The Preparation of Primary Mathematics Teachers in Singapore: Programs and Outcomes from the TEDS-M Study

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Abstract This paper describes five aspects of primary mathematics teacher education in Singapore: (a) the teaching profession in Singapore, (b) the structure of pre-service teacher education programs offered by the National Institute of Education, (c) self-reports of Singapore future primary mathematics teachers about the opportunities to learn mathematics-related contents offered by these programs, based on the TEDS-M (Teacher Education and Development Study in Mathematics) survey, (d) the performance of these future teachers in mathematics content knowledge and mathematics pedagogical content knowledge assessed by the TEDS-M study, and (e) the relationships of opportunities to learn with this performance. The paper concludes with some suggestions about how to improve the quality of initial teacher preparation in the areas of recruitment and training.

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1 Introduction and Main Aim of Paper

Recruiting, training, certifying, employing, developing and retaining well-qualified teachers are critical, inter-related issues confronted by policy-makers, education administrators, and teacher educators all over the world. These issues are premised on the claim that well-qualified teachers exert critical impact on student learning (Gopinathan et al. 2008; Izumi and Evers 2002; Schwille and Dembélé 2007; Wang et al. 2003). This claim is supported by studies that show that teachers' mathematical knowledge has positive effects on the mathematics achievement of primary pupils (e.g., Hill et al. 2005) and secondary students (e.g., Baumert et al. 2010). Citing a South Korean official, the authors of the McKinsey report on the best-performing school systems noted that "the quality of an education system cannot exceed the quality of its teachers" (Barber and Mourshed 2007). In an updated report, Auguste et al. (2010) stressed that high-performing school systems such as Singapore, Finland, and South Korea recruit their trainee teachers from the top third of the academic cohorts. The Singapore Ministry of Education (MOE) also accepts that teachers play a critical role in preparing students for the future and in implementing its curriculum for the 21st century. In 2009, it released the vision statement for its teachers: "Lead. Care. Inspire".[1] This vision goes beyond the subject mastery commonly associated with competent teachers to include qualities of leadership, care for the students, and the capacity to inspire students to achieve their potential.

The above-mentioned claims and expectations apply to mathematics teachers too. Several international studies have compared mathematics teacher education systems from around the world (Burghes 2008; Strässer et al. 2003). The recently completed project called Teacher Education and Development Study in Mathematics (TEDS-M) is the first international comparative study undertaken under the auspices of the International Association for the Evaluation of Educational Achievement (IEA). It involved about 14500 primary and 8600 secondary future mathematics teachers from 780 pre-service teacher education programs sampled from 490 teacher education institutes in 17 countries. The framework and objectives of the TEDS-M study, including selected questionnaires, can be found in Tatto et al. (2008). The main aim of this paper is to describe the pre-service training of future primary mathematics teachers in Singapore using relevant findings from the TEDS-M study and local documents, interpreted from our perspectives as Singapore mathematics teacher educators over many years. To achieve this aim, we will cover the following five areas:

1. National context for teacher preparation in Singapore, focusing on the recruitment of future teachers for Singapore government schools.

[1] http://www.moe.gov.sg/media/press/2009/08/vision-for-the-teaching-servic.php.

2. Pre-service primary mathematics teacher education at the National Institute of Education (NIE), the sole teacher education institute in Singapore. Details of the four relevant programs are described to enable meaningful international comparisons.
3. Self-reports about the opportunities to learn mathematics-related contents provided by the future teachers who participated in the TEDS-M study. These findings highlight areas of strengths and weaknesses of the Singapore teacher education programs that might be of interest to international readers.
4. Performance on measures of Mathematics Content Knowledge (MCK) and Mathematics Pedagogical Content Knowledge (MPCK) of the Singapore future primary mathematics teachers in the TEDS-M study, including detailed comments on four selected items. This discussion highlights the necessity to focus on the *mathematics* aspect of primary teacher education in comparative studies.
5. The relationships between the opportunities to learn mathematics-related contents and the performance of these future teachers in MCK and MPCK.

In the final Discussions and Conclusion section, we offer some suggestions about how to improve the quality of initial teacher preparation in the areas of recruitment and training. Relevant international findings will be mentioned only briefly because they are not the main focus of this paper.

2 The Singapore Context for Teacher Preparation

Singapore is a small country of area 710 km^2 with 5 million people. There are 172 primary schools with 13500 primary teachers and 265000 primary school pupils (Ministry of Education 2010). The average class size ranges from 30 in Grade 1 (age 6+) to 37 in Grade 6. Secondary schooling covers lower secondary (Grades 7 and 8) and upper secondary (Grades 9 and 10) levels. Post-secondary education is delivered through junior colleges (Grades 11 and 12), polytechnics, and other specialized institutes. Mathematics is a compulsory subject from Grade 1 to 10, and it is taught in English rather than the student's mother tongue.

The Ministry of Education (MOE) controls the recruitment, employment, and retention of teachers in government schools, whereas the training and certification of teachers are the responsibility of the NIE. With the exception of a small percentage of teachers recruited from overseas, teachers in Singapore schools generally receive their pre-service teacher education at the NIE. This clear demarcation of roles for the MOE and the NIE has been successful in the past sixty years in producing good quality teachers for Singapore because both organizations have worked very closely together for the same goal of producing a strong teaching force. In 2009, the education expenditure was S$8.70 billion, about 3.4 % of GDP of that year. About S$0.12 billion was spent on teacher education at the NIE, a considerable investment by the Singapore government in this area.

The main steps to becoming a fully qualified teacher in Singapore government schools are shown in Fig. 1. Prospective teachers can apply to the MOE for teaching positions all year round. The requisite academic qualifications vary with the

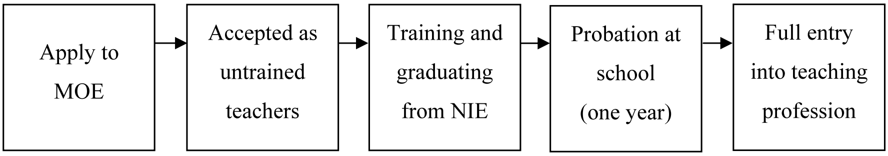


Fig. 1 From application to entry into the teaching profession in Singapore

teacher education programs. For the three concurrent programs (Diploma in Education, Bachelor of Arts with Education, and Bachelor of Science with Education, for which more details are provided in Sect. 3), the minimum requirement is good grades in the Singapore-Cambridge General Certificate of Education Advanced-Level (GCE A-Level) obtained by graduates from the junior colleges or a strong diploma from one of the five Singapore polytechnics. For the consecutive program (Postgraduate Diploma in Education), the applicant must hold a Bachelor degree from a recognized university. In addition, all applicants, irrespective of the programs, must have obtained the Singapore-Cambridge General Certificate of Education Ordinary-Level (O-Level) passes, or equivalent, in Mathematics and English Language. This underscores the importance of Mathematics in the training of all Singapore teachers. NIE programs are not expected to teach future teachers about mathematics that has been learned in secondary schools, as happens in some countries (Schwille and Dembélé 2007). Because English is the main medium of instruction in Singapore, all applicants, unless exempted, are required to pass an English Entrance Proficiency Test for oral and listening comprehension. Short-listed applicants are interviewed by teams comprising MOE officials and NIE faculty to assess the applicants' interest in teaching and their personal and leadership qualities.

Successful applicants are then appointed as untrained teachers (called General Education Officers) and sent to the NIE as student (or trainee) teachers. Applicants whose suitability for teaching requires further confirmation may be offered contracts to work in schools where the school leaders can provide additional screening. Those who are found to have the potential to become competent teachers are offered admission to the NIE.

The success rate for recruitment is fairly low with about one acceptance per eight applicants. Controlling entry to the profession prior to training ensures that only capable and committed applicants are selected for training and eventual entry into the teaching profession. This also reduces the "wastage" that occurs when too many teachers are trained, many of whom may have difficulty gaining teaching employment after graduation, thus becoming a problem as happened in some countries.

In recent years, the MOE has actively recruited applicants who wish to switch from their current careers (e.g., engineering or finance) to teaching, because they "can bring something new to the classroom, and share their wealth of knowledge from their previous careers with a generation eager to learn more".[2] These are called "mid-career" applicants. In the past three years, about 35 % of the recruits had prior working experience outside teaching (Tan 2011). With its competitive salaries (for example, beginning teacher salaries in Singapore are comparable to those of scientists, as noted by Carnoy et al. 2009) and many benefits, teaching is an attractive but also a demanding profession in Singapore.

Student teachers at the NIE receive full salary up to two years of their training and their tuition fees at the NIE are paid by the MOE. Under this unique recruitment-cum-training system, student teachers in Singapore are paid to receive teacher training. Furthermore, they do not have to worry about looking for future employment as teachers or face additional certification requirements after completion of their program. However, they need to serve a teaching bond (contract) from three to six years depending on the duration of the training programs. Those who fail to graduate are normally deemed to have broken the teaching bond and have to pay liquidated damages. This bond serves as a strong incentive for student teachers to graduate within the stipulated duration of their program; indeed, the failure rate at the NIE is less than 1 % per cohort. In recent years, about 1500 student teachers graduate annually from various NIE primary teacher education programs.

After graduation from the NIE, the beginning teachers are placed on probation for a year, during which they teach about 80 % of the normal teaching hours, are mentored at the school level and participate in the Structured Mentoring Programme conducted by the recently launched Academy of Singapore Teachers. At the end of this probation period, they are assessed by their school leaders, and upon satisfactory performance in teaching and professional duties, they are confirmed as teachers in the civil service. No official data are available about attrition at this stage, but it is understood that very few beginning teachers fail this probation assessment.

3 Pre-service Primary Mathematics Teacher Education at the NIE

This section describes the four NIE programs that train future primary mathematics teachers. The information in this section will help readers understand the perceptions reported by the future teachers who participated in the TEDS-M study, to be discussed in Sect. 4.

3.1 Four NIE Primary Teacher Education Programs

Future primary mathematics teachers are enrolled in one of the following four programs:

[2] http://www.moe.gov.sg/careers/teach/applying/mid-career/.

1. Diploma in Education (Dip Ed, with options A or C). This is a two-year concurrent program leading to the award of a diploma rather than a degree. It has two options: option A covers two teaching subjects (usually English and Mathematics), and option C covers three teaching subjects (usually English, Mathematics, and a third subject such as Science or Social Studies). This is the shortest program among the 780 international teacher education programs sampled in the TEDS-M study. Student teachers are assigned to the option by the MOE. However, in recent years, as schools generally regard teachers with three teaching subjects as easier to deploy, the MOE ceased assigning student teachers to option A from 2009.
2. Bachelor of Arts with Education, BA (Ed). This is a four-year concurrent program covering three teaching subjects: English, Mathematics, and a third teaching subject, which may be Social Studies, Art, Music, and Science (rare). These student teachers also take an academic (content) subject at university level and that subject may be chosen from English Language, English Literature, History, Geography, Art, Music, or Drama.
3. Bachelor of Science with Education, BSc (Ed). This is also a four-year concurrent program similar in structure to the BA (Ed). However, the third teaching subject for BSc (Ed) is usually Science and the academic subject is Biology, Chemistry, Physics, or Mathematics. In fact, it is the academic subject that determines whether the student teacher is in the BA (Ed) or BSc (Ed) program.
4. Postgraduate Diploma in Education (PGDE Primary). This is a one-year consecutive program for graduates, with options A or C, similar to those available in the Diploma in Education (above). Option A was also discontinued in 2009 for similar reasons about deployment.

3.2 Changes to the NIE Bachelor Programs

The two Bachelor programs were first introduced in 1991. Since then, their curriculum structures have been revised four times. A review conducted in 2003/2004 found that student teachers generally did not have the requisite content foundation for four teaching subjects (English, Mathematics, Science, and Social Studies), and hence, they could not cope well with learning to teach four subjects. Furthermore, the primary school curriculum has placed much heavier emphasis on English and Mathematics in recent years, and, to a lesser extent, Science, so that student teachers ought to focus on fewer teaching subjects. This resulted in changing the Bachelor programs from four teaching subjects (called the C-series) to three teaching subjects (the A-series), which is the current program described above. For the TEDS-M study, the Singapore data were collected in 2008 from future teachers who were the last cohort of the C-series. Hence, their self-reports of OTL may be different from those of the current future teachers in these Bachelor programs.

These changes to the NIE Bachelor programs illustrate the international trend of rapid changes made to teacher education programs in response to various external challenges and internal reviews. Thus comparative studies in teacher education may become obsolete from the time of data collection to publication of the findings. Nevertheless, these studies are still valuable to shed lights on the underlying factors and contexts that lead to such changes in structures and programs.

3.3 Curriculum Structures of NIE Primary Teacher Education Programs

To facilitate valid international comparisons, the TEDS-M Study organized primary teacher preparation programs into the following four program groups, according to the teaching role for which they would qualify:

- Lower Primary Generalist (Grade 4 maximum)
- Primary Generalist (Grade 6 maximum)
- Primary/Lower Secondary Generalist (Grade 10 maximum)
- Primary Mathematics Specialist

The four NIE programs were classified into Primary Generalist and Primary Mathematics Specialist, as shown in Table 1. Even though several NIE programs fell within the same program group, they actually have very different curriculum structures and durations (see Table 1). Thus, future teachers from the same program group may respond differently to the TEDS-M survey and tests.

3.4 Courses in the NIE Primary Teacher Education Programs

The courses undertaken in NIE programs are classified into Education Studies (ES), Curriculum Studies (CS), Subject Knowledge (SK), Academic Subjects (AS), Practicum (PRACT), and Others (miscellaneous courses and electives). The numbers of Academic Units (AU) of these courses are given in Table 1.

One AU is equivalent to 12 contact hours, and each course is to be completed within one semester. There are two semesters per academic year. These courses are briefly described below; the course outlines can be found in the website of the Office of Teacher Education of the NIE.[3]

Education Studies (ES) courses cover the Singapore education system and educational theories and practices. These courses help student teachers to understand the social, psychological, and technological contexts of schooling in Singapore as well as in general education.

The Curriculum Studies (CS) courses deal with the pedagogy (or methodology of teaching) of specific school subjects, such as Mathematics and English. The Mathematics CS courses are very similar across the four programs and cover the Singapore

[3] http://www.nie.edu.sg/programme-offices/office-teacher-education.

Table 1 Number of Academic Units (AU) of NIE primary teacher education programs by TEDS-M program groups

Program groups	NIE programs	Duration (years)	Types of Courses (AU)					PRACT (AU)	Total (AU)
			ES	CS Maths	SK Maths	AS	Others		
Primary Generalist (Grade 6 maximum)	Dip Ed (C)	2	8	8	6	None	32	15	69
	BA (Ed)	4	12	10	6	39	40	21	128
	BSc (Ed)	4	12	10	6	39	40	21	128
	PGDE (P) (C)	1	8	8	None	None	18	10	44
Primary Math Specialist	Dip Ed (A)	2	8	10	9	None	23	15	65
	PGDE (P) (A)	1	8	8	4	None	14	10	44

Notes. 1 AU = 12 contact hours. ES = Education Studies; CS = Curriculum Studies; SK = Subject Knowledge; AS = Academic Subject; Others: CS and SK courses for subjects other than Mathematics and various electives; PRACT = Practicum

mathematics curriculum, learning theories, lesson planning, assessment of mathematics learning, error analysis, and methods for teaching specific topics, such as whole numbers, fractions, geometry, and algebra. Since pedagogy is partially universal (e.g., practice mathematics skills) and partially culturally situated (e.g., Cai et al. 2009), NIE mathematics educators have integrated mathematics pedagogical principles from international research and practices with local contexts and lessons learned from local implementations, and they have published resource books to be used in Mathematics CS courses (Lee and Lee 2009; Yeap 2008). This enhances the links of these CS courses to the realities of local classroom teaching.

The Subject Knowledge (SK) courses help student teachers to gain a deeper understanding of the contents of the school subjects they are being prepared to teach. The Mathematics SK courses aim to enhance conceptual understanding and mastery of whole numbers (different numeration systems and divisibility), geometry (properties of geometric figures with proofs, tessellations, and use of dynamic geometry software), deductive and inductive reasoning, and statistical investigations. These topics are related to but go beyond the school mathematics curriculum. The SK courses exemplify international discussion about what is the appropriate mathematics that school teachers need to master (Ball et al. 2005; Kulm 2008). In addition to specific topics, the SK courses also cover problem solving, which is the major focus of the Singapore primary mathematics curriculum. This curriculum includes 12 problem solving heuristics, such as *draw a diagram, make a list, guess and check*, and *work backwards*, but the "model method" (Kho et al. 2009) is the most important one that student teachers need to learn to teach. This "model method" was developed by Singapore mathematics curriculum specialists in the 1980s to help primary school pupils solve word problems by drawing diagrams that reflect the underlying structures of the problems. This method has become the most distinctive feature of the so-called Singapore Math and it is now taught in several countries, including a US online adaptation called "Thinking blocks" (http://www.thinkingblocks.com/index.html). These SK courses underscore the strong alignment of teacher education with school curriculum and assessment. Furthermore, some SK activities such as hands-on activities and group discussions let student teachers "experience for themselves such sense-making from the perspective of learners of Mathematics" (Lim-Teo 2010, p. 210). This re-learning of familiar mathematics through new learning experiences with constructivist approaches emphasizing reasoning and sense-making aims to inculcate a deeper understanding of mathematics and to exemplify effective mathematics learning. However, feedback from the student teachers has been mixed (Lim-Teo 2010). There are also logistic constraints, for example, SK and CS courses are taught by different lecturers, sometimes resulting in weak connections between these courses (Lim-Teo 2009).

The Academic Subject (AS) courses are undergraduate courses that provide in-depth mastery of the contents of the respective disciplines. These are required for the Bachelor programs only, and the student teachers select only one academic discipline to study. AS Mathematics includes courses in Calculus, Linear Algebra, Statistics, Analysis, and others.

A large component of the "Others" category includes the CS and SK courses of teaching subjects other than Mathematics, such as English and Science. It also includes two compulsory courses specific to teaching. The first one is a communication skills course to equip student teachers with stronger skills of using English for teaching purposes. The second is a service learning project, which carries no AU or grade. The main aim of this group project is to develop in the student teachers a better appreciation of the needs to go beyond classroom teaching to serve the community. Past projects include engaging Institute of Mental Health patients in meaningful activities and helping young members of local charity organizations to develop leadership skills. Another aim of this group project is help student teachers to acquire the skills of project management that they can apply when they supervise similar projects in the schools in the future. This course provides yet another example of the close alignment of NIE programs with national and school contexts, thus enhancing the relevance of teacher preparation.

Most of the courses described above are delivered through a combination of lectures, tutorials, and group activities in a technology-enhanced teaching environment, including e-lectures and use of the Blackboard course management system. Student teachers are now provided with individual laptops during their training.

Most NIE courses are assessed using a combination of written tests, practical tests, essays, projects, micro-teaching, class participation, online forum, and so on. Most AS courses have final written examinations held at the end of each semester, but non-AS courses do not have major written examinations.

The practicum (field experience) is an essential component in most teacher education programs all over the world. In Singapore, it contributes between 15 % to 25 % of the requirements for NIE programs: 15 weeks in two semesters for Dip Ed; 22 weeks spread over three years for the Bachelor programs; 10 weeks in one semester for PGDE. During the practicum, every student teacher is assigned a School Coordinating Mentor (SCM), one or more Cooperating Teachers (CT) in their teaching subjects, and one NIE supervisor. The student teachers initially observe lessons conducted by their CTs and later plan and teach their own lessons, reflecting on feedback given by their CTs and the NIE supervisor. They teach about two thirds of the teaching hours of an average teacher so that they have more time to prepare engaging lessons based on what they have learned from their NIE courses. Formative and summative assessment of their teaching cover competencies in lesson preparation, lesson delivery, classroom management, assessment of learning, and professional qualities, such as showing care for their students, being responsive to feedback, and professional dressing. However, NIE student teachers are not required to write an extensive report or thesis about their practicum or to conduct research during their practicum. Their practicum grade (Distinction, Credit, Pass, or Fail) is determined by an assessment panel comprising the school principal, SCM, CTs, and the NIE supervisor. This panel allows for negotiation of the final grade from different perspectives:

- theory-practice link by the NIE supervisor,
- classroom teaching by the CTs, and
- professional activities and conduct in the school by the SCM and the principal.

This multi-party negotiation reduces the risk that the performance of the student teachers will be assessed using only a narrow set of criteria. Student teachers are usually appointed back to their practicum school after graduation.

To graduate, student teachers graduate must pass every course and obtain a Cumulative Grade Point Average (CGPA) of 2 out of a maximum of 5. Practicum is not included in the computation of the CGPA since it is awarded only a nominal grade. In general, the graduation rate is very high with less than 1 % not graduating with their cohort. The main reasons for non-graduation include inability to cope with the workload during practicum, weaknesses in content knowledge that become evident when the student teachers have difficulty in answering pupil questions, ineffective classroom management, and poor teacher-pupil rapport. Student teachers are given two chances to pass the final practicum.

4 Opportunities to Learn (OTL) Mathematics-Related Contents: Findings from TEDS-M Study

This section reports some findings from the TEDS-M survey about opportunities to learn mathematics-related contents offered by the NIE programs, as reported by the Singapore future primary mathematics teachers. For the rest of this paper, we will use the label FPMT to refer to these student teachers. We will begin this section with a brief description of the Singapore sample.

Table 2 NIE future primary mathematics teachers in the TEDS-M study

Program groups	NIE programs	Number	% Female	% mid-career
Primary generalist (Grade 6 maximum)	Dip Ed (C)	107	81.4	51
	BA (Ed)	31	71.0	19
	BSc (Ed)	36	61.2	11
	PGDE (P) (C)	89	77.6	56
Primary math specialist	Dip Ed (A)	45	66.7	49
	PGDE (P) (A)	72	72.3	63

Notes. Option (A) covers two teaching subjects and option (C) covers three teaching subjects

4.1 The Singapore Sample for TEDS-M Study

Since there is only one teacher training institute in Singapore, a census sample was taken for the TEDS-M study. All FPMT who had taken the Mathematics CS courses were requested to take the TEDS-M test and survey in May 2008 after their final practicum. The response rates ranged from 86 % to 96 %, and these satisfied the criteria set by the IEA.

The breakdown of the NIE sample by program groups and NIE programs is shown in Table 2. There were more females than males in all the NIE programs, and this is consistent with international trends. The percentages of mid-career FPMT varied from a low 11 % for BSc (Ed) (they were fresh graduates from junior colleges or polytechnics) to a high 63 % for PGDE (option A) (who already held a degree). This presents a challenge to NIE teacher educators, and we will discuss this in Sect. 7.2.

4.2 Opportunities to Learn Mathematics-Related Contents

The TEDS-M Future Teacher Questionnaire covers three aspects of opportunities to learn mathematics-related contents in the pre-service teacher education programs: tertiary level mathematics, school level mathematics, and mathematics pedagogy (Tatto et al. 2008).

To explore opportunities to learn tertiary level mathematics, FPMT were asked to indicate whether or not they had studied each of 17 topics in tertiary level mathematics, such as Calculus and Number Theory, either in their current or previous program. The mean proportions of topics studied as reported by the future teachers within each program group by country were used to compute a Tertiary Level Mathematics OTL scale, with values ranging from 0 to 1. NIE FPMT reported the lowest mean of 0.38 among the countries in the respective program groups, and this was due to the fact that tertiary level mathematics, as covered in the NIE AS courses, was optional for most of these FPMT. These tertiary level mathematics topics may constitute the "mathematics on the horizon" (Hill et al. 2008) to enrich the mathematical experiences of the future teachers. However, there is also the risk of "vertical disconnect" (Cuoco 2001) between what these future teachers need to know and what they have to teach.

In a similar way, the School Level Mathematics OTL scale was created based on responses to seven major topics: numbers, measurement, geometry, functions and relations, data representation, probability and statistics, and validation, structuring, and abstracting. NIE FPMT reported a moderate mean proportion of 0.62, which was lower than the means of most of the countries in the respective program groups. Most of these topics were covered in the NIE SK courses. The exception was the topics of validation, structuring, and abstracting; indeed, only 10 % of NIE FPMT reported having the opportunities to learn these topics.

The opportunities to learn mathematics pedagogy were evaluated in two ways. The first way, similar to the two scales above, was to create the Mathematics Pedagogy OTL scale by computing the mean proportions of how many of the eight mathematics education topics were studied. These eight topics were: foundations of mathematics, context of mathematics education, development of mathematical ability and thinking, mathematics instruction, development of teaching plans, mathematics teaching, mathematics standards and curriculum, and affective issues in mathematics. NIE FPMT reported a moderate proportion of 0.70 of coverage of these topics in their Mathematics CS courses. This suggests that the NIE programs were quite adequate in covering the mathematics pedagogy topics listed in the TEDS-M survey.

The second way to measure mathematics pedagogy OTL was to ask future teachers to indicate on a 4-point scale (never = 1, rarely = 2, occasionally = 3, often = 4) how frequently they were engaged in each of the 15 listed activities during the NIE Mathematics CS courses. On the basis of the mean scores of individual items, NIE FPMT reported that they frequently listened to lectures (3.57), worked in groups (with highest mean of 3.70), participated in whole class discussion (3.32), and solved mathematics problems using multiple strategies (3.23). The findings about these four activities show that the NIE Mathematics CS courses had provided these FPMT with learning experiences that covered both direct instruction and the constructivist approaches. At the other end of the scale, NIE FPMT reported that they rarely wrote mathematical proofs (with lowest mean of 1.88) and read about research on mathematics (2.17). Whether or not *primary* mathematics teachers need to know how to write proofs and to understand mathematics research is open for further discussion since they hardly encounter these two situations in their normal teaching.

Finally, 94 % of NIE FPMT rated their training program as highly effective or effective, compared to only 74 % internationally. Thus, NIE FPMT were more positive about their training than the international future teachers about their programs.

We will explore the relationships of these OTL measures with performance in MCK and MPCK in Sect. 6 below.

5 Performance in TEDS-M Primary MCK and MPCK Tests

The TEDS-M MCK framework covers four content knowledge domains (Number, Geometry, Algebra, and Data) and three cognitive domains (Knowing, Applying, and Reasoning). The MPCK framework covers three aspects: Mathematical curricular knowledge, Knowledge of planning for mathematics teaching and learning (pre-active), and Enacting mathematics for teaching and learning (interactive) (Tatto et al. 2008). Five different booklets were created to cover these test items, and each FPMT took only one of these booklets. This arrangement ensured that, across any sufficiently large group of respondents, the full range of content was tested.

5.1 Overall Performance in MCK and MPCK

The overall performance of NIE FPMT in the TEDS-M tests is given in Table 3. As a country, Singapore ranked first or second in MCK and MPCK in each program group, but individual NIE programs had different performance levels. The BSc (Ed) group had the best performance in both MCK and MPCK because some of them had the opportunity to study tertiary level mathematics. The Dip Ed (C) group had the lowest performance, and a plausible reason was that these student teachers, who did not qualify for a degree program, had to learn to teach three teaching subjects within two years. This may be quite challenging for some of them.

5.2 Relationship Between MCK and MPCK

The Pearson product-moment correlation coefficients between the MCK and MPCK scores, also given in Table 3, range from 0.32 (Singapore) to 0.54 (Poland). These values suggest that both types of tests had measured some common trait, likely to be mathematics in this case.

The average of the six correlations for countries in the Generalist group was 0.39, lower than the average of the six correlations for countries in the Specialist group (0.48). This may be because future teachers in the Specialist group tend to spend more time and effort working on MCK and MPCK for only one or two subjects, and this more focused experience is likely to mutually reinforce their development of MCK and MPCK. Although correlations do not necessarily imply causation, the fact that MPCK is built on mathematics suggests that a sufficient level of mathematics content knowledge is necessary for the development of sound pedagogical content knowledge.

In the case of Singapore, the correlations were moderated by the types of programs, from the lowest of 0.17 (PGDE, option C, generalist) to the highest of 0.42

Table 3 MCK and MPCK mean scores and correlations of future primary mathematics teachers

Program groups	Country/NIE programs	MCK Mean	MPCK Mean	Pearson r between MCK and MPCK
Primary Generalist (Grade 6 maximum)	**Singapore: BSc (Ed)**	**625**	**626**	**0.28**
	Chinese Taipei	623	592	0.43
	Singapore: PGDE(P) (C)	**593**	**596**	**0.17**
	Singapore (All)	**586**	**588**	**0.32**
	Singapore: BA (Ed)	**586**	**587**	**0.30**
	Singapore: Dip Ed (C)	**567**	**568**	**0.35**
	Switzerland	547	539	0.38
	USA	517	543	0.48
	Spain	481	492	0.41
	Philippines	439	457	0.34
Primary Mathematics Specialist	Poland	614	574	0.54
	Singapore: PGDE(P) (A)	**600**	**601**	**0.42**
	Singapore (All)	**599**	**603**	**0.40**
	Singapore: Dip Ed (A)	**598**	**607**	**0.39**
	Germany	555	552	0.52
	Thailand	528	506	0.50
	USA	519	544	0.47
	Malaysia	488	503	0.44

Notes. International mean: 500, standard deviation: 100. Special annotations about primary future teachers in various countries: (a) Poland: Combined participation rate between 60 % and 75 %; institutions with consecutive programs only were not covered. (b) Switzerland: Only institutions where German is the primary language of use and instruction were covered. (c) USA: Only public institutions were covered; combined participation rate between 60 % and 75 %. Caution about comparing findings across these countries. Singapore option (A) covers two teaching subjects and option (C) covers three teaching subjects

(PGDE, option A, specialist). These two extreme values are consistent with the above observation about the generalist-specialist divide and the different opportunities to learn how to teach two or three subjects required by these two different programs.

In the following sections, we have selected, from the items released by TEDS-M, two MCK and two MPCK items for discussion because they illuminate certain *mathematical* ideas that will become apparent later on. The overall results for these four items are given in Table 4.

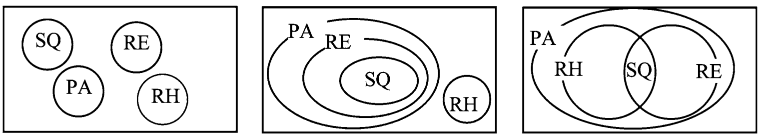
Table 4 Results of Singapore and international performance on two MCK and two MPCK items (Primary) (in percentages to 1 decimal place; correct answers in bold)

		A	B	C	D
MFC204	Singapore	3.9	30.3	**65.7**	
	International	11.2	25.1	**63.7**	
MFC412A	Singapore	**82.2**	4.0	3.3	10.5
	International	**60.0**	6.7	16.2	17.1
MFC412B	Singapore	13.7	**77.7**	8.0	0.7
	International	22.9	**54.8**	18.1	4.2
MFC108	Singapore	27.8	31.2	**32.9**	8.1
	International	30.6	28.5	**30.2**	10.7

		Correct	Partially Correct	Incorrect
MFC208A	Singapore	53.8	13.5	32.7
	International	25.6	15.8	60.4
MFC208B	Singapore	39.2	35.9	24.9
	International	23.0	23.3	58.3

5.3 *MFC204: MCK: Geometry, Knowing*

Three students have drawn the following Venn diagrams showing the relationships between four quadrilaterals: Rectangles (RE), Parallelograms (PA), Rhombuses (RH), and Squares (SQ).



[Tian] [Rini] [Mia]

Which student's diagram is correct? (A) [Tian] (B)[Rini] C [Mia]

This geometry knowledge item is an example of the specialized content knowledge for teaching (Graeber and Tirosh 2008; Hill et al. 2008), and similar Venn diagrams are found in many mathematics textbooks. About 66 % of NIE FPMT had chosen the correct option C. Although the 30 % choosing option B might not know the properties of rhombuses, they seemed to know the relationships among parallelograms, rectangles, and squares. Given that similar geometric relationships have been covered in the NIE SK courses and that this item is likely to be at only Level 3 (Abstraction) of the five levels of the van Hiele theory of geometry thinking (van Hiele 1986), we expect NIE FPMT to perform better in this task than the result reported here.

5.4 MFC412A and MFC412B: MCK: Algebra, Knowing

[Sam] wanted to find three consecutive EVEN numbers that add up to 84.
He wrote the equation $k + (k + 2) + (k + 4) = 84$.

(a) What does the letter k represent?

(A) The least of the three even numbers.
(B) The middle even number.
(C) The greatest of the three even numbers.
(D) The average of the three even numbers.

This is a straightforward test about interpreting the meaning of a letter used in a given equation. Hence, it is not surprising that 82 % of NIE FPMT had chosen the correct option. The small percentage of NIE FPMT (10 %) who had chosen option D may have been prompted by the surface feature of adding numbers together as part of the procedure to find an average.

Part (b) of the above item deals with *odd* numbers.

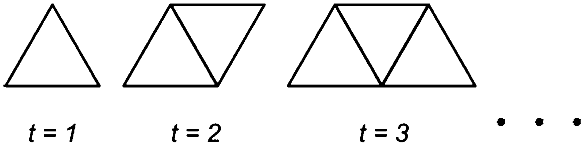
(b) Which of the following expressions could represent the sum of three consecutive ODD numbers?

(A) $m + (m + 1) + (m + 3)$
(B) $m + (m + 2) + (m + 4)$
(C) $m + (m + 3) + (m + 5)$
(D) $m + (m + 4) + (m + 6)$

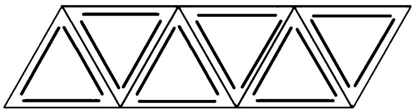
This item may be “tricky” because the correct option involves *even* rather than *odd* numbers. It challenges the future teachers to resolve this mathematical conflict, for example, by identifying the correct option even though it contains no odd numbers! Apparently about 22 % of NIE FPMT were affected by the distraction of seeing “ODD” in the stem and odd numbers in the options (A and C), although 78 % had chosen the correct option. The responses to both items taken together suggest the need to distinguish between surface and deep features of the symbols used in algebra.

5.5 *MFC108: MPCK: Enacting, Advanced Level*

[Amy] is building a sequence of geometric figures with toothpicks by following the pattern shown below. Each new figure has one extra triangle. Variable t denotes the position of a figure in the sequence.



In finding a mathematical description of the pattern, [Amy] explains her thinking by saying: I use three sticks for each triangle.



Then I see that I am counting one stick twice for each triangle, except the last one, so I have to remove those. Variable n represents the total number of toothpicks used in a figure. Which of the equations below best represent [Amy's] statement in algebraic notation?

(A) $n = 2t + 1$ (B) $n = 2(t + 1) - 1$ (C) $n = 3t - (t - 1)$ (D) $n = 3t + 1 - t$

Equivalent algebraic expressions can take different forms depending on how they are derived, especially when manipulatives are used in teaching. This is another form of specialized content knowledge for teaching.

Only 33 % of NIE FPMT could relate Amy's action to the correct expression (C). The other two popular options were B (31 %) and A (28 %); these two answers are mathematically correct, but the $2t$ term in these answers is not *directly* related to Amy's action of beginning with three sticks. As teachers are encouraged to use manipulatives to teach mathematics, it becomes imperative that they can link the kinesthetic actions of using the manipulatives to the underlying mathematics; otherwise, hands-on activities can degenerate into interesting "busy" work devoid of significant mathematics contents. The poor results above (both Singapore and international) suggest the need to find ways to help student teachers establish this critical mathematical-pedagogical link.

5.6 *MFC208A and MFC208B: MPCK: (A) Enacting, Intermediate Level; (B) Enacting, Advanced Level*

A crucial aspect of MPCK is the ability to recognize the mathematical nature of pupils' misconceptions and then to design appropriate remediation to help them

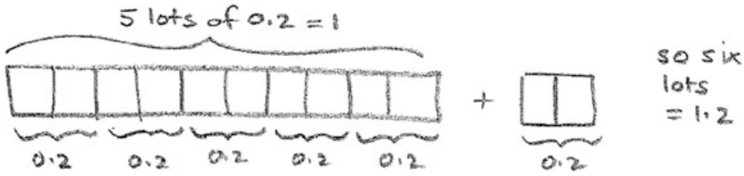


Fig. 2 A correct visual representation for 0.2 × 6

to overcome these misconceptions. At the primary level, visual representation is a powerful technique to help pupils learn, and this should be part of the repertoire of competent mathematics teachers. The following two items deal with the widely cited misconceptions of "multiplication makes bigger" and "division makes smaller" and how to deal with them.

> [Jeremy] notices that when he enters 0.2 × 6 into a calculator his answer is smaller than 6, and when he enters 6 ÷ 0.2 he gets a number greater than 6. He is puzzled by this, and asks his teacher for a new calculator!
> (a) What is [Jeremy's] most likely misconception?

Slightly more than half (54 %) the NIE FPMT could state both misconceptions, for example, "He thinks that when you multiply the answer should be larger and when you divide the answer should be smaller". About 14 % gave only one misconception, probably thinking that the word "is" in the item requires only one answer. What was truly surprising to us was that about one third of NIE FPMT could not recognize these misconceptions, giving irrelevant responses, such as "Jeremy did not know how to use calculator" or "did not understand decimals". These two misconceptions are covered in the NIE CS courses, and this poor result is of particular concern to us.

Part (b) tests whether future teachers could provide a visual representation to help Jeremy come to grips with one of the misconceptions.

> (b) Draw a visual representation that the teacher could use to model 0.2× 6 to help [Jeremy] understand WHY the answer is what it is?

A correct representation given in the TEDS-M marking scheme is shown in Fig. 2.

Partially correct responses include a correct diagram but not showing how 1.2 is obtained. About 40 % of NIE FPMT could give a correct response, and only 23 % gave the correct responses to both parts (a) and (b). These poor results are not satisfactory.

It is worthwhile at this stage to compare the above findings with two Singapore studies that investigated the multiplication misconception in a more direct way

(Cheang et al. 2007; Lim-Teo et al. 2007). The following item was used. It was scored on a 0–4 point scale.

> A pupil tells you that when you multiply two numbers together, the product is always larger than either of the two numbers. How do you respond to the pupil?

In the first study, a cohort of 80 Dip Ed student teachers responded to the item as a pretest in 2003 at the beginning of their program and two years later, 67 of them took it in a posttest at the end of their training. The mean increased significantly from 2.09 to 3.25. In the second study, a cohort of 113 PGDE (Primary) student teachers also performed significantly better between July 2005 and February 2006, with the mean increasing from 2.27 to 3.08. These two findings show that the NIE CS courses had been effective because the student teachers "showed more awareness in the post-test by quoting counter examples of multiplying by one or by zero" (Lim-Teo et al. 2007, p. 252). The TEDS-M items and this local item illustrate that the same misconception can be assessed differently.

A limitation about studying the performance of future teachers using the TEDS-M tests should be noted here. Doing well in these tests may indicate quality program outcomes, but future teachers who score highly in paper-and-pencil tests may or may not become competent mathematics teachers after graduation. This link between pre-service performance and enacted performance in the future is worthy of further investigation.

6 Relationships between OTL and Performance

This section explores the relationships between the OTL measures reported in Sect. 4.2 and the performance of FPMT in MCK and MPCK. Pearson product-moment correlation coefficients were computed between these two sets of variables, and the results are given in Table 5. The weak correlations suggest that further analysis is not warranted.

Performance in MCK was associated with Tertiary Level Mathematics OTL. This is not unexpected because more opportunities to learn higher level mathematics are likely to equip future teachers with stronger content knowledge, which was assessed by the MCK items. A similar positive relationship between MPCK and Tertiary Level Mathematics OTL may arise because the scoring of some of the MPCK items required correct mathematics in the answers. Thus, it appears that Tertiary Level Mathematics OTL could be a factor that led to the significant correlations between MCK and MPCK as reported in Table 3.

A plausible hypothesis is that MPCK is related to Mathematics Pedagogy OTL. However, the near-zero correlations between MPCK and the two measures of mathematics pedagogy OTL did not support this hypothesis. Furthermore, perceptions of effectiveness of the training program were not linked to performance in MCK and MPCK. These two issues require further investigation.

Table 5 Correlations between performance in MCK and MPCK and OTL scales (NIE FPMT)

	TM	SM	MP	Activity	Effective
Mean	0.38	0.62	0.70	2.81	3.14
MCK	0.160	−0.004	0.006	−0.035	−0.019
MPCK	0.153	0.016	−0.005	0.013	0.072

Notes. TM: Tertiary Level Mathematics OTL (0 = Not studied; 1 = Studied); SM: School Level Mathematics OTL (0 = Not studied; 1 = Studied); MP: Mathematics Pedagogy OTL (0 = Not studied; 1 = Studied); Activity: Activities engaged in during mathematics pedagogy course (1 = Never; 4 = Often); Effective: Perceptions of effectiveness of training program (1 = Very ineffective; 4 = Very effective)

7 Discussions and Conclusion

The search for effective strategies to place enough well-qualified mathematics teachers into schools is an ongoing challenge in many countries. Comparative findings about teacher education may suggest "good" practices that are taken for granted in one country but could stimulate other countries to re-examine their current practices leading to possible adaptations. We now offer several suggestions based on our interpretations of the Singapore practices and the TEDS-M findings reported in the earlier sections. These suggestions are discussed under the two broad areas of recruitment and training.

7.1 Recruitment Matters

The international community has stressed the importance of recruiting suitably qualified applicants with strong academic qualifications, communication skills, and appropriate motivations into the teaching profession. We wish to reiterate the point made by Auguste et al. (2010) that the top performing school systems recruit "the top third of the academic cohort" (p. 5) into the teaching profession. If the applicants were weak in subject matter knowledge, then "initial teacher preparation programmes are forced to teach mathematics that could have been learnt in secondary school" (Schwille and Dembélé 2007, p. 61), and this would not be productive use of the limited resources allocated to teacher training in some countries. One Singapore policy worthy of mention is that all primary future teachers must pass O-Level Mathematics, regardless of whether or not they are trained to teach it. This ensures minimum competency of mathematics among all Singapore teachers.

As noted earlier, Singapore has actively recruited qualified professionals who wish to make a mid-career switch to teaching because of their avowed altruistic reasons for becoming teachers, for example, to make a difference to the lives of children. This policy has opened up the education profession to many different types of applicants, and this will help to alleviate any shortage of teacher recruits from the traditional groups, namely fresh graduates from high schools, polytechnics, or universities. However, training these mid-career future teachers presents different challenges than training first-career future teachers. Some mid-career future teachers at the NIE require additional assistance to recall their mathematical content knowledge and skills. Nevertheless, they are more "mature" in understanding the complex contexts of teaching, being able to compare and contrast their previous professional experiences with the values, knowledge, and skills required for teaching. Teacher education institutes that intend to recruit these future teachers should be well prepared to offer different opportunities to learn for these adults, for example, incorporating techniques of andragogy (Knowles et al. 2005) into their course delivery.

A notable recruitment policy in Singapore is that all student teachers selected to attend the NIE draw full salaries for the first two years of training, with tuition fees paid by the MOE. They are able to devote full attention to the training without having to worry about living expenses and fees, as is the case of future teachers in countries that do not provide such generous financial support. This policy is costly, but the Singapore experience shows that it is worthwhile investment as it has produced the required number of qualified teachers for the country.

7.2 Training Counts

Well-qualified recruits require proper training to realize their potentials. Section 3 documents that the NIE, like many teacher education institutes around the world, has designed programs that vary in duration and curriculum structures to train different groups of future teachers. NIE programs have been revised regularly to ensure that they are responsive to both external changes such as recruitment numbers and education initiatives launched by the MOE and within-institution research and feedback from the student teachers about their training. Although such feedback has been gathered in the NIE on a regular basis, TEDS-M provides the first opportunity for the perceptions about the NIE programs to be compared across countries. Differences between NIE and international practices, some of which are briefly reported in Sect. 4, are appropriate starting points for future review. Further insights could be obtained from secondary analyses of the international dataset after it has been released in the public domain.

We mention in Sect. 3.4 that NIE student teachers are exposed to effective pedagogy that is a blend of global "best" practices with local experiences. This is illustrated by the training resources for Mathematics CS courses produced by NIE mathematics educators. This approach could have contributed to the overall positive perception of NIE FPMT to the effectiveness of their programs. On the other hand, the weak correlations of performance in mathematics pedagogical content knowledge with this effectiveness perception and opportunities to learn mathematics pedagogy as noted in Sect. 6 suggest that helping future teachers to develop effective pedagogy is more complex than merely providing opportunities to learn in specific courses conducted on campus.

Training that counts should include activities specially targeted for the specific subjects. For Mathematics, NIE student teachers are trained to strengthen their ability to solve and design challenging mathematics problems using the “model method” in the SK courses. The relatively high scores of NIE FPMT on the TEDS-M tests suggest that they have mastered much of the MCK and MPCK domains tested, but the detailed analyses of the four items given in Sect. 5 have highlighted some gaps in their mastery of the more advanced teaching scenarios. For example, it was noted above that as many as one third of NIE future teachers may need more opportunities to learn about ways to deal with pupils’ misconceptions in mathematics. Further analyses of the remaining items in the TEDS-M tests will inform mathematics educators of different teaching scenarios that they can discuss in their courses.

The released MCK and MPCK items from the TEDS-M study can be used as another training resource. The NIE TEDS-M team is preparing a book consisting of these released items, the scoring guides, the Singapore results against international benchmark, and samples of constructed responses from NIE FPMT. NIE lecturers can use these materials in their lessons with future cohorts of student teachers in a number of ways: explore strategies to remedy misconceptions, design classroom activities that mirror the scenarios described in the TEDS-M items, linking assessment items with the TEDS-M framework, and so forth. Thus, although the TEDS-M items were originally created as “assessment *of* teacher training” (summative), it can be used as “assessment *for* teacher training” (formative). Other countries might wish to develop similar materials from the rich data from the TEDS-M study.

At the NIE, mathematicians teach the SK and AS courses and mathematics educators teach the CS courses, but they belong to the same Academic Group (equivalent to department). Under this organization, there are many opportunities for mathematicians and mathematics educators to work in committees and projects that draw on their separate expertise to achieve the common goal of training competent mathematics teachers. They can also share information about the same student teachers who have taken these different types of courses. Furthermore, all NIE mathematicians learn to supervise practicum of student teachers at secondary schools through a process of informal mentoring, and this requirement provides an important opportunity for them to observe first-hand school mathematics teaching and to share their views as a subject specialist with the student teachers. Getting mathematicians actively involved within a well-defined structure in the training of future mathematics teachers is still not common in traditional teacher education institutes in many countries, but this could be a promising area to strengthen the discipline-pedagogy link as a factor to make pre-service training really count.

Although the TEDS-M study did not address in-service teacher education, it is necessary to consider pre-service and in-service teacher education along a continuum of life-long learning (Musset 2010). As the sole teacher education institute in Singapore, the NIE is in the prime position to establish this continuity by balancing the contents and delivery of both types of training, for example, to decide whether to include proofs and validation topics in initial teacher preparation or in-service professional development. In countries where these two types of training are offered by different institutes or where there is little collaboration among the ministry of education, schools, and teacher education institutes, providing this coherent transition can be challenging but this needs to be properly addressed.

To conclude, we hope to learn much more from the international reports of the TEDS-M study and comparative analyses of the TEDS-M data. Through participating in fruitful dialogues across different systems, teacher educators from around the world can work together to create quality teacher education programs so that well-qualified teachers are available to educate their pupils to lead meaningful lives in the 21st century.

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