathbf{U}'_x + \mathbf{U}_y\mathbf{U}'_y + \mathbf{U}_z\mathbf{U}'_z}{|\mathbf{U}|} - \frac{\cos\alpha}{|\cos\alpha|}\sin\alpha\alpha'|\mathbf{U}|\right]\end{aligned} \tag{4.206}$$

$$\begin{aligned}\mathbf{T}'_y = {} & |\mathbf{T}|' \frac{\mathbf{T}_y}{|\mathbf{T}|} + \frac{|\mathbf{T}|}{|\cos\alpha||\mathbf{U}|}(\cos\alpha \mathbf{U}'_y - \sin\alpha\alpha' \mathbf{U}_y) \\ & - \frac{\mathbf{T}_y}{|\cos\alpha||\mathbf{U}|}\left[|\cos\alpha| \frac{\mathbf{U}_x\mathbf{U}'_x + \mathbf{U}_y\mathbf{U}'_y + \mathbf{U}_z\mathbf{U}'_z}{|\mathbf{U}|} - \frac{\cos\alpha}{|\cos\alpha|}\sin\alpha\alpha'|\mathbf{U}|\right]\end{aligned} \tag{4.207}$$

$$\begin{aligned}\mathbf{T}'_z = {} & |\mathbf{T}|' \frac{\mathbf{T}_z}{|\mathbf{T}|} + \frac{|\mathbf{T}|}{|\cos\alpha||\mathbf{U}|}(\cos\alpha \mathbf{U}'_z - \sin\alpha\alpha' \mathbf{U}_z) \\ & - \frac{\mathbf{T}_z}{|\cos\alpha||\mathbf{U}|}\left[|\cos\alpha| \frac{\mathbf{U}_x\mathbf{U}'_x + \mathbf{U}_y\mathbf{U}'_y + \mathbf{U}_z\mathbf{U}'_z}{|\mathbf{U}|} - \frac{\cos\alpha}{|\cos\alpha|}\sin\alpha\alpha'|\mathbf{U}|\right]\end{aligned} \tag{4.208}$$

The derivative of the amplitude of the tangential force is

$$|\mathbf{T}|' = f\frac{1}{2}\rho C_d D l_0 |\mathbf{c}|^2 ([cos(\alpha)]^2)' \frac{d}{2} \tag{4.209}$$

which is

$$|\mathbf{T}|' = -\frac{d}{2} f\rho C_d D l_0 |\mathbf{c}|^2 cos(\alpha) sin(\alpha)\alpha' \tag{4.210}$$

4.3.3.5 Stiffness of the Normal and Tangential Forces on the V Twines

This evaluations are identical to the previous, but with V and β used in place of U and α.

4.3.4 Twine Flexionin Netting Plane

The resistance to twine bending in the plane of the net is also called the mesh opening stiffness (Fig. 4.13). In a first approximation, this stiffness is neglected, but the use of steeper nets makes it necessary to take this mechanical phenomenon into account in numerical models. Currently, only [15, 12] and the present model take this mesh opening stiffness into account.

In the present model, the half angle (α) between the twine vectors (**U** and **V**) could lead to a couple between twine vectors (**U** and **V**). This angle is calculated by

$$\alpha = \frac{1}{2} acos \left(\frac{\mathbf{U}.\mathbf{V}}{|\mathbf{U}||\mathbf{V}|} \right) \tag{4.211}$$

The couple on a knot due to the U twine is equilibrated by the couple of the V twine; otherwise the knot would not be in equilibrium. These couples are approximated in the model by

$$C_u = -C_v = H(\alpha - \alpha_0) \tag{4.212}$$

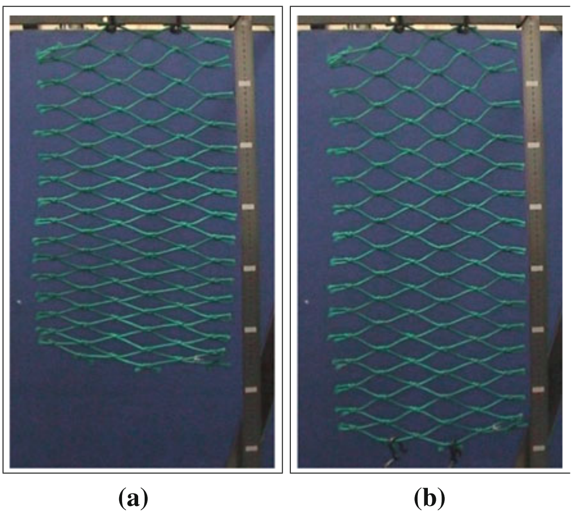


Fig. 4.13 Demonstration of mesh opening stiffness. Deformation remains limited despite the weight added to the bottom of the net on (**b**)

where α_0 is the angle between the unstressed twines (without couple on twines) and H is the mesh opening stiffness (N.m/Rad).

This couple varies linearly with the angle. [12, 15] suggest another formulation, since the twines are modelled as beams.

Forces at the vertices of the triangular element, mechanically equivalent to themesh opening stiffness, are calculated using the principle of virtual work:

If ∂x_1 is a virtual displacement along the x axis of vertex 1, then the external work (W_e) is

$$W_e = Fx_1 \partial x_1 \tag{4.213}$$

where Fx_1 is the effort along the x axis at vertex 1 of a triangular element.

This displacement creates a change in angle α, and therefore an internal work (W_i):

$$W_i = \frac{d}{2}(C_u \partial\alpha + C_v \partial\alpha) \tag{4.214}$$

$$d = (U_2 - U_1)(V_1 - V_3) - (U_3 - U_1)(V_1 - V_2) \tag{4.215}$$

where $d/2$ is the number of nodes in a triangular element.

Since the internal work is equal to the external work,

$$Fx_1 = C_u d \frac{\partial\alpha}{\partial x_1} \tag{4.216}$$

This gives, for all the force components at the vertices of the triangular element,

$$Fw_i = H(\alpha - \alpha_0) d \frac{\partial\alpha}{\partial w_i} \tag{4.217}$$

where $w = x, y$, and z, and $i = 1, 2$, and 3.

The derivative $\frac{\partial\alpha}{\partial w_i}$ of α relative to the coordinates w_i of vertices, which is necessary for calculating the forces, is

$$\frac{\partial\alpha}{\partial w_i} = \frac{\mathbf{V}_w v_i - \mathbf{U}_w u_i - \frac{\mathbf{U}_w(\mathbf{U}.\mathbf{V})v_i}{|\mathbf{U}|^2} - \frac{\mathbf{V}_w(\mathbf{U}.\mathbf{V})u_i}{|\mathbf{V}|^2}}{2dsin(\alpha)|\mathbf{U}||\mathbf{V}|} \tag{4.218}$$

where $w = x, y$, and z, and $i = 1, 2$, and 3.

The stiffness matrix $(-\mathbf{F}'(\mathbf{X}))$ is completed by calculating the derivative component of efforts related to the coordinates of the vertices of the triangular element:

$$-\frac{\partial F_w i}{\partial tj} \tag{4.219}$$

where as above, $w = x, y$, and z, and $i = 1, 2$, and 3, and $t = x, y$, and z, and $j = 1$, 2, and 3.

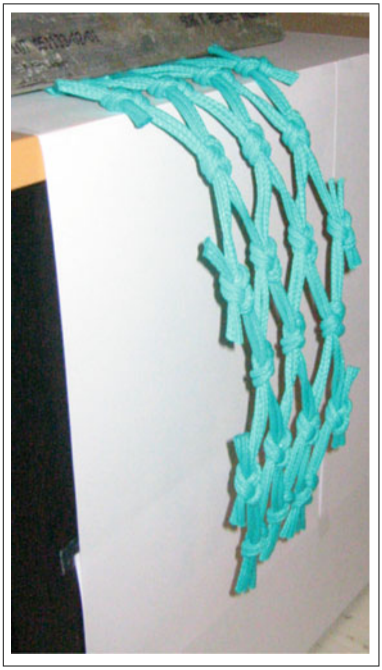


Fig. 4.14 The net bends under its own weight, which highlights the bending stiffness of the net

4.3.5 Twine Flexion Outside the Netting Plane

To our knowledge, no numerical model, except the present one, takes into account this mechanical property of the nets (Fig. 4.14). The angle between the U twine of a triangle ($\mathbf{U}_a$ in Fig. 4.15) and its neighbour ($\mathbf{U}_b$) is constant along the side common to the two triangular elements. This angle quantifies the bending of the twine.

The bending stiffness of the U twine tends to keep the twine straight. The equation governing the bending is as follows:

$$C = \frac{EI}{\rho} \tag{4.220}$$

C: bending couple on the U twine (Nm),
EI: flexural stiffness, which is Young's modulus by inertia (Nm^2),
ρ: radius of curvature of the U twine (m).

This couple is generated, in the present modelling, when two successive triangular elements are bent or, more precisely, when the U twine is bent to the passage of a triangular element with its neighbour. The couple will then generate forces on the vertices (1, 2, 3, 4 in Fig. 4.15) on the two adjacent triangular elements. Obviously the bending of the V twines also leads to a couple. In the following only the effect

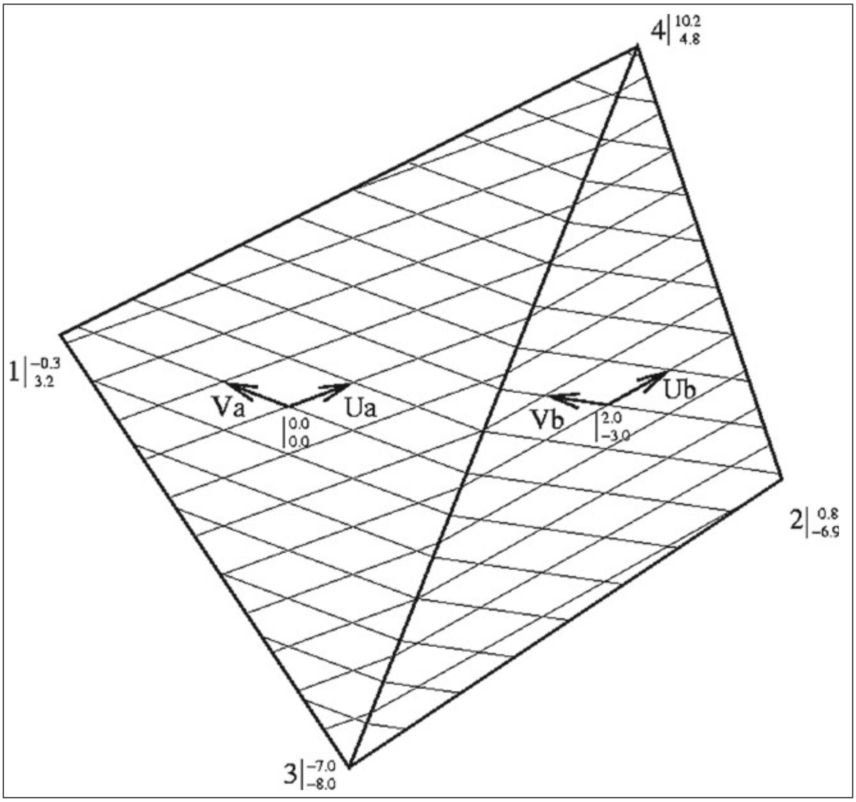


Fig. 4.15 Two triangular elements (134 and 243), the coordinates of which, in number of twines, are noted. The angle between the twine vectors $\mathbf{U_a}$ and $\mathbf{U}_b$ leads to a bending couple between the two triangular elements

of bending on the U twines is described; the bending on V twines has to be taken into account in the same way.

The radius of the curvature is estimated from the average lengths of twine U in each triangular element (Fig. 4.16). These average lengths are calculated using the average number of twine vectors ($\mathbf{U_a}$ and $\mathbf{U}_b$) by the U twine in the two triangular elements (n_a and n_b).

The twine vectors of the two triangular elements (see Sect. 4.2.1 p. 30) are as follows:

$$\mathbf{U}_a = \frac{V_4 - V_1}{d_a}\mathbf{13} - \frac{V_3 - V_1}{d_a}\mathbf{14} \tag{4.221}$$

$$\mathbf{V}_a = \frac{U_4 - U_1}{d_a}\mathbf{13} - \frac{U_3 - U_1}{d_a}\mathbf{14} \tag{4.222}$$

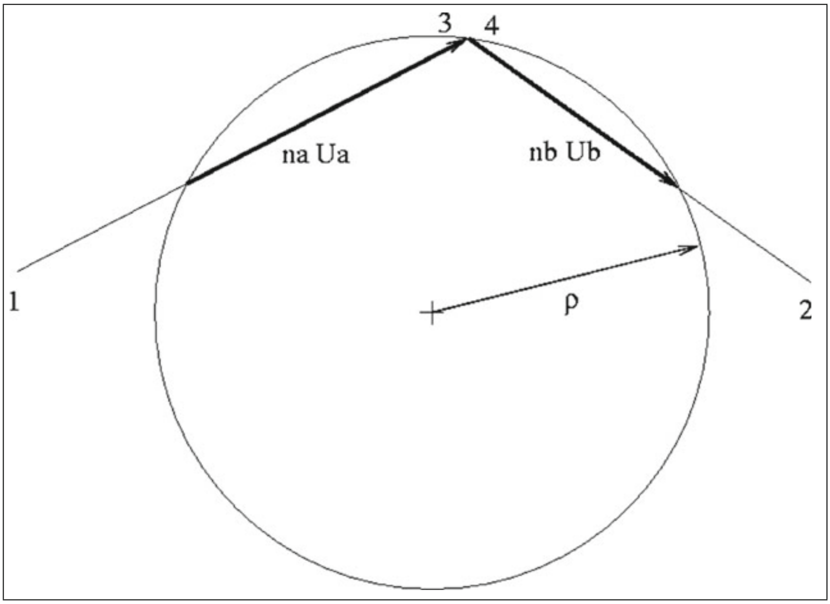


Fig. 4.16 Profile view of the two triangular elements. The radius of curvature (ρ) is estimated from the average length of twine vectors **U** in each triangle : $n_a\mathbf{U_a}$ and $n_b\mathbf{U}_b$

$$\mathbf{U}_b = \frac{V_3 - V_2}{d_b}\mathbf{24} - \frac{V_4 - V_2}{d_b}\mathbf{23} \tag{4.223}$$

$$\mathbf{V}_b = \frac{U_3 - U_2}{d_b}\mathbf{24} - \frac{U_4 - U_2}{d_b}\mathbf{23} \tag{4.224}$$

U_i, V_i: coordinates of vertex i in number of twines (twine coordinates).
With side vectors:

$$\mathbf{13} = \begin{vmatrix} x_3 - x_1 \\ y_3 - y_1 \\ z_3 - z_1 \end{vmatrix} \tag{4.225}$$

$$\mathbf{24} = \begin{vmatrix} x_4 - x_2 \\ y_4 - y_2 \\ z_4 - z_2 \end{vmatrix} \tag{4.226}$$

The numbers of twine vectors ($\mathbf{U}_a$ and $\mathbf{U}_b$) for the U twines in the two triangular elements are

$$d_a = (U_3 - U_1)(V_4 - V_1) - (U_4 - U_1)(V_3 - V_1) \tag{4.227}$$

$$d_b = (U_4 - U_2)(V_3 - V_2) - (U_3 - U_2)(V_4 - V_2) \tag{4.228}$$

The average numbers of twine vectors ($\mathbf{U}_a$ and $\mathbf{U}_b$) by U twine are calculated from the number of twine vectors in the triangular elements and the length of the common side in twine coordinates ($V_3 - V_4$):

$$n_a = \frac{d_a}{2|V_3 - V_4|} \tag{4.229}$$

$$n_b = \frac{d_b}{2|V_3 - V_4|} \tag{4.230}$$

The radius of the curvature (ρ) is calculated from the circumscribed circle in the triangle of sides $na\mathbf{U}_a$, $nb\mathbf{U}_b$ and $na\mathbf{U}_a + nb\mathbf{U}_b$, as shown in Fig. 4.16. The side lengths of the triangle are

$$A = |n_a\mathbf{U}_a| \tag{4.231}$$

$$B = |n_b\mathbf{U}_b| \tag{4.232}$$

$$C = |n_a\mathbf{U}_a + n_b\mathbf{U}_b| \tag{4.233}$$

The equations of the triangle, which can be obtained in a mathematical compendium, give the radius of curvature:

$$\rho = \frac{ABC}{4S} \tag{4.234}$$

where S and p, the surface and the half perimeter of the triangle, are

$$S = \sqrt{p(p-A)(p-B)(p-C)} \tag{4.235}$$

$$p = \frac{A+B+C}{2} \tag{4.236}$$

The forces on the vertices (1, 2, 3 and 4) of the two triangularelements due to the twine bending are calculated using the principle of virtual work. In case of the X component of the force on vertex 1 (F_{x1}), a displacement ($\partial x1$) is defined along X axis of vertex 1. This displacement generates an external work:

$$W_e = F_{x1}\partial x1 \tag{4.237}$$

This movement also causes a variation of angle ($\partial\alpha$) between the twine vectors ($\mathbf{U}_a$ and $\mathbf{U}_b$) of the two triangular elements. This variation induces an internal work:

$$W_i = C\partial\alpha(V_3 - V_4) \tag{4.238}$$

According to the principle of virtual work, these works are equal, which gives the following:

$$F_{wi} = \frac{EI}{\rho}\frac{\partial\alpha}{\partial wi}(V_3 - V_4) \tag{4.239}$$

w: directions x, y, and z,
i: vertices 1, 2, 3, and 4,
$V_3 - V_4$: number of twines involved in the bending.

The angle α between the two twine vectors ($\mathbf{U}_a$ and $\mathbf{U}_b$) of the two triangular elements is calculated with the dot product of twine vectors (Fig. 4.16):

$$cos(\alpha) = \frac{\mathbf{U}_a.\mathbf{U}_b}{|\mathbf{U}_a||\mathbf{U}_b|} \tag{4.240}$$

The 12 derivatives of α relative to the coordinates of the vertices of the two triangular elements ($\frac{\partial \alpha}{\partial wi}$) are therefore required to calculate the effort on the vertices. They are as follows:

$$\frac{\partial \alpha}{\partial w1} = (V_3 - V_4)\frac{(\mathbf{U}_a.\mathbf{U}_b)U_{aw} - U_{bw}|\mathbf{U}_a|^2}{|\mathbf{U}_a|^3|\mathbf{U}_b|d_a sin(\alpha)} \tag{4.241}$$

$$\frac{\partial \alpha}{\partial w2} = (V_4 - V_3)\frac{(\mathbf{U}_a.\mathbf{U}_b)U_{bw} - U_{aw}|\mathbf{U}_b|^2}{|\mathbf{U}_b|^3|\mathbf{U}_a|d_b sin(\alpha)} \tag{4.242}$$

$$\frac{\partial \alpha}{\partial w3} = (V_4 - V_1)\frac{(\mathbf{U}_a.\mathbf{U}_b)U_{aw} - U_{bw}|\mathbf{U}_a|^2}{|\mathbf{U}_a|^3|\mathbf{U}_b|d_a sin(\alpha)} + (V_2 - V_4)\frac{(\mathbf{U}_a.\mathbf{U}_b)U_{bw} - U_{aw}|\mathbf{U}_b|^2}{|\mathbf{U}_b|^3|\mathbf{U}_a|d_b sin(\alpha)} \tag{4.243}$$

$$\frac{\partial \alpha}{\partial w4} = (V_1 - V_3)\frac{(\mathbf{U}_a.\mathbf{U}_b)U_{aw} - U_{bw}|\mathbf{U}_a|^2}{|\mathbf{U}_a|^3|\mathbf{U}_b|d_a sin(\alpha)} + (V_3 - V_2)\frac{(\mathbf{U}_a.\mathbf{U}_b)U_{bw} - U_{aw}|\mathbf{U}_b|^2}{|\mathbf{U}_b|^3|\mathbf{U}_a|d_b sin(\alpha)} \tag{4.244}$$

Here, U_{aw} is the component along the w axis of $\mathbf{U}_a$. In this case w is the axis consisting of x, y, and z. Obviously, U_{bw} is the component along the w axis of $\mathbf{U}_b$.

The efforts on the four vertices of the two triangular elements due to the bending of the U twine between these two elements have been previously calculated.

The stiffness matrix ($-F'(X)$) is completed by calculating the derivative of the 12 components of the forces relative to the 12 coordinates of the vertices of the two triangular elements. The 144 components of this matrix are

$$-\frac{\partial F_{wi}}{\partial tj} \tag{4.245}$$

With, as above:
w: x, y, and z.
i: 1, 2, 3, and 4.

And more:
t: x, y, and z,
j: 1, 2, 3, and 4.

4.3.6 Fish Catch Pressure

The mechanical effect of caught fish (Fig. 4.17) in a net is estimated by a pressure [1]. This pressure is exerted directly on the triangular elements in contact with the fish. In the case of water speed relative to that catch:

$$p = \frac{1}{2}\rho C_d v^2 \tag{4.246}$$

p: pressure of the catch on the net (Pa),
ρ: density of water (kg/m^3),
C_d: drag coefficient,
v: current amplitude (m/s).

This pressure is then applied to the surface of the triangular element $\left(\frac{\mathbf{12}\wedge\mathbf{13}}{2}\right)$. The resultant force is directed perpendicular to the triangular element. The effort on each vertex is that force by 1/3.

$$\mathbf{F}_1 = \frac{\mathbf{12}\wedge\mathbf{13}}{2}\frac{p}{3} \tag{4.247}$$

$$\mathbf{F}_2 = \frac{\mathbf{12}\wedge\mathbf{13}}{2}\frac{p}{3} \tag{4.248}$$

$$\mathbf{F}_3 = \frac{\mathbf{12}\wedge\mathbf{13}}{2}\frac{p}{3} \tag{4.249}$$

With sides vectors:

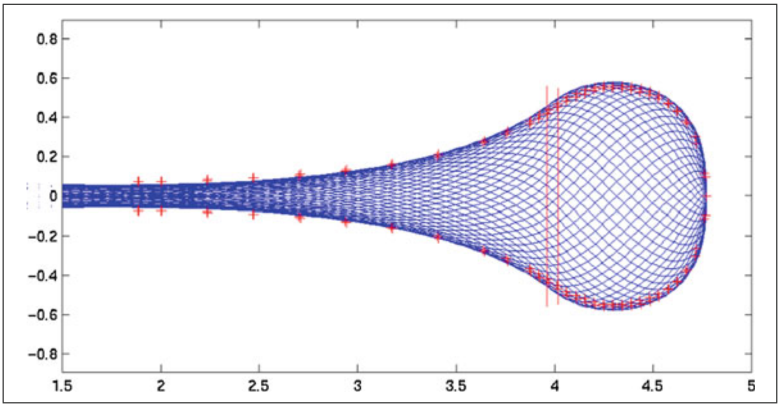


Fig. 4.17 Measurement in a flume tank tests (*cross*) and numerical modelling (*mesh*) for a scale (1/3) model of North Sea cod-end with 300 kg of catch

$$\mathbf{12} = \begin{vmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{vmatrix} \tag{4.250}$$

$$\mathbf{13} = \begin{vmatrix} x_3 - x_1 \\ y_3 - y_1 \\ z_3 - z_1 \end{vmatrix} \tag{4.251}$$

That gives:

$$\mathbf{F}_{1x} = \frac{p}{6}\left[(y_2 - y_1)(z_3 - z_1) - (z_2 - z_1)(y_3 - y_1)\right] \tag{4.252}$$

$$\mathbf{F}_{1y} = \frac{p}{6}\left[(z_2 - z_1)(x_3 - x_1) - (x_2 - x_1)(z_3 - z_1)\right] \tag{4.253}$$

$$\mathbf{F}_{1z} = \frac{p}{6}\left[(x_2 - x_1)(y_3 - y_1) - (y_2 - y_1)(x_3 - x_1)\right] \tag{4.254}$$

The contribution of this effect to the stiffness matrix is calculated through the derivatives of the forces. The derivatives of $\mathbf{F}_1$ is

$$\mathbf{F}_1' = (\mathbf{12}' \wedge \mathbf{13} + \mathbf{12} \wedge \mathbf{13}')\frac{p}{6} \tag{4.255}$$

The derivatives of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ are identical:

$$\frac{\partial \mathbf{F}_1}{\partial x_1} = \frac{p}{6}\begin{vmatrix} 0 \\ z_3 - z_2 \\ y_2 - y_3 \end{vmatrix} \tag{4.256}$$

$$\frac{\partial \mathbf{F}_1}{\partial y_1} = \frac{p}{6}\begin{vmatrix} z_2 - z_3 \\ 0 \\ x_3 - x_2 \end{vmatrix} \tag{4.257}$$

$$\frac{\partial \mathbf{F}_1}{\partial z_1} = \frac{p}{6}\begin{vmatrix} y_3 - y_2 \\ x_2 - x_3 \\ 0 \end{vmatrix} \tag{4.258}$$

$$\frac{\partial \mathbf{F}_1}{\partial x_2} = \frac{p}{6}\begin{vmatrix} 0 \\ z_1 - z_3 \\ y_3 - y_1 \end{vmatrix} \tag{4.259}$$

$$\frac{\partial \mathbf{F}_1}{\partial y_2} = \frac{p}{6}\begin{vmatrix} z_3 - z_1 \\ 0 \\ x_1 - x_3 \end{vmatrix} \tag{4.260}$$

$$\frac{\partial \mathbf{F}_1}{\partial z_2} = \frac{p}{6}\begin{vmatrix} y_1 - y_3 \\ x_3 - x_1 \\ 0 \end{vmatrix} \tag{4.261}$$

$$\frac{\partial \mathbf{F}_1}{\partial x_3} = \frac{p}{6} \begin{vmatrix} 0 \\ z_2 - z_1 \\ y_1 - y_2 \end{vmatrix} \tag{4.262}$$

$$\frac{\partial \mathbf{F}_1}{\partial y_3} = \frac{p}{6} \begin{vmatrix} z_1 - z_2 \\ 0 \\ x_2 - x_1 \end{vmatrix} \tag{4.263}$$

$$\frac{\partial \mathbf{F}_1}{\partial z_3} = \frac{p}{6} \begin{vmatrix} y_2 - y_1 \\ x_1 - x_2 \\ 0 \end{vmatrix} \tag{4.264}$$

4.3.7 Dynamic: Force of Inertia

The force of inertia is related to accelerations of the net and of the water particles just around the net. The calculation is done for each triangularelement in three parts, one for each vertex, since the acceleration is not constant over the entire surface of each triangular element. Under these conditions, the parameters are local parameters at each vertex, including the acceleration and the mass. The mass per vertex is considered the third of the total mass of netting of the triangular element.

The force of inertia on each vertex of a triangular element mesh is estimated by [7]:

$$\mathbf{F}_i = M_a(\gamma_h - \gamma) + \rho V \gamma_h - M\gamma \tag{4.265}$$

$\mathbf{F}_i$: inertial force on the vertex i (N),
M_a: added mass (kg) of $1/3$ of the triangular element,
M: mass of $1/3$ of the net (kg),
V: volume of $1/3$ of the net (m^3),
ρ : density of water (kg/m^3),
γ: acceleration of the vertex (m/s^2),
γ_h: acceleration of the water around the vertex (m/s^2).

The vertex speed is calculated as follows:

$$\mathbf{v} = \frac{\mathbf{x}_1 - \mathbf{x}}{\Delta t} \tag{4.266}$$

The acceleration of the vertex is

$$\gamma = \frac{\mathbf{v}_1 - \mathbf{v}}{\Delta t} \tag{4.267}$$

which gives

$$\gamma = \frac{\mathbf{x}_2 - 2\mathbf{x}_1 + \mathbf{x}}{\Delta t^2} \tag{4.268}$$

In this case, the contribution to the stiffness matrix, from the derivative of this inertia, is calculated by

$$-F' = -\frac{\partial \mathbf{F}_i}{\partial \mathbf{x}} \tag{4.269}$$

which leads to

$$-F' = (M + M_a)\frac{\partial \gamma}{\partial \mathbf{x}} \tag{4.270}$$

and

$$-F' = \frac{M + M_a}{\Delta t^2} \tag{4.271}$$

With: $\mathbf{x}$: position at t (m),
$\mathbf{x}_1$: position at $t - \Delta t$ (m),
$\mathbf{x}_2$: position at $t - 2\Delta t$ (m),
F': derivative of the force of inertia relative to the position (N/m),
Δt: time step (s).

4.3.8 Dynamic: Drag Force

The drag is related to the net and the relative speed of water particles just around the net. The calculation is done for each triangular element in three parts, one for each vertex, since this speed is not constant over the entire surface of each triangular element. Under these conditions the local parameters at each vertex are the vertex speed and one third of the number of twine vectors for the triangular element. The calculation is done for twines U and V.

The formulation for the twine drag is based on the assumptions of Landweber and Richtmeyer, as described earlier (Sect. 4.3.3, p. 46). The drag on the U twines applied on vertex i of the triangular element takes into account $1/3$ of the number of U twine vectors in the triangular element. This drag is as follows:

$$|\mathbf{F}_i| = \frac{d}{6}\frac{1}{2}\rho C_d D l_o(|\mathbf{c}_i| sin(\theta))^2 \tag{4.272}$$

$$|\mathbf{T}_i| = \frac{d}{6} f \frac{1}{2}\rho C_d D l_o(|\mathbf{c}_i| cos(\theta))^2 \tag{4.273}$$

F_i: normal force to the twines (N) on vertex i, this expression coming from the assumptions of Landweber,
T_i: tangential force (N) on vertex i, from Richtmeyer's assumption,
ρ: density of water (kg/m^3),
Cd: normal drag coefficient,
f: tangential coefficient,
D: diameter of twines U (m),
l_o: length of twine vectors U (m),

c_i: amplitude of the relative velocity of the water at vertex i (m/s),
θ: angle between the twine vectors U and the relative velocity (radians),
$\frac{d}{6}$: one third of the number of twine vectors U in the triangular element.

The angle θ between the twine vector **U** and the relative velocity is calculated by

$$cos(\theta) = \frac{\mathbf{c}_i \mathbf{U}}{|\mathbf{c}_i||\mathbf{U}|} \tag{4.274}$$

The directions of the drag in case of twine vector **U** are as follows:

$$\frac{\mathbf{F}_i}{|\mathbf{F}_i|} = \frac{\mathbf{U}}{|\mathbf{U}|} \wedge \frac{\mathbf{c}_i \wedge \mathbf{U}}{|\mathbf{c}_i||\mathbf{U}|} \tag{4.275}$$

$$\frac{\mathbf{T}_i}{|\mathbf{T}_i|} = \frac{\mathbf{F}_i}{|\mathbf{F}_i|} \wedge \frac{\mathbf{c}_i \wedge \mathbf{U}}{|\mathbf{c}_i||\mathbf{U}|} \tag{4.276}$$

The drag amplitude on twines V is calculated following the same scheme.

4.3.9 Buoyancy and Weight

Buoyancy and weight are vertical forces (along the z axis, if it is the vertical axis). Their expression is summed in the following:

$$F_z = d\pi \frac{D^2}{4} l_0 (\rho_{netting} - \rho) g \tag{4.277}$$

F_z: weight of the net once immersed (N),
d: number of twine vectors **U** and twine vectors **V** per triangular element,
ρ: water density (kg/m^3),
$\rho_{netting}$: net density (kg/m^3),
D: diameter of twines (m),
g: gravity of the Earth (around 9.81 m/s^2),
l_0: length of twine vectors (m).

The length of the twine vectors is approximated by the unstretched twine vector l_0, since the elongation is generally quite small.

There is a contribution of this force to the stiffness matrix when the netting crosses the water surface. In this case there is a variation of force with the immersion. This contribution is not described here.

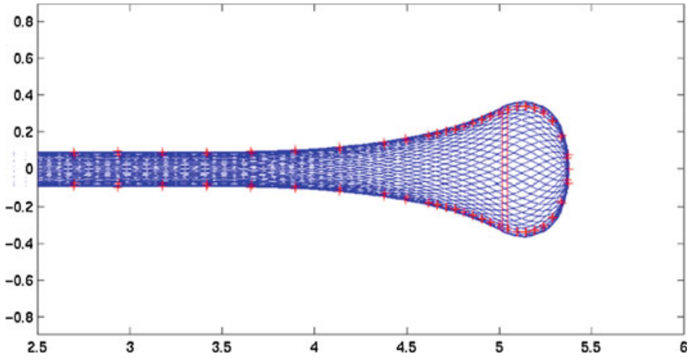


Fig. 4.18 Comparison between simulations (*net*) and flume tank tests (*crosses*) of trawl cod-ends [1]. Between 2.5 and 3.5 m the diameter is constant. This is due to contact between the nodes of the net

4.3.10 Contact Between Knots

It happens quite frequently that the nets are so close that the nodes come into contact with each other. This contact limits the closing of mesh (Fig. 4.18).

An effort similar to that described in Sect. 4.3.4 (p. 57) has been introduced to take into account this feature. This effort appears only when the twines are close enough, that is, when the angle between U and V twines is below a critical angle (α_{mini}). This angle is related to the node size and mesh side as follows (Fig. 4.19):

$$\alpha_{mini} = 2 \arcsin \left[\frac{knot_{size}}{2 mesh_{side}} \right] \tag{4.278}$$

α_{mini}: limit angle of contact between twines (rad),
$knot_{size}$: size of the node (m),
$mesh_{side}$: side of the mesh or length of twine vectors (m).

The $mesh_{side}$ could be the length of the twine vector along the U twine ($|\mathbf{U}|$) or the length of the twine vector along the V twine ($|\mathbf{V}|$). To avoid this choice (between $|\mathbf{U}|$ and $|\mathbf{V}|$), this length can be approximated by the unstretched length l_0 of the twine vector.

A couple is generated between the twines if the angle between them is less than the minimal angle:

$$\begin{cases} C = H(\alpha - \alpha_{mini}) \; if \; \alpha <= \alpha_{mini} \\ C = 0 \qquad\qquad\qquad if \; \alpha > \alpha_{mini} \end{cases} \tag{4.279}$$

C: couple between the twines due to the contact between knots (Nm),
α: angle between twines U and V (rad),

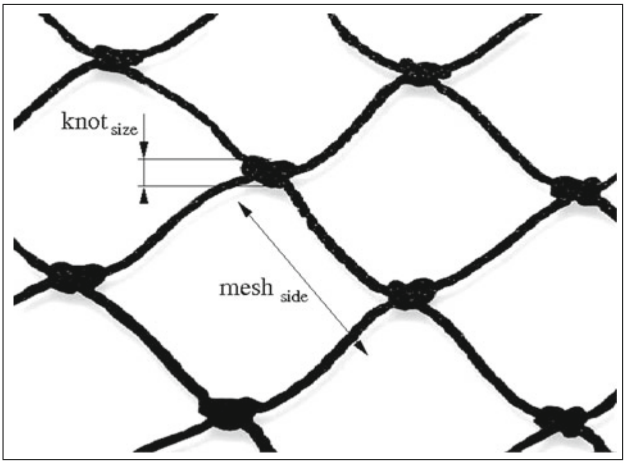


Fig. 4.19 The size of the knot limits the closure of the mesh. The minimal angle between twines is due to the size of the knot and the side of the mesh (which is also the length of twine vector)

H: stiffness (Nm/Rad).

This stiffness is not well known. Therefore, arbitrary values can be used, such as the following, proportional to the elongation stiffness of the twine (EA) (Fig. 4.19):

$$H = \frac{1}{100} \frac{mesh_{side}^2 EA}{knot_{size}} \tag{4.280}$$

A: section of the twine (m^2),
E: Young's modulus (Pa).

The forces on the vertices of triangular elements and the stiffness use the same expressions as those described in Sect. 4.3.4 (p. 57).