A Quasi-periodic Approximation Based Model Reduction for Limit Analysis of Micropile Groups

Zied Kammoun, Joseph Pastor, and Hichem Smaoui

Abstract The behavior of soils reinforced by micropile networks is still not fully understood due to the lack of accurate modelling capabilities. Particularly, the complex geometry of large soil-micropile systems makes accurate calculation of the bearing capacity of the reinforced soil a computational challenge. This complexity requires highly detailed and finely discretized models to achieve reasonable accuracy using direct numerical methods. Such models lead to large scale numerical optimization problems that are hardly tractable using a personal computer.

In the present paper a model reduction method is made capable of solving the numerical static limit analysis problem of soil reinforced by a group of micropiles according to a 2D plane strain model. The method has been successfully applied to the limit analysis problem of a soil reinforced by a large group of micropiles when resources did not permit solution of the full model.

1 Introduction

A micropile is a pile with a small diameter (generally in the range 75 to 200 mm) and high aspect ratio. Micropiles are used in soil reinforcement and foundation works beneath existing buildings. The micropile technique was developed as early as 1952 by the Fondedile company under the authority of F. Lizzi [1]. Micropiles were used for the first time in Italy in soil reinforcement of existing buildings and were then named root piles (pali radice). Within the timeframe of half a century, the technique

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has been applied all over the world [4] with micropile groups counting as many as 1100 micropiles in a landmark example in Neuchâtel, Switzerland.

Nevertheless, the behaviour of micropile groups is still not well understood, particularly because of the complex geometry of large soil-micropile systems that challenged the development of accurate modeling methods.

Various approaches are used for predicting the bearing capacity of micropile groups. Simplified analytical methods [3] are commonly used in engineering practice whereas elastoplastic analysis ([4] and [1]) is often applied in special application and in research. Another alternative is limit analysis [7] by direct methods. The merit of Limit Analysis (LA) is the rigorous underlying theoretical basis and the high level of accuracy that may be achieved.

Because of their complex geometry, raisonably accurate prediction for micropile groups of practical size by Limit Analysis requires finely discretized finite element models leading to numerical optimization problems that are too large to be directly tractable by available algorithms [7].

In an attempt to circumvent the problem size difficulty, different techniques have been devised to reduce the size of the numerical Limit Analysis problem to be solved. Among these techniques, homogenization methods [2] have been proposed. While successful in reducing the computational effort these methods do not provide a realistic description of the stress and strain fields in the heterogeneous medium, especially near the boundaries of the reinforced zone. Domain decomposition is another approach that is developed for solving large size LA problems. It converts the original numerical LA problem into a sequence of smaller LA like subproblems that are solved iteratively. This approach has proven to be successfull in solving problems that are untractable when solved directly ([8] and [10]).

In this work, an alternative technique is presented that aims at reducing the size of the numerical LA problem for uniformly spaced micropile groups by taking advantage of the periodicity of the geometry and structure of the reinforced zone. It is inspired from the case of fiber reinforced composites which is suited to modeling using periodic homogenization [5].

In this study, a two dimensional representation of the reinforced soil will be adopted to reduce the numerical problem dimension. Extension to the three dimensional problem will be possible because it is conceptually equivalent to the two dimensional problem.

The paper begins with a brief presentation of limit analysis followed by a description of the proposed periodic reduction method. The method is then tested and assessed by applying it to examples of soil-micropile systems.

2 Limit Analysis and the Static Method

According to Salençon (see [12, 13]), a stress tensor field σ is said to be statically admissible (SA) if equilibrium equations, stress vector continuity, and stress boundary conditions are verified. It is said to be plastically admissible (PA) if $f(\sigma) \leq 0$, where $f(\sigma)$ is the (convex) plasticity criterion of the material. A field σ that is SA and PA here will be said to be (fully) admissible.

Similarly, a strain rate tensor field v is kinematically admissible (KA) if it is derived from a continuous velocity vector field u such that the velocity boundary conditions are verified. It is said to be plastically admissible (PA) if the flow rule (1) is verified; the fields u and v, which are KA and PA, will be called admissible in the following.

$$v = \lambda \frac{\partial f}{\partial \sigma}, \quad f(\sigma) = 0,\ \lambda \geq 0. \tag{1}$$

The so-called associated flow rule (1) (or normality law) characterizes the standard material of LA. Equivalently, a standard material satisfies Hill's maximum work principle (MWP) [6], which states that:

$$\left(\sigma - \sigma^*\right) : v \geq 0 \quad \forall \text{PA } \sigma^*. \tag{2}$$

A solution to the LA problem is a pair of fields (σ, v) where σ and v are both admissible and associated by the normality law. Classically, these solutions can be found or approached using two optimization methods. The first one, involving only the stresses as variables, is the static (or lower bound) method. The second one, involving only the displacement velocities as variables, is the classical kinematical (or upper bound) method.

Let us assume that the virtual power of the external loads can be written as the scalar product of a loading vector $Q \in \mathbb{R}^{\mathrm{n}}$ and a generalized velocity vector $q = q(u)$, linear in u. A loading process linearly associated with a statically admissible stress field σ, $Q = Q(\sigma)$, is said to be admissible. The set of these admissible loadings forms a convex K in $\mathbb{R}^{\mathrm{n}}$ and the n components of Q are called loading parameters.

Finding the solution of the limit analysis problem consists in determining an admissible field σ together with an admissible strain rate field associated to σ by the normality law. In this case the loading $Q(\sigma)$ is a limit loading of the mechanical system. The set of the limit loadings is the boundary ∂K of the convex K: this boundary can be approached by solving the following optimization problems:

$$Q_{lim} = \left(Q_1^d, \ldots, \lambda_0 Q_i^d, \ldots, Q_n^d\right), \tag{3a}$$

$$\lambda_0 = \max\left\{\lambda,\ Q(\sigma) = \left(Q_1^d, \ldots, \lambda Q_i^d, \ldots, Q_n^d\right)\right\}, \tag{3b}$$

where σ is an admissible stress field and Q^d a given admissible loading. Then, by varying Q^d it is possible to construct various points on ∂K: the smallest convex envelope of these points gives an approximation of ∂K from inside. This is the static, or lower bound method of LA, as it will be used here.

3 Finite Element Formulation of the Static Problem

In the present work the problem is formulated in plane strain. The numerical static method is used as it was defined and detailed in [11].

Let us consider a triangular finite element discretization of the mechanical volume V in the global frame (x, y); the stress field is chosen as linearly varying in x, y coordinates in each triangular element and it can be discontinuous through any element edge. In plane strain, the von Mises or Tresca criterion is written as:

$$f(\sigma) = \sqrt{(\sigma_x - \sigma_y)^2 + (2\tau_{xy})^2} \leq 2c, \quad (4)$$

where c is the cohesion of the material. It is worth noting here that the proposed problem reduction method is valid for the Coulomb or Drucker-Prager criteria (provided the final optimization problem could be solved by efficient mathematical programming techniques).

In order to ensure static and plastic admissibility of the stress solution field, the following, briefly recalled conditions are imposed:

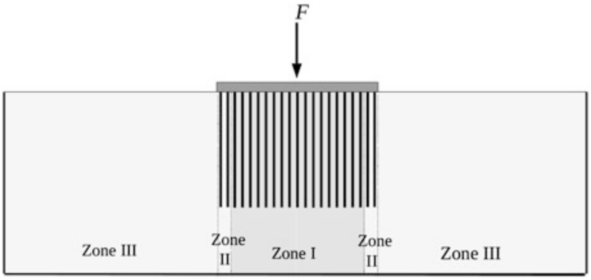
- In each element, the equilibrium equations $\sigma_{ij,j} + \gamma_i = 0$ expressed in the Cartesian frame, where γ is the specific weight vector.
- Continuity conditions: the stress vector is continuous across a discontinuity line: for each discontinuity segment of normal n, the continuity of the stress vector $T_i = \sigma_{ij} n_j$ is written at the apices defining this discontinuity segment.
- Boundary conditions: the stress vector verifies $\sigma_{ij} n_j = T_i^d$ at each apex of the boundary element sides where the stress vector T^d—linearly varying— is imposed.
- Definition of the functional from the power of external loads: for example, the integral of the normal stresses in the case of the footing under an imposed normal uniform velocity.
- Stress field plastic admissibility: imposed at each triangle apex. This ensures that it is verified over the total domain from the linear variation of the stress in a triangle and the convexity of the criterion (4).

By writing the criterion directly in the conic form $V = 2c \geq \sqrt{Y^2 + Z^2}$, where V is an auxiliary variable, the numerical optimization problem can be solved using the conic programming code MOSEK [9] as in [14].

4 Quasi-periodic Reduction Method

In large micropile groups the micropiles are usually arranged in a regular pattern with a periodic geometry and structure. When the loading is uniformly distributed among the reinforcements the reinforced soil tends to respond in a periodic mode, at least away from the boundaries of the reinforced zone. The proposed method takes advantage of this periodicity to reduce the size of the numerical limit analysis

Fig. 1 The problem of the rigid footing under a central force F



problem. It is inspired from the case of fiber reinforced composites for which a successful periodic homogenization approach was developed in [5].

Figure 1 shows a typical soil-micropile group configuration with a reinforced zone supporting a rigid foundation and the natural soil extending on its sides and beneath, all the way down to a rigid substrate. To apply the reduction procedure the domain is subdivided into three parts. The first is the central reinforced zone where the behavior is assumed to be periodic, denoted zone I. The second, the edge zone, denoted zone II, is a part of the reinforced soil separating the periodic zone from the domain occupied by the natural soil. Finally, the rest of the soil represents the zone III. Although geometrically and materialwise periodic, zone II (the transition zone) is treated as non periodic.

A representative volume element (RVE) is constituted by a micropile and half the width of soil on each side in addition to the underlying volume of soil.

Regardless of the number of micropiles it includes, the periodic zone is replaced by a single periodic representative volume element (PRVE) fulfilling built-in periodicity and inter-RVE continuity constraints.

The periodicity conditions imposed on the stress field are

$$\sigma^{left}.n = \sigma^{right}.n, \tag{5}$$

where n is the normal to the right (or the left) side of the PRVE. As the n_{pp} periodic RVEs are replaced by a PRVE, the loading F_R of the reduced problem, equivalent to the original load F (Figs. 2 and 3) is given by

$$F_R = F_T + n_{pp}.F_P \tag{6}$$

where F_T is the load supported by zone II and F_P is the load supported by the PRVE in the reduced problem.

This results in a considerable reduction in problem size at the cost of an approximation error. Interestingly, the error is on the conservative side, preserving the lower bound nature of the solution of the static problem. Edge zones are defined by a few RVE's on each side. The finite element mesh corresponding to these edge zones and the natural soil (Zone III) remains unchanged. Furthermore, the detailed modeling of the soil-micropile composite at the RVE level, both in the horizontal and vertical directions, has the merit of accounting for the toe and lateral effects on the bearing

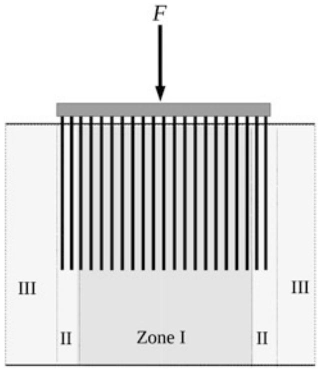


Fig. 2 Load in initial problem

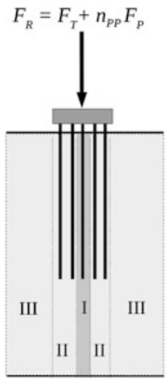


Fig. 3 Load in reduced problem

capacity. This is partly because the interaction effect between micropiles can be captured as a result of the full consideration of the soil layers underlying the reinforced zone.

5 Numerical Examples

The method is applied to some examples of soil reinforcement by groups of micropiles. The optimization problem is solved using the conic programming code MOSEK for both the direct problem, when possible, and the reduced problem, and performance is compared.

The LA problem considered (Fig. 4) is that of a Tresca soil reinforced by a group of n_p micropiles to support a weighless foundation slab of width b loaded at its middle by a force F. The soil cohesion is $C = 10$ kPa and depth is $H = 30$ m. The micropiles length is $h = 20$ m and width $d = 0.2$ m. The bearing capacity of the foundation is determined as the maximum load F that, together with a stress field σ, form a statically and plastically admissible pair. The associated numerical optimization problem is denoted P_0.

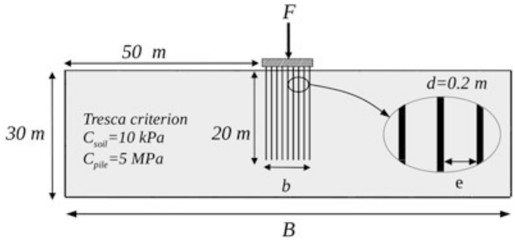


Fig. 4 Example of a soil reinforced with micropiles

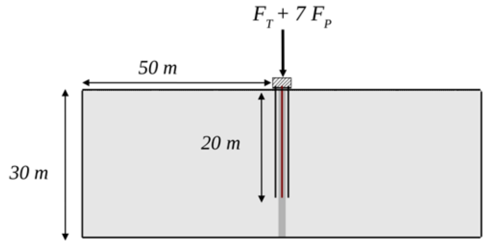


Fig. 5 Reduced problem of the considered example

In the reduced problem (Fig. 5), denoted P, Zone I is modeled using a single PRVE and the load is defined as the sum of the loads supported by the RVEs in Zone II and the load supported by the PRVE scaled up by the number of micropiles belonging to Zone I. Since the solution for the reduced problem is admissible for P_0, it provides a lower bound for the original LA problem. By limiting the number of transition micropiles to one on each side, the number of micropiles in the model decreased from n_p to only 3.

5.1 Effect of Load Transmission Mode

To illustrate the influence of the load transmission mode from the slab to the reinforced soil the problem is considered with two alternative transmission mechanisms and is modelled with the same degree of discretization. In the first, the foundation is assumed to be supported solely by the micropiles. In the second, it is supposed to rest on both the soil surface and the micropile tips. In both cases the kinematic and static bounds of the bearing capacity are first determined by solving the direct problem for a reinforcement with nine micropiles ($n_p = 9$). Furthermore, a static bound is estimated by solving the reduced problem resulting from the quasi-periodic approximation. Results are produced for a range of micropile spacings to assess the effects of spacing and surface load transmission mode.

5.1.1 Foundation Supported Solely by the Micropiles

The limit load (load-bearing capacity) F of the reinforced soil is determined in this case with the boundary conditions defined such that the load is carried only by the

Table 1 Number of elements for different values of micropile spacing

Spacing	1.8	2.8	3.8	4.8	5.8	6.8	7.8	8.8
Elements (×1000)	110	126	141	156	171	134	–	149

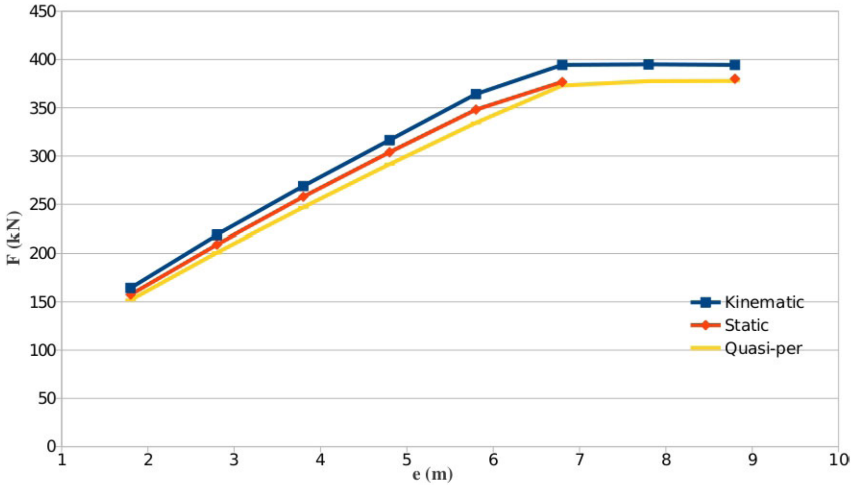


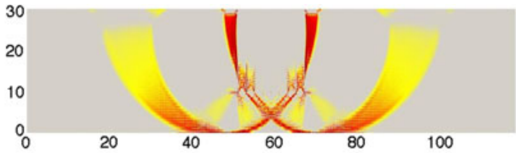
Fig. 6 Influence of spacing on bearing capacity. Direct and reduced model solutions

micropiles. The limit load is calculated for different values of micropile spacing. Figure 6 shows the limit load of the reinforced soil, calculated with different methods, as a function of spacing. The blue line represents the kinematic solution (upper bound) of the reference problem. The red line represents the static solution (lower bound) of the reference problem. The yellow line corresponds to the solution of the reduced model problem.

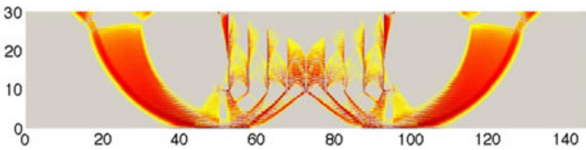
The results for $e < 6.8$ were all obtained with the same degree of discretization (elements size) which did not permit the direct solution beyond that spacing. Therefore, the problems with $e > 6.8$ were solved with fewer, larger elements. It should however be stated here that for the case $e = 7.8$ the results were not shown because the case was simply not treated. The reason was that it was not possible to create a regular mesh with the large element size because of the particular geometrical dimensions of the reinforced soil. The number of elements for each spacing is indicated in Table 1.

It may be noted from the results that:

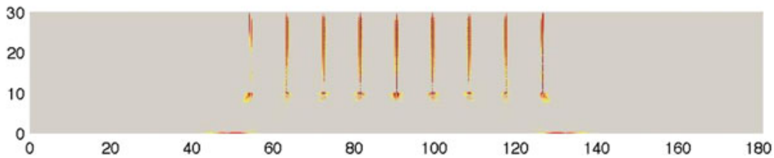
- The reinforced soil bearing capacity increases with spacing for spacing under 6.8 m. Beyond this value the bearing capacity saturates and remains nearly indifferent to spacing. The saturation should reflect the vanishing of the interaction among micropiles which tend to behave as isolated inclusions.
- The error between the direct static and the reduced model solution is relatively small (less than 4.2 %).



(a) piles charged $e = 1.8$



(b) piles charged $e = 4.8$



(c) piles charged $e = 8.8$

Fig. 7 Optimal stress field (direct solution)

The solution stress field is visualized in Fig. 7 for selected values of spacing. It can be seen that, for small spacing, the behavior of the reinforced area is reminiscent of a block mechanism (Fig. 7(a)). For large spacing (Fig. 7(c)), the elementary volumes tend to behave independently as if the micropiles were isolated. From the stress field in Fig. 7(c) a pattern can be seen that is characterized by a localization of the failure zone in a thin volume of soil surrounding the micropile.

5.1.2 Foundation Supported by Both the Soil and the Micropiles

The boundary conditions in this case are defined such that the load is carried at the soil surface by both the micropiles and the intersticial soil. Figure 8 shows the limit load calculated for different values of micropile spacing. It is observed that:

– The limit load always increases with spacing in contrast to the behavior observed with the loading supported solely by the micropiles. The reduced model solution increases linearly, whereas the direct static and kinematic limit load increases in a slightly bilinear pattern.
– The error increases with spacing to 9 % at $e = 8.8$ m.

The solution stress field is visualized in Fig. 9 for the same selection of spacing values. It shows that:

– For small spacing, the behavior of the reinforced zone is similar to that of a block mechanism.

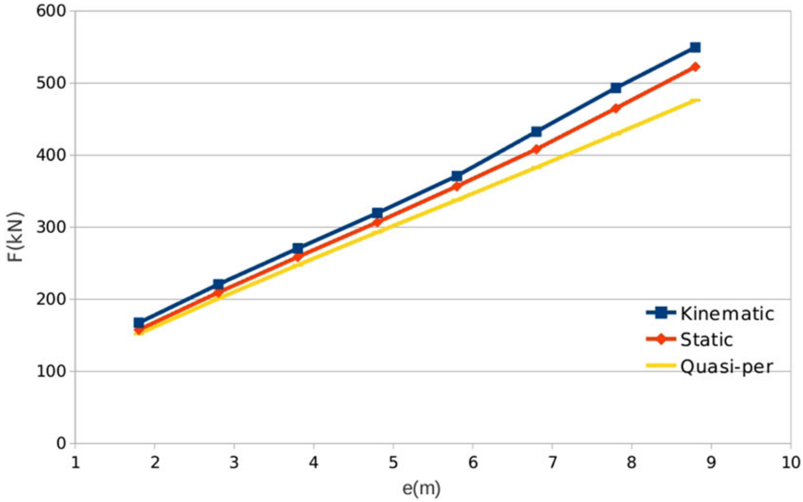


Fig. 8 Limit load for different values of spacing

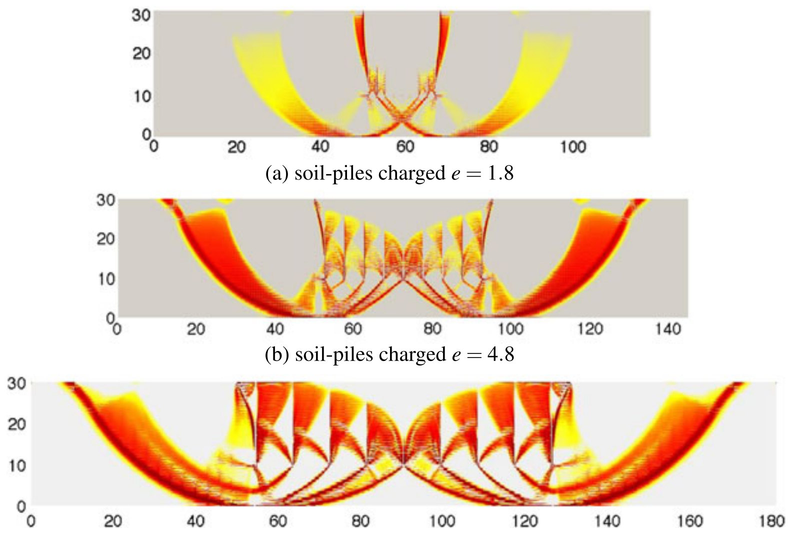


Fig. 9 Optimal stress field (direct solution)

– For large spacing, the behavior does not clearly reflect the assumption of periodicity. The stress distribution for $e = 8.8$ in Fig. 9(c) looks more like that in Fig. 7(b) (for $e = 5$) than the nearly periodic stress field shown in Fig. 7(c), obtained for the same spacing $e = 9$ when the load is supported by the micropiles only.

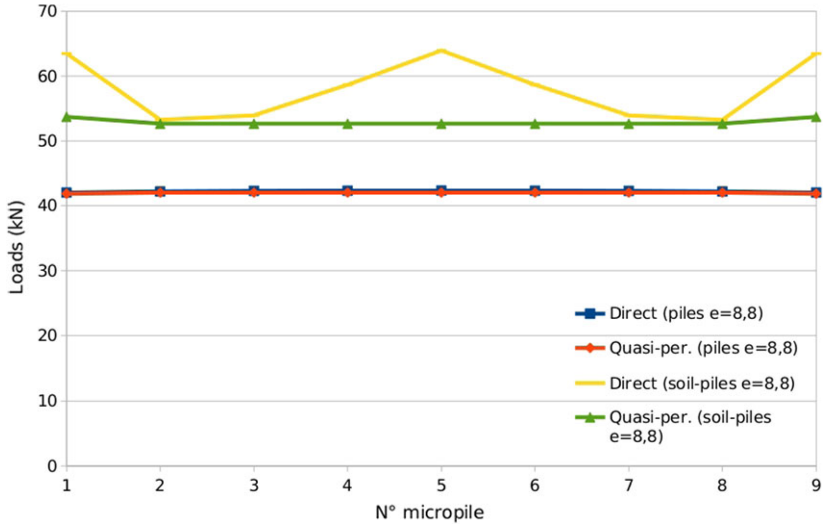


Fig. 10 Load distribution among elementary volumes

5.1.3 Results Interpretation

The details of the distribution of load among the elementary volumes are given in Fig. 10 for $e = 8.8$ m. For a load supported by both the soil and the micropiles, the distribution of the limit load obtained via the direct static solution shows significant fluctuations compared to the uniform distribution typical of the reduced model solution. The largest error on the load acting on an elementary volume is 17.65 %, it occurs at the center of the reinforced soil surface. The second largest error is found at the edge of the reinforced zone. These levels of errors are reasonable since they should be larger than the error of 9 % relative to the total load which is actually the integral of these elementary loads. For a load supported by the micropiles only, the distribution of limit load obtained by the direct static solution is almost uniform as expected since the micropiles have been shown to behave practically independently and, thus, to fulfill the periodicity assumption. When spacing is small, the failure occurs in a block mechanism mode regardless of the load transmission pattern. This explains the closeness of the limit loads evaluated using the Direct and Reduced formulations.

5.2 Effect of Micropile Number on Performance

To assess the performance gain of the reduction method for larger micropile group sizes (Fig. 11) the limit analysis problem is solved using the Direct (i) Static and (ii) Kinematic and the (iii) Reduced Model formulations with the number of micropiles varying from 1 to 31.

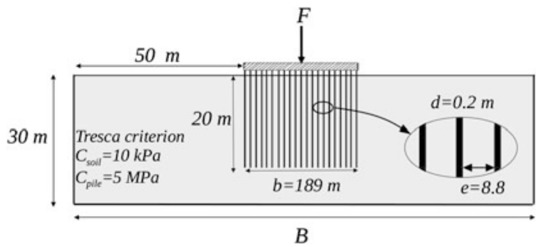


Fig. 11 Example for large number of micropiles

Table 2 Effect of micropile number

Nbr. of piles	Static			Quasi-periodic				Kinematic		
	Nbr elem.	F (kN)	CPU (s)	Nbr elem.	F (kN)	CPU (s)	Error (%)	Nbr elem.	F (kN)	CPU (s)
1	95760	419.7	249	–	–	–	–	98100	432.9	1078
3	103320	1263.4	232	–	–	–	–	114300	1304.7	1998
7	133560	2957.5	354	126000	2938.4	315	0.65	146700	3059.9	1845
9	148680	3800.3	204	126000	3779.2	273	0.56	162900	3944.1	1699
11	163800	4651.4	360	126000	4619.6	350	0.68	167160	4809.5	1948
15	194040	6342.8	402	126000	6301.0	352	0.66	197400	6580.7	1833
19	224280	8037.6	459	126000	7982.4	281	0.69	227640	8320.8	2299
21	239400	8884.6	446	126000	8822.9	257	0.69	–	–	–
31	–	–	–	126000	13026	327	–	–	–	–

The boundary conditions are such that the load is carried by the micropiles only. The same degree of discretization, i.e. in terms of size of finite elements, is used in all the models. The reduced model counts 126,000 finite elements regardless of the number of micropiles.

From the results, summarized in Table 2, it is seen that, as expected, the CPU time required by the reduced model solution has no clear tendency to increase with the number of micropiles, whereas the CPU time of the Direct solution increases with it nearly proportionally.

For a reinforcement with 21 micropiles (Table 3) the mesh of the Direct problem model amounts to 239,400 finite elements, nearly twice the number of elements in the reduced model, and the consumed CPU time is almost double the CPU of the reduced model solution for an accuracy gain of 0.7 %. This is the largest problem for which the Direct solution was possible with the Mac Pro 3 GHz machine used in this work.

For the same number of micropiles and using a finer mesh with 277200 elements in the Direct problem model, the Direct solution fails to converge whereas the reduced model solution converges in twice the CPU time and improves the "reduced" lower bound by 0.15 %.

The relative error between the Direct and the Reduced Model solutions is between 0.5 and 0.7 % and does not appear to increase with the number of micropiles.

Table 3 Reinforcement with 21 micropiles

Method	Nbr of elements	F (kN)	CPU (s)
Static	239400	8884.6	446
Quasi-periodic	126000	8822.9	257
Quasi-periodic	277200	8835.9	530

Consequently, it may be concluded that the reduction method provides a fairly accurate estimate for an unlimited number of micropiles within a nearly constant computational effort.

6 Conclusion

A model reduction method is proposed to solve the numerical static limit analysis problem of a composite medium, characterized by periodic reinforcement, embedded in a homogeneous domain, while preserving the fineness of the Finite Element description of the Representative Volume Element. The reduction method has been successfully applied to the Limit Analysis of a soil reinforced by a large group of micropiles when resources do not permit solution of the full model problem. Numerical results demonstrate that the reduction method provides a fairly accurate estimate of the limit load for an unlimited number of micropiles within a nearly constant computational time. Significant differences in behavior and bearing capacity are observed depending on the way the applied load is distributed between the soil and the micropiles. When the load is supported solely by the micropiles the reduced model results in terms of limit load of individual micropiles are very close to the reference solution (in confirmation of the periodicity assumption). When the load is supported by both the soil and the micropiles the error is larger than when only the micropiles carry the load. In a future work the reduction method will be extended to more general periodic representative volume elements by relaxing the symmetry requirement and allowing some forms of controlled variability of the stress field in the PRVEs. This will lead to more accurate solutions at the cost of a little extra computational effort. Another extension that might improve the accuracy of the reduction method, consists in limiting the length of the RVE to the height of the micropile allowing more degrees of freedom in the soil beneath the reinforced zone.

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