An Examination of Future Primary Teachers Attitudes About the Teaching of Mathematics: An International Perspective

Nathan Burroughs and William Schmidt

Abstract In this paper we explore the dynamics of teacher beliefs about mathematics with a special focus on future primary teachers. After reviewing earlier research about teacher beliefs, with special attention to the MT21 study and other work based on TEDS data, we examine the relationship among the different dimensions of teacher beliefs and the extent to which these beliefs are associated with teacher knowledge. We find considerable average variation in teacher beliefs about teaching and learning mathematics across countries, but find that most of the variation in beliefs is at the individual level. By contrast, teacher preparation programs appear to play little role in shaping beliefs. Employing multi-level modeling, we also find that teacher beliefs have a statistically significant and substantively important association with future primary teachers' knowledge of mathematics. Finally, our results raise questions about the cross-national validity of a sharp constructivist-traditionalist dichotomy.

Keywords Primary teacher · Attitude · Mathematics · TEDS-M · Belief · Teacher knowledge · Nature of mathematics · Content knowledge · National culture

Teachers are a key component of any educational reform. Most efforts to improve instruction have acknowledged the importance of teacher skills, organization, and support, but teacher *attitudes* also serve a critical role in student learning. Teacher attitudes influence the outcome of policies in relatively direct ways, of course. As principle stakeholders in schools and the ones most responsible for implementing curricular changes, their reactions can make or break a policy. A teaching force that is hostile to a policy, or accepts it only grudgingly, can spell disaster.

N. Burroughs
Michigan State University, Erickson Hall Room 236B, 620 Farm Lane, East Lansing, MI 48824, USA

W. Schmidt (✉)
Michigan State University, Erickson Hall Room 238, 620 Farm Lane, East Lansing, MI 48824, USA
e-mail: bschmidt@msu.edu

S. Blömeke et al. (eds.), *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn*, Advances in Mathematics Education,
DOI 10.1007/978-94-007-6437-8_13,

The attitudes of teachers about teaching itself can have major consequences too. What teachers believe about the content of what they teach, the best way of teaching it, and what students are capable of learning—all have powerful effects on what occurs in the classroom. Teachers are not passive instruments that neutrally convey information, but active participants in the process of educating students, and so their predispositions condition the success or failure of all educational reforms.

In this paper we explore the dynamics of teacher beliefs about mathematics with a special focus on future primary teachers. Secondary school mathematics instructors tend to be specialists in their fields. Given their more intensive exposure to mathematics, their attitudes about mathematics instruction may be quite different from those of primary school teachers, who are responsible for giving basic instruction in many subjects. As generalists with what may be only a smattering of math courses during their preparation to become teachers, the cultural background of primary school teachers could play an especially large role in shaping their beliefs. In addition, as their first exposure to formal mathematics, students' attitudes about math may be powerfully influenced by the beliefs of their elementary school teachers.

Our analysis is based on data drawn from the TEDS study, which contains a large international sample of future primary teachers and includes a series of questions about teacher beliefs. After reviewing earlier research about teacher beliefs, with special attention to the MT21 study and other work based on TEDS data, we examine the relationship among the different dimensions of teacher beliefs and the extent to which these beliefs are associated with teacher knowledge. Of major interest is evaluating how national culture might shape these relationships.

1 Previous Research on Teacher Beliefs

The study of teacher beliefs is fraught with difficulties. In some respects this is because studying beliefs of any sort (rather than behaviors) is an inherently tricky exercise. Beliefs are internal characteristics that people possess, and therefore very hard to measure validly. Asking people to describe their beliefs relies on the honesty, clarity, and self-knowledge of the respondent, while having people respond to previously defined categories risks having them mangle their (actual) beliefs in order to fit the framework.

Sometimes the expressed beliefs of teachers may seem contradictory or ill-formed, but as Leatham (2006) argues, we should accept this ambiguity and treat these beliefs as "sensible" if not necessarily coherent. Understanding the beliefs of teachers is a particular problem, not least of which because scholars have not always been clear what they mean by the term "belief" (Philipp 2007). As noted by Pajeres (1992) and Philipp (2007), there is not a clear consensus on how to define teacher beliefs. Philipp (2007) attempts to untangle the differences between the many similar terms (affects, beliefs, conceptions, knowledge, value, etc.), and in this paper we will try to follow his working definition:

> Psychologically held understandings, premises, or propositions about the world that are thought to be true. Beliefs are more cognitive, are felt less

intensely, and are harder to change than attitudes. Beliefs might be thought of as lenses that affect one's view of some aspect of the world or as dispositions toward action. Beliefs, unlike knowledge, may be held with varying degrees of conviction and are not consensual. Beliefs are more cognitive than emotions and attitudes.

Given the difficulties in defining "beliefs," it should be no surprise that there are a host of different means of conceptualizing beliefs about mathematics (Thompson 1992) and scales for measuring them (Philipp 2007). Ernest (1989) has developed an influential categorization of different sorts of beliefs: beliefs about the nature of mathematics, beliefs about the teaching of mathematics, and beliefs about the learning of mathematics. Beliefs about the nature of mathematics have been broken up into several distinct conceptions by Grigutsch et al. (1998) that reduce to four basic types. Quoting Schmidt et al. (2011)'s, mathematics is viewed as:

A *creative* science that consists of discovery and problem-solving
A *useful* science that can be applied to society and life
A *formal* and logical science that has an axiomatic basis and develops by deduction
An *algorithmic* science that represents a collection of terms, formulas, and rules

These four perspectives can be further collapsed into two broader conceptions: mathematics as a static perspective, characterized by a view of mathematics as a formal, exact science bound by set rules and procedures; and a dynamic perspective that sees mathematics as a process of problem-solving that can be readily applied in daily life (see for more details Chap. "The Cultural Dimension of Beliefs: An Investigation of Future Primary Teachers' Epistemological Beliefs Concerning the Nature of Mathematics in 15 Countries" by Felbrich et al. in this book).

Beliefs about the best means for teaching and learning mathematics involve a rich literature of mathematical pedagogy, which we will only brush upon here. In a review of the literature at the time, Kuhs and Ball (1986) laid out four basic approaches to the teaching of mathematics: learner-focused, content-focused with an emphasis on concepts, content-focused with an emphasis on performance, and classroom-focused literature. Much of the research on the pedagogical beliefs of mathematics teachers emphasizes a learner-focused approach, or what Peterson et al. (1989) dubs a "cognitively based perspective." This student-centered approach is closely related to the long-running debate about traditional/transmission vs. constructivist education (Barkatas and Malone 2005; Howard et al. 1997; Handal 2003; Raymond 1997), or the similar conceptual/calculational dichotomy of Thompson et al. (1994).This cognitive approach to studying beliefs is quite natural, given that the study of beliefs itself has a psychological orientation.

A presumption in studying teacher beliefs is that orientations towards the nature, teaching, and learning of mathematics may be related to one another or to educational practice. There is a plausible connection between a dynamic view of mathematics and a constructivist approach to teaching, for example. Empirical research suggests that there may be some link between what teachers believe about the nature of mathematics and what they believe about the teaching of mathematics

(Beswick 2005; Stipek et al. 2001; Barkatas and Malone 2005, but see Cross 2009, Yates 2006), although the only study focusing on future primary school teachers failed to find such a link (Yates 2006). However, beliefs about mathematics teaching is more directly connected to actual day-to-day instruction (Raymond 1997). Further, some researchers have found students with teachers adopting constructivist orientation may experience greater learning gains (Staub and Stern 2002; Peterson et al. 1989).

Assuming for the moment that the beliefs of teachers are related to their performance and ultimately to how their students learn, an important question is how malleable these beliefs are, and in particular whether teacher preparation programs can help foster the "right" beliefs about mathematics. Here the research suggests both good news and bad news. On a positive note, some research suggests that interventions can modify teacher attitudes. Hart (2002), Kajander (2007) and Gill et al. (2004) found that classroom-based interventions could move teacher beliefs in a more constructivist direction, while Akiba (2011) found that pre-service coursework could improve multicultural awareness. Field experiences may also encourage learner-centered perspectives (Ambrose 2004).

Despite these encouraging results, other scholars caution against expecting too much of pre-service interventions. Pajeres (1992) notes that attitudes about teaching and learning are formed early and are quite durable. Van Zoest et al. (1994) and Handal (2003) also highlight the contextual influences on teacher attitudes. Once teachers enter the workforce, their attitudes may revert to more traditional ones due to environmental pressure, or may find it difficult to translate their beliefs into practice. Notably, nearly all of these studies focused on future primary teachers.

On the whole the empirical literature on teacher beliefs about mathematics is rather thin. Much of it is based on relatively small sample sizes with limited geographic scope. The most important exception to these limitations are the Mathematics Teaching in the 21st Century (MT21) and the Teacher Education Study in Mathematics (TEDS-M). As described in Schmidt et al. (2007, 2011), MT21 surveyed approximately 2600 future teachers at 34 institutions across 6 countries. MT21 surveyed primary, middle, and lower secondary future teachers.

Along with testing teachers' content and pedagogical knowledge and asking about course-taking, the MT21 study also included a number of items regarding teachers' beliefs about the nature, teaching, and learning of mathematics. The study followed the 4-fold Grigutsch typology about the nature of mathematics, inquiring about teachers' orientation was algorithmic, useful, creative, or formal. Generally speaking, across countries future teachers inclined towards the "dynamic" (useful & creative) perspective, but found significant differences across countries. Taiwan, Korea, and Bulgaria future teachers adhered to all four conceptions at once, with the Taiwanese the most in favor of an algorithmic view of mathematics and German future teachers the least in favor. The US was something of an outlier, being the only country with more support for the algorithmic than formal conception. In addition, US elementary and middle school future teachers were more supportive of an algorithmic conception than were secondary future teachers.

With respect to beliefs about learning mathematics, the MT21 study asked questions tapping into five basic notions: the use of standard procedures, focusing on the right answer, mastery of skills, gaining understanding, and independent work by students. There was general agreement that students should try to gain understanding and work independently. Taiwanese future teachers had the strongest support for using standard procedures. The MT21 study also probed ideas about whether all students were capable of learning math. In most countries there was resistance to the idea that mathematics was based on natural ability (particularly in Germany the United States), and to the importance of gender and race (especially in Germany), with the exception of Taiwanese future teachers.

Building on MT21, the TEDS included a much larger sample of nearly 23,000 future teachers in 498 institutions across 16 countries (Tatto et al. 2012; Schmidt et al. 2013). The TEDS reduced the number of items related to teacher beliefs, compressing the number of dimensions into five: beliefs about the nature of mathematics (as a set of rules and procedures or as a process of inquiry), about learning mathematics (through teacher direction or through active involvement), and about mathematics achievement (whether mathematics is a fixed ability).

The TEDS found substantial variation across countries in beliefs about the nature of mathematics, and with primary, middle, and lower secondary teachers generally evincing similar beliefs within the same country. All countries' future teachers embraced the notion of mathematics as a process of inquiry and should be learned actively, and opposed the idea that mathematics is a fixed ability. There was much more variation across countries about whether mathematics is a set of rules and procedures and whether learning is best when directed by teachers. Using a simple correlational analysis, countries whose future teachers had more conceptual beliefs about mathematics (active learning, process of inquiry) generally had higher mean scores on the mathematical content knowledge (MCK) and mathematics pedagogical knowledge (PCK) tests than those with calculational perspectives (a set of rules and procedures, teacher direction).

Felbrich et al. (see Chap. "The Cultural Dimension of Beliefs: An Investigation of Future Primary Teachers' Epistemological Beliefs Concerning the Nature of Mathematics in 15 Countries" in this book) delved deeper into the TEDS data on future primary teachers' beliefs, with a specific emphasis on the static/dynamic dichotomy about the nature of mathematics. They noted a broad range of opinion within countries (as measured by standard deviations). After combining "math as a process of inquiry" and "math as a process of rules and procedures" into a single scale using ipsative values, the authors conducted a two-level analysis (country and individual) examining the dependence of teacher beliefs on mathematical content knowledge, previous school achievement, and the individualism of each country's culture. Higher-performing future teachers were found to have more dynamic attitudes about mathematics, while country individualism had a marginal effect (controlling for other factors).

2 Empirical Examination of Future Primary Teacher Beliefs

We add to these results by making use of the TEDS data set to explore the structure and impact of future primary teacher beliefs in a detailed way. As a preliminary step we describe the TEDS data to evaluate two different scales of beliefs and present descriptive data about these indices. Two main questions serve to structure our analysis. First, what is the relationship among the different dimensions of teacher beliefs? Second, what is the relationship between teacher beliefs and teacher knowledge? An important theme underlying both questions is the degree to which these relationships vary between and within countries.

2.1 Using TEDS Data to Examine Teacher Beliefs

A prerequisite to addressing all of these questions is resolving the problem of how to conceptualize teacher beliefs. The TEDS represents a considerable advance on earlier efforts given its large sample size and international character, but the design of the survey imposes certain limitations. The TEDS allows us to compare within and between country beliefs with a fairly high degree of precision. However, in designing the survey the authors of TEDS selected a smaller pool of items than existed within the previous MT21 study. The TEDS survey comprises the same three basic categories of beliefs as MT21 (nature of mathematics, learning of mathematics, beliefs about mathematics achievement), but reduced the number of distinct dimensions from twelve to five, and had only 33 belief items rather than the original 44 (a 25 % reduction).

In our analysis, we re-constructed the original MT21 scales using those items that remained in the TEDS. These scales are only rough estimates of the indexes as they would have manifested if the entire bank of MT21 belief items had been included. Some dimensions are at greater risk than others. For example, both of the mathematics achievement indices were essentially intact, but the formalism index (within the "Nature of Mathematics") had only one item as opposed to the original 5. It should therefore be no surprise that the reliabilities of some of the MT21 indices are lower than we would like. While the "nature of mathematics" indices (except for formalism, which had only one surviving item) and the "natural ability" element of mathematics achievement have Cronabch's alphas of about 0.8, the reliabilities of the "learning mathematics" beliefs are only about 0.6. The TEDS scales have a higher reliability (between 0.7 and 0.8), and also performed fairly well when re-created using MT21 data, but as simplified expressions of teacher beliefs may be missing more nuanced elements. While illuminating, both indices are only approximations of the structure of teacher beliefs.

A second restriction present in both the MT21 and TEDS studies is that they do not map perfectly onto the three types of beliefs as developed by Ernest and heavily employed in the literature. While TEDS and MT21 include the "nature of mathematics" category, the teaching of mathematics and learning of mathematics concepts have been partly combined into one group, while a component of learning mathematics has been separated into a different area related to beliefs about whether all students are capable of learning mathematics. While there it is certainly logical to posit a close connection between what a teacher believes about teaching and what he or she believes about learning, it is open to question whether this relationship is as tight as one might suppose. Any direct application to other empirical literature should therefore be treated with caution.

Mean values of both sets of indices are presented for all nations and by country in Table 1. Each index is the mean of responses to the items within each dimension, weighted by respondent. Each item posed a question rated on a 1 to 6 scale, with higher scores indicating greater agreement. We reproduced the TEDS scales using means rather than IRT scaling because of data limitations, but there was a very high correlation between the two (over 0.9). The mean scores for the TEDS scales are of course quite similar to that presented in the TEDS report, with more agreement with the concept of mathematics as a process of inquiry acquired through active learning. Math as a fixed ability and learning through teacher direction received much less support. Interestingly, this method of aggregating responses found nearly as much endorsement of math as a system of rules as it did for a process of inquiry.

The MT21 report sampled all three populations of future teachers and the report presented pooled results across grades, so it is difficult to make precise comparisons between the MT21 and TEDS samples for only future primary teachers. A few countries (Taiwan, Germany, and the United States) participated in both studies, and the mean responses using the smaller-item indexes in the TEDS sample are fairly close, despite the fact that it compares primary teachers in one sample to all teachers in the other. For the reproduced MT21 scales, we found considerable support for all four conceptions of the nature of mathematics (global means ranging from 4.4 to 4.9), the importance using different approaches (4.7) and student understanding (5.1). There was much less support for other beliefs. For both sets of scales there was appreciable variation in average beliefs by country, with a range in mean responses between 1.1 and 1.8.

Country-level averages reinforce the point that primary future teacher beliefs are partly conditioned by cultural context (see Chap. "The Cultural Dimension of Beliefs: An Investigation of Future Primary Teachers' Epistemological Beliefs Concerning the Nature of Mathematics in 15 Countries" by Felbrich et al. in this book). There is considerable within-country variation in future teacher beliefs as well. The variation in beliefs within countries was about one standard deviation for the pooled sample (and a cross-country mean standard deviation of 1.5). The US was a clear outlier at around 4 standard deviations, but this might be due in part to its much larger sample size. Differences in teacher preparation programs, either through selection effects or a different approach for training future teachers, could also account for the variation in teacher beliefs. In an attempt to sort out how much variation is due to country and institution-level effects, we performed a three-level variance decomposition analysis for the 5 TEDS and 11 MT21 belief scales.

The results presented in Table 2 suggest that although country-level effects have a substantial influence on the variation in future primary teacher beliefs (explaining

Table 1 Mean beliefs about mathematics

Beliefs dimension	Scales	All countries	Botswana	Chile	Georgia	Germany	Malaysia	Norway	Philippines	Poland	Russsia	Singapore	Spain	Switzerland	Taiwan	Thailand	USA
Nature of mathe-matics	Algorithmic	4.4	5.2	4.5	4.4	3.8	5.0	3.9	5.5	4.4	4.3	4.6	4.4	3.9	4.4	5.1	4.6
	Usefulness	4.9	5.5	5.4	3.9	4.5	5.3	5.0	5.4	4.7	4.7	5.2	5.2	4.5	4.8	5.2	5.3
	Creative	4.8	5.3	5.1	3.9	4.6	5.2	4.9	5.5	4.5	4.6	5.0	4.9	4.8	5.1	5.3	4.9
	Formalism	4.6	4.5	4.5	4.3	3.8	4.8	4.2	5.2	4.8	5.2	4.9	4.7	3.7	5.0	4.9	4.3
Teaching mathe-matics	Products	2.4	2.3	2.7	3.4	2.0	3.6	1.9	3.7	2.3	2.5	2.0	2.0	1.9	2.2	2.2	2.2
	Different approaches	4.7	4.9	5.1	4.0	5.0	4.3	4.7	4.7	4.8	4.8	4.5	4.5	5.1	5.0	4.6	4.5
	Algorithms	3.2	3.4	3.0	4.0	2.8	4.1	2.7	4.4	3.4	3.6	3.3	2.8	2.6	2.8	2.9	3.1
	Standard procedures	2.6	2.8	2.9	3.5	2.2	3.9	2.0	3.8	2.7	2.6	2.3	2.3	2.1	2.2	2.6	2.0
	Understanding	5.1	5.1	5.3	4.3	5.1	4.8	5.3	5.3	5.1	5.0	5.3	5.2	5.2	5.1	5.3	5.4
Nature of ability	Categorical differences	2.8	2.9	2.3	3.8	2.2	3.6	2.5	3.7	3.0	2.9	2.4	2.0	2.2	3.4	3.6	2.3
	Natural ability	3.6	3.8	3.3	4.2	3.2	4.4	2.9	4.7	4.1	4.0	3.2	3.3	2.9	3.5	4.1	2.8
Learning mathe-matics	Enquiry	4.9	5.5	5.2	4.0	4.7	5.2	5.0	5.5	4.7	4.7	5.1	5.1	4.8	5.0	5.3	5.1
	Rules	4.5	5.1	4.5	4.4	3.8	5.0	4.0	5.4	4.5	4.5	4.6	4.4	3.8	4.5	5.1	4.6
	Active	4.9	5.0	5.2	4.1	5.1	4.6	5.0	5.0	5.0	4.9	4.9	4.9	5.2	5.1	4.9	5.0
	Directions	2.9	3.0	3.1	3.8	2.4	4.0	2.3	4.1	3.0	3.1	2.7	2.6	2.3	2.6	2.6	2.6
	Fixed	3.2	3.4	2.8	4.0	2.8	4.1	2.7	4.1	3.6	3.6	2.9	2.7	2.6	3.2	3.7	2.5
	N	14633	86	657	506	1032	576	551	592	2112	2266	380	1093	936	923	660	2263

Table 2 Variance decomposition of teacher beliefs

	Institution	Country	Individual
Enquiry	4.1 %	21.5 %	74.4 %
Rules	6.5 %	27.7 %	65.8 %
Active	4.8 %	14.6 %	80.6 %
Directions	7.4 %	38.1 %	54.5 %
Fixed	4.5 %	35.2 %	60.3 %
Algorithmic	7.2 %	28.0 %	64.8 %
Usefulness	3.3 %	19.1 %	77.6 %
Creative	4.6 %	20.3 %	75.1 %
Formalism	2.3 %	16.0 %	81.7 %
Products	5.0 %	24.0 %	71.0 %
Different approaches	6.0 %	13.8 %	80.2 %
Algorithms	8.4 %	27.4 %	64.2 %
Standard procedures	4.9 %	27.2 %	67.9 %
Understanding	3.1 %	13.5 %	83.5 %
Categorical differences	4.9 %	21.6 %	73.5 %
Natural ability	4.0 %	30.4 %	65.6 %

from 14 % to 38 % of the total variance), most of the variation was in fact attributable to student-level differences (55 % to 84 % of total variance). The impact of teacher preparation institutions was slight (2 % to 7 %). One salient finding is that the "constructivist" beliefs tended to have far more of the variation explained at the individual level, whereas "traditional" beliefs tended to have a greater proportion of variance explained by country-level influences.

2.2 The Relationship Among Beliefs

The relationship among dimensions of primary teacher beliefs includes two different considerations. First, there is the methodological concern about the extent to which the MT21 and TEDS indexes tap into the same phenomena—in short, whether the simplified TEDS typology adequately captures the range of teacher beliefs. Second, and more substantively important, the connection of different categories of beliefs to each other touches upon one of the most contested issues in mathematics education research, as well as the validity of a great deal of research related to teacher beliefs: the distinction between a more constructivist or more traditional approach to mathematics education.

Some relationship between the MT21 and TEDS beliefs scales is to be expected, given that the TEDS indices were based on the MT21 approach and include many of the same items, but also maintain the integrity of MT21 concepts: although the number of items were slimmed down, MT21 belief dimensions were not broken up across TEDS categories. For the nature of math indices, the algorithmic and formalist views were combined into the math as rules and procedures concept, while math as a creative science and math as a useful activity were combined into the concept math as a process for inquiry. Among the two "beliefs about learning math" indexes, "teacher direction" included elements of the "algorithms," "focus on products," and "standard procedures" scales; "active learning" comprised "different approaches" and "understanding" items. Finally, the "math as fixed ability" index in TEDS incorporated questions from the "categorical differences" and "natural ability" scales.

Correlation analysis indicated that most of the MT21 scales are strongly related to the relevant TEDS scale (see Table 3). "Math as a Process of Inquiry" was strongly correlated to the Usefulness and Creative scales (0.84, 0.88), as was math as active learning to the understanding (0.78) and different approaches (0.87). The association of fixed beliefs about mathematics with categorical differences (0.77) and natural ability (0.92), and of a directive orientation with standard procedures (0.73), products (0.77), and algorithms (0.84) was also quite strong. Finally, there was almost perfect collinearity between a rule-based outlook and an algorithmic perspective (0.98), but less overlap with formalism (0.62). However, because the MT21 items were truncated, the index scores could be somewhat biased towards alignment with TEDS scales.

As should be evident from the literature review, the study of teacher beliefs has been closely connected to the debate over whether a broadly constructivist or traditional approach to mathematics instruction is to be preferred. A plurality of the researchers studying teacher beliefs appears to support the idea of a more active, learner-centered, cognitive pedagogical strategy. Underlying this debate is the assumption that dynamic attitudes about the nature of mathematics and the belief that math is best learned through a process of active learning exists at the opposite end of a continuum from beliefs that mathematics is a static discipline that should be taught under the direction of teachers. The presumption therefore is that individuals (or countries) that generally support one sort of belief will oppose the other.

However, the relationship between the two different beliefs about the nature of math (math as rules and math as inquiry) do not appear to be contrary, at least according to TEDS data. In fact, there was virtually no relationship between the two dimensions, with a (very weakly) positive correlation of 0.06. When responses for the entire TEDS sample of 16 countries were pooled together, static and dynamic conceptions appeared to be orthogonal to each other, rather than inversely related to each other.

The weak relationship between static and dynamic conceptions of mathematics was replicated using MT21 scales: formalistic and algorithmic beliefs about the nature of mathematics had the same very low correlation with the usefulness (0.01, 0.11) and creative (0.04, 0.02) dimensions. Further, the relationship between the concepts of mathematics learning and the nature of mathematics are broadly "conceptual"—the belief that mathematics requires active learning and that it is a process of inquiry, were only moderately related (0.45). The correlation between the "calculational beliefs"—math as rules and learning through teacher direction—was identical (0.45). These results suggest a link between beliefs about the nature

Table 3 Correlations among teacher beliefs

	Algo-rithmic	Usefulness	Creative	Formal-ism	Products	Different approaches	Algorithms	Standard procedures	Under-standing	Categorical differences	Natural ability	Enquiry	Rules	Active	Direc-tions	Fixed
Algorithmic	1.00	0.11	0.02	0.45	0.19	−0.17	0.53	0.21	0.09	0.21	0.28	0.06	0.98	−0.06	0.43	0.26
Usefulness	0.11	1.00	0.55	0.01	−0.13	0.20	−0.12	−0.18	0.40	−0.14	−0.26	0.84	0.10	0.34	−0.16	−0.25
Creative	0.02	0.55	1.00	0.04	−0.10	0.34	−0.16	−0.14	0.39	−0.09	−0.17	0.88	0.03	0.44	−0.16	−0.16
Formalism	0.45	0.01	0.04	1.00	0.11	−0.03	0.34	0.19	0.04	0.19	0.33	0.04	0.62	0.01	0.30	0.31
Products	0.19	−0.13	−0.10	0.11	1.00	−0.09	0.47	0.46	−0.26	0.30	0.38	−0.14	0.19	−0.20	0.77	0.43
Different approaches	−0.17	0.20	0.34	−0.03	−0.09	1.00	−0.16	−0.14	0.37	−0.04	−0.03	0.32	−0.15	0.87	−0.17	−0.04
Algorithms	0.53	−0.12	−0.16	0.34	0.47	−0.16	1.00	0.42	−0.12	0.32	0.47	−0.17	0.54	−0.17	0.84	0.49
Standard procedures	0.21	−0.18	−0.14	0.19	0.46	−0.14	0.42	1.00	−0.29	0.34	0.44	−0.20	0.23	−0.25	0.73	0.50
Understanding	0.09	0.40	0.39	0.04	−0.26	0.37	−0.12	−0.29	1.00	−0.17	−0.20	0.45	0.09	0.78	−0.25	−0.25
Categorical differences	0.21	−0.14	−0.09	0.19	0.30	−0.04	0.32	0.34	−0.17	1.00	0.56	−0.13	0.23	−0.11	0.41	0.77
Natural ability	0.28	−0.26	−0.17	0.33	0.38	−0.03	0.47	0.44	−0.20	0.56	1.00	−0.24	0.32	−0.13	0.55	0.92
Enquiry	0.06	0.84	0.88	0.04	−0.14	0.32	−0.17	−0.20	0.45	−0.13	−0.24	1.00	0.06	0.45	−0.19	−0.24
Rules	0.98	0.10	0.03	0.62	0.19	−0.15	0.54	0.23	0.09	0.23	0.32	0.06	1.00	−0.05	0.45	0.30
Active	−0.06	0.34	0.44	0.01	−0.20	0.87	−0.17	−0.25	0.78	−0.11	−0.13	0.45	−0.05	1.00	−0.25	−0.16
Directions	0.43	−0.16	−0.16	0.30	0.77	−0.17	0.84	0.73	−0.25	0.41	0.55	−0.19	0.45	−0.25	1.00	0.60
Fixed	0.26	−0.25	−0.16	0.31	0.43	−0.04	0.49	0.50	−0.25	0.77	0.92	−0.24	0.30	−0.16	0.60	1.00

Table 4 Correlation between static and dynamic beliefs by country

Country	Correlation
Botswana	0.19
Chile	0.08
Georgia	0.80
Germany	−0.23
Malaysia	0.78
Norway	−0.25
Philippines	0.47
Poland	0.09
Russian Fed.	0.35
Spain	−0.10
Switzerland	−0.30
Taiwan	0.16
Thailand	0.40
USA-All	−0.12
Singapore	0.19
All	0.06

of mathematics and beliefs about teaching and learning mathematics, with a more conceptual and more calculational approach as distinct families of beliefs, but ones that are not in opposition to one another.

Within-country correlations indicate that the relationship between beliefs about the nature of mathematics varied dramatically across national units (see Table 4). In some countries the relationship between these two beliefs reflected the international average, with very low correlations between math as a process of inquiry and math as a set of rules: Botswana, Taiwan, the US, Singapore, Poland, Chile, and Spain all posted correlations of 0.2 or less. However, there were a few countries—in particular Georgia—that suggested a strong positive correlation between a static and dynamic view of mathematics. The two kinds of beliefs about the nature of mathematics were modestly negatively correlated in a few countries; Switzerland, Germany, and Norway all saw negative correlations of between 0.2 and 0.3.

2.3 The Relationship Between Beliefs and Knowledge

As a study directed strictly at mathematics teacher preparation, TEDS data did not include K-12 student data, and therefore cannot be used to directly measure the impact of teacher beliefs on K-12 student performance. However, given the strong link between the mathematics knowledge of primary teachers and student learning gains (Hill et al. 2005), there may be an indirect effect of teacher beliefs on student learning via teacher knowledge. The relationship between beliefs on the one hand

and mathematical content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK) assessment results on the other hand could give a hint as to the ultimate impact of beliefs in the classroom. The connection between teacher beliefs and teacher knowledge is of particular concern at the primary school level, as elementary school teachers are rarely mathematics specialists.

We conducted a statistical analysis using a two-level model using PROC MIXED, measuring future teacher characteristics at the individual level and controlling for country-level clustering in the second level. We experimented with two outcome variables (MCK and PCK), but the high degree of correlation between these two measures led to substantively similar results and we therefore present only MCK results. We ran a series of regressions, with each belief index (both MT21 and TEDS) as the main independent variable for each regression. Treating teacher beliefs as a fixed or random effect had substantively identical results, with minimal change to parameter estimates and significance levels.

Our analysis builds on several previous studies. First, the TEDS report presented the country-level correlations between MCK and PCK on the one hand and teacher beliefs on the others. Our work therefore adds an additional level of sophistication to this analysis by incorporating country-level and individual effects into one model and incorporating a number of control variables. These control variables included a number of student-level measures. Following Felbrich et al. (Chapter "The Cultural Dimension of Beliefs: An Investigation of Future Primary Teachers' Epistemological Beliefs Concerning the Nature of Mathematics in 15 Countries" in this book), we included the student's self-reported typical class ranking in secondary school as a proxy for a student's mathematical knowledge before he or she entered a teacher preparation program. As an additional background characteristic, we included the average number of books the student reported in the home, standardized within each country to adjust for differences in wealth across countries. Finally, following Schmidt et al. (2011), we controlled for the effects of program coursework by including the percentage of mathematics and general pedagogy courses taken by the student.

The results of our analysis are presented in Table 5. Our results suggest that teacher beliefs have a statistically significant and substantively important association with future primary teachers' knowledge of mathematics. Only one of the indices (formalism) failed to register a significant effect, which may be due to it being limited to a single item in the TEDS study. Consistent with the TEDS report, the more "conceptually" oriented beliefs (math as inquiry, active learning) were associated with higher mathematics knowledge scores, and the "calculational" oriented beliefs with lower MCK scores (math as rules, directive learning), as is belief in fixed abilities.

The MT21 scales produced similar results, although with slightly smaller coefficients. Beliefs in mathematics as algorithmic, focused on the right answer, using standard procedures, and ability as categorical or natural were negatively related with the mathematics knowledge of future primary teachers. By contrast, the belief that mathematics is useful, creative, requires different approaches, and student understanding were associated with higher MCK scores. Again, these findings are

Table 5 Multilevel model of beliefs' relationship to MCK

Effect	Estimate	StdErr	Probt
Enquiry	16.15	0.92	0.000
Rules	−15.88	0.83	0.000
Active	15.26	1.05	0.000
Directive	−18.94	0.89	0.000
Fixed	−14.60	0.81	0.000
Algorithmic	−17.00	0.79	0.000
Usefulness	12.49	0.77	0.000
Creative	12.24	0.83	0.000
Formalism	−1.10	0.61	0.072
Products	−10.11	0.64	0.000
Different approaches	10.33	0.79	0.000
Algorithms	−11.42	0.70	0.000
Standard procedures	−12.75	0.68	0.000
Understanding	11.61	1.02	0.000
Categorical differences	−8.91	0.54	0.000
Natural ability	−12.46	0.68	0.000

unchanged if one uses PCK rather than MCK as an outcome, or if the relationship of beliefs to MCK is permitted to vary by country.

Estimates and significance levels for the control variables are not shown, but were statistically significant, quite consistent across models, and replicated previous results. The number of books in the home and previous performance in school were associated with higher MCK scores. The estimates for opportunity-to-learn (OTL) measures in our multi-country model were virtually identical to the US-only analysis by Schmidt et al. (2011), with each additional percentage of mathematics courses resulting in an extra half-point in MCK scores, and each percentage of general pedagogy associated with a 0.9 point decline in performance. These results strengthen the findings of the Schmidt et al. (2011) piece and suggest that OTL has a similar effect across different educational systems.

Once again we found that beliefs have disparate dynamics in different countries. In Table 6 we present the results of within-country regression analyses employing the same set of independent variables as used in the previous analysis. Although the direction of the relationship between beliefs and mathematical knowledge was consistent for those countries in which it is statistically significant, the size of the coefficients varied quite a bit across countries. In addition, there were some instances (in particular Botswana) where beliefs appeared to have little relationship with MCK scores. There was certainly a link between the beliefs and mathematical knowledge of future primary teachers, but the strength of this association was apparently conditioned by national cultures and institutions.

Table 6 Regression of the relationship between beliefs and MCK, by country

Country	Enquiry	Rules	Active	Directive	Fixed	Algo-rithmic	Useful	Creative	Forma-lism	Products	Different approaches	Algo-rithms	Standard procedures	Under-standing	Cat. diff.	Nat. ability
Botswana	7.56	−8.06	11.34	−0.01	−10.2	−6.6	0.3	7.6	−6.2	1.8*	4.5	−0.9	−0.8	9.5	−2.2	−15.7*
Chile	12.32*	−9.29*	10.59*	−15.47*	−8.5*	−9.9*	9.1*	10.3*	−1.3	−5.0*	3.5	−8.7*	−9.6*	13.7*	−5.9*	−5.8*
Georgia	2.46	3.57	4.15	2.02	0.2	3.7	1.4	3.1	1.4	−2.4	3.7	1.2	4.4	3.5	1.2	−0.4
Germany	20.06*	−9.93*	18.82*	−16.53*	−3.3	−11.5*	14.6*	13.3*	1.3	−8.1*	14.9	−7.2*	−12.1*	15.0*	−6.1*	−1.1
Malaysia	6.47*	0.76	5.96*	−5.89*	1.2	0.6	6.3*	5.6	0.8	−4.3*	2.2	−0.6	−6.1*	7.4*	0.8	0.0
Norway	26.84*	−28.77*	21.50*	−21.70*	−11.7*	−30.1*	14.4*	22.3*	−2.1	−9.2*	19.1	−12.1*	−17.8*	10.8*	−6.6*	−10.1*
Philippines	19.90*	−0.90	9.42*	−14.64*	−8.7*	−2.2	11.2*	18.0*	1.8	−7.1*	0.5	−7.5*	−10.1*	14.2*	−3.2	−7.1*
Poland	14.45*	−26.12*	16.93*	−32.18*	−18.4*	−27.6*	8.4*	13.8*	−0.8	−16.1*	15.6	−23.6*	−16.0*	10.7*	−10.2*	−14.1*
Russian Fed.	15.67*	−4.73	16.81*	−17.27*	−14.8*	−8.0*	14.0*	11.6*	14.2*	−12.5*	12.1	−8.9*	−14.4*	12.5*	−5.7*	−12.9*
Singapore	−0.59	−16.38*	−4.17	−17.36*	−9.7*	−16.6*	1.9	−3.1	−2.2	−8.9*	−6.1	−12.9*	−7.5	3.7	−8.1*	−5.9
Spain	9.89*	−13.26*	9.18*	−12.30*	−8.0*	−13.3*	6.9*	5.7*	−1.5	−6.8*	5.3	−5.4*	−7.9*	8.8*	−3.9*	−7.4*
Switzerland	9.77*	−13.27*	−4.84	−5.43	−9.9*	−13.8*	3.5	11.6*	−1.2	−7.7*	−7.1	4.7	−8.5*	−0.2	−6.1*	−6.8*
Taiwan	17.82*	−8.89*	18.98*	−24.64*	−12.2*	−8.9*	10.2*	14.8*	0.4	−14.4*	16.7	−14.1*	−14.0*	11.9*	−7.0*	−7.8*
Thailand	12.74*	−20.88*	8.46	−37.40*	−23.3*	−21.7*	5.2	6.4	−4.9	−22.9*	4.8	−28.0*	−19.8*	7.8	−14.0*	−16.2*
USA	15.27*	−17.62*	15.57*	−15.81*	−15.1*	−18.6*	13.0*	11.2*	−3.5*	−7.9*	10.0	−9.6*	−11.9*	11.3*	−10.1*	−13.7*

3 Discussion

Beliefs do not exist in a vacuum. They are both shaped by and interact with individual and social context. But given the considerable autonomy possessed by teachers in the classroom, how teachers conceive of mathematics—what mathematics is, how it is best taught and learned, and who is capable of learning it—could have a substantial influence on how their students ultimately approach mathematics. The earlier these beliefs are instilled, the greater the potential long-term effects, and hence the critical importance of understanding the beliefs of elementary school mathematics instructors.

Our analysis has yielded two principal insights. First, properly modeling beliefs is a devilishly tricky task. Drawing the proper conceptual boundaries around ideas so that they are mutually exclusive and exhaustive is a difficult enterprise in any field, but are doubly so when the meanings of these ideas vary so much across cultural contexts. The dynamic-static dichotomy about the nature of math, and the constructivist-traditional dichotomy about mathematics pedagogy, were developed in very specific cultural milieus. Our results raise questions about whether these categories are quite so distinct. It is important to remember that simply because two beliefs may be logically opposed doesn't not mean that people aren't fully capable of subscribing to both simultaneously. What might be seen from one perspective as a battle between "good" and "bad" conceptions of mathematics may in fact simply be an example of a fruitful tension between two different, worthwhile approaches.

Secondly, differences in national culture and teacher preparation programs to shaping teacher beliefs should not be overdrawn. Most variation in the beliefs of future primary teachers lies not at the national or program level, but with individuals. Further, although the link between different dimensions of belief differs across societies, there are relatively stable connections between beliefs about mathematics and mathematical knowledge. In virtually every country those future teachers who see mathematics as an engaging discipline and emphasized student understanding tended to know more about mathematics and mathematics pedagogy. Those future teachers who adopted a more rigid, didactic approach to mathematics and hewed to an essentialist view of human characteristics tended to know less. In developing interventions to improve mathematics instruction and teacher preparation, the idea that math is a living discipline rather than a collection of facts is most common among the brightest teachers, something that policymakers and researchers should probably keep in mind.

References

Akiba, M. (2011). Identifying program characteristics for preparing pre-service teachers for diversity. *Teachers College Record*, *113*(3), 658–697.

Ambrose, R. (2004). Initiating change in prospective elementary school teachers' orientations to mathematics teaching by building on beliefs. *Journal of Mathematics Teacher Education*, *7*, 91–199.

Barkatas, A., & Malone, J. (2005). A typology of mathematics teachers' beliefs about teaching and learning mathematics and instructional practices. *Mathematics Education Research Journal*, *17*(2), 69–90.

Beswick, K. (2005). The beliefs/practice connection in broadly defined contexts. *Mathematics Education Research Journal*, *17*(2), 39–68.

Center for research in math and science education (2007). *Breaking the Cycle: An International Comparison of US Mathematics Teacher Preparation*. East Lansing: Michigan State University.

Cross, D. I. (2009). Alignment, cohesion and change: Examining mathematics teachers' belief structures and their influence on instructional practices. *Journal of Mathematics Teacher Education*, *12*, 325–346.

Ernest, P. (1989). The impact of beliefs on the teaching of mathematics. In P. Ernest (Ed.), *Mathematics Teaching: The State of the Art* (pp. 249–254). New York: Falmer Press.

Felbrich, A., Kaiser, G., & Shmotz, C. (2012). The cultural dimension of beliefs: An investigation of future primary teachers' epistemological beliefs concerning the nature of mathematics in 15 countries. *ZDM Mathematics Education*, *44*(3), 355–366.

Gill, M. G., Ashton, P. T., & Algina, J. (2004). Changing preservice teachers' epistemological beliefs about teaching and learning in mathematics: An intervention study. *Contemporary Educational Psychology*, *29*, 165–185.

Grigutsch, S., Raatz, U., & Türner, G. (1998). Einstellungengegenuber Mathematik bei Mathematiklehrern. *Journal fur Mathematik-Didaktik*, *19*, 3–45.

Handal, B. (2003). Teachers' mathematical beliefs: A review. *The Mathematics Educator*, *13*(2), 47–57.

Hart, L. C. (2002). Preservice teachers' beliefs and practice after participating in an integrated content/methods course. *School Science and Mathematics*, *102*(1), 4–14.

Hill, H. C., Rowan, B., & Ball, D. (2005). Effects of teachers' mathematics knowledge for teaching on student achievement. *American Education Research Journal*, *42*(2), 371–406.

Howard, P., Perry, B., & Lindsay, M. (1997). Secondary mathematics teacher beliefs about the learning and teaching of mathematics. In *Proceedings of the 20th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 231–238).

Kajander, A. (2007). Unpacking mathematics for teaching: A study of preservice elementary teachers' evolving mathematical understandings and beliefs. *Journal of Teaching and Learning*, *5*(1), 33–54.

Kuhs, T. M., & Ball, D. L. (1986). *Approaches to Teaching Mathematics: Mapping the Domains Knowledge, Skills, and Dispositions*. East Lansing: Michigan State University.

Leatham, K. (2006). Viewing mathematics teachers beliefs as sensible systems. *Journal of Mathematics Teacher Education*, *9*, 91–102.

Pajeres, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research*, *62*, 307–332.

Peterson, P. L., Fennema, E., Carpenter, T. P., & Loef, M. (1989). Teachers' pedagogical content beliefs in mathematics. *Cognition and Instruction*, *6*(1), 1–40.

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 257–315). Charlotte: Information Age.

Raymond, A. M. (1997). Inconsistency between beginning elementary school teacher's mathematics beliefs and teaching practice. *Journal for Research in Mathematics Education*, *28*(5), 550–576.

Schmidt, W. H., Blomeke, S., & Tatto, M. T. (2007). *Teacher Education Matters*. New York: Teachers College Press.

Schmidt, W. H., Cogan, L., & Houang, R. (2011). The role of opportunity to learn in teacher preparation: An international context. *Journal of Teacher Education*, *62*(2), 138–153.

Schmidt, W. H., Cogan, L., & Houang, R. (2013, this volume). Emphasis and balance among the components of teacher preparation: comparing US practice with international benchmarks from TEDS-M. In S. Blömeke, F. Hseih, G. Kaiser, & W. Schmidt (Eds.), *Teacher knoweldge,*

teacher beliefs and opportunities to learn in 16 countries: Findings from the IEA TEDS-M study on primary and lower secondary teaching. New York: Springer.
Staub, F. C., & Stern, E. (2002). The nature of teachers' pedagogical content beliefs matters for students' achievement gains: quasi-experimental evidence from elementary mathematics. *Journal of Educational Psychology*, *94*, 344–355.
Stipek, D. J., Givvin, K. B., Salmon, J. M., & MacGyvers, V. L. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education*, *17*, 213–226.
Tatto, M. T., Schwille, J., Senk, S. L., Ingvarson, L., Rowley, G., Peck, R., Bankov, K., Rodriguez, M., & Reckase, M. (2012). *Policy, Practice, and Readiness to Teach Primary and Secondary Mathematics in 17 Countries*. Amsterdam: International Association for the Evaluation of Educational Achievement.
Thompson, A. (1992). Teachers' beliefs and conceptions: a synthesis of the research. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 127–146). New York: Macmillan Co.
Thompson, A. G., Philipp, R. A., & Thompson, P. W. (1994). Calculational and conceptual orientations in teaching mathematics. In A. Coxford (Ed.), *1994 Yearbook of the NCTM* (pp. 79–92). Reston: NCTM.
Van Zoest, L. R., Jones, G. A., & Thornton, C. A. (1994). Beliefs about mathematics teaching held by pre-service teachers involved in a first grade mentorship program. *Mathematics Education Research Journal*, *6*(1), 37–55.
Yates, S. (2006). Primary teachers' mathematics beliefs, teaching practices and curriculum reform experiences. In *Proceedings of the Australian Association for Research in Education Conference*.