Chapter 45
Problem Posing: A Possible Pathway to Mathematical Modelling

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Abstract Problem posing is an important component of learning mathematics as is problem solving, and it is an essential part of mathematical modelling. This chapter reports two small studies conducted in a Year 1–2 class and a Year 3–4 class, respectively. The purpose of each study was to examine the extent to which the use of real world artefacts provide a stimulus for young students to pose problems that can be investigated using mathematical modelling. The students in both studies had no prior experience in problem posing. However, they generated a range of problems linked to the real world albeit some in a superficial sense, whereas others gave genuine rise to mathematical modelling.

1 Problem Posing: What Is It? Why Use It?

Problem posing involves both the generation of new problems and reformulation of given problems (English 2003; Silver 1994; Whitin 2006). Bonotto (2010b) suggests that mathematical problem posing is the process by which "students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems" (p. 109). English (2003) goes further in suggesting that problem posing "is a fundamental part of learning and doing mathematics" (p. 187) as is problem solving to mathematical modelling. Others concur with English and argue that problem posing is a key component of mathematical exploration (Bonotto 2010b; Cai and Hwag 2002). Christou et al. (2005) argue that problem posing is "an integral part of modelling cycles, which require the mathematical idealization of real world phenomenon" p. 149. For this reason, Bonotto (2010a) considers "problem

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posing is of central importance in the discipline of mathematics and in the nature of mathematical thinking and it is an important companion to problem solving" (p. 404).

Constructing problems is only one facet of problem posing. When students are applying problem posing processes they are "actively engaged in challenging situations that involve them in exploring, questioning, constructing, and refining mathematical ideas and relationships" (English 2003, p. 197). Bonotto (2008) argues that immersing students in situations related to their real-life experience and meaningful sense–making is a way to "deepen and broaden their understanding of the scope and usefulness of mathematics as well as learning ways of thinking mathematically that are supported by mathematising situations" (p. 14).

Problem posing can be initiated using diagrams, definitions, questions, statements, equations, concrete materials, real world situations, children's literature, or objects (Brown and Walter 2005; English 1998; Whitin 2004). Bonotto (2008) suggests the use of real world artefacts that are relevant and meaningful to students, such as weekly TV guides and supermarket dockets, bridges the gap between formal school mathematics and informal out-of-school mathematics.

Although problem posing is a natural occurrence in real life it does not receive sufficient attention in the mathematics classroom. According to English (1998) educators need to

> broaden the types of problem experiences we present to children … and, in so doing, help children "connect" with school mathematics by encouraging everyday problem posing (Resnick et al. 1991). We can capitalize on the informal activities situated in children's daily lives and get children in the habit of recognizing mathematical situations wherever they might be (p. 100).

Bonotto (2008) concurs with English (1998) and argues that school mathematics should more closely relate to the experiences students have in their everyday life, to make it relevant to students. By doing so, students are enabled to view mathematics as a "means of interpreting and understanding reality" (Bonotto 2010b, p. 110), rather than purely as a set of abstract, formal structures.

As stated earlier, English (2003) indicates that problem posing underpins students' mathematical learning and involves skills such as questioning, describing, constructing, justifying, and explaining, all of which are evident in mathematical modelling. It could be argued that problem posing provides the foundation for not only students' development of problem solving skills, but also for mathematical modelling. Stillman (in press) argues, "finding and posing problems are essential ingredients in any education program in schools promoting mathematical modelling". One of the features of mathematical modelling is that "modelers find and pose their own problems to solve" (Stillman in press).

Mathematical modelling refers to a process in which a real world situation has to be "problematised and understood, translated into mathematics, worked out mathematically, translated back into the original situation, evaluated and communicated" (Bonotto 2010b, p. 108). It "involves using mathematical concepts, structures and relationships to describe and characterise, or model, a real world situation in a way that it captures its essential features" (Stillman in press). As is the case in real-life situations, "modelling activities often comprise information that might be incomplete, ambiguous, or undefined, with too much or too little data" (English et al. 2005, p. 156).

Mathematical modelling provides rich learning experiences that extend students problem solving and problem-posing abilities (English et al. 2005). Making conjectures and predictions, justifying and refuting arguments and resolving conflicts are components of mathematical modelling situations (English et al. 2005). Posing problems and questions occurs naturally throughout mathematical modelling situations because they "evoke repeated asking of questions and posing of conjectures" (p. 156).

Traditionally, mathematical modelling has been a topic for secondary school students. However, English et al. (2005) argue that younger students would benefit from rich learning experiences that could promote their problem-solving and problem-posing abilities. These abilities are necessary "to function effectively in a world that is demanding more flexible, creative, and future-orientated mathematical thinkers and problem solvers" (p. 156).

From a research perspective, little is known about students' ability to create their own problems in numerical and non-numerical contexts, or the extent to which these abilities are linked to their problem-solving competency (English 1998). English (1997) found Year 5 students who participated in specific problem posing programs exhibited greater facility in creating solvable problems than those who did not. Silver and Cai (1996) found that there was a high correlation between students' problem posing performance and their problem solving performance.

The studies reported in this chapter will attempt to demonstrate that young students (ages 6–9) can pose problems using real world contexts, such as buttons, children's literature and photographs that can be investigated using mathematics. Furthermore, these real-life situations are more conducive to young students' development of mathematical understanding and problem solving skills than formal contexts, which are quite removed from their natural real world experiences (Bonotto 2008).

2 Methods

This chapter reports on two studies in the form of teaching experiments, one in a Year 1–2 (6–7 year olds) class in their second and third year of schooling and the other in a Year 3–4 (8–9 year olds) class. Within some schools the classes are multi-age in structure, hence the notion of Year 1–2. Neither class had any formal experience with posing problems during mathematics lessons. The Year 3–4 students' problem solving experiences were limited to word problems for which they could apply a known procedure or follow a clearly defined pathway (Brown and Walters 2005). The purpose of the studies was to explore the extent to which young students, with no prior experience, could pose problems that could be investigated using mathematics, in particular mathematical modelling.

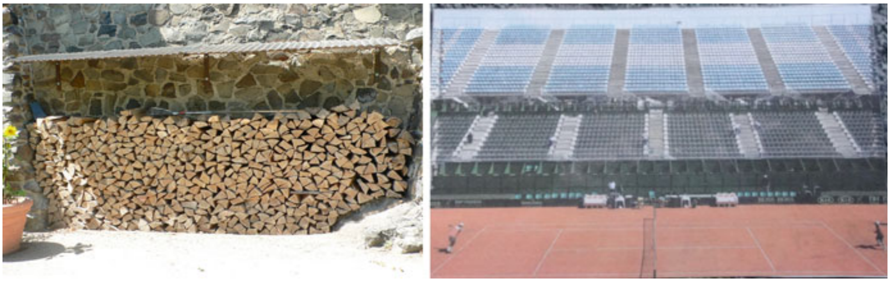

Fig. 45.1 Examples of real world photographs used for problem posing stimuli

These teaching experiments are situated within a larger project, Contemporary Teaching and Learning of Mathematics Project (CTLM), the aim of which is to improve the teaching and learning of mathematics by providing teacher professional development and in-school classroom support. These two studies are part of the in-school classroom demonstration lesson component of the CTLM project. They were conducted in two different schools, both in their second year of participation in the project.

Each teaching experiment consisted of a single lesson (approximately 90 min). The lessons were conducted by the researcher and observed by several classroom teachers. Data collection occurred in February (start of the school year) in the Year 1–2 class (12 boys, 13 girls) and in June (5 months into the school year) in the Year 3–4 class (19 boys, 7 girls). Data collected included student work samples, photographs of student engagement at various stages during the lesson, audio recording of student conversations during the task, field notes, and post lesson discussion with observing teachers.

2.1 The Tasks

In both classes real world artefacts (drawing on the work of Bonotto 2008, 2010b) were used as the main stimulus for students' problem posing activities. One task was implemented with the Year 1–2 class and two tasks with the Year 3–4 class. In the Year 1–2 lesson, the real world artefact was a very old tin containing a collection of recycled buttons of various shapes, sizes, textures, and hole configurations. In the Year 3–4 lesson the students first explored a numerical situation that drew on the work of English (1998): "What problems could you pose about the number 20?" Following English (1998) and Whitin (2006), students then selected one of their questions to compose a real-life word problem for it. Real world photographs were the stimulus for the second problem posing activity. Real world photographs used included a wood stack and a tennis stadium (see Fig. 45.1).

2.2 Implementation

As the tin of buttons was introduced to the Year 1–2 students, they were immediately engaged in problem posing and reasoning about the contents of this tin, an object that they could associate with the real world (Bonotto 2008). Fluency, flexibility, and originality (English 1998; Silver 1994) were evident in the responses. As the tin was shaken the students began to eliminate some of the suggestions with justification such as, 'If there were chocolates inside, it would not make this much noise.' Novel responses included socks, pasta, toy cars, and money. The categories included food, toys, natural objects (e.g., rocks, sand, pebbles), jewellery, money, and classroom equipment (e.g., scissors, pencils, paper clips). It took some time for buttons to be considered.

The buttons were then distributed to the tables where students worked in pairs to explore the buttons for 5 min. Students were then asked to think of questions they could pose that we could investigate as a class. During this time, the observing teachers were encouraged to observe, listen and assist with recording of questions (only if necessary) and to refrain from prompting.

The lesson in the Year 3–4 class began by indicating to the students my interest in their mathematical thinking and the type of problems they might pose for the class to investigate. Students were asked to think about some problems they could pose about the number 20. They worked individually then some of the problems posed were shared with the class. All the problems posed were in numerical form. Students were then asked if the questions could be made more challenging which led to open-ended problems such as $\_\times\_=20$ and $\_$ (+, −, ×, or ÷) $\_=20$. A brief discussion ensured about the kind of thinking they needed to do when problem posing compared to finding a solution. Some students indicated that it was more difficult to pose a problem because "you had to work backwards from the answer." Students then had to make up some word problems for $4\times5=20$. These were discussed briefly to highlight the need for the problems posed to be clear, make sense and include a question.

This activity provided the groundwork for the main part of the lesson, the use of real world photographs to pose a problem that could be investigated using mathematics. The students worked in self-selected pairs, all single gender except for one pair. Each pair had 20 min to generate a range of problems, test them out and refine them to ensure they met the following criteria: (1) it is a genuine problem to investigate using mathematics; (2) the problem is clear; (3) the problem requires more than a calculation of the given information; and (4) solving the problem requires the extra-mathematical world to be considered. The students also needed to think about how they might solve the problem and the mathematics they might use to solve it.

In sharing their problems the students were required to explain and justify how the problem posed addressed the criteria. During this time, other students were encouraged to contribute their thoughts. For example, some students did not consider *The Stamps Problem* (see Fig. 45.3) to be a real world problem because one-cent coins are no longer in circulation. Others suggested that the cost would be rounded up to the next multiple of ten like they do in the supermarket, while others said you could use a credit card!

At the conclusion of the lesson challenges faced in having to pose problems rather than solve them were discussed. Common responses included: “Making sure the problems make sense”, “Trying to translate the problem that someone else has made”, “Making sure the problem is clear for you and those who have to solve it”, and “Coming up with a problem that is challenging”.

3 Findings

3.1 Year 1–2 Problem Posing

The questions posed for the *Buttons Task* by the Year 1–2 class ranged from “How many buttons in the collection?” to “What are buttons made of?” The list of questions generated from the lesson is included in Fig. 45.2. The majority of problems posed related to quantity and exploration of number and some data representation. However, other problems led to more in-depth investigations relating to pattern, shape and size. The ‘what’ and ‘why’ problems the students posed link to the real world and provide opportunities for mathematical modelling. In contrast, the ‘how many’ problems posed may appear trivial from a modelling and applications point of view but are important mathematically in terms of quantifying, sorting and classifying. The ‘what’ and ‘why’ problems posed are messy and complex and require more thought to begin to solve them than the ‘how’ problems posed. Brown and Walter (2005) suggest that it is important in the initial stages to accept all questions even when some appear irrelevant or nonsensical.

How many holes in each button?
How many buttons have just two holes?
How many buttons have just one hole at the back?
How many holes on all the buttons?
How many buttons of each colour?
How many buttons altogether?
How many buttons fit in the tin?
How many buttons cover my page?
How far will the buttons go, if we made a line of them?
What size are the buttons?
What kinds of patterns are on the buttons?
What kinds of shapes are the buttons?
Why are most buttons round?
How do you know if something is a button?
What are buttons made of?

Fig. 45.2 Problems posed from the button exploration

Table 45.1 Examples of the word problems posed for $4 \times 5 = 20$

Word problem	Connection between reality and mathematical world
Patrick and Ryan go to the shops and spent $20. They bought 4 things. How much did each thing cost?	Yes, using a routine problem with result unknown
There are 20 cupcakes and five girls at a party. How many cupcakes did each girl get?	
Declan, Aidan, Luke and Dan each gave Liam 5 lollys [sic]	Yes, but assumes each gets the same amount
How many did Liam have altogether?	
Dad put 20 apples in boxes. Each box had the same amount	Yes, partially used the context to match the situation but more information is required
How many boxes did he get and how many apples in each?	
Patrick had 40 marballs [sic]. He shared 10 to Ryan and Daniel	None
How many did Patrick have left? (4×5)	
I had a box of 20 cookies and dropped it. Nine were broken and I ate 3. How many are left?	None

The observing teachers were surprised by the amount of mathematics the students used in the lesson, their engagement and the range of problems they posed particularly from some of the students who normally struggle in mathematics and are reluctant to contribute. In the follow-up lesson, which the class teacher conducted, the students chose one of the problems to work on in pairs.

3.2 *Year 3–4 Problem Posing – Story Problems for 20*

There were a range of problems generated for the equation $4 \times 5 = 20$. Most of the problems posed indicate students' use of the real world (contexts such as shopping, games, lollies) to demonstrate their understanding of the mathematical world. Some of the problems reflected insufficient information while others indicated little consideration of the equation. Examples of the different word problems posed are included in Table 45.1.

The students' problems posed for 20 were classified according to the connection between reality and the mathematical world and whether they reflected the original equation. There were 26 student responses and the results of the analysis of these are presented in Table 45.2.

From this analysis it is reasonable to infer that students were familiar with solving routine problems and so were able to draw on their experience and formulation of problems of this nature. It was not surprising, based on research by others (e.g., English 1998) that some students had difficulty in posing problems that related to the equation and to the real world.

Table 45.2 Frequency of word problems types posed for 20 ($N=26$)

Problem type	Frequency
Connection between reality and mathematical world using routine problem	9
Connection between reality and mathematical world with missing element	1
Partial connection between reality and mathematical world with additional information required	4
No connection between reality and mathematical world, nor does the problem match equation	8
Not formulated as a word problem	4

3.3 Year 3–4 Problem Posing from Real World Photographs

From an analysis of all 13 problems posed, most were based on the literal interpretation of the picture initially then as the lesson developed some students explored the possibilities of more challenging problems. This was facilitated by the problems posed being presented to the class and discussed in relation to the criteria in Sect. 2.2. Some students generated more than one problem relating to their picture while others had difficulty in moving beyond a pure computation type problem. Each pair of students was able to produce a problem that could be investigated using mathematics, regardless of the level of complexity involved. Examples of problems posed and real world photographs are included in Fig. 45.3.

Some of the problems generated could be considered too challenging for 8 and 9 year old students to solve, however they do give genuine rise to problem solving, making some assumptions and lead to mathematical modelling, such as the *Windows Problem* and the *Grains of Rice Problem*. The other problems (*Strawberries*, *Jellybeans*, and *Stamps*) are routine type word problems, which suggests the students possibly posed the problems based on a literal interpretation of the photographs. These problems do not reflect engagement with the real world. Such problems can be solved using prior knowledge and skills and do not require any problem solving. However, for some students the *Strawberries Problem* is a genuine problem, as it requires them to make some assumptions.

Two observations made by the classroom teachers watching the lesson give some insight into possible reasons for the nature of the problem posing. First, some of the boys were very competitive when the problems were shared and wanted to work them out rather than offer suggestions. This could suggest that they were focused on the solution rather than the problem posing. Second, some of the more mathematically capable students struggled initially with generating a problem beyond just the literal interpretation of the picture, possibly because of the need to think differently. It appears that the students' view of mathematics or their experiences of mathematics is as a purely abstract subject. It could be argued that from a young age students need to develop views of mathematics as being both abstract and connected to the real world.

Windows problem
How many windows in the building?
How long would it take someone to clean the windows?
How many rooms might be in the building?

Grains of rice problem
If my dad grew fifteen rice plants, how much rice would he have in a year?
If me and Flynn eat 50 grams of rice each day, how many grams of rice would we eat in a year? How many grains of rice would that be?

Stamps problems
There are 50 stamps. Tom buys 10 on Monday, 3 on Tuesday.
How many stamps does he have left? Each stamp costs 3 cents.
He has $3.00. He has until Friday to buy all 50 how many will he buy on the other three days? How much money will he have left when he buys all 50 stamps?

Jellybean candies problem
A man in a lolly shop put jellybeans in a container. There were 12 rows and 18 in each row. How many jellybeans are there?

Strawberries problem
Me and Declan picked 60 strawberries. I ate ¼ of them. How many are left?
We had to pack 60 strawberries into containers.
How many containers did we need and how many did we pack in each?

Fig. 45.3 Problems posed from photographs of real world situations

4 Discussion and Conclusions

Silver and Cai (1996) have reported that student engagement in problem posing experiences has a positive influence on their problem solving achievement or their disposition towards mathematics. The students in both teaching experiments were engaged and the teachers gained insights into their learning that they were not necessarily aware of prior to the lesson. In the Year 3–4 class students of various ability levels persevered with the challenge of formulating questions and wanted to produce something for their efforts. The observing teachers found the use of real world artefacts beneficial and some were surprised at the level of engagement of the Year 1–2 students and their understanding of number.

The findings suggest that students as young as 6 years of age, with no prior experience with problem posing, are capable of generating questions that can be investigated using mathematical modelling. As stated earlier, mathematical modelling involves skills in problem posing and problem solving so it is reasonable to argue that providing students with opportunities to develop expertise in problem posing may enhance their capabilities as mathematical modelers (English 2003; English et al. 2005; Stillman in press). Furthermore, the use of real world artefacts that are meaningful to students enabled them to construct meaningful mathematical problems as suggested by Bonotto (2008).

While acknowledging the studies reported in this chapter are small, they provide evidence to suggest that the use of problem posing and real world artefacts in the early years of primary school could provide the foundation for use of real life modelling and applications in the classroom. Further research into the extent to which the use of real world artefacts support young students' ability to pose rich mathematical problems that can be investigated using mathematical modelling, and develop their problem solving and reasoning skills needs to be conducted particularly with respect to the influence of the views of mathematics that students appear to be developing at such a young age.

References

Bonotto, C. (2008). On the relationships between informal out-of-school mathematics and formal in-school mathematics in the development of abstract mathematical knowledge [regular lecture]. In M. Niss (Ed.) *ICME10 2004, Proceedings of tenth international congress on Mathematical Education* [CD]. Roskilde: IMFUFA.

Bonotto, C. (2010a). Realistic mathematical modeling and problem posing. In R. Lesh, P. Galbraith, C. R. Haines, & A. Hurford (Eds.), *Modeling students' mathematical competencies* (pp. 399–408). New York: Springer.

Bonotto, C. (2010b). How it is possible to make real-world mathematics more visible: Some results from two Italian projects. In A. Araújo, A. Ferandes, A. Azevedo, & J. F. Rodrigues (Eds.), *Conference proceedings of EIMI 2010; Educational interfaces between mathematics an industry* (pp. 107–116). Lisbon/Bedford: Centro Internacional de Matemática/COMAP.

Brown, S., & Walter, M. (2005). *The art of problem posing* (3rd ed.). Mahwah, NJ: Lawrence Erlbaum.

Cai, J., & Hwag, S. (2002). Generalized and generative thinking in US and Chinese students' mathematical problem solving and problem posing. *Journal of Mathematical Behaviour, 21*, 401–421.

Christou, C., Mousoulides, N., Pittalis, M., Pitta-Pantazi, D., & Sriraman, B. (2005). An empirical taxonomy of problem posing processes. *Zentralblatt für Didaktik der Mathematik, 37*(3), 149–158.

English, L. D. (1997). The development of fifth-grade children's problem-posing abilities. *Educational Studies in Mathematics, 34*(3), 183–217.

English, L. (1998). Children's problem posing within formal and informal contexts. *Journal for Research in Mathematics Education, 29*(1), 83–106.

English, L. D. (2003). Engaging students in problem posing in an inquiry-oriented mathematics classroom. In F. K. Lester Jr. & R. I. Charles (Eds.), *Teaching mathematics through problem solving, Prekindergarten-Grade 6* (pp. 187–196). Reston, VA: National Council of Teachers of Mathematics.

English, L. D., Fox, J. L., & Watters, J. J. (2005). Problem posing and solving with mathematical modelling. *Teaching Children Mathematics, 12*(3), 156–163.

Silver, E. A. (1994). On mathematical problem posing. *For the Learning of Mathematics, 14*(1), 19–28.

Silver, E. A., & Cai, J. (1996). An analysis of arithmetic problem posing by middle school students. *Journal for Research in Mathematics Education, 27*(5), 521–539.

Stillman, G. (in press). Problem finding and problem posing for mathematical modelling. In N. H. Lee & K. E. D. Ng (Eds.), *Mathematical modelling—From theory to practice.* Singapore: World Scientific.

Whitin, D. (2004). Building a mathematical community through problem posing. In R. Rubinstein & G. W. Bright (Eds.), *Perspectives on the teaching of mathematics* (pp. 129–140). Reston, VA: NCTM.

Whitin, D. J. (2006). Problem posing in the elementary classroom. *Teaching Students Mathematics, 13*(1), 14–18.