New Algorithm for Variable Speed Gear Generation Process

Marius Vasie, Domenico Mundo and Laurenţia Andrei

Abstract The paper is focused on the generation process of a particular type of noncircular gears, i.e. the variable speed gears. A curiosity in the old gear industry, the noncircular gears have become a challenge and an opportunity for the computer graphics and technology. In the attempt of generalizing the gear generation process, the Gielis supershape is proposed for the gear pitch curve. Limited to proper variation ranges, the supershape defining parameters generate symmetric-asymmetric, convex-concave, close-open pitch curves. Based on the rolling method, using either a rack cutter or a shaper, the further gear generation is simulated in AutoCAD environment, importing Matlab data. Original algorithms are created to configure the main stages of the variable speed gear generation process: (1) the pitch curve modeling and analysis, (2) the selection of the proper cutting tool and kinematics, (3) the simulation of the tooth generation.

Keywords Noncircular gear · Variable speed gear · Noncircular pitch curve · Supershape · Gear generation

List of Symbols

r_i, θ_i	Polar coordinates for the driving ($i = 1$) and driven ($i = 2$) gears
m_{12}	Gear instantaneous transmission ratio
D	Center distance
ω_{ij}	Angular speed for gear blank ($ij = rr$) or shaper ($ij = cr$)

M. Vasie (✉) · L. Andrei
University Dunarea de Jos of Galati, Galati, Romania
e-mail: v_marius_gl@yahoo.com

L. Andrei
e-mail: landrei@ugal.ro

D. Mundo
University of Calabria, Cosenza, Italy
e-mail: d.mundo@unical.it

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v_{ij}	Velocity of translational motion for gear blank ($ij = rr$) or rack cutter ($ij = ct$)
x_{rt}, y_{rt}, y_{ct}	Displacements of gear blank (rt) and rack cutter (ct)
γ_r, γ_c	Rotational angles of gear blank (r) and shaper (c)
m	Gear modulus
z_i	Number of teeth for the driving ($i = 1$) and driven ($i = 2$) gears

1 Introduction

Noncircular gears are a special category of geared transmissions that can be used in those applications that require a variable output motion. Noncircular gears are similar to cams, chains and sprockets or linkages, but from other points of view, they are better (compact sized, there is no gross separation or decoupling between elements, possibility of rotation in both directions, large variety of shapes) as mentioned by Dooner and Seireg (1995).

The first manufacture attempts were made with tools that used master noncircular racks, methods that proved to be complex and inefficient. After the enveloping method was proposed by Litvin (1994), noncircular gears could be manufactured like any other gear, performing a pure rolling of the tool's centrode over the gear's centrode. In the design of noncircular gears, some scientists focused on the synthesis of the centrodes, some concentrated on profile generation, while others found various applications for this type of gears. Yang and Tong (1998) generated identical noncircular centrodes, using a monotonically increasing function for the driving pitch curve. In order to generate teeth for a planetary gear train, Mundo (2006) used a numerical approach, by integrating a differential equation describing the contact point displacement along the line of action, during meshing. Instead of deducing and solving complicated meshing equations specific to enveloping method, Li et al. (2007) proposed a method for generating noncircular gears' teeth based on the real shaping process. Jing (2009) introduced a pressure angle function to characterize the mathematical model for the geometry and geometric characteristics of noncircular gears' tooth profiles.

The authors develop a general procedure for the variable speed gear generation, based on the simulation of the gear cutting process by rolling method. The Gielis supershape (Gielis et al. 2003) is used for the modeling of gear pitch curve and requires optimal selection of its six defining parameters, in order to lead to potential gear pitch curves. Once the pitch curve is modeled, the generation of the gear teeth follows the traditional algorithm of the noncircular gear cutting process, simulating the rolling between the gear blank and the selected tool, i.e. a rack cutter or a shaper, for convex or convex-concave pitch curves, respectively.

2 Gear Pitch Curve Modeling Process

The noncircular gear generation procedure starts with the design of conjugate pitch curves that perform assigned transmission function or correspond to predesigned geometries. The modeling of mating pitch curves manipulates two essential data, i.e. the constant center distance and the pure rolling motion. As regards to the hypothesis of desired noncircular driving pitch curve, several designs have been reported: elliptical shapes (Litvin and Fuentes 2004; Bair et al. 2009), N-lobe and polinomial smooth profiles (Yang and Tong 1998; Figliolini and Angeles 2005), deformed limacon shapes (Dawei and Tingzhi 2011) and curves aproximated by Fourier series (Tsay and Fong 2005) or generated with monotonically increasing functions.

The authors also developed the modeling process of mating pitch curves, starting from a desired driving pitch curve. The significant steps considered for the modeling process are as follows (Fig. 1):

Step 1. Driving pitch curve modeling and analysis. In the attempt of generalizing the noncircular gear design, the authors propose the Gielis' superformula (Andrei and Vasie 2010) for the driving noncircular pitch curve polar expression:

$$r_1(\theta_1) = \left[\left|\frac{1}{a}\cdot\cos\frac{n\theta_1}{4}\right|^{n_2} + \left|\frac{1}{b}\cdot\sin\frac{n\theta_1}{4}\right|^{n_3}\right]^{-\frac{1}{n_1}} \tag{1}$$

where a, b are the semilengths of the major axes, n–a real number that determines the number of lobes of the shape; n_1, n_2 and n_3–real numbers that leads to pinched, bloated or polygonal, symmetric or asymetric forms.

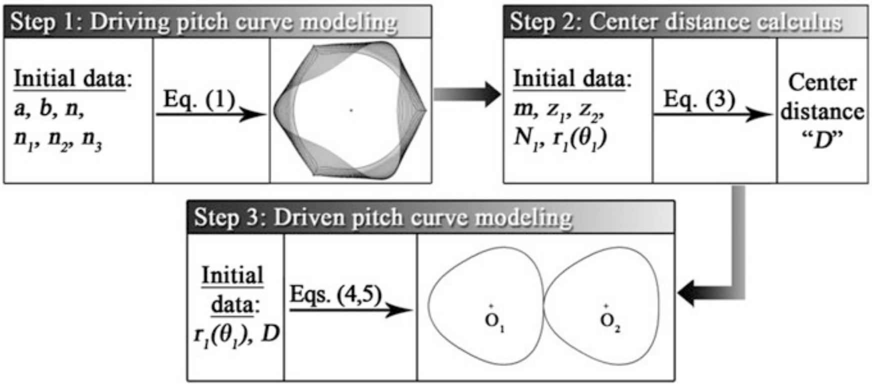

Fig. 1 Algorithm of mating pitch curves design

Analyzing the supershape family, a proper selection for the defining parameters is imposed in order to generate adequate pitch curves that would not lead to further undercutting.

Step 2. Determination of constant center distance. Considering the general case, in which the driving pitch curve performs N_1 revolutions for one revolution of the driven pitch curve, in order to get a closed form curve for the driven centrode, the following condition must be considered (Litvin 1994):

$$\frac{2\pi}{N_1} = \int_0^{2\pi} \frac{1}{m_{12}(\theta_1)} d\theta_1 = \int_0^{2\pi} \frac{r_1(\theta_1)}{r_2(\theta_2)} d\theta_1 = \int_0^{2\pi} \frac{r_1(\theta_1)}{D - r_1(\theta_1)} d\theta_1 \tag{2}$$

Equation (2) enables the center distance D to be determined through numerical integration and iterative procedure. The accuracy for the above calculus is considered as:

$$\Delta = \frac{2\pi}{N_1} - \int_0^{2\pi} \frac{r_1(\theta_1)}{D - r_1(\theta_1)} d\theta_1 \le 10^{-6} \tag{3}$$

Step 3. Driven pitch curve modeling. Based on the driven pitch curve and the constant center distance, the driven pitch curve is represented by:

$$r_2(\theta_2(\theta_1)) = D - r_1(\theta_1) \tag{4}$$

$$\theta_2(\theta_1) = \int_0^{\theta_1} \frac{1}{m_{12}} d\theta_1 = \int_0^{\theta_1} \frac{r_1(\theta_1)}{D - r_1(\theta_1)} d\theta_1 \tag{5}$$

Using Gielis' superformula for the pitch curve definition, six defining parameters are influencing the shapes of the mating pitch curves. A brief analysis on the influence the supershape parameters have on the pitch curve geometry and gear ratio is presented in Figs. 2, 3, and 4. The pitch curves are scaled in order to correspond to a gear with modulus 2 mm and number of teeth 36. It can be noticed that:

- as the number of lobes (n) is increased (Fig. 2), the pitch curves change from eccentric circular shapes ($n = 1$) to elliptical ($n = 2$) and rounded polygonal shapes ($n = 3$), while the gear ratio keeps constantly its amplitude but gets periodical variation;
- as the exponent n_1 is increased (Fig. 3), the shape changes from convex–concave to convex shapes and the amplitude of the gear ratio is reduced; an increase of 500 % of the n_1 value leads to a reduction of 88 % of the gear ratio amplitude;

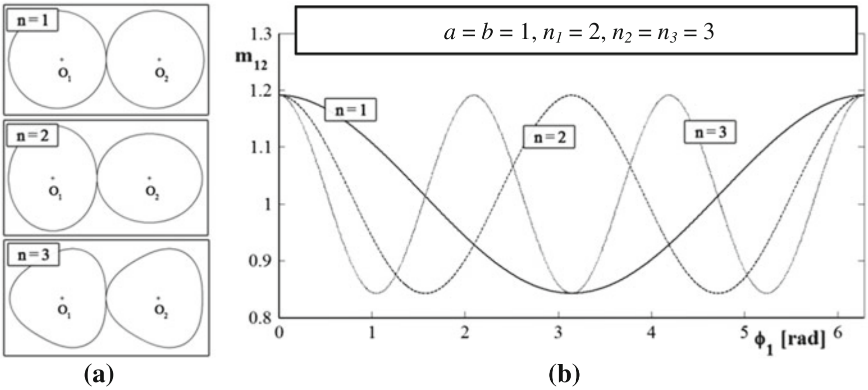

Fig. 2 Influence of parameter n on pitch curve geometry (**a**) and on gear ratio (**b**)

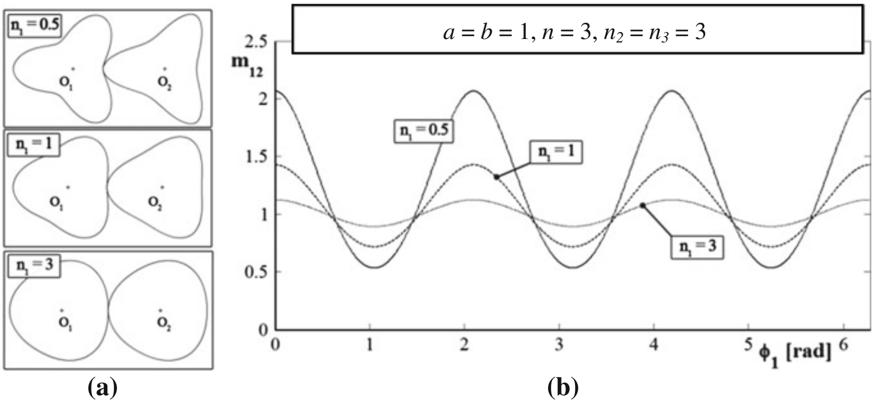

Fig. 3 Influence of parameter n_1 on pitch curve geometry (**a**) and on gear ratio (**b**)

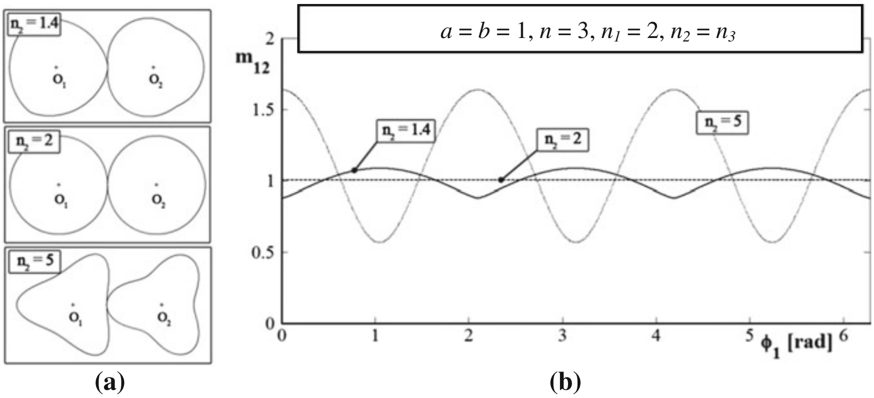

Fig. 4 Influence of parameter n_2 on pitch curve geometry (**a**) and on gear ratio (**b**)

- as the exponent n_2 is increased (Fig. 4), the shape is changed from convex-concave ($n_2 < 2$) to circular shapes ($n_2 = 2$) and back to convex–concave shapes ($n_2 > 2$). The gear ratio variation is slightly reduced as the pitch curves get close to circular shapes (no variation for m_{12}, as known) and increased by high values of n_2;

If one of the major axis length is increased, the elliptical shapes are generated and an increase of their eccentricity is reported.

3 Generation of Variable Speed Gear

The noncircular gears are usually manufactured by a rack cutter, a shaper, a hob, or by wire EDM. Considering the tooth generation by rolling method, the rack cutter and the shaper are the most common tools dedicated to convex and convex-concave pitch curves, respectively. As the authors are using Gielis' supershape for the driving pitch curve, the choice of the cutting tools and kinematics is directly influenced by the supershape defining parameters. The simulation of the gear generation is developed in AutoCAD environment, using imported Matlab data.

3.1 Generation by a Rack Cutter

To enable noncircular gear generation by a rack cutter, an analysis on pitch curve convexity is firstly developed. Therefore, the selection of the supershape defining parameter is focused on the variation/sign of pitch curvature radius, calculated by:

$$\rho(\theta) = \frac{\left[r^2(\theta) + \left(\frac{dr(\theta)}{d\theta}\right)^2\right]^{\frac{3}{2}}}{r^2(\theta) + 2\left(\frac{dr(\theta)}{d\theta}\right)^2 - r(\theta)\frac{d^2r(\theta)}{d\theta^2}} \tag{6}$$

As long as $\rho > 0$, the convexity of the pitch curve recommends the gear generation by a rack cutter. The simulation of the gear generation is based on the following coordinate systems (Fig. 5):

- a fixed system, $O_fX_fY_f$, with the origin O_f in the initial position of the contact point, T_0, and with the Y_f axis along the common tangent, and
- two movable coordinate systems, $O_1X_1Y_1$ and $O_cX_cY_c$ rigidly attached to the noncircular gear blank and to the rack cutter, respectively.

The rack cutter, defined by a standard geometry, is translated with velocity v_{ct}, along the common tangent (t) to the centrodes. The current position is defined by:

$$y_{ct}(\theta_1) = -s(\theta_1) \pm r(\theta_1) \cdot \cos\mu(\theta_1) \tag{7}$$

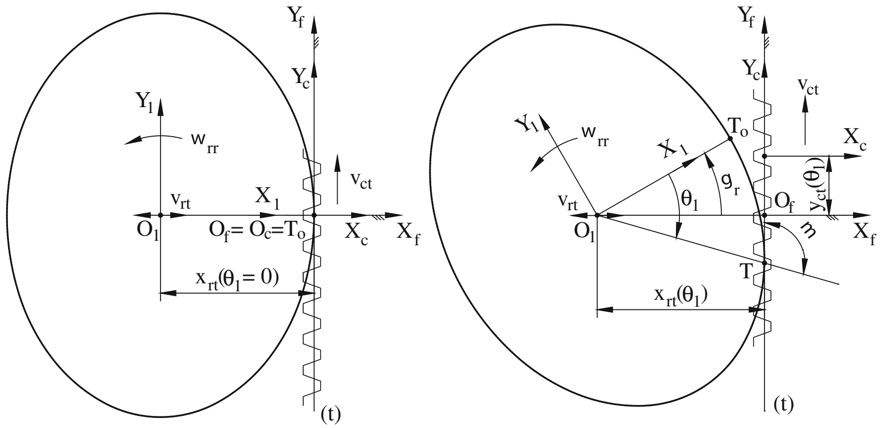

Fig. 5 Kinematics of noncircular gear generation process by rack cutter

where $\mu(\theta_1)$ defines the orientation of the tangent (t) to the gear centrode, at contact point T,

$$\mu(\theta_1) = arctg \frac{r(\theta_1)}{\frac{dr(\theta_1)}{d\theta_1}} \tag{8}$$

$s(\theta_1)$ is the rolling distance, respectively the length of the arc T_0T:

$$s(\theta_1) = T_0T = TO_c = \int_{-\theta_1}^{0} \frac{r(\theta_1)}{\sin\mu(\theta_1)} d\theta_1 \tag{9}$$

The gear is rotated about its center of rotation O_1, with the angular velocity ω_{rr}, and is translated along the perpendicular to (t) direction, with velocity v_{rt}. The current position is defined by:

$$\gamma_r(\theta_1) = \theta_1 + \mu(\theta_1) - \pi/2 \tag{10}$$

$$x_{rt}(\theta_1) = -r(\theta_1) \cdot \sin\mu(\theta_1) \tag{11}$$

3.2 Generation by Shaper Cutter

For convex-concave noncircular gears, the generation of the gear teeth is developed by simulating the manufacture using a shaper cutter. The range of the supershape defining parameters variation that leads to lobed pitch curves is further

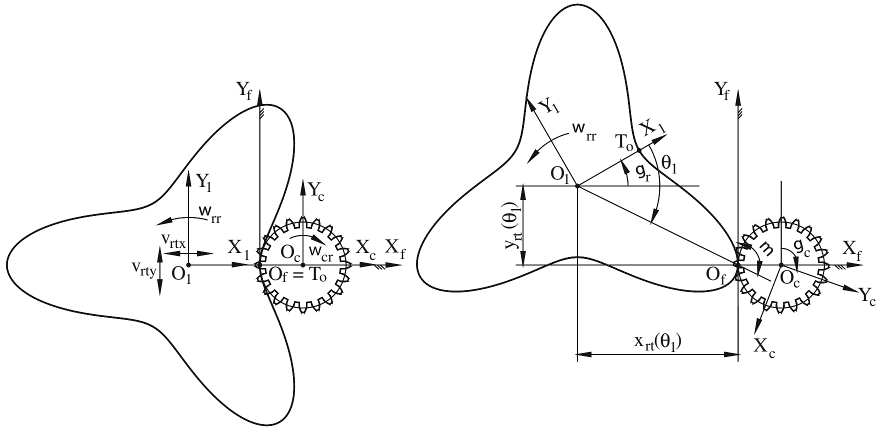

Fig. 6 Kinematics of noncircular gear generation process by shaper

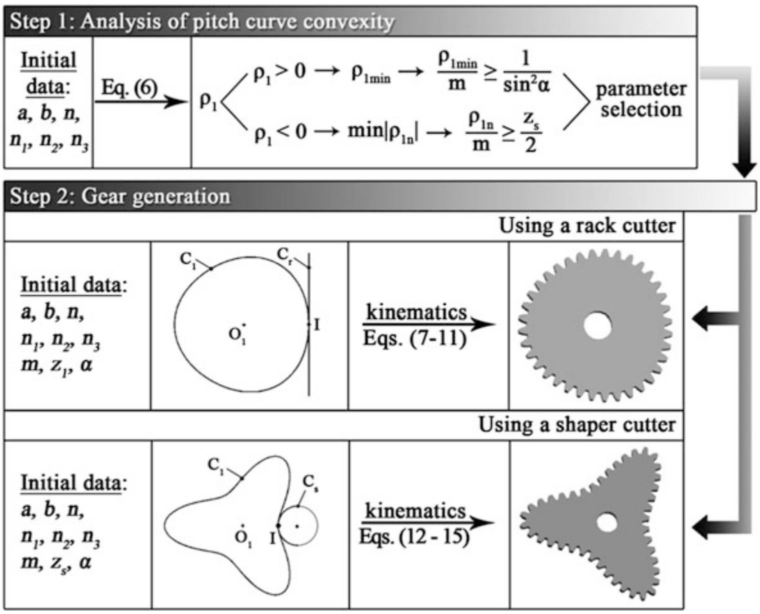

Fig. 7 Algorithm of variable speed gear generation

limited in order to avoid undercutting. Therefore, a new requirement is analyzed, i.e. the minimum curvature radius at concave sectors should be larger than the shaper pitch radius. The simulation of the gear generation by a shaper is based on the following coordinate systems (Fig. 6): a fixed system, $O_fX_fY_f$, with the origin

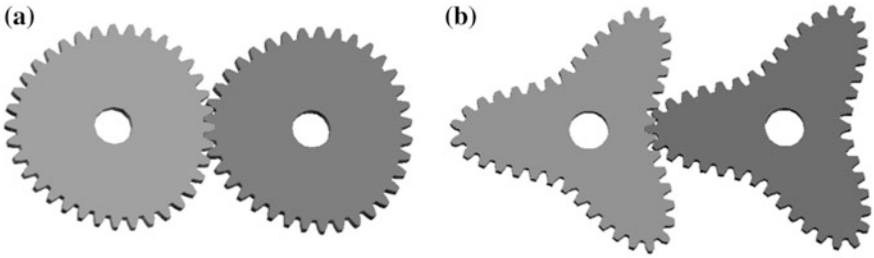

Fig. 8 Variable speed gears generated by rack cutter (**a**) and shaper cutter (**b**)

in the initial contact point, T_0 and two movable coordinate systems, $O_1X_1Y_1$ and $O_sX_sY_s$, rigidly attached to the noncircular blank and shaper, respectively.

The shaper, defined by a standard geometry of m modulus, pressure angle $\alpha = 20^\circ$ and z_s number of teeth, is rotated about its center of rotation O_c, by:

$$\gamma_s(\theta_1) = \frac{s(\theta_1)}{R_s} \tag{12}$$

The gear is rotated about its center, O_1, and translated along the common tangent (t) and along the perpendicular to (t) direction. The current position is defined by:

$$\gamma_r(\theta_1) = \theta_1 + \mu(\theta_1) - \pi/2 \tag{13}$$

$$x_{rt}(\theta_1) = r(\theta_1) \cdot \sin\mu(\theta_1) \tag{14}$$

$$y_{rt}(\theta_1) = r(\theta_1) \cdot \cos\mu(\theta_1) \tag{15}$$

Figure 7 illustrates the algorithm developed for the gear generation, based on the driving pitch curve defined by Gielis superformula and gears "manufactured" by a rack cutter and a shaper, as pitch curve shapes required. Pairs of variable speed gears are presented in Fig. 8.

4 Conclusions

The latest developments in computer graphics and manufacture technologies have enabled new approaches to noncircular gear design and generation. As a special type of noncircular gears, the variable speed gears are the main objective of the paper. Based on the hypothesis of the predesigned driving pitch curve, modeled by Gielis' superformula, algorithms for mating pitch curves generation and tooth manufacture simulation are developed by the interference Matlab-AutoLISP applications. The generalization of the gear generation process is based on the proper selection of the supershape defining parameters that lead to suitable pitch curves and avoid undercutting during the applied process of gear generation.

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