Recent Development of Non-Involute Cylindrical Gears

Gorazd Hlebanja and Jože Hlebanja

Abstract The paper represents an overview over the development of gears aiming to improve contact circumstances during meshing. It is well known fact that classical involute gears form a convex–convex contact during meshing and that the curvature radius value is very low in the vicinity of the basic circle. Corresponding research was focused in development of gears with convex-concave contact. Therefore, a gradual development of gears with a curved path of contact is presented below. Furthermore, attempts to extend these concepts to various gear types and applications are also presented in the paper.

Keywords Gears · Power transmission · Non-involute gears · Curved path of contact · S-gears

1 Introduction

Gears are necessary in accurate power and rotation transmission from a motor (power supply) to a work machine. Uncountable work machines have been invented; however, they govern numerous processes with various rotational speeds. Also numerous power machines have been developed with ever increasing speed. Increasing input speed and variety of output speeds of pairs power-work machines imply more sophisticated geared transmissions each new generation.

Requirements, to which power-transmissions should comply and which imply their functionality, are becoming higher every day, so wide spread scientific

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research is necessary and resulting progress notable. In this respect are many research institutions, which have human resources and specialized equipment, also financially funded by manufacturers and sometimes even by governmental sources.

We should also point out some basic development and economic trends:

- working power of energy supplying machines and demands for working machines are increasing;
- rotational speed and variety of speeds of such machine pairs are increasing;
- capacity of working machines and automation production are higher;
- power flow is increasing;
- higher loads and more demanding working circumstances of such gears;
- gears endurance for the entire life cycle is becoming a law;
- requirements towards lower vibrations and less noise and uniform running are becoming higher;
- production capacities all over the world are high, prices of gear transmissions are lowering.

All above directs to the search of new ways to improve competitive abilities of gear manufacturers.

2 Basic Principles

The basic principles of contemporary gearings were discovered in 1733 by Charles-Étienne Louis Camus (1699–1768), a mathematician and professor from Paris (Complete Dictionary of Scientific Biography, "Camus, Charles-Étienne-Louis," 2008) and independently by famous Leonhard Euler in 1752. Camus defined the conditions that have to be fulfilled for a pair of gears in his work "Sur la figure des dents des roués et des ailes des pignons pour rendre les horologes plus parfaits". This condition is defined by the following: if, in uniform rotation, power is to be transmitted via a pair of teeth, then the normal to the teeth flanks at the contact point P on the path of contact must pass through the pitch point C (Matschoß 1940), as illustrated in Fig. 1, which is exactly the law of gearing known today.

Leonhard Euler (1707 Basel–1783 St. Petersburg) sought the most advantageous shape of gear teeth flanks, and according to Jacobi, (The Euler Archive 2012) in 1752 he was the first to publish a treatise on the usefulness of the involute for the shape of gear teeth flanks (Euler 1760). His concept of involute gearing is in common use today. He also showed how to graphically determine the radii of curvature as represented in Fig. 2.

Cycloids, involutes and rolling curves can be used to form gear teeth according to the law of gearing.

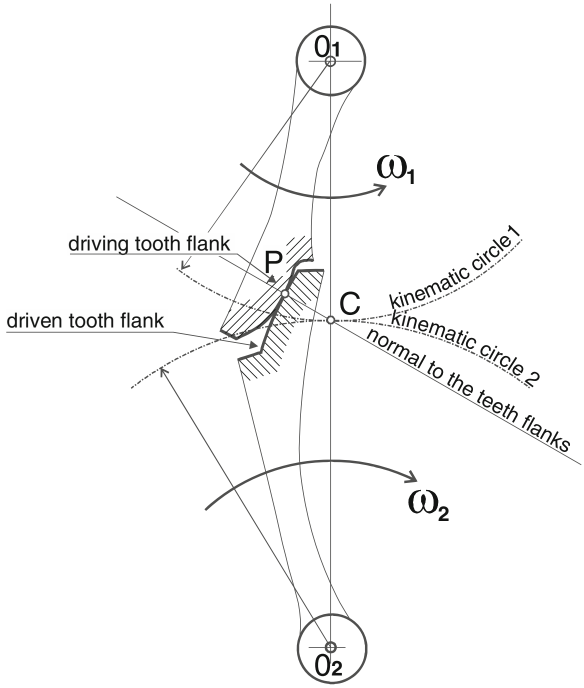


Fig. 1 The law of gearing

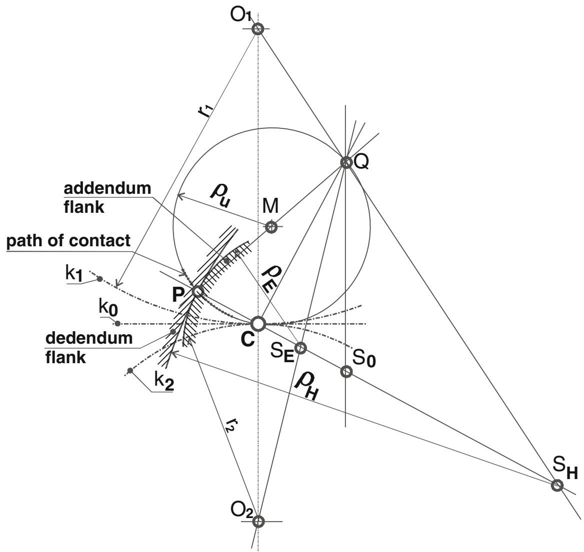


Fig. 2 Graphical determination of the radius of curvature after Euler-Savary

3 Curved Path of Contact

Gradual development evolved in the perfect, optimal shape of involute gears which transmit power by the convex–convex contact. However, the intrinsic property of the involute gear is their curvature radius function in the dedendum part when approaching the base circle. Values in general are small and limit to zero at the base circle and therefore imply high contact loads in this area. Additionally, for gears with low number of teeth the dedendum flank is comparatively short thus invoking excessive sliding and friction losses. Yet another problem is undercutting of the dedendum area. This was why numerous gear developers sought new solutions to make the teeth flanks of the driving and the driven gears fit together better. In view of this requirement, the concave-convex pair seems an obvious solution, which was precisely what researchers and inventors suggested. However, this paper is focused on gradual development of S-gear shape.

The aim was to define tooth flanks of adequate characteristics. This was achieved by the curved path of contact which implies a concave-convex fit of the meshing gear teeth flanks. The path of contact is a sequence of contact points of the meshing gear pair, which transmits rotation, and each contact point should comply with the law of gearing at the same time. The path of contact should warrant sufficiently high contact ratio. The contact load in the kinematic pole C depends on the pressure angle α_C. The starting pressure angle α_A should be also limited. The sector defined by points A, C, 1 (Fig. 3) represents the zone of possible paths of contact. The condition for manufacturing gears of the same module with arbitrary number of teeth by the same tool profile is the half-symmetrical path of contact.

Gears are designed according to their root strength and flank durability. The path of contact shape and the root fillet influence the root thickness. However, the flank shape essentially influences its durability. The most important factors influencing flank durability are the reduced radius of curvature and sliding. The higher radii imply lower Hertzian pressure. The sliding circumstances are essentially improved in the case of convex-concave contact. The research showed that

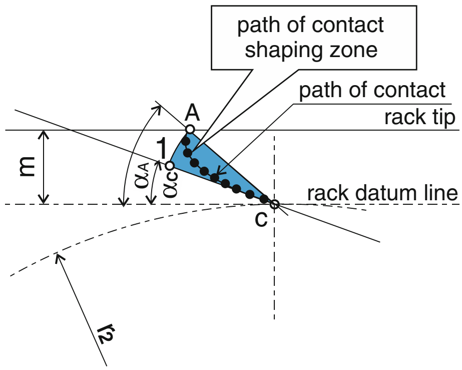


Fig. 3 Path of contact shaping zone (Hlebanja 1976)

areas of the path of contact with a higher curvature imply lower sliding and higher reduced radii of curvature. Due to necessity of the stronger oil film in the meshing start zone the path of contact curvature in that area should be higher and the path of contact gets a distinctive S-form.

This tooth flank form was used in grooved roller gear for rolling mills. It was installed in Sisak rolling mill. It transmitted 1500 kW at 80 up to 160 RPM. Material was alloy steel 30CrMoV9. The tooth profile is illustrated in Fig. 4. Initially the involute gearing was installed, which suffered severe scuffing in the gear teeth dedendum and addendum areas. S-formed gearing was an essential improvement. This gear form was also reported in Niemann and Winter (1989).

4 S-Shaped Path of Contact

Subsequent research was oriented to define the S-gear path of contact in such a way to ensure smaller flank pressure, better lubrication and less sliding and which would be linear or almost linear in the vicinity of the kinematic pole C. Figure 5

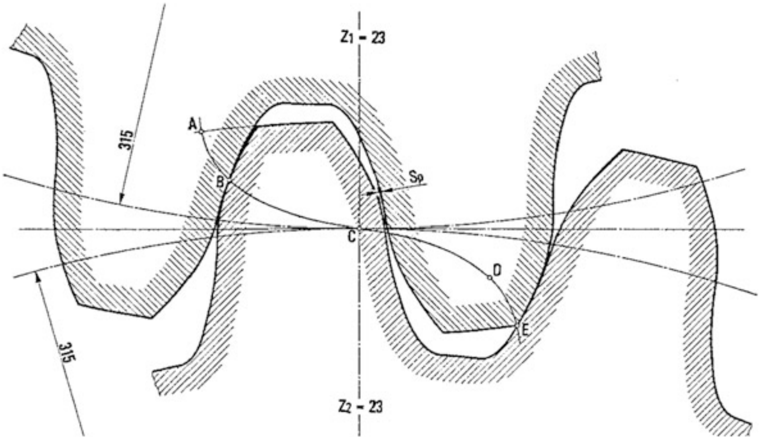
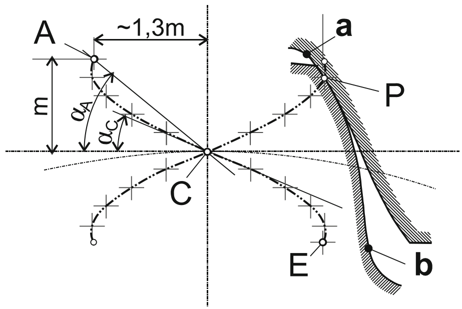


Fig. 4 Industrial implementation (Hlebanja 1976)

Fig. 5 Path of contact with gradually increasing curvature



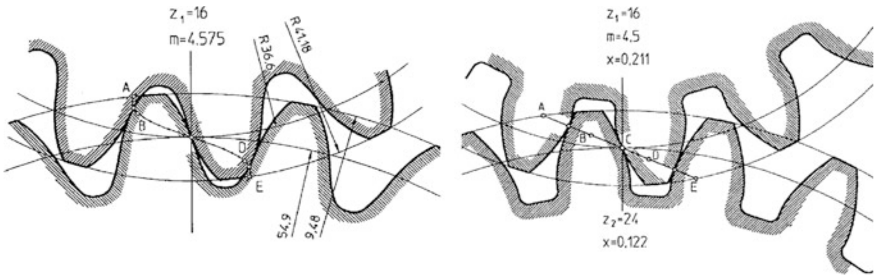


Fig. 6 Gear pairs for comparison: (*left*) S-gears, (*right*) E-gears

represents such a path of contact, with the basic characteristic of gradual increase of curvature towards the meshing start and end and corresponding gear and pinion flanks.

Furthermore, the research was conducted in order to compare S- and E-gears. First, typical characteristics have been calculated for both gear types and later experiments on the FZG-machine were carried out. Both gear pairs are illustrated in Fig. 6.

Therefore, S- and E- gears are to be compared regarding damages, which might appear during operation: (a) fracture due to overloading; (b) damages of tooth flanks due to pitting; (c) cold and hot scuffing; and (d) wear of tooth flanks.

Regarding endurance against tooth fracture, the measure of such endurance is the tooth root stress, which further depends on the tooth root thickness and fillet radius. It was demonstrated in this particular case that the S-gear teeth are stronger in the root, as well as their fillet radius is larger, which also imply that S-type teeth are stronger for approximately 20 %.

The contact load of teeth flanks is evaluated by Hertzian pressure σ_{H}. For cylindrical surfaces we have:

$$\sigma_H = \sqrt{\frac{F \cdot E}{2\pi(1-\nu^2)\, l} \cdot \frac{1}{\rho_{red}}} = 0,418\sqrt{w \cdot E'} \cdot \sqrt{\frac{1}{\rho_{red}}} \tag{1}$$

However, Hertzian pressure may not exceed maximal allowable limit σ_{Hdop}. The contact force F, the tooth width l and the reduced radius of curvature ρ_{red} are those design parameters which decisively influence gear durability. Calculated values for σ_{H} are represented for both, i.e. S- and E-gears on Fig. 7. It should be pointed out that the Hertzian pressure and pressure angle α_{C} in the kinematic pole C are equal for both gear types.

The measure of endurance against damages of tooth flanks is oil film thickness h_0, which is defined by the Dowson-Higginson's (1977) equation:

$$h_0 = 1,6 \cdot \alpha^{0,6} \cdot \eta_0^{0,7} \cdot E^{0,03} \cdot w^{-0,13} \cdot \rho_{red}^{0,43} \cdot u^{0,7} \tag{2}$$

Factors in the above Eq. (2) are: α—the pressure coefficient of oil; η_0—dynamic viscosity; E—module of elasticity; w—line load of teeth, i.e. F/l; ρ_{red}—reduced

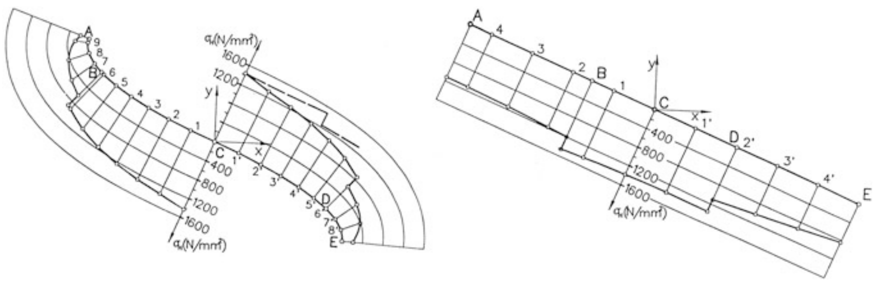


Fig. 7 Hertzian pressure (*left*) S-gears, (*right*) E-gears

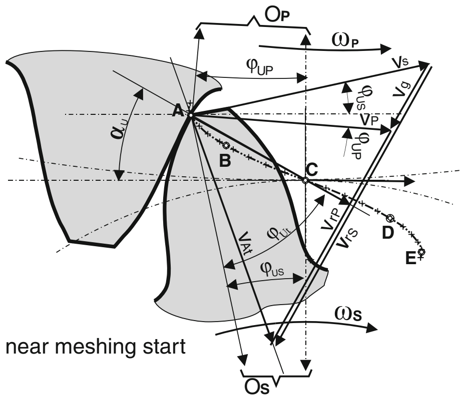
radius of curvature; and u—sum of relative velocities. Having equal loads of S- and E-gears, equal lubrication means, the same rotational speeds and equal gear materials, the only remaining parameters influencing the oil film thickness are the reduced radius of curvature ρ_{red} and the sum of relative velocities u. Both have better characteristics for S-gears (i.e. higher relative velocities, higher reduced radii of curvature) and therefore imply better lubrication and thicker oil film. Figure 8 represents velocity circumstances in the vicinity of the meshing start.

Another important issue is a temperature characteristic of a meshing gear pair. Blok's temperature concept (Blok 1963), represented in Eq. 3 is adopted here.

$$\vartheta_{fl} = 0,62\mu\, w^{0,75}\left(\frac{E\prime}{\rho_{red}}\right)^{0,25}\frac{|v_{rS} - v_{rP}|}{\sqrt{B_{MS}v_{rS}} + \sqrt{B_{MP}v_{rP}}} \tag{3}$$

Symbols in the equation are: ϑ_{fl}—"flash" temperature; μ—friction coefficient; w—contact load (line load); ρ_{red}—reduced radii of curvature; v_{rS}, v_{rP}—relative velocities of the flanks in contact; B_{MS}, B_{MP}—property of material; $B_M = \lambda\ \rho\ c$, where λ stands for thermal conductivity, ρ for material density and c for specific heat. The friction coefficient is given by (Niemann and Winter 1989)

Fig. 8 Velocities in the vicinity of the meshing start $u = v_{rP} + v_{rS}$



$$\mu = 0,12 \cdot \left(\frac{w \cdot R_a}{\eta \cdot u \cdot \rho_{red}} \right)^{0,25} \tag{4}$$

where additional factors are u—sum of the relative velocities and R_a—average surface roughness. Figure 9 illustrates behaviour of the flash temperature along the path of contact.

5 Experiments

Experiments have been conducted in order to examine endurance against:

- Pitting of tooth flanks
- Scuffing of tooth flanks
- Surface heating, wear and efficiency.

Experimental work has been carried out on a FZG testing machine, so values for the module and the centre distance were adapted accordingly. The selected material was alloy steel 42CrMo4, which was heat treated to 28–30 HRC prior to toothing. Experimental lots comprised (a) hardened and tempered gears "V" and (b) hardened, tempered and plasma nitrided gears "NV". Technical data of gears are collected in Table 1.

Experimental results with regard to pitting are summarised in Woehler diagrams, Fig. 10. Results indicated that the load capacity for hardened and tempered S-gears was slightly increased, whereas for hardened, tempered and nitrided S-gears the load capacity was bigger as that for E-gears. However, more experiments would have been necessary.

Hardened, tempered and nitrided S- and E-gears have been tested with regard to scuffing. Experiments were conducted according to the load levels prescribed by DIN 51354. E-gears exhibited scuffing damages of the addendum part of the flanks already at the load level 11, whereas S- gears did not have damages even at the level 12. Oil ISO VG 100 without additives was used in both cases. Lubrication

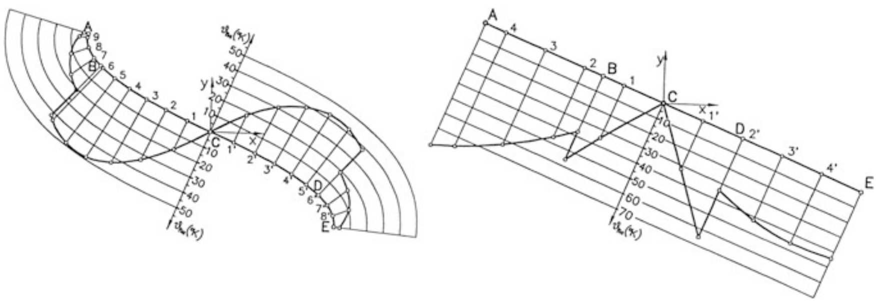


Fig. 9 Flash temperatures along the path of contact (*left*) S-gears, (*right*) E-gears

Table 1 Data of the gears used in experiments

	Designation	S-gears		E-gears	
Module	m_n [mm]	4,575		4,5	
Number of teeth	z	16	24	16	24
Coefficient of the profile shift	x	–	–	+0,233	+0,12
Kinematic circle diameter	d_w[mm]	73,2	109,8	73,2	109,8
Pressure angle in C	α_C [^{0}]	22		22,438	
Face width	b [mm]	20		20	
Centre distance	a [mm]	91,5		91,5	
Rotational speed	n [$\min^{-1}$]	2100	1400	2100	1400

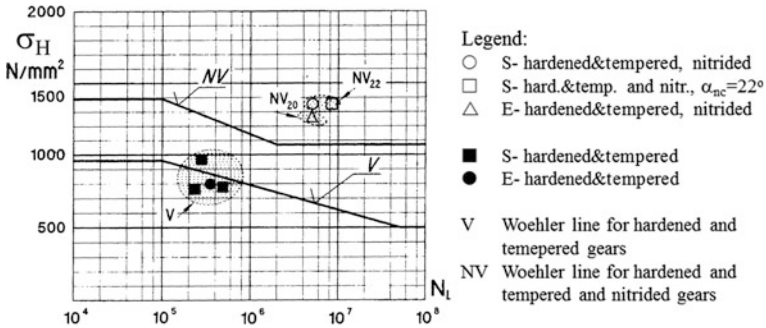


Fig. 10 Woehler diagrams of the pitting experiments

procedure was with gears dipped into the oil. The oil temperature in the casing was 90 °C in both cases. Each loading level lasted for 15 min. The oil temperature in the casing during this time increased due to friction losses of the operating gears. Measurement of temperatures clearly indicated that oil in the casing was less heated in the case of S-gears, which implies that losses in this case were lower. The difference in the temperature increase between S- and E-gear type was approximately 10 %.

Wear of gear tooth flanks was determined during the pitting tests by weighting gear pairs in a precision balance. Weighting was conducted before and after experiments for both gear types. The experiment on the FZG loading level 9 lasted 4×10^6 rotations of the pinion. Results are summarised in Table 2. It can be concluded that the wear of S-gears is approximately half of that in E-gears, which can be attributed to the ticker oil film. More information can be found in Hlebanja and Okorn 1999.

Design work with the S-type path of contact, which was first used for spur gears, continued in the direction of other gear types in order to explore their usability (Hlebanja and Hlebanja 2005):

Table 2 Wear measured by a balance

Gear type	Load level according to DIN 51354	Wear [mg]		
		Pinion	Gear	Sum
S-	9	17	16	33
S-	9	15	8	23
E-	9	41	21	62

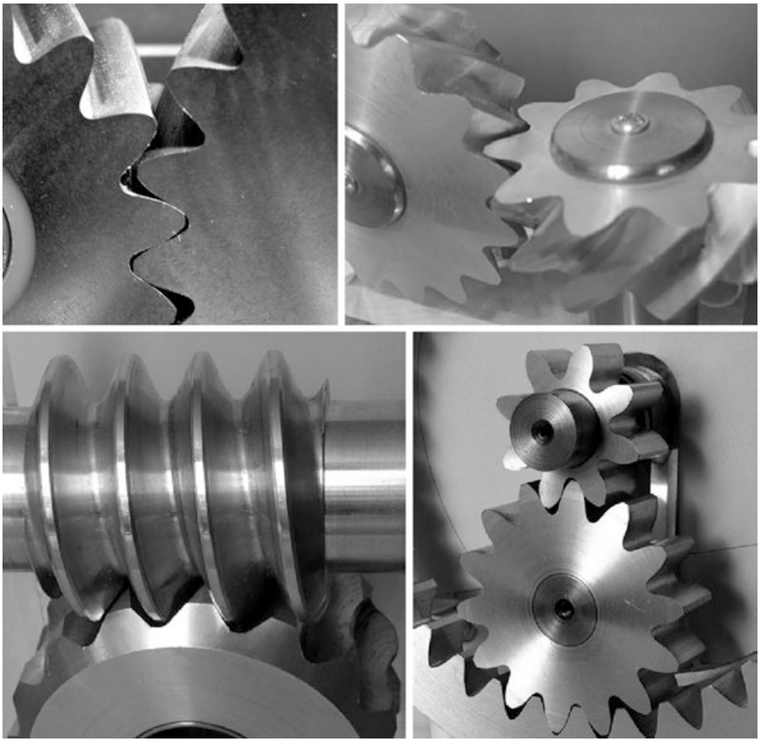


Fig. 11 Working models for various S-type gearings **a** helical; **b** crossed helical; **c** ZS worm drive;**d** planetary arrangement

- Helical gears
- Crossed helical gears
- ZS worm drives
- Internal S-gears for planetary gear trains.

Working prototypes or models were produced for the each type, as revealed from Fig. 11.

6 Recent Development

Another proposal are cylindrical worm-gearings with a progressively curved shape of teeth flanks, the so called parabolic worm gear drives (Hlebanja et al. 2009), Fig. 12. The proposed approach in cylindrical worm gearings design is based on the mathematically defined worm tooth profile in the worm axial plane, wherefrom the worm gear tooth profile derives and profiles in any parallel plane can be calculated and in this way teeth flanks are defined. The primary feature of the proposed teeth flanks is their progressive curvature and continual concave-convex contact. Worm and worm gear meshing of such an arrangement generates better lubricating oil film, resulting in better EHD lubrication conditions, therefore reduced energy losses and lower wear damages are anticipated. An experimental worm-gearing loaded under working conditions verified theoretical considerations. Computer simulation also confirmed contact theory of the proposed gearing.

G. and J. Hlebanja (2009) proposed a new version of WN gears, UPT (uniform power transmission) gears illustrated in Fig. 13. According to this proposal, the tooth flank profile is comprised of three circular arcs, with the first arc forming

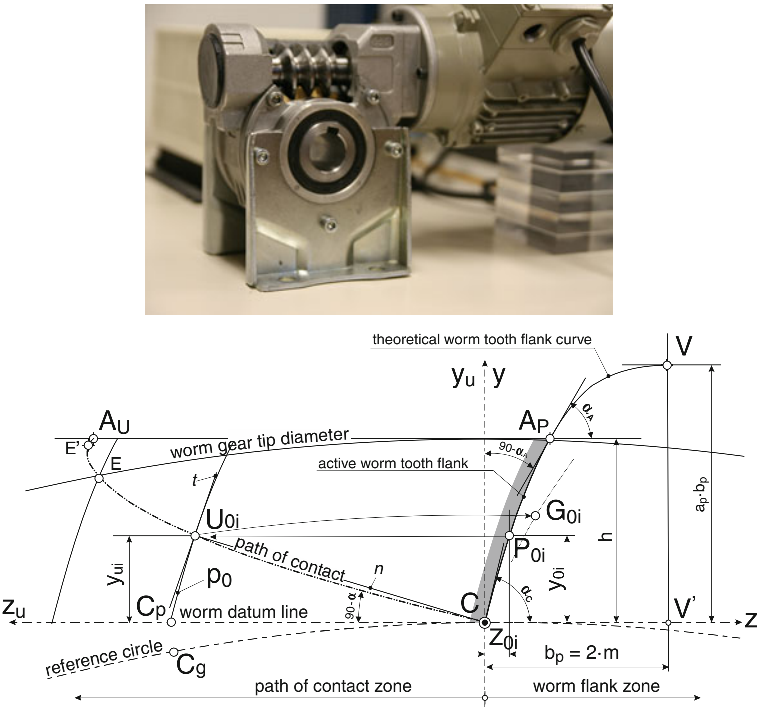


Fig. 12 Parabolic worm-gearing; industrial housing after an observation window had been cut off (*above*); worm tooth basic profile and the corresponding path of contact in the axial section (*below*)

the addendum; the second forming the dedendum; the third arc forming the intermediate section, which prevents the flanks from touching as they rotate around the kinematic pole C. The advantage of this solution is a simple flank geometry, which is easier in terms of tools, while the relative rotation of one gear vis-à-vis its pair is similar to the movement of the shaft in the bearing. The essential element of the UPT gears is the absence of a pitch line and the gear-teeth contact in the transverse plane. In addition, there is no sliding between the teeth flanks in the transverse plane. Power is mainly transmitted by the rolling of the teeth flanks at both contact points, with the simultaneous sliding of the teeth flanks around the pitch point C. The contact load is divided into two contact points. Better lubrication conditions can be expected as a result of the thicker oil-film thickness and lower heat generation. And the most important features of the UPT gears are non-intermittent sliding and power transmission. These features indicate that UPT gears can be used with heavy loads in non-stop operating condition, for example, in the power transmission of wind turbines, gear units for refinery services, and similar applications.

Hlebanja and Hlebanja (2010); Hlebanja (2011) developed S-gear concept from the S-shaped path of contact to the S-shaped rack profile. This method of designing gears based on the definition of the basic rack tooth profile. It is defined by a mathematical function and according to the fundamental law of gearing for each point P_i of the base rack profile a single point U_i on the path of contact can be assigned and based on that a unique point G_i on the gear tooth flank. The basic rack profile is defined as an analytic curve, Eq. (5).

$$y_{Pi} = a_p(1 - (1 - x_{Pi})^n) \tag{5}$$

In this way we have a clearly defined cutting tool and arbitrary gears defined through this rack profile by proper coordinate transformations. The size factor a_p and the exponent n can also be optimised in order to achieve required results, e.g. ticker root, or the size of convex-concave area, as it can be seen from Fig. 14. The

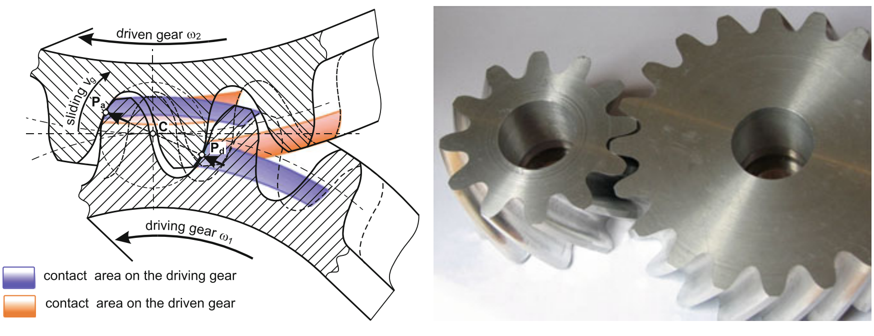


Fig. 13 UPT gears proposed by Hlebanja; contact areas (*left*); UPT gear pair with $m_n = 5$ mm; $z_1 = 12$, $z_2 = 20$

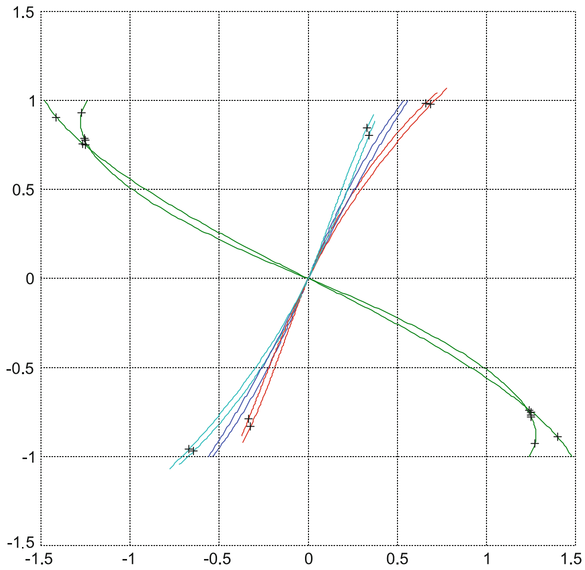


Fig. 14 Influences of parameter changes on gear and path of contact shapes

diagram shows how the change of vital parameters influences the path of contact, gear and pinion tooth flank shape and the rack profile.

The crucial characteristics of the S-gear concept are summarised in the following paragraph. The mating gears exhibit convex-concave contact in the vicinity of the contact start and contact end. The minimal teeth number of S-gears can be as low as four. The S-gear tooth flank profile assures higher comparative curvature radii, and thus lower contact load and higher relative velocities of the contact surfaces which imply better lubrication. Due to their S-shape, the velocity characteristics of mating gears are improved, especially in both external areas with high relative velocities and low sliding velocity. The meshing start zone in involute gears represents potential danger of micro-pitting, whereas S-gears exhibit advantage in this context due to the ticker oil film in this area, which diminishes possibility of damage. Another important feature of the S-gears is more evenly distributed contact point density, which causes lower sliding and less power losses. The dedendum flank of pinion is not substantially smaller as that of gear addendum even for low number of teeth.

7 Conclusion

Let us summarise this paper with some advantages of the S-gears and dissimilarities from the E-gears.

The list of improvements over the E-gears starts with a higher root strength of the S-gears due to thicker teeth in the root and bigger fillet radius. A higher strength of flanks can be attributed to the S gears, which is due to the convex-concave contact, implying lower Hertzian stress. Thicker oil film implies less wear, lower losses and better efficiency. Due to convex-concave contact and high relative velocities in the meshing start area the thicker oil film forms there, preventing scuffing and makes tip relief in S-gears unnecessary. For many common applications nitriding may be sufficient procedure, which makes special heat treatment and subsequent grinding unnecessary. S-gear gaps are thinner, which implies less cut material. The contact point density of the S-gears is more evenly distributed, which causes lower sliding and less power losses.

However, the acting force varies in accordance with pressure angle deviations along the curved path of contact, which is dissimilar to the involute case. The centre distance depends on the number of teeth and the module. The profile corrections are limited in comparison to the E-gears. And due to the curved path of contact there is some sensitivity to centre distance deviations. However, quite many industrial applications incorporating gears with the curved path of contact ran successfully for many years. An attempt has been made to construct the path of contact with a small linear part in the vicinity of the kinematic pole C. And even for the path of contact without such a linear segment it can be observed, that its mid-part near C is almost linear.

We can also state that the S-gears are new, without routine production experience and standards and with little practical experience. However, already mentioned advantages make authors believe that this gear type can be a successful substitute for involute gears for diverse applications, like gear-boxes for wind power-plants on the large scale and miniature plastic gears for domestic appliances.

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