Chapter 26
Teachers' Self-Perceptions of Their Pedagogical Content Knowledge Related to Modelling – An Empirical Study with Austrian Teachers

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Abstract Empirical research into teachers' views and pedagogical content knowledge related to modelling in the mathematics classroom is still relatively scarce, such as in the domain of self-perceptions of modelling-specific pedagogical content knowledge. Consequently, this study concentrates on such views of an Austrian sample of pre-service and in-service mathematics teachers. Both quantitative and qualitative methods were used to explore how teachers perceive their knowledge about possibilities of providing students with specific help in the modelling process and how they see their professional development at university with respect to modelling. The findings show that the mean self-perceptions in both of these areas were not positive, indicating a need for intensified professional development support.

1 Introduction

There is a consensus that modelling is an integral part of mathematics and that it should play an important role in mathematics education. For this reason modelling is agreed to be a big idea for mathematics as a scientific discipline with high relevance for mathematical literacy. Consequently, teachers should be aware of this big idea, know how modelling relates to a variety of curricular content, and

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master instructional strategies linked to modelling. However, empirical research into in-service teachers' knowledge and views associated with modelling is still relatively scarce, despite its importance.

The project ABCmaths ("Awareness of Big Ideas in Mathematics Classrooms", www.abcmaths.net) therefore concentrates on pre-service and in-service teachers' views related to modelling, including self-perceptions held by the teachers of their modelling-specific professional knowledge. In this study, we analyse quantitative and qualitative data from a questionnaire survey on modelling in the mathematics classroom. The results give insight into how teachers think about their own professional development related to mathematical modelling at the university level, as well as into their views about their own abilities of supporting students in modelling processes. The results indicate differences in the teachers' perceptions, which point to a need for intensifying both learning opportunities at the university level and ongoing professional development focusing on modelling.

2 Theoretical Framework

2.1 Modelling – A Big Idea for the Mathematics Classroom

The idea of modelling (Blum 1996; Blum et al. 2007) is an aspect of mathematics that is not only important within mathematics (Siller et al. 2011), but also is crucial for the competencies of mathematically literate citizens (OECD 2003). For instance, national standards in many countries (AECC 2008; KMK 2004; NCTM 2000) and assessment studies in the domain of mathematical literacy (OECD 2003) emphasise the significance of modelling as an aspect of mathematical competency.

Process aspects are in the focus of the idea of modelling, such as the descriptions of the modelling cycle (Blomhøj and Jensen 2003; Blum and Leiß 2005). However, empirical observations have shown that the modelling process is individual and does often not follow the cyclic model (Borromeo Ferri 2006). In the process of modelling, translation processes between the real world context and mathematics (in both directions) play a central role (Blum 2007), an aspect that tends to increase the complexity of modelling problems and to cause many difficulties of learners. Consequently, support for learners often focuses on meta-knowledge about modelling and an increased awareness of the modelling process (Blum 2007; Breitenbücher and Kuntze 2010; Maaß 2006). Further, an appropriate task culture (Neubrand 2002) can enhance instructional quality by a focus on tasks with modelling requirements, on a level of difficulty adapted to the specific groups of learners. Siller (2010) suggests learning opportunities for student-centred classroom settings that aim at fostering modelling competencies. An increasing body of publications with suggestions of "good classroom practice" associated with modelling (Blum et al. 2007) points to a need for developing corresponding professional knowledge of mathematics teachers. For the domain of professional knowledge, we will outline the theoretical background of this study.

2.2 Professional Knowledge and Instruction-Related Views of Mathematics Teachers

Creating cognitively activating learning opportunities related to modelling and helping students when dealing with modelling requirements presupposes professional knowledge linked to modelling in the classroom. Such specific professional knowledge includes knowledge about modelling in mathematics, meta-knowledge about the modelling process (Blomhøj and Jensen 2003; Blum and Leiß 2005; for empirical results see Maaß and Gurlitt 2009; Siller et al. 2011) and implications for teaching, knowledge about specific settings for modelling in the mathematics classroom, knowledge about technology use in the modelling process (Siller and Greefrath 2010), about possibilities of support of students in the modelling process, and views related to tasks (Kuntze 2011; Kuntze and Zöttl 2008). Hence, specific professional knowledge of mathematics teachers should cover a range of aspects – including those mentioned in the previous section. These aspects are necessary for an awareness of modelling as a big idea with relevance for the mathematics classroom.

As an over-all framework for professional knowledge, we refer to Shulman's (1986) categories of "pedagogical content knowledge" (PCK), "content knowledge" and "pedagogical knowledge". For designing learning opportunities in the mathematics classroom, PCK is expected to play a key role. In a more detailed model of professional knowledge (Kuntze 2011), we integrate the spectrum between knowledge and epistemological beliefs/convictions (Törner 2002) as well as the spectrum of so-called levels of globality (Törner 2002) as additional dimensions. For example, the teachers' views about the significance of modelling for the mathematics classroom (Siller et al. 2011) can be described according to this model: they can either be global (general significance of modelling) or rather content-specific (e.g., views about the significance of modelling for certain content areas such as functions). Among such views are also self-perceptions of the teachers' PCK related to modelling, a component of professional knowledge that merits attention, as empirical evidence in this area is scarce.

2.3 Teachers' Self-Perceptions of Pedagogical Content Knowledge Related to Modelling

Self-perceptions of PCK, that is, views of teachers concerning their own PCK, are likely to play a mediating role for the development of professional knowledge. Moreover, they may be considered as indicators of the teachers' PCK, in an analogous way to the predictive role of self-efficacy for competency (Dweck 1986). Further, such self-perceptions afford insight into the teachers' views about their personal needs of professional development. In this sense, these self-perceptions can be considered as "local" individual views, which might frame somewhat "larger" epistemological beliefs, as understood by Törner (2002).

In a first approach to self-perceptions of PCK related to modelling, empirical indicators can use the perspective of a direct reference, that is self-perceptions of PCK when relevant for teacher-student interactions related to modelling in the classroom, or the perspective of a more indirect reference, that is, perceptions of the professional development at university against the background of the requirements of instructional practice. For both of these complementary perspectives, qualitative explanations for the teachers' answers can support the validity of the data. Hence, open questions focusing on corresponding PCK and on views about teacher professional development at the university can help to frame empirical findings related to self-perceptions of PCK in the domain of modelling.

3 Research Questions

Consequently, in order to find out about teachers' self-perceptions of PCK in the domain of modelling, the following two research questions are at the centre of this study:

(a) What self-perceptions of their PCK related to modelling do Austrian pre-service and in-service teachers hold?
(b) What explanations for these self-perceptions can be found in the teachers' answers to open questions related to their modelling-specific PCK?

4 Sample and Methods

In this study, 38 Austrian pre-service and 48 Austrian in-service teachers were asked to answer a pencil-and-paper-questionnaire. All pre-service teachers and 20 in-service teachers answered the questionnaire. The 20 in-service teachers (14 female, 6 male) had a mean age of 32.6 years ($SD=9.97$ years) and had been teaching for a mean time of 6.7 years ($SD=9.51$ years) at academic-track secondary schools. The 38 pre-service teachers (30 female, 8 male) had a mean age of 23.5 years ($SD=3.55$ years) and were preparing to teach in academic-track secondary schools.

The questionnaire consists of several sub-sections. For the research questions above, a multiple-choice section about self-perceptions complemented with open questions about providing help to students in the modelling process, hence focusing on specific PCK, and views related to possibilities of improving modelling-specific professional development at the university. The multiple-choice part of the questionnaire was designed to consist of four indicator-like scales; the data were analysed with quantitative methods based on the answers given in the format of four point Likert scales. The answers to the open questions were analysed with qualitative methods in a bottom-up approach. The scope of this qualitative analysis was to explore possible backgrounds to the quantitative results and to check their validity, according to research question (b).

5 Results

5.1 Self-Perceptions of PCK Related to Modelling

Research question (a) focuses on the teachers' self-perceptions of their PCK related to modelling. As mentioned at the end of Sect. 1, these self-perceptions were explored from the perspectives of views about PCK relevant for modelling-specific teacher-student interactions ('direct reference') and views about the teachers' satisfaction with what they had learned at university concerning ways of fostering modelling abilities ('indirect reference'). For both of these perspectives, two scales had been conceived for the questionnaire, covering more than only one aspect per perspective. The 'direct reference' scales focused on self-perceptions of diagnostical knowledge related to the modelling process and on providing modelling-specific help to students. Sample items are given in Table 26.1. The 'indirect reference' items focused on self-perceptions about modelling-specific PCK learned at university and about PCK relevant for technology use in the modelling process. The teachers' answers were coded with values from 1 ("I do not agree at all") to 4 ("I fully agree"). The reliability values for all scales were good, given the low number of items per scale.

Figure 26.1 displays mean values and standard errors for the two sub-samples. The mean values are close to the centre of the scale or in the spectrum of negative answers. The difference between the mean answers of pre-service and in-service teachers for the self-perceptions of PCK about technology use is significant. The data in Fig. 26.1 show that the indicators for self-perceptions of PCK related to modelling do not show high values, neither for pre-service nor for in-service teachers.

Table 26.1 Scales about self-perceptions of PCK and reliability values

Scales Positive self-perception concerning PCK …	Sample item	Number of items	α (Cronbach)
in the domain of diagnostical knowledge related to the modelling process	I know how to connect the origins of difficulties of learners with (particular) phases of the solution process of modelling tasks.	2	.86
relevant for giving specific help to students in the area of modelling	I can give hints to students which phases of the modelling cycle have to be gone through again in order to improve the quality of the final result.	2	.79
about modelling learned at university	During my professional development at the university I have been able to build up a good knowledge base for teaching modelling tasks.	4	.85
about technology use in modelling processes learned at university	As far as the use of technological tools for solving modelling tasks in the classrooms is concerned, I see my preparation at the university as satisfactory.	2	.76

Table 26.2 Correlations between the scales displayed in Table 26.1

Positive self-perception concerning PCK …	(2)	(3)	(4)
in the domain of diagnostical knowledge related to the modelling process (1)	.80***	.64***	.06
relevant for giving specific help to students in the area of modelling (2)		.65***	.10
about modelling learned at university (3)			.45***
about technology use in modelling processes learned at university (4)			

*** p<0.001

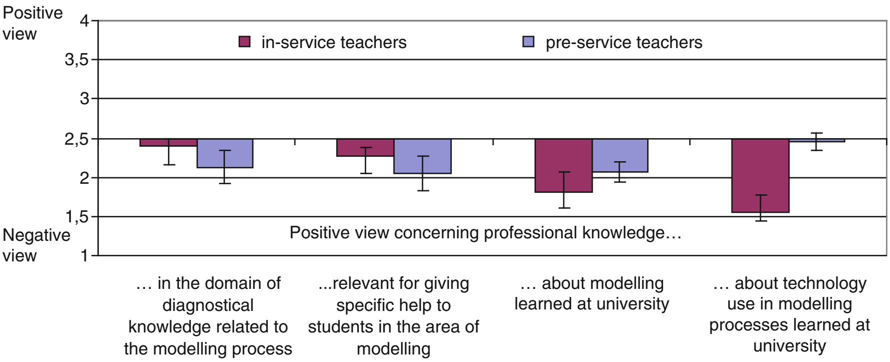

Fig. 26.1 Teachers' self-perceptions of their PCK related to modelling

Especially the in-service teachers saw their PCK development at the university as rather negative.

Table 26.2 shows correlations between the scales considered in this study. The values indicate a relatively high interdependence between the 'direct reference' scales, suggesting that the constructs may not be empirically different. These scales correlate with the self-perceptions of modelling-specific PCK learned at university. This expected correlation appears to underpin the significance of professional development at the university for modelling-specific PCK. Another expected correlation is the interdependence between the 'indirect reference' scales referring to the perceived PCK learning outcomes of professional development at university.

5.2 *Explanations of Self-Perceptions of PCK Related to Modelling*

In order to evaluate whether the findings in Fig. 26.1 correspond to a low modelling-specific PCK, research question (b) focuses on an analysis of the open questions. Relatively closely linked to the first two scales, the teachers were asked which help they would provide students with who are facing problems in the modelling process.

Table 26.3 Examples of answers to the open question on modelling-specific PCK

Teacher	Question: Which possible teacher reactions to potential difficulties of students when working on modelling tasks can you think of?
Claire's answers (pre-service teacher):	Repeating contents again, with which problems have occurred Addressing particular weaknesses of students Using tasks with easily accessible solution path (→ i.e. using also solutions that "make sense")
Fred's answers (in-service teacher):	Re-considering/refining situational model Help when interpreting the mathematical solution Finding further possibilities of solving the problem mathematically

We observed that those in-service and pre-service teachers who were able to give an answer to this open question (about 74 % of all pre-service and 45 % of all in-service teachers) frequently focused on rather general suggestions instead of specific help linked to different phases of the modelling process. Examples of typical answers are given in Table 26.3. Claire's suggestions, for instance, appear to stay on a relatively superficial level, with an emphasis on the teacher's role and little awareness of specific aspects of the modelling process. In contrast, Fred's answers contain aspects specific to modelling and hence suggest that he has corresponding PCK.

Practical experience might play a role: There is a tendency that in-service teachers noted more diverse strategies when asked to do so in the questionnaire, in line with the findings related to research question (a). For example, in-service teachers referred to strategies like using different models, working with analogies, asking well-directed questions or helping students interpreting the results. In contrast, pre-service teachers predominantly listed strategies like giving hints (which were mostly not explained further) or just showing them how they could translate a real-life problem into a mathematical one. In-service teachers also answered that they would help students by giving them help with respect to the content of the problem. None of the pre-service teachers suggested this form of intervention, perhaps a result of a lack of experience with students' work on modelling problems in the classroom.

Qualitative evidence linked to the 'indirect reference' scales can be seen in the sample answers to the question displayed in Table 26.4. For instance, in several answers by in-service and pre-service teachers, there appears to be a demand for more emphasis on practical experience like active work with students in class on modelling tasks. The work on modelling tasks is seen as a valuable preparation activity for building up PCK about modelling by several teachers. This might reflect the process focus and the task culture aspect of modelling-specific PCK. Moreover, meta-knowledge about modelling ("what modelling is") plays a role.

6 Discussion and Conclusions

The teachers' self-perceptions of their PCK related to modelling suggest that there is a need for professional development not only as far as PCK related to modelling is concerned, but also related to the aspect of a pedagogical modelling-specific

Table 26.4 Examples of answers to the open question on modelling-specific PCK

	Question: How can professional development at university be improved concerning the work with modelling tasks in the mathematics classroom?
Answers by pre-service teachers	Practice is lacking not only in the area of modelling in the classroom
	Practice-orientation instead of knowledge orientation
	Putting in place special university courses that put the focus on modelling
	Teacher education: dealing with modelling (for more than one hour)
Answers by in-service teachers	Presenting what modelling is, producing examples and discussing them
	Solving tasks, that can be used also in the school context
	Work on such tasks (university level), for sharing the situation and developing solution strategies

self-efficacy of teachers. This self-efficacy may be supported by positive experiences of the teachers with modelling tasks in the classroom. In particular, professional development support for teachers related to the use of technology in the modelling process might be helpful according to the in-service teachers' self-perceptions. Even if the results have to be interpreted with care, given the limited sample size, the results related to research question (a) indicate that there might be differences between pre-service and in-service teachers concerning the perception of their education at university. These differences might stem from recent developments in the professional development program at the university that include the idea of modelling in relation with mathematical contents and mathematical education goals. Moreover, the pre-service teachers' comparatively high self-efficacy concerning the application of technology when dealing with modelling tasks might not be fully justified – it could be seen as a result of their lack of practice in the mathematics classroom. The pre-service teachers' assessment concerning their relatively good knowledge towards the use of technology might change when they need to use technology in the setting of modelling tasks with all the possible impeding conditions.

However, intensified professional development in the domain of modelling-specific PCK appears to be needed, both in initial teacher education at the university level and at the ongoing professional development level for in-service teachers, even in the case of existing modelling courses at universities. The relatively non-optimistic self-perceptions of the teachers' PCK point to a need even seen from the teachers' own perspective. These findings are in line with prior empirical results about professional knowledge related to modelling (Kuntze 2011; Maaß and Gurlitt 2009; Siller et al. 2011).

The results related to research question (b) raise the issue of the role of instructional practice and classroom experience. Classroom experience might shift the perspective not only of learning at the university, but also of the teachers' PCK.

Hence, the development of self-perceptions of PCK against the experience of pre-service and in-service teachers merits deepened attention in further research, for example, in deepening qualitative studies.

There appear to be differences in the answering patterns of the pre-service and the in-service teaches. Even if the data in Table 26.3 suggest that there are in-service teachers who have developed modelling-specific PCK, the relatively low answering rate of the in-service teachers indicates that there are other in-service teachers who might not have been able to describe any modelling-specific help to learners in the classroom.

As far as the pre-service teachers are concerned, the self-perceptions of modelling-specific PCK tended to be lower, in line with less specific answers to the open question in Table 26.3. However, the pre-service teachers showed a less negative view of their PCK development related to the use of technology for solving modelling tasks experienced at university. These findings appear to draw a picture of professional knowledge in a phase of development. With intensified and specific opportunities of professional learning and support when linking this knowledge to classroom experience, there is the hope that these teachers will continue to build up modelling-specific professional knowledge. However, pre-service learning in the area of modelling should be combined with continuous professional development activities, as the findings related to providing specific help in classroom situations suggest.

Generally, the teachers' answers to the open questions appear to support the validity of the quantitative indicator-like scales. For example, the answers that stay on a relatively superficial level, with an emphasis on the teacher's role and little awareness of specific aspects of the modelling process can be interpreted as non-optimal modelling-specific PCK, in line with the non-optimistic self-perceptions reported by the teachers.

As a conclusion for practice, not only pre-service teachers' professional knowledge but also in-service teachers' professional knowledge concerning modelling should be developed further. In order to meet this demand, one field of activity in the project ABCmaths (www.abcmaths.net) aims at providing teachers with specific learning opportunities connected to modelling. The findings also indicate a need for further research into the structure of professional teacher knowledge concerning modelling. Such research could provide an empirical base for the conception of sustainable professional development activities for teachers about modelling.

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