# Chapter 8 Optimising Bioenergy Villages' Local Heat Supply Networks

Anke Daub, Harald Uhlemair, Volker Ruwisch, and Jutta Geldermann

Abstract Bioenergy villages' local energy facilities produce electricity and heat for their inhabitants. This electricity is fed into the public grid with the heat distributed to the households via a local hot water grid. We use a linear mathematical model to simultaneously optimise the course of the heat supply network and the selection of households to be connected to the grid. In a first step, the heat distribution system is economically optimised. In a second step, we analyse the impacts of including social criteria and of varying parameters (e.g., prices). The model is applied to a small village with 24 households.

Keywords Bioenergy village • biogas plant • heat supply network • optimisation model • sensitivity analysis

## 8.1 Introduction

This chapter deals with the economic optimisation of local heat supply networks for bioenergy villages. Potential heat customers could be private households, public buildings, farms, industrial buildings, hotels and recreational facilities such as swimming pools or gyms. It is assumed that an independent operating company runs the district heat supply system. The required amount of heat is purchased from a local bioenergy plant, which comprises a combined heat and power biogas plant, a central heat station burning wood chips and an additional oil-based heat generator

V. Ruwisch

A. Daub  $(\boxtimes) \cdot H$ . Uhlemair  $\cdot J$ . Geldermann

Chair of Production and Logistics, University of Göttingen, Platz der Göttinger Sieben 3, Göttingen, Germany 37073,

e-mail: [anke.daub@wiwi.uni-goettingen.de](mailto:anke.daub@wiwi.uni-goettingen.de)

Interdisciplinary Centre for Sustainable Development, University of Göttingen, Goldschmidtstr. 3, Göttingen, Germany 37077,

<span id="page-1-0"></span>to ensure the heat supply during very cold periods of the year. In Sect. 8.2, a linear mathematical model is presented and applied to a village with 24 potential heat customers. The model calculates which potential heat customers should be connected to the heat supply network and what the pipeline's optimal course would be. In Sect. [8.3,](#page-11-0) the results of a sensitivity analysis are shown. This analysis incorporates different scenarios regarding the villagers' willingness to be connected to the network. We also investigate how much the heat customers' initial fixed costs can be reduced by examining the profits after the optimisation. In addition, we calculate how high the price that the operating company pays for heat can rise before the network becomes unprofitable. In Sect. [8.4,](#page-18-0) we summarise the findings and describe further research steps.

### 8.2 The Optimisation Model

### 8.2.1 Components of Mathematical Optimisation Models

To set up a resource-efficient and cost-efficient heat distribution system, the planning process should be mathematically modelled. Models are a means to reduce de facto complex relationships to their essential structures in order to identify the important components, the dependencies between them and the effects that changing data have.

Mathematical planning and optimisation models consist of three basic components: the decision field (the model's variables), the planning target (the model's objective function) and the planning framework (the model's constraints) (Hillier and Lieberman [2010,](#page-19-0) p. 25 ff.). The decision variables describe the decision-maker's scope of action; by assigning a specific value to each variable, one of all the possible decisions is chosen.

On the one hand, the scope of possible actions is determined by the variables' domain (e.g., binary variables for potential heat customers – whether connected or not connected to the heat supply network –, or nonnegative real numbers for an energy plant's capacity (kW)). On the other hand, the scope of action may be restricted by constraints that should not be violated. These could be the available amount of biomass, a financial budget, or the plant's capacity. These constraints frame the set of all feasible solutions, i.e. the decision variables' region of permissible values in which all constraints are met.

An objective function has to be formulated to measure different solutions' quality. This function represents the decision-maker's preferences and consists of the performance measure (e.g., the local heat supply network's profits or the amount of emissions resulting from the biogas station's energy generation) and the direction into which these should develop (e.g., maximisation or minimisation). The optimal solution is the one with the most favourable performance measure value.

Linear optimisation models are characterised by the variables not being squared, cubed or multiplied by one another in the objective function or in the constraints, etc. Whether the region of feasible solutions is convex or non-convex depends, among others, on the variables' domain (Hu [1969](#page-19-0)). The simplex algorithm (Murty [1976\)](#page-19-0), interior-point methods (Domschke and Drexl [2007\)](#page-18-0) and branch-and-boundbased algorithms (Murty [1976](#page-19-0)) can be used to solve linear optimisation problems.

Before we develop an optimisation model for a specific village, we describe its structures and characteristics.

### 8.2.2 Selected Bioenergy Village

The village on which the following analysis is based is located in southern Lower Saxony. The villagers want to use locally produced bioenergy in future and have therefore supplied information on, for instance, their individual heat demands. The village structure and the other necessary parameters used in this analysis are real data collected in this village. Accordingly, the model described below can be used as a decision support tool to help ensure a bioenergy project's success.

This village has 24 households, each with its own heating system, which will in future receive heat from a local heat supply network. Consequently, a local hot water grid has to be installed. A local energy plant comprising a combined heat and power biogas plant will generate the required amount of heat. Electricity is fed into the national grid and heat distributed to the villagers via a local hot water grid. An additional heating system burning wood chips and an oil-based peak load boiler ensure heat supply on very cold winter days. It is assumed that there will always be sufficient energy to supply the villagers with heat. An independent operating company – such as the cooperative of farmers and villagers found in Jühnde<sup>1</sup> – will run the heat supply network. It will buy the heat from the bioenergy plant (at a set price of  $0.03$  euro per kWh<sub>th</sub>) and sell it to the heat consumers (at a set price of 0.059 euro per  $kWh_{th}$ ). Most of the households have signed a contract, in which they declare their willingness to be connected to the heat supply grid, with the network operating company; for various personal reasons some households have not signed this contract.

The decision situation is depicted in Fig. [8.1.](#page-3-0) The black lines show the possible course of the heat supply grid and the 24 potential heat recipients are represented by the nodes  $x_1, \ldots, x_{24}$ . Three nodes  $(x_{25}, x_{26}, x_{27})$  – representing the crossroads branch points – have been introduced.

We will formulate a decision model and design a distribution system for this village for an economically optimal heat supply to the households. We do not consider the producing and selling of electricity, nor is the production system part of the planning and the decision model. Consequently, the energy biogas plant's capacity as well as its configuration and location, is considered as given.

 $<sup>1</sup>$  See Chap. [2](http://dx.doi.org/10.1007/978-94-007-6642-6_2) in this book.</sup>

<span id="page-3-0"></span>

Fig. 8.1 Potential heat recipients and possible grid segments

# 8.2.3 Linear Optimisation Model for a Local Heat Supply Network

### 8.2.3.1 The Model's Variables

The three components of an optimisation model (variables, objective function, constraints – see Sect.  $8.2.1$ ) have to be explained and defined in terms of the bioenergy village described above.

Two decisions have to be made to construct the local heat supply grid:

- Which objects (private households, public buildings, industrial enterprises, etc.) will be linked to the grid?
- What is the grid's optimal course?

The variables can be divided into two groups: variables relating to the households<sup>2</sup> and those relating to the heat supply grid. The first group consists of all possible objects located within the village, irrespective of whether or not the homeowners have signed a heat supply contract. Since connecting a household to the local heat supply grid is a mere yes or no decision, the relevant variables (in this context called  $x_i$ ) have to be defined as binary variables and can therefore only have the value 0 or 1. Since the village consists of 24 households, the variables  $x_1, x_2, \ldots$ ,

 $2$  Here, "households" include public buildings, industrial enterprises, etc.

#### <span id="page-4-0"></span>Box 8.1 Net Present Value (NPV)

The net present value is one of the basic key figures for investment appraisal. There are various models that support investment decisions. They can, for example, be categorised according to time (static and dynamic) and certainty (deterministic and stochastic). On the one hand, examples include the comparative cost or profit method, the comparative profitability method, or the static amortisation method. On the other hand, there are dynamic models such as the net present value method, the annuity method, the internal rate method and the dynamic amortisation method. The net present value method is used often, because it is easy to deploy and suitable to evaluate whether an investment is absolutely or relatively advantageous. More information on methods and different performance figures can be found in Götze et al.  $(2007)$  $(2007)$  $(2007)$ .

 $x_{24}$  represent the decision whether or not a specific household will be integrated into the grid (supplemented by the variables  $x_{25}$ ,  $x_{26}$ ,  $x_{27}$  for the crossroads).

In the second group of variables (the grid variables), all possible courses of the grid have to be considered. The grid is divided into different network segments described by the variables  $y_{ij}$ . Each segment  $y_{ij}$  represents the link between household *i* and *j*. Contrary to the households variables, the grid variables  $y_{ij}$  are not given beforehand. Instead, the grid's different technically feasible courses have to be identified and variables have to be assigned to all the potential network segments.

### 8.2.3.2 Objective Function

In this case, the objective function's performance measure is the whole system's net present value (NPV, see Box 8.1). This is the value of all of an investment's present and future cash flows at the start of the planning horizon. Since the future payments cannot be compared with the current payments due to issues such as inflation, uncertainties and alternative investment possibilities, they have to be discounted by using a plausible internal discount rate. A positive net present value indicates that an investment is profitable and better than a financial investment based on a specific interest rate (Götze et al. [2007\)](#page-18-0).

The local heat supply network's net present value consists of all the (usually positive) present values of the households and all the (negative) present values of the network segments.

From the operating company's perspective, the households' net present values comprise the following positive or negative payments:

• annual revenues from selling heat (product of the individual heat demand  $(w_i)$ ) and the difference between the selling price  $(p_s)$  and the buying price  $(p_B)$  per  $kWh<sub>th</sub>$ ) and an annual basic fee which the households have to pay

- one-time payments (positive payments such as the connection fee, the capital contribution, a government grant and the negative payment for the individual grid connections)
- negative annual payments for maintenance (dependent on the individual grid connections) and support.

The model for optimising the local heat supply network is based on the following assumptions:

- The planning horizon is 20 years.
- It is calculated with a 3 % internal discount rate.
- All payments are net payments.
- The problem of self-financing vs. external financing is not explicitly addressed (at least concerning the households).
- There are no heat losses when heat is conveyed through the grid.

Using the annuity present value factor to discount the (constant) payments to the start of the planning horizon (see the quotient at the end of the formula), the net present value for household  $i$  (NPV<sub>i</sub>) can be calculated as follows (the figures in brackets below the formula show the parameters' values in this village):

$$
NPV_i = (c + g + cc - h_i) + [w_i \cdot (p_S - p_B) + b - m \cdot h_i - a] \cdot \frac{(r + 1)^T - 1}{r \cdot (r + 1)^T}
$$

with:



Table [8.1](#page-6-0) lists the individual heat demands and different installation costs<sup>3</sup> of connecting the households to the grid.

The net present values for all the households can be calculated by using the above-mentioned formula; for example, the net present value of household 1 amounts to:

<sup>&</sup>lt;sup>3</sup> Although payments are sometimes called "costs" here, "payments" is the correct term in economics theory, because only cash-effective amounts are considered.

household		$\overline{2}$	3	$\overline{4}$	5	6		8
heat demand installation costs	40,000	10,000	20,000	32,000	18,000	16,000	26,000	25,000
(euro)	8,700	11,700	21,700	33,700	19,700	17,700	27,700	26,700
household	9	10	11	12	13	14	15	16
heat demand installation costs (euro)	18,000 19,700	28,000 29,700	22,000 23,700	12,000 13,700	31,000 32,700	4.000 5,700	28,000 29,700	14,670 16,370
household	17	18	19	20	21	22	23	24
heat demand installation costs	18,000	26,000	31,333	34,000	16,000	14,000	8,000	24,000
(euro)	19,700	27,700	33,033	35,700	17,700	15,700	9,700	25,700

<span id="page-6-0"></span>Table 8.1 Parameters for the households

Table 8.2 Coefficients for the households

$x_i$								
$c_i$	17,671	8.699	12,988	19,431	13,427	10,624	16,599	15,910
$x_i$	9	10	11	12	13	14	15	16
$c_i$	11,611	14,861	15,402	8,649	19,391	7,164	16,159	11,480
$x_i$	17	18	19	20	21	22	23	24
$c_i$	13,817	17,637	19,404	19,510	12,310	11,063	5,766	14,573

$$
NPV_1 = (2,000 + 1,800 + 2,500 - 8,700) + [40,000 \cdot (0.059 - 0.03) + 420.17 - 0.02 \cdot 8,700 - 50] \cdot \frac{1.03^{20} - 1}{0.03 \cdot 1.03^{20}} = 17,671
$$

All the net present values, which are simultaneously the coefficients  $c_i$  for the variables  $x_1$  up to  $x_{24}$  in the objective function, are shown in Table 8.2.

To complete the objective function, i.e. to add the coefficients  $c_{ii}$  of the network segments, the corresponding net present values have to be calculated. The payments for installing the grid segments vary with the length of the single segment (in metres) and the soil type (street, grass strip, meadow, etc.), where the strip of pipeline has to be laid. Without considering the government grant of 80 euro per metre,<sup>4</sup> Table [8.3](#page-7-0) lists the lengths and costs per metre for the various segments  $y_{ii}$ between the nodes i and j.

<sup>4</sup> This grant is addressed in the formula below.

segment	$0 - 1$	$1 - 3$	$2 - 3$	$0 - 4$	$4 - 5$	$5 - 6$	$6 - 7$
length of the segment	35	120	15	63	25	$\mathbf{0}$	32
costs per metre (euro)	200	250	250	250	350	$\theta$	350
segment	$7 - 25$	$8 - 25$	$8 - 9$	$9 - 10$	$10 - 11$	$11 - 26$	$23 - 26$
length of the segment	24	22	14	25	12	9	35
costs per metre (euro)	350	350	350	350	400	400	350
segment	$23 - 24$	$22 - 26$	$21 - 22$	$20 - 21$	$21 - 27$	$17 - 27$	$18 - 27$
length of the segment	16	18	12	$\bf{0}$	13	38	11
costs per metre (euro)	350	350	350	$\theta$	400	300	400
segment	$18 - 19$	$16 - 19$	$12 - 26$	$12 - 14$	$14 - 15$	$12 - 13$	$13 - 25$
length of the segment	6	7	8	12	18	23	34
costs per metre (euro)	400	400	400	300	250	300	200

<span id="page-7-0"></span>Table 8.3 Parameters for the segments

Contrary to the payments that have to be made to connect the households, it is assumed that a credit amount, which will be paid back at a constant rate per year (interest plus redemption), is needed to install the whole grid. This annuity is the result of the net payment for the segment (the segment cost minus the government grant) multiplied by the inverse of the annuity present value factor on the basis of the credit interest rate. Furthermore, annual payments have to be considered for maintenance; these payments amount to 2 % of the payments for the main grid as a whole.

The net present values of the network segments ij  $(NPV_{ii})$  can be calculated as follows:

$$
NPV_{ij} = \left[ -l_{ij} \cdot k_{ij} \cdot m - (l_{ij} \cdot k_{ij} - gn \cdot l_{ij}) \cdot \frac{f \cdot (f+1)^{T}}{(f+1)^{T}-1} \right] \cdot \frac{(r+1)^{T}-1}{r \cdot (r+1)^{T}}
$$

with:



The net present values for all the segments can be calculated by means of this formula. For example, the net present value of the segment between node  $x_1$  and  $x_3$ amounts to:

$x_i - x_i$	$0 - 1$	$1 - 3$	$2 - 3$	$0 - 4$	$4 - 5$	$5 - 6$	$6 - 7$
$c_{ii}$	7,097	33,280	4,160	17,472	10,662	$\mathbf{0}$	13,647
$x_i - x_j$	$7 - 25$	$8 - 25$	$8 - 9$	$9 - 10$	$10 - 11$	$11 - 26$	$23 - 26$
$c_{ij}$	10,235	9,382	5,971	10,662	6,012	4,509	14,926
$x_i - x_j$	$23 - 24$	$22 - 26$	$21 - 22$	$20 - 21$	$21 - 27$	$17 - 27$	$18 - 27$
$c_{ij}$	6,824	7,676	5,118	$\theta$	6,513	13,372	5,511
$x_i - x_j$	$18 - 19$	$16 - 19$	$12 - 26$	$12 - 14$	$14 - 15$	$12 - 13$	$13 - 25$
$c_{ii}$	3,006	3,507	4,008	4,223	4,992	8,094	6,894

Table 8.4 Coefficients for the segments

$$
NPV_{13} = \left[ -120 \cdot 250 \cdot 0.02 - (120 \cdot 250 - 80 \cdot 120) \cdot \frac{0.05 \cdot (1.05)^{20}}{(1.05)^{20} - 1} \right]
$$

$$
\cdot \frac{1.03^{20} - 1}{0.03 \cdot 1.03^{20}} = 33,280
$$

Table 8.4 shows all the net present values.

#### 8.2.3.3 Optimisation Model

The optimisation problem regarding the heat supply networks can be described as a Steiner tree problem and modelled as a mixed integer program (MIP) (Uhlemair et al. [2010](#page-19-0)). In accordance with Fig. [8.1,](#page-3-0) the biogas plant  $(x_0)$  and all the potential heat customers  $(x_1, \ldots, x_{24})$  are treated as nodes in the heat supply network. Three additional nodes  $(x_{25}, x_{26}, x_{27})$  are introduced as crossroads branch points where network segments from several different directions come together. Their coefficients in the objective function are zero. As mentioned above, the heat supply grid is divided into segments  $y_{ij}$ , which link two adjoining nodes i and j. The following variables are used in the model:

$$
x_i = \begin{cases} 1, if \text{ object } i \text{ is connected to the grid} \\ 0, else \\ |x_0| = number \text{ of nodes connected to the grid} \\ y_{ij} = \begin{cases} 1, if \text{ segment } ij \text{ is installed} \\ 0, else \end{cases} \end{cases}
$$

Accordingly, the objective function can be described as follows:

$$
\left(\sum_{i=1}^{n} c_i x_i + \sum_{i=0}^{n} \sum_{j=0}^{n} c_{ij} y_{ij}\right) \Rightarrow \max \tag{8.1}
$$

The constraints are:

$$
x_0 + \sum_{i=1}^{n} x_i = 0 \tag{8.2}
$$

$$
\sum_{j=0}^{n} f_{ji} - \sum_{j=0}^{n} f_{ij} = x_i \qquad i = 0, \dots, n \qquad (8.3)
$$

$$
n \cdot y_{ij} \ge f_{ij} \qquad i,j=0,\ldots,n \qquad (8.4)
$$

$$
x_0 \le 0 \tag{8.5}
$$

$$
x_i \in \{0, 1\} \qquad i = 1, \dots, n \tag{8.6}
$$

$$
y_{ij} \in \{0, 1\}
$$
  $i, j = 0, ..., n$  (8.7)

$$
f_{ij} \ge 0 \qquad i,j = 0,\ldots,n \qquad (8.8)
$$

The objective function maximises the local heat supply network's overall net present value. It sums up the net present values of all objects  $x_i$  ( $n = 27$  in the village) and network segments  $y_{ij}$ , which, according to the model's result, constitute the network's optimal course.

Constraint (8.3) is equivalent to the flow conservation equation of a network flow problem (Hamacher and Klamroth [2006](#page-19-0)). This constraint ensures that there is a flow to every object  $x_i$  connected to the network  $(x_i = 1)$ . Constraint (8.4) ensures that for every flow between nodes  $i$  and  $j$ , a pipeline segment is built to transport heat from *i* to *j*. The variable  $f_{ii}$  represents this flow. It is not necessary to use the actual heat flow in kWh. Constraint  $(8.2)$  guarantees that constraint  $(8.3)$  can always be fulfilled. Constraint (8.5) requires a negative demand for heat for the production system, i.e. the bioenergy plant is the heat supplier. Initially, it is assumed that enough heat is generated in the bioenergy plant for every possible solution of the heat supply network optimisation model. Constraints  $(8.6)$ ,  $(8.7)$  and  $(8.8)$  are integer and non-negativity constraints.

As Steiner tree problems are NP-hard, it will be difficult to solve the problem for an increasing number of nodes and segments within a reasonable running time.<sup>5</sup>

In the next section, the model is applied to the village. The model can be solved by means of a branch-and-bound-based algorithm.

<sup>&</sup>lt;sup>5</sup> For definitions of complexity and "NP-hard", see Eiselt et al. [\(1987](#page-18-0)) or Garey and Johnson ([1979\)](#page-18-0). Lists of the running times of different types of models can be found in Ahuja et al. ([1989\)](#page-18-0).

$x_i$			3			$\mathbf{o}$		8
		$\theta$	$\theta$					
$x_i$	9	10	11	12	13	14	15	16
$x_i$	17	18	19	20	21	22	23	24
							$\Omega$	$\theta$

**Table 8.5** Optimal values for the household variables  $x_i$ 

Table 8.6 Optimal values for the segment variables y<sub>ii</sub>

$x_i - x_j$	$0 - 1$	$1 - 3$	$2 - 3$	$0 - 4$	$4 - 5$	$5 - 6$	$6 - 7$
		$\theta$	$\theta$				
$x_i - x_j$	$7 - 25$	$8 - 25$	$8 - 9$	$9 - 10$	$10 - 11$	$11 - 26$	$23 - 26$
				$\theta$			$\theta$
$x_i - x_j$	$23 - 24$	$22 - 26$	$21 - 22$	$20 - 21$	$21 - 27$	$17 - 27$	$18 - 27$
	$\theta$						
$x_i - x_j$	$18 - 19$	$16 - 19$	$12 - 26$	$12 - 14$	$14 - 15$	$12 - 13$	$13 - 25$

 $(y_{ij} = 1)$ : pipeline between node  $x_i$  and node  $x_j$  is built;  $y_{ij} = 0$ : pipeline between node  $x_i$  and node  $x_i$  is not built)

#### 8.2.3.4 Optimal Solution of the Model

The software programme Xpress is used to optimise the heat supply grid shown in Fig. [8.1.](#page-3-0) The results are presented in Tables 8.5 and 8.6.

The figures show that almost all the households are part of the optimised heat network. Only four households are not connected. Figure [8.2](#page-11-0) clarifies why households 2, 3, 23 and 24 are not included in the grid. They are situated further afield; fairly long and expensive pipeline segments would therefore need to be installed to connect them to the grid. The revenues from heat sales are not high enough to compensate the costs of the required grid segments.

Dotted lines indicate households and network segments not incorporated into the grid. For instance, the link between nodes  $x_9$  and  $x_{10}$  is not part of the network; network segment  $y_{9:10}$  is unnecessary to supply both households with heat. In terms of the objective function, it is more cost effective to transport heat to household 10 via a pipeline that starts at the biogas plant und turns into the direction of household 13 at branch point 25 and into the direction of household 10 at branch point 26. If segment  $y_{9:10}$  were also installed, a circle  $(25-13-12-26-11-10-9-8-25)$  would be generated in the network. On the basis of graphs theory, the optimal grid is

<span id="page-11-0"></span>

Fig. 8.2 Optimal solution

therefore a so called tree, which does not include circles (Uhlemair et al. [2010\)](#page-19-0). Furthermore, it is clear that segment  $y_{9:10}$  is unnecessary, which will prevent the (negative) payment for building this segment. The net present value of this solution amounts to 134,219 EUR. This is the highest possible value that the objective function can achieve.

### 8.3 Post-optimal Analysis

### 8.3.1 Overview

The solution shown in Fig. 8.2 assumes that all villagers want to be connected to the heat supply grid. It is also assumed that excluding those households whose connection to the grid would be unprofitable (from the operating company's perspective) would not be problematic.

This assumption of a "free optimisation" seems problematic, because in real life (as is the case in this village) some households do not want to receive local bioenergy. However, the calculated net present value of this "free optimum" can be used as a benchmark and the variables' values can be used as a starting point for calculating new solutions when changing the assumptions.

Making statements about the effects of changed premises based on an existing solution is called post-optimal analysis. It enables a decision-maker to avoid recalculating a problem completely when certain parameters or assumptions

<span id="page-12-0"></span>change. We consider two types of a post-optimal analysis here: suboptimal analysis and sensitivity analysis.<sup>6</sup>

Sensitivity analyses examine the extent to which parameters' values can vary without having structural effects on the pre-existing solution. This allows existing uncertainties concerning procurement costs or demand for heat to be considered. Statements can also be made about the critical price threshold that would put the system's profitability at risk. Furthermore, the scope for different connection fee rates can be analysed more closely. Reducing the connection fee could make the usage of locally produced bioenergy more attractive for some villagers.

In contrast, *suboptimal analysis* concentrates on the consequences of deviating from a specific pre-existing optimal solution. Referring to the planning situation specified here, suboptimal analysis seeks to estimate which net present value will apply if, for example, households without a contract for heat are excluded. Furthermore, potential heat customers whose connection to the grid is economically unviable in terms of the target function could then be connected regardless. Suboptimal analysis can show the extent to which the local heat supply grid's course and its net present value could change.

### 8.3.2 Suboptimal Analyses

#### 8.3.2.1 Planning Scenario 1

In planning scenario 1, it is assumed that all households are connected to the grid, irrespective of a heat supply contract. In this case it is not considered that some households do not want to participate in the local heat supply network nor that some objects cannot be connected profitably (for instance, outlying households).

Looking at the model's formulation, a 100 % connection quota can be realised by inserting an appropriate constraint into the original model or by assigning the value 1 to all variables referring to households  $(x_1, \ldots, x_{24})$ .

Although the resulting solution is suboptimal compared to a free optimisation, it is the optimum with respect to the given restrictions. This solution, shown in Fig. [8.3,](#page-13-0) leads to a net present value of 117,055 EUR.

Figure [8.3](#page-13-0) shows that the main network structure is the same as in Fig. [8.2](#page-11-0) (same pipeline course with a break between nodes  $x_9$  and  $x_{10}$ ). Additionally, households 2, 3, 23 and 24 are connected to the grid, and the model selects the corresponding network segments.

<sup>&</sup>lt;sup>6</sup> In the literature, other types are also mentioned, such as the interpretation of the optimum solution or parametric programming; see Dinkelbach [\(1969](#page-18-0)), Eiselt et al. [\(1987](#page-18-0)) or Hillier and Lieberman ([2010\)](#page-19-0).

<span id="page-13-0"></span>

Fig. 8.3 Optimal local heating grid for planning scenario 1

### 8.3.2.2 Planning Scenario 2

In planning scenario 2, only those households with a signed heat supply contract are considered potential heat recipients. Households 13, 14, 18 and 19 do not want to use local bioenergy and therefore did not sign a heat supply contract. Consequently, these households are excluded and a value of 0 (not connected to the grid) is assigned to their binary variables  $(x_{13}, x_{14}, x_{18})$  and  $x_{19}$ ). They are therefore no longer part of the set of (changeable) variables, and the optimisation process concentrates on the remaining households. All the other households have signed a heat supply contract and are therefore treated as potential heat customers. However, potential heat customers are not automatically connected to the heat supply grid. The optimisation model selects profitable heat customers and excludes unprofitable households. Whether a certain household can be profitably integrated into the network is mainly a question of heat demand and the costs of installing the necessary pipeline segments (see the net present values for the households and network segments in Sect. [8.2.3.2\)](#page-4-0).

Figure [8.4](#page-14-0) shows a situation in which only a route from the biogas plant straight through the village to household 17 (supplemented by the branches to object 1 and object 15) will be realised. In contrast to the previous planning scenarios, the link between nodes  $x_9$  and  $x_{10}$  will be built, but the route section  $x_{25}-x_{13}-x_{12}$  seems too expensive and will be omitted. In advance, it had not been certain whether the branch to node  $x_{15}$  would be part of the grid, because household 14 had not signed a contract. However, object 15's heat demand is apparently high enough to make the pipeline from node  $x_{12}$  to node  $x_{15}$  profitable, although household 14 is not provided with heat.

<span id="page-14-0"></span>

Fig. 8.4 Optimal local heating grid for planning scenario 2

The local heat supply grid in Fig. 8.4 leads to a net present value of 81,561 EUR. A comparison of this result with planning scenario 1's net present value is particularly interesting in this case. If all households are connected to the heat supply grid (planning scenario 1), including those that cannot be profitably integrated into the heat supply network, this results in a remarkably higher net present value than when only those with a heat supply contract (planning scenario 2) are considered. This leads to the conclusion that it would be extremely worthwhile convincing indecisive homeowners, or even those households that still prefer not to use local bioenergy, to become part of the group of local heat consumers.

#### 8.3.2.3 Planning Scenario 3

Finally, it is considered that – following the idea of a bioenergy village – all those who have signed a heat supply contract will be offered the opportunity to receive bioenergy from the local heat supply grid. Whether or not this is economically viable in specific cases (from the operating company's perspective) is not taken into consideration. A value of 0 is assigned to the binary variables of households 13, 14, 18 and 19, because they do not want to be connected to the heat supply grid. Since all the other households have signed a heat supply contract, a value of 1 is assigned to their variables. Figure [8.5](#page-15-0) shows the optimised heat supply grid with a net present value of 57,785 euro.

The grid's course is similar to that in planning scenario 2; the two networks differ only in the connection of the (unprofitable) households 2, 3, 23 and 24 in planning scenario 3.

<span id="page-15-0"></span>

Fig. 8.5 Optimal local heating grid for planning scenario 3

Planning solution 3, rather than planning solutions 1 and 2, is expected to be characterised by the lowest net present value, because the values of the variables for all the households are determined prior to the optimisation and only the network's course is left to be optimised. However, it is interesting to compare this planning solution's net present value with that of planning solution 1 (Sect. [8.3.2.1](#page-12-0)). Planning solution 1 does not reflect reality, since several objects (nodes  $x_{13}$ ,  $x_{14}$ ,  $x_{18}$  and  $x_{19}$ ) are part of the heat supply grid although their owners have not signed a contract. Nevertheless, the calculated solution can be used as a benchmark for further analysis. In fact, the difference between the two net present values indicates the financial scope of increasing the households' willingness to be integrated into the local heat supply network (e.g., by reducing the connection fee).

### 8.3.3 Sensitivity Analyses

#### 8.3.3.1 Reducing the Connection Fee

On comparing planning solution 1, in which all the households form part of the network, with planning solution 3, in which a heat supply contract is a precondition for integration into the grid, the respective net present values reveal a large difference (117,055 EUR – 57,785 EUR = 59,270 EUR). Hence, it would be desirable with respect to the idea of the bioenergy village and for economic reasons to convince the remaining villagers to consider signing a contract.

If they are not convinced, efforts should be made to point out the positive impacts of a local heat supply grid and the concept of a bioenergy village.<sup>7</sup> If they have economic reasons for not wanting to be connected to the grid, it is important to have financial incentives (e.g., reducing fees) to convince these villagers to sign a contract.

We now analyse how far the connection fee (for all the villagers)<sup>8</sup> could be cut without overly reducing the operating company's profit. The difference between the net present values in planning scenarios 1 and 3 can be used as a maximum financial margin, because the lower one (57,785 euro, planning scenario 3) is the best that can be achieved if the unwilling owners do not wish to be connected to the grid.

Following this argument, a financial scope of  $(59,270 \text{ EUR}/24 =) 2,470 \text{ EUR}$  per household can be used as an incentive to use locally produced bioenergy. This implies that the connection fee for each household in the village can be decreased by 2,000 EUR. Without a connection fee, planning scenario 1's local heat supply network would lead to a net present value of 69,055 EUR.<sup>9</sup>

#### 8.3.3.2 Variation in the Buying Price

It is assumed that the necessary amount of heat is available at 0.03 euro per kWh. Nevertheless, there may be changes to the price the operating company pays for the heat. Although rising prices are taken into account by using the internal discount rate (which can contain a certain risk surcharge, among others), $10$  their impact on profitability should be analysed separately.

The starting point for the following sensitivity analysis is planning solution 3, in which only those households with a signed contract are part of the heat supply network. Without going into detail – many factors influence the buying price –, it is crucial to identify the critical price above which the system would no longer be profitable.

When profitability is considered, the net present value is again the key figure: If this value becomes negative, the system will be unprofitable. Therefore, there is a financial scope of up to 57,785 EUR (planning solution 3's net present value, based on a buying price of 0.03 EUR per kWh) that can compensate for potential price increases.

The net present value can be divided into those components that vary with the buying price (summand 1) and those components that do not depend on the buying price (summand 2):

 $<sup>7</sup>$ Chap. [10](http://dx.doi.org/10.1007/978-94-007-6642-6_10) deals with bioenergy villages' acceptance.</sup>

<sup>&</sup>lt;sup>8</sup> For reasons of fairness, the other villagers cannot be excluded from the fee reduction.

<sup>9</sup> This analysis does not consider the question of liquidity. As noted, it is assumed that the capital needed to connect the households to the grid is completely self-financed. Decreasing the connection fee may therefore require some external financing.

<sup>&</sup>lt;sup>10</sup> The various functions of the internal discount rate are described by Götze et al.  $(2007)$  $(2007)$ .

net present value  $(NPV)$  = price-dependent components + fixed components

The second summand consists of all the payments for building the grid ( $\Sigma_i \Sigma_j$ )  $NPV_{ii}$ ), the one-time payments for connecting the objects to the grid ( $c + g + cc +$  $\Sigma_i h_i$ ), and the constant annual payments such as the basic fee and the payments for maintenance and support (discounted to the beginning of the planning horizon). The first summand includes the heat demand, the difference between the selling price and the buying price for heat, and the annuity present value factor for discounting the payments (last quotient in the formula below). In detail, it looks as follows:

summand 
$$
1 = \sum_{i \in I^*} w_i \cdot (p_S - p_B) \cdot \frac{(r+1)^T - 1}{r \cdot (r+1)^T}
$$

with:  $I^*$ : set of households with a heat supply contract

When looking at that critical price  $p_{B,crit}$ , which leads to a net present value of 0, the analysis can concentrate on summand 1, because rising buying prices only affect this summand. Consequently, when answering the question of how high the buying price can go without leading to a negative net present value (planning scenario 3), the critical buying price at which the difference between the current value of summand 1 (using a buying price of 0.03 EUR per kWh) and the value of summand 1 using  $p_{B,crit}$  equals the net present value of 57,785 EUR needs to be calculated. $11$ 

The equation below describes these considerations.

$$
NPV = \sum_{i \in I^*} w_i \cdot (p_S - p_B) \cdot \frac{(r+1)^T - 1}{r \cdot (r+1)^T} - \sum_{i \in I^*} w_i \cdot (p_S - p_{B,crit.}) \cdot \frac{(r+1)^T - 1}{r \cdot (r+1)^T}
$$

Filling in planning scenario 3's data, the following equation provides:

$$
57,785 = 423,670 \cdot (0.059 - 0.03) \cdot \frac{1.03^{20} - 1}{0.03 \cdot 1.03^{20}} - 423,670 \cdot (0.059 - p_{B,crit})
$$

$$
\cdot \frac{1.03^{20} - 1}{0.03 \cdot 1.03^{20}}
$$

When solving the equation for  $p_{B,crit}$ , one can see that the critical buying price is 0.0392 EUR per kWh. Thus, based on the initial price, an increase of up to 30 % can be dealt with without descending into unprofitability.

These findings can be used to evaluate the risk of the heat supply network becoming unprofitable if the price that the operating company pays for heat varies. Clearly, there are other uncertainties (especially with regard to the long planning

 $11$  At the same time, this buying price leads to a net present value of 0 if both summands are taken into account.

<span id="page-18-0"></span>horizon of 20 years) that may lead to negative impacts in terms of the heat supply network's profitability. Technical problems in the energy station and in the heat supply network as well as biomass availability problems regarding the energy facility are examples of the insecurities that need to be analysed in future research.

### 8.4 Conclusion

We have developed an optimisation model for a local heat supply network. In terms of the objective function, the best grid course has been identified and the profitable heat recipients have been selected. In addition, optimal solutions were calculated for different scenarios regarding people's willingness to use bioenergy conveyed by a local heat supply network. Further, the consequences of changing parameters (e.g., the price of heat and the connection fee) have been analysed and the breakeven point at which the investment would lose its profitability has been calculated. We will carry out further sensitivity analyses regarding governmental grants and the internal discount rate used in the calculations of the net present values. It may be reasonable to increase the internal discount rate so that the calculation of the system's profitability follows the principle of caution.

In future research, the distribution system will be enhanced by the production system. So far, it has been assumed that enough heat will be available. The production facility was not considered. The next research steps will be to develop a model that simultaneously optimises the distribution system (as described in this chapter) and the bioenergy facilities' capacities and configuration. The combined heat and power biogas plant will be supplemented by further heating stations and a peak load boiler to ensure heat supply over the coldest days of the year. When integrating the production system into the optimisation model, important factors such as sustainable energy crop cultivation, the impact on biodiversity when using biomass as a renewable energy source, general biomass availability and the special logistics issues associated with biomass usage should also be considered.

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