Theoretical Investigation of the Energy Efficiency of Planetary Gear Trains

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Abstract The paper presents an investigation in the area of planetary gear trains. Based upon previous investigations of the author, methodology for the determination of geometrical boundaries for planetary gear trains is systemized. Precise analytical relationships are analyzed. The objectives of the presented investigation are: to summarize and improve the existing methodology for the determination of the geometry boundaries of internal gear trains; to analyze the influence of the tribological parameters upon the functional capacity of such gear trains, to define and precise the influence of the geometry in internal meshing upon the energy efficiency in such transmission elements and to indicate the approach for finding optimal design solutions considering the energy efficiency of gear trains. The results of the investigation of internal gear trains with small difference between the teeth number of the pinion and the teeth number of the ring (from 1 to 10) give the options for a quick and precise determination of the optimal addendum modification coefficients for the pinion and the ring considering the criteria contact strength and energy efficiency of the gear meshing. Conclusion and recommendations concerning the energy efficiency of planetary gear trains are suggested by the author.

Keywords Internal meshing · Geometry boundaries · Energy efficiency · Friction · Heating

List of Symbols

a	Centre distance
d_{a1}	Addendum diameter of pinion
d_{a2}	Addendum diameter of ring
d_{b1}	Base diameter of pinion
d_{b2}	Base diameter of ring
d_{f1}	Root diameter of pinion

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$h_a{}^*$	Coefficient of the addendum of the tooth depth
m	Module
m_n	Normal module
u	Gear ratio of pair
W	Ratio characterizing special kind of interference
x_0	Addendum modification coefficient of cutter
x_1	Addendum modification coefficient of pinion
z_0	Teeth number of cutter
z_1	Teeth number of pinion
z_2	Teeth number of ring
α_0	Angle of the standard basic tooth profile
α_{at1}	Profile angle of tooth at addendum pinion circle
α_{at2}	Profile angle of tooth at addendum ring circle
α_{w0min}	Minimal pressure angle during tooth generation
α_{wt}	Pressure angle of meshing
θ_{1D}	Angle decribing inteference, Eq. (5)
θ_{2D}	Angle decribing inteference, Eq. (6)
μ	Coefficient of friction
v	Angle decribing inteference, Eq. (8)
ψ	Angle decribing inteference, Eq. (7).

1 Introduction

For certain design layouts of gear trains and transmissions it is possible to realize great values of gear ratios in cases when some of the gear stages have internal meshing and especially those internal gear trains with small difference between the teeth number of the pinion and the teeth number of the ring (from 1 to 10).

It is well known that a scientific team with leader Assoc. Prof. E. Angelova and Prof. P. Nenov from the University of Ruse elaborates software programs for geometrical and strength calculation of gear trains with the help of geometry and power blocking contours as described by Angelova et al. (2010, 2011).

For different kinds of internal gear trains, there are graphical presentations of the geometry boundaries, represented in DIN 3993. For certain gear trains with great values of gear ratios and in some cases with internal meshing with small difference between the teeth number of the pinion and the teeth number of the ring, those graphical presentations of the geometry boundaries can not always be applied.

The objectives of the presented investigation are: to summarize and improve the existing methodology for the determination of the geometry boundaries of internal gear trains; to analyze the influence of the tribological parameters upon the functional capacity of such gear trains, to define and precise the influence of the geometry in internal meshing upon the energy efficiency in such transmission

elements and to indicate the approach for finding optimal design solutions considering the energy efficiency of gear trains.

2 Methodology for Determination of Geometry Boundaries for Internal Gear Trains

The theoretical research of planetary gear trains with special applications is based upon the systemized and additionally elaborated methodology for the determination of the geometry boundaries of internal gear pairs, which is described in details by Dobreva (2011).

The geometry boundaries of the internal meshing have to fulfill the following conditions: avoiding the cutter interference of the gear pair; additional specific condition for the internal meshing, non-admission of meshing interference in the area outside the plane of action; ensuring a minimal allowable contact ration and providing a sufficient length of the space width at the root diameter of the ring with internal teeth.

The solutions of those several conditions for the geometry boundaries of the internal meshing are presented usually in a graphical form and they are used as general bases for the geometrical design of cylindrical internal involute gear pairs.

The geometry conditions, which the internal gear pair has to fulfill, are to be defined through the following approach:

2.1 Avoiding Cutter Interference

The first geometry boundary condition is defined according to DIN 3960 in Eq. (1):

$$x_{1\min} = \frac{(inv\alpha_{w0\min} - inv\alpha_0).(z_1 + z_0)}{2.tg\alpha_0} - x_0. \tag{1}$$

The meaning of the parameters, used in Eq. (1) is as follows: $x_{1\min}$—minimal permissible addendum modification coefficient of the pinion; α_0—angle of the standard basic tooth profile; z_0—teeth number of the cutter; z_1—teeth number of the pinion; x_0—addendum modification coefficient of the cutter. One of the parameters from Eq. (1) is described through the following equation:

$$\alpha_{w0\min} = arctg\{z_0/(z_1 + z_0).tg[arccos(m.z_0.cos\alpha_0/(m.z_0 + 2h_a * +2x_0)]\}. \tag{2}$$

The additional parameters, used in Eq. (2) are: m—module of the gear pair; h_a*—coefficient of the addendum of the tooth depth.

The graphical solution of the Eqs. (1) and (2) are presented through the line 1, shown on Fig. 1.

2.2 Additional Specific Condition for Internal Meshing

A very important additional specific condition for the internal meshing is describing the boundary case, when the addendum (tip) circle of the ring is crossing the approach contact of the active part of the path of meshing contact at the base diameter of the pinion.

The geometry relationships for this case are defined precisely by Linke (2010) as a special type of interference by internal meshing. In order to satisfy this condition, it is necessary according to the literature source mentioned, the following inequality to be fulfilled:

$$|d_{a2}| \geq \sqrt{d_{b2}^2 + \left(2|a| \sin \alpha_{wt} + \sqrt{d_{f1}^2 - d_{b1}^2}\right)^2}. \tag{3}$$

The following parameters are applied in Eq. (3): d_{a2}—addendum diameter of the ring with internal teeth; d_{b2}—base diameter of the ring with internal teeth; a—centre distance; α_{wt}—pressure angle of the meshing; d_{f1}—root diameter of the pinion; d_{b1}—base diameter of the pinion.

The graphical solution of inequality (3) is based upon several other well known geometry relationships of the internal meshing. It is visualized on Fig. 1, line 2.

2.3 Interference Outside the Plane of Action

The non-admission of meshing interference in the area outside the plane of action is especially important for internal meshing in cases when the difference between the teeth numbers of the pinion z_1 and of the ring z_2 is small, for example: between one and 10. For that kind of eccentric planetary gear trains, an impact between the addendum parts of the pinion and of the ring is possible to occur in the area outside the plane of action.

Fig. 1 Geometry boundaries of internal gear pair as a function of addendum modification coefficient of pinion x_1 and of the sum modification x_Σ in accordance to DIN 3993
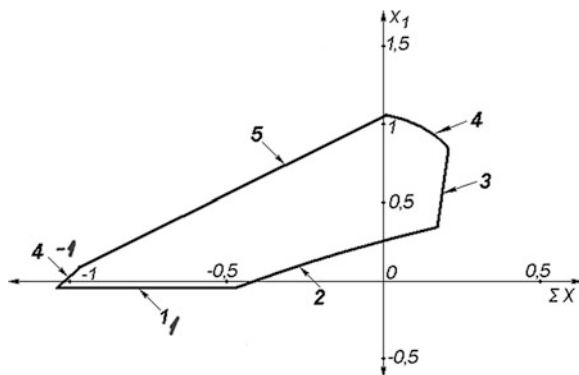
In order to avoid that type of interference, the following inequality has to be fulfilled mentioned in literature (Linke 2010):

$$W = \frac{\theta_{1D} + \psi}{u.(\theta_{2D} - v)} \geq 1. \tag{4}$$

The designations used in (4) are cooresponding to those applied by Linke (2010): W—the ratio, which is characterizing this kind of interference; u—gear ratio of the gear pair ($u = |z_2|/z_1$). The angles θ_{1D}, θ_{2D}, ψ and v are to be calculated according to the relationships in Eqs. (5), (6), (7) and (8):

$$\theta_{1D} = \arccos \frac{d_{a_2}^2 - d_{a_1}^2 - 4a^2}{4 d_{a_1}.|a|} \tag{5}$$

$$\theta_{2D} = \arccos \frac{d_{a2}^2 - d_{a1}^2 + 4a^2}{4|d_{a2}|.|a|} \tag{6}$$

$$\psi = inv\alpha_{at1} - inv\alpha_{wt} \tag{7}$$

$$v = inv\alpha_{wt} - inv_{at2}. \tag{8}$$

The angles, which are included in the Eqs. (7) and (8), are calculated through the addendum and base diameters as shown in the Eqs. (9) and (10):

$$\alpha_{at1} = \arccos \frac{d_{b_1}}{d_{a_1}} \tag{9}$$

$$\alpha_{at2} = \arccos \frac{d_{b_2}}{d_{a_2}}. \tag{10}$$

The geometry parameters from Eqs. (4) to (10) are presented precisely in a graphical way by Linke (2010).

2.4 Influence of the Contact Ratio

The necessary condition to be fulfilled concerning the contact ratio is well known. The following relationship has to be satisfied: $\varepsilon_\alpha \geq 1.1$.

2.5 Space Width at the Root Circle of the Ring

The space width at the root circle of the ring with internal teeth is recommended to be assumed at least equal to 0, 2. m_n. It is calculated according to DIN 3960. The graphical solution of that relationship is presented through line 5 on Fig. 1.

The solutions of the relationships, described briefly in that part of the paper, are shown in a graphical way on Fig. 1. Each one point from the area inside the contour is representing a solution of a given problem considering only the geometrical point of view.

3 Investigation of Tribological Parameters of Internal Gear Trains

Based upon a contract between the University of Ruse and a Bulgarian enterprise, it was necessary to develop several versions for eccentric planetary gear trains with small difference between the teeth number of the pinion and the teeth number of the ring. The activities of the contract included the theoretical analyses of those developed versions and a carried out comparison between the preliminary design layouts of the elaborated gear trains.

According to the results of the implemented research the best possible verions for eccentric planetary gear trains are those with difference between the teeth number of the pinion and the ring equal to two or four teeth. In both cases, the ring of the planetary gear train was designed with 110 teeth.

The geometry boundaries for eccentric planetary gear trains have been solved according to the methodology, presented in the second part of this paper. The graphical solutions for those two possible gear ratios are shwon on Fig. 2.

Based upon methods, described in details by Linke (2010) and Dobreva (2011), a tribological investigation is carried out concerning the energy efficiency of variety of possible planetary gear trains with a teeth difference between z_1 and z_2, equal to two and four.

The results of the theoretical analyses of the energy efficiency of the investigated planetary gear trains are shown on Fig. 3. The values of the zone factor Z_H, which takes into account the flank curvatures at the pitch point and transforms the tangential load at the reference cylinder to tangential load at the pitch cylinder, are calculated according to ISO 6336-2: 2006(E). The obtained results for Z_H and for the transverse contact ratio are shown in a graphical way as a function of the sum modification x_Σ for the eccentric planetary gear train with the following teeth numbers: $z_1 = 106$ and $z_2 = 110$.

As a second stage, an analysis is carried out concerning the influence of the contact stresses upon the energy efficiency. The following conclusion is deduced: the influence of the contact stresses upon the energy efficiency for that kind of planetary gear trains is considerably greater in comparison with the influence of the bending stresses at the root diameters of the pinion and the ring.

The geometry of the internal meshing influences mainly the relative and the specific sliding along the path of the path of contact and respectively the efficiency coefficient in gear meshing.
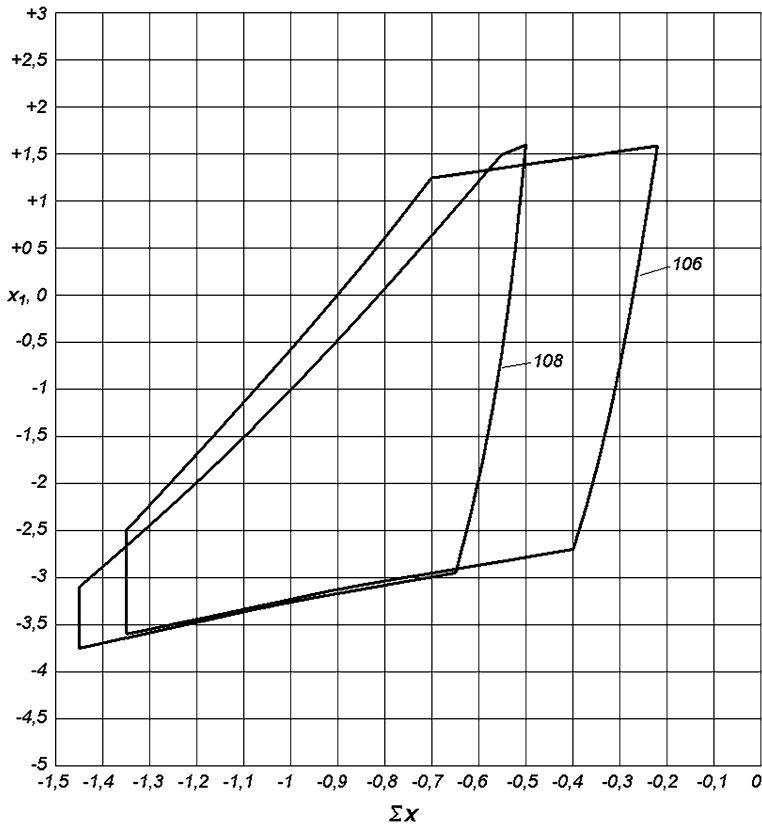
Fig. 2 Geometry boundaries for eccentric planetary gear trains ($z_1 = 106$ and $z_2 = 110$; $z_1 = 108$ and $z_2 = 110$)

Based upon the implemented theoretical research, the author recommends that the planetary gear train with a difference between z_1 and z_2, equal to 4, i.e. $z_1 = 106$ and $z_2 = 110$ has to be produced for the needs of the Bulgarian enterprise for gear trains and reducers taking into account certain technological considerations. Concerning the energy efficiency, the best solution would be the specific gear pair with the following parameters: a sum modification $\Sigma x = -0,9$ and an addendum modification coefficient for the pinion $x_1 = -1$. The selected geometry for the gear pair has the following advantages: the contact strength is secured with a sufficient safety coefficient and the value of the contact ratio ensures a minimal friction coefficient along the active path of meshing contact.

With the exception of the pitch point, the teeth profiles in meshing area are moving relatively to each other through out the length of the path of contact. The relative movement between the pinion and the ring and the sliding cause increasing the temperatures in the lubrication film. This process changes the thickness and the loading capacity of the lubrication film and it has a considetable
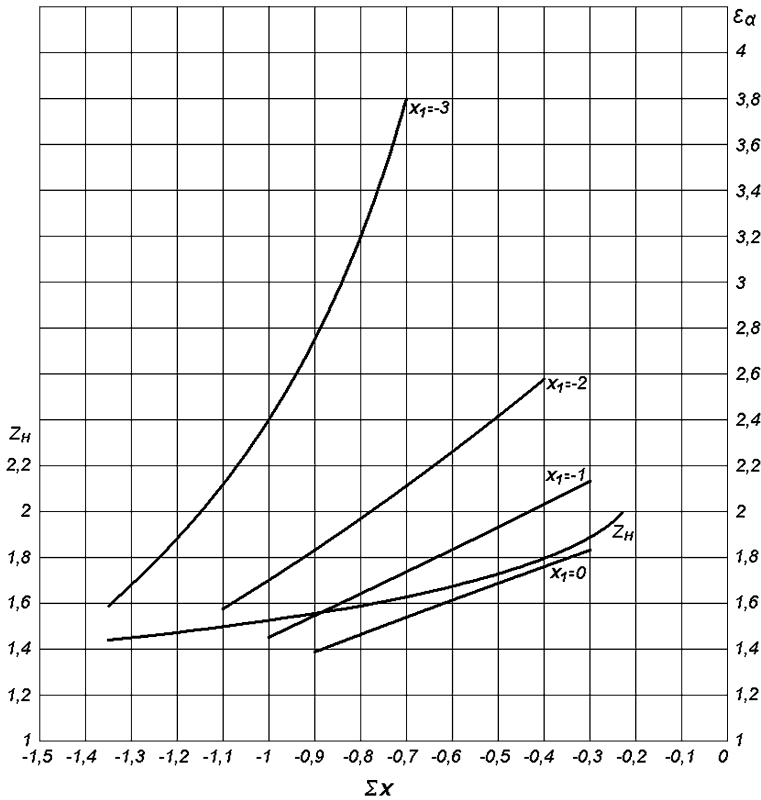
Fig. 3 Tribological parameters for eccentric planetary gear trains as a function of sum modification x_Σ

influence upon the magnitude of the friction forces in the meshing. The value of the temperature in the meshing contact depends considerably from the friction coefficient μ, which is changing with the magnitude of the sliding.

Besides, the high surface temperatures of the gears lead to reducing the strength of the lubrication film and therefore to a secondary increasing of the friction in the meshing area. This process, based upon the mutual dependance between temperature and friction causes pitting, breakage of teeth and other damages.

For planetary and eccentic gear trains, which ensure great gear ratios, the relative sliding achieves great values and therefore, the strength calculations according to the criteria contact strength and bending is not enough for ensuring the normal functioning of that kind of internal gear trains.

It is also necessary a calculation of the heating temeperatures to be carried out, because it is possible that internal gear trains which are calculated in a perfect way according to the main criteria: contact strength (pitting) and bending and also possessing meshing geometry entirely corresponding with the geometry boundaries of the described methodology can be overheated. Therefore, that type of

internal gear trains may loose their functional capacity earlier in comparison with the envisaged preliminary calculated durability. A precise methdology for calculation of the heating parameters concerning planetary and eccentric gear trains is described by Dobreva and Dobrev (1993).

4 Conclusions

The presented investigation of internal and planetary gear trains gives the reasons to deduce the follwoing conclusions:

1. The results of the investigation of internal gear trains with small difference between the teeth number of the pinion and the teeth number of the ring (from 1 to 10) give the options for a quick and precise determination of the optimal addendum modification coefficients for the pinion and the ring considering the criteria contact strength and energy efficiency of the gear meshing.
2. Based upon precisely defined geometry boundaries for internal gear trains with small difference between the teeth number of the pinion and the teeth number of the ring, the value of the contact ratio is an important indicator for the selection of a gear pair considering the optimal operational parameters of the gear trains.
3. For planetary and eccentric gear trains ensuring great gear ratios, it is necessary to implement calculation of the heating parameters as well.

References

Angelova E, Ronkova V, Nenov P (2010) Increasing load capacity of cylindrical gear by optimizing their geometric parameters, "INMATEH—Agricultural Engineering", May-August, 31(2), National Institute of Research-Development for Machines and Installations Designed to Agriculture and Food Industry-INMA Bucharest, ISSN:2068-2239, ISSN:2068-4215, pp 40–46

Angelova E, Ronkova V, Tiufektchian A (2011) Computer-aided design of involute cylindrical gear drives for portable electric tools. In: 2nd internacional scientific and practical conference "technology, materials, transport and logistics: development prospects" TMTL'11, Yalta (Crimea), Ukraine, The Scientific Journal 12(166) ISSN 1998-7927, pp 14–18

DIN 3960. Begriffe und Bestimmungsgroessen fuer Stirnraeder (Zylinderraeder) und Stirnpaare (Zylinderpaare) mit Evolventenverzahnung

DIN 3993. Geometrische Auslegung von zylindrischen Innenradpaaren

Dobreva A (2011) Improving tribological characteristics of gear trains with internal meshing. In: Proceedings of scientific conference of University of Ruse, vol 50. Book 4, ISSN:1311-3321, pp 175–180

Dobreva A, Dobrev V (1993) Improving the tribological characteristics of heavy loaded gear boxes. In; Proceedings of the first balkan conference on tribology "Balkantrib'93", vol 2.3. Sofia, pp 166–170

ISO 6336 (2006)(E) Calculation of load capacity of spur and helical gears

Linke H (2010) Stirnradverzahnung. Hanser Verlag, ISBN 978-3-446-41464-8