The Cultural Notion of Teacher Education: Future Lower Secondary Teachers' Beliefs on the Nature of Mathematics, the Learning of Mathematics and Mathematics Achievement

Shu-Jyh Tang and Feng-Jui Hsieh

Abstract The paper aims to highlight the cultural notion of mathematics-teaching related beliefs of future lower secondary teachers at the end of their training. The analysis is based on the data of TEDS-M and the belief scores produced by the authors through employing a Rasch partial credit model. Six belief scales were created: beliefs that the nature of mathematics is open and creative or conservative and rigorous, beliefs that the learning of mathematics should be guided by student initiative, teacher instruction (TI) or utilitarianism in Teaching (UT), and beliefs that mathematical ability is natural and fixed (NF). UT is a new scale created for this paper and different from the original TEDS-M scales. Cultural country groups adapted from Blömeke and Kaiser (ZDM, Int. J. Math. Educ. 44(3):249–264, 2012) are used for comparing the outcomes of our study. A common pattern among all groups exhibits the belief that the nature of mathematics is open and creative on the one hand and conservative and rigorous on the other hand. Furthermore, it is stressed that mathematics is best learned through considering student initiatives such as figuring out own solutions regardless of the time consumed. The least approved belief among all country groups is an utilitarianism in teaching such as learning works best through memorization or other non-time-consuming ways. The cultural groups may be further classified into two classes using TI and NF as dividers. The first class holding negative views on both TI and NF contains Developed Europe (including Germany, Norway and Switzerland), Confucian Asia (including Singapore and Taiwan), and the American group (including Chile and the USA). The other class holding positive views on both TI and NF contains Developing Asia (including Malaysia, the Philippines and Thailand) and East Europe (including Poland, Russia, and Georgia). Breaks within cultural groups may also be seen, which prompts a need for further studies on cultural classification.

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1 Introduction

Many researches have revealed that teachers' beliefs plays an important role in mathematics teaching (Ernest 1991; Nespor 1987; Op't Eynde et al. 2002; Pajares 1992; Philipp 2007; Thompson 1992; Törner 1997; Wilson and Cooney 2002). When we talk about teacher performance in class, beliefs represent dispositions toward action and a bridge between teacher knowledge and actual teaching (Felbrich et al. 2012; Rokeach 1960; Philipp 2007). Therefore, future teachers' beliefs may be crucial to their perception of classroom situations and their decision on what kind of knowledge to draw on or how to react. Beliefs are generally harder to change than so-called "attitudes" though (Philipp 2007; Raymond 1997; Skott 2001; Stipek et al. 2001; Thompson 1984). This is an important point for those who care about teacher education.

"Beliefs" can be defined as "psychologically held understandings, premises, or propositions about the world that are thought (or felt) to be true" (Philipp 2007; Richardson 1996). In spite of a lack of an agreed definition of beliefs, the definition above reflects a rather broad approach. According to this definition, the belief construct relates to one's cognition, knowledge, or affect, and additionally, it can be regarded as having an experiential and context-bound nature based on the social context in which one's beliefs developed (Beswick 2005; Felbrich et al. 2012; Hoyles 1992; Op't Eynde et al. 2002; Philipp 2007; Schoenfeld 1998). This also means that beliefs might be a culturally shaped mental construct. This idea is necessary for understanding the deeper meaning of beliefs especially when comparing beliefs of teachers, including future teachers, from many countries with different historical traditions or educational identities that vary significantly among countries (Felbrich et al. 2012).

The "Mathematics Teaching in the 21st Century" study (MT21) was a pioneer study of the "Teacher Education and Development Study in Mathematics" (TEDS-M). It compared future secondary teachers' beliefs in six countries, namely Bulgaria, Germany, Mexico, Taiwan, South Korea and the USA. The results have shown cross-country patterns and country-specific differences in the beliefs of future teachers (Schmidt et al. 2011a). For example, one of its findings was that "it is remarkable how homogeneously future teachers from South Korea and Taiwan view most of the issues", and this indicates that "some shared values about mathematics, teaching, and learning exist in South Korea and Taiwan, which probably reflects a relatively high level of social and cultural homogeneity in these two countries as compared with other countries" (Schmidt et al. 2011a, p. 187).

Furthermore, the MT21 study identified significant country differences in addition to the cultural split between East Asian and Western countries. It noted substantial differences between South Korea and Taiwan: the future teachers in South Korea showed "only a neutral view on the design of school life as a fundamental goal of teachers" but "the Taiwanese strongly supported this goal (Schmidt et al. 2011a, p. 187)." These findings remind us that the cultural homogeneity and heterogeneity in teaching-related beliefs of future teachers at the secondary-school level is subtle.

Culture is basically a collection of traits defining particular groups of people or a category specified by geographical boundaries, so that the members of this group or category can be distinguished from others (Hofstede 2001; Markus and Hamedani 2007). Properly speaking, culture consists of explicit and implicit patterns of historically derived or selected ideas including belief and value systems, and their embodiment in institutions or practices. These cultural patterns might be simultaneously considered to be products of action as well as conditioning elements of future action (Markus and Hamedani 2007).

On the other hand, culture involves shared understandings that serve as a medium through which individual human minds interact in communication with one another (Stenhouse 1967). This is why some people said culture can shape "the way things are done" and our understanding of reasons how things should be. The beliefs, as a kind of culturally shaped mental constructs, may serve as a means for our understanding of the way things are done and should be. Revealing beliefs may thus reflect not only the understanding and dispositions of the belief holders but also the influence caused by culture. This study takes the chance of the national-representative large-scale samples in TEDS-M to explore the patterns of mathematics-related beliefs across countries. The patterns of beliefs may help to reveal meaningful cultural images within or between countries.

2 Study Design

2.1 The Basic Aim

This paper is based on earlier studies of the authors (Tang and Hsieh 2012a, 2012b) and aims at highlighting the cultural notion of mathematics teacher education by analyzing beliefs of future lower secondary teachers.[1] The major questions of this study are as follows: (1) What patterns of beliefs about the nature of mathematics and about teaching and learning mathematics do future lower secondary teachers hold at the end of their preparation? (2) What cultural meanings do the above patterns of beliefs imply?

2.2 Theoretical Framework

Because many studies have suggested that beliefs about mathematics and mathematics learning that beginning teachers hold might influence how they teach and

[1]The data set used in this paper is TEDS_MS_DRAFT_IDB_20101103_v31.

subsequently how their students learn, TEDS-M gathered data about the following three aspects of mathematics future teachers' teaching-related beliefs: (1) beliefs about the nature of mathematics; (2) beliefs about the learning mathematics; and (3) beliefs about mathematics achievement (Tatto et al. 2012).

Generally speaking, there are several major views of mathematics, mathematics learning, and mathematics ability. In terms of the nature of mathematics, the dichotomy between "dynamic" and "static" has often been discussed. The former perceives mathematics as being continually undergoing change and revision, or a creative and generative process. But the latter views mathematics as a static and immutable unified entity, maybe including the accumulation of facts, rules, and skills (Ernest 1988; Stipek et al. 2001).

In terms of learning mathematics, the "transmission/traditional" view and "learner-centredness/constructivist" view are two common and important categories (Barkatsas and Malone 2005; Cross 2009; Perry et al. 1999). The first perspective tends to allot the role of transmitting mathematics to a teacher, and the role of carefully obeying teachers' instruction to a student. The second perspective tends to emphasize the initiative of a student and that teachers should establish a learner-focused environment for students as well as providing them opportunities to construct their own meanings.

When mentioning mathematics achievement in teaching or learning, the "ability" construct plays a critical role. Theorists usually make a distinction between the "entity" and the "incremental" views (Stipek et al. 2001). The former considers "ability" to be stable and not very amenable to change. Hence, any efforts to enhance the achievement are usually limited. The latter is opposite to the former.

Beliefs have been viewed as a component in culture that identifies people as belonging to the same or to different collectives (Hofstede 1984; Tylor 1889). How to distinguish countries by cultural identities is still an open question. It is an appropriate beginning to distinguish mostly by geographical region or historical traditions (Blömeke and Kaiser 2012; Leung 2006). This paper adopts the schemes of Blömeke and Kaiser, and Leung's in distinguishing countries into different cultural groups for probing probable cultural meaning relative to mathematics teacher education at the secondary school level. The classification of cultural groups will be delineated in the data analysis section below.

2.3 The Sample

The sampling plan of TEDS-M followed a stratified multistage probability sampling design (Tatto et al. 2009). The target populations were the future lower secondary teachers in their last year of training to teach mathematics. Although the International Association for the Evaluation of Educational Achievement (IEA) set a minimum requirement of a 75 % combined participation rate as a threshold, samples having a participation rate of 60–75 % were also suitable for use, according to the IEA's criterion, with an annotation of low participation rates.[2] Our analyses included data from fifteen participating countries covering Botswana, Chile, Georgia, Germany, Malaysia, Norway, Oman, the Philippines, Poland (concurrent type of institution only), the Russian Federation (Russia), Singapore, Switzerland (German-speaking cantons), Taiwan, Thailand, and the United States of America (the USA, public institutions only). 8207 future lower secondary teachers participated in total.

2.4 Instrument

TEDS-M surveyed the future teachers' beliefs about the nature of mathematics, about learning mathematics, and about mathematics achievement (Tatto et al. 2012). These teaching-related topics are considered as three indicators of the cultural meanings behind the future teachers' mathematics-teaching views in this paper. Mathematics teaching is one kind of common and important cultural activity in modern human society. How to view mathematics and its learning/transitive process is crucial in this context.

There were 12, 14, and 8 items capturing the beliefs about the nature of mathematics, about learning mathematics, and about mathematics achievement respectively in the TEDS-M questionnaires. The future teachers were asked to choose one from six response alternatives in each item: 1—strongly disagree; 2—disagree; 3—slightly disagree; 4—slightly agree; 5—agree; 6—strongly agree (Tatto et al. 2012).

2.5 Data Analysis

Originally TEDS-M analyzed the date from primary and secondary future teachers as a whole. Because of the differences in mathematical materials and some pedagogical goals between primary and secondary levels, our study decided to develop new scales only for future lower secondary teachers by factor analysis and Rasch modeling (employing IRT). The study conducted factor analyses to examine the factor structure of the future teachers' responses on each topic with the software SPSS, and continued to use the partial credit model (Masters 1982), one model out of the family of Rasch models, to complete the Rasch scaling on each factor with the software Winsteps.

Every scale was treated as one indicator to depict the future teachers' teaching-related beliefs. When calibrating the items for each of the scales, those cases with

[2]The combined participation rates of Chile and Poland were between 60 % and 75 %. Of Norway, it was 58 % (Tatto et al. 2012). The other participating countries like the USA and Germany suggested to make an exception and to include the national data of Norway (e.g. Blömeke et al. 2011; Schmidt et al. 2011b).

50 % or more missing responses on the items for the scale were excluded from the analysis. In addition, the sampling weights were transformed linearly so that each country contributed equally to the calibration and analysis scale by scale. During each analysis, this study set the Rasch score of 10 as neutral belief, i.e. neither positive nor negative. A value more than 10 implied a positive belief or endorsement, and on the contrary, a value less than 10 implied a negative belief or disagreement.

This study also classified the TEDS-M countries roughly into several groups with specific educational cultures or backgrounds according to the scholarly distinction (Blömeke and Kaiser 2012; Leung 2006). Notwithstanding that each of them may have its own complicated educational culture, it should be an appropriate beginning to distinguish mostly by geographical region or historical traditions: (1) Confucian Asia: including the two Asian countries, Singapore and Taiwan, having a Confucian tradition. (2) Developing Asia: including the three countries, Malaysia, the Philippines and Thailand, belonging to the Association of Southeast Asian Nations. (3) East Europe: including three countries, Poland and Russia of East Europe and Georgia of Central Asia, coming from the former Eastern European block led by the Soviet Union. (4) Developed Europe: including Germany, Norway and Switzerland, belonging to the traditional developed group. (5) American group: including Chile and the USA. (6) The others: including Botswana of Africa and Oman of West Asia. The last group will not be discussed as a whole in this paper due to not being able to identify their similarity by geographical region or historical tradition. However, their individual results will still be reported. The other groups are called "cultural groups".

In some cases, the percentages of approval or disapproval toward a belief item are used in this paper. The approval includes responses checking 4—slightly agree, 5—agree, and 6—strongly agree. The disapproval includes responses checking 1-strongly disagree, 2—disagree, and 3—slightly disagree. The percentages were calculated by applying total sample weights of the future teachers. A percentage of a belief for a country was the average over all individuals in that country. The international percentage of a belief was obtained by averaging over the percentages of all countries so that every country was weighted equally in this paper. This approach of weighting also applied to calculating the averages of any group of countries. The percentages of approval are denoted by *perA* and the percentages of disapproval are denoted by *perD*.

3 Results

3.1 Overall Factors

With respect to the nature of mathematics, the analysis yielded two principal factors explaining a total of 52.1 % of the variance in the belief items (Table 1). The first factor was labeled "Open and Creative nature (OC)" due to the high loadings

Table 1 Factor loadings of the belief items about the nature of mathematics

Item	Factor Loadings		Communality
	OC	CR	
I. Many aspects of mathematics have practical relevance	0.750	–	0.573
D. In mathematics many things can be discovered and tried out by oneself	0.724	–	0.530
F. If you engage in mathematical tasks, you can discover new things (e.g., connections, rules, concepts)	0.708	–	0.531
C. Mathematics involves creativity and new ideas	0.695	–	0.502
J. Mathematics helps solve everyday problems and tasks	0.691	–	0.495
H. Mathematical problems can be solved correctly in many ways	0.686	–	0.479
E. When solving mathematical tasks you need to know the correct procedure else you would be lost	–	0.760	0.577
A. Mathematics is a collection of rules and procedures that prescribe how to solve a problem	–	0.757	0.575
B. Mathematics involves the remembering and application of definitions, formulas, mathematical facts and procedures	–	0.743	0.565
L. Mathematics means learning, remembering and applying	–	0.737	0.557
K. To do mathematics requires much practice, correct application of routines, and problem solving strategies	–	0.685	0.546
G. Fundamental to mathematics is its logical rigor and preciseness	–	0.517	0.316
Eigenvalue	4.135	2.112	6.247
% of total variance	34.46 %	17.60 %	
Total variance	34.46 %	52.06 %	

Note. Only the loadings with the absolute value over 0.400 were shown in this table. OC = Open & Creative nature; CR = Conservative & Rigorous nature. Source: "Beliefs of Future Secondary Mathematics Teachers", (in Chinese) by S.-J. Tang & F.-J. Hsieh, 2012, in F.-J. Hsieh (Ed.), *Taiwan TEDS-M 2008: Teacher Education and Development Study in Mathematics* (in Chinese; pp. 221–252). Department of Mathematics, National Taiwan Normal University. Adapted with permission

of the items about openness and creativity as characteristics of the nature of mathematics. The next factor was labeled "Conservative and Rigorous nature (CR)". This was due to the high loadings of the items about accepted procedures or rules as characteristics of the nature of mathematics. The OC factor explains 34.5 % of the variance. It is about twice as much as the variance explained by the CR factor.

Table 2 Factor loadings of the belief items about learning mathematics

Item	Factor Loadings			Communality
	SI	TI	UT	
N. It is helpful for pupils to discuss different ways to solve particular problems	0.737	–	–	0.571
M. Teachers should encourage pupils to find their own solutions to mathematical problems even if they are inefficient	0.726	–	–	0.605
H. Teachers should allow pupils to figure out their own ways to solve mathematical problems	0.724	–	–	0.560
L. Pupils can figure out a way to solve mathematical problems without a teacher's help	0.655	–	–	0.498
G. In addition to getting a right answer in mathematics, it is important to understand why the answer is correct	0.555	–	−0.422	0.567
K. Time used to investigate why a solution to a mathematical problem works is time well spent	0.555	–	–	0.415
B. Pupils need to be taught exact procedures for solving mathematical problems	–	0.756	–	0.579
E. Pupils learn mathematics best by attending to the teacher's explanations	–	0.695	–	0.553
A. The best way to do well in mathematics is to memorize all the formulas	–	0.655	–	0.484
D. To be good in mathematics you must be able to solve problems quickly	–	0.556	0.416	0.482
J. Hands-on mathematics experiences aren't worth the time and expense	–	–	0.708	0.544
C. It doesn't really matter if you understand a mathematical problem, if you can get the right answer	–	–	0.661	0.501
F. When pupils are working on mathematical problems, more emphasis should be put on getting the correct answer than on the process followed	–	–	0.644	0.538
I. Non-standard procedures should be discouraged because they can interfere with learning the correct procedure	–	0.423	0.468	0.399
Eigenvalue	3.506	2.666	1.125	7.296
% of total variance	25.04 %	19.04 %	8.03 %	
Total variance	25.04 %	44.08 %	52.12 %	

Note. Only the loadings with the absolute value over 0.400 were shown in this table. SI = learning through Student Initiative; TI = learning by following Teacher Instruction; UT = Utilitarianism in Teaching. Source: "Beliefs of Future Secondary Mathematics Teachers" (in Chinese) by S.-J. Tang and F.-J. Hsieh, 2012, in F.-J. Hsieh (Ed.), *Taiwan TEDS-M 2008: Teacher Education and Development Study in Mathematics* (in Chinese; pp. 221–252). Department of Mathematics, National Taiwan Normal University. Adapted with permission

Table 3 Factor loadings of the belief items about mathematics achievement

Item	Factor loadings	Communality
	NF	
C. Mathematics is a subject in which natural ability matters a lot more than effort	0.752	0.565
B. To be good at mathematics you need to have a kind of "mathematical mind"	0.731	0.535
D. Only the more able pupils can participate in multi-step problem solving activities	0.712	0.506
F. Mathematical ability is something that remains relatively fixed throughout a person's life	0.632	0.400
H. Some ethnic groups are better at mathematics than others	0.620	0.385
E. In general, boys tend to be naturally better at mathematics than girls	0.618	0.381
G. Some people are good at mathematics and some aren't	0.607	0.369
A. Since older pupils can reason abstractly, the use of hands-on models and other visual aids becomes less necessary	0.533	0.284
Eigenvalue	3.425	3.425
% of total variance	42.82 %	
Total variance	42.82 %	

Note. Only the loadings with the absolute value over 0.400 were shown in this table. NF = Natural & Fixed mathematical ability. Source: "Beliefs of Future Secondary Mathematics Teachers", (in Chinese) by S.-J. Tang and F.-J. Hsieh, 2012, in F.-J. Hsieh (Ed.), *Taiwan TEDS-M 2008: Teacher Education and Development Study in Mathematics* (in Chinese; pp. 221–252). Department of Mathematics, National Taiwan Normal University. Adapted with permission

For the beliefs on the learning of mathematics, the analysis yielded three factors explaining a total of 52.1 % of the variance in the items (Table 2). The first principal factor was labeled "Learning through Student Initiative (SI)" because most of the items with high loading for this factor emphasized the students' priority and initiative in learning processes. The next factor was labeled "Learning by following Teacher Instruction (TI)" due to the high loadings of the corresponding items. More than 40 % of the variance can be explained by these two factors: the SI factor explains 25.0 % and the TI explains 19.0 %. The third factor was labeled "Utilitarianism in Teaching (UT)" and explains 8.0 % of the variance. Those items with higher loadings for the UT factor usually contained utilitarian calculations about learning or teaching.

With respect to the beliefs on mathematics achievement, the analysis yielded only one principal factor for the set of items. This factor explains a total of 42.8 % of variance (Table 3). It was labeled "Natural and Fixed mathematical ability (NF)" because the items with high loading for this factor involve how to interpret the ability behind mathematics achievement.

3.2 Descriptive Results

The majority of future teachers of the TEDS-M sample (about 8,000 lower-secondary mathematics teachers in their final year of teacher education from 15 countries) tended to agree with the belief items of the OC factor about the nature of mathematics and the SI factor about learning mathematics. All approval percentages of the belief items in the OC factor were high and exceeded 88 %. The most popular belief items were as follows (*perA* > 90 %, *perD* < 10 %):

- *Mathematical problems can be solved correctly in many ways* (*perA* = 93.7 %, *perD* = 3.5 %).
- *Many aspects of mathematics have practical relevance* (*perA* = 91.6 %, *perD* = 5.3 %).
- *If you engage in mathematical tasks, you can discover new things* (*e.g., connections, rules, concepts*) (*perA* = 91.5 %, *perD* = 5.7 %).

The high *perA* of the second belief item means that for most future teachers mathematics is not abstract, rather it has practical relevance. The high *perA* of the third belief is a rather surprising result, especially in the Confucian Asia countries where the difficulty level of their mathematics curricula barely endorses a discovery of new rules or concepts. In contrast to the OC factor, all the approval percentages of the belief items in the CR factor were smaller than 85 % except item K (*perA* = 88.7 %, *perD* = 8.5 %), which emphasizes the heavy practice and correct application of routines in mathematics. The fact that all but one belief item in the OC factor received more percentages of approval than that of item K implies that most of the future teachers also held strong beliefs of some "temperament" of mathematics beyond the beliefs of practicing or applying routines in mathematics. More than about a quarter of future teachers disapproved the following items in the CR factor:

- *When solving mathematical tasks you need to know the correct procedure else you would be lost* (*perA* = 64.3 %, *perD* = 33.0 %).
- *Mathematics is a collection of rules and procedures that prescribe how to solve a problem* (*perA* = 73.2 %, *perD* = 24.2 %).

Overall, all belief items about the nature of mathematics were approved by more than 60 % of the future teachers. This suggests that future lower-secondary mathematics teachers perceive mathematics as a multifaceted entity.

In terms of the beliefs about learning mathematics, all the percentages of the items in the SI factor were above 80 %. These items earned at least a 20 % higher approval than the rest of the items in terms of the beliefs about learning mathematics. The most popular belief statements emphasizing an initiative of learners were as follows (*perA* > 90 %, *perD* < 10 %):

- *It is helpful for pupils to discuss different ways to solve particular problems* (*perA* = 93.4 %, *perD* = 3.6 %).
- *Teachers should allow pupils to figure out their own ways to solve mathematical problems* (*perA* = 92.5 %, *perD* = 4.6 %).

- *In addition to getting a right answer in mathematics, it is important to understand why the answer is correct* (*perA* = 92.3 %, *perD* = 5.0 %).

The results also showed the future teachers' tendency to disapprove the items in the factors TI and UT. Almost all the approval percentages were below 60 % and the percentages of disapproval were above one third except the item B which emphasized exact procedures for solving mathematical problems (*perA* = 61.3 %, *perD* = 35.7 %). Most of the approval percentages for utilitarian items were especially low such that the most unpopular statements of all items fell in the UT factor, which were as follows ($perA < 33$ %, $perD > 70$ %):

- *When pupils are working on mathematical problems, more emphasis should be put on getting the correct answer than on the process followed* (*perA* = 25.4 %, *perD* = 71.6 %).
- *Hands-on mathematics experiences aren't worth the time and expense* (*perA* = 21.0 %, *perD* = 75.4 %).
- *It doesn't really matter if you understand a mathematical problem, if you can get the right answer* (*perA* = 17.4 %, *perD* = 79.5 %).

For the belief items in the TI factor, the approval percentages were between 35 % and 61 %. The beliefs of the future teachers were split in two rather equal groups. To summarize, most of the future teachers consistently emphasized the students' priority and initiative, and tended to reject utilitarian schemes in learning mathematics, whereas the consensus on whether to stress teachers' instruction was relatively low.

Regarding the beliefs about mathematics achievement, the future teachers seemed to have no consensus on seven of the eight belief items. The percentages of approval for these seven items spread from 33.5 % to 59.0 %. Only three items obtained approval of more than 50 %:

- *Some people are good at mathematics and some aren't* (*perA* = 73.4 %, *perD* = 22.7 %).
- *To be good at mathematics you need to have a kind of "mathematical mind"* (*perA* = 59.0 %, *perD* = 37.7 %).
- *Mathematical ability is something that remains relatively fixed throughout a person's life* (*perA* = 54.8 %, *perD* = 41.1 %).

Each percentage of disapproval with respect to the other items was over 60 % except the item H referring to ethnic issues (*perA* = 36.7 %, *perD* = 59.2 %).[3] In general, most future teachers believed that natural mathematical ability or mind was something relatively fixed such that some people were good at mathematics or not. But when gender,[4] ethnicity,[5] and equality or opportunities to learn[6] became an

[3]The item H is "*Some ethnic groups are better at mathematics than others*".

[4]As the item E: "*In general, boys tend to be naturally better at mathematics than girls*".

[5]As the item H.

[6]As the item A: "*Since older pupils can reason abstractly, the use of hands-on models and other visual aids becomes less necessary*", and the item D: "*Only the more able pupils can participate in multi-step problem solving activities*".

issue of a statement, their responses were affected. This result may be helpful for our understanding of the "differentiation" of beliefs of future secondary teachers (Rokeach 1960).

3.3 Factors and Distribution

The central tendency of the future teachers' beliefs from the different countries and the distribution of their beliefs about the nature of mathematics, learning mathematics, and mathematics achievement can be seen at a glance in the following tables (see Tables 4, 5 and 6) and figures (see Figs. 1 to 5).

The Belief of the Nature of Mathematics With respect to the nature of mathematics, the means of each country were significantly greater than the neutral value 10 on both the OC and CR scales (Table 4). This means that future teachers in all

Table 4 Means of beliefs about the nature of mathematics

Country	OC		Country	CR	
	M	(SE)		M	(SE)
Philippines	13.34[a,b]	(0.13)	Philippines	12.76[a,b]	(0.11)
Oman	13.20[a,b]	(0.09)	Thailand	12.06[a,b]	(0.04)
US-public	12.85[a,b]	(0.11)	Malaysia	11.79[a,b]	(0.07)
Thailand	12.83[a,b]	(0.05)	Botswana	11.74[a,b]	(0.16)
Botswana	12.74[b]	(0.15)	Oman	11.62[a,b]	(0.05)
Chile	12.69[a,b]	(0.08)	Georgia	11.54[b]	(0.15)
Malaysia	12.45[b]	(0.09)	Chile	11.32[a,b]	(0.04)
Taiwan	12.43[b]	(0.07)	US-public	11.30[a,b]	(0.16)
IA	12.43		IA	11.24	
Germany	12.33[a,b]	(0.08)	Singapore	11.19[b]	(0.06)
Poland	12.24[a,b]	(0.09)	Taiwan	11.09[a,b]	(0.05)
Singapore	12.10[a,b]	(0.06)	Russia	10.81[a,b]	(0.04)
Switzerland	12.08[a,b]	(0.10)	Poland	10.63[a,b]	(0.06)
Norway	12.03[a,b]	(0.06)	Norway	10.57[a,b]	(0.03)
Russia	11.77[a,b]	(0.06)	Switzerland	10.20[a,b]	(0.05)
Georgia	11.32[a,b]	(0.15)	Germany	10.04[a,b]	(0.03)

Note. IA = international average; OC = Open & Creative nature; CR = Conservative & Rigorous nature. Source. "Beliefs of Future Secondary Mathematics Teachers" (in Chinese) by S.-J. Tang and F.-J. Hsieh, 2012, in F.-J. Hsieh (Ed.), *Taiwan TEDS-M 2008: Teacher Education and Development Study in Mathematics* (in Chinese; pp. 221–252). Department of Mathematics, National Taiwan Normal University. Adapted with permission

[a]Mean ≠ IA, significant ($p < 0.05$)

[b]Mean ≠ 10, significant ($p < 0.05$)

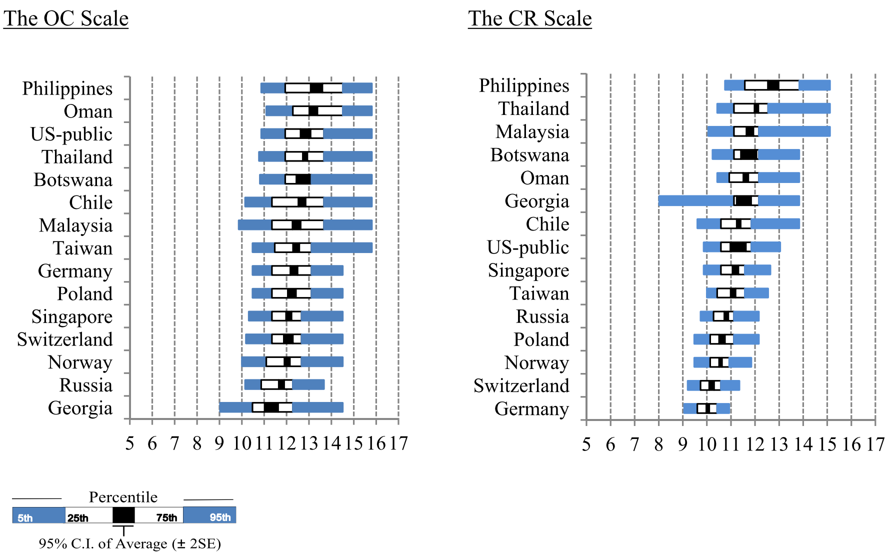

Fig. 1 Distribution of belief scores about the nature of mathematics (*two scales*). From "Taiwanese Future Secondary Mathematics Teachers' Beliefs under the Perspective of International Comparison", (in Chinese) by S.-J. Tang and F.-J. Hsieh, 2012, in F.-J. Hsieh (Ed.), *Taiwan TEDS-M 2008 Secondary Analysis* (in Chinese). Department of Mathematics, National Taiwan Normal University. Adapted with permission

countries approved that mathematics is an open, practical and creative field on the one hand, as well as rigorous and requiring fixed procedures on the other hand. However, the OC beliefs were significantly stronger than the CR beliefs for each country. An interesting result is shown in Fig. 1: those countries that scored higher tended to have wider spreads of scores for both the OC and CR scales.

Regarding the cultural notions, we can see from Table 4 that Developing Asia and the American groups always have larger means than the international average and the other cultural groups, so they tend to hold stronger beliefs on the nature of mathematics than the other groups. Table 4 and Fig. 1 also show that some cultural groups have their own characteristics or specific foci on the nature of mathematics. For the countries of the Confucian Asia group, all the means are not significantly different from the international averages. It seems to exist a "Doctrine of the Mean" (middle path) of Confucianism for mathematics. On the other hand, the ranges of belief scores of Developed Europe and East Europe countries (excluding Georgia) are the narrowest on the CR scale and their means are lower than the other cultural groups. This result may reflect the general situation that the future secondary teachers' cognition on conservative and rigorous nature of mathematics was quite alike in any one of these countries.

The Belief of Leaning of Mathematics The international averages in Table 5 show that the central tendencies of the participating countries on the three scales

Table 5 Means of belief scores in the scale about learning mathematics

Country	SI		Country	TI		Country	UT	
	M	(SE)		M	(SE)		M	(SE)
Chile	12.55[a,b]	(0.08)	Philippines	11.13[a,b]	(0.08)	Malaysia	10.24[a,b]	(0.04)
Switzerland	12.39[a,b]	(0.12)	Georgia	10.79[a,b]	(0.12)	Philippines	10.04[a,b]	(0.08)
Taiwan	12.26[a,b]	(0.05)	Malaysia	10.70[a,b]	(0.06)	Georgia	9.61[a,b]	(0.11)
Germany	12.23[a,b]	(0.06)	Oman	10.54[a,b]	(0.05)	Oman	9.51[a,b]	(0.04)
US-public	12.11[a,b]	(0.09)	Botswana	10.18	(0.15)	Botswana	9.42[a,b]	(0.12)
Poland	12.08[a,b]	(0.11)	Russia	9.99[a]	(0.04)	Chile	9.30[a,b]	(0.04)
Oman	11.95[b]	(0.07)	Singapore	9.99	(0.05)	IA	9.32	
IA	11.92		Chile	9.95[b]	(0.04)	Thailand	8.96[b]	(0.05)
Philippines	11.85[a,b]	(0.13)	IA	9.92		Russia	8.94[a,b]	(0.04)
Thailand	11.85[a,b]	(0.05)	US-public	9.62[a,b]	(0.12)	Singapore	8.87[a,b]	(0.06)
Russia	11.79[a,b]	(0.06)	Poland	9.49[a,b]	(0.06)	Poland	8.63[a,b]	(0.08)
Botswana	11.78[b]	(0.14)	Thailand	9.48[a,b]	(0.05)	Taiwan	8.39[a,b]	(0.06)
Norway	11.73[a,b]	(0.05)	Taiwan	9.37[a,b]	(0.05)	Germany	8.38[a,b]	(0.08)
Singapore	11.49[a,b]	(0.05)	Norway	9.23[a,b]	(0.03)	Norway	8.38[a,b]	(0.05)
Georgia	11.44[a,b]	(0.19)	Switzerland	9.19[a,b]	(0.08)	Switzerland	8.38[a,b]	(0.10)
Malaysia	11.31[a,b]	(0.06)	Germany	9.13[a,b]	(0.06)	US-public	8.38[a,b]	(0.14)

Note. IA= international average; SI = learning through Student Initiative; TI = learning by following Teacher Instruction; UT = Utilitarianism in Teaching. From "Beliefs of Future Secondary Mathematics Teachers", (in Chinese) by S.-J. Tang and F.-J. Hsieh, 2012, in F.-J. Hsieh (Ed.), *Taiwan TEDS-M 2008: Teacher Education and Development Study in Mathematics* (in Chinese; pp. 221–252). Department of Mathematics, National Taiwan Normal University. Adapted with permission

[a]Mean ≠ IA, significant ($p < 0.05$)

[b]Mean ≠ 10, significant ($p < 0.05$)

are different about the learning of mathematics. A tendency of endorsing students' initiative and ignoring teacher instruction and utilitarian ideas in teaching can be seen. All means are significantly greater than 10 for belief items in the SI scale, but only so for a few means on the other scales. On the scale of TI and UT, the ranges of mean scores among countries are wider than the range on SI. The UT means though spread out but almost entirely located under 10 (Table 5), which may reveal a general idealism.

Regarding the distribution of the belief scores in each country, Fig. 2 shows that in general SI scores spread out wider than TI and UT scores. It means that though SI gains higher approval from future teachers, the beliefs among future teachers in a country are more varied. Further, for the SI scale, Fig. 2 shows that those countries scored higher tended to have wider spreads of scores about the belief for SI, whereas narrower about the belief for TI.

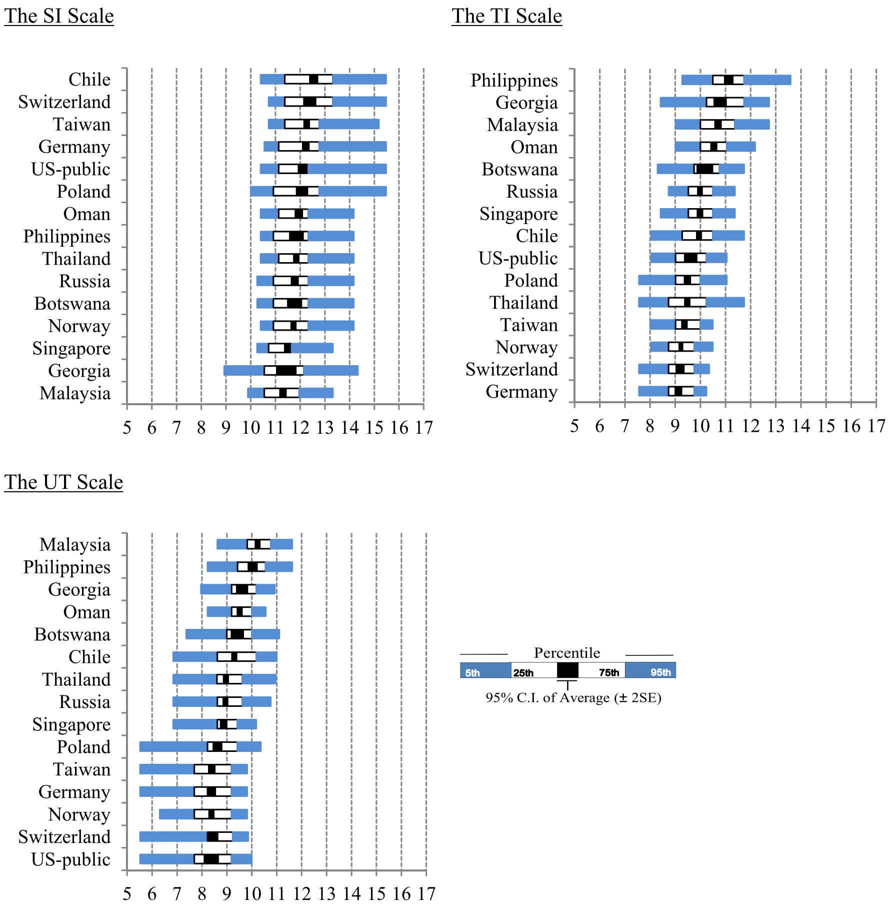

Fig. 2 Distribution of belief scores about learning mathematics (*three scales*). From "Taiwanese Future Secondary Mathematics Teachers' Beliefs under the Perspective of International Comparison", (in Chinese) by S.-J. Tang and F.-J. Hsieh, 2012, in F.-J. Hsieh (Ed.), *Taiwan TEDS-M 2008 Secondary Analysis* (in Chinese). Department of Mathematics, National Taiwan Normal University. Adapted with permission

Regarding the cultural notion, Fig. 3 shows that for some of the cultural groups, beliefs of some scales among countries are similar but are not similar in other scales or to other cultural groups. The most consistent group is the Developed Europe, where TI and UT are almost identical among countries. American group and Confucian Asia are similar in terms of the high SI and below-neutral TI and UT. Countries of Developing Asia express a larger variation and so do those of East Europe. Countries in Developing Asia generally have narrower ranges than the other groups. If we concentrate on the spots of scores for all scales of every country, we can see that the USA, Taiwan, Switzerland, and Germany have quite similar distributions. This is a break of cultural group and needs further study.

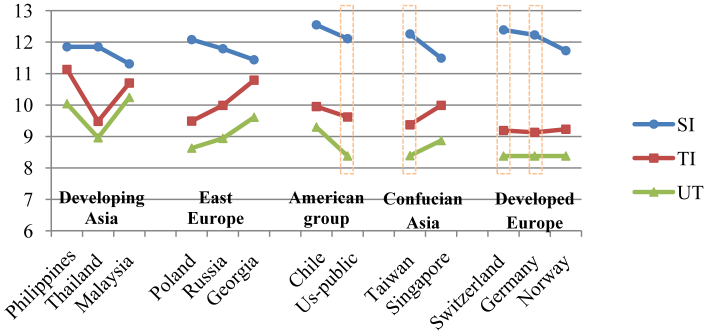

Fig. 3 Plots of mean logit scores on the scales of the beliefs about learning mathematics by cultural groups

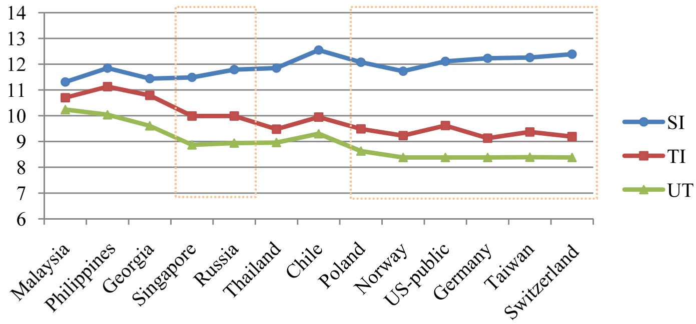

Fig. 4 Plots of mean logit scores on the scales of the beliefs about learning mathematics according to the size of ranges of all three scales

The variance among countries in some cultural groups prompted this study to check other combinations of countries in terms of beliefs about leaning mathematics. Figure 4 shows that the USA, Poland, and Taiwan join the Developed Europe group if we consider the threshold that none of the differences between any two countries on any scale were bigger than 0.5 points (except the 0.66 for Norway and Switzerland on the SI scale). For both the USA and Poland a relation with Europe can be identified in terms of geographical region or historical tradition. The USA has a root from England, a developed European country. Poland is located in central Europe. It seems reasonable if they share similar patterns of learning beliefs with the Developed Europe. However, in the case of Taiwan as an East Asian country embedded in the Confucian ideology, its consistency with Developed Europe is a mystery that needs further study.

The Belief of Natural and Fixed Mathematical Ability According to the international average in Table 6, the central tendency of the belief scores of all participating countries is below the value of 10 on the NF scale. In spite of the tendency of disapproval, Mean of most countries of the Developing Asia and East Europe

Table 6 Means of belief scores in the scale about mathematics achievement

Country	NF	
	M	(SE)
Malaysia	10.58[a,b]	(0.05)
Philippines	10.53[a,b]	(0.07)
Georgia	10.37[a,b]	(0.09)
Thailand	10.33[a,b]	(0.02)
Botswana	10.13[a]	(0.10)
Oman	10.09[a,b]	(0.05)
Russia	10.06[a,b]	(0.02)
Poland	9.89[a,b]	(0.04)
IA	9.84	
Taiwan	9.83[b]	(0.04)
Singapore	9.73[a,b]	(0.04)
Chile	9.35[a,b]	(0.04)
Norway	9.34[a,b]	(0.03)
Switzerland	9.22[a,b]	(0.06)
Germany	9.13[a,b]	(0.04)
US-public	9.02[a,b]	(0.16)

Note. IA = international average; NF = Natural & Fixed mathematical ability. From "Beliefs of Future Secondary Mathematics Teachers" (in Chinese) by S.-J. Tang and F.-J. Hsieh, 2012, in F.-J. Hsieh (Ed.), *Taiwan TEDS-M 2008: Teacher Education and Development Study in Mathematics* (in Chinese; pp. 221–252). Department of Mathematics, National Taiwan Normal University. Adapted with permission

[a]Mean ≠ IA, significant ($p < 0.05$)

[b]Mean ≠ 10, significant ($p < 0.05$)

(Poland excluded) reveals that they tend to agree with the statements included in this scale. Contrarily, the American group and Developed Europe tend to disapprove the idea of natural and stable nature of mathematical ability. The range of the USA is the widest (see Fig. 5).

3.4 Cultural Notion Across Scales

(1) *Common Ground*. The results showed that there are common trends of mathematics-related teaching beliefs internationally. Over 80 % of future secondary teachers (82.9–93.7 %) approved (including slightly agree, agree, and strongly agree) three quarters of the belief items[7] about the nature of mathematics, and all belief items in the OC (open and creative nature) factor were included therein. This represents that the open and creative nature of mathematics is a prevalent belief across countries.

Regarding the items about the belief of learning mathematics, all, and only these, belief items of the SI (about student initiative in learning) factor received

[7]The *perA* of the items about the nature of mathematics are as follows: H (94 %), I (92 %), F (92 %), J (89 %), D (89 %), K (89 %), C (88 %), G (85 %), and B (83 %). See Table 1 for the content.

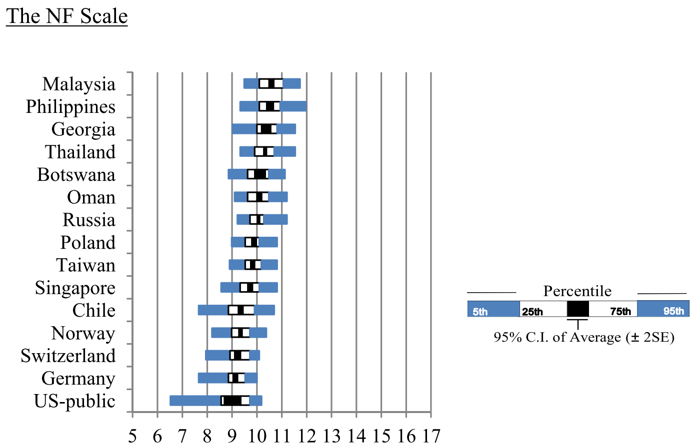

Fig. 5 Distribution of belief scores about mathematics achievement (*one scale*). From "Taiwanese future secondary mathematics teachers' beliefs under the perspective of international comparison", (in Chinese) by S.-J. Tang and F.-J. Hsieh, 2012, in F.-J. Hsieh (Ed.), *Taiwan TEDS-M 2008 Secondary Analysis* (in Chinese). Department of Mathematics, National Taiwan Normal University. Adapted with permission

approval from over 80 % of future teachers (81.2–93.4 %),[8] which accounted for 43 % of the total items about learning mathematics. On the other hand, although all the individual disapproval percentages do not exceed 80 %, some of them are either close to or not far from it. Many future teachers disapproved that understanding a mathematical problem didn't really matter if you could get the right answer ($perD = 79.5$ %), hands-on mathematics experiences were not worth the time and expense ($perD = 75.4$ %), and getting the correct answer deserved more emphasis than the working process ($perD = 71.6$ %).[9] Generally speaking, it seemed that most of the future secondary teachers tended to reject utilitarianism and hold the ideas of openness and "learning first" in mathematics teaching even though the nature of content involved logical rigor and preciseness, the remembering and correct application of formulas, and mathematical routines and procedures. The existence of common beliefs suggests that a part of the beliefs is shaped by some matters; a possible one is the wave of globalization, but it is still a hypothesis to test.

(2) *Cultural Patterns*. The concept of logit scores allows a comparison of means across different scales. Figure 6 shows the mean logit scores of all scales of mathematics-teaching related beliefs by cultural groups. Among all the scales, SI receives the smallest dispersion which means that the belief about students' initiative gains strongest agreement among the cultural groups. On the contrary, CR

[8]The *perA* of the items about the learning of mathematics are as follows: N (93 %), H (93 %), G (92 %), K (87 %), M (84 %), and L (81 %). See Table 2 for the content.

[9]These items were on the UT scale.

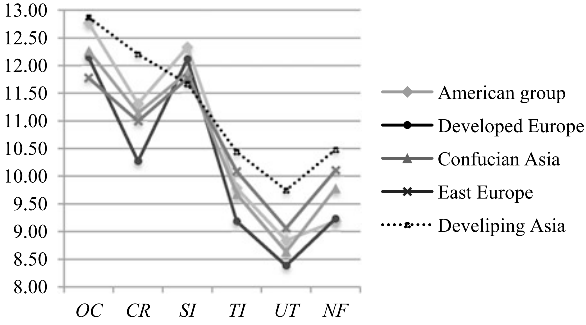

Fig. 6 Plots of mean logit scores of all scales of mathematics-teaching related beliefs by cultural groups. OC = Open & Creative nature; CR = Conservative & Rigorous nature; SI = learning through Student Initiative; TI = learning by following Teacher Instruction; UT = Utilitarianism in Teaching; NF = Natural & Fixed mathematical ability

receives the largest dispersion which means that the cultural groups has little consensus on the conservation and rigor nature of mathematics. Figure 6 also shows that the two groups, East Europe and Confucian Asia exhibit the most closed patterns on all the scales, while Developing Asia has the most distinct pattern than other groups. A rough pattern among all groups except Developing Asia is that they have the strongest beliefs on OC and SI, the second on CR, the third on TI and NF, and the last on UT.

If we consider the neutrality of the beliefs among cultural groups, we found that all groups approved both scales of the nature of mathematics and the SI of mathematics learning, and also disapproved the UT of mathematics learning. But on TI and NF scales, the cultural groups split into two classes. The first class includes Developed Europe, Confucian Asia, and the American group which has negative views on both TI and NF. The other class includes Developing Asia and East Europe that hold positive beliefs on both TI and NF (see Table 7).

The analysis of neutrality provides a comparison of cultural groups with an absolute measure, while an analysis of above or below international averages provides the comparison with a relative measure. The results of the latter analysis are shown in Table 7.

In general, the two classes probably reflect a kind of classification by the relative ideology of the future teachers. The key dividers of these two classes include the scales about whether to approve (relatively) on the natural and fixed mathematical ability (NF) and emphasize (relatively) the role of teacher instruction (TI) and whether to focus (relatively) on getting right answers in a non-time-consuming way (UT). For the first class, a relative negative view on the scales of key dividers can be seen, while the second class has a relative positive view. The views about nature of mathematics and initiative of students are the advanced dividers within classes.

Table 7 Patterns of relative positive or negative views of two classes of cultural groups

Classification	Nature of mathematics		Learning mathematics			Mathematics achievement
	OC	CR	SI	TI	UT	NF
Class 1						
American group	+	+	+	−	−	−
Developed Europe	−	−	+	−	−	−
Confucian Asia	−	−	−	−	−	−
Class 2						
East Europe	−	−	−	+	+	+
Developing Asia	+	+	−	+	+	+

Note. OC = Open & Creative nature; CR = Conservative & Rigorous nature; SI = learning through Student Initiative; TI = learning by following Teacher Instruction; UT = Utilitarianism in Teaching; NF = Natural & Fixed mathematical ability. "+" = the group average is higher than international average; "−" = the group average is lower than international average

4 Conclusion

The future lower-secondary teachers' beliefs about the nature of mathematics, learning mathematics, and mathematics achievement were analyzed in this paper. The analysis was based on six scales gained by factor analyses and by running a procedure employing a Rasch partial credit model (Masters 1982) for all the lower-secondary samples of TEDS-M. In comparison with the original TEDS-M factors, a factor "Utilitarianism in Teaching" (UT) was extracted from the "teacher direction" factor in TEDS-M. The remaining items were labeled as teacher instruction (TI).

Regarding cultural notions, this paper adapted the classification of Blömeke and Kaiser (2012) to divide the participating TEDS-M countries into five cultural groups. The results reveal that there are common patterns as well as distinct patterns across cultural groups. A common one is that all groups believe that the nature of mathematics is open and creative (OC) on the one hand and conservative and rigorous (CR) on the other hand; this kind of mathematics is best learned through considering student initiatives (SI) such as figuring out own solutions regardless of the time consumed. The existence of common beliefs suggests that a part of the beliefs is shaped by an underlying tendency; a possible one is the wave of globalization, but this is still a hypothesis to test.

Comparing the patterns of all cultural groups across all scales about mathematics-teaching related beliefs, this study found that all groups but one (the group of Developing Asia) shared a rough pattern. This pattern has the strongest beliefs on an open and creative nature of mathematics and student initiatives of learning mathematics, the second strongest beliefs on the conservative and rigorous nature of mathematics, and the third on the importance of teacher instruction and explanation to students (NF). In this pattern, the least approved belief is the utilitarianism in teaching such as learning is best through memorization or other non-time-consuming ways.

When considering neutrality, all cultural groups gain consistent positions in terms of positive or negative views on the OC, CR, and SI scales but not on the TI and NF scales. When the latter two scales are used as dividers, the cultural groups can be divided into two classes: one with negative views on both TI and NF, including Developed Europe, Confucian Asia, and the American group, the other with positive views on both TI and NF, including Developing Asia and East Europe.

Though this study analyzed cultural patterns according to the aforementioned cultural groups, breaks within groups may also be seen. For the beliefs on learning mathematics, the USA, Poland, and Taiwan join the Developed Europe pattern. This situation prompts a need for further studies of cultural group classifications such as whether the USA should join the Developed Europe group due to historical roots or whether Poland should join this group for its geographical location. Besides, why Taiwan abandons Singapore to join the Developed Europe pattern remains an open question that requires further study.

The most valuable cultural meaning of the belief patterns may be the presentation of diversity in mathematics teaching and the beauty of regular patterns. In fact, all the so-called "high-achieving" countries, the means of which were higher than the international average of 500 test points in the MCK or MPCK surveys of TEDS-M, come from almost every culture group in this paper. It is therefore difficult to say which belief pattern or culture of mathematics teaching should be best for teacher education. Examining and realizing where we are is always the first policy in terms of belief or culture.

References

Barkatsas, A. T., & Malone, J. (2005). A typology of mathematics teachers' beliefs about teaching and learning mathematics and instructional practices. *Mathematics Education Research Journal*, *17*(2), 69–90.

Beswick, K. (2005). The beliefs/practice connection in broadly defined contexts. *Mathematics Education Research Journal*, *17*(2), 39–68.

Blömeke, S., & Kaiser, G. (2012). Homogeneity or heterogeneity? Profiles of opportunities to learn in primary teacher education and their relationship to cultural context and outcomes. *ZDM—The International Journal on Mathematics Education*, *44*(3), 249–264.

Blömeke, S., Suhl, U., & Kaiser, G. (2011). Teacher education effectiveness: Quality and equity of future primary teachers' mathematics and mathematics pedagogical content knowledge. *Journal of Teacher Education*, *62*(2), 154–171.

Cross, D. I. (2009). Alignment, cohesion, and change: examining mathematics teachers' belief structures and their influence on instructional practices. *Journal of Mathematics Teacher Education*, *12*(5), 325–346.

Ernest, P. (1988). The impact of beliefs on the teaching of mathematics. http://www.people.ex.ac.uk/PErnest/impact.htm.

Ernest, P. (1991). *The philosophy of mathematics education*. London: Falmer Press.

Felbrich, A., Kaiser, G., & Schmotz, C. (2012). The cultural dimension of beliefs: an investigation of future primary teachers' epistemological beliefs concerning the nature of mathematics in 15 countries. *ZDM—The International Journal on Mathematics Education*, *44*(3), 355–366.

Hofstede, G. (1984). National cultures and corporate cultures. In L. A. Samovar & R. E. Porter (Eds.), *Communication between Cultures* (p. 51). Belmont: Wadsworth

Hofstede, G. (2001). *Culture's consequences: comparing values, behaviors, institutions, and organizations across nations* (2nd ed.). London: Sage.

Hoyles, C. (1992). Mathematics teaching and mathematics teachers: a meta-case study. *For the Learning of Mathematics*, *12*(3), 32–44.

Leung, K. S. F. (2006). Mathematics education in east Asia and the West: does culture matter? In F. K. S. Leung, K.-D. Graf, & F. J. Lopez-Real (Eds.), *Mathematics Education in Different Culture Traditions—A Comparative Study of East Asia and the West, the 13th ICMI Study* (pp. 21–46). New York: Springer.

Markus, H. R., & Hamedani, M. G. (2007). Sociocultural psychology: the dynamic interdependence among self systems and social systems. In S. Kitayama & D. Cohen (Eds.), *Handbook of Cultural Psychology* (pp. 3–39). New York: Guilford.

Masters, G. (1982). A Rasch model for partial credit scoring. *Psychometrika*, *47*(2), 149–174.

Nespor, J. (1987). The role of beliefs in the practice of teaching. *Journal of Curriculum Studies*, *19*(4), 317–328.

Op't Eynde, P., De Corte, E., & Verschaffel, L. (2002). Framing students' mathematics-related beliefs. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: a hidden variable in mathematics education?* (pp. 13–37). Dordrecht: Kluwer Academic.

Pajares, M. F. (1992). Teachers' beliefs and educational research: cleaning up a messy construct. *Review of Educational Research*, *62*(3), 307–332.

Perry, B., Howard, P., & Tracey, D. (1999). Head mathematics teachers' beliefs about the learning and teaching of mathematics. *Mathematics Education Research Journal*, *11*(1), 39–53.

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 257–315). Charlotte: Information Age.

Raymond, A. M. (1997). Inconsistency between a beginning elementary school teacher's mathematics beliefs and teaching practice. *Journal for Research in Mathematics Education*, *28*(5), 550–576.

Richardson, V. (1996). The role of attitude and beliefs in learning to teach. In J. Sikula, T. Buttery, & E. Guyton (Eds.), *Handbook of research on teacher education*, New York: Macmillan Co. (2nd ed., pp. 102–119).

Rokeach, M. (1960). *The open and closed mind: investigations into the nature of belief systems and personality systems*. New York: Basic Books.

Schmidt, W. H., Blömeke, S., Tatto, M. T., Hsieh, F.-J., Cogan, L. S., Houang, R. T., & Schwille, J. et al. (2011a) *Teacher education matters: a study of middle school mathematics teacher preparation in six countries*. New York: Teacher College Press.

Schmidt, W. H., Cogan, L., & Houang, R. (2011b). The role of opportunity to learn in teacher preparation: an international context. *Journal of Teacher Education*, *62*(2), 138–153.

Schoenfeld, A. H. (1998). Toward a theory of teaching-in-context. *Issues in Education*, *4*(1), 1–94.

Skott, J. (2001). The emerging practices of a novice teacher: the roles of his School mathematics images. *Journal of Mathematics Teacher Education*, *4*, 3–28.

Stenhouse, L. (1967). *Culture and education*. New York: Weybright & Talley.

Stipek, D. J., Givvin, K. B., Salmon, J. M., & MacGyvers, V. L. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education*, *17*, 213–226.

Tang, S.-J., & Hsieh, F.-J. (2012a). Beliefs of future secondary mathematics teachers (in Chinese). In F.-J. Hsieh (Ed.), *Taiwan TEDS-M 2008: Teacher Education and Development Study in Mathematics*. Taipei: National Taiwan Normal University (pp. 221–252).

Tang, S.-J., & Hsieh, F.-J. (2012b). Taiwanese future secondary mathematics teachers' beliefs under the perspective of international comparison. In F.-J. Hsieh (Ed.), *Taiwan TEDS-M 2008 Secondary Analysis* (in Chinese). Taipei: National Taiwan Normal University.

Tatto, M. T., Schwille, J., Senk, S., Rodriguez, M., Bankov, K., Reckase, M. D., & Meinck, S. (2009). *Teacher education study in mathematics (TEDS-M): technical summary*. East Lansing: Michigan State University.

Tatto, M. T., Peck, R., Schwille, J., Bankov, K., Senk, S. L., Rodeiguez, M., & Holdgreve-Resendez, R. (2012). *Policy, practice, and readiness to teach primary and secondary mathematics in 17 countries: findings from the IEA teacher education and development study in Mathematics (TEDS-M)*. http://www.iea.nl/fileadmin/user_upload/Publications/Electronic_versions/TEDS-M_International_Report.pdf.

Thompson, A. G. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics*, *15*, 105–127.

Thompson, A. G. (1992). Teachers' beliefs and conceptions: a synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics learning and teaching* (pp. 127–146). New York: Macmillan Co.

Törner, G. (1997). Views of German mathematics teachers on mathematics. In J. A. Dossey, J. O. Swafford et al. (Eds.), *Science, Mathematics and Environmental Education:* Vol. *I. Proceedings of the 19th annual meeting of the north American chapter of the international group for the Psychology of Mathematics Education*. Columbus: ERIC Clearinghouse for for Science (pp. 275–281). http://logistik.math.uni-duisburg.de/pdf/PME-NA-19.pdf.

Tylor, E. B. (1889). *Primitive culture: researches into the development of mythology, philosophy, religion, language, art, and custom*. New York: Holt.

Wilson, M. S., & Cooney, T. J. (2002). Mathematics teacher change and development: the role of beliefs. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: a hidden variable in mathematics education?* (pp. 127–147). London: Kluwer Academic.