Advances in Mathematics Education

Sigrid Blömeke
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William H. Schmidt *Editors*

International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn

TEDS-M Results

Springer

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ISSN 1869-4918 ISSN 1869-4926 (electronic)
Advances in Mathematics Education
ISBN 978-94-007-6436-1 ISBN 978-94-007-6437-8 (eBook)
DOI 10.1007/978-94-007-6437-8
Springer Dordrecht Heidelberg New York London

Library of Congress Control Number: 2013949543

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Series Preface

The sixth volume of the series Advances in Mathematics Education differs from other books in this series in several respects. Based on an issue of ZDM published recently the book offers results from the international comparative study TEDS-M 2008 (Teacher Education and Development Study—Learning to Teach Mathematics). TEDS-M is a comparative study of teacher education examining the preparation of teachers of mathematics at the primary and lower secondary levels at the end of their study. The study was carried out under the auspices of the International Association for the Evaluation of Educational Achievement (IEA).

TEDS-M focuses on the connections between teacher education policies, practices and outcomes. The main goal of TEDS-M is to show whether and how much teacher preparation policies, programs, and practices across the world contribute to the capability to teach mathematics well in primary and lower secondary schools. TEDS-M analyses teacher education under three following perspectives: at the level of the country context, that comprises studies of teacher policies, programs and practices on the national level; at the institutional level analyses of curricula and practices of teacher preparation, including standards and expectations for teacher learning and at the individual level the impact of teacher preparation on the knowledge, skills and dispositions acquired by future teachers.

These three-folded goals in connection with nationally representative samples of primary and lower secondary mathematics teachers in their final year of teacher training from 16 countries as well as representative samples of teacher educators and training institutions made this study to a real challenge. The papers in the sixth volume of Advances in Mathematics Education describe the theoretical framework of the study, design and test instruments and results at different levels and from different perspectives. The book samples papers, which had already been printed at other places and combines them with newly written chapters based on new data analyses.

The book provides an insightful overview on the efficiency and effects of teacher education internationally, which the reader will hopefully find interesting.

Hamburg, Germany — Gabriele Kaiser
Missoula, USA — Bharath Sriraman

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Part I
Introduction

Framing the Enterprise: Benefits and Challenges of International Studies on Teacher Knowledge and Teacher Beliefs—Modeling Missing Links

Sigrid Blömeke

Abstract This book presents a collection of the most important papers that examined—based on data from the "Teacher Education and Development Study: Learning to Teach Mathematics (TEDS-M)"—the outcomes of mathematics teacher education in terms of knowledge and beliefs, the relationship between opportunities to learn (OTL) during teacher education and outcomes, as well as the relationship between the future teachers' background and teacher education outcomes. As an introduction, in this chapter the challenges of taking on an enterprise like TEDS-M are discussed. Firstly, the value-added of international studies and their methodological limits are reflected. Second, different approaches to examine teacher education outcomes over time and across countries are presented. In a third step, missing links on the continuum of teacher learning from teacher education through induction up to continuous professional development are modeled. Thus, the state of research on teacher knowledge and teacher beliefs is summarized in a new way. Finally, the practical relevance of studies such as TEDS-M is demonstrated by using their instruments as tools for learning during teacher education. The objective of these four parts is to frame the book by placing its results in the broader context.

Keywords Teacher education · Teacher induction · Continuous professional development (CPD) · Teacher competence · Teacher knowledge · Teacher beliefs · Teaching performance · Generalizability · Validity · Large-scale assessment · International comparison

In a first review of the state of teacher-education research for the "ZDM—The International Journal on Mathematics Education" in 2008, we summarized the state as follows: "Teacher-education research lacks a common theoretical basis, which prevents a convincing development of instruments and makes it difficult to connect studies to each other" (Blömeke et al. 2008a). Since then, research on future and practicing teachers has developed. The "Teacher Education and Development

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S. Blömeke et al. (eds.), *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn*, Advances in Mathematics Education,
DOI 10.1007/978-94-007-6437-8_1,

Study: Learning to Teach Mathematics (TEDS-M)" was particularly important in this context.

TEDS-M was the first study in which primary and lower secondary mathematics teachers' competence in their last year of teacher education was examined with direct measures, and this with representative samples and across countries (Blömeke et al. 2011, 2012; Tatto et al. 2008, 2012). TEDS-M was carried out under the supervision of the "International Association for the Evaluation of Educational Achievement (IEA)".[1] The ranking of the countries and teacher education programs provided benchmarks to evaluate the quality and effectiveness of teacher education in 16 countries.

The pioneering work of TEDS-M has paved the way for a special ZDM issue in 2012 that in turn provided the basis for this book. It presents a collection of the most important papers that examined—based on TEDS-M data—the outcomes of mathematics teacher education in terms of knowledge and beliefs, the relationship between opportunities to learn (OTL) during teacher education and outcomes as well as the relationship between the future teachers' background and teacher education outcomes. Besides the ZDM papers, core articles from other journals were included if they covered crucial research questions and if the copyright regulations allowed us to do so. All papers were adjusted for the purpose of this book to develop a coherent reading.

As an introduction, we discuss the challenges of taking on an enterprise like TEDS-M. We reflect firstly on the value added of international studies and on their methodological limits (Sect. 1). Second, we present different approaches to examine teacher education outcomes over time and across countries (Sect. 2). In a third step, we model some missing links by placing TEDS-M as a study on teacher education into the continuum of teacher learning from teacher education through induction up to continuous professional development (Sect. 3). Thus, the state of research on teacher knowledge and teacher beliefs is summarized in a new way. Finally, we demonstrate the practical relevance of studies like TEDS-M by using their instruments as tools for learning during teacher education (Sect. 4). The objective of these four parts is to frame the book by placing its results against the broader context.

1 Benefits and Challenges of International Studies on Teacher Education

1.1 Value-Added Through International Comparisons[2]

During the past two decades, the interest in international comparative studies on teachers, in particular on mathematics teachers has increased (Cochran-Smith and

[1]TEDS-M was funded by the IEA, the National Science Foundation (REC 0514431), and the participating countries. In Germany, the German Research Foundation funded TEDS-M (DFG, BL 548/3-1). The views expressed in this book are those of the authors and do not necessarily reflect the views of the IEA, the participating countries or the funding agencies.

[2]Based on Blömeke and Paine (2008).

Zeichner 2005; Darling-Hammond 2000). Mathematics teachers play a central role in the preparation of future generations' K-12 students. An examination of mathematics teacher education is therefore an important step to ascertain school quality.

The open question is why such studies should be carried out in a comparative way. What is to be learned from an international perspective? Whereas large-scale assessments like TIMSS or PISA regularly examine K-12 achievement as part of monitoring the education system, comparative studies are a rather new approach in teacher research. The idea that there might be global processes that are influencing policies and practices pertaining to teacher education has not been a substantive focus for prior inquiries. However, transnational actors in recent years have grown to become major players in the conversations about teaching and teacher development. Global or transnational agencies such as the World Bank or OECD frame teachers in particular ways through indicator studies, policy briefs, and surveys (Lauder et al. 2012; Robertson 2012). Whether in Germany, China or the USA, the link between school quality in terms of student achievement and teacher quality as teacher education outcome has become a driving force for reform (Takayama 2012).

It becomes therefore imperative to examine teacher education beyond national borders and review research internationally in order to discuss such issues in an evidence-based way. Large-scale cross-national studies can provide information about teacher learning, teacher competence and teaching practices from different countries. They have the power to indicate both global and regional patterns of similarity and difference in these characteristics (Blömeke 2012). The results of comparative studies provide, thus, benchmarks of what level and quality of teacher knowledge can be achieved during teacher education and which country-specific strengths and weaknesses exist.

In many countries, the results of such studies on K-12 student achievement have led to fundamental reforms of the school system. The publication of the PISA 2000 results in Germany, for example (Baumert et al. 2001), one of the first international studies the country took again part in after a long time, and the realization that Germany performed at only a mediocre level—in contrast to the country's self-image—came as a shock. Heated debates among policymakers, researchers, and lay people finally resulted in changes. Similarly, the USA implemented significant reforms in its mathematics school curricula after the so-called "Sputnik shock" and the country's weak performance was confirmed in comparative studies such as SIMS (Pelgrum et al. 1986) and TIMSS (Mullis et al. 1997). Thus, comparative studies of student achievement provided the chance to understand educational phenomena in a new way. Research on teacher education across countries may produce similar effects.

But international perspectives are useful not only for benchmarking. International comparisons also allow us to ask questions in new ways. For example, international research allows us to analyze cultural dimensions of teaching practices and teacher knowledge. By developing international studies, many matters are questioned which may remain unquestioned in national studies. The structure and the content of teacher education depend on a deeper rationale which is a result of factors which may be at least partly cultural. Like the water in the fish's tank, such cultural givens are too often invisible—and international comparisons provide the chance to move beyond the familiar, and to see with a kind of "peripheral vision" (Bateson 1994).

Like everyone else, researchers are embedded in their own culture, and so they often overlook matters of culture. This is particularly the case for teaching and teacher education, given the unique way in which it incorporates or touches upon many different levels of education and stands at the intersection of education and other social, economic, and political forces. The embedded character of the system of teaching and teacher development in a country makes looking beyond that country's experience mandatory in order to recognize the assumptions which drive it and which are all too often taken for granted. The investigation of another teaching system in a foreign country, for example, and the discovery that it is possible to organize the training differently, sheds new light on domestic systems (LeTendre 1999).

It is a methodological challenge to assess teacher competence from a comparative perspective though. Research perspectives have to be adjusted across borders and deeply-rooted educational traditions. Furthermore, it is a challenge to assess the development of knowledge among prospective teachers in the context of a differentiated tertiary education system. Not only do a variety of institutions, teacher training programs and job requirements exist, but also the outcome is hard to define and even harder to measure.

As such, language problems become important in comparative studies as well and are far more demanding to resolve than a "simple" translation of instruments or responses (National Research Council 2003). At one level, language problems are a common, familiar and well-studied aspect of cross-cultural studies, for which there are widely-used conventions of translation and back translation (Hambleton 2002). In teacher education, however, more language-related challenges exist that require attention. They are a problem of cultural boundaries.

In some countries, the language of schooling may vary from the language of the home for many students. Many terms from native languages cannot be translated because adequate English terms are missing and vice versa. It is even difficult to name the process by which future teachers learn their profession: is it teacher education, is it teacher training or is it perhaps teacher preparation? These questions relate to deeper and often tacit assumptions about schooling, teaching, and learning to teach. They are worth examining in detail (for further discussions on the relationship of culture and teacher education see the chapter "Learning from the Eastern and the Western debate: the case of mathematics teacher education" by Kaiser and Blömeke in this book).

1.2 Methodological Challenges: Validity and Generalizability

The theoretical models underpinning teacher education assessments like TEDS-M decompose teacher competence, as the outcome of teacher education, into several facets like content knowledge, pedagogical content knowledge and general pedagogical knowledge (Shulman 1985). The future teachers' achievement in these facets is measured with different tests that allow for rankings on the country level as part of monitoring the teacher education systems.

As sophisticated as these approaches are nowadays and as valuable as decomposing competence into manageable components to facilitate judgments is, the act of decomposition can obscure how a teacher would juggle the various bits together to form a coherent whole. Shavelson (2012) unpacks competence as a complex ability construct closely related to real-life performance. He exemplifies how to make it amenable to measurement in a holistic way by research from business, military, and education in contrast to analytic approaches. It may be worthwhile to follow this line with research projects that compare the results of analytic and holistic teacher assessments.

Assessments intended to capture real-life performance are an issue that has long been discussed (Kane 1992). It seems to be difficult to generalize results from one real-life situation to another, that is, problems with the reliability of empirical results exist (Brennan and Johnson 1995). How representative are, for example, the situations to be worked on in an assessment for the situations to be coped with in real life? TEDS-M is a good example for the difficulties of such questions. Although its conceptual framework looks convincing, a comparison with how California evaluates its pre-service mathematics teachers' knowledge (Wu 2010) reveals that different approaches can be taken. California's Teacher Performance Assessment (TPA) depicts classroom situations according to the state's "Teaching Performance Expectations". Four tasks have to be dealt with: connecting instructional planning to student characteristics, assessment, lesson design, and reflection. These have to be applied to (only) two groups of learners which are not present in the TEDS-M framework at all: English language learners and special education students. This difference reveals different visions of what mathematics teachers are supposed to know and be able to do.

2 Different Approaches to Examine Teacher Education Outcomes

2.1 Historical Development of Studies on Teacher Education Outcomes[3]

Different visions of what teachers are supposed to know and be able to do have driven the different approaches to examine teacher education outcomes over the past decades too. In several international and comparative studies, the intention was to examine teacher competence as the outcome of teacher education. Sometimes this construct was labeled "competence", other times "teacher quality" and sometimes "professional knowledge". The nature of this construct has each time changed.

[3] Based on Blömeke and Delaney (2012).

A first important model that characterized the process of pre-service teacher education and its outcomes can be labeled as "teacher learning". This model included approaches such as learning by observation in a kind of apprenticeship (Zeichner 1980), learning by planning, application, and reflection (Schön 1983) and teacher learning as a craft (Brown and McIntyre 1983). The concept's starting point for modeling teachers' competence was teachers' existing classroom practices.

Similar to this concept was a second one, prominent in the 1990s, in which the cognitive basis of teachers' pedagogical practices started to emerge. The first small-scale comparative studies based on this concept were carried out in the field of mathematics teaching (Pepin 1999; Kaiser 2002). Several studies on—mainly mathematics—teachers and teacher education followed (e.g., An et al. 2004; Ma 1999; Burghes 2008). Important steps were also the ICMI study on teacher education (Even and Ball 2009) and the Topic Study Group on mathematics knowledge for teaching at ICME-11 in Mexico (Adler and Ball 2009). About 50 colleagues from a broad range of countries presented their approaches to measuring (future) mathematics teacher competence (e.g., Kristjánsdóttir 2008; Naik 2008). Much of the teacher research, however, neglected the content domain, focused on beliefs (Bramald et al. 1995; Calderhead 1996) or intended to capture competence by self-reports. Studies including direct measures and cross-country studies are still needed (Brouwer 2010).

More recently, teacher-education research and research on practicing teachers has started to focus on the content-related base of teachers' classroom practice. Besides the studies already mentioned, this paradigm included studies by Rowland et al. (2005), Chick et al. (2006) and the chapters in Rowland and Ruthven (2010). Similar but more analytical is the most recent approach that underpinned also TEDS-M and LMT. This approach was elaborated with respect to the field of mathematics by, for example, Niss (2002) and Schoenfeld and Kilpatrick (2008). In-the-moment decision making in well-practiced, knowledge-intensive domains like teaching can according to them be regarded "a function of their orientations, resources, and goals" (Schoenfeld 2010, p. 187). Mathematics content knowledge (MCK) and mathematical pedagogical content knowledge (MPCK) are the most important resources of mathematics teachers in this context.

Whereas only some differences exist across countries how precisely to define MCK, much more differences exist with respect to MPCK and, in particular, with respect to further facets of teacher competence that are not cognitive. Affective-motivational facets such as orientations and goals or meta-cognitive facets like self-regulation are in some studies supposed to be decisive in the teaching process because they provide orientation how to perceive and analyze a classroom situation whereas they do not at all get recognized in others. These differences in research methodology reflect differences in the views on teaching outcomes, whether they are long term or short term, whether they are focused on factual student knowledge or include complex cognitive skills like problem solving or affective characteristics like student motivation.

2.2 *The Role of Teacher Beliefs*[4]

Research suggests that beliefs are a crucial part of mathematics teachers' competence (Calderhead 1996; Richardson 1996). As beliefs are thought to guide perception and actions, they can be regarded crucial for the application of knowledge in classroom situations (Leder et al. 2002; Thompson 1992) and they can be conceptualized as a bridge between knowledge and teaching (Stipek et al. 2001; Voss et al. 2011). Furthermore, some studies reveal that teachers' beliefs are relevant for the outcomes of teaching in terms of student achievement in mathematics (Dubberke et al. 2008; Staub and Stern 2002). Teacher beliefs and meta-cognitive dispositions have probably to be included in order to develop a full model of teacher competence and to increase the validity of empirical studies.

Despite the extensive debate on beliefs, a precise definition of the belief construct, as well as clear-cut differentiations from other concepts such as convictions, attitudes or perceptions, have not yet been established (Hofer and Pintrich 2002; Pajares 1992). Richardson (1996, p. 103) developed a widely-followed although broad definition, in which beliefs are seen as "psychologically held understandings, premises, or propositions about the world, that are felt to be true". Comparative large-scale assessments in the context of (future) teachers, like MT21 or TEDS-M, are based on this definition. They understand beliefs in addition as socially and culturally shaped mental constructs, which are acquired in educational settings with different historical traditions that vary significantly between countries. Thus, cultural patterns are expected that are related to overall models of relationships in a society. Hofstede (1986), for example, distinguishes between collectivistic and individualistic societies. In individualistic societies, learners are perceived more strongly as autonomous subjects acquiring knowledge mainly independently on their own than in collectivistic countries where familial relationships are an important driving force for learning (Triandis 1995).

Empirical studies of beliefs have primarily focused on students (Grigutsch 1996; Leder et al. 2002) and on practicing teachers of primary and secondary schools (Dubberke et al. 2008; Peterson et al. 1989). The study "Mathematics Teaching in the 21st Century" (Blömeke et al. 2008b; Schmidt et al. 2011) was the first study to compare future lower-secondary teachers' beliefs in several countries, namely Bulgaria, Germany, the USA, Mexico, Taiwan and South Korea. The MT21 results revealed country-specific patterns in the teachers' beliefs. The "Teaching and Learning International Survey (TALIS)" (OECD 2009) examined practicing teachers' epistemological beliefs on teaching and learning pointed in the same direction. In individualistically oriented societies, for example Australia and Northwest European countries, constructivist beliefs on teaching and learning were more prevalent. In contrast, in collectivistically oriented societies, such as Malaysia and South American states, transmission views have more strongly been articulated by teachers. Beliefs of Eastern European teachers and South Korean teachers were situated between both groups of countries (Klieme and Vieluf 2009; Vieluf and Klieme 2011).

[4]Based on Felbrich et al. 2008.

TEDS-M continued examining the beliefs of future teachers across countries (for more information see the chapter "The Cultural Notion of Teacher Education: Future Primary Teachers' Beliefs on the Nature of Mathematics" by Felbrich, Kaiser and Schmotz in this book).

3 Modeling Missing Links

3.1 The Continuum of Teacher Learning After Pre-service Teacher Education: Teacher Induction[5]

Initial teacher education sets only a first tone for the development of teacher competence. The next years as beginning teachers are regarded decisive for further professional development (Feiman-Nemser and Parker 1990; Veenman 1984). Beginning teachers have to cope with an almost overwhelming task: applying the knowledge gained during teacher education to different and complex classroom situations with multidimensional challenges occurring at high speed (Sabers et al. 1991). Developing teaching quality during these first years is therefore an important task for all education systems. Evidence suggests that the quality of the school environment is important at this stage of a teaching career (Darling-Hammond and McLaughlin 1995). However, it is widely unknown which characteristics of the school environment are relevant and how they are related to different indicators of teaching quality.

Induction is a necessary phase in learning to teach. It marks the period following pre-service teacher education. Beginning teachers take for the first time full responsibility for regular classes of elementary or secondary students. This experience of being a novice and learning how to teach within an established community of practice may be different from school system to school system and be labeled and understood differently, but it is all induction.

Of the 25 countries reviewed for "Teachers Matter" (OECD 2005), only ten had mandatory induction programs (Australia, England and Wales, France, Greece, Israel, Italy, Japan, Korea, Switzerland, and Northern Ireland). In six countries, schools can elect to offer induction programs. In the other countries, there were no induction programs at all. A similar variation was found in terms of where these were housed. Induction programs were most commonly offered by schools, but in four countries (Israel, Japan, Switzerland and Northern Ireland) induction programs were provided jointly by teacher education institutions and schools. The length of the programs varied from seven months in South Korea to two years in Switzerland. An ETS review of induction in 8 countries revealed further variation depending on the amount of practical experiences during pre-service teacher education (Wang et al. 2003, p. 28). Some teacher education programs, such as those in Switzerland, provided extensive field experience; others provided little time in the field (South Korea and France). Induction complements these approaches and thus varies.

Several purposes for induction exist: connecting theory to practical experiences where this had not happened during pre-service education, improving teaching, re-

[5] Based on Blömeke and Paine (2009), Paine and Schwille (2010).

ducing attrition, supporting novice teachers' well-being and responding to mandates and practical needs of educational systems (Huling-Austin 1990). Induction appears to occupy a special place, uniquely influenced by looking backward to pre-service teacher education and forward to the career of teaching (Feiman-Nemser 2001). Paine et al. (2003) argue teacher induction is about building "something desirable: effective teachers, a strong teaching force, a vital profession, and optimum learning for students in schools" (p. 80).

One important finding is the dual commitment to focusing on both improving teaching quality and personal development. Britton et al. (2003) identified seven categories of content that induction programs offered—with variation in the specifics of time and focus: effective subject-matter teaching; understanding and meeting pupils' needs; assessing pupil work and learning; reflective and inquiry-oriented practice; dealing with parents; understanding school organization and participating in the school community; and understanding oneself and current status in one's career.

It is very common that a beginning teacher has the opportunity to work one-on-one with an experienced teacher through their induction program. The OECD (2005) report found that in 15 countries where programs of induction occur, 13 have mentors as either the key person with whom the novice works or as one of the main people responsible for supporting them. The programs share the assumption that one does not learn to teach in isolation. Programs work to help the novice tap into collective experience of the profession through close and sustained contact with a more experienced teacher.

3.2 Continuous Professional Development

Teacher learning continues during the teachers' professional life. The extent of systematic support varies greatly though. While some form of a teacher-learning continuum exists in all educational systems, national teaching forces rely on very different assumptions and structural arrangements to support that learning (Ingersoll et al. 2007; Barber and Mourshed 2007). What is expected of pre-service teachers' competence at the end of their training in some countries, would be seen as part of practicing teachers' learning in others (Paine et al. 2003).

In many countries, national standards for student achievement have been launched by the Ministers of Education during the past 10 years. New demands for teachers emerged (Blum et al. 2006). Continuous professional development that enables the teachers to cope with such a context of change has thus become an important issue. To establish an evidence-based organization of continuous professional development (CPD) is challenging though. We do not have much empirical research. The professional knowledge of pre-service teachers has been researched in depth and from different perspectives. Corresponding research is missing in the field of CPD (Lipowsky 2004; Sowder 2007).

Content-focused coaching (West and Staub 2003) is a model of professional development that assists teacher learning on the job. Expert teachers work as coaches individually or with groups of classroom teachers to design, implement, and reflect on lessons that promote student learning. This is an approach particularly prominent in East Asia. Two of the models' central elements are an emphasis on collaborative lesson planning in pre-lesson conferences and a suggested framework of core issues for the planning and reflection of lessons that aim to focus on pivotal aspects of lesson design in relation to content-specific processes of learning. Quasi-experimental intervention studies in different settings in Switzerland (e.g., Kreis and Staub 2011; Vogt and Rogalla 2009) and in the US (Matsumura et al. 2012) provide evidence on effects of such an approach.

Several countries have launched new approaches, among others Austria (IMST), England (NCETM), Germany (DZLM) and Sweden (NCM). The approaches share that they combine the fostering of CPD activities for mathematics teachers through new types of national CPD institutions with research on the effectiveness of different types of CPD. In 2011 for example, the "Deutsche Telekom Stiftung" launched the German Center for Mathematics Teacher Education (DZLM) to contribute to mathematics teachers' CPD in Germany. A consortium of eight universities that combine research expertise from the fields of mathematics, mathematics education and the educational sciences has established the DZLM. The main objective is to approach CPD from a systemic point of view.

The DZLM aims at implementing a cascade of CPD. In this respect, the training of mentor teachers is considered a core issue, as they are expected to pass on their in-depth knowledge and expertise to fellow teachers. A second core issues is a qualification programs for out-of-field teachers. Another activity concentrates on empowering teacher inquiry and research through supporting local teacher working groups and networks. Such "lesson-study" types of CPD were firstly introduced after the TIMSS 1995 video study as a tool to examine collaboratively as a group of teachers one's own mathematics lessons in order to improve teaching performance and to teach more effectively (Lewis 2002).

In Austria, the IMST initiative has addressed enhancing school quality on a systemic level. IMST is characterized by a consequent bottom-up approach (Krainer 2007). Empirical results reveal that in particular approaches of research-based learning CPD are effective (Krainer et al. 2009). The CPD programs examined followed an approach focused on "action research" (Altrichter and Posch 2007). A four-semester university program "Pedagogy and Subject Didactics for Teachers" (PFL) and the project "Innovations in Mathematics, Science and Technology Teaching" (IMST) increased teachers' self-reported competence and their analytical abilities as measured through video components. Teachers' motivation to teach and students' learning motivation do not change over time.

4 Research on Teacher Competence as a Tool to Improve Teacher Education

Wong, Boey, Lim-Teo, and Dindyal (in this book) make an important point with respect to the practical relevance of studies such as TEDS-M: the released MCK and MPCK items can be used as a training resource. In fact, the Singaporean TEDS-M team prepared a book consisting of these released items, the scoring guides, the Singapore results against international benchmarks and samples of constructed responses. Teacher educators can now use these materials with future teachers by, for example, exploring strategies to remedy misconceptions, designing classroom activities that mirror the scenarios described in the TEDS-M items and linking the assessment items to the TEDS-M framework and thus analyzing conceptions of teacher knowledge. Although the TEDS-M items were originally created as a summative assessment of teacher knowledge at the end of their training, they can so be used as a formative assessment of teacher knowledge.

Another practical use of TEDS-M may be within-country comparisons and further evaluations of local teacher education programs. Whereas the international comparison provides an overall picture that reveals what can be achieved in general, local comparisons may point to features of teacher education easier to transfer from one institution to another. TEDS-M revealed that within most countries, huge between-program disparity existed. This means that within the same cultural context some institutions are more effective than others. They may represent a benchmark. A closer examination of these programs' structure, their mathematics and mathematics pedagogy curriculum, the teaching methods, their selection criteria may put the corresponding features at lower-performing institutions to the test.

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Theoretical Framework, Study Design and Main Results of TEDS-M

S. Blömeke and G. Kaiser

Abstract The comparative "Teacher Education and Development Study: Learning to Teach Mathematics (TEDS-M)", carried out under the supervision of the International Association for the Evaluation of Educational Achievement (IEA), provided the opportunity to examine the outcomes of teacher education in terms of teacher knowledge and teacher beliefs both across countries and specifically with respect to mathematics for the first time. This chapter describes the conceptual framework that guided TEDS-M and its study design. The instruments used to measure teacher knowledge and beliefs as well as opportunities to learn (OTL) are described. In addition, core descriptive results, previously only published in German (see Blömeke et al. "Cross-national comparison of the professional competency of and learning opportunities for future primary school teachers", 2010a; "Cross-national comparison of the professional competency of and learning opportunities for future secondary school teachers of mathematics", 2010b (in German)), are described. These results serve as the basis for the other chapters in this monograph. It turns out that teacher education institutions structure their provision of OTL in a way that is consistent with their particular philosophy of what teachers need to know and be able to do. The need to strengthen teachers' content knowledge is one of the dominant ideas that has guided reform efforts in many countries over the past 20 years. The results of TEDS-M which are reported in this chapter are therefore crucial for policymakers. In addition, international comparisons provide benchmarks for national teacher education systems. Countries that do better in TEDS-M may have more effective teacher training programs than countries at the bottom end of the ranking.

Keywords Mathematics content knowledge (MCK) · Pedagogical content knowledge (PCK) · Comparative study · Teacher competence · Teacher beliefs · Opportunities to learn · Teacher education outcomes

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S. Blömeke et al. (eds.), *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn*, Advances in Mathematics Education,
DOI 10.1007/978-94-007-6437-8_2,

The comparative “Teacher Education and Development Study: Learning to Teach Mathematics (TEDS-M)”, carried out under the supervision of the International Association for the Evaluation of Educational Achievement (IEA), provided the opportunity to examine the outcomes of teacher education in terms of teacher knowledge and teacher beliefs both across countries and specifically with respect to mathematics for the first time (Blömeke et al. 2011, 2012; Tatto et al. 2008, 2012).[1] TEDS-M was the first large-scale assessment of higher education that included direct testing of outcomes; graduates from 16 countries were surveyed. With this ambitious design, TEDS-M broadens existing research in many respects, which will be elaborated in this chapter.

Teacher education institutions structure their provision of opportunities to learn (OTL) in a way that is consistent with their particular philosophy of what teachers need to know and be able to do. The need to increase teachers’ content knowledge is one of the dominant ideas that has guided reform efforts in many countries over the past 20 years (Shulman 1987). Evaluating whether these reforms have been successful is an important step towards assuring the professional quality of those working in teaching. The results of TEDS-M which we will report in this paper are there crucial for policy makers.

In addition, international comparisons provide benchmarks for national teacher education systems. Countries that do better in TEDS-M may have more effective teacher training programs than countries at the bottom end of the ranking. Studying teacher education in an international context is a challenge though. Differences in the structure and content of teacher education include the risk that the data gathered in different countries may not be comparable. At the same time, such differences are precisely that what makes comparative research so valuable. The variety of implementations makes hidden national assumptions visible (for more details on the value added of international comparisons see chapter “Framing the Enterprise: Benefits and Challenges of International Studies on Teacher Knowledge and Teacher Beliefs—Modeling Missing Links” in this book).

The present chapter describes the conceptual framework that guided TEDS-M and its study design. These descriptions have been part of several of our papers in similar versions; most recently they have been part of Blömeke (2012b) with respect to teacher competence as outcome of teacher education and the instruments used to measure teacher knowledge and beliefs as its facets. With respect to the opportunities to learn during teacher education and the instruments to gather data on them, we point to Blömeke (2012a) as well as to Blömeke and Kaiser (2012). For the purpose of this chapter, we revised and adjusted these parts. In addition, we present core descriptive results, which serve as central basis for the other chapters in this monograph, that were previously only published in German (see Blömeke et al. 2010a, 2010b).

[1]TEDS-M was funded by the IEA, the National Science Foundation (REC 0514431) and the participating countries. In Germany, the German Research Foundation funded TEDS-M (DFG, BL 548/3-1). The instruments are copyrighted by the TEDS-M International Study Center at MSU (ISC). The views expressed in this chapter are those of the authors and do not necessarily reflect the views of the IEA, the ISC, the participating countries or the funding agencies.

1 Theoretical Framework

Teacher Competence as Outcome of Teacher Education The TEDS-M concept of teacher education outcomes is based on the notion of "professional competence". Competence is defined as those latent dispositions that enable professionals to master their job-related tasks (see, e.g., Weinert 2001). These dispositions include cognitive abilities—in TEDS-M, this is the future teachers' professional knowledge—as well as convictions and values, in TEDS-M these are the future teachers' professional beliefs. Teacher competence underlies teaching performance in the classroom.

Teacher knowledge as one facet of competence can further be subdivided into different sub-facets which have been frequently discussed in the literature (Shulman 1985; Blömeke 2002; Baumert and Kunter 2006). In his seminal work, Shulman identified three content-related facets and one generic facet, namely content knowledge, pedagogical content knowledge, curricular knowledge and general pedagogical knowledge. A teacher has to develop all four of these to be able to deal effectively with the various challenges of her job: classroom management, assessment, supporting students' social and moral development, counseling and participating in school activities.

The four facets were reduced to three and defined as follows in TEDS-M (for further details, see Tatto et al. 2008):

(1) *Content knowledge* is future primary and lower-secondary teachers' mathematics content knowledge (MCK). MCK includes fundamental mathematical definitions, concepts, algorithms and procedures.

(2) *Pedagogical content knowledge*—including the Shulman facet "curricular knowledge"—is mathematics pedagogical content knowledge (MPCK). This includes knowledge about how to present fundamental mathematical concepts and methods to students adapted to their prior knowledge. Lesson planning knowledge is essential before mathematics instruction in the classroom can begin. The mathematics content must be selected appropriately, simplified and connected to teaching strategies taking into account possible learning difficulties or learning barriers caused amongst others by misconceptions of central mathematical concepts and methods. Knowledge about the way in which students learn should be taken into account when selecting a teaching strategy as well. Such knowledge requires teachers in turn to review students' answers, verbal or written, in the context of the tasks or questions given to them. Teachers should ask questions of varying complexity, identify misconceptions, provide feedback and react with appropriate scaffolding or intervention strategies. Teachers have to consider curricular issues such as the order of topics in primary or lower-secondary curriculum and need to develop their lesson planning in accordance with curricular requirements (Goos et al. 2007; Vollrath 2001). Pedagogical content knowledge may depend on the teaching and learning philosophy of the pedagogical context a teacher is working in and other cultural influences such as differences between Eastern and Western educational traditions (for more details see the final chapter in this book by Kaiser and Blömeke).

MCK and MPCK both cover mathematics, but from different perspectives. Studies by Schilling et al. (2007) and Krauss et al. (2008) demonstrate that while it is possible to distinguish between MCK and MPCK, the two knowledge facets are closely related (for more theoretical reflections on nature of mathematical subject knowledge in teaching and its relation pedagogical content knowledge see Rowland and Ruthven 2011).

(3) According to Shulman (1987) *general pedagogical knowledge* involves, "broad principles and strategies for classroom management and organization that transcend subject matter" (p. 8), as well as generic knowledge about learners and learning, assessment and educational contexts and purposes. Future mathematics teachers need to draw on this range of knowledge and transform it into coherent understanding and skills if they are to become competent in dealing with what McDonald (1992) calls the "wild triangle" that connects learner, subject matter and teacher in the classroom.

Beliefs are in TEDS-M—following a definition developed by Richardson (1996)—understood as "understandings, premises or propositions about the world that are felt to be true" (Richardson 1996, p. 103). This broad understanding is challenged by other approaches emphasizing the experiential and context-bound nature of beliefs though (Schoenfeld 1998). If beliefs are looked at alongside both the subject being taught and the professional task of teaching which needs to be mastered, evidence suggests that there is a link between teacher beliefs and the actual teaching in the classroom (Staub and Stern 2002; Voss et al. 2011). Several studies point out that beliefs are a crucial aspect of a teacher's perception of teaching situations and her choice of teaching methods (Leinhardt and Greeno 1986; Leder et al. 2002). Thus, they may also serve as an indicator of the type of teaching methods the future teachers will use in the classroom. In addition, empirical evidence exists that beliefs of the teachers influence students' achievement (Dubberke et al. 2008; Peterson et al. 1989).

Despite the rich literature about beliefs, they are not a well-defined construct. Clear distinctions between terms such as attitudes, perceptions or conceptions on the one hand and cognitive features on the other hand are rare and there exists no consensus about the various definitions and borderlines between these concepts (Goldin et al. 2009). With respect to teachers the distinction towards knowledge—in particular towards pedagogical content knowledge and general pedagogical knowledge—is more heuristic than that it can strictly be kept up (Furinghetti and Morselli 2009).

Several efforts have been made to categorize the belief systems of teachers (Thompson 1992; Op 't Eynde et al. 2002), for example epistemological beliefs on the nature of mathematics and the genesis of mathematical knowledge or beliefs on teaching and learning processes. Regarding the beliefs on the nature of mathematics, various definitions exist, which share a common ground (Liljedahl et al. 2007). An early classification by Ernest (1989) differentiates between three fundamental views of mathematics: the instrumentalist, the Platonist, and the problem solving view, which is similar to a conception by Dionne (1984), who distinguishes between a traditional view on mathematics (similar to Ernest's instrumentalist view), a formalist perspective (connected to the Platonist view by Ernest) and a constructivist perspective on mathematics (with similarities to the problem-solving view by

Ernest). Another well-known distinction by Grigutsch et al. (1998) distinguishes between a dynamic and a static view on mathematics, which are further differentiated as follows: static views on mathematics are either formalism-oriented or scheme-related views, the dynamic view on mathematics is either process-related or as new approach, application-oriented.

TEDS-M follows the latter approach and distinguishes between static and dynamic beliefs about the nature of mathematics referring to the sub-classification by Grigutsch et al. (1998). In addition, TEDS-M examines beliefs about the teaching and learning of mathematics separating transmission beliefs from constructivist views as developed by Peterson et al. (1989), and beliefs about teacher education and professional development. Self-related beliefs were not covered in TEDS-M.

With respect to the relationship between teacher knowledge and teacher beliefs, there are theories on the importance of MCK and MPCK when it comes to epistemological beliefs on the nature of mathematics (Schmidt et al. 2011). A certain level of MCK and MPCK may be needed before it is possible to see the dynamic nature of mathematics. These epistemological beliefs, in turn, probably influence beliefs on the teaching and learning of mathematics. The more a teacher is able to see the dynamic nature of mathematics, the more she may prefer student-oriented teaching methods in which students explore mathematics by themselves rather than just listening to the teacher.

Opportunities to Learn During Teacher Education TEDS-M followed the IEA tradition of connecting educational opportunity and educational achievement to determine whether cross-national differences in teacher competence were caused by differences in the teachers' opportunities to learn (OTL) during teacher education (McDonnell 1995). OTL are based on culturally influenced norms on education and intentionally developed by educational policy makers and teacher-education institutions. National and program specifications of OTL therefore reflect particular visions of what future primary and lower secondary teachers are expected to know and be able to do in a classroom and how teacher education should be organized to foster the competence necessary to master these tasks (Stark and Lattuca 1997; Schmidt et al. 2008).

The current state of research points to distinct educational philosophies that influence schooling and teacher education in different countries. Alexander (2001), in his seminal comparative study of primary school education in England, France, India, Russia and the USA, illustrated the subtle and long-term relationship between culture and pedagogy. Tobin et al. (1989, 2009) confirmed these findings with respect to early childhood education in China, Japan and the USA. Leung et al. (2006) were able to demonstrate similar cultural differences with respect to mathematics education in the East and the West.

In the same manner, data from a first comparative study on lower-secondary mathematics teacher-education programs in six countries, the "Mathematics Teaching in the 21st Century (MT21)" study (Blömeke et al. 2008; Schmidt et al. 2011), indicated that heterogeneous OTL profiles exist and that these may have been influenced by context characteristics. In five out of six countries examined, the multiple institutions where teacher education took place tended to cluster together with respect to the OTL offered, suggesting agreement *within* countries but distinct visions *between* countries, thereby reflecting a cultural effect (Schmidt et al. 2008).

OTL are probably related to teacher education outcomes. However, we know already that pure structural features, such as program or degree type, do not appear to have significant effects on short-term outcomes, such as teacher competence, or long-term outcomes, such as teacher retention or student achievement (Goldhaber and Liddle 2011). In contrast, especially in the case of mathematics teachers the evidence increasingly suggests that the *quality* of programs does have an impact on teacher outcomes (Boyd et al. 2009; Constantine et al. 2009).

Content courses in mathematics are assumed to be effective in the literature, as they deliver background knowledge and the body of deep conceptual and factual knowledge necessary to present mathematics topics to learners in a meaningful way and to connect the topics to one another as well as to the learner's prior knowledge and future learning objectives (Cochran-Smith and Zeichner 2005; Wilson et al. 2001).

Knowing the content, however, provides only a foundation for mathematics teaching. Student achievement is higher if strong content knowledge is combined with strong educational credentials (Clotfelter et al. 2007). The importance of *professional preparation*, specifically the understanding of how learners acquire mathematical knowledge, how to teach racially, ethnically and linguistically diverse students and using a wide array of instructional strategies, represents another robust finding of teacher-education research across various studies (Constantine et al. 2009; NRC 2010). Another robust finding on the impact of OTL on the outcomes of teacher education is the quality of the *teaching methods* experienced, in particular, the opportunity to engage in actual teaching practices, such as planning a lesson or analyzing student work, rather than only listening to lectures (Boyd et al. 2009).

Corresponding with these findings, OTL in TEDS-M were framed as content coverage on the one hand, specifically, as "the content of what is being taught, the relative importance given to various aspects" (Travers and Westbury 1989, p. 5). On the other hand, the concept of OTL included quality indicators, such as the teaching methods experienced. Both types of OTL were surveyed via self-reports of the future teachers. The results about how the OTL during mathematics teacher education were shaped in the TEDS-M countries and which effects they had on outcomes are presented in Chaps. 14 through 18 in this book.

It is urgent to discuss such issues of teacher education curriculum in an evidence-based manner (Blömeke and Paine 2008) rather than relying solely on anecdotal experience. For policy makers, the TEDS-M results provide information with respect to where reform is necessary *and* if it is possible to implement changes. For theory development, the results enable us to better understand the nature of teaching and teacher education.

Table 1 Participating countries in the TEDS-M primary and lower-secondary studies

Botswana	Chile	Germany	Georgia
Malaysia	Norway	Oman (lower-secondary school only)	Philippines
Poland	Russia	Spain (primary school only)	Switzerland
Singapore	Taiwan	Thailand	USA

2 Sampling

The target groups of TEDS-M were defined as future teachers in their final year of teacher education who were studying to teach mathematics in primary or lower secondary schools (Tatto et al. 2008). A teacher training program was identified as primary school level if the qualification included one of the grades 1 to 4 (primary or basic education, cycle 1; UNESCO 1997) and as lower secondary level if the qualification included grade 8 (basic education, cycle 2; UNESCO 1997).

In a two-stage process, random samples were drawn from these target groups in each participating country. The samples were organized according to important teacher education features such as the type of program (consecutive vs. concurrent programs), the school level to be taught (grade range included in the qualification, e.g. grades 1 to 4 vs. grades 1 to 10), the attention paid to OTL (in particular with or without mathematics) and the region where a teacher education institution was based (for example, federal states) to reflect accurately the future teachers' characteristics at the end of their training.

In 2008, about 14,000 future primary and 8,000 future lower-secondary teachers from 16 countries (see Table 1) were tested on their MCK and MPCK (and in three countries also on their GPK) with a standardized paper-and-pencil assessment. All countries had to meet the IEA quality requirements. These included controlling of the translation, monitoring test situations and meeting participation rates. If a country missed the participation benchmark only slightly, its results are reported with the annotation "combined participation rate less than 75 %".

In most countries, TEDS-M covered the full target population. Only Switzerland, Poland and the USA had to limit their studies for budgetary reasons: Switzerland limited its participation to German-speaking regions, Poland limited its participation to institutions with concurrent programs (90 % of all institutions), and the USA limited its participation to public universities. The situation was particularly complex in Norway. Two data sets were available that were likely to overlap. While information about the extent of a possible overlap was not available, several TEDS-M countries realized that using only one subsample would lead to strongly biased estimates for this country. After an examination of the Norwegian literature on teacher training, combining TEDS-M data with publicly available evaluation data from Norway (NOKUT 2006), and having sought the opinion of experts, these countries decided to combine the two subsamples in order to present the future Norwegian teachers' knowledge as accurately as possible. However, the results should be regarded as an approximation only.

Scaled scores in TEDS-M were created separately for MCK and MPCK in one-dimensional models using item response theory (Tatto et al. 2012). The data were analyzed on two levels of aggregation: (1) Due to the traditional policy orientation of IEA's large-scale assessments, TEDS-M focused on the country level. This approach stressed the overall educational effectiveness of a country, regardless of the structure of its education system. In this perspective, with regard to international competitiveness, it considered what a nation accomplishes as a whole—and differences in the structure of teacher-education systems between countries represented a function of differences in their educational policy. (2) Additional information was gained by looking into program types. Thus, it was possible to learn about pathways to success within countries without confounding variables like cultural or societal features. However, one has to bear in mind that the relatively small sample sizes on the country level became even smaller when types of programs were examined and that the precision of estimates was probably lower. The results of these analyses have therefore interpreted with caution.

3 Instruments

Testing MCK, MPCK and GPK TEDS-M sought to measure future teachers' MCK and MPCK in all participating countries (as mentioned GPK was only a national option, see below). For this purpose, a 60-minute paper-and-pencil assessment had to be completed during a standardized and monitored test session. The items were intended to depict classroom performance as closely as possible (see, e.g., National Council of Teachers of Mathematics (NCTM) 2000). The primary assessments consisted of five booklets with 104 items in total: 72 mathematics items and 32 mathematics-pedagogy items. The lower-secondary assessments consisted of three booklets with 103 items in total: 76 mathematics items and 27 mathematics-pedagogy items. The items were assigned to booklets following a balanced-incomplete-block design to capture the desired breadth and depth of teacher knowledge.

The mathematics items included the content areas of number, algebra (including a few items on functions and calculus) and geometry, with each set of items having roughly equal weight, as well as a small number of items about data (as that part of probability and statistics most common and relevant for teachers). The mathematics pedagogy items included aspects of curricular and planning knowledge and knowledge about how to teach mathematics. These two sets of items were given approximately equal weight. The items covered areas such as establishing learning goals, knowing different assessment formats or linking teaching methods and instructional designs, and identifying different approaches for solving mathematical problems. The items relating to knowledge about how to teach mathematics covered, for example, diagnosing typical student responses, including misconceptions, explaining or presenting mathematical concepts or procedures, and providing appropriate feedback.

The majority of items were complex multiple-choice items. Some were partial-credit items. In addition, both tests covered three cognitive dimensions: knowing (recalling and remembering), applying (representing and implementing), and reasoning (analyzing and justifying). Another feature that led the development of the items was their level of difficulty (novice, intermediate and expert). Scaled scores were created using item response theory. The achievement scores were transformed to a scale with an international mean of 500 test points and a standard deviation of 100 test points.

The items were developed among others based on the MT21 study (Schmidt et al. 2011), as well as the two Michigan studies "Knowing Mathematics for Teaching Algebra" (Ferrini-Mundy et al. 2005) and "Learning Mathematics for Teaching" (Hill et al. 2008). Released items are available on request by e-mailing tedsm@msu.edu. For more details see Tatto et al. (2008, 2012).

The instrument measuring general-pedagogical knowledge of future teachers in Germany, Taiwan and the USA consisted of 85 test items. These included dichotomous and partial-credit items as well as open-response (about half of the test items) and multiple-choice items. The items were fairly equally distributed across different teacher tasks like lesson planning, dealing with heterogeneity, motivation, classroom management and assessment. Following the MCK and MPCK test design, five or three booklets in a balanced-incomplete-block design were used.

Surveying the Future Teachers' Beliefs The future primary and lower-secondary teachers' beliefs about the nature of mathematics were surveyed using an instrument developed by Grigutsch et al. (1998). This instrument originally consisted of 75 items, but due to time constraints it was reduced to 12 items. These were selected according to both the highest factor loadings on each scale in the original study and high-scale reliability in the TEDS-M pilot studies. The items' two-dimensional structure represented a static and a dynamic view on the nature of mathematics. This structure was confirmed through explorative and confirmatory factor analysis. The future teachers had to express their agreement on a six-point Likert scale (1 = strongly disagree, 6 = strongly agree). The raw data were scaled using a partial-credit IRT model (Tatto et al. 2012). For the sake of clarity, individual scores were transformed to a scale with a mean value of 10, which represents a neutral view.

A dynamic view of mathematics sees the subject as a process of enquiry. The scale consists of six items which emphasize the process- and application-related character of mathematics, for example, "in mathematics you can discover and try out new things by yourself" or "many aspects of mathematics are of practical use". A static view of mathematics sees the subject as a set of rules and procedures. This scale consists of six items which stress the importance of definitions, formulae and mathematical facts and procedures, for example, "mathematics is a collection of rules and procedures that prescribe how to solve a problem" or "logical rigor and precision are fundamental to mathematics".

The future teachers' beliefs about the teaching and learning of mathematics were surveyed with two scales from instructional research (Peterson et al. 1989). The first scale represented a constructivist view. Strong agreement meant that teachers regarded mathematics learning as an active process in which students conduct their own enquiries and develop approaches to problem solving. Two examples of these items are: "In addition to getting the right answer, it is important to understand why the answer is correct"; and "Teachers should allow pupils to develop their own ways of solving mathematical problems".

In contrast, teachers who agreed strongly on the second scale tended to see mathematics learning as teacher-centered with the students' role being to follow instructions given. Two examples of these items are: "The best way to do well in mathematics is to memorize all the formulae"; and "Pupils need to be taught exact procedures for solving mathematical problems". The scaling happened in the same way as with respect to the nature of mathematics.

Surveying OTL TEDS-M intended to describe opportunities to learn during teacher education across countries. The topics listed in the survey were generated so as to be exhaustive of the content exposures in mathematics, mathematics pedagogy and general pedagogy in the participating countries. The future teachers indicated whether they had "studied" or "not studied" these topics. Their responses were prompted by three initial requests "Consider the following topics in university level mathematics (or mathematics pedagogy or general pedagogy respectively). Please indicate whether you have studied each topic."

Nineteen topics in mathematics were included as well as eight topics in mathematics pedagogy and eight topics in general pedagogy. For mathematics, these topics included categories such as "linear algebra", "abstract algebra", "analytic geometry" or "probability". In consultation with mathematicians in each of the countries and through a series of pilot and field studies, these categories were found to have essentially the same meaning across countries. Mathematics pedagogy included categories such as "mathematics standards and curriculum", "development of mathematics ability and thinking", or "developing teaching plans". The history, philosophy and sociology of education were included under general pedagogy as were topics related to assessment, teaching and the theory of schooling. National expert reviews and pilot studies ascertained the cultural validity of these items in all participating countries.

10 items captured how well the future teachers were prepared for specific professional challenges: the diversity of students in a mathematics class and the need for continuous professional development. The items had to be rated on 4-point Likert scales ranging from "never" to "often" after the initial request "In your teacher preparation program, how often did you have the opportunity to learn to do the following?" Examples of items were "Develop specific strategies and curriculum to teach pupils from diverse cultural backgrounds", "… with behavioral and emotional problem" or "… gifted pupils" on the one side and "Develop strategies to reflect upon the effectiveness of your teaching" or "… upon your professional knowledge" on the other side.

The teaching methods experienced at university had to be rated on the same type of 4-point Likert scales. Again the items listed were generated so as to be the union of methods exposures across the participating countries. The 15 items covered typical teaching methods used in most programs at a university (e.g. "Listen to a lecture" or "Make presentations to the rest of the class") but also methods typical for teacher education only (e.g. "Teach a class session using methods demonstrated by the instructor") or methods typical for mathematics programs only (e.g. "Solve a given mathematics problem using multiple strategies" or "Write mathematical proofs"). The research aspect of university programs was covered as well (e.g. "Read about research on mathematics education").

4 Results

Detailed information and complex analyses are reported from Chap. 4 in this book. The main function of the present chapter is to provide an overview of the most important descriptive results on the country level to frame the later in-depth results. As far as we can see, this basic information has not yet been published in English but only in German (Blömeke et al. 2010a, 2010b) since the official TEDS-M report is limited to program types as the unit of analysis (Tatto et al. 2012).

Structure of Primary and Lower-Secondary Teacher Education Primary school covers grades 1 through 6 in many TEDS-M countries. Correspondingly, teacher education prepares for teaching in these grades. In most countries, the primary teachers examined in TEDS-M were prepared as generalists either for grades 1 through 3 (e.g., in Poland and Taiwan) or up to grade 6 (e.g., in the Philippines or Spain). The role of generalists means that as head (or class or form) teachers they will have to teach most subjects in one class. During teacher education the future primary teachers had opportunities to go into more depth with respect to the content of three or four subjects, among others in mathematics.

Germany is an exception as primary school in most federal states includes only four grades and teacher education either prepares for teaching in these (as generalists) or for teaching up to grade 10 (then prepared as specialists in two subjects, in the context of TEDS-M one of these would have been mathematics). Most countries offer two pathways into teaching: a 4-year concurrent and a consecutive route with a basic Bachelor degree followed either by a teaching license or a Master degree. The majority of future teachers were enrolled in a concurrent program. Also in this respect Germany is an exception as its teacher education system combines important features of both approaches ("hybrid system").

Lower-secondary school in most of the TEDS-M countries consists of the grades 7 to 9 (Tatto et al. 2012). Teacher education prepares for the teaching of one or two subjects in either in these grades only (e.g., in Taiwan) or in grades 7 through 12 (e.g., in Georgia). In the context of TEDS-M, one of the subjects would be mathematics. Else, the characteristics of the teacher education system are similar to primary teacher education.

Background of Future Primary and Lower-Secondary Teachers at the End of Their Training Seen across all TEDS-M countries, *a typical primary teacher at the end of her training* was on average 24 years old and female. Her parents typically had a degree on the UNESCO (1997) classification levels 3 or 4 (educational degree from an upper- or post-secondary institution) and there was on average a medium amount of books in her parents' homes (between 26 and 100). Typically, there was a computer in these homes as well. The teacher's prior knowledge from schooling was on average high: 12 years of mathematics and good or even very good grades across all school subjects compared to her age cohort. The language of teacher education typically fit to the language spoken at home. Intrinsic pedagogical motives dominated the decision to become a teacher much more than extrinsic status motives but also more than intrinsic intellectual motives.

Not surprisingly there was huge *variation* between countries with respect to these average characteristics of future primary teachers. It seems as if teachers from consecutive programs were older than those from concurrent programs. And whereas future primary teachers in the Philippines and Georgia were on average only 21 years old at the end of their training, teachers from Germany were already 27 years old. This high age at the end of their training was an accumulated consequence of many different societal, schooling and teacher education features. In none of the TEDS-M countries males represented the majority of primary teachers at the end of their training. However, a tendency existed that their proportion increased if their program required more mathematics or if they had to teach higher grades.

In many TEDS-M countries the educational background of the primary teachers' mothers and fathers was roughly equal. This did not apply to all countries though. In Germany, Switzerland and Spain mothers on average had lower, in Russia, Poland and Georgia mothers had higher degrees than fathers. These differences are probably related to the role of women in these societies (Hradil 2001; UNICEF 1999).

The number of books in the parents' homes varied between countries as well. In Germany and Norway the future primary teachers' cultural capital was especially high. Strikingly high was also the cultural capital of teachers in Georgia and Russia given their rank on the UN Human Development Index. This result might reflect high educational aspirations in these societies (Alexander 2001). In general, one has to notice that in most countries the teachers' cultural capital was higher than the cultural capital of K-12 students. The much lower number of books reported by the latter group (for example, in TIMSS; Mullis et al. 2008) points to a selection effect.

With respect to the language spoken at home compared to the official language in teacher education (i.e. the test language of the TEDS-M tests and surveys), a distinct difference between two groups of countries existed that is not reflected in the portray of a typical primary teacher presented above. In one group that included Botswana, Malaysia, and the Philippines, future teachers were tested in English whereas this was the language spoken at home only for a small minority. We also found substantial proportions of teachers speaking a different language at home compared to teacher education in Singapore (Malay, Chinese or Tamil vs. English), Thailand (several different languages and dialects, among others Kadai or Chinese, vs. Thai) and Taiwan (Taiwanese vs. Mandarin). In contrast, in many countries almost every future primary teacher spoke the official test language at home—although we sometimes found substantial proportions of language diversity in these countries as well (e.g., in Germany and the USA).

Interesting variation between countries existed also with respect to the motivation why the future teachers went into teacher education. Primary teachers in the USA, Switzerland, Norway, Germany, Spain, and Chile stated particularly strongly pedagogical motives in relation to intellectual or extrinsic motives. This result might be related to the long-standing tradition of child-orientated pedagogy in these countries. In contrast, future teachers in the Asian and Eastern European countries stressed particularly strongly the intellectual challenge of teaching. This result might be related to the high value of mathematics in these countries and in the East Asian countries in addition to their Confucian heritage and its valuing of teachers. With the teaching of higher grades and the study of more mathematics, the intellectual motive was on average more strongly supported.

One more split between countries existed with respect to the extent future primary teachers felt limited by financial or familial constraints during their studies. On the one side, we found countries where future teachers stressed family obligations more strongly than financial worries. This applied to all Asian countries in TEDS-M as well as to Botswana and Chile. On the other side, we had the Western countries and Poland where financial limitations dominated in relation to familial issues. It is probably not far-fetched to relate this result to cultural differences as they were expressed by the Hofstede (2001) continuum of collectivism and individualism.

A typical lower-secondary teacher at the end of teacher training showed many similarities with primary teachers in her background characteristics. She was typically aged 24 and female. The teacher's parents had on average a degree at level 3 or 4 of the UNESCO (1997) ISCED classification and they had between 26 and 100 books at home. They usually had a computer as well. The typical future lower-secondary teacher had completed 12 years of mathematics classes and had good or even very good grades compared to her peers. The language of teacher education was typically the language spoken at home. On average, the future teachers had entered teacher education for intrinsic pedagogical reasons. They were less interested in extrinsic status reasons or intrinsic intellectual reasons.

Also in this group of future teachers huge *variation* between countries existed. Teachers in consecutive programs were on average older than those in concurrent programs. Although in most TEDS-M countries the majority of lower-secondary teachers in their final training year were women, in three countries—Botswana, Taiwan and Switzerland—the majority were men. In Germany, the mothers of the future teachers had on average reached a lower level of higher education than the fathers, whereas in Russia and Poland the mothers hold higher-level degrees.

In Germany, Norway and Switzerland the teachers' cultural capital was especially high. The cultural capital of teachers in Georgia was high given this country's rank on the UN Human Development Index. Also with respect to future lower-secondary teachers, the cultural capital is higher than that of their students. The language split between countries was for future lower-secondary teachers very similar to the grouping of countries with respect to primary teachers. In Oman, where only lower secondary teachers took part in TEDS-M, significant proportions of teachers spoke a different language at home to the one used in teacher education (Persian or Indian vs. English).

Future lower-secondary teachers in the USA, Switzerland, Norway, Germany, Chile and Singapore particularly strongly selected pedagogical motives over intellectual or extrinsic motives to explain their career choice. In contrast, future teachers in Poland, Russia and Oman stressed more strongly the intellectual challenge of teaching compared to other reasons that had motivated their choice of career. In Taiwan, the Philippines, Malaysia, Georgia and Thailand extrinsic motives dominated the reasons given for becoming a teacher.

Similar to the results of primary teachers, future lower-secondary teachers from all Asian countries in TEDS-M as well as from Botswana and Chile stressed particularly strongly family obligations over financial worries when describing factors that limited their study. On the other hand, in the Western European countries and the USA financial limitations dominated over family issues. These results can once again be explained by looking at cultural differences as expressed by the Hofstede continuum of collectivism and individualism (Hofstede 2001).

Opportunities to Learn (OTL) During Primary and Lower-Secondary Teacher Education The extent of OTL in mathematics varied a lot between the TEDS-M countries. In Thailand where they trained specialists for this school subject even on the primary level, future teachers have covered the most topics. Germany is one of the countries where the extent of OTL in mathematics was significantly below the international average. This result was mainly a function of one program type in which mathematics was neglected (primary and lower-secondary teachers without specialization in mathematics). Graduates from the other three types covered significantly more mathematical topics during their training.

It is possible to identify an international profile of OTL in mathematics: Number was a dominant field of study in primary teacher education followed by data and within certain limits geometry. Calculus was in most countries of significantly lower importance. Another commonality across countries was the relatively high amount of OTL taken in general pedagogy, and this with respect to theoretical as well as practical topics. There seemed to be a consensus that general pedagogy had to be a vital part of teacher knowledge. Less agreement existed with respect to mathematics pedagogy, specifically with its theoretical part. Germany was one of the countries with the lowest extent of OTL in this field.

Teacher educators play an important role in providing OTL. On average more than half of the teacher educators in the TEDS-M countries were female. The proportion of teacher educators with a degree on ISCED level 6 (at least PhD) varied between the countries: between 0 % in Botswana and 82 % in Georgia.

Lower-secondary teacher education was also characterized by considerable variation in the OTL in mathematics, mathematics pedagogy and general pedagogy between the TEDS-M countries. At the end of their training, future teachers in Germany, Poland, Russia, Georgia, Taiwan, Oman and Thailand indicated more OTL in mathematics compared to mathematics pedagogy and general pedagogy. In contrast, lower-secondary teacher education in Norway, the USA, Chile and Botswana focused particularly strongly on pedagogical topics. In the first set of countries the focus was obviously on the content, whereas in the second set the teaching of the content was considered most important.

In Botswana, Singapore, Georgia, Malaysia, Oman and Taiwan there were particularly many OTL in calculus compared to number, geometry and data. This result suggest that mathematics teacher education in these countries focused on the higher grades of lower-secondary school. In Norway, Switzerland, the USA and Chile the OTL in calculus were low, which suggested an orientation towards the lower grades.

Overall, lower-secondary mathematics teachers, who were also qualified to teach at the upper-secondary level, had significantly more OTL in mathematics than their peers who were intending to teach at the lower-secondary level. In Norway and Chile, where lower-secondary teachers were trained as generalists, and in Germany and Singapore, where they were trained in two subjects, the future teachers reported the fewest OTL in mathematics.

MCK, MPCK and GPK as Outcomes of Primary Teacher Education Significant mean differences in teacher-education outcomes in terms of MCK, MPCK and GPK existed between the countries involved in TEDS-M. The data revealed a wide range of what was accomplished in primary teacher education. The ranking of countries and teacher education programs according to these outcomes provided international benchmarks to evaluate the effectiveness of the education that future primary teachers received.

Taiwan and Singapore achieved the best results with respect to MCK and MPCK (see Tables 2 and 3). The difference to the international mean of 500 test points was large, at approximately one standard deviation which is a highly relevant difference (Cohen 1988). Switzerland, Norway and the USA achieved results significantly above the international mean in both facets as well while primary teachers from Georgia, Chile, Botswana, the Philippines, Spain and Poland were significantly below the international mean in both facets (for further details, see Blömeke et al. 2011, 2012).

Interesting differences exist with respect to achievement in MCK and MPCK which require more research. Whereas Singapore was behind Taiwan in case of MCK, the countries were on the same level in case of PCK. With respect to MPCK, Norway and the USA were only one half of a standard deviation behind the two East Asian countries whereas the difference was up to one standard deviation with respect to MCK. Malaysia scored around the international mean in MPCK whereas the country scored below the mean in MCK. Russia, Thailand, and Germany performed significantly lower in MPCK than in MCK. These differences are worth to be examined in detail. They may point to country-specific strengths and weaknesses.

Table 2 MCK of future primary teachers at the end of their training by country (M = mean, SE = standard error, SD = standard deviation)

Country	M	SE	SD
Taiwan	623	4.2	84
Singapore	590	3.1	74
Switzerland[*]	543	1.9	66
Russia	535	9.9	91
Thailand	528	2.3	75
Norway[a,n]	519	2.6	73
USA[**,a,b]	518	4.1	69
Germany	510	2.7	83
International	500	1.2	100
Poland[***,a]	490	2.2	98
Malaysia	488	1.8	54
Spain	481	2.6	57
Botswana	441	5.9	48
Philippines	440	7.7	52
Chile[a]	413	2.1	65
Georgia	345	3.9	85

[*] Pedagogical universities in German-speaking cantons

[**] Public universities

[***] Institutions with concurrent teacher-educations programs

[n] Sample meets the TEDS-M definition only partly, deviation from the IEA report

[a] Combined participation rate <75 %

[b] Substantial proportion of missing values

Table 3 MPCK of future primary teachers at the end of their training by country (M = mean, SE = standard error, SD = standard deviation)

Country	M	SE	SD
Singapore	593	3.4	71
Taiwan	592	2.3	68
Norway[a,n]	545	2.4	64
USA[**,a,b]	544	2.5	68
Switzerland[*]	537	1.6	64
Russia	512	8.1	83
Thailand	506	2.3	70
Malaysia	503	3.1	67
Germany	502	4.0	92
International	500	1.3	100
Spain	492	2.2	63
Poland[***,a]	478	1.8	101
Philippines	457	9.7	67
Botswana	448	8.8	75
Chile[a]	425	3.7	90
Georgia	345	4.9	100

Annotations are explained above (Table 2)

With respect to the achievement of primary teachers coming from different program types, MPCK is taken as an example in this summary (with respect to MCK

Table 4 Mathematics knowledge in grade 4, in grade 8 and at the end of primary teacher education (M = mean, d = Cohen's d)

Country	TIMSS 2007—Grade 4		TIMSS 2007—Grade 8		TEDS-M 2008	
	M	d	M	d	M	d
Taiwan	576	+0.9	598	+1.0	623	+1.3
Singapore	599	+1.1	593	+1.0	590	+1.0
Russia	544	+0.5	512	+0.1	535	+0.4
Norway[a,n]	473	−0.3	469	−0.4	519	+0.2
USA[**,a,b]	529	+0.3	508	+0.1	518	+0.2
Germany	525	+0.3	–	–	510	+0.1
International	500	0.0	500	0.0	500	0.0
Georgia	438	−0.7	410	−0.9	345	−1.7

Annotations are explained above (Table 2)

see Blömeke et al. 2010a). Not surprisingly specialists show the best performance. No MPCK mean of any program type was significantly below the international mean of 500 test points. Single results of teachers from other programs were more striking though. In Taiwan, Singapore, and Norway future teachers from non-specialist programs showed high achievement in MPCK, too. At the same time we have to notice huge differences within countries, for example in Poland and Germany. In these two countries it is possible to teach mathematics in primary schools either with a license from a generalist or a specialist program. The average MPCK achievement of these programs differed by about a full standard deviation.

The achievement of future primary teachers from countries which, according to the UN Human Development Index (HDI), were classified as developed or highly developed, was often above the international mean. This did not apply to Germany, Poland and Spain though so that for these three countries it seems to be necessary to examine in detail potential problems of their mathematics teacher-education systems. In contrast, given their positions on the HDI, the performance of teachers from Russia and Thailand (and partly also form Malaysia) was remarkably good.

For seven countries, comparisons between the TEDS-M results and the TIMSS results of grades 4 and 8 (Mullis et al. 2008) are possible on the country level. The effect size "Cohen's *d*" represents the deviation of a country's score on each scale from the respective international mean. Conclusions have, of course, to be drawn only very cautiously because of the complex relationship between student achievement and teacher achievement. But all in all, the results show astonishingly clear similarities of the country-level results for grade 4, grade 8 and primary teacher education (see Table 4). The same countries, namely Singapore and Taiwan, show outstandingly high achievements in all large-scale assessments with roughly the same effect sizes. Likewise, the achievement of Russia, Germany and

Table 5 Correlations between future primary teachers' MCK and MPCK by country (Pearson's *r* and standard errors)

Country	*r*	SE
Poland[***,a]	0.68	0.01
Germany	0.62	0.03
Russia	0.58	0.05
Norway[a,n]	0.53	0.03
Thailand	0.50	0.03
USA[**,a,b]	0.48	0.03
Chile[a]	0.46	0.03
Malaysia	0.44	0.05
Taiwan	0.43	0.04
Spain	0.41	0.03
Georgia	0.38	0.03
Switzerland[*]	0.38	0.03
Singapore	0.34	0.04
Philippines	0.34	0.04
Botswana	0.28	0.11

Annotations are explained above (Table 2)

the USA was each time higher than the international mean while Georgia scored significantly below the mean. Only with respect to Norway we have to note a gap between the students‘ and the teachers‘ results. The future primary teachers performed, relatively speaking, better than the students in both K-12 assessments.

The analyses done so far have revealed that the country rankings for MCK and MPCK were similar. Indeed, MCK and MPCK conceptually overlap as MCK must be regarded a precondition for mastering tasks that require MPCK. Nevertheless, only a few countries showed very high correlations between MCK and MPCK (see Table 5) while the correlations differed between the countries participating in TEDS-M: In Poland and Germany, both knowledge facets co-varied strongly so that we can speak of closely related facets. In contrast, in Botswana, the Philippines, Singapore, Georgia and Switzerland low correlations existed. At present, it is unclear what might be the reason for these differences between the countries, as neither only countries with top-performing teachers showed high correlations (as an opposite example see, e.g., Poland) nor countries with low-performing teachers showed low correlations only (as an opposite example see, e.g., Switzerland). Similarly, neither only European countries showed high correlations (as an opposite example see, e.g., Thailand) nor non-European countries showed low correlations only (as an opposite example see, e.g., Botswana).

If one compares the countries according to their relative strengths in MCK vs. MPCK, three groups are distinguishable: In the Asian countries Taiwan and Thailand and the four European countries Russia, Poland, Germany and Switzerland future primary teachers performed better in MCK in relationship to MPCK. In contrast, teachers in Norway, the USA, Spain and Chile as well as in Malaysia and the Philippines were characterized by their relative strengths in MPCK compared to MCK. In Georgia, Singapore and Botswana balanced profiles can be observed. These knowledge profiles did not correlate with the absolute levels of achievement. This result shows that there is no single ideal way to gain strong achievement in both knowledge facets (for details and a discussion of this result see the chapter by Kaiser and Blömeke in this book).

However, it may be that cultural traditions play a role for shaping the profiles. In East-Asian countries, subject-based knowledge is given high value (Leung 2001). A teacher is regarded an expert of a subject (Leung et al. 2006) but subject-related knowledge plays a significant role in Continental and Eastern Europe, too (Alexander 2001; Kaiser et al. 2006). This tradition contrasts with the child-oriented concept prevalent in Scandinavia as well as in North and South America.

The aggregated MCK score does not show the teachers' strengths or weaknesses in subdomains like number, algebra or geometry. Therefore, based on the proportion of correct responses, the relative achievement in these subdomains was examined (for details how these relative scores were estimated see Blömeke et al. 2010a). The primary teachers solved correctly on average 62 % of the number and the algebra items and 59 % of the geometry items. The range was between 25 and 31 % in Georgia to 79 and 85 % in Taiwan.

Future teachers in Taiwan, Thailand, Switzerland and the USA showed a relative strength on number items and relative weaknesses in geometry and algebra. This profile meets the demands of the lower primary grades. In four countries, including Germany, the future primary teachers showed a relative strength in algebra but weaknesses in number and geometry. Such a profile indicates an orientation at teaching on the lower secondary level. This matches for instance with Germany's teacher education where about half of the future primary teacher population consists of teachers trained for teaching in grades 1 through 10. Teachers of the third group showed a balanced profile across the three subdomains.

The international TEDS-M team developed cut scores in order to describe different performance levels in MCK and MPCK (for details how this was done see Tatto et al. 2012). For MCK, two thresholds and thus three groups of future primary teachers could be distinguished. The best-performing group, positioned above the second threshold, consisted of teachers who had extensive MCK, could solve standard problems with a high probability and who, in order to give an example,were able to identify irrational numbers with a probability higher than 70 %. Across all TEDS-M countries, about two fifths of the teachers belonged to this group. While in Taiwan and Singapore more than 80 % of the future primary teachers were part of this group, in other countries like Georgia, Chile, the Philippines and Botswana only less than 10 % fall into this group. In Germany, the USA and Norway approximately 50 % of the primary teachers fall into this highest-performing group.

Future primary teachers in the middle group, between the first and the second threshold, were equipped with a basic understanding of natural and whole numbers, but they experienced difficulties when they had to apply number theory-related concepts. They were able to construct and interpret two- and three-dimensional geometric forms and to calculate its surface area, but they had difficulties with geometric forms in a representation of coordinates. In algebra, they were familiar with variables and could execute equivalence transformation, but they had difficulties in recognizing square and functional exponential relationships. In most of the TEDS-M participating countries around 30 to 50 % of primary teachers belong to this group.

Low MCK was reported for one fifth of the future primary teachers that belonged to the third group below the first threshold. These teachers suffered from a deeper understanding and they faced problems with example-related argumentation. They had problems in dealing with natural and rational numbers. In algebra, for instance, they did not succeed in carrying out visually represented equivalence transformation. Likewise, it was difficult for them to correlate various mathematical concepts and to develop argumentative proofs. Only in Taiwan, Singapore and Switzerland less than five % of the teachers belonged to this group. In contrast, 88 % of all teachers in Georgia and 60 % of the teachers in Chile had such a low MCK while in Botswana and the Philippines still around 40 % represented this level of knowledge. In Germany, Russia, Thailand, Norway, the USA and Malaysia between 7 and 12 % of future primary teachers belonged to this group. These results point out that in these countries primary teachers showed clear deficits.

A brand new field of research is the assessment of teachers' GPK. TEDS-M was the first comparative study that addressed this dimension. Germany and Taiwan assessed the knowledge of future primary teachers about lesson planning, classroom management, motivation, dealing with heterogeneity and assessment—each dimension was subdivided into three cognitive tasks (recalling, understanding and creating). The main result was that German future primary teachers significantly outperformed US teachers. The difference was about one standard deviation overall as well as within respect to each subdimension and it was therefore highly relevant. Within German graduates from pure primary programs performed significantly better than students from combined primary and lower-secondary programs.

MCK, MPCK and GPK as Outcomes of Lower-Secondary Teacher Education With respect to MCK, by far the best result was achieved by future lower-secondary teachers in Taiwan (see Table 6). Their MCK was more than 1.5 standard deviations higher than the international mean. In addition, Taiwan exceeded the achievement of teachers from the second-best country, Russia, by more than half a standard deviation. Even the lowest achievers from Taiwan had better results than the best results achieved in Chile, Georgia, Botswana, the Philippines, Norway and Oman, as indicated by the respective 5^{th} or 95^{th} percentiles.

Russia together with Singapore, Poland, Switzerland and Germany belonged to a group of countries where the MCK was significantly higher than the international mean. It is remarkable that with Poland and especially Russia two countries belong to this group whose developmental level (HDI) was lower than that of the other countries. With respect to Switzerland, we have to point out that only lower-secondary teachers participated in TEDS-M who were educated at Pedagogical Universities. If teachers educated at universities for teaching at the upper-secondary

Table 6 MCK of future lower-secondary teachers (M = mean, SE = standard error, SD = standard deviation)

	M	SE	SD
Taiwan	667	3.9	75
Russia	594	12.8	96
Singapore	570	2.8	61
Poland[***,a]	540	3.1	66
Switzerland[*]	531	3.7	50
Germany	519	3.6	94
USA[**,a,c]	505	9.7	67
International	500	1.5	100
Malaysia	493	2.4	51
Thailand	479	1.6	59
Oman	472	2.4	47
Norway[b,n]	444	2.3	63
Philippines	442	4.6	49
Botswana	441	5.3	39
Georgia[a]	424	8.9	84
Chile[a]	354	2.5	84

Annotations are explained above (Table 2)

level (grades 10 to 12) were included, the country might have achieved even better results.

The MCK of future lower-secondary teachers from the USA and Malaysia did not differ significantly from the international mean. Significantly below the international mean were the results of Thailand, Oman, Norway, the Philippines, Botswana, Georgia and Chile. The MCK in the latter country was 1.5 standard deviations below the international mean. According to its HDI, Chile is similarly developed like Poland but much higher than Thailand, Georgia, the Philippines or Botswana. However, even more worrying was the achievement of Norway, one of the highest-developed countries in the world.

As with respect to primary teachers, all in all astonishingly similarities of the TEDS-M results with the TIMSS results at grade 8 can be noted (see Table 7). In all countries where the teacher population performed above the international mean, the K-12 student achievement was higher as well and vice versa. Also the country ranking came out quite similar in both studies.

The TEDS-M results with respect to MPCK were comparable with those to MCK. Again Taiwan and Chile represented the best and lowest performing countries. However, the deviation from the international mean was lower in the case of MPCK than MCK. In general, the results of the participating countries did not vary so much. Similar to MCK, five countries performed significantly higher than the international mean (see Table 8): Russia, Singapore, Switzerland, Germany and Poland. Once more, it must be pointed out that especially the MPCK results of Russia, a country which according to its HDI is classified as a relatively low developed country, were remarkable. This might indicate strength of the East-European

Table 7 MCK in grade 8 and at the end of lower-secondary teacher education (M = mean, d = Cohen's d)

Country	TIMSS 2007—Grade 8		TEDS-M 2008	
	M	d	M	d
Taiwan	598	+1.0	667	+1.9
Russia	512	+0.1	594	+1.0
Singapore	593	+1.0	570	+0.8
USA[**,a,c]	508	+0.1	505	+0.1
International	500	–	500	–
Malaysia	474	−0.3	493	−0.1
Thailand	441	−0.6	479	−0.3
Norway[b,n]	469	−0.4	444	−0.7
Philippines	378*	−1.3	442	−0.7
Botswana	364	−1.5	441	−0.8
Georgia[a]	410	−0.9	424	−0.8
Chile[a]	387*	−1.2	354	−1.6

Annotations are explained above (Table 2)

tradition of education. In contrast, it must be stated again that Norway, a highly-developed country, fell far behind the international mean. Although there were similarities in the MCK and MPCK results, it is at the same time important to distinguish between the two facets. Whereas Malaysian teachers scored only slightly below the international mean in MCK, they had much lower scores when it came to MPCK, for example. Such differences are worth examining in detail. They may point to specific strengths and weaknesses in teacher education in the different countries.

Comparable to the primary results, the conceptual overlap of MCK and MPCK led to varying correlations between these (see Table 9). While in Germany, Russia, Poland and the USA it was almost not possible anymore to separate the two facets, in Botswana a systematic correlation did not exist at all. For the moment, it is not yet clear, what the reason might be for these remarkable differences.

Country-specific profiles with respect to relative strengths and weaknesses in MCK and MPCK can be recognized. Three groups of countries can be distinguished: In the three Western European countries Germany, Switzerland and Norway together with Chile and Georgia, future teachers showed relative strength in MPCK compared to MCK. In contrast, future teachers in the East-European countries Russia and Poland, in the Asian countries Singapore, Taiwan, Malaysia and also in Botswana performed relatively better in MCK than in MPCK. In the third group, consisting of the USA, Thailand, Oman and the Philippines, an even result for both knowledge areas came out. The profiles varied independently from the absolute performance level. The profiles might rather reflect cultural traditions. For instance, in East-Asian countries which are strongly influenced by Confucian philosophy (e.g., Singapore and Taiwan) teachers are regarded as experts of the content and they are given the role of "scholar-teachers" (Leung et al. 2006, p. 43). Therefore, a great proportion of teacher education consists of subject-related components. In East-European countries, subject-based knowledge plays an important

Table 8 MPCK of future lower-secondary teachers (M = mean, SE = standard error, SD = standard deviation)

Land	M	SE	SD
Taiwan	649	5.2	95
Russia	566	10.1	96
Singapore	553	4.7	84
Switzerland[*]	549	5.9	72
Germany	540	5.1	96
Poland[***,a]	524	4.2	81
USA[**,a,c]	502	8.7	75
International	500	1.6	100
Thailand	476	2.5	64
Oman	474	3.8	66
Malaysia	472	3.3	61
Norway[b,n]	463	3.4	72
Philippines	450	4.7	60
Georgia[a]	443	9.6	79
Botswana	425	8.2	59
Chile[a]	394	3.8	88

Annotations are explained above (Table 2)

role as well (Alexander 2001). In contrast, since the era of the Reform Pedagogic, learner-focused and constructivist approaches have existed in Continental Europe but also in Chile, a country strongly influenced by European traditions (for details see the chapter by Kaiser and Blömeke in this book).

The aggregated proportions of correct solutions by content domain revealed interesting strengths or weaknesses as well. The future lower-secondary teachers solved correctly 47 % of the algebra, 52 % of the number and geometry items as well as 55 % of the MPCK items related to issues of curriculum and planning and 59 % of the MPCK items related to interaction in classroom. The algebra test was obviously more difficult than other tests but the differences were small. Relative strengths in geometry combined with relative weaknesses in number and algebra were shown by future lower-secondary teachers in Norway and Malaysia. In contrast, a relative weakness in geometry combined with relative strengths in number and algebra were revealed for Germany, the Philippines, Oman, Botswana, Taiwan and Poland. Switzerland, Thailand and Singapore demonstrated a relative weakness in algebra combined with relative strengths in number and geometry. In the remaining countries, the teachers displayed a knowledge profile largely corresponding with the international mean.

With respect to MCK, like in the primary study, two thresholds were identified that distinguished three groups of future lower-secondary teachers (490 and 560 test points). Due to the small number of MPCK items only two levels could be distinguished here (510 test points). The test items located at the thresholds describe for each competence level the minimum of existing knowledge and the not-existing

Table 9 Manifest correlations between future lower-secondary teachers' MCK and MPCK by country (Pearson's r and standard errors)

Land	r	SE
Germany	0.70	0.03
Russia	0.68	0.04
Poland[***,a]	0.67	0.05
USA[**,a,c]	0.64	0.03
Georgia[a]	0.56	0.11
Singapore	0.55	0.04
Norway[b,n]	0.53	0.04
Malaysia	0.52	0.04
Chile[a]	0.51	0.03
Thailand	0.50	0.03
Taiwan	0.45	0.04
Oman	0.44	0.04
Switzerland[*]	0.40	0.08
Philippines	0.37	0.10
Botswana	0.18	0.14

Annotations are explained above (Table 2)

knowledge (for a detailed description how the levels were found and item examples see Blömeke et al. 2010b).

In Taiwan, almost all future lower-secondary teachers reached the highest competence level. With a probability of more than 70 % they were able to apply university-level definitions, theorems and algorithms in calculus, algebra and higher geometry. They had a profound knowledge of elementary and complex operations and they were also able to apply abstract definitions and formalisms. Further, they knew how to solve abstract algebraic or geometric problems by referring to axiomatic definitions. In Russia and Singapore, the majority of the teachers performed also at this high competence level.

In contrast, a group of nine countries had the largest proportion of teachers—or even the majority of teachers—on the lowest competence level: the USA, Thailand, Malaysia, Georgia, Oman, Norway, the Philippines, Botswana and Chile. Teachers on this level had only basic knowledge of rational numbers, and, at a limited degree, they were able to execute simple calculations, such as solving linear or simple quadratic equations, especially by applying trial-and-error methods. They were also able to solve problems with whole numbers. Further, they were able, at a limited degree, to deal with fundamental two- and three-dimensional geometric figures, as well as they were able to recognize and produce simple geometric figures. All in all, these teachers' knowledge was limited to school knowledge of the secondary level and its application to known types of problems.

It is interesting to note that the TEDS-M data did not necessarily support the hypothesis that teachers in consecutive programs did better than teachers in concurrent programs. Another important outcome was that test results improved when future lower-secondary teachers had more OTL in mathematics. German lower-secondary teachers who were trained to teach on the upper-secondary level as well (up to grade 12) showed an outstanding level of MPCK, for example. In contrast, German mathematics teachers qualified to teach up to grade 10 did less well.

Germany, Taiwan and the USA assessed their lower-secondary teachers' GPK as well. The German and Taiwanese teachers significantly outperformed their US counterparts.

Beliefs as Outcomes of Teacher Education Finally, beliefs were captured as teacher-education outcomes in TEDS-M. There was huge variation between and within countries—however, it was possible to identify profiles which seemed to be influenced by cultural features, specifically on the Hofstede continuum of individualism and collectivism. In individualistic countries like Germany, future primary teachers specifically stressed dynamic aspects of mathematics in relation to static aspects and constructivist principles of teaching and learning in relation to transmission-orientated principles. In contrast, in collectivistic countries the support of static and transmission aspects was relatively high compared to the support of dynamic and constructivist aspects. Countries which seemed to be moving from collectivism to individualism according to Hofstede's index were positioned in the middle of the TEDS-M countries as well. If a country deviated in TEDS-M from Hofstede's index (e.g., Poland), the special tradition of mathematics might be an explanation. Within Germany the profile of beliefs varied according to program types. The more mathematics a future teacher had taken, the more she supported dynamic and constructivist beliefs.

The results for lower-secondary future teachers were much the same. Whereas future teachers in Germany, Switzerland, Poland and Norway either had a neutral view of mathematics or even denied its static nature, teachers in the Philippines, Thailand, Malaysia and Botswana agreed with statements that mathematics mainly involves algorithms. There was more agreement between teachers in the different countries when it came to the dynamic nature of mathematics. In all countries, future teachers reacted positively to statements that stressed the creativity and usefulness of mathematics.

When comparing how strongly teachers agreed with both notions in relation to each other, certain profiles appeared. Future teachers from countries like Malaysia and Thailand expressed much more agreement with static beliefs than with dynamic beliefs. In contrast, teachers from countries like Germany and Switzerland agreed more strongly with dynamic beliefs than with static beliefs. These results can be linked to Hofstede's index of individualism and collectivism (Hofstede 2001).

With respect to constructivist and transmission beliefs on the teaching and learning of mathematics, lower-secondary teachers in Germany, Switzerland and Norway rejected teacher-led learning, whereas teachers in the Philippines and Malaysia supported it. In contrast, agreement with statements that support student orientation was high in all countries. In line with the results on the nature of mathematics, a relationship to the countries' positions on Hofstede's scale of individualism and collectivism was revealed when comparing the relative endorsement of constructivist and transmission views. In Switzerland, Germany, Norway, the USA and Poland—countries characterized by individualism—the future teachers stressed the importance of student orientation over teacher orientation particularly strongly. In contrast, teachers in Russia, Singapore, the Philippines and Malaysia stressed teacher orientation particularly strongly compared to student orientation. The OECD Teaching and Learning International Study (TALIS; OECD 2009) of practicing teachers produced similar results.

5 Summary

If one tries to summarize the main aspects we learned from TEDS-M, there are methodological and substantive aspects to be mentioned. TEDS-M showed that studies in the field of higher education are challenging and difficult to do. Several levels of aggregation are to be considered—and each one has its own benefits and limits. From a substantive point of view, we learned that achievement in different domains of teacher knowledge (MCK, MPCK, GPK) can differ a lot. And the achievement of teachers from different programs within a country can differ a lot as well. Here, we can learn the most for policy efforts within countries to improve the effectiveness of a system. Overall, teacher knowledge does not seem to be an exclusive function of societal features, of features of incoming students or of the length, the structure or the content of teacher education only but a complex amalgam of these characteristics. Complex and detailed analyses will shed more light on these issues in the following chapters.

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Learning About and Improving Teacher Preparation for Teaching Mathematics from an International Perspective

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Abstract This chapter highlights what readers can expect to learn from reading this book, which goes beyond a direct reporting of the TEDS-M results. Four important contributions are summarized first, including the significance of learning from the first-ever large-scale international study on teacher preparation programs and teacher learning outcomes, as well as theoretical contributions and policy implications made possible by contributors of this volume. Three aspects are then discussed as possible extensions of the study to help us move further forward in learning from and improving the preparation and professional development of teachers from an international perspective.

Keywords International study · Mathematics teacher · Teacher beliefs · Teacher knowledge · Teacher preparation

This is a wonderful book, a book that provides timely and important findings that resulted from a recent large-scale international study on mathematics teacher preparation ("Teacher Education and Development Study—Learning to Teach Mathematics", or TEDS-M) under the auspices of the International Association for the Evaluation of Educational Achievement (IEA). This book contains 21 chapters in the follow-up five parts contributed by 33 scholars from four educational systems (i.e., Germany, Singapore, Taiwan, USA). Although many chapters have been published elsewhere through various outlets (e.g., *Journal of Teacher Education* and *ZDM—The International Journal on Mathematics Education*), this volume pieces together different individual and group contributions to share with readers some of the different aspects of what pre-service teachers may learn through program studies in 16 individual educational systems. As a collection, this volume presents the mathematics education community with a systematic, in-depth examination of teachers' knowledge, beliefs and their opportunities to learn through an international perspective.

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S. Blömeke et al. (eds.), *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn*, Advances in Mathematics Education,
DOI 10.1007/978-94-007-6437-8_3,

Given the existence of several large-scale international studies related to school mathematics and science, this book can also be easily overlooked as a collection of chapters about another large-scale international study. Partially, this is due to the fact that the results from several large-scale international studies shared a similar pattern of students from certain education systems consistently outperforming their counterparts in other educational systems. Such high performing education systems are typically those located in East Asia; including Japan, Singapore, South Korea and Taiwan. While the public in the West may get tired of reading about another similar story, what makes this volume unique is the fact that the TEDS-M study focused on pre-service teachers and teacher preparation programs but not students in schools. Although the quality of teachers and their teaching is commonly recognized as key to the improvement of students' achievement in school mathematics (e.g., Leung and Li 2010; NRC 2010; Sowder 2007), it has not been clear to the mathematics education community whether teachers have received the expected quality training to develop the skills and beliefs that are needed for teaching mathematics. This volume fills the knowledge gap in helping us learn what pre-service teachers know and are able to do at the end of their program studies as well as the possible influences of institutional practices and culture on their learning outcomes both within and across educational systems. This volume goes beyond a direct reporting of TEDS-M results to include several important contributions. With the expanded collection of chapters based on the TEDS-M study, I will highlight the following four important contributions that this volume provides.

1. *This volume builds upon the first-ever large-scale international study on teacher preparation programs and teacher learning outcomes.*

Examining and understanding teacher education from an international perspective is not a new idea (e.g., Jaworski et al. 1999; Leung and Li 2010; Li and Lappan 2002; Tisher and Wideen 1990). However, discussions about teacher education programs and practices were often limited by their scopes and the resources available to be devoted to different studies. Previous studies also carried various focuses and used different research methodologies, which likely increased the difficulty of making possible cross-study comparisons and connections. The growing interest in examining teacher education in an international context, as related to the sustained interest in documenting students' mathematics achievement cross-nationally, naturally called for stronger and more systematic international collaborations. In fact, the TEDS-M study was the first-ever large-scale international study on teacher preparation programs and teacher learning outcomes. As reported in various chapters in this volume, readers should find much valuable information about pre-service teachers' knowledge, beliefs and their opportunities to learn across 16 educational systems.

The significance of the TEDS-M study goes beyond documenting pre-service teachers' knowledge and beliefs as related to their program studies. As discussed in my previous commentary article (Li 2012), the TEDS-M study made it possible for researchers to (1) build upon international collaborative efforts in conceptualizing and examining mathematics teachers' knowledge needed for teaching (see chapters in Part II); (2) examine and compare pre-service teachers' beliefs (see chapters in

Part III); and (3) connect teacher preparation policies and program features with pre-service teachers' knowledge and beliefs as learning outcomes (see chapters in Part IV).

2. *This volume makes theoretical contributions through reflecting on teacher knowledge conceptions, the relationship between teacher knowledge and beliefs, and cultural influences on the assessment and development of teacher knowledge and beliefs*.

After reading this book, few would disagree that this is a research volume. It contains chapters that go beyond descriptions of teacher preparation programs to include systematic analyses of empirical data and careful interpretations of research findings. Furthermore, I want to point out that this volume also makes theoretical contributions through reflecting on teacher knowledge conceptions, the relationship between teacher knowledge and beliefs, and cultural influences on the assessment and development of teacher knowledge and beliefs.

In the TEDS-M study, teachers' knowledge needed for teaching mathematics is conceptualized as consisting of at least two essential components: mathematics content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK). Further specifications for MCK follow the assessment framework for mathematics content from the Trends in Mathematics and Science Studies (TIMSS; Mullis et al. 2007). The MPCK conception is developed based on a literature review, findings from a previous study, and critical reviews by international experts in the field. Besides MCK and MPCK, general pedagogical knowledge (GPK) is also discussed and accepted as part of teachers' knowledge needed for teaching (e.g., Shulman 1986, 1987). Assessing pre-service teachers' GPK was provided as an option for participating education systems in the TEDS-M study. Although such conceptualizations of teachers' knowledge needed for teaching has been a topic of many studies (e.g., Even and Ball 2009; Hill et al. 2007; Shulman 1986), it has been explored mainly within education systems, not in an international context. The adoption of this knowledge conception in TEDS-M suggests a general agreement of those knowledge components important to teachers' competence in teaching mathematics, and is one important step in conceptualizing teacher knowledge needed for teaching mathematics in an international context.

At the same time, researchers did not simply take this teacher knowledge conception for granted (see Blömeke et al. 2013a; Döhrmann et al. 2013). Along with the rich empirical data collected in the TEDS-M study, they used multidimensional approaches to modeling teacher knowledge (Blömeke et al. 2013a) and verified that the nature of teacher knowledge, as measured by the knowledge tests in the TEDS-M study as designed, is multidimensional. At the same time, Döhrmann et al. (2013) took a further look at TEDS-M knowledge tests as designed and several specific test items. Their analyses and discussions led to the understanding of the reliability and validity of the tests in general as well as great challenges in separating different knowledge components in the tests and addressing various knowledge components across different education systems. The limitations of the two knowledge components (i.e., MCK and MPCK) as specified in the TEDS-M tests are acknowledged, and the complexity of conceptualizing teacher knowledge needed in teaching mathematics in different educational systems (e.g., both the measurement invariance and culturally-based specificity) suggests mores research needed in the future.

It is commonly acknowledged that both knowledge and beliefs have close relationships with teachers' performance. However, possible relationships between knowledge and beliefs are not well understood. In the TEDS-M study, researchers conceptualize the teacher competency with knowledge and beliefs as two essential components. Through analyzing TEDS-M data on pre-service primary teachers, Schmidt and Burroughs (2013) contributed an interesting chapter that was not available in the ZDM thematic issue published in 2012 (Li 2012). They found that teachers' beliefs and knowledge of mathematics have a close and substantial association. At the same time, the strength of this association, as demonstrated by results obtained from the specific tests in TEDS-M, varies across educational systems, likely influenced by culture and institutional practices.

Conducting such a large-scale international study like the TEDS-M is an advantage, as we can learn important lessons from others. However, it also presents many challenges (Blömeke 2013). Understanding possible cultural influences on the development of teacher knowledge and beliefs is a topic area that lends opportunities for both learning and challenging. In Parts II to V, readers should be able to find multiple chapters related to the issue of cultural influences. These chapters' contributions go beyond simple acknowledgment and advance our understanding of cultural influences on the assessment and development of teacher knowledge and beliefs.

3. *This volume illustrates the potential and ways of exploring possible connections between teacher preparation and student achievement*.

It has been clear that the TEDS-M study was designed to examine primary and lower secondary teachers' competence (knowledge and beliefs) in their last year of a teacher preparation program study across 16 educational systems. The study further intended to explore possible influence of system policies and institutional practices on teachers' performance documented in the tests, however, a focus on teacher preparation itself did not provide enough justification for the motivation behind this large-scale international study. As I pointed out at the beginning, understanding possible connections between teacher education and students' achievement in school mathematics across educational systems should provide a strong incentive for developing and conducting studies on the preparation and professional development of teachers, like the TEDS-M.

Blömeke and Kaiser (2013) exemplified such possibilities as exploring potential connections between teacher preparation and student achievement. By comparing teachers' MCK scores obtained in the TEDS-M study with IEA's Trends in International Mathematics and Science Study's results (TIMSS 2007, see Mullis et al. 2008), they found 'astonishing' similarities in the system-level results between pre-service teachers' MCK in TEDS-M and corresponding grade-level students' mathematics achievement reported in TIMSS 2007. The importance of teachers' MCK for students' high mathematics achievement overall is likely supported, but many other factors are also needed to ensure students' successful learning remain unexamined. Although Blömeke and Kaiser provide a strong cautious note about drawing any quick conclusion from their brief comparisons, the consistence identified with results from multiple education systems suggests the potential of this analysis and ways of exploring possible connections further in the future.

4. *This volume contains policy implications for evaluating teacher preparation and program quality*.

Based on such a large-scale international study and related findings, researchers tend to make possible connections with policy in evaluating teacher preparation and program quality. The same is true for this volume, as it includes many chapters that look at the system-based cases such as Singapore (Wong et al. 2013), Taiwan (Hsieh et al. 2013b), the United States (Schmidt et al. 2013a, 2013b), as well as the evaluation of teacher preparation program quality in general (Hsieh et al. 2013a).

In these chapters on system-based cases, readers should notice that different education systems have developed and used various policies and programs in their teacher preparation practices. For example, Singapore has a highly centralized education system that emphasizes recruitment, training, certification, employment, and retention of teachers with special financial support and specific quality control (Lim-Teo 2010; Wong et al. 2013). Such a system is different from the education system in Taiwan with both department-based and center-based teacher preparation institutions (Hsieh et al. 2013b), and even more different from the United States (Schmidt et al. 2013b). These chapters remind us not only of the complexity of examining possible factors contributing to teacher preparation, but also of the importance of discussing and understanding possible policy implications for teacher preparation in a system context.

Building upon the TEDS-M study, Hsieh et al. (2013a) also proposed a conceptual framework for evaluating the quality of teacher preparation programs in general. Five components are included in their proposed framework: future teacher achievement, instructor effectiveness, teaching coherence between universities and schools, courses/content arrangement, and overall effectiveness of teacher education programs. They further used the TEDS-M data to conduct the initial analysis and discussion of teacher preparation program quality for each participating education system in terms of these components. Indeed, their compilation of these five components should promote further discussion about conceptualizing teacher preparation program quality and its evaluation in the future.

With the TEDS-M as the first large-scale international study on teacher education, this volume is not positioned to provide answers to many questions that we may have. Instead, this volume furthers the on-going international conversation about what we do in teacher education and different questions and issues we face (e.g., Even and Ball 2009; Li and Even 2011). When reading this volume, it is equally important for us to think about and discuss the next steps. Here, I would like to suggest the following three aspects that could help us move further forward in learning from and improving the preparation and professional development of teachers.

1. *Focusing on teacher preparation practice*.

The TEDS-M study collected data through tests and surveys on a large scale. The value and significance of these test results and findings are self-evident through reading this book. However, learning what pre-service teachers know and are able to do in the last year of their program study and their OTL may only be part of the whole picture. How teachers experience their program study is not only the function of the possible OTL being provided, but also how such OTL is managed and implemented. As an example, we know that different instructors in the teacher preparation program can offer and teach the same course quite differently, with different content focuses and pedagogy. This calls for special attention to teacher preparation practice. Different research methods and approaches can possibly be developed for examining and documenting teacher preparation practice. One method is the classroom instruction video study as used in TIMSS (Stigler and Hiebert 1999), which can provide holistic information for further analyses. Often the difficulty in analyzing and documenting teacher educators' practices is well related to the lack of a shared articulation of different pedagogical actions and approaches that are used by different instructors. The identification and specification of different pedagogical actions and approaches can certainly benefit from broad international collaborations and discussions in the future.

Teacher preparation practices can also be characterized in terms of the background and training of teacher educators. Sample questions can include who these instructors are and which course is offered by which department for pre-service teachers. Hsieh et al. (2013b) examined similar factors at the system, major, and degree option levels. As pre-service teachers learn through course taking, further examination and analyses down to the course and instructor levels would be important to have.

2. *Developing and conducting a longitudinal study to examine possible effects of program studies on pre-service teachers' learning outcomes*.

Possible effects of pre-service teachers' prior knowledge on their performance at the end of their program study are well recognized by several researchers in this book (e.g., Blömeke et al. 2013b; Schmidt et al. 2013a). At the same time, it is important to recognize that sampled pre-service teachers' prior knowledge in the TEDS-M study was collected through alternative measures as a proxy. For general prior knowledge, pre-service teachers' perceived high-school achievement as compared with their age cohort was measured as a proxy using a five-point Likert scale (1: "generally below average" through 5: "always at the top"). For domain-specific prior knowledge, the number of mathematics classes taken by pre-service teachers during K-12 schooling was taken as a proxy with a five-point Likert scale (1: "below year 10" through 5: "year 12 (advanced level)"). Although such information about pre-service teachers' prior knowledge can be helpful, its utility can be very restricted, as its validity can be questioned. The challenge of documenting possible learning effects from teacher preparation program studies is thus apparent with the lack of adequate measures of pre-service teachers' prior knowledge in the TEDS-M study. A possible solution is to design and conduct a longitudinal study to examine and document what pre-service teachers may know at the beginning of their program study, what they learn during the program study, and what they know at the end of the program study. Such a longitudinal study would call for long-term commitments from selected education systems with well-coordinated collaborations in the future, and the outcomes and implications could be remarkable.

3. *Identifying and learning about different policies, practices and approaches that prove to be effective in specific education systems.*

Educational research should not be the end by itself, but provide ways and suggestions for educational improvement (Burkhardt and Schoenfeld 2003). Likewise, international collaboration and study can and should go beyond documenting possible differences and similarities in students' and teachers' performance to explore different policies, practices and approaches that led to such performance differences. The TEDS-M study can be a very good starting point for researchers to identify and examine specific policies, practices and approaches that prove to be effective in different education systems. For example, Wong et al. (2013) highlighted the importance of both the recruitment and training provided to pre-service teachers in Singapore. In particular, all applicants for teacher education programs in Singapore are required to go through a sequence of screening steps after obtaining the Singapore-Cambridge General Certificate of Education Ordinary-Level (O-Level) passes in mathematics and English language. The success rate for recruitment is low, with about one acceptance per eight applicants. A selective process for teacher education programs also exists in Taiwan (Lin 2010), where most students enrolled in teacher preparation programs are outstanding academically and the very competitive teacher job market has further elevated expectations for graduates from teacher preparation programs. For a decentralized education system like the US, it is easy to notice dramatic variations in the recruitment/selection and program preparation of teachers. However, Schmidt et al. (2012) analyzed the TEDS-M data, and found that both "recruiting/selecting more mathematically able students" and "providing key mathematics and mathematics pedagogy OTL courses" have important connections with pre-service primary teachers' MCK and MPCK in the US. Such cross-system consistency in emphasizing recruitment/selection could well be examined further for a policy recommendation and implementation.

Recognizing and learning about the different policies and practices used in different education systems help us not only understand possible teacher performance differences across educational systems better, it also provides us with a hint to identifying policies, practices and approaches that prove to be effective in specific education systems. Such identification and learning often requires us to go extra miles after learning the possible differences in teacher knowledge and beliefs across educational systems. The international community could work together to build upon the momentum with the TEDS-M study to identify and learn effective policy and practice, and to make such efforts a focus of collaboration within and across educational systems.

When I wrote a commentary article for the ZDM thematic issue on the TEDS-M study (Li 2012), I highly recommended that thematic issue to readers. After reading this volume, I found myself gaining more insights about pre-service teachers' knowledge and beliefs, as well as their program studies as derived from the TEDS-M study. Readers may find that this chapter builds upon my previous commentary article, after learning more from this volume. Not surprisingly, I continue to highly recommend this volume to readers for the reasons that you already know. I hope your reading of this volume will also promote you to further research and discussion about mathematics teacher preparation and its quality improvement both within and across educational systems.

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Part II
Knowledge at the End of Teacher Education: International Perspectives

Knowledge of Future Primary Teachers for Teaching Mathematics: An International Comparative Study

Sharon L. Senk, Maria Teresa Tatto, Mark Reckase, Glenn Rowley, Ray Peck, and Kiril Bankov

Abstract This article reports the results of the Teacher Education and Development Study in Mathematics (TEDS-M) that are related to prospective primary teachers' knowledge for teaching mathematics. TEDS-M was conducted under the auspices of the International Association for the Evaluation of Educational Achievement with additional support from the US National Science Foundation and the participating countries. In 2008 more than 15,000 future primary teachers, enrolled in about 450 institutions that prepare future primary teachers, were surveyed. Two domains of knowledge for teaching mathematics were assessed using items that had been developed and validated in a cross-national field trial. Large differences in the structure of teacher preparation programs are reported. Differences in mathematical content knowledge (MCK) and mathematical pedagogical content knowledge (MPCK) were also observed both within and between programs and countries. Anchor points on the MCK and MPCK scales are used to describe qualitative characteristics of knowledge for teaching mathematics.

Keywords International · Mathematics · Content knowledge · Pedagogical content knowledge · Primary teacher education · Comparative education

1 Introduction

The aim of this article is to describe the recent research conducted by the Teacher Education and Development Study in Mathematics (TEDS-M) pertaining to the

Originally published in ZDM—The International Journal on Mathematics Education.

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S. Blömeke et al. (eds.), *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn*, Advances in Mathematics Education,
DOI 10.1007/978-94-007-6437-8_4,

knowledge (content and pedagogical) of future primary teachers in 17 countries. Section 2 provides a brief overview of the study, including the assessment framework; Sect. 3 describes the methodology used, including populations and samples; and Sect. 4 describes the instruments. Section 5 presents results and describes anchor points illustrated with some sample items; and Sect. 6 provides a final summary and discussion.

TEDS-M is the first cross-national study of teacher preparation based on nationally representative probability samples. Hence, it provides an international comparative perspective on preparation for teaching mathematics and addresses many of the methodological concerns identified by Blömeke and her colleagues (Blömeke et al. 2008a, 2008b).

Across the world, policy makers are concerned about potential shortfalls in both the number and the quality of teachers being produced. That is, they worry whether enough teachers with the knowledge and skills needed to meet the needs of society in the 21st century can be produced (OECD 2005). Concerns about preparing professionals who can teach mathematics effectively are particularly acute.

The quality of teachers is of universal interest because research has begun to identify ways in which teachers' characteristics are related to how well their students learn. For instance, research in the United States has shown that students' achievement in primary mathematics is influenced by the knowledge, skill, and understanding of their teachers (AERA Research Points 2004; Hill et al. 2005). Research in Germany has shown that the more a teacher of lower secondary mathematics knows about how instructional content can be made accessible to students, the more challenging the students perceive their instruction to be (Baumert et al. 2010).

Scholars have also shown that teachers' knowledge and their teaching practices vary considerably both within and across countries. For example, Ma (1999) uncovered dramatic differences in understanding of mathematics between primary school teachers in China and the United States. An et al. (2004) found differences in approaches by secondary mathematics teachers in China and the United States to developing conceptual understanding. Stigler and Hiebert (1999) argue that teaching practices are a "cultural activity"; in particular, they claim that the ways in which teachers and students interact in Grade 8 classrooms in Germany, Japan, and the United States varies much more between countries than it does within countries.

Researchers and policy makers have recently turned their attention to the initial preparation of teachers (e.g. National Research Council 2010). However, as Blömeke et al. (2008a) point out, much research on mathematics teacher education has been characterized by small specialized samples, often drawn from within the researchers' own institution, and has generally lacked a theoretical basis (p. 719). Even and Ball (2009) called for more cross-cultural exchanges of knowledge and information about the education of teachers of mathematics in the hope of being able to inform research, theory, practice, and policy in mathematics teacher education, both locally and globally. A recent study by Schmidt et al. (2011a) made theoretical and methodological advances in studying mathematics teacher preparation across six countries. But the generalizability of the study is limited because of the use of convenience samples.

2 Overview of TEDS-M

TEDS-M was conducted under the auspices of the International Association for the Evaluation of Educational Achievement (IEA), with additional financial support from the US National Science Foundation (NSF) and the participating countries.[1] TEDS-M is the first IEA study of higher education.[2] It builds on the tradition of cross-national studies of mathematics achievement in primary and secondary school begun by Husen (1967) to gather empirical evidence about mathematics teacher preparation for both primary and lower secondary grades. Seventeen countries participated in TEDS-M: Botswana, Canada[3] (four provinces only), Chile, Chinese Taipei, Georgia, Germany, Malaysia, Norway, Oman[4], the Philippines, Poland, Russia, Singapore, Spain (primary teacher education only), Switzerland (German-speaking cantons only), Thailand, and the USA (public institutions only). All data were collected in 2008.

Although TEDS-M was designed to answer many questions about policies, programs, and practices in mathematics teacher preparation for both primary and lower secondary grades, this article addresses only the following research questions:

1. What are the level and depth of the knowledge for teaching mathematics attained by prospective primary teachers?
2. How does this knowledge vary across countries by program group?

Explaining why future teachers in some programs have achieved better on the knowledge assessment than others is beyond the intention and scope of this paper but reference is made later in the article to some analyses that attempt to do so.

In order to place the responses to research questions 1 and 2 in context, some discussion of the overall design of the study is provided, and background information about characteristics of the sample is reported. The TEDS-M Conceptual Framework (Tatto et al. 2008) is available at http://teds.educ.msu.edu/framework/.

2.1 Frameworks for Knowledge for Teaching Mathematics

It is generally accepted that knowledge for teaching consists of at least two components: content knowledge and pedagogical content knowledge (Shulman 1987;

[1]NSF grant number REC 0514431. The views expressed in this article are those of the authors and do not necessarily reflect the views of the NSF or the IEA.

[2]TEDS-M is directed by a Joint Management Committee consisting of Maria Teresa Tatto (chair), John Schwille and Sharon L. Senk of Michigan State University; Lawrence Ingvarson, Ray Peck, and Glenn Rowley of the Australian Council *for* Educational Research; and *ex-officio* members from the IEA and Statistics Canada.

[3]Canada was unable to satisfy the minimum sample size requirements set by the study and consequently results for Canada do not appear in Sect. 5.

[4]For Oman, only future secondary teachers participated in the TEDS-M study.

National Research Council 2010). In the past several decades, scholars around the world have tried to describe how these two constructs might be identified and assessed in mathematics education, and how they are put into effect by primary and secondary mathematics teachers in instruction (e.g. Ball and Bass 2000; Ball et al. 2001; Even and Ball 2009; Hill et al. 2007; Ma 1999; Krauss et al. 2008a; Pepin 1999; Schmidt et al. 2007).

Ball et al. (2005), using a practice-based approach, observed American primary teachers at work in order to describe mathematical knowledge for teaching. Their definition included planning lessons, evaluating students' work, writing and grading assessments, explaining class work to parents, making and managing homework, etc. Scenario-based multiple-choice items (unlike TEDS-M, the items developed by Ball et al. did not include any constructed response items) were developed primarily in the number and operations domain, but also the domains of patterns, functions, algebra and geometry. The framework defined mathematical content knowledge for teaching as being composed of two key elements: "common" knowledge of mathematics, that any well-educated adult should have; and mathematical knowledge that is "specialized" to the work of teaching and that only teachers need know. One finding from the study with 700 first- and third-grade primary teachers was that teacher knowledge was significantly related to student gain scores even after controlling for other variables such as socioeconomic status.

The COACTIV project on "Professional Competence of Teachers, Cognitively Activating Instruction, and the Development of Students' Mathematical Literacy", directed by Jürgen Baumert, Werner Blum and Michael Neubrand, and funded by the German Research Foundation (DFG) from 2002 to 2006, also assessed teacher knowledge for teaching, but only for secondary teachers already placed in schools. The COACTIV MPCK assessment was presented as 'scenarios' and contained three subscales: knowledge of the multiple solution paths of mathematical tasks (4 items), knowledge of student misconceptions and difficulties (7 items), and knowledge of mathematics-specific instructional strategies (11 items). The COACTIV MCK assessment consisted of 13 items lying between the school-level mathematical knowledge that school students ought to have and the mathematical knowledge that is taught at university that does not overlap with the content of the school curriculum. A significant finding of the COACTIV study relevant to the TEDS-M study was that, 'Because no positive correlation was found between years of teaching practice and the two knowledge categories, teacher training can be assumed to be at the core of the development of the two knowledge categories. Thus, our results support current efforts to improve teacher education by placing a stronger emphasis on subject-based pedagogical content knowledge. Future research may provide deeper insights into the acquisition of PCK and CK during teacher training' (Krauss et al. 2008a, 2008b).

As described in Senk et al. (2008), development of frameworks for assessing knowledge for teaching mathematics in TEDS-M built on earlier research in this area using an iterative process involving researchers across the world. Two constructs were identified for study: Mathematics Content Knowledge (MCK) and

Mathematics Pedagogical Content Knowledge (MPCK). The assessment frameworks for mathematics content from the Trends in Mathematics and Science Studies (TIMSS) (e.g. Mullis et al. 2007; Garden et al. 2006) were used as starting points in the development of the content framework for TEDS-M. Thus, in TEDS-M, MCK consists of four sub-domains: number and operations, algebra and functions, geometry and measurement, and data and chance; and each MCK item is also classified by cognitive sub-domain (knowing, applying or reasoning).

In the TEDS-M framework most of the content to be assessed was designed to be at the level that the future teacher was being prepared to teach, but some content was selected from two or three years beyond that. Given the limited testing time available, it was not possible to assess primary teachers at higher levels although this was considered. Assessing MCK at grade levels beyond the future teachers' intended teaching is justified for several reasons. First, as is evident from Table 2, there is no universal definition of what grades constitute primary school. What is a primary grade in one country may be considered a lower secondary grade in another. In fact, in some countries, teachers for primary and lower secondary grades are prepared in the same programs. Second, curricula vary across the world. What is taught at one grade in one country may be taught in a higher or lower grade in another (e.g. Schmidt et al. 2001; Son and Senk 2010). Third, students' interests and achievement in mathematics vary. Ideally, each teacher should be prepared to challenge even the talented students who may be able to learn content normally taught beyond their current grade.

The framework for Mathematical Pedagogical Content Knowledge (MPCK) in TEDS-M was developed after a review of the literature and informed by the framework used by the Mathematics Teaching in the 21st Century Project (MT21), a study of mathematics teacher preparation for lower secondary grades in six countries that was originally designed to be a precursor to TEDS-M (Blömeke et al. 2008b; Schmidt et al. 2011a). The final version of the MPCK framework was arrived at following a critical review by international experts in the field. The TEDS-M MPCK framework consists of three sub-domains: curricular knowledge, knowledge of planning for teaching, and knowledge of enacting teaching. Because each MPCK item is situated in a classroom context, it can also be classified by mathematics content and curricular level. Table 1 presents examples of the types of activities that characterize each of these three sub-domains.

Several of the MPCK items in the TEDS-M study required respondents to construct their responses. Detailed coding guides were developed for these items and coder training workshops were conducted. Figures 3, 6 and 7 below provide examples of three constructed-response items used in the study. The coding guides for the constructed response items in this article are provided in the Appendix. The complete set of 39 TEDS-M primary released items with coding guides, at the time of publication, is available on the ACER TEDS-M website http://www.acer.edu.au/research/projects/iea-teacher-education-development-study-teds-m/.

Table 1 Mathematics Pedagogical Content Knowledge (MPCK) framework

Sub-domain	Sample Characteristics
Mathematical curricular knowledge	Knowing the school mathematics curriculum
	Establishing appropriate learning goals
	Identifying key ideas in learning programs
	Selecting possible pathways and seeing connections within the curriculum
	Knowing different assessment formats and purposes
Knowledge of planning for mathematics teaching and learning	Selecting appropriate activities
	Predicting typical students' responses, including misconceptions
	Planning appropriate methods for representing mathematical ideas
	Linking didactical methods and instructional designs
	Identifying different approaches for solving mathematical problems
	Choosing assessment formats and items
Enacting mathematics for teaching and learning	Explaining or representing mathematical concepts or procedures
	Generating fruitful questions
	Diagnosing students' responses, including misconceptions
	Analyzing or evaluating students' mathematical solutions or arguments
	Analyzing the content of students' questions
	Responding to unexpected mathematical issues
	Providing appropriate feedback

3 Method

The target population of future primary teachers consists of those persons enrolled in the last year of a teacher preparation program that is explicitly intended to prepare teachers qualified to teach mathematics in any of the grades at primary school. Some teacher preparation programs are consecutive; that is, they consist of a first phase of academic studies that leads to a degree or diploma, followed by a second phase of professional studies in pedagogy and practical experience that leads to a separate degree or credential. Other teacher preparation programs are concurrent; they consist of a single program that includes academic studies in the subject(s) the future teachers may eventually teach, professional studies of pedagogy, and practical experience.

A two-stage sampling plan was designed. First, in each country a national probability sample was selected from the population of teacher preparation institutions that offer teacher preparation programs to the target population. Once an institution was selected, all programs, whether consecutive or concurrent, associated with primary teacher preparation were included in the survey. Then, within these institutions (and programs), samples of future teachers would be selected randomly. All countries participating in TEDS-M were encouraged to provide complete national coverage of their target populations and to meet IEA sampling standards.

After consultation with the National Research Coordinators, a sampling plan was developed for each participating country so that institutions, programs, and future teachers were selected in ways that would eventually yield accurate estimates of key characteristics of the target population in each country.[5] In smaller countries, all teacher preparation institutions were selected to participate in TEDS-M, and in some countries, all eligible future teachers in the sampled institutions were surveyed. In five countries (Botswana, Georgia, Norway, Singapore, and Thailand), censuses of institutions, programs, and future primary teachers were conducted. When complete coverage was not achieved or participation rates did not meet the IEA sampling standards, annotations are made to the relevant tables that appear later in this article.

Results from the surveys of institutions reveal that most teacher preparation programs are concurrent (Tatto et al. 2013). However, all programs preparing primary teachers in Germany are consecutive, as are some in Canada, Malaysia, Singapore, Thailand, and the United States. Primary school teacher preparation programs vary in duration from a two-year concurrent program in Singapore to five and a half year consecutive programs in Germany (that involve three and a half years of university study followed by two years in a second institution operated by one of the state governments). Teacher preparation programs were also found to vary by the intended grade levels and the extent of specialization in mathematics.

In all, 15,163 future teachers drawn from 451 institutions in 16 countries participated in the TEDS-M study of primary teacher preparation. In order to recognize the variation that exists in the structure of primary teacher preparation programs internationally and to make cross-national comparisons fairer, results are reported in this article for four program groups that were determined in consultation with the National Research Coordinators after data were collected:

- Lower Primary Generalists (Grade 4 maximum)
- Primary Generalists (Grade 6 maximum)
- Primary/Lower Secondary Generalists (Grade 10 maximum)
- Primary Mathematics Specialists.

Table 2 presents a summary of key organizational aspects of teacher preparation programs by country and program group. Notice that some countries (e.g. the Russian Federation, Chinese Taipei, and the Philippines) have programs that fall within only one program group; whereas other countries, such as Germany, Poland, and Switzerland, have programs that fall into several program groups.

Table 3 gives information about the age and gender of the sample in each country. The youngest teachers, with a mean age of about 21 years, were in Georgia and the

[5]The software package WinW3S was provided by the IEA and used in each National Research Center to select the samples of future teachers. This software also allows reliable documentation of the whole sampling process. Sampling errors were computed using a well-established re-sampling method, specifically BRR (balanced half-sample repeated replication).

Table 2 Duration and grade span of primary teacher preparation programs by country and program group

Program group	Country	Duration (years)[a]	Grade span	
			Low	High
1. Lower primary generalist (Grade 4 maximum)	Georgia	4	1	4
	Germany	3.5 + 2 consecutive	1	4
	Poland	3 or 5	1	3
	Russian Federation	5	1	4
	Switzerland	3	K	2 or 3
2. Primary generalist (Grade 6 maximum)	Canada	4 concurrent, 4 + 1 or 4 + 2 consecutive	1	6
	Chinese Taipei	4.5	1	6
	Philippines	4	1	6
	Singapore	4 concurrent, 4 + 1 or 4 + 2 consecutive	1	6
	Spain	3	1	6
	Switzerland	3	K, 1 or 3	6
	USA	4	1	3, 4, or 5
		4 + 1 consecutive	1	3, 4, or 5
3. Primary or lower secondary generalist (Grade 10 maximum)	Botswana	3	1	7
	Canada	4 + 1 consecutive	4	10
	Chile	4	1	8
	Norway	4	1	10
4. Primary mathematics specialist	Germany	3.5 + 2 consecutive	1	9 or 10
	Malaysia	3 or 4 concurrent, or 4 + 1 consecutive	1	6
	Poland	3	4	9
		5	4	12
	Singapore	2 or 4 + 1 consecutive	1	6
	Thailand	5 concurrent or 4 + 1 consecutive	1	12
	USA	4 or 4 + 1 consecutive	4 or 5	8 or 9

[a] All programs are concurrent unless otherwise indicated

Philippines; the oldest, with an average age of at least 27 years, were in Germany, Norway (ALU+), and Singapore (mathematics specialists).

In all countries the majority of future primary teachers were female. Botswana had the largest percentage of males among its future primary teachers. Readers are

Table 3 Future primary teachers' age and gender at time of the survey by program group and country

Program group	Country	N^{f}	Age (years)			% Female	% Male
			Mean	SE	% Missing		
1. Lower primary generalist (Grade 4 maximum)	Georgia	502	21.3	0.1	0.8	100	0
	Germany	935	27.5	0.2	0.7	93	7
	Poland[a]	1,811	25.6	0.2	0.0	98	2
	Russian Fed.[b]	2,232	24.2	0.5	1.2	94	6
	Switzerland	121	23.4	0.3	0.0	96	4
2. Primary generalist (Grade 6 maximum)	Chinese Taipei	921	23.2	0.1	0.3	72	28
	Philippines	591	20.9	0.2	0.3	81	19
	Singapore	263	26.3	0.3	0.0	76	24
	Spain	1,093	23.6	0.4	0.0	81	19
	Switzerland	815	23.7	0.1	0.2	83	17
	USA[c]	1,309	25.4	0.4	0.1	90	10
3. Primary or lower secondary generalist (Grade 10 maximum)	Botswana	86	26.0	0.7	0.0	59	41
	Chile[d]	636	23.6	0.1	3.3	85	15
	Norway (ALU)[e]	389	24.2	0.3	0.6	76	24
	Norway (ALU+)[e]	159	28.8	0.6	0.0	68	32
4. Primary mathematics specialist	Germany	97	27.1	0.4	1.1	82	18
	Malaysia	568	25.9	0.1	1.4	63	37
	Poland[a]	299	23.5	0.1	0.3	78	22
	Singapore	116	27.4	0.4	0.8	70	30
	Thailand	659	22.3	0.0	0.2	75	25
	USA[c]	190	25.7	0.8	0.4	82	18

[f]The obtained sample was 13,871

Notes: Table 3 must be read with awareness of the limitations annotated in Figs. 4 and 8 in Sect. 5. These annotations refer to the data footnoted in Table 3. The shaded areas identify data that, for reasons explained in these annotations, cannot be compared with confidence to data from other countries

referred to the forthcoming Technical Report (Tatto 2013, in press) for greater detail on methodology.

4 Instruments

All TEDS-M items had undergone pilot testing and several rounds of review by international expert panels and the National Research Centers; the better items were tested in a field trial in both English and non-English speaking countries. The MCK and MPCK items field-tested included not only items that had been written especially for TEDS-M and managed by the ACER team, but also items developed for other studies of mathematics for teaching, including items developed by Hill and Ball (2004) and the MT21 project (Schmidt et al. 2011a). Following the field trial, other expert panels of mathematicians, mathematics educators, psychometricians, and statisticians helped select the items for the instruments to be used in the main study, and comments and approval were solicited from researchers in participating countries. Instruments were translated into the languages of the participating countries. Translation verification was managed by the IEA Secretariat.

The field trial indicated that, on average, respondents were able to answer about 30 knowledge items in about 60 minutes. Feedback on the field trial also indicated that no more than 90 minutes of testing time would be available for future teachers to answer any questions. To ensure adequate coverage of both MCK and MPCK in limited testing time, a rotated block (matrix sampling) design (Mazzeo et al. 2006) was used. Five blocks of items were assembled, each with 12–15 questions, several with multiple parts. Each primary future teacher received a booklet with two of the blocks of items about knowledge for teaching mathematics. In addition, each form of the Future Teacher Questionnaire contained questions about the background of respondents, opportunities to learn, knowledge for teaching mathematics, and beliefs about mathematics and teaching, to be completed in a 90-minute period.

In all, 70 questions (130 items) about knowledge for teaching mathematics were administered with approximately two-thirds assessing MCK and one-third assessing MPCK. Some of these 70 questions were complex multiple choice with multiple parts. Some constructed response items allowed partial credit, permitting 2 score points (full credit) or 1 score point (partial credit). In total there were 130 knowledge items in the primary pool. On average, because of the rotated block design, each future teacher answered 40 % of the item pool, or about 50 items, and each item was responded to by at least 5000 future teachers. To enable valid comparisons across countries, only items with good psychometric properties across the range of participating countries were retained, so that, in the scales finally used there was little item-country interaction. In the end, the two primary knowledge scales were built from 74 MCK items and 32 MPCK items. The balance for MCK was: number (34 %), algebra (29 %), geometry (29 %) and data (8 %). For MPCK, the balance was curriculum and planning (50 %), and enacting (50 %). Three item formats were used: multiple choice (MC), complex multiple choice (CMC), and constructed response (CR). Sample items are included later in this article.

For the scaling of knowledge of mathematics content and mathematics pedagogical content, calibration was carried out using item response models from the Rasch family. For the dichotomous items, the standard Rasch model (Rasch 1980) was used and for polychotomous items the partial credit model (Masters 1982) was used to fit the matrix of item scores. Both item types were analyzed simultaneously using the ACER Conquest software (Wu et al. 2007), with weights applied so that each participating country contributed equally to the calibration.

Standardization was carried out using the data from the calibration. The estimates (in logits) were standardized to a mean of 500 and a standard deviation of

100 following the TIMSS procedure in which all countries were weighted so that they contributed equally to the standardization sample. This process was repeated for each of the four key measures: MCK (Primary), MCK (Secondary), MPCK (Primary) and MPCK (Secondary).

Once standardization was completed, scores were computed for all participants for whom MCK and MPCK estimates could be obtained, including those not included in the final sample. The mean of 500 and standard deviation of 100 apply therefore to the calibration sample rather than to the complete set of scores.

The separation reliability indices for these scales are 0.73 (MCK) and 0.64 (MPCK). For each of the MCK and MPCK scales the international mean is 500 and the standard deviation is 100. Although the TEDS-M research team originally hoped to be able to report scores for various sub-domains, the number of items used in the final instruments was not sufficient to permit reliable measures of sub-scores.

5 Results

In order to help readers interpret the knowledge scales, TEDS-M researchers identified key points on the scales called *anchor points*. The anchor points do not represent *a priori* judgments of what are good or bad scores. Rather, they are descriptions of the performance of future teachers with scores at specific points on the scale. The number of anchor points possible depended on the number and distribution of items available to develop the descriptions of performance. There were sufficient MCK items to reliably identify two anchor points for the MCK scale, but only sufficient MPCK items to reliably identify one anchor point for the MPCK scale. On the MCK scale, anchor point 1 represents a lower level of knowledge; whilst anchor point 2 represents a higher level.

Descriptions of performance at the anchor points depend on items identified by a specific probability that a person with that score will answer the item correctly. That is, future teachers with scores at an anchor point were able to provide correct answers to some of the items in the survey with a probability of 0.70 or greater. The contents of these items were used to develop descriptions of what future teachers at (or above) that point were *likely to achieve*. The set of items that future teachers with scores at the anchor point were likely to answer correctly with a probability of less than 0.50 were considered *difficult to achieve*. An expert panel of mathematicians and mathematics educators analyzed the items in each of these sets for an anchor point and formulated descriptions of the knowledge of future teachers at each point. This process was followed separately for each anchor point.

5.1 Mathematics Content Knowledge

Anchor point 1 represents a lower level of MCK and corresponds to a scaled score of 431; anchor point 2 represents a higher level of knowledge corresponding to a score of 516.

Primary MCK Anchor Point 1 Future teachers scoring at anchor point 1 on the Primary MCK scale were likely to correctly answer items involving basic computations with whole numbers, identifying properties of operations with whole numbers, and reasoning about odd or even numbers. They were generally able to solve straightforward problems using simple fractions. Future teachers at this anchor point were likely to achieve success at visualizing and interpreting standard 2-dimensional and 3-dimensional geometric figures, and solving routine problems about perimeter. They could generally understand straightforward uses of variables and equivalence of expressions, and solve problems involving simple equations.

Future teachers at anchor point 1 tended to over-generalize and had difficulty solving abstract problems and those requiring multiple steps. They had limited knowledge of proportionality, multiplicative reasoning, and least common multiples, and had difficulty solving problems that involved coordinates and problems about relations between geometric figures. Future teachers at anchor point 1 were likely to have difficulty reasoning about multiple statements and relationships among several mathematical concepts, such as understanding that there are an infinite number of decimal numbers between two given numbers, finding the area of a triangle drawn on a grid, and identifying an algebraic representation of three consecutive even numbers.

Primary MCK Anchor Point 2 Future teachers who scored at anchor point 2 could solve the mathematics tasks that could be done by future teachers at anchor point 1. In addition, they were more successful than future teachers at anchor point 1 at using fractions to solve story problems, and at recognizing examples of rational and irrational numbers. They were likely to know how to find the least common multiple of two numbers in a familiar context, and to recognize that some arguments about whole numbers are logically weak. They were generally able to determine areas and perimeters of simple figures, and had some notion of class inclusion among polygons. Future teachers at anchor point 2 also had some familiarity with linear expressions and functions.

However, even though future primary teachers at anchor point 2 could solve some problems involving proportional reasoning, they often had trouble reasoning about factors, multiples, and percentages. Applications of quadratic or exponential functions were challenging, and they had limited success applying algebra to geometric situations; e.g., writing an expression for the reflection image of the point with coordinates (a, b) over the x-axis, identifying a set of geometric statements that uniquely define a square, or describing properties of the function defined by the ratio of the area and circumference of a circle.

Overall, future teachers at anchor point 2 tended to do well on items testing knowing, and on standard problems about numbers, geometry, and algebra classified as applying, but they had more difficulty answering problems that require more complex reasoning in applied or non-routine situations.

For instance, in Fig. 1 the items in parts A, B, and C assess whether the commutative and associative properties are true for the operations of addition, subtraction or division of whole numbers. They illustrate items on which future teachers with

Indicate whether each of the following statements is true for the set of all whole numbers *a*, *b* and *c* greater than zero.

Check one box in each row.

		True	Not True
A.	$a - b = b - a$	☐	☐
B.	$a \div b = b \div a$	☐	☐
C.	$(a + b) + c = a + (b + c)$	☐	☐
D.	$(a - b) - c = a - (b - c)$	☐	☐

Fig. 1 Complex multiple choice MCK items MFC202A-D about number and operations. International average: MFC202 *A* (81 %), *B* (86 %), *C* (92 %), *D* (64 %)

scores at anchor point 1 or above had high probabilities of achieving success. In contrast, part D presents an incorrect generalization of the associative property.

Although the international average for this item is 64 % correct, future teachers with scores at anchor point 1 found this item difficult to achieve; i.e., they had a less than 50 % chance of responding correctly. However, future primary teachers with scores at or above anchor point 2 had higher probabilities of selecting the correct answer.

The TEDS-M Item Almanacs produced by the IEA Data Processing and Research Centre in Hamburg indicate the difficulty of each item for each participating country. Because of space considerations, only one example (item MFC202A) is provided in this article as shown in Table 4.

Even though item MFC202A was one of the easiest items in the pool, the percentage of future primary teachers correctly responding to the item ranged from 41 % (Georgia) to 98 % (Singapore).

Figure 2 shows a geometry item that requires respondents to calculate the area of a triangle in which neither the base nor the height is indicated. Future primary teachers with scores at or above anchor point 2 on the MCK scale were likely to respond correctly to this item, but future teachers scoring at anchor point 1 were not.

Figure 3 asks a non-routine algebra question about two expressions, in which the underlying mathematics involves the solution of an inequality. Only about a third of the international sample of future primary teachers earned any credit on this item. Even future teachers with scores at anchor point 2 had less than a 50 % chance of responding correctly to this MCK item, either partially or completely.

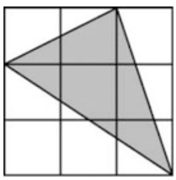
Figure 4 gives descriptive statistics for MCK for future primary teachers by program group. It also contains plots showing the achievement of future teachers in each of the program groups that took the primary survey. Anchor point 1 (431) and anchor point 2 (516) are marked by vertical lines on the display.

Table 4 Percent correct by country for item MFC202A from the Item Almanac

Country	*N*	Valid *N*	Percent correct	1	2[a]	Omitted	Not reached
Botswana	86	34	91.6	5.5	91.6	3.0	0.0
Chile	657	252	52.4	38.4	52.4	8.9	0.4
Chinese Taipei	923	369	92.2	4.6	92.2	3.1	0.0
Georgia	506	208	41.1	36.5	41.1	22.4	0.0
Germany	1032	408	87.5	7.3	87.5	5.0	0.3
Malaysia	576	219	82.1	15.6	82.1	2.3	0.0
Philippines	592	234	67.1	30.4	67.1	2.4	0.0
Poland	2112	863	83.6	13.4	83.6	2.1	0.8
Russian Federation	2266	901	86.8	10.5	86.8	2.7	0.1
Singapore	380	151	98.0	2.0	98.0	0.0	0.0
Spain	1093	440	81.4	16.5	81.4	2.0	0.2
Switzerland	936	383	93.9	4.3	93.9	1.8	0.0
Thailand	660	267	93.3	4.5	93.3	2.2	0.0
United States of America	1501	425	88.9	10.7	88.9	0.4	0.0
International average	**951**	**368**	**81.4**	**14.3**	**81.4**	**4.2**	**0.1**
Norway (ALU)	392	165	86.1	10.7	86.1	3.2	0.0
Norway (ALU+)	159	58	87.5	6.0	87.5	6.5	0.0

[a]Key

The area of each small square is 1 cm^2.



What is the area of the shaded triangle in cm^2?

Check one box.

A. 3.5 cm^2 ☐

B. 4 cm^2 ☐

C. 4.5 cm^2 ☐

D. 5 cm^2 ☐

Fig. 2 Multiple choice MCK item MFC408 about geometry and measurement. International average: 60 %

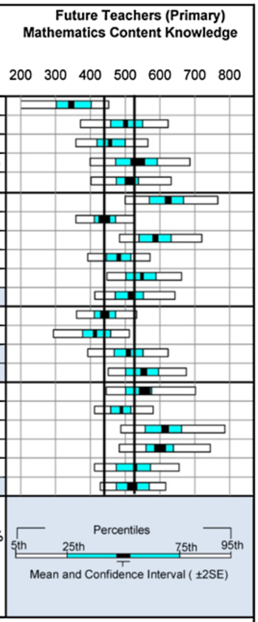
Students who had been studying algebra were asked the following question:

For any number n, which is larger, $2n$ or $n + 2$?

Give the answer and show your reasoning or working.

Fig. 3 Constructed response MCK item MFC509 about algebra and functions. International average: full credit (12 %), partial credit (21 %)

Program Group	Country	Sample Size	Valid Data (N)	Percent Missing (Weight'd)	Percent at or above Anchor Point 1 (SE)	Percent at or above Anchor Point 2 (SE)	Scale Score Mean (SE)
1. Lower Primary (Grade 4 Maximum)	Georgia	506	506	0.0	11.9 (1.4)	0.9 (0.5)	345 (4)
	Germany	935	907	2.4	86.4 (1.3)	43.9 (2.1)	501 (3)
	Poland [a]	1812	1799	0.9	67.9 (1.3)	16.8 (1.2)	456 (2)
	Russian Federation [b]	2266	2260	0.2	89.7 (2.3)	57.3 (4.6)	536 (10)
	Switzerland [c]	121	121	0.0	90.5 (2.7)	44.2 (5.4)	512 (6)
2. Primary (Grade 6 Maximum)	Chinese Taipei	923	923	0.0	99.4 (0.3)	93.2 (1.4)	623 (4)
	Philippines	592	592	0.0	60.7 (5.1)	6.3 (0.9)	440 (8)
	Singapore	263	262	0.4	100.0	82.5 (2.3)	586 (4)
	Spain	1093	1093	0.0	83.4 (1.6)	26.2 (1.6)	481 (3)
	Switzerland	815	815	0.0	97.2 (0.6)	70.6 (1.4)	548 (2)
	† USA [d]	1310	951	28.6	92.9 (1.2)	50.0 (3.2)	518 (5)
3. Primary/ Secondary (Grade 10 Maximum)	Botswana [e]	86	86	0.0	60.6 (5.3)	7.1 (2.8)	441 (6)
	Chile [f]	657	654	0.4	39.5 (1.8)	4.0 (0.7)	413 (2)
	Norway (ALU) [g]	392	392	0.0	88.5 (1.5)	46.9 (2.3)	509 (3)
	Norway (ALU+) [g]	159	159	0.0	96.5 (1.4)	68.7 (3.1)	553 (4)
4. Primary Mathematics Specialists	Germany	97	97	0.0	96.0 (2.1)	71.7 (7.0)	555 (8)
	Malaysia	576	574	0.4	88.7 (1.1)	28.1 (1.3)	488 (2)
	Poland [a]	300	300	0.0	97.9 (1.0)	91.0 (1.6)	614 (5)
	Singapore	117	117	0.0	98.3 (1.2)	87.3 (2.8)	600 (8)
	Thailand	660	660	0.0	91.7 (0.9)	56.2 (1.4)	528 (2)
	† USA [d]	191	132	33.2	94.9 (1.7)	48.1 (6.5)	520 (7)



Notes

1. This table and chart must be read with awareness of the limitations annotated below.
2. The symbol (†) is used to alert readers to situations where data were available from less than 85% of respondents.
3. The shaded areas identify data that, for reasons explained in the annotations, can be compared with data from other countries only with great caution.
4. The solid vertical lines on the chart show the two anchor points (431 and 516).

Country Annotations

a. **Poland:** Reduced coverage: institutions with consecutive programs only were not covered. Combined participation rate between 60 and 75%.
b. **Russian Federation:** Reduced coverage: secondary pedagogical institutions were excluded.
c. **Switzerland:** Reduced coverage: the population covered includes only institutions where German is the primary language of use anc instruction.
d. **USA:** Reduced coverage: public institutions only. Combined participation rate between 60% and 75%. An exception was made to accept data from two institutions because, in each case, one additional participant would have brought the response rate above the 50% threshold. Although the participation rate for the complete sample meets the required standard, the data contain records that were completed using a telephone interview, when circumstances did not allow administration of the full questionnaire. Of the 1501 recorded as participants, the full questionnaire was administered to 1185. Bias may arise in the data because of the number of individuals who were not administered the full questionnaire.
e. **Botswana:** The sample size is small (N=86), but arises from a census of a small population.
f. **Chile:** Combined participation rate between 60% and 75%.
g. **Norway:** Combined participation rate between 60% and 75%. An exception was made to accept data from one institution because one additional participant would have brought the response rate above the 50% threshold. Program types ALU and ALU+ are reported separately because the two populations partly overlap; these program types cannot be aggregated.

Fig. 4 Descriptive statistics for Mathematics Content Knowledge, by program group

Across all program groups and within each participating country, future teachers' scores on the Mathematics Content Knowledge items varied widely. Differences between the maximum mean score and the minimum mean score in each program group ranged between 100 and 200 score points; that is, between one and two standard deviations of the full population. The distributions in the four parts of Fig. 4 also overlap considerably. That is, even in the lower scoring countries, there are some future teachers who outperformed some future teachers in the higher scoring countries.

Among the five countries with programs that prepare teachers for lower primary grades, future teachers in the Russian Federation and Switzerland earned the highest mean scores, but the Russian Federation was the only country in which more than half the sample achieved scores at or above anchor point 2.

Among the six countries that prepare primary generalists to teach through Grade 6, future teachers in Chinese Taipei earned the highest mean and more than 90 % of the future teachers scored at or above anchor point 2. In fact, the mean MCK score of the generalist teachers in Chinese Taipei was higher than the mean MCK scores of specialist mathematics teachers in other countries. Performance was also strong in this group for future teachers in Singapore and Switzerland.

Respondents in Botswana and in Chile in programs preparing future teachers for both primary and lower secondary grades generally found the MCK items difficult. Performance in the two Norwegian general teaching programs (ALU and ALU+) was higher, with future teachers in the smaller ALU+ programs achieving somewhat higher MCK scores than those in the ALU programs—a result to be expected, given that students in the ALU+ programs undertake additional studies in Mathematics compared to students in the ALU programs.

Future teachers in programs for primary mathematics specialists generally performed well compared to the international sample, with all but one country achieving a mean score greater than 500. Future teachers from Poland and Singapore achieved the highest mean MCK scores, and almost all future teachers in both samples scored at or above anchor point 2. Even in Malaysia, the lowest scoring country in this group, almost 30 % reached anchor point 2.

5.2 Mathematics Pedagogical Content Knowledge

Because of the relatively small number of items measuring Mathematics Pedagogical Content Knowledge, only one MPCK anchor point was able to be reliably defined at the primary level. It represents a score of 544 on the MPCK scale.

Primary MPCK Anchor Point Future primary teachers who scored at this anchor point were generally able to recognize the correctness of a teaching strategy for a particular concrete example, and to evaluate students' work when the content was conventional or typical of primary grades. They were likely to identify the arithmetic elements of single-step story problems that influence their difficulty.

A <Grade 1> teacher asks her students to solve the following four story problems, in any way they like, including using materials if they wish.

Problem 1: [Jose] has 3 packets of stickers. There are 6 stickers in each pack. How many stickers does [Jose] have altogether?

Problem 2: [Jorgen] had 5 fish in his tank. He was given 7 more for his birthday. How many fish did he have then?

Problem 3: [John] had some toy cars. He lost 7 toy cars. Now he has 4 cars left. How many toy cars did [John] have before he lost any?

Problem 4: [Marcy] had 13 balloons. 5 balloons popped. How many balloons did she have left?

The teacher notices that two of the problems are more difficult for her children than the other two.

Identify the **TWO** problems which are likely to be more **DIFFICULT** to solve for <Grade 1> children.

Problem _______ and Problem _______

Fig. 5 Constructed response MPCK item MFC505 about planning a number task. International average: full credit (77 %), partial credit (20 %)

Although future primary teachers at the primary MPCK anchor point were likely to be able to interpret some students' work, their responses were often unclear or imprecise. In addition, future teachers at the anchor point were unlikely to use concrete representations to support students' learning or to recognize how a student's thinking is related to a particular algebraic representation. They generally were unlikely to understand some measurement or probability concepts needed to reword or design a task. These future teachers also were unlikely to know why a particular teaching strategy made sense, if it would always work, or whether a strategy could be generalized to a larger class of problems. They were unlikely to be aware of common misconceptions or to conceive useful representations of numerical concepts.

Figure 5 shows a primary level constructed response item (MFC505) tapping pedagogical content knowledge about curriculum and planning. For this item, future teachers were required to compare four verbal problems, each of which can be

[Jeremy] notices that when he enters 0.2 × 6 into a calculator his answer is smaller than 6, and when he enters 6 ÷ 0.2 he gets a number greater than 6. He is puzzled by this, and asks his teacher for a new calculator!

(a) What is [Jeremy's] most likely misconception?

(b) Draw a visual representation that the teacher could use to model 0.2 × 6 to help [Jeremy] understand **WHY** the answer is what it is?

Fig. 6 Constructed response item MFC208A-B about enacting a number task. International average: MFC208A full credit (20 %), partial credit (12 %); MFC208B full credit (16 %), partial credit (16 %)

solved using a single arithmetic operation with whole numbers. Future primary teachers with scores at or above the MPCK anchor point had at least a 70 % chance of correctly responding to this item. Virtually all the international sample recognized one or both of the more difficult problems—namely Problem 1, which requires multiplication or repeated addition, or Problem 3, a 'separate/start unknown' problem (Carpenter et al. 1999).

However, future teachers at or below the MPCK anchor point found the items about enacting mathematics teaching (MFC208) shown in Fig. 6 difficult to achieve. They had less than a 50 % chance of identifying the common misconception about multiplication in part (a), i.e. 'that multiplication makes things bigger' or, more formally, that the product results in a larger number than either factor. Nor were future teachers at the MPCK anchor point able to draw a representation to help children dispel this misconception in part (b) of this item. The coding guides for these two items are reproduced in the Appendix.

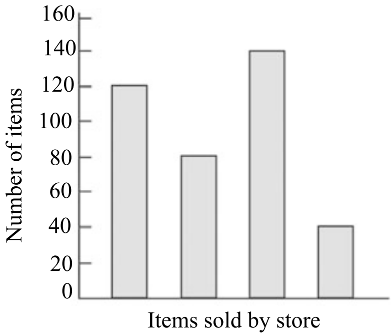
Figure 7 shows an item about data in which part (a) (a 2003 Grade 8 TIMSS item with 67.6 % international facility) tests MCK and part (b) tests MPCK. Future teachers tended to get part (a) correct (85 % facility), but part (b) incorrect (23 % facility for full credit and 51 % for partial credit). The coding guide for part (b) is shown in the Appendix.

Again, within each country, for MPCK there are large differences between the lowest and highest scoring future teachers in each program group as shown by Fig. 8.

Also, in each program group, the range of mean scores by country within program groups is from about 100 points (among the primary mathematics specialists) to 150 points (among primary generalists). Thus, as was the case for MCK, even after controlling for intended grade level and degree of specialization in mathematics, there are often large differences in MPCK among future primary teachers in different program groups in different countries.

The following problem was given to children in <primary> school.

The graph shows the number of pens, pencils, rulers and erasers sold by a store in one week.



The names of the items are missing from the graph. Pens were the item most often sold. Fewer erasers than any other item were sold. More pencils than rulers were sold.

Check one box.

(a) How many pencils were sold?

A. 40 ☐$_1$

B. 80 ☐$_2$

C. 120 ☐$_3$

D. 140 ☐$_4$

(b) Some <primary> students would experience difficulty with a problem of this type. What is the main difficulty you would expect? Explain clearly with reference to the problem.

Fig. 7 Items MFC 502A, a multiple choice MCK item about interpreting a data representation, and MFC502B, a constructed response MPCK item about planning to enact a data task. International average: MFC502A (85 %), MFC502B full credit (23 %), partial credit (51 %)

6 Summary and Discussion

TEDS-M makes several important contributions to research in mathematics education. First, it documents large structural variations in how teachers are prepared to teach mathematics across the world. Second, it provides the first cross-national evidence based on national probability samples of how future primary teachers' knowledge for teaching mathematics varies between and within countries. Third, anchor points on the mathematics content knowledge and mathematics pedagogical content knowledge scales give qualitative descriptions of what future teachers with scores at those points know and can do.

Program Group	Country	Sample Size	Valid Data (N)	Percent Missing (Weight'd)	Percent at or above Anchor Point (SE)	Scale Score Mean (SE)
1. Lower Primary (Grade 4 Maximum)	Georgia	506	506	0.0	0.6 (0.2)	345 (5)
	Germany	935	907	2.4	25.9 (2.0)	491 (5)
	Poland [a]	1812	1799	0.9	11.9 (1.3)	452 (2)
	Russian Federation [b]	2266	2260	0.2	31.6 (4.1)	512 (8)
	Switzerland [c]	121	121	0.0	31.6 (4.2)	519 (6)
2. Primary (Grade 6 Maximum)	Chinese Taipei	923	923	0.0	77.0 (1.3)	592 (2)
	Philippines	592	592	0.0	5.9 (1.6)	457 (10)
	Singapore	263	262	0.4	74.9 (2.5)	588 (4)
	Spain	1093	1093	0.0	17.5 (1.3)	492 (2)
	Switzerland	815	815	0.0	44.0 (1.5)	539 (2)
	† USA [d]	1310	951	28.6	47.6 (1.7)	544 (3)
3. Primary/ Secondary (Grade 10 Maximum)	Botswana [e]	86	86	0.0	6.2 (2.8)	448 (9)
	Chile [f]	657	654	0.4	4.9 (1.0)	425 (4)
	Norway (ALU) [g]	392	392	0.0	42.2 (2.9)	539 (3)
	Norway (ALU+) [g]	159	159	0.0	58.7 (3.8)	564 (6)
4. Primary Mathematics Specialists	Germany	97	97	0.0	59.6 (3.4)	552 (7)
	Malaysia	576	574	0.4	23.4 (1.9)	503 (3)
	Poland [a]	300	300	0.0	67.3 (2.3)	575 (4)
	Singapore	117	117	0.0	81.1 (3.9)	604 (7)
	Thailand	660	660	0.0	26.4 (1.5)	506 (2)
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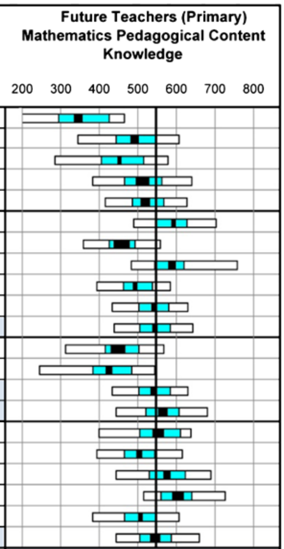

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3. The shaded areas identify data that, for reasons explained in the annotations, can be compared with data from other countries only with great caution.
4. The solid vertical line on the chart shows the single anchor point (544).

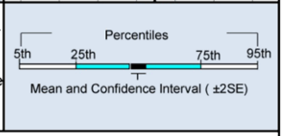

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Fig. 8 Descriptive statistics for Mathematics Pedagogical Content Knowledge by program group

To honor differences in the extent to which programs are designed to prepare future teachers for teaching children of different ages or to become mathematics specialists rather than generalist teachers, TEDS-M grouped teacher preparation programs into four groups. Thus, unlike other IEA studies, TEDS-M does not rank countries from highest to lowest in relation to MCK or MPCK scores.

Not surprisingly, TEDS-M research indicates that knowledge for teaching mathematics varies considerably among individuals within a given country, and that primary teachers prepared to be mathematics specialists tend to score higher on measures of MCK or MPCK than those prepared to be generalists. However, differences in mean scores of the highest and lowest achieving countries within each program group are also striking, with differences in mean MCK scores ranging from about 100 to 200 score points, or one to two standard deviations of the population. Differences in MPCK are somewhat smaller, ranging from about 100 to 150 score points.

The relative performance of countries that have more than one program type in relation to other countries is not fixed. For example, the mean MCK score of future lower primary teachers in program group 1, grade 4 maximum, in Poland is below the international mean and fourth among the five countries in that program group, but the mean MCK score of future primary mathematics specialists in program group 4 in Poland is more than 100 points above the international mean and the highest among all countries in that program group.

It is natural to ask what accounts for differences in knowledge even within a single country. The answer to this question requires additional analyses, and is beyond the scope of this paper. However, in a recent analysis, Blömeke et al. (2011b) found that gender and opportunity to learn were significant explanatory variables for the knowledge scores of future primary teachers in the TEDS-M study. In a second analysis, Blömeke et al. (2011a) also investigated the effects of program and language on the MCK and MPCK scores of the future primary teachers in the TEDS-M study. See also other recent analyses by Hsieh et al. (2011), Schmidt et al. (2011b), Tatto et al. (2013) and Blömeke et al. (2011a). Other factors contributing to cross-national differences in knowledge may be differences in selection standards used to admit applicants to various programs and differences in policies about teacher certification or quality assurance (Tatto et al. 2013).

For each participating country and institution, the results of TEDS-M serve as a baseline for further investigation. For example, content experts may look at the descriptions of the anchor points for MCK and MPCK and the percent of the future teachers graduating from their program or country who reach each anchor point and study how changes in curriculum may lead to improved performance. Policy makers may want to investigate policies that can be implemented to encourage more talented secondary school graduates to select teaching as a career or how teacher preparation programs of the same duration can lead to higher levels of MCK and MPCK.

Acknowledgements The authors of this article gratefully acknowledge Michael Rodriguez, University of Minnesota; Jean Dumais, Statistics Canada; Falk Brese, Ralph Carstens, Barbara Malak, Sabine Meinck, and Hans Wagemaker of the IEA; and our colleagues from the National Research Centers in the 17 participating countries.

Appendix

This appendix shows the coding guides for the five constructed response sample items reproduced in this article. The items can be identified by their item IDs.

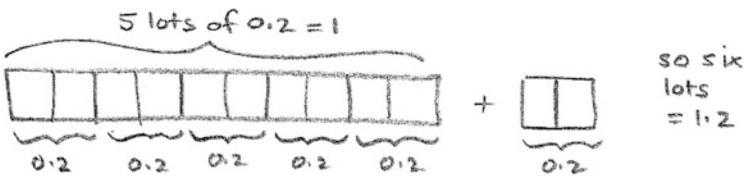
Code	Response	Item: MFC208A
	Correct response	
20	Responses that suggest the misconception is that multiplication always gives a larger answer **and** that division always gives a smaller answer *Example*: • *He thinks that when you multiply the answer should be larger and when you divide the answer should be smaller*	
	Partially correct response	
10	Responses that suggest the misconception is that multiplication always gives a larger answer **or** that division always gives a smaller answer **but not both** *Examples*: • *He thinks that when you multiply the answer should be larger than either/both numbers* • *He thinks that division should give an answer that is smaller than the numbers you started with*	
11	Responses that suggest that Jeremy considers 0.2 as a whole number *Example*: • *He thinks he is multiplying and dividing by 2 rather than by 0.2*	
	Incorrect response	
70	Responses relating to understanding of decimal numbers, decimal multiplication/division or use of a calculator *Example*: • *He doesn't understand decimal multiplication* (*or division*) • *He doesn't know how to use his calculator* • *Mathematical operations* • *The decimal point*	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task)	
	Non-response	
99	Blank	

Code	Response	Item: MFC208B
	Correct response	
20	A suitable visual representation that clearly shows why 0.2×6 is 1.2 *Example*: • *6 lots of 0.2 making it clear that 5 lots of* $0.2 = 1$, *probably with some annotation. See Pictures 1, 2, 3 and 4 below*	
	Partially correct response	
10	A visual representation that shows 6 lots of 0.2 but does NOT make it clear how this equals 1.2. Accept 0.2 shown as one-fifth or as two-tenths *Example*: *See Picture 5 below*	

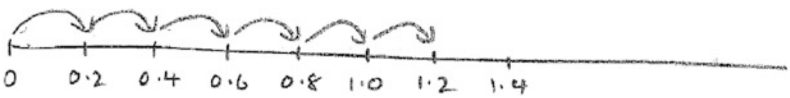
Code	Response	Item: MFC208B
11	A visual representation that shows how 5 lots of 0.2 make a whole but does NOT make it clear how 6 lots of 0.2 equals 1.2 *Example*: *See Picture 6 below*	
12	A visual representation of an equation $0.2 \times 6 = 1.2$ without showing why it is true *Example*: *See Picture 7 below* • $0.2 + 0.2 + 0.2 + 0.2 + 0.2 + 0.2 = 1.2$	
	Incorrect response	
70	A visual representation showing 6 lots of 0.2 without showing what 0.2 is or how 5 lots of 0.2 equals 1 *Example*: *See Picture 8 below*	
71	An example in words suggesting counting in lots of 0.2 *Example*: • "*Count 6 lot's of 0.2 as follows: 0.2, 0.4, 0.6, 0.8, 1.0, 1.2*" Note: This is a good teaching strategy but is not a visual representation	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task) *Example*: *An equation or written calculation of the form* $0.2 \times 6 = 1.2$	
	Non-response	
99	Blank	

Correct Responses (Code 20)

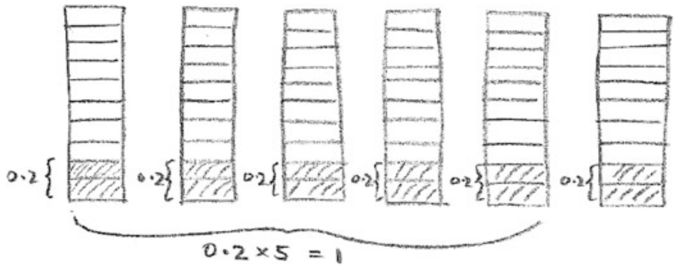
Picture 1



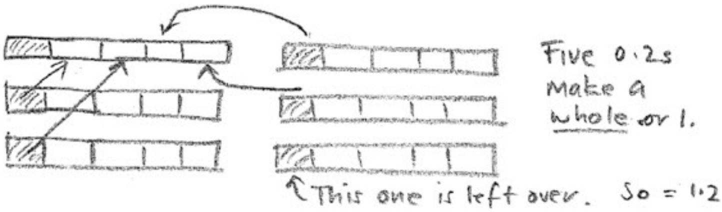
Picture 2



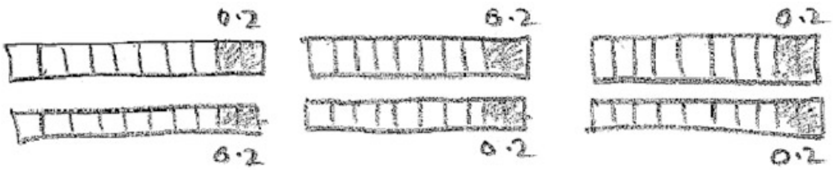
Picture 3



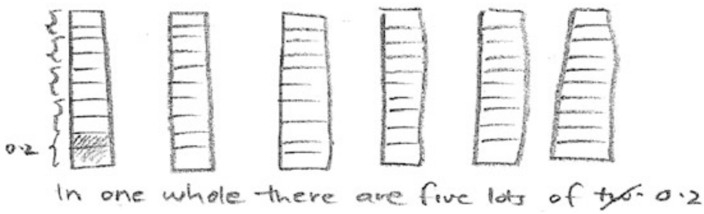
Picture 4



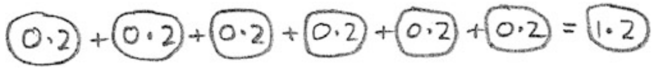
Picture 5 (Code 10)



Picture 6 (Code 11)

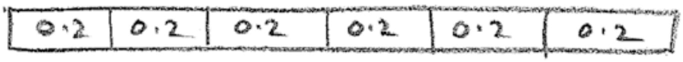


Picture 7 (Code 12)



Incorrect response (Code 70)

Picture 8



Code	Response	Item: MFC502B

Correct response

20 Responses that refer to reading and comprehension difficulties related to the **complexity of the language** used in the question with **reasons** and/or **references to specific examples**
Examples:

- *The language used is quite challenging. Example*, "*fewer than any other*" *and* "*more pencils than rulers*"
- *Students would be challenged by the difficulty/complexity of the wording in the question such as* '*most often*', '*fewer*'. *There is a considerable load on their* '*higher order*' *skills as they are required to organise, interpret and relate back to the graph*
- *The items described in the text are listed in a different order to the bars on the graph creating logistic or sequencing challenges*

Partially correct response

10 Less detailed responses that recognize that the language is likely to be a difficulty for children but **without reasons or examples**
Examples:

- *They would have trouble with the language used in the question*
- *Reading and comprehending the text would be difficult for many children*
- *There is a considerable amount of information to read, organize, sequence and relate to the graph*

11 A statement describing difficulties attributable to the graph rather than the text
Examples:

- *They would have trouble reading the graph*
- *The names are missing from the graph and they wouldn't have experienced this before*

12 A statement attributing difficulties to the level of **problem-solving** or **analysis** required without explaining how/why
Examples:

- *They would have trouble analyzing the information in the problem*
- *The problem requires problem-solving strategies and they would have trouble with that*

Incorrect response

79 Incorrect (including crossed out, erased, stray marks, illegible, or off task)

Non-response

99 Blank

Code	Response	Item: MFC505
	Correct response	
20	Problem 1 and Problem 3 (or Problem 3 and Problem 1)	
	Partially correct response	
10	Problem 1 only correct (with or without Problems 2 and 4) *Examples*: • *Problem 1 and Problem 2 (or 2 and 1)* • *Problem 1 and Problem 4 (or 4 and 1)* • *Problem 1 and Problem _ (blank)*	
11	Problem 3 only correct (with or without Problems 2 and 4) *Examples*: • *Problem 3 and Problem 2 (or 2 and 3)* • *Problem 3 and Problem 4 (or 4 and 3)* • *Problem 3 and Problem _ (blank)*	
	Incorrect response	
70	At least one problem selected but **neither Problem 1 nor Problem 3** *Examples*: • *Problem 2 and Problem 4 (or 4 and 2)* • *Problem 2 and Problem _ (blank)* • *Problem 4 and Problem _ (blank)*	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task)	
	Non-response	
99	Blank	

Code	Response	Item ID: MFC509
	Correct response	
20	**A correct general solution** written in words or using inequalities *Examples*: • *Correct inequality statements* — *OR* ***without*** *the* $n = 2$ *case* *If* $n > 2$ *then* $2n > n + 2$ — *If* $n > 2$ *then* $2n > n + 2$ *If* $n = 2$ *then* $2n = n + 2$ — *If* $n < 2$ *then* $2n < n + 2$ *If* $n < 2$ *then* $2n < n + 2$ • *In words, such as*, "$n + 2$ *is larger when n is less than* 2 *and* $2n$ *is larger when n is greater than* 2"	
21	**A correct general solution** using graphs • *Responses that construct a graph of* $y = n + 2$ *and* $y = 2n$ *AND show on the graph where one is greater than the other OR conclude in words that* $n + 2 > 2n$ *when* $n < 2$ *and* $2n > n + 2$ *when* $n > 2$	

Code	Response	Item ID: MFC509

22 **A correct, ordered, specific-value solution**
Examples:

- *A table (or sequential list of ordered pairs) with values of n and evaluations of $2n$ and $n+2$ AND from the table/list conclude that $n+2>2n$ when $n<2$ and $2n>n+2$ when $n>2$*

n	$2n$	$n+2$
1	2	3
2	4	4
3	6	5
4	8	6

"*The table shows that $2n$ is less than $n+2$ when n is less than 2 and that $2n$ is greater than $n+2$ when n is greater than 2*"

Partially correct response

10 **General responses** that are 'on the right track' but incomplete or are limited in some way
Examples:

- *One correct inequality without the other*
  e.g. If $n>2$, then $2n>n+2$
  e.g. $2n$ is greater than $n+2$ when n is greater than 2
- *Two inequalities but only one is correct*
  e.g. (a) *If $n<2$, then $2n>n+2$ (incorrect) and if $n>2$, then $n+2<2n$ (correct)*
  e.g. (b) *If $n<2$, $n+2$ is larger (correct) and if $n>2$, $n+2$ is larger (incorrect)*

11 **Graphical solutions** that are 'on the right track' but incomplete or are limited in some way
Examples:

- *Two correct graphs **without** showing on the graph where one is greater than the other OR **without** concluding in words that $n+2>2n$ when $n<2$ and $2n>n+2$ when $n>2$*
- *Two graphs but **only one is correct**. The conclusion or annotation with the graphs must be correct for the two graphs shown*

12 **Specific-value solutions** that are 'on the right track' but incomplete or are limited in some way
Examples:

- *Responses that use trial-and-error and **more than one** specific value of n but **do not generalize** them into the same categories as shown under code 20*
- *Responses that say it **depends on the value of** n with **more than one** supporting example. For example, "It depends. When $n=1$, $n+2$ is larger, when $n=5$, $2n$ is larger"*

Incorrect response

70 Responses that indicate that:

- it cannot be known which is larger because the value of n is not known; *or*
- 'it depends on the value of n', **with no (or only one)** supporting example or with no other valid argument

Code	Response	Item ID: MFC509
71	One correct inequality only and an additional error *Examples*: • $2n > n + 2$ *when* $n > 1$ • $n + 2$ *is greater than* $2n$ *when* n *is* 1 *or less* (Has assumed n is integral)	
72	Conclusion reached on the basis of **only one specific value of** n *Example*: *If* $n = 10$, $2n = 20$ *and* $n + 2 = 12$ *so* $2n > n + 2$	
73	Responses that select $2n$ with no correct qualifying inequality (e.g. without 'when $n > 2$')	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task)	
	No response	
99	Blank	

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Teacher Education Effectiveness: Quality and Equity of Future Primary Teachers' Mathematics and Mathematics Pedagogical Content Knowledge

Sigrid Blömeke, Ute Suhl, and Gabriele Kaiser

Abstract The effectiveness of teacher education was examined by taking two indicators into account (Creemers and Kyriakides, "The dynamics of educational effectiveness: a contribution to policy, practice and theory in contemporary schools", 2008): future teachers' mean achievement on a paper-and-pencil test as an indicator of *quality* and the variability of teacher achievement due to background characteristics as an indicator of *equity*. In detail, the effects of gender and language on mathematics content knowledge and mathematics pedagogical content knowledge were examined. The analyses were embedded in IEA's "Teacher Education and Development Study in Mathematics" (TEDS-M) and they referred to primary teachers from 15 countries in their final year of teacher education. The study revealed significant cultural differences in the effectiveness of teacher education. Gender and language effects could be decomposed into *direct* and *indirect* effects. The latter represented a combination of differential choices of teacher education programs according to background characteristics and differential achievement of teachers from these programs. Implications for educational policy are discussed.

Keywords Direct effect · Indirect effect · Educational effectiveness · Teacher quality · Comparative study · Large-scale assessment · Teacher knowledge · Primary effect · Secondary effect

The effectiveness of education systems can be examined with respect to several indicators. K-12 school effectiveness research revealed that two important indicators

Adjusted version of Blömeke et al. (2011).

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S. Blömeke et al. (eds.), *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn*, Advances in Mathematics Education,
DOI 10.1007/978-94-007-6437-8_5,

are the *mean* and the *variability* of student achievement (Creemers and Kyriakides 2008).

The national *mean* of student achievement provides important information with respect to international competitiveness of a school system at a specific point in time (e.g. grade 8 in TIMSS)—regardless of which decision policy makers had taken about the structure of this system. In this sense, national means state a matter of fact and serve as an anchor to get an idea about a country's achievement whereas reasons for country differences are to be examined in further analyses.

How successfully a school system is able to reduce social disparities in student achievement represents a second indicator of educational effectiveness. Small or in the best case no significant proportions of *variance* due to background characteristics like gender or language represent an important educational objective (Lerman 2000; Skovsmose and Valero 2001). In a modern democracy, educational achievement should be independent of a person's origin and everybody should have the same chance to gain the knowledge and skills necessary to participate in a community or in the labor market ("education for all"; see e.g. UN's Universal Declaration of Human Rights 1948). Differential success due to demographics (Bourdieu and Passeron 1990: "reproduction" of social inequality) can hardly be justified in a democracy. Systematic social disparity in achievement related to background characteristics is in this sense to be regarded as educational injustice (Gates and Vistro-Yu 2003).

Based on data from the comparative study TEDS-M ("Teacher Education and Development Study in Mathematics"),[1] this approach of school effectiveness research to consider *quality* as well as *equity* as indicators of educational effectiveness is applied to mathematics teacher education in 15 countries in order to learn

(1) about the level of mathematics content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK) of future primary teachers in these countries assessed with a paper-and-pencil test at the end of their training (*quality*),
(2) to what extent differential effects of demographic characteristics on the acquisition of MCK and MPCK exist (*equity*),
(3) whether these differences in MCK and MPCK are due to *direct* effects of these demographic characteristics or were mediated—at least partly—by differential choices of teacher education programs (*indirect* effects), and finally
(4) to what extent similarities and differences in these effects exist across countries.

[1]TEDS-M was funded by the International Association for the Evaluation of Educational Achievement (IEA), the National Science Foundation (REC 0514431), and the participating countries. In Germany, the German Research Foundation funded TEDS-M (DFG, BL 548/3-1). The analyses prepared for this report and the views expressed are those of the authors but do not necessarily reflect the views of the IEA, the NRCs of the participating countries or the funding agencies.

1 Theoretical Framework and State of Research

Gender-Related Achievement Differences in Mathematics During K-12 schooling, equity in achievement is rarely accomplished. With respect to *gender*, mathematics has been regarded as a male-dominated subject for a long time (Burton 2001). For Western countries like the USA, longitudinal and trend studies revealed that even though differential mathematics achievement of boys and girls has decreased over the past decades, females still show lower achievement compared to their male counterparts on mathematics tests in higher school grades and college (Fan et al. 1997; Hyde et al. 2008). Since mathematics is not only a core subject during schooling but also a gate keeper with respect to successful participation in the labour market, especially with respect to high-prestige professions (Freudenthal 1983), such achievement gaps represent a serious disadvantage for females.

Even in countries with a long tradition of gender equity like Sweden, mathematics appears to have the strongest gender imbalance of almost all educational and professional fields from upper-secondary school on (Brandell 2008). Explanations for such inequity point either to biological differences (mainly in early publications, e.g. Benbow and Stanley 1980) or to socio-psychological aspects. In the latter perspective, females receive less support and encouragement from teachers and parents and they have fewer formal and informal opportunities to learn mathematics (Henrion 1997).

Drawing on Boudon's well-known model of education and social inequality developed with respect to socio-economic status (Boudon 1974), such a gender effect could be called a "primary" effect. In addition, differential "secondary" gender effects exist. These are revealed, for example, in the choice of classes during K-12 schooling. Despite a comparable level of achievement, the proportion of female students opting for advanced classes in mathematics is usually significantly lower than the proportion of male students. If the class level is controlled, gender differences in achievement are less apparent (Stockdale 1995). This result points to the necessity of decomposing gender effects into *differential achievement* (primary effects) and *differential choices* (secondary effects).

One of the few international studies on gender effects in teacher education that used direct achievement measures, the six-country study "Mathematics Teaching in the 21st Century (MT21)" carried out with future lower secondary mathematics teachers (Schmidt et al. 2011), revealed that gender-related achievement differences in mathematics also exist in teacher education in some countries. Male future lower secondary teachers from Germany significantly outperformed their female counterparts on the mathematics tests. In contrast, the USA seemed to be more successful to avoid such achievement gaps.

The differences in Germany could partly be identified as primary effects directly related to gender characteristics (Blömeke et al. 2010): Even within groups of future teachers graduating from the same type of teacher-education program with similar opportunities to learn, the mathematics achievement of male teachers was higher than that of females. A differential secondary effect due to program choice was relevant as well though. As a matter of self-selectivity males had preferred to enrol in those programs leading to a lower-secondary teaching licence heavily loaded with mathematics. In contrast, females had preferred to enrol in programs more oriented towards pedagogy. Since the opportunities to learn mathematics and, as a result, also the mathematics achievement was significantly lower in the latter than in the former, part of the achievement gap between men and women was mediated by such differential program choices.

Language-Related Achievement Differences Besides gender, differences in *language* are a characteristic known to be associated with K-12 student achievement. In many countries, those students performed significantly worse whose opportunities to learn occurred in their second language compared to those students whose opportunities to learn occurred in their first language (Walter and Taskinen 2007). This problem applied even to those countries that are generally able to reduce social disparities during schooling like the Nordic countries (Kobarg and Prenzel 2009). Classroom discourse plays a major role in this context as Schütte and Kaiser (2011) could show with respect to German primary students with a different first language than German. The language disadvantages result from the difference in language skills sufficient for communication at home or with peers and the language proficiency necessary for school success (Cummins 1983; Council of Chief State School Officers 1990).

Even with support through several kinds of interventions like bilingual instruction, it takes a long time before students with a different first language than that used in instruction reach the 50th percentile of an achievement distribution (Ramírez et al. 1991; McLaughlin 1985). Thomas' and Collier's (1997) large-scale study with more than 700,000 minority language students in the USA can be taken as an example for important findings in this context. The study confirmed the achievement gap between students with English as their first language and students with another first language. In addition, the study pointed to cumulative effects in the sense that language effects increased in higher grades of schooling. Regarding TEDS-M as a study on higher education this is an important finding and leads to the question whether a match of a teacher's first language and the official language of instruction in teacher education is related to higher outcomes.

Thomas and Collier (1997) also revealed differential effects of opportunities to learn. This result points once more to a distinction of primary language effects and secondary effects due to choices in education. Heath and Brinbaum (2007) had, in fact, found systematic inequalities in achievement in several European countries. Second-language learners were not only at a disadvantage in terms of test results or grades (primary language effect) but also in terms of continuation rates into higher education by controlling for the level of achievement compared to first-language learners (secondary effect).

Significance of the Study and Hypotheses Based on this state of research, future primary teachers' mean achievement in mathematics and mathematics pedagogy—assessed at the end of their teacher education in a paper-and-pencil test—is presented as a first (*quality*) indicator of teacher education effectiveness in 15 countries.

MCK and MPCK represent core facets of primary teachers' professional competencies. As head teachers they will have to teach mathematics. To be able to master the challenges related to the rigorous mathematics standards for primary schools more and more countries have implemented in recent years, they need a deep understanding of mathematics and mathematics pedagogy.

As a second (*equity*) indicator of teacher education effectiveness, we examine whether future primary teachers' achievement in mathematics or mathematics pedagogy significantly varies due to gender or language background. Derived from the K-12 state of research, we hypothesize significant differences in favor of male compared to female future teachers and in favor of teachers whose first language matches the official language of instruction in teacher education compared to others. If we, in fact, find evidence for such inequalities, these should be of major concern for policy makers.

Our follow-up hypothesis is that the differences in MCK and MPCK represent a combination of *primary* and *secondary* background effects. Differential choices of teacher-education programs may be a mediating factor that explains variance in primary teachers' achievement. From a policy point of view, it is important to distinguish between these two types of effects because addressing education inequality would require different measures depending on which effect dominates.

All analyses were done by country to find out which similarities or differences in primary teacher education effectiveness exist. Within a country, we often take for granted that educational outcomes must be the way they are. International comparisons can reveal whether this assumption is really true.

2 Study Design

The target population of the present study was defined as future teachers in their final year of teacher education who would receive a license to teach mathematics in primary schools (Tatto et al. 2008). A teacher education program was identified as preparing primary teachers if the license would include one of the grades 1 through 4 as the common denominator of education level 1 in the "International Standard Classification of Education" (primary or basic education, cycle 1; UNESCO 1997).

In a two-stage process, random samples were drawn from this target population in each participating country. The samples were stratified according to important teacher education features like "route" (consecutive vs. concurrent programs), "type" of program (grade span the license includes, e.g. grades 1 through 4 vs. 1 through 10), "focus" of opportunities to learn (with or without extensive mathematics) or "region" (e.g. federal states) in order to reflect accurately the distribution of future primary teachers characteristics at the end of their training.

In 2008, about 14,000 future primary teachers from 15 countries (see Table 1) were tested on their MCK and MPCK in a standardized paper-and-pencil assessment. If a country missed the participation benchmark only slightly, its results are reported in an annotated way ("Combined Participation Rate < 75 %"). On the

Table 1 Participating countries in TEDS-M 2008 (primary study)

Botswana	Chile[1]	Germany	Georgia
Malaysia	Norway[1,n]	Philippines	Poland[**,1]
Russia	Spain	Switzerland[*]	Singapore
Taiwan	Thailand	USA[***,1,2]	

[*]Colleges of Education in German speaking regions

[**]Institutions with concurrent programs

[***]Public Universities

[n]Results for Norway are reported by combining the two data sets available in order to present an accurate country mean

[1]Combined Participation Rate <75 %

[2]High proportion of missing values

(Canada) Country had to be excluded

primary level, this applies to Chile, Norway, Poland and the USA. In most countries, TEDS-M covered the full target population. Only Switzerland, Poland and the US had to limit their study for economic or other reasons (for more details see Chap. "Theoretical Framework, Study Design and Main Results of TEDS-M" in this book).

3 Instruments

Background of Future Teachers Data about gender and language background of future teachers were gathered with multiple-choice items. The *gender* variable was dichotomous with two values (male/female). Corresponding to other IEA studies, the *language spoken at home in contrast to the test language*—which was the official language of instruction in teacher education—was captured with four values (always/almost always/sometimes/never). For the purpose of this study we summarized these four categories into two (always/almost always or sometimes/never).

Teacher Knowledge TEDS-M sought to measure future teachers' MCK and MPCK at the end of teacher education. For this purpose, a 60-minute paper-and-pencil assessment had to be completed during a standardized and monitored test session. The items were supposed to depict classroom performance as closely as possible. Many of them therefore represent problems and situations constitutive for mathematics teaching (NCTM 2000; NBPTS 2003).

In order to capture the desired breadth and depth of teacher knowledge, a matrix design was applied. Five test booklets were developed that had rotated blocks of items ("Balanced Incomplete Block Design"). Scaled scores were created using Item Response Theory. The achievement scores were transformed to a scale with an international mean of 500 test points and a standard deviation of 100 test points. The 76 items of the mathematics test covered number, algebra, geometry and to a small extent also data. In addition, three cognitive dimensions were covered: knowing, applying and reasoning. A third heuristic that led the development of mathematics items was the levels of difficulty (novice, intermediate, and expert). For more information about the TEDS-M test see Chap. “Theoretical Framework, Study Design and Main Results of TEDS-M” in this book. The full set of released items is available at tedsm@msu.edu.

The 32 items of the mathematics pedagogy test covered two subdimensions: pre-active curricular and planning knowledge which is necessary before a teacher enters the classroom and interactive knowledge about how to enact mathematics for teaching and learning. In line with the mathematics test, three levels of difficulty and four content areas were distinguished.

The item development was mainly informed by the MT21 study (Schmidt et al. 2011) as well as by the two Michigan studies “Knowing Mathematics for Teaching Algebra” (KAT; Ferrini-Mundy et al. 2005) and “Learning Mathematics for Teaching” (LMT; Hill et al. 2008). Three item formats were used: multiple choice, complex multiple choice, and open constructed response.

Data Analysis Parameter estimations were done using the “International Data Base Analyzer” provided by IEA. This includes that all results are based on data appropriately weighted (taking unequal selection probabilities into account as well as non-response adjustments) and with appropriate estimations of standard errors (taking the complex sample design and the weights into account by using the “Balanced Repeated Replication” technique).

Regression analyses were used to estimate whether country and group differences were statistically significant. Whether the difference between two values was not only significant but practically relevant was evaluated with Cohen’s parameter d. This indicator takes the standard deviation of the values into account. Differences were regarded as practically relevant if they exceeded a quarter of a standard deviation.

Primary and secondary background effects were statistically estimated in regression analyses as direct and indirect effects separately for each of the 14 or 13 countries respectively. Georgia had to be eliminated from the gender-related analyses because 100 % of the future primary teachers were female; due to very small coverage in one group Botswana and Chile had to be eliminated from the language-related analyses.

As predictors gender ($0 =$ female, $1 =$ male) or language respectively ($0 =$ the official language of instruction in teacher education was rarely or never spoken at home, $1 =$ the official language of instruction was almost always or always spoken at home) and if applicable the teacher education program (dichotomized as explained below) were used. Gender and language did not vary completely independently of each other but in all countries the correlations were either below $r = 0.12$ or not significant at all. MCK and MPCK served as dependant variables.

6 or 5 countries respectively offered only *one* teacher education program for future primary teachers. In these countries, achievement differences according to background represented direct effects—in the context of this paper interpreted as

primary effects as conceptualized by Boudon (1974). Secondary effects due to program choice did not exist. 8 countries offered *different* programs for future primary teachers. In these countries, background effects after program choices were controlled represented the direct effects. Additional indirect effects—in the context of this paper interpreted as secondary effects—could occur as a combination of effects due to differential program choices and effects due to differential achievement in these programs. Whether these indirect effects were significant was tested with the Goodman (1960) test.

To estimate secondary effects this way has to be regarded as limited compared to the standard procedure. However, in TEDS-M—as often in cross-country studies—only relatively few variables were available. Future teachers' MCK and MPCK knowledge level prior to their entrance into teacher education were missing. Conceptually, the main intention of Boudon (1974) of distinguishing between primary and secondary effects should still be met by this procedure.

If a country offered different teacher education programs for future primary teachers, these were dichotomized as follows: We distinguished between program *types* according to the grade span teachers will have to cover in school (up to grade 4, 6 or 10) or to the role they are supposed to take over in class (generalists with many different teaching subjects or specialists with only one or two teaching subjects). In five countries, two such types of programs were offered (in Germany, Poland, Singapore, Switzerland, and the USA).

In addition, we distinguished between different *foci* of programs within one program type if opportunities to learn differed widely. In Norway, primary teachers for the same grade span could be trained in a program with or without a focus on mathematics. In Malaysia and Thailand, primary teachers for the same grade span could be trained in a concurrent or a consecutive route. In order to facilitate the reading we label these distinctions as teacher-education "programs".

By taking into account teacher-education programs as predictors, this paper intends to contribute to more clarity about benefits and limits of analyzing the TEDS-M data below the country level. The traditional policy orientation of studies like TIMSS and PISA focuses on the evaluation of educational *systems*. This is a valuable approach because it stresses the overall effectiveness on the national level and this perspective is necessary with respect to international competitiveness in education. The advantage of this perspective is a high precision of estimates due to large sample sizes. Additional information is gained by taking the structure of teacher education into account.

4 Results

Outcomes of Teacher Education Systems With respect to MCK, future primary teachers from Taiwan achieved the best result of all TEDS-M countries (see Table 2). The difference to the international mean of 500 test points was huge—more than one standard deviation, which is a highly relevant difference. The achievement

Table 2 MCK of future primary teachers by country

Country	Mean	SE
Taiwan	623	4.2
Singapore	590	3.1
Switzerland*	543	1.9
Russia	535	9.9
Thailand	528	2.3
Norway[1,n]	519	2.6
USA**,[1,2]	518	4.1
Germany	510	2.7
International	**500**	**1.2**
Poland**,[1]	490	2.2
Malaysia	488	1.8
Spain	481	2.6
Botswana	441	5.9
Philippines	440	7.7
Chile[1]	413	2.1
Georgia	345	3.9

Annotations are explained below Table 1

of primary teachers from the USA was slightly above the international mean and roughly on the same level as the achievement of teachers in Germany and Norway. The difference to the international mean was significant but of low practical relevance. If one takes into account the *Human Development Index* used by the UN in order to indicate the social, economic and educational developmental state of a country, the high performance of teachers from Russia and Thailand is striking.

With respect to MPCK, the achievement of future primary teachers from the USA was roughly on the same level as the achievement in Norway and it was significantly above the international mean (see Table 3). In this case, the difference to the international mean was also of practical relevance. The difference to the achievement of teachers from Singapore and Taiwan was, however, still highly relevant.

Variability According to Demographic Characteristics How important it is to distinguish between MCK and MPCK becomes apparent when *gender* differences in teacher knowledge were examined. Whereas in many countries pronounced achievement gaps in MCK existed at the end of teacher education, this did not apply to the same extent to MPCK.

The USA belonged to a group of 10 countries where gender differences in *MCK* were statistically significant. As hypothesized, in all cases the differences operated in favor of males (see Fig. 1; to facilitate the reading standard errors were left out; for the precise size of the differences see also Table 8, right column "total gender effect"). With about 130 test points, the effect was the largest in Poland. In Russia,

Table 3 MPCK of future primary teachers by country

Country	Mean	SE
Singapore	593	3.4
Taiwan	592	2.3
Norway[1,n]	545	2.4
USA**,[1,2]	544	2.5
Switzerland*	537	1.6
Russia	512	8.1
Thailand	506	2.3
Malaysia	503	3.1
Germany	502	4.0
International	**500**	**1.3**
Spain	492	2.2
Poland**,[1]	478	1.8
Philippines	457	9.7
Botswana	448	8.8
Chile[1]	425	3.7
Georgia	345	4.9

Annotations are explained below Table 1

the USA, Thailand, Singapore, Spain and Norway males scored higher than females between almost 30 and more than 40 points. Such a difference of about one third of a standard deviation is of high practical relevance. In only four TEDS-M countries, there were no gender differences. This applied to Malaysia, the Philippines and (due to the large standard error in one of the subgroups) to Botswana and Germany.

Compared to the overall ranking of the countries, no linear relationship of future teachers' mean achievement on the national level and a teacher education system's ability to avoid a gender gap existed. In case of the acquisition of MCK in teacher education, the two main objectives of an educational system—*quality* as indicated by high achievement and *equity* as indicated by low variability according to background (Creemers and Kyriakides 2008)—seemed to vary widely independently of each other. None of the TEDS-M countries was successful on both objectives, although Taiwan came close with high achievement and a significant but relatively small gender effect.

The situation was different with respect to MPCK (see Fig. 1; for the precise size of the differences see also Table 9, right column "total effect"). In only four countries a significant achievement difference in favor of male future teachers existed and with about 15 test points even this advantage was of low practical relevance in Spain, Thailand and Norway. Only in Poland where the difference was the largest (83 test points) a practically relevant difference by gender existed.

In most countries, the difference between male and female future teachers was below 10 points and not statistically significant. It seems as if it was easier to avoid gender inequality in this facet of teacher knowledge. In one country, Malaysia, fe-

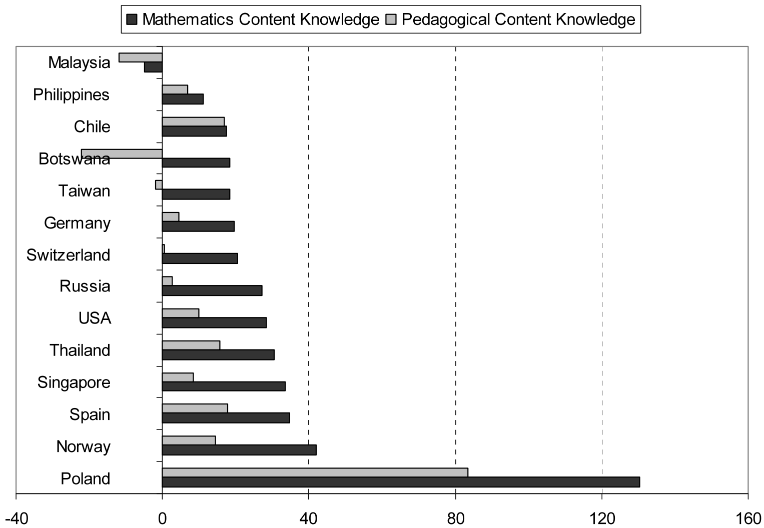

Fig. 1 Gender differences in MCK and MPCK of future primary teachers in favor of males (without Georgia where 100 % were female; for sample limitations see annotations below Table 1)

male teachers even outperformed male teachers at the end of teacher education (by about 12 test points; due to the large standard error the same tendency in Botswana was not statistically significant).

In line with our inference in the case of MCK, we can infer from the ranking of the countries that in the case of MPCK the mean achievement on the country level and the magnitude of gender differences varied independently from each other. Two countries met both objectives of educational effectiveness: high achievement and gender equity, and these were Taiwan and Singapore. Switzerland and the USA can also be regarded as relatively successful with gender equity and achievement well above the international mean.

With respect to differential effects of the *language* used in teacher education versus the language spoken at home, the USA is part of a group of three countries where significant differences occurred in favor of those teachers whose first language matched the language of instruction and where the group difference applied to mathematics as well as to mathematics pedagogy (see Fig. 2; for the precise size of the differences see also Tables 10 and 11, right columns "total language effect"). Besides the USA, the group consisted of Germany and Thailand. The language gap was the largest in Germany (65 or 60 points respectively) but it was of high practical relevance in the USA as well (39 or 33 points respectively). It seems as if the education systems in these countries set students with a different language than the language of instruction in teacher education at a disadvantage.

In all other countries, the differences were either only significant in one of the two dimensions tested (Norway: MCK; Georgia, Taiwan: MPCK) or they were not statistically significant at all (although this sometimes occurred only due to small

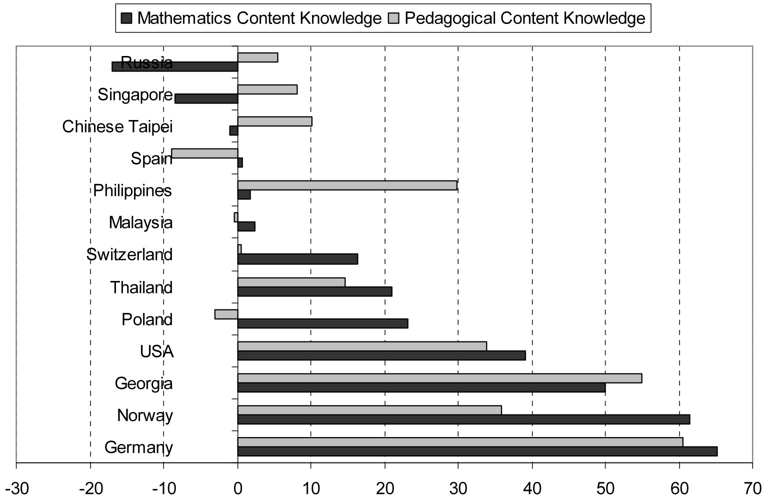

Fig. 2 Language-related differences in future primary teachers' MCK and MPCK in favor of those tested in their first language (due to very small coverage in one group, Botswana and Chile could not be included; for sample limitations see annotations below Table 1)

sample sizes in one subgroup, e.g. in the Philippines or Russia). These countries seemed to be more successful in avoiding differential language effects than the USA, Germany and Thailand. In the case of Malaysia, this result is particularly remarkable as 90 % of the future teachers did not speak the language of their teacher education program (English) at home.

Regarding teacher education effectiveness, the achievement of future teachers from Singapore and within limits from Switzerland was remarkable. Their results were not only (very) high on average but at the same time language equity existed—and this in mathematics as well as in mathematics pedagogy. In Taiwan, Norway and Russia, quality and equity were achieved on one of the two subdimensions of teacher knowledge.

Decomposition of Gender and Language Inequity into Direct and Indirect Effects In the next step, we examined whether the gender and language inequity documented on the national level was directly caused by these background characteristics or whether they were mediated by differential program choices in those 8 countries where several teacher education programs for future primary teachers were offered in parallel to each other.

The TEDS-M data revealed for the latter countries that the proportion of males and females varied by teacher-education program. The differences were not always statistically significant but the tendency offered a clear finding: The proportion of females was higher in generalist programs compared to mathematics specialist programs (in particular in Poland, less apparent in the USA), in programs for lower grades of primary school compared to programs which include higher grades as

Table 4 Slope, standard error, and t-value of program and gender effects on MCK and variance explained per country

Country	Program effect			Gender effect			R^2
	b	SE	t	b	SE	t	
Poland**,1	149	5.8	25.8	46	11.2	4.1	45.1
Norway1,n	41	5.2	7.8	39	8.3	4.7	11.9
Thailand	60	9.7	6.2	29	7.1	4.1	8.4
Switzerland*	34	6.9	4.9	17	5.6	3.0	4.1
Germany	53	8.3	6.5	no significant effect			6.3
Malaysia	−28	6.2	−4.4	no significant effect			2.2
Singapore	no significant effect			33	8.1	4.1	4.4
USA***,1,2	no significant effect			28	12.4	2.3	1.3

Poland, Germany, Singapore, USA: 0 = generalists up to grade 4 or 6, 1 = specialists; Norway: 0 = generalists up to grade 10 without a focus on math, 1 = generalists up to grade 10 with a focus on math; Malaysia, Thailand: 0 = specialists concurrent route, 1 = specialists consecutive route; Switzerland: 0 = generalists up to grade 4, 1 = generalists up to grade 6; Gender effect: 0 = female future primary teachers, 1 = male future primary teachers; Annotations are explained below Table 1

well (Switzerland), in programs without a focus on mathematics compared to programs with such a focus (Norway), and in concurrent routes compared to consecutive routes (in particular in Malaysia, less apparent in Thailand).

The variation in the proportions of future teachers with different language background was relatively low. Only in three countries statistically significant differences existed and this was in Germany, Switzerland, and Thailand. Here, the proportion of teachers with a mother tongue different from the official language of instruction in teacher education was higher in generalist programs up to grade 4 than in mathematics specialist programs (Germany), in generalist programs up to grade 6 than in programs up to grade 4 (Switzerland) and in concurrent programs than in consecutive programs (Thailand). In the other five countries, language effects were not important when it came to the choice of teacher-education programs.

The question is now whether in those countries with several programs leading to a license for teaching at primary schools, differences in achievement due to background still existed if the program distribution was controlled.

With respect to *gender* effects on MCK, in six countries—in Norway, Poland, Switzerland and the USA as well as in Singapore and Thailand—this characteristic was still significant (see Table 4). The effects were partly very large and always operated in favor of males. In Poland almost half of the variance in MCK was explained by gender and program choice, which is an exceptionally large amount.

The situation was different with respect to MPCK. In only two countries—Norway and Thailand—, gender effects were still significant if the program was controlled (see Table 5). It seems as if the risk of females to fall behind was much lower in MPCK compared to gender inequity in MCK.

Table 5 Slope, standard error, and t-value of program and gender effects on MPCK and variance explained per country

Country	Program effect			Gender effect			R^2
	b	SE	t	b	SE	t	
Thailand	38	9.4	4.0	15	6.8	2.2	3.4
Norway[1,n]	24	6.3	3.8	13	5.8	2.2	3.5
Poland**,[1]	120	4.7	25.7	no significant effect			25.2
Germany	62	8.7	7.2	no significant effect			6.6
Switzerland*	21	6.1	3.4	no significant effect			1.2
Malaysia	no significant effect			no significant effect			
Singapore	no significant effect			no significant effect			
USA***,[1,2]	no significant effect			no significant effect			

The coding of program and gender effects is explained below Table 4. Annotations are explained below Table 1

Table 6 Slope, standard error, and t-value of program and language effects (in favor of those tested in their first language) on MCK and variance explained per country

Country	Program effect			Language effect			R^2
	b	SE	t	b	SE	t	
Norway[1,n]	45	5.1	8.9	71	22.6	3.1	8.0
Germany	67	7.6	8.7	57	12.8	4.4	9.3
Switzerland*	37	6.8	5.4	20	8.9	2.3	3.8
Thailand	58	9.7	6.0	17	5.1	3.3	6.7
Poland**,[1]	158	5.0	31.4	16	8.0	2.1	43.9
USA***,[1,2]	no significant effect			39	17.7	2.2	0.5
Singapore	no significant effect			no significant effect			–
Malaysia	−28	64	−4.4	no significant effect			2.1

The coding of program effects is explained below Table 4. Annotations are explained below Table 1. Language effect: 0 = official language of instruction in teacher education rarely or never spoken at home, 1 = official language of instruction almost always or always spoken at home

An interesting picture emerged with respect to language effects on MCK (see Table 6). There were six TEDS-M countries with significant effects due to future teachers' language background if the program choice was controlled. This group of countries included all four Central European countries (Norway, Germany, Switzerland, the USA and Poland—here with a huge effect size) as well as Thailand. To speak a different language than the official language of teacher education always proved to be a disadvantage. Only in Singapore was neither a significant effect of language background nor of program choice found. This country has obviously developed strategies to avoid the reproduction of language-related disadvantages in teacher education.

Table 7 Slope, standard error, and *t*-value of program and language effects (in favor of those tested in their first language) on MPCK and variance explained per country

Country	Program effect			Language effect			R^2
	b	SE	*t*	*b*	SE	*t*	
Germany	66	9.8	6.8	−52	19.7	−2.7	7.5
Thailand	36	10.2	3.5	−12	5.7	−2.1	3.2
USA***,1,2	no significant effect			−34	16.2	−2.1	0.4
Poland**,1	123	4.5	27.5	no significant effect			25.0
Norway1,n	26	6.2	4.2	no significant effect			3.5
Switzerland*	21	6.1	3.4	no significant effect			1.2
Singapore	no significant effect			no significant effect			–
Malaysia	no significant effect			no significant effect			–

The coding of program and language effects is explained below Tables 4 and 6. Annotations are explained below Table 1. Language effect: 0 = official language of instruction in teacher education rarely or never spoken at home, 1 = official language of instruction almost always or always spoken at home

Differential language effects were of lower relevance with respect to MPCK compared to MCK (see Table 7). Fewer countries were affected and the variance explained was lower. This result is interesting insofar as the acquisition of MPCK could be regarded as more dependent on language proficiency than MCK due to its pedagogical component. In the case of MCK, perhaps a long-lasting effect of schooling has to be noted.

In Germany, the USA and Thailand significant effects due to future teachers' language background existed if the program choice was controlled. In Singapore and Malaysia, neither a significant effect of language background nor of program choice was found. The data revealed for the other countries significant effects of differential program choices though.

As a final exploratory step, the relative weight of direct background effects and—if several programs were offered in a country—indirect background effects due to program choices were estimated in order to learn about possible reasons for inequity.

As pointed out before (see section "Variability according to demographic characteristics"), significant *total gender effects* on the acquisition of MCK existed in the majority of countries (see Table 8). With the exception of Poland, *direct* gender effects explained either all gender inequity among future mathematics teachers in these countries or almost all of it.

Indirect gender effects on MCK were in TEDS-M obviously of lower importance. Even in Norway and Switzerland, where statistically significant indirect gender effects were found, the differences in outcome caused by a higher proportion of females in the lower-achieving teacher education programs were small compared to the direct gender effects. Thus, only in Poland did an important indirect gender effect exist. The proportion of females in the Polish specialist program that was heavily loaded with mathematics was much lower than the proportion

Table 8 Total gender effects as well as test point differences and proportions of inequity caused by direct and indirect gender effects on MCK (without Georgia where 100 % were female)

Country	Direct effect due to gender		Indirect effect due to differential program choice		Total gender effect (corresponds to Fig. 1)	
	Test point difference	Proportion of total effect	Test point difference	Proportion of total effect	Test point difference	Proportion of total effect
Poland**,1	46	35 %	85	65 %	130	100
Norway1,n	39	92 %	3	8 %	42	100
Switzerland*	17	83 %	4	17 %	21	100
Singapore	33	98 %	ns	ns	33	100
Thailand	29	96 %	ns	ns	31	100
USA***,1,2	28	99.8 %	ns	ns	29	100
Malaysia	ns	ns	−2	–	ns	ns
Germany	ns	ns	ns	ns	ns	ns
Spain	35	100 %	na	na	35	100
Russia	27	100 %	na	na	27	100
Taiwan	18	100 %	na	na	18	100
Chile1	17	100 %	na	na	17	100
Botswana	ns	ns	na	na	ns	ns
Philippines	ns	ns	na	na	ns	ns

Above the line: countries with several teacher education programs; below the line: countries with only one program. Indirect effects represent the combined effect of differential program choice and differential achievement according to teacher education programs. Annotations are explained below Table 1. na: not applicable, ns: not significant

of males and consequently males achieved a better test result. This large indirect effect added to a large direct effect in this country with the result that the overall gender inequity in Poland was by far the largest compared to all other countries.

The situation with respect to MPCK was different. Both direct and indirect gender effects existed in none of the TEDS-M countries (Table 9). Furthermore, in more than half of the countries neither the former nor the latter existed. Gender inequity seemed to be much less of an issue with respect to MPCK compared to MCK. Only in Poland substantial gender inequity existed and this was almost exclusively caused by differential program choices and females opting more often for programs with lower achievement in mathematics pedagogy.

Small gender effects existed in Spain, Thailand and Norway and these represented almost exclusively direct effects. Malaysia and Switzerland were special cases. In Malaysia, none of the separate direct and indirect effects were statistically significant but the insignificant inequity added up to a significant total effect. In Switzerland a significant indirect effect due to differential program choice was

Table 9 Total gender effects as well as test point differences and proportions of inequity caused by direct and indirect gender effects on MPCK

Country	Direct effect due to gender		Indirect effect due to differential program choice		Total gender effect (corresponds to Fig. 1)	
	Test point difference	Proportion of total effect	Test point difference	Proportion of total effect	Test point difference	Proportion of total effect
Poland**,1	ns	ns	68	–	83	100
Thailand	15	95 %	ns	ns	16	100
Norway1,n	13	87 %	ns	ns	15	100
Malaysia	ns	ns	ns	ns	−12	100
Switzerland*	ns	ns	2	–	ns	100
Singapore	ns	ns	ns	ns	ns	ns
USA***,1,2	ns	ns	ns	ns	ns	ns
Germany	ns	ns	ns	ns	ns	ns
Spain	18	100 %	na	na	18	100
Russia	ns	ns	na	na	ns	ns
Taiwan	ns	ns	na	na	ns	ns
Chile1	ns	ns	na	na	ns	ns
Botswana	ns	ns	na	na	ns	ns
Philippines	ns	ns	na	na	ns	ns

Above the line: countries with several teacher education programs; below the line: countries with only one program. Indirect effects represent the combined effect of differential program choice and differential achievement according to teacher education programs. Annotations are explained below Table 1. na: not applicable, ns: not significant

suppressed by a small but not significant direct effect. So, these two countries have to deal with inequity issues as well.

The results with respect to language background were again particularly striking. At the beginning of this paper, we had stated that on the national level significant effects on MCK due to language existed in only four countries. Table 10 shows, however, that direct language effects were found in six countries. In Poland and Switzerland, significant disadvantages of future teachers with a different language than the one spoken in teacher education did not mount up to total language effects because they were suppressed either by the lack of significant indirect effects (Poland) or by positive indirect language effects (Switzerland). In seven countries, language inequity was neither an issue with respect to direct nor to indirect effects.

In the case of MPCK, inequalities due to language effects were also mainly direct effects (see Table 11). These were significant in five countries, among others in the US. Significant indirect effects due to program choice were only found in Germany and Thailand.

Table 10 Total language effects as well as test point differences and proportions of inequity caused by direct and indirect language effects on MCK (due to very small coverage in one group, Botswana and Chile could not be included)

Country	Direct effect due to gender		Indirect effect due to differential program choice		Total gender effect (corresponds to Fig. 2)	
	Test point difference	Proportion of total effect	Test point difference	Proportion of total effect	Test point difference	Proportion of total effect
Germany	57	87 %	8	13 %	65	100 %
Norway[1,n]	71	115 %	ns	ns	61	100 %
USA***,[1,2]	39	99.6 %	ns	ns	39	100 %
Thailand	17	80 %	4	20 %	21	100 %
Switzerland*	20	123 %	−4	−23 %	ns	100 %
Poland**,[1]	16	70 %	ns	30 %	ns	100 %
Malaysia	ns	ns	ns	ns	ns	ns
Singapore	ns	ns	ns	ns	ns	ns
Georgia	ns	ns	na	na	ns	ns
Philippines	ns	ns	na	na	ns	ns
Spain	ns	ns	na	na	ns	ns
Taiwan	ns	ns	na	na	ns	ns
Russia	ns	ns	na	na	ns	ns

Above the line: countries with several teacher education programs; below the line: countries with only one program. Indirect effects represent the combined effect of differential program choice and differential achievement according to teacher education programs. Annotations are explained below Table 1. na: not applicable, ns: not significant

Validating the Results by Controlling for General Ability Since it is plausible to assume that achievement in teacher education is not only influenced by gender or language background or different opportunities to learn in teacher education programs but also by the future teachers' general cognitive ability, all analyses were repeated controlling for this. As a proxy, self-reported estimations of their average level of high-school achievement across all subjects compared to their age cohort were used.

Whereas there were only very few cases with changes in the effects, the variance explained strongly increased in most countries. Except in Botswana and the Philippines, general ability seemed to be an important predictor of achievement at the end of teacher education. The effects always pointed in the same direction: the better a future teacher regarded her high-school achievement, the higher her MCK and MPCK as measured in our tests was at the end of teacher education.

The few cases with changes in the effects when high-school achievement was controlled occurred in Poland and the USA. Here significant direct gender effects on MPCK were found then. In contrast, the language disadvantage in Poland and Switzerland on MCK disappeared when high-school success was controlled.

Table 11 Total language effects as well as test point differences and proportions of inequity caused by direct and indirect language effects on MPCK (due to very small coverage in one group, Botswana and Chile could not be included)

Country	Direct effect due to gender		Indirect effect due to differential program choice		Total gender effect (corresponds to Fig. 2)	
	Test point difference	Proportion of total effect	Test point difference	Proportion of total effect	Test point difference	Proportion of total effect
Germany	52	86 %	8	14 %	60	100 %
USA***,1,2	34	99.9 %	ns	ns	34	100 %
Thailand	12	82 %	3	18 %	15	100 %
Switzerland*	ns	ns	2	–	ns	ns
Singapore	ns	ns	ns	ns	ns	ns
Malaysia	ns	ns	ns	ns	ns	ns
Poland**,1	ns	ns	ns	ns	ns	ns
Norway1,n	ns	ns	ns	ns	ns	ns
Georgia	55	100 %	na	na	55	100 %
Taiwan	10	100 %	na	na	10	100 %
Philippines	ns	ns	ns	ns	ns	ns
Russia	ns	ns	ns	ns	ns	ns
Spain	ns	ns	ns	ns	ns	ns

Above the line: countries with several teacher education programs; below the line: countries with only one program. Indirect effects represent the combined effect of differential program choice and differential achievement according to teacher education programs. Annotations are explained below Table 1. na: not applicable, ns: not significant

5 Summary and Conclusions

Educational effectiveness is commonly evaluated in terms of mean achievement (*quality*) and in terms of achievement distribution according to background characteristics (*equity*; see Creemers and Kyriakides 2008). If inequity is found, primary background effects and secondary effects due to educational choices have to be distinguished (Boudon 1974). The present paper examined across 15 countries to what extent primary teacher education can be regarded as effective and possible reasons of inequity.

Quality of Teacher Achievement and Policy Recommendations Based on the mean achievement of future primary teachers as a first indicator of teacher education effectiveness, it is possible to infer that the systems in Taiwan and Singapore were particularly effective and this with respect to MCK as well as to MPCK. The achievement of future teachers from Switzerland (on both facets of teacher knowledge) as well as from Norway and the USA (only on MPCK) was well above the international mean, too.

Thus, regarding the *quality* of teacher education, this result points on the one side to a strength of US primary teacher education. This strength includes enabling teachers to plan mathematics lessons, to apply different teaching methods, to identify student misconceptions and to give appropriate feedback to students and parents. At the same time, the result points to a less favorable outcome in the USA with respect to MCK. However, given what we know from results of student assessments like TIMSS or PISA, it is probably not far fetched to assume that a lower level of US teacher candidates' mathematics knowledge prior to teacher education compared to candidates from Taiwan, Singapore or Switzerland may at least be partly responsible for this worrying outcome.

Since MCK can be regarded as an important precondition for applying MPCK successfully in class (Baumert et al. 2010), efforts seem to be meaningful to enhance the mathematics knowledge of US primary teachers. Our data point to two possible measures although we have to be careful with final conclusions due to the limitations of the TEDS-M data set: Universities and federal states may have to look at their teacher education curricula and licensure procedures and try to find ways to strengthen the opportunities to learn mathematics in their teacher education programs. In addition, they may want to strengthen the admission criteria in order to select teacher candidates from a pool of students with stronger prior knowledge of mathematics.

Equity of Teacher Achievement and Policy Recommendations With respect to equity in teacher education, our analyses supported our gender-related hypothesis. In most countries, significant achievement differences existed in favor of male compared to female future teachers. A linear relationship of future teachers' mean achievement in MCK and MPCK on the national level and a teacher education system's ability to avoid a gender gap was not found but it turned out to be important to distinguish between MCK and MPCK.

Whereas in many countries, pronounced gender gaps in MCK existed at the end of teacher education, this did not apply to the same extent to MPCK. The gender effect on MCK was the largest in Poland whereas only in Malaysia, the Philippines, Botswana and Germany no significant differences occurred. In contrast, with respect to MPCK we found significant achievement differences in favor of male future teachers only in Poland (the largest effect), Norway, Spain and Thailand. In most countries, the difference between male and female future teachers was not statistically significant. In Malaysia, female teachers even outperformed male teachers.

The striking difference between MCK and MPCK does not correspond with our initial hypothesis which stated similar gender effects for both facets of teacher knowledge. Exploring possible reasons of this unexpected result reveals differences in the nature of MCK and MPCK: On the one side has MCK been built over a long period of schooling and primary teacher education whereas MPCK was only taught at university. Gender effects in the case of MCK may therefore be cumulative effects since K-12 disadvantages in mathematics achievement of girls compared to boys are well documented (Hyde et al. 2008). On the other side includes MPCK more than only mathematics. The pedagogical nature of MPCK may reduce disadvantages of females given their motivation to become primary teachers. Studies revealed that female future teachers tend to support pedagogical motives stronger than male future teachers, specifically in comparison to subject-specific motives (Eberle and Pollak 2006).

Gender effects varied not only by knowledge dimension but also by country. This latter difference indicates that teacher achievement reflects cultural patterns but not properties inherent to gender. People in many countries, may have gotten used to disadvantages of girls in K-12 mathematics and recognize this phenomenon now with respect to future primary teachers' MCK. However, our results reveal that such disadvantages do not occur in all countries. There are several examples (like Malaysia or the Philippines) where it was possible to avoid gender inequalities. These examples do not only point to possibilities for educational policies. They are also a request not to give up in our efforts to create a democratic society.

For the USA, the challenge is to overcome the gender gap in MCK. One possibility would be to look at the curriculum of teacher education and systematically to implement discourses about gender and mathematics. In TEDS-M, only half of the primary teachers from the USA indicated that they had studied such a topic during their training (Blömeke et al. 2010). Given the sensitivity of such an issue for a nation, this is certainly not enough.

Language was the second background characteristic examined in this paper. In Germany (the largest gap), the USA and Thailand differences of high practical relevance occurred and this in MCK as well as in MPCK. The differences were always in favor of those future primary teachers whose first language matched the official language of instruction in teacher education. It seems as if the education systems in these countries set students with a different language at a disadvantage. This inequity should be of major concern—specifically given the fact that there were seven countries which seemed to be successful in avoiding differential language effects.

Decomposing Background Effects As hypothesized, it was possible to decompose gender and language effects into *primary* and *secondary* effects. Taking the limitations of the TEDS-M data into account, primary effects were estimated as direct effects in regression analyses after controlling for program choice in those 8 countries where several teacher education programs were offered. Secondary effects were estimated in these countries as indirect effects combining gender or language effects on program choice and differential program outcomes. In countries with only one teacher education program, gender and language effects represented primary effects since they could not have been caused by program choice.

Overall, direct background effects were much more important for the gender- and language-related variability of the TEDS-M results than indirect effects due to program choice. Gender effects in Poland represented an exception from this picture. Here, large differences between male and female teacher achievement due to program choices were found. The two Polish teacher education programs (K-3 generalist teachers versus 4–9 mathematics specialist teachers) offered obviously such different opportunities to learn and job opportunities that these resulted in a significant gender split. At this point of research it has to remain an open question why the gender effects were specifically strong in Poland although some other countries, e.g. Germany, had a similar structure of teacher education without similar gender effects. The large secondary effect added in Poland to a large primary effect which led to the by far highest overall extent of gender inequity of all TEDS-M countries.

Challenges for Future Research From a methodological point of view, several remarks have to be made which in turn point to future research needs. An accurate estimation of background effects would only be possible in a longitudinal design, of course, whereas TEDS-M was a cross-sectional study. Thus, we can present only tentative results. Furthermore, results of regression analyses depend on the models specified and these depend on the variables available. In TEDS-M, we did not have measures of cognitive ability or characteristics of future teachers prior to teacher education. In this sense, direct gender and language effects represent composites of a long history of growing up and education. They cannot be related back only to teacher education. A true longitudinal study design would allow for decomposing the different effects. We had controlled for self-reported estimations of high-school success though. With only a few exceptions we did not find substantial changes in the results. This can be taken as a first indicator of the validity of our findings.

Summarizing the results about quality and equity of teacher achievement, we have to point out that none of the TEDS-M countries was successful on both indicators of teacher education effectiveness with respect to both background characteristics, gender as well as language. Singapore and Taiwan may be regarded as the most effective teacher education systems with high achievement and gender equity on MPCK and high achievement and language equity on MCK as well as in the case of Singapore on MPCK. For future research these results indicate the necessity to examine in more detail what characteristics in detail led to these favorable outcomes. We knew beforehand that the two East Asian are remarkably successful on the K-12 achievement level (Mullis et al. 2008). TEDS-M revealed now for the first time that the same success occurred on the teacher education level—and that it went along with strong gender equity. Presumably, not everybody would have expected this result beforehand.

Switzerland can also be regarded as relatively successful with achievement well above the international mean and gender equity on MPCK and language equity on MCK as well as on MPCK. These results are worthwhile to be pointed out because the country's context conditions are very much comparable to other Western countries which makes Swiss teacher education an interesting case for these.

Further studies should extend the present analyses of quality and equity as indicators of teacher education effectiveness to socio-economic status. In TEDS-M, SES was measured by the number of books available at home and by parents' education so that valid indicators exist. These analyses could shed light on the results presented here as well because the language spoken at home is correlated to SES. These kinds of analyses may therefore provide even stronger insight into background effects in tertiary education.

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In-depth Analyses of Different Countries' Responses to MCK Items: A View on the Differences Within and Between East and West

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Abstract This chapter looks into the MCK performance of future teachers by in-depth analyses which go beyond the overall MCK scores provided by the international TEDS-M study. The purpose is to identify factors that may describe the differences and similarities of performance between countries. Many new findings are revealed through a multifaceted analysis of cognitive subdomains and individual items on both the country and the cultural level. Our analysis identified six performance patterns based on the relative achievement in knowing, applying, and reasoning as cognitive subdomains. The performance distribution has a tendency to cluster culturally similar countries in the same group, but exceptions do appear.

We constructed a variable that models the difficulty of the cognitive subdomains. Based on this model, we identified the impact of cognitive elements on countries' performance. For example, we found that the two developed European countries, Norway and Switzerland, and almost all Eastern countries are strong on the reasoning element of items, which indicates a focus of their mathematics teacher education on reasoning.

The in-depth item analysis reveals new findings as well. Russia and the Philippines tend to employ uniform methods to solve problems, while the United States, Germany, Norway, Poland, and Taiwan tend to employ multiple methods. A tendency that the Western culture embodies an open and creative nature in their mathematics education is inferred. This study also finds a different philosophy in math-

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This chapter includes certain content from two articles: (1) Hsieh, F.-J., Lin, P.-J., & Wang, T.-Y. (2012). Mathematics related teaching competence of Taiwanese primary future teachers: Evidence from the TEDS-M. *ZDM—The International Journal on Mathematics Education, 44*(3), 277–292. doi:10.1007/s11858-011-0377-7; (2) Hsieh F.-J., & Wang, T.-Y. (2012). 中學數學職前教師之數學知能 [Mathematics competence of future secondary mathematics teachers]. In F.-J. Hsieh (Ed.), 臺灣數學師資培育跨國研究—*Taiwan TEDS-M 2008* (pp. 93–118). Taipei: Department of Mathematics at National Taiwan Normal University.

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S. Blömeke et al. (eds.), *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn*, Advances in Mathematics Education,
DOI 10.1007/978-94-007-6437-8_6,

ematics education relating to the rigor and formalism of acceptable mathematics solutions between the Eastern and the Western countries.

Keywords TEDS-M · Mathematics content knowledge (MCK) · Teacher education · International comparison · Cognitive effect · Cultural comparison

1 Introduction

Previous research has shown that teacher quality is a significant school-related factor influencing students' performance and learning in the classroom (Cobb et al. 1991; Rice 2003), but identifying and measuring the characteristics that constitute a qualified teacher remains a significant problem (Baumert et al. 2010; Hill et al. 2007). Many attempts have drawn on theoretical views, for instance the construction of conceptual frameworks of teacher quality for evaluation purposes (Ball et al. 2008; Baumert et al. 2010; Hill et al. 2004; Schmidt et al. 2011a). Different domains of teacher knowledge, such as pedagogical knowledge and content knowledge, have been pointed out as instructional determinants of student learning and applied to studies in many fields including mathematics (Ball and Bass 2003; Grossman and McDonald 2008; Hill et al. 2008; Krauss et al. 2008; Shulman 1986, 1987).

During the past two decades scholarly interest in international comparisons of mathematics teachers has increased (An et al. 2004; Ma 1999). Studies such as "Mathematics Teaching in the 21st Century" (MT21) have shown that different countries' future teachers achieved different results in their teaching knowledge and also had different opportunities to learn (Blömeke et al. 2008; Schmidt et al. 2011b). The "Teacher Education and Development Study in Mathematics" (TEDS-M) was the first data-based international study about mathematics teacher education with national representative samples. It provided participating nations with the opportunity to take an international perspective on their teacher education systems in areas such as future teachers' knowledge (Blömeke et al. 2011; König et al. 2011), their opportunities to learn (Schmidt et al. 2011b), and the quality of mathematics teacher education (Hsieh et al. 2011).

TEDS-M showed that the Taiwanese and Singaporean future teachers' achievement in mathematics content knowledge (MCK) ranked either first or second; however, the difference of their scores was large and statistically significant. This large difference is contrary to the situation that the countries achieved the same level with respect to their primary and lower-secondary students' performance in TIMSS. This phenomenon initiated an in-depth investigation into whether future teachers' responses from different countries or from countries with different cultural identities are different.

This chapter compares the MCK performance of future mathematics teachers from different countries and investigates the patterns emerging in terms of the relative strengths and weaknesses in the cognitive subdomains of MCK, namely, knowing, applying, and reasoning. The patterns are discussed from a cultural lens. In-depth analyses are applied to several items where the underlying cultural notion is examined to see whether they can be used to account for the MCK performance differences in the participating countries.

2 Conceptual Framework

This study utilizes the TEDS-M frameworks for MCK which include three cognitive subdomains: knowing, applying, and reasoning; as well as four content subdomains: number and operations, data and probability, geometry and measurement, and algebra and functions (Tatto et al. 2012).

According to TEDS-M, the subdomain "knowing" assesses future teachers' knowledge which includes recalling definitions and properties, carrying out algorithmic procedures, and retrieving information from graphics and tables. The subdomain "applying" assesses future teachers' abilities to apply the knowledge they possess in the first domain. For instance, future teachers should be able to solve routine or familiar types of problems, or generate representations for a given mathematical entity or relationship. The subdomain "reasoning" assesses more complex and deeper abilities of future teachers, such as combining various mathematical procedures, making connections between different elements of knowledge and related representations, and solving non-routine problems especially in unfamiliar or complex contexts (Tatto et al. 2012).

3 Research Method

3.1 Participants

This paper focuses on future primary and lower-secondary teachers in their last year of training from 15 countries, drawn from TEDS-M. The TEDS-M sampling plan followed a stratified multistage probability sampling design (Tatto et al. 2009). A minimum requirement of 75 % combined participation rate was set by the International Association for the Assessment of Educational Achievement (IEA) as a threshold. Samples having a participation rate of 60–75 % were also suitable for use but the low participation rate has to be annotated. Based on this criterion, our analyses included data from the following countries: Botswana, Chile, Germany, Georgia, Malaysia, Norway, the Philippines, Poland, Russia, Spain (participating only in the primary study), Switzerland, Singapore, Taiwan, Thailand, the USA, and Oman (participating only in the lower-secondary study).[1] Across countries, or even within

[1]In the primary study, the combined participation rates of Chile, Poland, Norway, and the United States were between 60 % and 75 %. Analyses for Norway were conducted by combining the two data sets available. In the lower-secondary level, the combined participation rates of Chile, Georgia, Poland, and the United States were between 60 % and 75 %. The combined participa-

Table 1 Distributions of test items by cognitive subdomains

Subdomain	Primary	Lower secondary
Knowing	32	24
Applying	29	34
Reasoning	12	18
Total	73	76

one country, separate programs existed, resulting in various definitions of "teaching grades for the primary level". For example, in Thailand there were two programs for teaching grades 1–12, while in Switzerland there was a program for teaching grades 1–2 exclusively. A thorough description regarding these grade spans can be found in the TEDS-M technical report (Tatto et al. 2009).

3.2 Measures

The TEDS-M study generated two future teacher questionnaires—one for the primary level and the other for the lower-secondary level—that included MCK tests. After deleting inappropriate items and accounting for combinations of items, TEDS-M used 73 or 76 MCK items in the primary or secondary study, respectively (Tatto et al. 2012). The distribution of test items by cognitive subdomain is shown in Table 1. It can be seen that the number of items in the reasoning subdomain is lower than in the other domains, especially in the primary level.

3.3 Data Processing and Analyses

Participants' responses to the MCK items were coded and scored according to the Item Scoring Guide developed by the TEDS-M consortium (Tatto et al. 2008). The scoring system for each constructed response item is a two-digit code. The first digit, either a 1 or a 2, indicates a correct or partially correct response and also signifies the number of score points given to that response. The second digit captures different approaches used by the future teachers.[2]

tion rate of Norway was 58 %, which only slightly missed the threshold of 60 % and therefore was still included. Datasets of four Norwegian program types were available, which were combined for analysis in an attempt to accurately represent the situation in Norway. Poland limited its participation to institutions with concurrent programs. Switzerland limited its participation to German-speaking regions. The United States limited its participation to public universities.

[2]For example, a response with a code 20 or 21 was scored as 2 points, whereas a code 10 or 11 was scored as 1 point.

For this chapter, several variables were either adopted directly or derived from the TEDS-M test. For each test item that scored one point, the percentage of correct answers in each country was computed (along with its standard error) and this statistic was called "item percent correct". For constructed-response items that scored two points, the item percent correct is the sum of the percentage of answers receiving the two points plus half of the percentage of answers scored as one point. For a set of items, the item percent corrects were averaged over this set to obtain an average percent correct. This statistic is called "percent correct" for that set of items. The international average percent correct was obtained by averaging over the percent corrects of all participating countries. When comparing two measures of a country or a measure of two countries, dependent or independent t-tests were applied at the $p < 0.05$ level.

For the cultural analysis, we roughly classified the countries into "the West" and "the East". This classification was based on cultural divisions as well as geographical areas that shaped a common educational culture. Each group was further divided into subgroups (Blömeke and Kaiser 2012; Leung 2006).The East cluster was grouped (1) by cultural demarcations forming Confucian Asia, including the two Asian countries, Singapore and Taiwan, having a Confucian tradition; and (2) by geographical areas as Developing Asia, including the three countries Malaysia, the Philippines, and Thailand, belonging to the Association of Southeast Asian Nations. The West group was divided into: (1) East Europe, including three countries Poland and Russia of East Europe and Georgia of Central Asia, coming from the former Eastern European bloc led by the Soviet Union; (2) Developed Europe, including Germany, Norway, Spain, and Switzerland belonging to the traditional developed European group; (3) American group, including Chile and the USA grouped according to geographical area; and (4) the others, including Botswana of Africa and Oman of West Asia. The last group will not be discussed as a whole in this chapter due to our not being able to identify their similarity by geographical region or historical tradition.

We acknowledge that this identification is not well defined and countries in different groups may still share common cultural heritages; for example, the Philippines were strongly influenced by the Spanish culture and also probably by the US culture because of colonization. Another example is that Malaysia, the Philippines, and Singapore share a common feature that uses English as the official language. But even without a theoretically based structure for classifying countries, our approach should be an acceptable beginning to distinguish mostly by geographical region or historical tradition.

4 Results and Discussion

Throughout the chapter, we adopt two approaches to present or interpret our data: one including the results of all participating countries and one including only the "higher achieving countries"—those countries that achieved results on the full MCK and MPCK scales above the international mean of 500.[3] The first approach is used when there is a need for providing a global view and the second is used to make a more focused interpretation by analyzing countries that performed well and may demonstrate important paradigms of mathematics teacher preparation.

4.1 Future Teachers' Achievement

4.1.1 Lower-Secondary Level

The comparison of percent corrects in cognitive subdomains among countries can illustrate their relative strengths and weakness. We are aware that the difficulty level of the content (e.g., tertiary-level content may be more difficult than secondary-level content) required to solve problems in different cognitive subdomains may differ and hence the differences of percent corrects between cognitive subdomains may partially arise from the content difference. However, a country with a higher percent correct in one subdomain than in another indicates a relative strength compared with the margin of another country between the two subdomains.

In general, the percent corrects in the reasoning subdomain were lower than those of the other two subdomains (see Table 2). For all countries, the difference between reasoning and applying was significant. The difference between reasoning and knowing was also significant in all countries except Norway and Switzerland. By contrast, for certain countries the percent corrects in knowing were higher than in applying, and for others those in applying were higher instead. When ordering the percent correct differences between knowing and reasoning, or between applying and reasoning, the countries did not form meaningful classes. However, when we ordered the countries by the percent correct differences between knowing and applying, countries formed four meaningful classes each with a unique pattern (see Table 2).

The distribution of percent corrects across countries revealed a tendency to cluster countries into cultural groups, but exceptions did appear. The first class contained countries from Developing Asia without Malaysia. This pattern (Pattern 1) was denoted by three statistically significant, gradually decreasing percent corrects from knowing, to applying, and to reasoning (see the upper-left graph in Fig. 1). The approach (relative descending or ascending) of the pattern deviated from the international pattern (shown by the dotted line) with descending from knowing to applying. The second class contained East Europe and Taiwan. This pattern (Pattern 2) exhibited a significant lower percent correct of reasoning but no significant difference between knowing and applying (see the upper-right graph in Fig. 1). This pattern was the pattern that matched the international pattern the best.

[3]The higher-achieving countries include Taiwan, Russia, Singapore, Poland, Switzerland, Germany, and the USA at the secondary level; and Taiwan, Singapore, Norway, Switzerland, Russia, Thailand, the USA, and Germany at the primary level.

Table 2 Percent corrects of cognitive subdomains of MCK (lower-secondary level)

Countries	Knowing	Applying	Reasoning	Diff KA	Diff AR	Diff KR	pKA	pAR	pKR	Pattern
Philippines	40 %	35 %	24 %	5.2 %	10.6 %	15.9 %	0.00	0.00	0.00	1
Thailand	48 %	43 %	35 %	4.8 %	7.8 %	12.6 %	0.00	0.00	0.00	1
Poland	62 %	60 %	44 %	1.8 %	16.2 %	18.1 %	0.10	0.00	0.00	2
Georgia	35 %	34 %	25 %	0.9 %	9.1 %	10.0 %	0.67	0.00	0.00	2
Taiwan	82 %	82 %	78 %	−0.1 %	3.8 %	3.7 %	0.94	0.00	0.00	2
Russia	68 %	70 %	55 %	−2.4 %	14.9 %	12.5 %	0.15	0.00	0.00	2
Singapore	64 %	66 %	55 %	−2.5 %	10.8 %	8.2 %	0.00	0.00	0.00	3
Malaysia	47 %	50 %	36 %	−2.9 %	14.2 %	11.3 %	0.00	0.00	0.00	3
Chile	20 %	25 %	14 %	−4.6 %	10.7 %	6.1 %	0.00	0.00	0.00	3
US-Public	48 %	52 %	40 %	−4.8 %	12.1 %	7.4 %	0.05	0.00	0.00	3
Germany	51 %	56 %	46 %	−5.1 %	10.3 %	5.1 %	0.00	0.00	0.00	3
Switzerland	52 %	57 %	51 %	−5.3 %	6.4 %	1.0 %	0.00	0.00	0.55	4
Norway	31 %	40 %	32 %	−9.3 %	8.5 %	−0.8 %	0.00	0.00	0.26	4
Oman[a]	49 %	44 %	27 %	5.0 %	17.1 %	22.1 %	0.00	0.00	0.00	NA
Botswana[a]	35 %	38 %	19 %	−3.0 %	19.2 %	16.2 %	0.14	0.00	0.00	NA
IA	49 %	50 %	39 %	−1.5 %	11.5 %	10.0 %				

IA = international average of percent correct; Diff KA = percent correct of knowing—applying, pKA = *p*-value for the significance of the difference between knowing and applying. Diff AR, pAR, Diff KR, pKR are abbreviated in the same manner with K standing for knowing, A for applying, and R for reasoning

[a]Country does not belong to any cultural group and was not included in classification

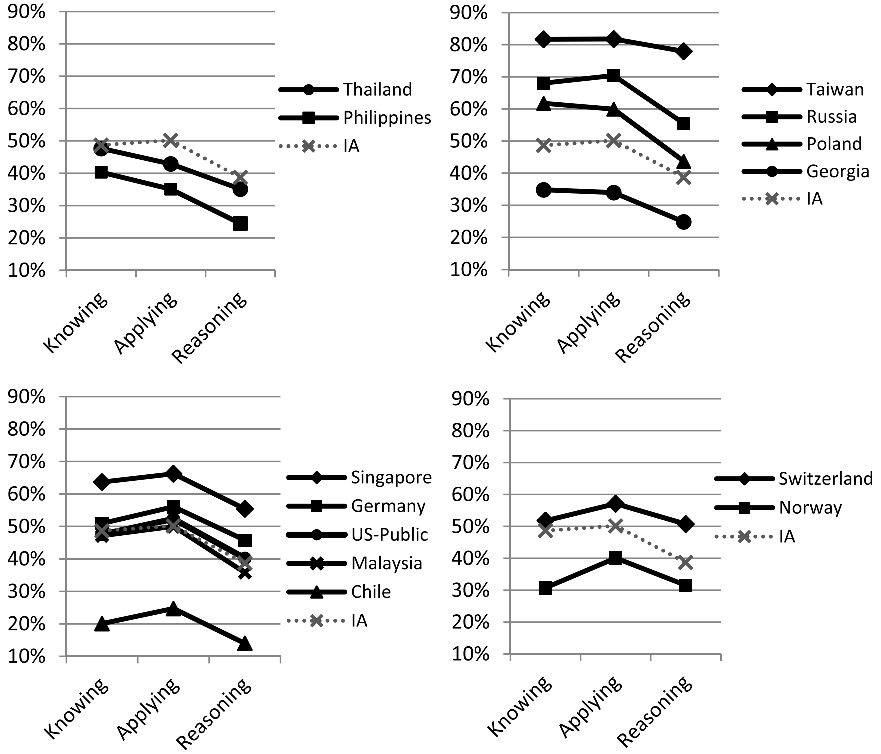

Fig. 1 Four classes of countries by percent corrects between cognitive subdomains at the secondary level. IA = international average of percent corrects

The third class was the largest one, consisting of the American group and two joint countries from Eastern Asia, Malaysia and Singapore, and Germany. Future teachers in this pattern (Pattern 3) significantly performed best in applying, then in knowing, and worst in reasoning (see the lower-left graph in Fig. 1). The approach of this pattern deviated from the international pattern by there being a significant ascent from knowing to applying, rather than a slight ascent. The last class contained two countries in Developed Europe. Future teachers in this pattern (Pattern 4) also performed best in applying as Pattern 3 but significant differences between knowing and reasoning were not seen (see the lower-right graph in Fig. 1). This pattern deviated from the international pattern by there being no descents from knowing to reasoning.

4.1.2 Primary Level

Also on this level, reasoning had the lowest international average of percent corrects (see Table 3). When ordering the percent correct differences between knowing

Table 3 Percent corrects of cognitive subdomains of MCK (primary level)

Countries	Knowing	Applying	Reasoning	Diff KA	Diff AR	Diff KR	pKA	pAR	pKR	Pattern
Singapore	79 %	76 %	70 %	2.5 %	6.1 %	8.6 %	0.01	0.00	0.00	5
Switzerland	71 %	67 %	62 %	4.3 %	5.6 %	9.8 %	0.00	0.00	0.00	5
Thailand	68 %	64 %	58 %	4.3 %	5.6 %	9.9 %	0.00	0.00	0.00	5
Taiwan	84 %	80 %	75 %	3.8 %	4.7 %	8.5 %	0.00	0.00	0.00	5
Germany	63 %	61 %	58 %	2.6 %	2.5 %	5.0 %	0.00	0.02	0.00	5
Russia	69 %	63 %	62 %	5.5 %	0.8 %	6.3 %	0.00	0.71	0.00	6
US-Public	65 %	61 %	61 %	4.5 %	0.1 %	4.5 %	0.00	0.96	0.00	6
Philippines	48 %	43 %	43 %	5.7 %	−0.6 %	5.1 %	0.01	0.83	0.07	6
Chile	43 %	37 %	38 %	5.5 %	−1.4 %	4.1 %	0.00	0.14	0.00	6
Spain	59 %	51 %	53 %	7.2 %	−1.6 %	5.6 %	0.00	0.23	0.00	6
Malaysia	56 %	61 %	51 %	−4.7 %	9.3 %	4.7 %	0.00	0.00	0.00	NA
Norway	64 %	64 %	60 %	0.2 %	3.7 %	3.9 %	0.81	0.00	0.00	NA
Botswana[a]	47 %	47 %	44 %	0.1 %	2.5 %	2.6 %	0.95	0.37	0.44	NA
Georgia	28 %	26 %	28 %	1.8 %	−1.8 %	0.0 %	0.07	0.11	0.98	NA
Poland	59 %	53 %	56 %	5.7 %	−3.0 %	2.7 %	0.00	0.00	0.00	NA
IA	60 %	57 %	55 %	3.3 %	2.2 %	5.4 %				

IA = international average of percent correct; Diff KA = percent correct of knowing—applying, pKA = *p*-value for the significance of the difference between knowing and applying. Diff AR, pAR, Diff KR, pKR are abbreviated in the same manner with K standing for knowing, A for applying, and R for reasoning

[a] Country does not belong to any cultural group and was not included in classification

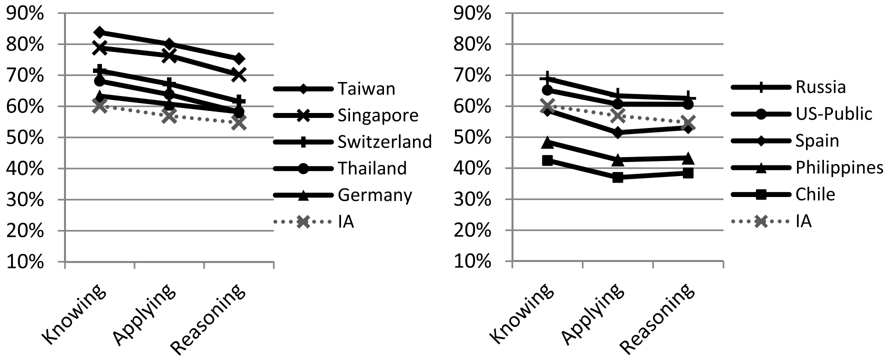

Fig. 2 Two classes of countries by differences of percent corrects between cognitive subdomains at the primary level. IA = international average of percent corrects

and applying or between knowing and reasoning, the countries did not form meaningful classes. However, when we ordered the countries by the percent correct differences between knowing and reasoning, countries formed two meaningful classes with unique patterns, with the exceptions of Malaysia and Norway.

The first class contained five countries, including the two Confucian Asia countries and two countries from Developed Europe,[4] Switzerland and Germany. This pattern (Pattern 5) was denoted by three statistically significant, gradually decreasing percent corrects from knowing, to applying, and to reasoning (see the left graph in Fig. 2). This pattern is similar to Pattern 1 at the secondary-level study. All these countries belonged to the higher-achieving country group. It is possible that high performance was a factor that shaped these countries' cognitive pattern. The approach of this pattern matched with that of the international average.

The second class also contained five countries, including the two in the American group. Among the remaining three countries,[5] only the Philippines was regarded as an East country. However, as a country colonized by Spain for more than 300 years and colonized by the USA for about 50 years, Philippine culture has a strong Western culture heritage. It is possible that the shared cultural tradition was a factor shaping these countries' patterns (Pattern 6). The approach of this pattern deviated from the international trend by exhibiting an ascending back from applying to reasoning (see the right graph in Fig. 2).

[4]Norway, though not strictly adhering to this pattern, was close to it by a non-significant deviation between the percent corrects of knowing and applying.

[5]Though the Philippines did not show a significant difference between knowing and reasoning at the 0.05 level, the $p = 0.07$ was close to 0.05 and was regarded as acceptable for the purpose of testing our approach.

4.1.3 Cognitive Effects for Both Primary and Secondary Level Studies

One may argue that knowing should be the easiest level of cognition and the higher percent correct in applying in the third and fourth patterns seemed unreasonable. Similarly, since reasoning deals with non-routine problems, especially in unfamiliar or complex contexts, it was expected to be more difficult than knowing, which conflicted with the pattern in the fourth class. The conflicting results prompted us to look into the underlying content.

Our purpose was to determine whether the percent correct differences across cognitive subdomains were, in fact, a consequence of cognitive elements required in different subdomains rather than merely an effect of the content difficulty. We were aware that it was impossible to actually calculate a "pure content difficulty degree" of an item because once it was tested the item difficulty was determined by its whole characteristics, including those relating to cognition, content, wording, and readiness to students. To resolve this problem, we used an idea from mathematics modeling. A measure was developed to model (represent) the content difficulty of the cognitive subdomains. Our model used the concept that mathematics is a field with a rigorous structure, that the understanding of new content must be based on the mastery of content learned previously. This hierarchical nature may best be modeled by the trajectory of the content; and the curricular level may be a rough but practical measure of the trajectory considering the feasibility of the TEDS-M data. The idea was that the difficulty of the content gradually increased from the primary, to lower-secondary, to upper-secondary, to the tertiary level.

The new measure was named "content percent correct estimate" (CPCE). For every country, its CPCE was calculated as follows. For each cognitive subdomain, the proportion of items at different curricular levels was obtained, representing the weight of items on the different curricular levels. A country's percent correct for the four curricular levels was obtained, representing the content difficulty degrees of the four curricular levels. Then a country's CPCE for a cognitive subdomain was obtained by calculating the country's percent corrects of the subdomain weighted with the proportions of items by curricular levels (see a formula explanation in the footnote[6]). The CPCE was an expected value standing for the content difficulty degree of that cognitive subdomain. In this manner, when a country's actual percent correct of a specific cognitive subdomain differed from its expected CPCE, we regarded this as an indicator of the cognitive effect.

All the MCK items were first re-classified into four categories according to curricular levels (by Taiwan's definition of "curricular levels") by three experts of the

[6] Suppose a country's percent corrects for items across the four curricular levels are: IA_p, IA_l, IA_u, and IA_t for primary, lower-secondary, upper-secondary, and tertiary; and the percentages of items for a cognitive subdomain, say knowing, for the four curricular levels are P % (dividing number of items in primary level by number of total items), L %, U %, and T %. The CPCE for the knowing subdomain for this country is obtained as $IA_p \times P\,\% + IA_l \times L\,\% + IA_u \times U\,\% + IA_t \times T\,\%$, which represents the content difficulty degree of the knowing subdomain for this particular country.

Taiwanese mathematics curriculum. The results showed that, for the secondary-level study, the percent corrects across curricular levels decreased from the primary, lower-secondary, upper-secondary, to the tertiary levels, being 53 %, 50 %, 45 %, and 39 % respectively. For the primary-level study, the percent corrects also decreased from primary to upper-secondary, being 61 %, 54 %, and 44 % respectively.[7]

For the secondary-level study, the CPCE (at the international level) for knowing, applying, and reasoning subdomains were 44 %, 47 %, and 46 % respectively; those for the primary-level study were consistent across all subdomains as 56 %, 56 %, and 56 % respectively.[8]

One-sample *t*-tests were used to test if the actual percent corrects of the cognitive subdomains were significantly different from the CPCEs. Table 4 shows that the cognitive effects were stronger on the primary level than on the lower-secondary level. Only seven countries were influenced by either one or two cognitive elements at the lower-secondary level, but 11 countries were influenced by one to three cognitive elements at the primary level. Reasoning often functioned as a factor promoting future teachers' performance, but applying often lowered their performance. A possible explanation was that for some countries, future teachers were taught in ways requiring reasoning activities such as transformation of representations or solving non-routine problems. For the primary level, the performance of future teachers in many countries was raised when a knowing element was included. By contrast, Norway and the Philippines at the secondary level tended to respond worse when a knowing element was presented. This may be a reflection of a shortage of the coverage of content tested in these two countries' curricula, so hindering future teachers' concept recalls or carrying out of procedures.

Regarding the cognitive patterns mentioned, countries in the same patterns tended to be affected by similar cognitive elements. However, individual differences exist within patterns. The two Developed Europe countries, Norway and Switzerland, and all Eastern countries at both levels, except Philippines and Taiwan (at secondary level), had higher performance on the reasoning element, which may show a focus of their mathematics education for teachers on reasoning. The fact that Taiwan was not affected by any cognitive elements at the secondary level may be a reflection of its high achieving on all items; thus no matter what cognitive elements were embedded in the items, the items were not difficult for Taiwanese future teachers. By contrast, the countries achieving lowest MCK also tended not to be affected by any cognitive elements, suggesting that the items were all too difficult for them.

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[7]There were no tertiary-level MCK items in the primary-level study.

[8]This result is consistent with the chi-square test result expressed in Hsieh et al. (2012).

Table 4 Positive or negative effects of cognitive elements on the performance of future teachers in different countries

Country	Knowing	Applying	Reasoning	Pattern
Secondary				
Norway	–		+	4
Switzerland			+	4
Thailand		–	+	1
Philippines	–			1
Chile		+		3
Singapore			+	3
Malaysia			+	3
US-Public				3
Germany				3
Taiwan				2
Georgia				2
Poland				2
Russia				2
Oman				NA
Botswana				NA
Primary				
Switzerland	+	–	+	5
Singapore	+	–	+	5
Taiwan	+	–	+	5
Germany		–	+	5
Thailand		–	+	5
Norway	+	–	+	NA
Malaysia		–	+	NA
Poland		+	–	NA
Botswana			+	NA
Georgia			–	NA
Spain	+			6
Chile				6
US-Public				6
Russia				6
Philippines				6

+ refers to a higher performance on the corresponding cognitive subdomain; – refers to a lower performance on the corresponding cognitive subdomain

4.2 In-depth Analysis of Secondary Future Teachers' Responses on MCK Items

We now turn to the in-depth analyses to examine whether the cultural effect played a role in the future teachers' responses. Most items we picked were from the TEDS-M released items, with which readers are likely to be familiar. Thus the selection was somewhat limited. Some of the descriptions were available from a paper written by Hsieh and Wang (2012) in Chinese. The balance among the cognitive subdomains was not a concern in this section.

4.2.1 A Lower-Secondary Level Item Example

#604A is an algebra item and its cognitive subdomain is applying (see Fig. 3). TEDS-M future teachers were required to solve A1 and A2. To obtain the correct answers, future teachers had to find out the relations between the three people. In A1, future teachers could use David as the unit to measure Peter and James in order to set up algebraic equations in a relatively straightforward manner. In A2, there was no such obvious unit and if future teachers applied a similar method by using Wendy as the unit, then fractions appeared and resulted in a more tedious calculation.

The international average of the percent correct of A1 was higher than that of A2 by 23.4 %, which meant that A2 was much more difficult than A1 (see Fig. 3). Taiwan was the only country that showed no significant difference between A1 and A2—the complexity raised by A2 seemed not to bother Taiwanese future teachers.

Regarding the methods adopted by future teachers, the proportions of codes received among those who correctly solved the problems are shown in Fig. 3. Russia, Georgia, Philippines, and Botswana employed the same method to solve A1; the former two used the same method to solve A2.[9] By contrast, many countries employed multiple methods to solve both A1 and A2, showing the open and creative approach of their mathematics education. These countries included the USA, Germany, Norway, Poland, and Taiwan. All of them but Taiwan are Western countries.[10] About 10 % or a little more of the future teachers in Malaysia, Taiwan, and Chile used the ratios of the three objects (code 14) to solve the problems. The ratio method was not as procedural as the methods involving setting up equations. It demands fewer abstract symbols than the equation method but requires an ability to shorten the reasoning process (Krutetskii 1976; see Fig. 4 for examples of responses from Taiwan).

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[9] For A1, more than 90 % of the future teachers from the mentioned countries employed the method receiving code 11; for A2, more than 80 % of the mentioned countries receiving code 11.

[10] For both A1 and A2, countries in this group had less than 70 % same-method responses. For A2, all Western countries in this group had less than 55 % same-method responses.

Item	Content Domain	Cognitive Domain
#604A	Algebra	Applying

The following problems appear in a mathematics textbook for lower secondary school

A1. Peter, David, and James play a game with marbles. They have 198 marbles altogether. Peter has 6 times as many marbles as David, and James has 2 times as many marbles as David. How many marbles does each boy have?

A2. Three children Wendy, Joyce and Gabriela have 198 zeds altogether. Wendy has 6 times as much money as Joyce, and 3 times as much as Gabriela. How many zeds does each child have?

Solve each problem.

<Rubric>（partial）

Codes for full credit：

11：Using one variable, setting one equation and solving the problem correctly.

12：Using more than one variable, establishing a system of equations, performing substitutions and solving the problem correctly.

13：Using trial and error or guess and check and solving the problem correctly.

14：Using ratio or other arithmetic methods and solving the problem correctly.

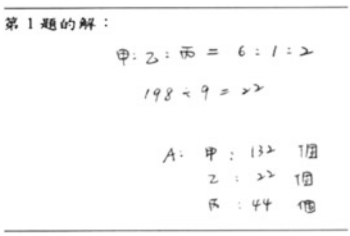
15：Using representation or diagram and solving the problem correctly.

Country	PC %	Proportion of Code (%)				
		11	12	13	14	15
A1						
Taiwan	98	69	**21**	0	**9**	0
Singapore	96	73	8	0	5	**14**
Switzerland	94	74	**26**	0	0	0
Russia	91	**98**	1	0	0	0
Poland	86	65	**34**	1	0	0
Germany	80	59	**39**	0	1	0
Malaysia	79	74	9	1	**16**	0
US-Public	72	56	**30**	**10**	3	0
Oman	69	46	**52**	1	1	0
Norway	67	62	12	**13**	5	1
Botswana	62	**90**	5	0	5	0
Philippines	54	**89**	8	2	1	0
Georgia	50	**92**	4	0	3	0
Thailand	50	83	16	0	0	0
Chile	31	75	3	**13**	9	0
IA	72	73	19	2	4	1
A2						
Taiwan	97	64	**22**	0	**11**	0
Singapore	89	60	12	1	6	**21**
Russia	81	**91**	9	0	0	0
Switzerland	71	65	**31**	3	0	0
Poland	61	47	**52**	1	0	0
Germany	59	49	**48**	1	2	0
Malaysia	52	46	**39**	2	**13**	0
US-Public	49	41	**46**	5	4	0
Norway	35	55	16	**11**	3	1
Oman	32	16	**81**	2	0	0
Georgia	31	77	7	**10**	5	0
Thailand	30	64	**22**	0	**11**	0
Botswana	21	60	12	1	6	21
Philippines	12	**91**	9	0	0	0
Chile	8	65	**31**	3	0	0
IA	48	47	52	1	0	0

Fig. 3 Future teachers' performance in #604A. PC = percent correct; IA = international average of all participating countries. The Proportion of Code for a specific code was obtained by dividing the amount of that code by the total number of correct codes and changing the unit to %

For the trial and error method, none of the Eastern countries exceeded 2 %. However, future teachers from the USA and Norway favored this method the most compared with other countries. In Taiwan, this method is only encouraged at the problem-solving process stage but not at the writing out solution stage since it is considered as informal and not rigorous. The two Western countries' views were therefore quite different from these Eastern countries.

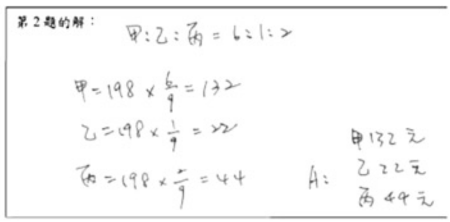
As for Singapore, another East Asian country performing equally to Taiwan in TIMSS, their future teachers had a greater tendency than other countries to use representation or diagram to solve the problem (code 15). This is an interesting issue for further investigation.



Translation:
Solution for A1

$$A:B:C=6:1:2$$
$$198\div 9=22$$

Answer: A: 132 marbles
B: 22 marbles
C: 44 marbles



Translation:
Solution for A2

$$A:B:C=6:1:2$$
$$A=198\times\frac{6}{9}=132$$
$$B=198\times\frac{1}{9}=22$$
$$C=198\times\frac{2}{9}=44$$

Answer: A:132 zeds
B: 22 zeds
C: 44 zeds

Fig. 4 Taiwanese future teachers' original responses to #604A1 and #604A2 with the translations

4.2.2 A Tertiary-Level Item Example

The above item #604 assessed future teachers' ability to transform information into abstract symbols, and to manipulate them suitably, somewhat in a routine manner. Item #708 assessed the ability to apply an abstract concept to determine if it applied in a different context. It was relatively non-routine and demanded more abstract thinking. Indeed, #604 is a lower-secondary level item and #708 is a tertiary-level item.

#708 asked future teachers to determine whether the four relations given were equivalent relations or not (see Fig. 5). The objects used in options A, C, and D were related to mathematical objects while option B concerned real-life objects. Taiwan and Russia outperformed all other countries by about 10 average percent corrects: Taiwan outdid other countries in options A, C, and D while Russia outdid other countries in A and B. This difference revealed the possibility of different focuses for different countries in mathematics education. Taiwan's percent correct in B fell dramatically to seventh place, indicating less emphasis on real-life connection in tertiary-level mathematics. The countries' differences of percent correct in A and B ranged from 34 for Taiwan to −2 for Germany. Seven countries had a difference bigger than 15, while eight had a difference smaller than 10. All the Eastern countries belonged to the former group except Singapore, indicating these countries focus lesser on real-life situations than on certain mathematics context. A common approach of the East was seen here with the exception of Singapore, whose data may result from their "practical" approach to teacher education which de-emphasizes formal and abstract mathematics (Hsieh et al. 2013).

Item	Content Domain	Cognitive Domain
#708	Number and operations	Applying

<Description of the item>

This item requires future teachers to determine whether each of the four relations given is an equivalent relation or not. The relations are spread over different contexts.

A. An equivalent relation in geometry.

B. An equivalent relation in the real-life context.

C. A non-equivalent relation in a number set.

D. An equivalent relation in a number set (the relation and the number set are both different from C).

Country	Percent Correct (%)				
	A	B	C	D	CA
Taiwan	83	49	71	70	68
Singapore	**48**	**46**	**42**	**40**	**44**
Thailand	*67*	*51*	*37*	*47*	51
Philippines	*76*	*49*	*20*	*44*	47
Malaysia	*64*	*40*	*22*	*51*	44
Germany	**48**	**50**	**50**	**47**	**49**
Switzerland	40	32	40	46	39
Norway	43	14	26	26	27
US-Public	**61**	**59**	**50**	**52**	**56**
Chile	33	28	16	23	25
Russia	86	69	60	53	67
Poland	**64**	**63**	**51**	**54**	**58**
Georgia	36	28	25	9	24
Oman	64	37	57	40	49
Botswana	35	25	18	25	26
IA	57	43	39	42	45

Fig. 5 Future teachers' performance in #708. PC = percent correct; IA = international average of all participating countries; CA = country's average of all items as each weighted 1

In contrast to the sharp drops of percent corrects in some options, several countries exhibited a relatively equal level of percent corrects across the four options. These countries were Singapore from the group of Confucian Asia, Germany from Developed Europe, the USA from American group, and Poland from East Europe (see Fig. 5). This result may suggest that the content overwhelmed the cultural impact on the focus of mathematics education in the teacher preparation period in many cultural groups. Countries in Developing Asia shared a common pattern with a relatively high percent correct in A, a large drop in B, again a large drop in C, and then a rebound in D.

4.2.3 An Upper-Secondary Item Example

Due to the different patterns of performance in handling abstract content versus dealing with real-life situations showed in the tertiary-level item #708, we selected item #710 to further investigate the ability related to real-life mathematical modeling in the upper-secondary level curriculum.

Item	Content Domain	Cognitive Domain
#710	Algebra	Applying

Indicate whether the following situations can be modeled by an exponential function.

B. The amount of money *A* in a bank after *w* weeks, if each week *d* zeds are put in the bank.

C. The value *V* of a car after *t* years if it depreciates *d* % per year.

ANSWER KEYS:

B. No.

C. Yes.

Country	PC-B %	PC-C %	Diff CB %
US-Public	29	75	47
Philippines	28	66	39
Taiwan	59	95	36
Thailand	27	59	32
Norway	35	61	26
Malaysia	40	62	22
Botswana	24	45	21
Oman	32	52	20
Georgia	24	40	17
Switzerland	59	74	15
Chile	15	28	12
Germany	61	72	11
Poland	35	46	11
Russia	45	54	9
Singapore	68	70	3
IA	39	60	21

Fig. 6 Future teachers' performance in #710B and #710C. PC-B = percent correct for option B; PC-C = percent correct for option C; Diff CB = difference obtained by subtracting PC-B from PC-C; IA = international average of all participating countries

#710B and #710C both assessed abilities involving mathematical modeling, and both were set in a similar financial context (see Fig. 6). The situation in option B may be modeled with $w \times d$ for the interest after w weeks, a very simple expression without using exponents, while the expression for modeling C includes $(1 + d\ \%)^t$, a much more complicated exponential expression. However, all countries performed better in C than in B. The percent correct differences of these two options for the participating countries ranged from 3 % to 47 % with an international average of 21 %. We were interested in what might be the factors accounting for the inverted performance of these two options in terms of the content difficulty.

As for Taiwan, it is quite common to use compound interest as an illustrative example in the teaching of exponential functions. It is therefore not surprising if Taiwan future teachers mistook #710B as a compound interest situation. On the other hand, with the emphasis on real-life mathematics and the tradition of free market capitalism, the US future teachers should also be familiar with the idea of compound interest and happened to make an unexpected error in #710B. It is interesting to notice that the percentage of error in #710B, $100\ \% - 28.5\ \% = 71.5\ \%$, was quite close to the percentage of correctness in #710C, 75 %. Although both the USA and Singapore particularly emphasize real-life mathematics, Singapore managed to avoid this error. Does this mean Singapore future teachers are more cautious? Or does this support the finding in item #708 that Singapore's de-emphasis on abstract content such as exponential functions and focus on practical mathematics has enabled the future teachers to connect mathematics content and real-life situations? These are still open questions requiring further investigation.

Item	Content Domain	Cognitive Domain
#509	Algebra and Functions	Reasoning

Students who had been studying algebra were asked the following question:

For any number n, which is larger, 2n or n + 2?

Give the answer and show your reasoning or working.

<Rubric> (partial)

Type A

20: General arguments either with words or inequalities.

10: On the right track of 20, but incomplete or limited.

Type B

22: Correct, ordered, specific-value checking and making general conclusions.

12: On the right track of 22, but incomplete or limited.

Country	PC	% received each code			
		20	22	10	12
Taiwan	42	29	8	0	15
Singapore	42	15	10	3	39
Malaysia	39	17	10	4	26
Thailand	28	13	14	1	13
Philippines	9	0	0	0	18
Norway	37	11	13	5	28
Switzerland	35	20	17	1	10
Germany	30	17	20	0	6
Spain	7	5	3	0	1
US-Public	24	5	9	1	29
Chile	5	0	1	2	6
Russia	27	15	11	0	11
Poland	17	7	8	1	9
Georgia	2	1	1	0	1
Botswana	5	4	0	0	3
IA	23	11	8	1	14

Fig. 7 Future teachers' performance in #509. PC = percent correct; IA = international average of all participating countries

Figure 6 also shows that the higher-achieving countries in the East Europe group performed relatively weakly. Whether their performance relates to the economic philosophy during the Soviet Union-led period remains an open question. Another noticeable result also shown in Fig. 6 is that the magnitudes of the differences between percent corrects in C and in B were all larger than the international average for all the Eastern countries except Singapore. This finding is consistent with the finding in item #708, indicating a lesser emphasis in Eastern countries on real-life connection with mathematics in both tertiary- and secondary-level mathematics.

4.3 In-depth Analysis of Primary Future Teachers' Responses on MCK Items

4.3.1 An Item in the Reasoning Subdomain

Item #509 was chosen to exhibit future teachers' competence in reasoning mathematically, including devising formal and informal mathematical arguments, and transforming heuristic arguments to valid proofs (see Fig. 7).

Item #509 displays three types of correct answers, though only two of them are particularly worthy of note, namely, Type A (code 20 and 10) and Type B (code 22 and 12). Taiwanese future teachers provided a greater number of correct or partially correct Type A solutions than Type B ones. In contrast, Singapore, the other Confucian heritage country, had more Type B responses than Type A. One could argue that no Type B response should be awarded full credit because it lacks generalization to rigorously validate the reasoning; however, Type B responses do successfully show the responders' chain of reasoning and thus, if reasoning is valued over rigorous proof, the value of Type B responses can be seen.

An examination of the preference between Type A and Type B for the countries along cultural groupings showed that the disparity in Confucian Asia was also found in all other cultural groups. This may suggest that the content of teacher preparation overwhelmed the cultural impact. However, for the commonality between Singapore and the USA (both preferring Type B), cultural factors may be involved. Studies have documented that the role played by the English language in the mathematics textbooks and the roots of the Singaporean mathematics curriculum from American and British literature cause Singaporean textbooks to be somewhat like those used in the USA (Li and Ginsburg 2006; Yeap et al. 2006).

The lower percentage of Type B responses from Taiwanese future teachers again confirmed that the Taiwanese system is one that values formalism and closely associates it with the explicit expression of one's reasoning processes. This conclusion gains support when one examines the percentage of attempts to answer this item, with at least partial success. While Taiwan ranked first in overall percent corrects, Taiwanese future teachers had fewer attempts (54 %), whereas other countries such as Norway and Singapore had more (58 % and 67 %, respectively). In other words, when incapable of providing formal proofs, Taiwanese future teachers tended not to try a more natural heuristic approach to show their reasoning. Figure 8 provides four examples of Taiwanese future teachers' answers to show their Type A (Example 1: code 20; Example 4: code 10) and Type B answers (Example 2: code 22; Example 3: code 12).

4.3.2 An Item in the Knowing Subdomain

In order to exemplify future teacher handling and manipulation of statements or expressions containing symbols and formulae, the authors chose item #207 (see Fig. 9).

This item involves translation from natural language to symbols; however, a direct translation does not lead to successfully solve the problem. There are two keys needed to successfully solve the problem. First, problem solvers have to transform between quantities to correctly express the quantitative relationship of objects. Second, x and y should be viewed as variables representing numbers rather than the labels of objects A and B. If a future teacher fails to do this, she might make a "reversal error" (Clement 1982), which entails seeing x and y as labels and the quantities as adjectives to describe the unknowns. Taiwanese future teachers performed significantly better than all participating countries (see Fig. 9). However, there were still 29.7 % of future teachers that made a reversal error (A2 and A3). Though this percentage was high, it was still the lowest among all the participating countries—all other countries ranged between 45 % and 74 % and the international average was large at 53 %. An examination of the wordings of this item

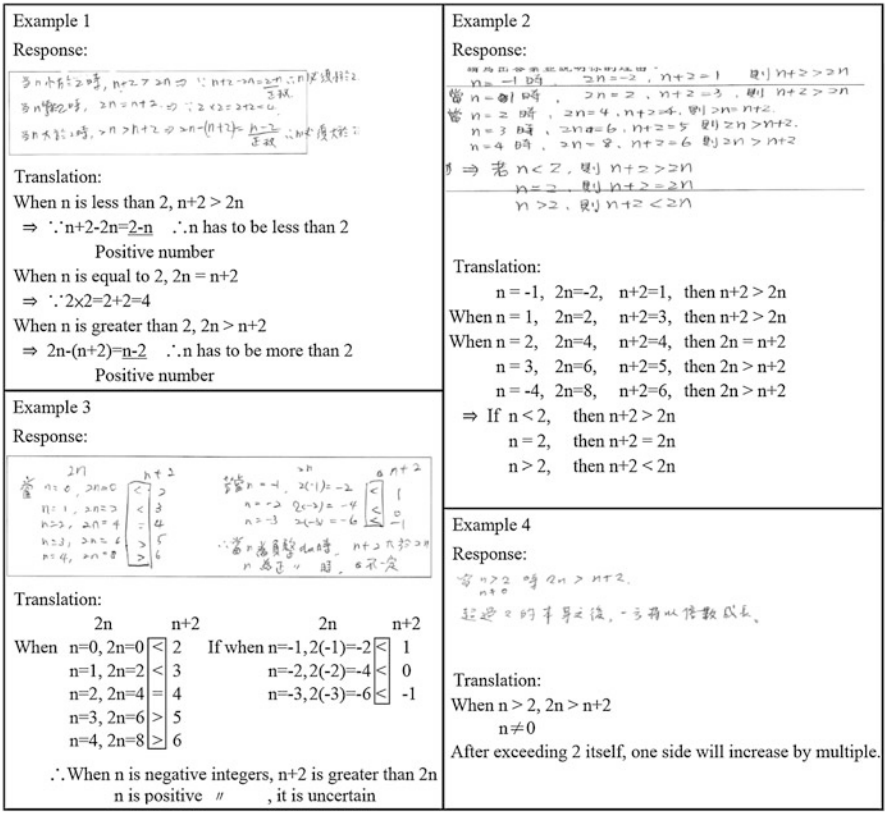


Fig. 8 Taiwanese future teachers' original responses to #509 and the translations

in the English and Chinese versions revealed syntax structure dissimilarities which changed the relative orders of the quantities and variables. Whether this kind of variance affects the solutions of this type of problem may require further investigation.

Both Confucian heritage countries outperformed almost all other countries, including their Western and Developing Asia counterparts. This may relate to the algebra curriculum. A study examining the curricula of Taiwan, Singapore, and the USA found that Taiwanese and Singaporean algebra curricula emphasized cultivating lower secondary students' abilities to write expressions or equations according to the context of problems, while this was not the main focus of the American curriculum (Chen and Yang 2010). For comparison with other Western countries and Developing Asia, further studies are needed to examine the differences embedded in the curriculum that possibly reflect the cultural influence.

<table>
<tr><th>Item</th><th>Content Domain</th><th>Cognitive Domain</th><th>Country</th><th colspan="4">Percentage</th></tr>
<tr><td>#207</td><td>Algebra</td><td>Knowing</td><td></td><td>A1</td><td>A2</td><td>A3</td><td>A4</td></tr>
<tr><td colspan="3" rowspan="16"><Description of the item>
A quantity relationship of two objects, say A and B, is given with a certain percentage in the stem. The symbols x and y are assigned to represent the numbers of A and B respectively. Future teachers were asked to choose a correct algebraic equation from four options to represent the quantity relationship.
<Rubric> (partial)
A1: Correctly transform the quantity in the stem, and correctly see x and y as variables not labels.
A2: Correctly transform the quantity in the stem, but see x and y as labels not variables.
A3: See x and y as labels not variables, and directly use the quantity in the stem without transforming it.
A4: See x and y as variables not labels, but directly use the quantity in the stem without transforming it.</td><td>Taiwan</td><td>64</td><td>11</td><td>19</td><td>5</td></tr>
<tr><td>Russia</td><td>41</td><td>12</td><td>33</td><td>13</td></tr>
<tr><td>Singapore</td><td>40</td><td>31</td><td>18</td><td>11</td></tr>
<tr><td>Switzerland</td><td>29</td><td>36</td><td>16</td><td>18</td></tr>
<tr><td>Malaysia</td><td>29</td><td>20</td><td>27</td><td>21</td></tr>
<tr><td>Norway</td><td>25</td><td>31</td><td>27</td><td>15</td></tr>
<tr><td>Poland</td><td>22</td><td>21</td><td>38</td><td>13</td></tr>
<tr><td>Germany</td><td>21</td><td>30</td><td>19</td><td>23</td></tr>
<tr><td>US-Public</td><td>19</td><td>17</td><td>44</td><td>20</td></tr>
<tr><td>Georgia</td><td>17</td><td>24</td><td>29</td><td>9</td></tr>
<tr><td>Thailand</td><td>16</td><td>10</td><td>44</td><td>26</td></tr>
<tr><td>Botswana</td><td>14</td><td>12</td><td>42</td><td>26</td></tr>
<tr><td>Spain</td><td>10</td><td>13</td><td>47</td><td>24</td></tr>
<tr><td>Philippines</td><td>8</td><td>21</td><td>53</td><td>16</td></tr>
<tr><td>Chile</td><td>6</td><td>19</td><td>37</td><td>8</td></tr>
<tr><td>IA</td><td>24</td><td>21</td><td>33</td><td>17</td></tr>
</table>

Fig. 9 Future teachers' performance in #207. A1 is the correct answer. PC = percent correct; IA = international average of all participating countries

5 Conclusions

Going beyond the overall MCK scores, this chapter has looked into the performance by cognitive subdomains and individual items. The purpose is to identify factors that may explain the differences and similarities of performance between countries. Many new findings have been manifested through a multifaceted analysis of cognitive subdomains and an in-depth analysis of individual items on both the country and the cultural level. Countries have been roughly classified as West or East countries, and two cultural subgroups for each of the West and the East have been further identified.

5.1 Impact of Cognitive Elements

This paper has identified six performance patterns based on the countries' relative achievement in knowing, applying, and reasoning as cognitive subdomains; four patterns in the secondary-level study and two in the primary-level study. The distribution of countries had a tendency to cluster countries into cultural groups, but exceptions appeared. For the secondary level, the East Europe countries belong to the same group accompanying Taiwan; the remaining East Asian countries split into three classes. The pattern was characterized by three gradually decreasing percent corrects from knowing, to applying, and to reasoning (Pattern 1). Although this sounds like a "natural" pattern, it contains only two East Asian countries, Thailand

and the Philippines. This pattern is similar to a primary-level pattern, Pattern 5, that contains more countries, especially those higher-achieving countries.

The reason for the gradual descent may be that a high performance in knowing is not sufficient for equal performance in the other cognitive domains. Pattern 5 is probably formed by high achievement in MCK (and MPCK). In contrast to the achievement factor, countries in the other pattern (Pattern 6) at the primary level may be shaped by cultural traditions, because it includes only one East Asian country, Philippines, which has a strong cultural heritage from Spain and the USA.

To deal with the possible effects due to the item content difficulty in different cognitive subdomains, this study uses an idea from mathematics modeling. A variable, named "content percent correct estimate" (CPCE) was developed to represent the content difficulty of each cognitive subdomain for each country. By comparing the actual percent corrects of any cognitive subdomain to its CPCE, we have identified various impacts of cognitive elements on countries' performance. For example, we found that the two developed European countries, Norway and Switzerland, and all Eastern countries at both levels, except the Philippines and Taiwan (at secondary level), benefit from the reasoning element, which may show a focus of their mathematics education for teachers on reasoning. The fact that Taiwan and the Philippines are not impacted by the cognitive elements may be a reflection of their achievements; Taiwan achieves high, thus no matter what cognitive elements an item embeds, they still feel easy. By contrast, the Philippines achieves low, thus may feel difficulty in all items with different cognitive elements.

5.2 Insights from the In-depth Item Analysis

Previous studies with in-depth item analyses have often been confined to a domestic scope. The in-depth analysis with a complement of international comparisons conducted in this chapter provides the researcher with the opportunity to uncover some unique insights.

An analysis of item #604 A1 and A2 has revealed that two countries, Russia, and Georgia tend to employ the same method to solve both items. By contrast, countries such as the USA, Germany, Norway, Poland, and Taiwan employ multiple methods. The latter group contains all Western countries, except for Taiwan. The use of multiple methods indicates an open and creative approach of their mathematics education. It is an open question what factors embodies an open and creative nature in their mathematics education. The analysis also reveals another distinction between the West and the East. None of the Eastern countries use the trial and error method, but future teachers from the USA and Norway in particular favor this method. This may show a different philosophy in mathematics education that relates to the rigor and formalism of acceptable mathematics solutions. More research is required to examine these different philosophies and their impact on student learning as well as on teacher education.

An analysis of item #207, examining the ability of handling and manipulation of statements or expressions containing symbols and formulae, revealed that internationally more than half of future primary mathematics teachers made a "reversal error" (Clement 1982). Even half of the Singaporean future primary teachers made this error. This is 20 % higher than the number of Taiwanese future teachers. This result raises a question concerning the construction of international tests of the TEDS-M-type: Did the wording of this item in English (as administered to future teachers in Singapore and many other countries) and in Chinese (as administered in Taiwan) change the syntax of the statements and thus result in different types of potential errors for responders? Further research on this question is required if more international tests of this scope are to be employed by mathematics pedagogy researchers.

A common phenomenon has been found by the in-depth analyses across different items and different study levels. This finding indicates that the East carries out an approach with less emphasis on real-life connection with mathematics, though Singapore is an exception. Singaporean data reveals emphasis on "practical" issues in teacher education practice and de-emphasis on formal and abstract mathematics. However, whether this distinction of Singapore from its East counterparts is a prevalent condition or not is still an open question requiring further investigation.

On the other hand, what makes the Western countries superior to their Eastern counterparts in dealing with the connection between real-life situations and mathematics is an interesting problem to investigate. Is it due to the recruiting practice, or teacher preparation program, or in-service professional development, or the utilitarian tradition in the Western culture? In order to answer these questions, further research is required. Due to the limitations of the TEDS-M dataset, we have had to be careful with drawing final conclusions. However, our data have provided us with an initial approach. More data from all countries are needed to further investigation.

Traditionally, Western/Greek cultures appreciate formalism more than Eastern/Confucian cultures in the sense that the axiomatic method was established by Euclid's *Elements* and this totally influences the way mathematics has developed. But nowadays, there seem to be more Eastern teachers, at least at the secondary and primary levels, who appreciate more the formalism or abstract thinking rather than the applications which mathematics can offer. On the other hand, Western teachers tend to appreciate real-life mathematics more than their Eastern counterparts. The boundaries made by different cultural values seem to be more blurred when they share and appreciate one another.

Acknowledgements We gratefully acknowledge the following: the IEA, the International Study Center at Michigan State University, the Data Processing Center, the ACER, the U.S. NSF, the Taiwan TEDS-M team, and all TEDS-M national research coordinators for sponsoring the international study and providing information and data. Taiwan TEDS-M 2008 was supported by the National Science Council and Ministry of Education.

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Why Did Taiwan Excel: Hot Topics and Pressing Issues

Feng-Jui Hsieh, Pei-Chen Wu, and Ting-Ying Wang

Abstract TEDS-M displayed the results of an international comparison of primary and lower-secondary future teachers' performances on MCK and MPCK. Taiwanese future teachers excelled on both levels. This study probes into the reasons by analyzing future teachers' performance corresponding to three different background factors: different systems of teacher preparation, different majors and different academic degrees. Overall, eight different models based on these factors existed. The results from paired comparison revealed that teachers from a department-based system performed significantly better than from a center-based system on both MCK and MPCK. Mathematics majors performed significantly better than non-math majors on both MCK and MPCK. For academic degrees, future teachers who had majored in mathematics with a master or doctoral degree were not better than those who had majored in mathematics with a bachelor degree. Regard the eight different models, we discovered that "major" is an important criterion influencing teachers' performance on MCK and MPCK. In addition, this study examined the relationship between Taiwan's excellent performance and two other factors, namely the OTL provided in tertiary level mathematics and the highest level of school mathematics. However, checking the percent correct on MCK and MPCK items our study reveals that Taiwanese primary and lower secondary level future teachers did not perform as well as expected according to our deeply-rooted standard. This stunning result has raised great concerns among teacher educators in Taiwan.

Keywords Mathematics content knowledge (MCK) · Mathematics pedagogical content knowledge (MPCK) · Department-based teacher education · Center-based teacher education · Mathematics major · Master degree · Ph.D.

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S. Blömeke et al. (eds.), *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn*, Advances in Mathematics Education,
DOI 10.1007/978-94-007-6437-8_7,

1 Introduction

As a participating country in TEDS-M study, Taiwan had the chance to examine how its teachers perform in mathematics content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK) compared with other countries. Using national representative samples, the outcomes of TEDS-M can be applied quite accurately to national populations. The results of the MCK and MPCK achievement of future teachers from Taiwan impressed the world: How can a small island train such high-quality future teachers? The results sometimes also surprised Taiwan scholars: Taiwan's percentages of correct answers for some "primary items with low-level of difficulty" were low (Krainer et al. 2012).

2 Literature Background and Research Questions

Ever since Shulman had introduced the concept of subject-matter content knowledge, pedagogical content knowledge and curricular knowledge, teacher knowledge has drawn many researchers' attention (Grossman 1995; Koirala et al. 2008; Shulman 1987; Wilson et al. 1987). In the field of mathematics, researchers have the same interest about the issue: whether teachers are equipped with sufficient MCK and MPCK or not. TEDS-M looked into this research question.

Many scholars have probed into "teacher knowledge" based on empirical studies. For MCK, Leinhardt and Smith (1985) pointed out the novice mathematics teachers' lack of knowledge about the structure of mathematics as well as the connections among mathematical concepts that students are going to learn. Capraro et al. (2005) found that some mathematics teachers were inadequately prepared for an understanding of mathematical concepts. For MPCK, Borko et al. (1992) found that some mathematics teachers in their study could not present mathematical concepts with correct representations or use appropriate mathematical representations for students. They even could not correctly explain certain computation procedures. Capraro et al. (2005) discovered that some teachers in their study could not distinguish and use appropriate pedagogical strategies to help students to structure their knowledge.

These researches showed thus the mathematics teachers' insufficiency in MCK or MPCK and they pointed out that this affects their teaching. Leinhardt and Smith (1985) discovered that teachers' mathematics knowledge influences their understanding of the lesson structure as well as the selection of examples, the formulation of explanations, and demonstrations. Cankoy (2010) indicated that if a teacher lacks MPCK he teaches students mainly procedural, fostering memorization. Therefore, it is important to explore the level of teachers' MCK and MPCK as well as which factors affect the performance of teachers.

TEDS-M displayed the performance of future primary and lower-secondary teachers on MCK and MPCK. Surprisingly, the difference between the highest country score and the lowest one is more than 2.5 standard deviations (Hsieh et al. 2010). This gap shows the difference of the levels of teacher knowledge between countries.

No matter whether it is the primary or the lower-secondary level, Taiwan always comes out on the top. In this paper, we examine why Taiwan excels.

Every country may have different viewpoints on which level of MCK future teachers should reach. In order to understand why Taiwan excels, we probed into the level of MCK from elementary to high-school level, or tertiary level mathematics, from Taiwan's perspective. According to that, every teacher should be familiar with not only the mathematics content of his teaching level but also with the notion that "teachers must know in detail and from a more advanced perspective the mathematical content they are responsible for teaching and both prior to and beyond the level they are assigned to teach" (National Mathematics Advisory Panel 2008, p. 21). Teachers with higher level mathematics knowledge can provide students with an advanced standpoint on mathematics. Just like climbing hills, the leader who is located on higher ground can give the followers clearer directions. In our understanding, the teachers are the leaders and the students are the followers.

There have been broad discussions in the literature about a teachers' background necessary for teaching but some of them were not especially focused in TEDS-M such as whether teachers have a master or a higher academic degree (Betts et al. 2003; Wayne and Youngs 2003) or whether teachers major in mathematics (Klecker 2008; Wenglinsky 2002). In Taiwan, after the Teacher Education Act (TEA) was enforced, the teacher education system has been divided into two branches: one is a "department-based system", originally placed at traditional normal universities or teacher colleges, and the other one is a "center-based system" which is set in comprehensive universities. Secondary school teachers can be from either system. With respect to mathematics teachers' majors, they can major in mathematics or other subjects. With respect to academic degrees, they are regulated to graduate with a bachelor degree by the TEA, but some future teachers are still studying for or have already gotten a master or a doctoral degree. Hence, we are going to discuss how future teachers' MCK and MPCK relate to these different background factors: different teacher-education systems, different majors and different academic degrees.

In addition, in the literature the opportunities to learn (OTL) are a commonly discussed aspect for the development of teacher knowledge. TEDS-M also investigated the OTL of future teachers, including the courses they took in their teacher preparation programs such as the OTL of tertiary level mathematics and mathematics education courses, and the past learning experiences such as the highest-grade level of school mathematics. Hsieh et al. (2010) found that the OTL of tertiary level mathematics and the highest-grade level of secondary mathematics were two important factors influencing future teachers' performances. The present paper examines the relationship between these factors and Taiwan's excellent performance.

Unlike future secondary teachers, future primary teachers in Taiwan need to study all fields in their teacher preparation program, including Chinese, English, science, mathematics, even music and art. Secondary future teachers are trained by specialists. Lower-secondary mathematics teachers focus on mathematics. After graduation, they can teach either at lower-secondary schools or upper-secondary schools depending on their screening exam. We hypothesize that the specialization,

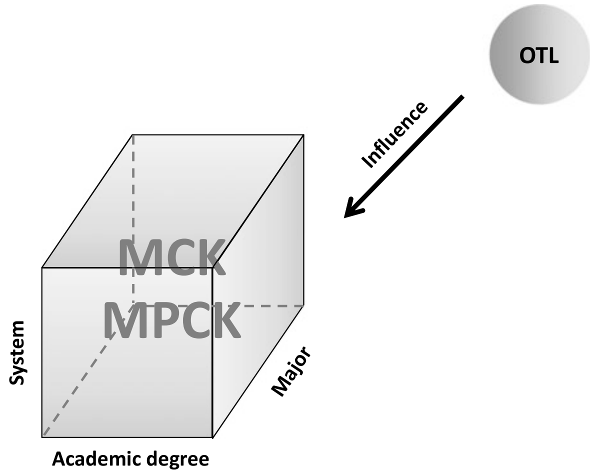

Fig. 1 The conceptual framework of this chapter

the major, might have great impact on the performance in MCK and MPCK. Therefore, we focus in the following on the performance of secondary school teachers. The conceptual framework of our paper is shown above (see Fig. 1).

3 Research Method

3.1 Participants

This paper uses the data from TEDS-M which has been processed by the IEA Data Processing and Research Center (IEA DPC). The data is from the following countries: Botswana, Chile, Germany, Georgia, Malaysia, Norway, Philippines, Poland, Russia, Spain (participating only in the primary study), Switzerland, Singapore, Taiwan, Thailand, the United States and Oman (participating only in the secondary study).[1]

19 teacher preparation institutes in Taiwan participated in TEDS-M. Three of them were department-based systems, while the other 16 were center-based. 365 out of 375 sampled teachers participated, and the participating rate was 97.3 %. The numbers of participants in the different models are listed in Table 1.

[1]The combined participation rates of Chile and Poland were between 60 and 75 %. Poland limited its participation to institutions with concurrent programs. Switzerland limited its participation to German speaking regions. The United States limited its participation to public universities. The combined participation rate of future teachers in Norway was 58 %, which was just slightly below the threshold set by the IEA for direct comparisons with other countries and therefore was still taken into account by this study. Data sets of four Norwegian program types are available, which were combined for analysis in an attempt to accurately represent the situation in Norway.

Table 1 Number of participants in the different models (weights of future teachers were applied)

Factor 1: System	Department-based system	Center-based system	Sum
	167	208	375
Factor 2: Major	Math department	Non-math department	Sum
	329	46	375
Factor 3: Academic degree	Bachelor degree	Master or doctoral degree	Sum
	109	266	375

Note: Adapted from Taiwan TEDS-M 2008 Secondary Analysis (Chaps. 6 and 7), by F.-J. Hsieh (Ed.), 2012, Taipei: Department of Mathematics, National Taiwan Normal University. Copyright 2012 by Department of Mathematics, National Taiwan Normal University. Adapted with permission

3.2 Measures

This study uses the MCK and MPCK items in the TEDS-M future teacher questionnaires. The questionnaires contain 73 MCK items and 27 MPCK items for the secondary level. For the primary level, the questionnaires contain 73 MCK items and 32 MPCK items. From the viewpoint of content domain, both MCK and MPCK are distinguished into four fields: number and operations, geometry and measurement, algebra and functions, data and probability. As for cognitive domain, MCK contains three levels: knowing, applying, and reasoning; while MPCK contains two levels: Knowledge of mathematical curriculum and of planning for mathematics teaching and learning (CT), and Enacting mathematics for teaching and learning (ET).

Concerning OTL of participants, TEDS-M applied same questions on secondary and primary future teachers. For tertiary level mathematics, TEDS-M asked future teachers to mark on a list of 19 topics if they had studied in teacher preparation program. In this paper, the topics studied stands for an OTL in the tertiary level. We also see the highest grade level of secondary mathematics taken by future teachers as an indicator of their OTL in the secondary level. In order to know this OTL, future teachers were asked to choose a proper item to fit their situation from the following options: 12th grade advanced level, 12th grade level, 11th grade level, 10th grade level and below 10th grade level.

3.3 Data Analysis

The international TEDS-M study adopted the partial-credit model of the item-response theory for scaling the MCK and MPCK data with the same weight for every country. The international mean was set at 500 test points and a standard deviation of 100 points (Tatto et al. 2009). The TEDS-M items covered four content and three cognitive domains. Whereas the international study did not estimate

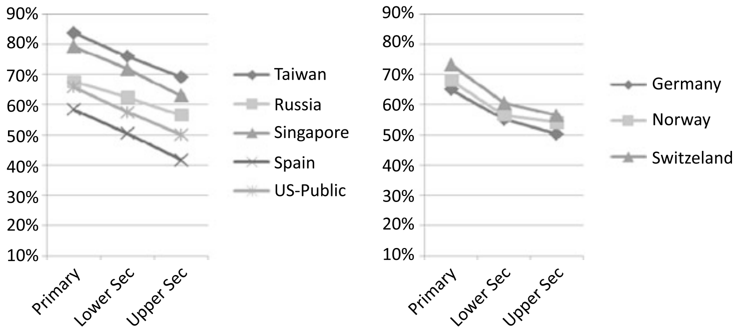

Fig. 2 Percentage of correct answers for MCK items across different curricular levels in the TEDS-M primary study. *Note*: Adapted from "The TEDS-M-plenary panel at ICME-12: important issues, results and question," by Krainer et al. (2012, p. 11). Copyright 2012 by 12th International Congress on Mathematical Education. Adapted with permission

scores for these, our study computed the Rasch logit scores of these to probe into details of the Taiwan future teachers' performance on MCK. We adopted the same partial-credit model (Masters 1982) as TEDS-M, we used the same weights and we transformed the logits on an international mean of 500 and a standard deviation of 100 points (Cook and Eignor 2005). We do not report data and probability though because of its low scale reliability (Bond and Fox 2007).

The MCK items were in addition categorized according to four curricular levels: primary, lower secondary, upper secondary or tertiary. We computed the average percent correct for each level. With respect to partial-credit items, the percent correct estimate is the sum of the percentage of answers receiving two points plus half of the percentage of answers scored one point. When comparing means, t tests were applied.

4 Results

4.1 MCK and MPCK Performance Around the World

Primary Level Future Teachers Several countries are displayed in Figs. 2 and 3 according to their MCK performance pattern on three of the curricular levels we had classified the items into: primary, lower secondary and upper secondary (cf. the TEDS-M plenary panel at ICME-12; Krainer et al. 2012). The performance of Taiwan future primary teachers was ahead of other countries. They achieved good results on primary-level MCK items as well as on lower-secondary and upper-secondary items. This result may demonstrate that Taiwan recruits high-achieving students for primary teacher education programs. However, unlike lower-secondary teachers, Taiwanese primary teachers do not surpass other countries a lot.

Taiwan may have demonstrated superior performance especially on the lower-secondary level items because of the following reasons: Taiwan teaches more topics in both school- and tertiary-level mathematics than other countries, and future

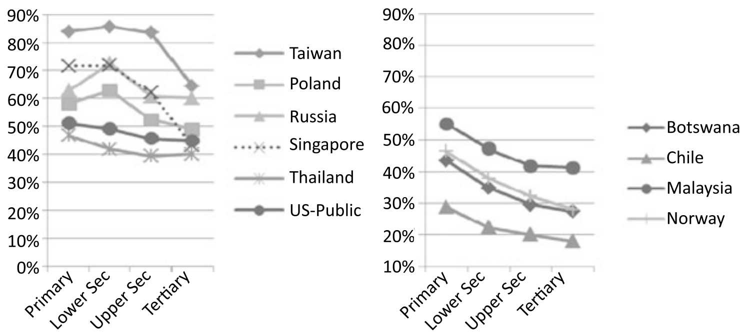

Fig. 3 Percentage of correct answers for MCK items across different curricular levels in the TEDS-M lower secondary study (Sec = secondary). *Note*: Adapted from "The TEDS-M-plenary panel at ICME-12: important issues, results and question," by Krainer et al. (2012, p. 11). Copyright 2012 by 12th International Congress on Mathematical Education. Adapted with permission

Taiwanese teachers have increased opportunities to perform challenging problems (thought-oriented); this is consistent with findings from analyses of relationships between the opportunities to learn (OTL), MCK and MPCK (Hsieh et al. 2012), which are shown in the following sections.

According to deeply-rooted concepts in Taiwan, a teacher should be able to solve all mathematics problems of her teaching level. However, although Taiwan excels other countries, Taiwan's future primary teachers cannot reach a 80 %-percent correct threshold on 36 % of the primary-level MCK items and 83 % of the primary-level MPCK items. Teacher educators from Taiwan were surprised and worried about these results of future teachers who still have problems in solving primary level mathematics despite their preparation on these.

Lower-Secondary Level Future Teachers With respect to the performance of lower-secondary teachers, we found one pattern for Taiwan and Singapore, one for Russia and Poland, one for Thailand and the United States, and one pattern for Botswana, Chile, Malaysia and Norway. It would be interesting to discuss the similarity and homogeneity of these paired countries. Here we concentrate on Taiwan's performance though.

Taiwan future lower-secondary teachers show high achievement on items of the primary, lower secondary and upper secondary curricular level but a sharp decline in tertiary mathematics. For all school-level MCK items, Taiwan outperformed the second place, Singapore, by 10–20 percent and by even more the other countries. Especially in upper secondary level mathematics, Taiwan is ahead of Singapore and other countries by more than 20 %. However, with respect to tertiary level mathematics Taiwan's and Russia's performance was close.

This result may demonstrate that Taiwan recruits high-achieving students for lower secondary teacher education programs. The results also explain why Taiwan performed well on lower secondary MPCK items. However, we had the same concerns about those secondary teachers who could not answer correctly and completely primary level mathematics problems as before. What's worse, these future lower-secondary teachers did not reach the 80 %-threshold on 30 % of the primary and lower secondary level MCK items and on 33 % of the MPCK items.

The result has to be a warning for teacher educators. Almost all future secondary teachers were trained in mathematics departments. It should be impossible that these future teachers whose mathematics grades reached high standards in college entrance exams cannot solve primary level mathematics problems.

It has been speculated in Taiwan that some future teachers were not earnest while answering questionnaires. This suspicion might not be true yet since some items have up to 99 percent correct. Therefore, it is necessary to probe into the education system in Taiwan, and to find out the problem, such as whether future teachers from the non-mathematics departments under the current system may decrease the average percent correct of some items. Taiwan's TEDS-M study used a census on the lower secondary level. All mathematics future teachers in all teacher preparation institutions were included in the study, except a few that trained fewer than five future teachers. Thus, we can look into different systems and shift the concern to domestic hot topics.

4.2 Hot Topics and Pressing Issues

4.2.1 The Current Pre-service Teacher Education System in Taiwan

Teacher education in Taiwan is a nationally policy-driven system. The current pre-service teacher-education system is regulated mainly by the national Teacher Education Act (TEA) and the Teacher Education Act Enforcement Rules (TEAER), enacted in 1994, 1995, and last amended in 2005, 2011, respectively. These regulations established the targets, institutions, recruitment, curricula, and accreditation of the teacher education institutes. The institutes include (1) normal universities or universities of education respectively, (2) universities with departments affiliated to teacher education (majors), and (3) universities with teacher education centers. Teacher education programs are separated into two levels: primary teachers who teach grades 1–6 for various subjects and secondary teachers who teach grades 7–9 or 10–12 for a single subject.

Future teachers must complete the teacher education curriculum from any institute and finish a practicum, before they take a national teacher assessment held yearly. The average passing rate of the assessment for the years of 2007–2010 was 67.4 %. After passing the assessment, future teachers need to undergo a public, competitive, on-site screening process administered by a school district or individual school. The average passing rate of this screening were at the primary, lower secondary and upper secondary levels 3.5 %, 11.9 % and 6.5 %, respectively during 1997–2010 (Hsieh et al. 2012). The screening is thus very competitive and ensures that Taiwan can recruit better qualified teachers and influence in turn the thinking and the inner quality of K-12 students.

Table 2 MCK and MPCK in different teacher-education systems

	MCK Mean (SE)	MPCK Mean (SE)	Difference of performance
Department-based system	684 (5.1)	662 (6.4)	22**
Center-based system	646 (5.3)	632 (8.0)	14
Difference of systems	38**	30**	

Note: (SE) = standard error. Difference = MCK-MPCK. Adapted from *Taiwan TEDS-M 2008 Secondary Analysis* (Chap. 7), by F.-J. Hsieh (Ed.), 2012, Taipei: Department of Mathematics, National Taiwan Normal University. Copyright 2012 by Department of Mathematics, National Taiwan Normal University. Adapted with permission. $^{*}p < 0.05$, $^{**}p < 0.01$

In the following we discuss which factors influenced the outcomes of teacher education. Most of the figures and tables are from Hsieh (2012) or from Wang and Hsieh (2012), the Taiwan TEDS-M 2008 secondary analysis report. We discuss the performance on MCK and MPCK with respect to three factors: the teacher-education system, the major and the academic degree. Besides these factors, we consider the connection between OTL (future teachers' opportunity to learn of tertiary level math and the highest-grade level of secondary mathematics) and MCK, MPCK.

4.2.2 Difference of MCK and MPCK by Teacher-Education Systems

Ever since the enforcement of TEA, the mathematics teacher preparation system has been divided into two branches. One is the "department-based system", originally from traditional normal universities or teacher colleges. The teacher education curriculum and courses are planned, carried out and managed by the related department. For example, future mathematics teachers come from the mathematics department. The other system is a "center-based system", which is a new product of TEA. The teacher education center is placed in a comprehensive university, and it recruits and enrolls anyone who wants to be a teacher from all departments. The "department-based system" stands for a traditional type of teacher education whereas the "center-based system" is a reform product.

The MCK performance of the department-based system is significantly better than the center-based system and this by 38 points. Regarding MPCK, the mean score of the future teachers from the department-based system is 662 points whereas the mean score from the center-based system is 632 points. The difference of 30 points is significant as well (see Table 2). These results reveal that, in Taiwan, the system is a factor influencing future teachers' performance in MCK and MPCK.

Table 2 also shows that the performance in MCK is better than in MPCK no matter in which system. In the department-based system, the MCK is significantly higher than MPCK by 22 points, and in the center-based system, the MCK is higher by 14 points although this difference is not significant.

Table 3 OTL of tertiary level math and highest grade of secondary school mathematics for different teacher-education systems

	OTL of tertiary level mathematics Mean (SE)	Highest grade of secondary mathematics[a]		
		12th grade advanced level	12th grade	11th grade
Department-based system	0.92 (0.01)	98 %	2 %	0 %
Center-based system	0.87 (0.01)	82 %	17 %	1 %
Difference	0.06^{**}	$\chi^2(2, N = 375) = 27.77$, $p = 0.00$		

Note: OTL = Number of tertiary level mathematics courses that future teachers had taken divided by the number of tertiary level mathematics courses listed on the questionnaire. Difference = Department-based system – center-based system. Adapted from *Taiwan TEDS-M 2008 Secondary Analysis* (Chap. 6), by F.-J. Hsieh (Ed.), 2012, Taipei: Department of Mathematics, National Taiwan Normal University. Copyright 2012 by Department of Mathematics, National Taiwan Normal University. Adapted with permission. [a]Percentages, $^{*}p < 0.05$, $^{**}p < 0.01$

What causes the significant differences between the performances of the two systems? Some researchers suggest that the OTL may be the reason. Hsieh et al. (2010) indicated that, for Taiwanese future secondary teachers, OTL of tertiary level math is related to MCK and MPCK, with $r = 0.25$ and 0.21 respectively. They also found that the highest-grade level of secondary mathematics was related to MCK with $r = 0.31$.

Almost all future teachers from the department-based system major in mathematics. On average, the OTL of tertiary level math in the department-based system is significantly higher than that in the center-based system. This result is consistent with the fact that MCK and MPCK are higher in the department-based system than in the center-based system.

With respect to the highest grade of secondary mathematics taken in school, future teachers had mathematics either up to 12th grade (advanced level), 12th grade or 11th grade. The ratio for teachers from universities with a department-based system is 98 %, 2 % and 0 % respectively. In contrast, the ratio for teachers from a center-based system is 82 %, 17 % and 1 % respectively. The difference of this pattern is significant according to a Chi-square test. It might be the reason why the MCK of universities with a department-based system is higher than the other one (see Table 3).

4.2.3 Difference of MCK, MPCK Between Math Department and Non-Math Department

Before the Teacher Education Act was enforced, there were two sources of middle-school mathematics teachers. One is a major in a mathematics department in normal universities or universities of education, and the other one is a major in education-related department with a minor in mathematics. After the act was enforced, the training of mathematic teachers was not limited to mathematics departments only.

Table 4 MCK and MPCK in different departments

	MCK Mean (SE)	MPCK Mean (SE)	Difference of performance
Math department	676 (4.3)	657 (5.6)	20**
Non-math department	604 (10.7)	595 (14.2)	9
Difference of department	72**	62**	

Note: (SE) is standard error. Difference of performance = MCK-MPCK. Difference of department = math department – non-math department. Adapted from *Taiwan TEDS-M 2008 Secondary Analysis* (Chap. 7), by F.-J. Hsieh (Ed.), 2012, Taipei: Department of Mathematics, National Taiwan Normal University. Copyright 2012 by Department of Mathematics, National Taiwan Normal University. Adapted with permission. $^{*}p < 0.05$, $^{**}p < 0.01$

Those who were not from a mathematics department were still qualified to take the national common Teacher Qualification Assessment as long as they finish the teacher education curriculum (TEC) regulated by the Ministry of Education (Hsieh et al. 2009).

The question is whether a difference on MPCK and MPK between these two groups exists. In the following, we refer to "mathematics department future teachers" in case of those who are from a mathematics department or an applied mathematics department (i.e., with a major in mathematics or applied mathematics). Those from other departments are regarded as non-mathematics departments. In TEDS-M, 27 % of this latter group come from science and technology departments, 45 % from education departments, and 28 % from other departments such as foreign language or economy. There are 328 mathematics department and 47 non-math department future teachers.

Mathematics-department future teachers perform better in MPCK than those of non-math departments by 62 points, which exceeds 0.6 standard deviation, and they perform also better on MCK by 72 points, which is about 0.7 standard deviation. (see Table 4). The table also shows that the performance on MCK is better than on MPCK no matter in which department. In mathematics departments, the MCK is significantly higher than the MPCK and this by 20 points, and in non-mathematics departments, MCK is higher by 9 points but this difference is not significant.

MCK contains three content and three cognitive domains. Mathematics-department future teachers perform better than non-math department teachers on the overall MCK as well as on three content and three cognitive domains.

In the cognitive domains, the smallest difference between the two groups exists on the "application" level (see Fig. 4). This implies that the performance of non-math department future teachers is close to that of mathematics-department teachers when it comes to questions with more familiar scenarios. The greatest difference is on the "reasoning" level. This result indicates that non-math department teachers perform not as well as mathematics department teachers when it comes to more complicated questions or questions which need to be solved with deeper thinking. In the content domains, mathematics department future teachers are much better at

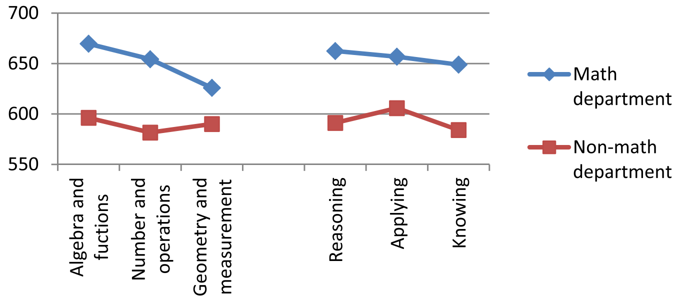

Fig. 4 Performance on content and cognitive domain for mathematics-department and non-math department teachers. *Note*: Adapted from *Taiwan TEDS-M 2008 Secondary Analysis* (Chap. 6), by F.-J. Hsieh (Ed.), 2012, Taipei: Department of Mathematics, National Taiwan Normal University. Copyright 2012 by Department of Mathematics, National Taiwan Normal University. Adapted with permission

Table 5 OTL of tertiary level math and the highest-grade level of secondary mathematics for math department and non-math department future teachers

	OTL of tertiary level math	The highest-grade level of secondary mathematics[a]		
		12th grade advanced level	12th grade level	11th grade level
Math department	0.92 (0.00)	99 %	1 %	0 %
Non-math department	0.75 (0.02)	37 %	59 %	5 %
Difference	0.17**	$\chi^2(2, N = 375) = 186.37$, $p = 0.00$		

Note: OTL = the number of tertiary level math courses that future teachers have taken divided by the number of tertiary level math courses listed on the questionnaire. Difference = math department − non-math department. Adapted from *Taiwan TEDS-M 2008 Secondary Analysis* (Chap. 6), by F.-J. Hsieh (Ed.), 2012, Taipei: Department of Mathematics, National Taiwan Normal University. Copyright 2012 by Department of Mathematics, National Taiwan Normal University. Adapted with permission. [a]The percentage of people who study the different highest-grade level of secondary mathematics, $^{*}p < 0.05$, $^{**}p < 0.01$

algebra and functions, but not as much better at geometry and measurement. Significant difference exists in all these domains though. Non-math department teachers perform quite equally in the three domains.

Again the question arises what causes the difference. Table 5 shows that the OTL of tertiary level math for math department future teachers is prominently higher than non-math department. This result is consistent with the fact that math department teachers do better in MCK and MPCK. It might be the reason why math department future teachers have better understanding and competence in various math fields and cognitive domain.

As for the highest-grade level of secondary mathematics, regarding as an indicator of OTL in secondary level, future teachers studied highest to 12th grade advanced level or 12th grade level or 11th grade level math. The ratio of these three levels for teachers from math department is 99 %, 1 % and 0 % respectively. However, the

Table 6 Comparison in MCK and MPCK between different academic degrees in math and non-math department

	Sample size	MCK	MPCK	Difference (MCK − MPCK)
Math department with bachelor degree (A)	240	676 (4.3)	659 (6.3)	17**
Math department with master or doctoral degree (B)	88	676 (8.7)	650 (11.0)	26**
Non-math department with bachelor degree (C)	25	588 (12.3)	583 (20.6)	5
Non-math department with master or doctoral degree (D)	21	622 (17.0)	609 (14.9)	13
Difference (A − B)		1	9	
Difference (C − D)		−34	−26	
Difference (A − C)		88**	76**	

Note: (value) is standard error. Adapted from *Taiwan TEDS-M 2008 Secondary Analysis* (Chaps. 6 and 7), by F.-J. Hsieh (Ed.), 2012, Taipei: Department of Mathematics, National Taiwan Normal University. Copyright 2012 by Department of Mathematics, National Taiwan Normal University. Adapted with permission. $^{*}p < 0.05$, $^{**}p < 0.01$

ratio for teachers from non-math department is 37 %, 59 % and 5 % respectively. The model is significantly different for these two categories under Chi-square test. The ratio of studying more difficult math level for math department future teacher is higher than non-math department. This learning experience could be the reason that math department teachers do better in MCK.

4.2.4 Difference of MCK, MPCK Between Bachelor and Master/Doctoral Degree, Math Department and Non-math Department

Nowadays, more and more teachers would like to pursue further education for better reputation, better pay, and of course, professional development. Does the further pursuance of education really help to improve teacher's teaching? If the teachers would like to improve teaching competence, which degrees should they get? We might be able to find out the answer by looking into the correlation between the performance in MCK, MPCK and academic degrees. Here, bachelor degree is one category, and the further academic degree such as doctoral and master degree is another category.

The study wants to find the relation between academic degrees and MCK MPCK, for both math department and non-math department future teachers. Surprisingly, from Table 6, we discovered that there is no significant difference on MCK and MPCK between bachelor degree and higher academic degree in math department and non-math department. This shows that whether future teachers study further education or not, their performance on MCK and MPCK doesn't have significance difference.

Table 6 also shows the difference between MCK and MPCK within modes. In math department, future teachers with bachelor or higher academic degrees both show better performance on MCK than MPCK. In non-math department, there's no significant difference between MCK and MPCK for teachers with bachelor, or higher academic degrees.

As a result, there's no significant difference between bachelor degree and higher degrees for math department future teachers. The differences in individual field of content domain range from 1 to 13 points (see Table 7). For non-math department future teachers, even there's also no big difference between bachelor degree and higher degrees, the differences in paired fields are quite a lot. The performances of master or doctoral degree are 10 to 53 points higher than bachelor degree.

What make the difference? We checked the OTL of tertiary-level math and the highest-grade level of secondary mathematics. For math department future teachers, there's no significant difference between bachelor degree and master or higher degree on the aspect "OTL of tertiary level math" and "the highest-grade level of secondary mathematics" (see Table 8). This finding is identical to the outcome of MCK. Even the OTL of these two groups are similar, teachers with master or doctoral degree should have more chances to learn more difficult math and be more experienced in math research. Nevertheless, these extra experiences don't make them excel the teacher with bachelor degree in MCK. We can infer that teachers with math department bachelor degree are already equipped with necessary MCK via math leaning in college.

In the case of non-math department future teachers, the OTL of tertiary level math of teachers with master or doctoral degree is remarkably higher than teachers with bachelor degree. We cannot detect the difference of MPCK, so this prominent difference in OTL is not strong enough to cause the difference in MPCK. Furthermore, the percentages of highest-grade level of secondary mathematics are very different. Most of the teachers with bachelor degree took 12th grade level math, while most of higher academic degrees teachers took 12th grade advanced level.

4.2.5 Comprehensive Comparison: Three Influential Factors

We have discussed three potentially influential factors on MCK and MPCK, which are teacher-education systems (department-based system vs. center-based system), majors (mathematics department vs. non-math department), and academic degrees (bachelor degree, master or doctoral degree). In Table 9, we compared these 8 models and ranked their MCK and MPCK from the highest to the lowest. For the MCK ranking, there is only one sample in the model "Non-math department master or doctoral degree from a department-based system" and two samples in the model "Non-math department bachelor degree from a department-based system". For the MPCK ranking, there are only two samples in the model "Non-math department

Table 7 Difference in MCK between different academic degrees in math and non-math department

	Content domain		
	Numbers and computation	Geometry and measurement	Algebra and functions
Math department with bachelor degree (A)	658 (5.1)	627 (4.6)	667 (4.6)
Math department with master or doctoral degree (B)	645 (10.9)	623 (8.2)	676 (7.6)
Non-math department with bachelor degree (C)	558 (21.1)	580 (15.1)	586 (12.5)
Non-math department with master or doctoral degree (D)	610 (22.2)	602 (18.2)	608 (18.2)
Difference (A − B)	13	4	−8
Difference (C − D)	−53	−22	−22
Difference (A − C)	100**	47**	81**
	Cognitive Domain		
	Understanding	Application	Reasoning
Math department with bachelor degree (A)	647 (5.0)	655 (4.4)	666 (5.1)
Math department with master or doctoral degree (B)	655 (8.8)	663 (10.8)	653 (6.6)
Non-math department with bachelor degree (C)	579 (19.5)	588 (13.7)	577 (14.1)
Non-math department with master or doctoral degree (D)	590 (16.9)	627 (17.5)	608 (20.4)
Difference (A − B)	−8	−8	13
Difference (C − D)	−10	−40	−31
Difference (A − C)	67**	67**	89**

Note: (value) is standard error. International mean and standard deviation is assumed to be 500 and 100, respectively. Adapted from *Taiwan TEDS-M 2008 Secondary Analysis* (Chap. 6), by F.-J. Hsieh (Ed.), 2012, Taipei: Department of Mathematics, National Taiwan Normal University. Copyright 2012 by Department of Mathematics, National Taiwan Normal University. Adapted with permission. $^{*}p < 0.05$, $^{**}p < 0.01$

bachelor degree from a department-based system" and only one sample in the model "Non-math department master of doctoral degree from a department-based system". We skipped these models due to their small sample size.

With respect to MCK, future teachers from mathematics departments in department-based systems performed the best. The second place goes to mathematics departments of center-based systems, and then come the non-math departments of center-

Table 8 OTL of tertiary level math and the highest-grade level of secondary mathematics for future teachers with different academic degrees and in different departments

	OTL of tertiary level math	The highest-grade level of secondary mathematics[a]		
		12th grade advanced level	12th grade level	11th grade level
Math department with bachelor degree (A)	0.92 (0.01)	99 %	1 %	0 %
Math department with master or doctoral degree (B)	0.90 (0.01)	99 %	1 %	0 %
Non-math department with bachelor degree (C)	0.68 (0.02)	13 %	83 %	4 %
Non-math department with master or doctoral degree (D)	0.84 (0.03)	65 %	29 %	5 %
Difference (A – B)	0.02	$p = 1.00$, Fisher's exact test[b]		
Difference (C – D)	-0.16^{**}	$\chi^2(2, N = 46) = 15.22$, $p = 0.00$		

Note: OTL = the number of tertiary level math courses that pre-serving teachers have taken divided by the number of tertiary level math courses listed on the questionnaire. Adapted from *Taiwan TEDS-M 2008 Secondary Analysis* (Chap. 6), by F.-J. Hsieh (Ed.), 2012, Taipei: Department of Mathematics, National Taiwan Normal University. Copyright 2012 by Department of Mathematics, National Taiwan Normal University. Adapted with permission. [a]The percentage of people who study the different highest-grade level of secondary mathematics. [b]There are only two level of math, 12th grade advanced level and 12th grade level math. Since two of these percentages are 5 smaller than the expected value, we apply Fisher's exact test, $^{*}p < 0.05$, $^{**}p < 0.01$

based systems. In these three categories, future teachers with master or doctoral degrees performed better than those with bachelor degrees.

With respect to MPCK, among the top three models, we can see that the factor "mathematics department" is related to most of them. Clearly, a mathematics department is a very important factor influencing MPCK. Math-department future teachers perform better than non-math department teachers. If the teachers are not from mathematics departments, they should be recommended to pursue higher academic degrees. In the center-based system, the best performance goes to those who get math-department bachelor degrees, the next goes to those who get math-department master or doctoral degrees.

This result contradicts the deeply-rooted concept that it is better to pursue higher education, because the MPCK of teachers with a higher academic degree is not higher than that of those with a bachelor degree. But in the department-based system, those with master or doctoral degrees perform better. Among the top three models, two are from department-based systems. Comparatively speaking, teachers from department-based system work better in MPCK than those from center-based teacher preparation education systems.

In the following, we use a 3D graph to show the relative ranking on MCK and MPCK among the eight models (see Figs. 5, 6). Three perpendicular edges represent

Table 9 Comparisons in MCK and MPCK for future teachers with different academic degrees in different departments in different systems

Rank in MCK	Modes of future teachers	MCK	Rank in MPCK	Modes of future teachers	MPCK
1	Math department master or doctoral degree of department-based system	704 (20.1)	1	Math department master or doctoral degree of department-based system	713 (23.2)
–	Non-math department master or doctoral degree of department-based system	690 (0.0)	2	Math department bachelor degree of center-based system	664 (13.4)
2	Math department bachelor degree of department-based system	683 (5.2)	3	Math department bachelor degree of department-based system	658 (7.0)
3	Math department master or doctoral degree of center-based system	668 (9.1)	–	Non-math department bachelor degree of department-based system	644 (15.4)
4	Math department bachelor degree of center-based system	653 (8.1)	4	Math department master or doctoral degree of center-based system	632 (12.9)
5	Non-math department master or doctoral degree of center-based system	618 (17.4)	5	Non-math department master or doctoral degree of centert-based system	610 (15.8)
–	Non-math department bachelor degree of department-based system	599 (24.1)	–	Non-math department master or doctoral degree of department-based system	597 (0.0)
6	Non-math department bachelor degree of center-based system	587 (13.2)	6	Non-math department bachelor degree of center-based system	578 (22.3)

Note: (value) is standard error. The modes without ranking are those with small sample size. Adapted from *Taiwan TEDS-M 2008 Secondary Analysis* (Chaps. 6 and 7), by F.-J. Hsieh (Ed.), 2012, Taipei: Department of Mathematics, National Taiwan Normal University. Copyright 2012 by Department of Mathematics, National Taiwan Normal University. Adapted with permission

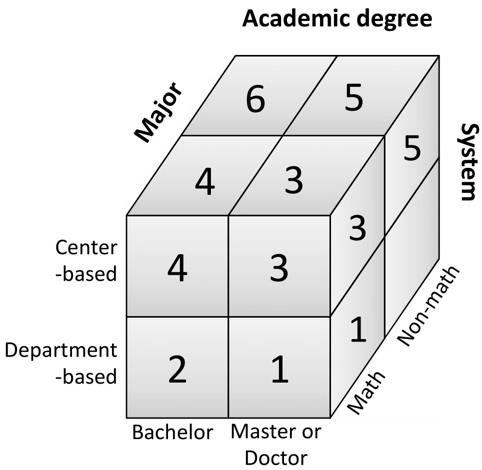

Fig. 5 3D graph of relative performances on MCK for eight modes. *Note*. Adapted from *Taiwan TEDS-M 2008 Secondary Analysis* (Chap. 6), by F.-J. Hsieh (Ed.), 2012, Taipei: Department of Mathematics, National Taiwan Normal University. Copyright 2012 by Department of Mathematics, National Taiwan Normal University. Adapted with permission

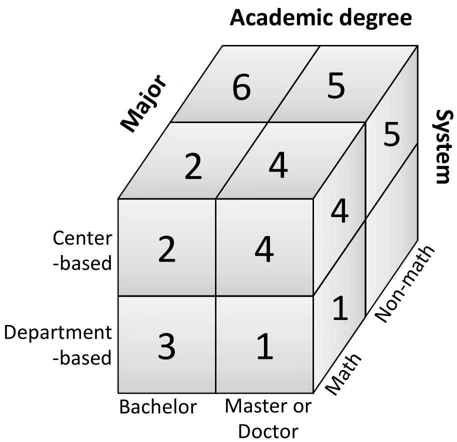

Fig. 6 3D graph of relative performances on MPCK for eight modes. *Note*. Adapted from *Taiwan TEDS-M 2008 Secondary Analysis* (Chap. 7), by F.-J. Hsieh (Ed.), 2012, Taipei: Department of Mathematics, National Taiwan Normal University. Copyright 2012 by Department of Mathematics, National Taiwan Normal University. Adapted with permission

three individual factors. The numbers labeled on the cube refer to the ranking in the table above. For example, cube number 1 in Fig. 5 represents that teachers who get a math-department master or doctoral degree from a department-based system get the first place in MCK performance. Since we did not consider the models with small sample sizes, there are only six models in the figures. In Fig. 5, MCK 3D graph, the first four modes out of six locate on the front face, which indicates that math department is a key factor for good MCK performance. The first two modes locate in the second parallel layer, which indicate that department-based education system influences the performance of MCK. In Fig. 6, MPCK 3D graph, the first four also locate on the front face, math department. Obviously that "math department" is a crucial element for good MCK and MPCK performance.

5 Conclusions and Reflections

We have seen many comparisons with different factors. To conclude, we found that both on MCK and MPCK, the performance of teachers from department-based system is better than center-based system; math department is better than non-math department. But there is no significant difference between bachelor degree and master or higher academic degrees. The performance of bachelor degree holders is not worse than master or doctoral degree.

Speaking of the comparison between MCK and MPCK within each mode, mostly MCK is better than MPCK. But there's no significant difference within center-based system and within non-math department. When we put every factor together to compare, we discovered that "major" is a very important element to influence teachers' performance on MPCK. The results of this study show the difference between two systems, the difference between MCK and MPCK within same system. These comparisons would be very helpful for our Ministry of Education when they establish or modify the teacher cultivation framework. We also found that the OTL of tertiary level math and the highest-grade level of secondary mathematics are indeed consistent with some performances in MCK and MPCK. Indeed, we have discovered some significance in some comparisons. This OTL issue needs to be examined and reviewed when MOE modify the teacher education curriculum for a better future.

Since Shulman posted thoughts on teacher knowledge, the idea that mathematics pedagogical content knowledge has deep influence on teachers' teaching efficiency has been broadly accepted. Some researchers found that student's mathematics achievement improves by improving teachers' mathematical knowledge (Hill et al. 2005). For mathematics teachers, it is not enough to have abundant mathematics content knowledge; we need rich MPCK to precede teaching.

How do we teach students? How do we teach this subject? How can we do to help students understand this subject efficiently? These questions related to MPCK are even more important for a mathematics teacher. However, the fact that future teachers perform better in MCK, not MPCK, has raised our concerns. Is the current curriculum enough for these future teachers or not? Which mathematics pedagogy content knowledge and competence they need to learn and understand before their graduation from teacher preparation program? This issue needs more endeavors in the future to improve our MPCK.

With these outcomes of MCK and MPCK, we not only analyzed the education curriculum and systems in Taiwan, but also examined how future teachers perform and found the weaknesses and strengths for Taiwan future teachers' teaching competence comparing to other countries. In the first section, we point out the excellence of Taiwan, especially lower secondary future teachers. Meanwhile, we also discover some attention-getting truths. In Taiwan, future teachers are expected to be knowledgeable and to master the concepts and skills in the field that they intend to teach. With such high prospect from our society, it is expected that at least 80 % of future teachers can answer correctly for any items at or below their teaching level. However, data of Taiwan's MCK and MPCK showed that, in both lower secondary and primary level study, large percentages of MCK and MPCK items did not meet the desired 80 % threshold. These results are strong warnings for the Taiwan teacher education system. Taiwan needs to examine our teacher education curriculum carefully, and improve our math competence. As a reflection on this problem, a wave has proceeded in Taiwan. Taiwan government has decided to add the subject of Mathematics Teaching Materials and Method in the Teacher Qualification Assessment, starting 2013 and the course "teaching practice in mathematics" has become required course to improve future teachers' competence in MPCK.

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The Preparation of Primary Mathematics Teachers in Singapore: Programs and Outcomes from the TEDS-M Study

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Abstract This paper describes five aspects of primary mathematics teacher education in Singapore: (a) the teaching profession in Singapore, (b) the structure of pre-service teacher education programs offered by the National Institute of Education, (c) self-reports of Singapore future primary mathematics teachers about the opportunities to learn mathematics-related contents offered by these programs, based on the TEDS-M (Teacher Education and Development Study in Mathematics) survey, (d) the performance of these future teachers in mathematics content knowledge and mathematics pedagogical content knowledge assessed by the TEDS-M study, and (e) the relationships of opportunities to learn with this performance. The paper concludes with some suggestions about how to improve the quality of initial teacher preparation in the areas of recruitment and training.

This project was funded by the Centre for Research in Pedagogy and Practice, National Institute of Education, Nanyang Technological University, Singapore (EP No. 5/06 WKY). The released TEDS-M items are copyrighted by the International Association for the Evaluation of Educational Achievement (IEA). The analyses prepared for this paper and the views expressed in this paper are those of the authors' and do not necessarily reflect the views of the IEA, the TEDS-M Study Centers (Michigan State University and the Australian Council for Educational Research), the Centre for Research in Pedagogy and Practice, the National Institute of Education, or the Singapore Ministry of Education.
We wish to thank Dr. Glenn Rowley and the anonymous reviewers for their helpful comments.

Originally published in ZDM—The International Journal on Mathematics Education.

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S. Blömeke et al. (eds.), *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn*, Advances in Mathematics Education,
DOI 10.1007/978-94-007-6437-8_8,

Keywords Singapore · Primary mathematics teacher education · Mathematics content knowledge · Mathematics pedagogical content knowledge · Opportunities to learn (OTL)

1 Introduction and Main Aim of Paper

Recruiting, training, certifying, employing, developing and retaining well-qualified teachers are critical, inter-related issues confronted by policy-makers, education administrators, and teacher educators all over the world. These issues are premised on the claim that well-qualified teachers exert critical impact on student learning (Gopinathan et al. 2008; Izumi and Evers 2002; Schwille and Dembélé 2007; Wang et al. 2003). This claim is supported by studies that show that teachers' mathematical knowledge has positive effects on the mathematics achievement of primary pupils (e.g., Hill et al. 2005) and secondary students (e.g., Baumert et al. 2010). Citing a South Korean official, the authors of the McKinsey report on the best-performing school systems noted that "the quality of an education system cannot exceed the quality of its teachers" (Barber and Mourshed 2007). In an updated report, Auguste et al. (2010) stressed that high-performing school systems such as Singapore, Finland, and South Korea recruit their trainee teachers from the top third of the academic cohorts. The Singapore Ministry of Education (MOE) also accepts that teachers play a critical role in preparing students for the future and in implementing its curriculum for the 21st century. In 2009, it released the vision statement for its teachers: "Lead. Care. Inspire".[1] This vision goes beyond the subject mastery commonly associated with competent teachers to include qualities of leadership, care for the students, and the capacity to inspire students to achieve their potential.

The above-mentioned claims and expectations apply to mathematics teachers too. Several international studies have compared mathematics teacher education systems from around the world (Burghes 2008; Strässer et al. 2003). The recently completed project called Teacher Education and Development Study in Mathematics (TEDS-M) is the first international comparative study undertaken under the auspices of the International Association for the Evaluation of Educational Achievement (IEA). It involved about 14500 primary and 8600 secondary future mathematics teachers from 780 pre-service teacher education programs sampled from 490 teacher education institutes in 17 countries. The framework and objectives of the TEDS-M study, including selected questionnaires, can be found in Tatto et al. (2008). The main aim of this paper is to describe the pre-service training of future primary mathematics teachers in Singapore using relevant findings from the TEDS-M study and local documents, interpreted from our perspectives as Singapore mathematics teacher educators over many years. To achieve this aim, we will cover the following five areas:

1. National context for teacher preparation in Singapore, focusing on the recruitment of future teachers for Singapore government schools.

[1] http://www.moe.gov.sg/media/press/2009/08/vision-for-the-teaching-servic.php.

2. Pre-service primary mathematics teacher education at the National Institute of Education (NIE), the sole teacher education institute in Singapore. Details of the four relevant programs are described to enable meaningful international comparisons.
3. Self-reports about the opportunities to learn mathematics-related contents provided by the future teachers who participated in the TEDS-M study. These findings highlight areas of strengths and weaknesses of the Singapore teacher education programs that might be of interest to international readers.
4. Performance on measures of Mathematics Content Knowledge (MCK) and Mathematics Pedagogical Content Knowledge (MPCK) of the Singapore future primary mathematics teachers in the TEDS-M study, including detailed comments on four selected items. This discussion highlights the necessity to focus on the *mathematics* aspect of primary teacher education in comparative studies.
5. The relationships between the opportunities to learn mathematics-related contents and the performance of these future teachers in MCK and MPCK.

In the final Discussions and Conclusion section, we offer some suggestions about how to improve the quality of initial teacher preparation in the areas of recruitment and training. Relevant international findings will be mentioned only briefly because they are not the main focus of this paper.

2 The Singapore Context for Teacher Preparation

Singapore is a small country of area 710 km^2 with 5 million people. There are 172 primary schools with 13500 primary teachers and 265000 primary school pupils (Ministry of Education 2010). The average class size ranges from 30 in Grade 1 (age 6+) to 37 in Grade 6. Secondary schooling covers lower secondary (Grades 7 and 8) and upper secondary (Grades 9 and 10) levels. Post-secondary education is delivered through junior colleges (Grades 11 and 12), polytechnics, and other specialized institutes. Mathematics is a compulsory subject from Grade 1 to 10, and it is taught in English rather than the student's mother tongue.

The Ministry of Education (MOE) controls the recruitment, employment, and retention of teachers in government schools, whereas the training and certification of teachers are the responsibility of the NIE. With the exception of a small percentage of teachers recruited from overseas, teachers in Singapore schools generally receive their pre-service teacher education at the NIE. This clear demarcation of roles for the MOE and the NIE has been successful in the past sixty years in producing good quality teachers for Singapore because both organizations have worked very closely together for the same goal of producing a strong teaching force. In 2009, the education expenditure was S$8.70 billion, about 3.4 % of GDP of that year. About S$0.12 billion was spent on teacher education at the NIE, a considerable investment by the Singapore government in this area.

The main steps to becoming a fully qualified teacher in Singapore government schools are shown in Fig. 1. Prospective teachers can apply to the MOE for teaching positions all year round. The requisite academic qualifications vary with the

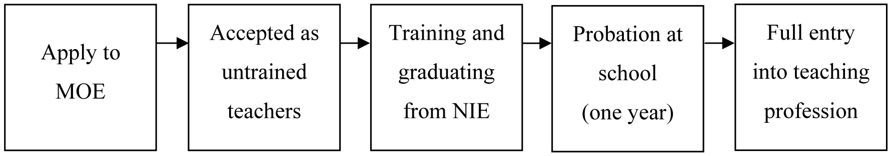


Fig. 1 From application to entry into the teaching profession in Singapore

teacher education programs. For the three concurrent programs (Diploma in Education, Bachelor of Arts with Education, and Bachelor of Science with Education, for which more details are provided in Sect. 3), the minimum requirement is good grades in the Singapore-Cambridge General Certificate of Education Advanced-Level (GCE A-Level) obtained by graduates from the junior colleges or a strong diploma from one of the five Singapore polytechnics. For the consecutive program (Postgraduate Diploma in Education), the applicant must hold a Bachelor degree from a recognized university. In addition, all applicants, irrespective of the programs, must have obtained the Singapore-Cambridge General Certificate of Education Ordinary-Level (O-Level) passes, or equivalent, in Mathematics and English Language. This underscores the importance of Mathematics in the training of all Singapore teachers. NIE programs are not expected to teach future teachers about mathematics that has been learned in secondary schools, as happens in some countries (Schwille and Dembélé 2007). Because English is the main medium of instruction in Singapore, all applicants, unless exempted, are required to pass an English Entrance Proficiency Test for oral and listening comprehension. Short-listed applicants are interviewed by teams comprising MOE officials and NIE faculty to assess the applicants' interest in teaching and their personal and leadership qualities.

Successful applicants are then appointed as untrained teachers (called General Education Officers) and sent to the NIE as student (or trainee) teachers. Applicants whose suitability for teaching requires further confirmation may be offered contracts to work in schools where the school leaders can provide additional screening. Those who are found to have the potential to become competent teachers are offered admission to the NIE.

The success rate for recruitment is fairly low with about one acceptance per eight applicants. Controlling entry to the profession prior to training ensures that only capable and committed applicants are selected for training and eventual entry into the teaching profession. This also reduces the "wastage" that occurs when too many teachers are trained, many of whom may have difficulty gaining teaching employment after graduation, thus becoming a problem as happened in some countries.

In recent years, the MOE has actively recruited applicants who wish to switch from their current careers (e.g., engineering or finance) to teaching, because they "can bring something new to the classroom, and share their wealth of knowledge from their previous careers with a generation eager to learn more".[2] These are called "mid-career" applicants. In the past three years, about 35 % of the recruits had prior working experience outside teaching (Tan 2011). With its competitive salaries (for example, beginning teacher salaries in Singapore are comparable to those of scientists, as noted by Carnoy et al. 2009) and many benefits, teaching is an attractive but also a demanding profession in Singapore.

Student teachers at the NIE receive full salary up to two years of their training and their tuition fees at the NIE are paid by the MOE. Under this unique recruitment-cum-training system, student teachers in Singapore are paid to receive teacher training. Furthermore, they do not have to worry about looking for future employment as teachers or face additional certification requirements after completion of their program. However, they need to serve a teaching bond (contract) from three to six years depending on the duration of the training programs. Those who fail to graduate are normally deemed to have broken the teaching bond and have to pay liquidated damages. This bond serves as a strong incentive for student teachers to graduate within the stipulated duration of their program; indeed, the failure rate at the NIE is less than 1 % per cohort. In recent years, about 1500 student teachers graduate annually from various NIE primary teacher education programs.

After graduation from the NIE, the beginning teachers are placed on probation for a year, during which they teach about 80 % of the normal teaching hours, are mentored at the school level and participate in the Structured Mentoring Programme conducted by the recently launched Academy of Singapore Teachers. At the end of this probation period, they are assessed by their school leaders, and upon satisfactory performance in teaching and professional duties, they are confirmed as teachers in the civil service. No official data are available about attrition at this stage, but it is understood that very few beginning teachers fail this probation assessment.

3 Pre-service Primary Mathematics Teacher Education at the NIE

This section describes the four NIE programs that train future primary mathematics teachers. The information in this section will help readers understand the perceptions reported by the future teachers who participated in the TEDS-M study, to be discussed in Sect. 4.

3.1 Four NIE Primary Teacher Education Programs

Future primary mathematics teachers are enrolled in one of the following four programs:

[2] http://www.moe.gov.sg/careers/teach/applying/mid-career/.

1. Diploma in Education (Dip Ed, with options A or C). This is a two-year concurrent program leading to the award of a diploma rather than a degree. It has two options: option A covers two teaching subjects (usually English and Mathematics), and option C covers three teaching subjects (usually English, Mathematics, and a third subject such as Science or Social Studies). This is the shortest program among the 780 international teacher education programs sampled in the TEDS-M study. Student teachers are assigned to the option by the MOE. However, in recent years, as schools generally regard teachers with three teaching subjects as easier to deploy, the MOE ceased assigning student teachers to option A from 2009.
2. Bachelor of Arts with Education, BA (Ed). This is a four-year concurrent program covering three teaching subjects: English, Mathematics, and a third teaching subject, which may be Social Studies, Art, Music, and Science (rare). These student teachers also take an academic (content) subject at university level and that subject may be chosen from English Language, English Literature, History, Geography, Art, Music, or Drama.
3. Bachelor of Science with Education, BSc (Ed). This is also a four-year concurrent program similar in structure to the BA (Ed). However, the third teaching subject for BSc (Ed) is usually Science and the academic subject is Biology, Chemistry, Physics, or Mathematics. In fact, it is the academic subject that determines whether the student teacher is in the BA (Ed) or BSc (Ed) program.
4. Postgraduate Diploma in Education (PGDE Primary). This is a one-year consecutive program for graduates, with options A or C, similar to those available in the Diploma in Education (above). Option A was also discontinued in 2009 for similar reasons about deployment.

3.2 Changes to the NIE Bachelor Programs

The two Bachelor programs were first introduced in 1991. Since then, their curriculum structures have been revised four times. A review conducted in 2003/2004 found that student teachers generally did not have the requisite content foundation for four teaching subjects (English, Mathematics, Science, and Social Studies), and hence, they could not cope well with learning to teach four subjects. Furthermore, the primary school curriculum has placed much heavier emphasis on English and Mathematics in recent years, and, to a lesser extent, Science, so that student teachers ought to focus on fewer teaching subjects. This resulted in changing the Bachelor programs from four teaching subjects (called the C-series) to three teaching subjects (the A-series), which is the current program described above. For the TEDS-M study, the Singapore data were collected in 2008 from future teachers who were the last cohort of the C-series. Hence, their self-reports of OTL may be different from those of the current future teachers in these Bachelor programs.

These changes to the NIE Bachelor programs illustrate the international trend of rapid changes made to teacher education programs in response to various external challenges and internal reviews. Thus comparative studies in teacher education may become obsolete from the time of data collection to publication of the findings. Nevertheless, these studies are still valuable to shed lights on the underlying factors and contexts that lead to such changes in structures and programs.

3.3 Curriculum Structures of NIE Primary Teacher Education Programs

To facilitate valid international comparisons, the TEDS-M Study organized primary teacher preparation programs into the following four program groups, according to the teaching role for which they would qualify:

- Lower Primary Generalist (Grade 4 maximum)
- Primary Generalist (Grade 6 maximum)
- Primary/Lower Secondary Generalist (Grade 10 maximum)
- Primary Mathematics Specialist

The four NIE programs were classified into Primary Generalist and Primary Mathematics Specialist, as shown in Table 1. Even though several NIE programs fell within the same program group, they actually have very different curriculum structures and durations (see Table 1). Thus, future teachers from the same program group may respond differently to the TEDS-M survey and tests.

3.4 Courses in the NIE Primary Teacher Education Programs

The courses undertaken in NIE programs are classified into Education Studies (ES), Curriculum Studies (CS), Subject Knowledge (SK), Academic Subjects (AS), Practicum (PRACT), and Others (miscellaneous courses and electives). The numbers of Academic Units (AU) of these courses are given in Table 1.

One AU is equivalent to 12 contact hours, and each course is to be completed within one semester. There are two semesters per academic year. These courses are briefly described below; the course outlines can be found in the website of the Office of Teacher Education of the NIE.[3]

Education Studies (ES) courses cover the Singapore education system and educational theories and practices. These courses help student teachers to understand the social, psychological, and technological contexts of schooling in Singapore as well as in general education.

The Curriculum Studies (CS) courses deal with the pedagogy (or methodology of teaching) of specific school subjects, such as Mathematics and English. The Mathematics CS courses are very similar across the four programs and cover the Singapore

[3] http://www.nie.edu.sg/programme-offices/office-teacher-education.

Table 1 Number of Academic Units (AU) of NIE primary teacher education programs by TEDS-M program groups

Program groups	NIE programs	Duration (years)	Types of Courses (AU)					PRACT (AU)	Total (AU)
			ES	CS Maths	SK Maths	AS	Others		
Primary Generalist (Grade 6 maximum)	Dip Ed (C)	2	8	8	6	None	32	15	69
	BA (Ed)	4	12	10	6	39	40	21	128
	BSc (Ed)	4	12	10	6	39	40	21	128
	PGDE (P) (C)	1	8	8	None	None	18	10	44
Primary Math Specialist	Dip Ed (A)	2	8	10	9	None	23	15	65
	PGDE (P) (A)	1	8	8	4	None	14	10	44

Notes. 1 AU = 12 contact hours. ES = Education Studies; CS = Curriculum Studies; SK = Subject Knowledge; AS = Academic Subject; Others: CS and SK courses for subjects other than Mathematics and various electives; PRACT = Practicum

mathematics curriculum, learning theories, lesson planning, assessment of mathematics learning, error analysis, and methods for teaching specific topics, such as whole numbers, fractions, geometry, and algebra. Since pedagogy is partially universal (e.g., practice mathematics skills) and partially culturally situated (e.g., Cai et al. 2009), NIE mathematics educators have integrated mathematics pedagogical principles from international research and practices with local contexts and lessons learned from local implementations, and they have published resource books to be used in Mathematics CS courses (Lee and Lee 2009; Yeap 2008). This enhances the links of these CS courses to the realities of local classroom teaching.

The Subject Knowledge (SK) courses help student teachers to gain a deeper understanding of the contents of the school subjects they are being prepared to teach. The Mathematics SK courses aim to enhance conceptual understanding and mastery of whole numbers (different numeration systems and divisibility), geometry (properties of geometric figures with proofs, tessellations, and use of dynamic geometry software), deductive and inductive reasoning, and statistical investigations. These topics are related to but go beyond the school mathematics curriculum. The SK courses exemplify international discussion about what is the appropriate mathematics that school teachers need to master (Ball et al. 2005; Kulm 2008). In addition to specific topics, the SK courses also cover problem solving, which is the major focus of the Singapore primary mathematics curriculum. This curriculum includes 12 problem solving heuristics, such as *draw a diagram, make a list, guess and check*, and *work backwards*, but the "model method" (Kho et al. 2009) is the most important one that student teachers need to learn to teach. This "model method" was developed by Singapore mathematics curriculum specialists in the 1980s to help primary school pupils solve word problems by drawing diagrams that reflect the underlying structures of the problems. This method has become the most distinctive feature of the so-called Singapore Math and it is now taught in several countries, including a US online adaptation called "Thinking blocks" (http://www.thinkingblocks.com/index.html). These SK courses underscore the strong alignment of teacher education with school curriculum and assessment. Furthermore, some SK activities such as hands-on activities and group discussions let student teachers "experience for themselves such sense-making from the perspective of learners of Mathematics" (Lim-Teo 2010, p. 210). This re-learning of familiar mathematics through new learning experiences with constructivist approaches emphasizing reasoning and sense-making aims to inculcate a deeper understanding of mathematics and to exemplify effective mathematics learning. However, feedback from the student teachers has been mixed (Lim-Teo 2010). There are also logistic constraints, for example, SK and CS courses are taught by different lecturers, sometimes resulting in weak connections between these courses (Lim-Teo 2009).

The Academic Subject (AS) courses are undergraduate courses that provide in-depth mastery of the contents of the respective disciplines. These are required for the Bachelor programs only, and the student teachers select only one academic discipline to study. AS Mathematics includes courses in Calculus, Linear Algebra, Statistics, Analysis, and others.

A large component of the "Others" category includes the CS and SK courses of teaching subjects other than Mathematics, such as English and Science. It also includes two compulsory courses specific to teaching. The first one is a communication skills course to equip student teachers with stronger skills of using English for teaching purposes. The second is a service learning project, which carries no AU or grade. The main aim of this group project is to develop in the student teachers a better appreciation of the needs to go beyond classroom teaching to serve the community. Past projects include engaging Institute of Mental Health patients in meaningful activities and helping young members of local charity organizations to develop leadership skills. Another aim of this group project is help student teachers to acquire the skills of project management that they can apply when they supervise similar projects in the schools in the future. This course provides yet another example of the close alignment of NIE programs with national and school contexts, thus enhancing the relevance of teacher preparation.

Most of the courses described above are delivered through a combination of lectures, tutorials, and group activities in a technology-enhanced teaching environment, including e-lectures and use of the Blackboard course management system. Student teachers are now provided with individual laptops during their training.

Most NIE courses are assessed using a combination of written tests, practical tests, essays, projects, micro-teaching, class participation, online forum, and so on. Most AS courses have final written examinations held at the end of each semester, but non-AS courses do not have major written examinations.

The practicum (field experience) is an essential component in most teacher education programs all over the world. In Singapore, it contributes between 15 % to 25 % of the requirements for NIE programs: 15 weeks in two semesters for Dip Ed; 22 weeks spread over three years for the Bachelor programs; 10 weeks in one semester for PGDE. During the practicum, every student teacher is assigned a School Coordinating Mentor (SCM), one or more Cooperating Teachers (CT) in their teaching subjects, and one NIE supervisor. The student teachers initially observe lessons conducted by their CTs and later plan and teach their own lessons, reflecting on feedback given by their CTs and the NIE supervisor. They teach about two thirds of the teaching hours of an average teacher so that they have more time to prepare engaging lessons based on what they have learned from their NIE courses. Formative and summative assessment of their teaching cover competencies in lesson preparation, lesson delivery, classroom management, assessment of learning, and professional qualities, such as showing care for their students, being responsive to feedback, and professional dressing. However, NIE student teachers are not required to write an extensive report or thesis about their practicum or to conduct research during their practicum. Their practicum grade (Distinction, Credit, Pass, or Fail) is determined by an assessment panel comprising the school principal, SCM, CTs, and the NIE supervisor. This panel allows for negotiation of the final grade from different perspectives:

- theory-practice link by the NIE supervisor,
- classroom teaching by the CTs, and
- professional activities and conduct in the school by the SCM and the principal.

This multi-party negotiation reduces the risk that the performance of the student teachers will be assessed using only a narrow set of criteria. Student teachers are usually appointed back to their practicum school after graduation.

To graduate, student teachers graduate must pass every course and obtain a Cumulative Grade Point Average (CGPA) of 2 out of a maximum of 5. Practicum is not included in the computation of the CGPA since it is awarded only a nominal grade. In general, the graduation rate is very high with less than 1 % not graduating with their cohort. The main reasons for non-graduation include inability to cope with the workload during practicum, weaknesses in content knowledge that become evident when the student teachers have difficulty in answering pupil questions, ineffective classroom management, and poor teacher-pupil rapport. Student teachers are given two chances to pass the final practicum.

4 Opportunities to Learn (OTL) Mathematics-Related Contents: Findings from TEDS-M Study

This section reports some findings from the TEDS-M survey about opportunities to learn mathematics-related contents offered by the NIE programs, as reported by the Singapore future primary mathematics teachers. For the rest of this paper, we will use the label FPMT to refer to these student teachers. We will begin this section with a brief description of the Singapore sample.

Table 2 NIE future primary mathematics teachers in the TEDS-M study

Program groups	NIE programs	Number	% Female	% mid-career
Primary generalist (Grade 6 maximum)	Dip Ed (C)	107	81.4	51
	BA (Ed)	31	71.0	19
	BSc (Ed)	36	61.2	11
	PGDE (P) (C)	89	77.6	56
Primary math specialist	Dip Ed (A)	45	66.7	49
	PGDE (P) (A)	72	72.3	63

Notes. Option (A) covers two teaching subjects and option (C) covers three teaching subjects

4.1 The Singapore Sample for TEDS-M Study

Since there is only one teacher training institute in Singapore, a census sample was taken for the TEDS-M study. All FPMT who had taken the Mathematics CS courses were requested to take the TEDS-M test and survey in May 2008 after their final practicum. The response rates ranged from 86 % to 96 %, and these satisfied the criteria set by the IEA.

The breakdown of the NIE sample by program groups and NIE programs is shown in Table 2. There were more females than males in all the NIE programs, and this is consistent with international trends. The percentages of mid-career FPMT varied from a low 11 % for BSc (Ed) (they were fresh graduates from junior colleges or polytechnics) to a high 63 % for PGDE (option A) (who already held a degree). This presents a challenge to NIE teacher educators, and we will discuss this in Sect. 7.2.

4.2 Opportunities to Learn Mathematics-Related Contents

The TEDS-M Future Teacher Questionnaire covers three aspects of opportunities to learn mathematics-related contents in the pre-service teacher education programs: tertiary level mathematics, school level mathematics, and mathematics pedagogy (Tatto et al. 2008).

To explore opportunities to learn tertiary level mathematics, FPMT were asked to indicate whether or not they had studied each of 17 topics in tertiary level mathematics, such as Calculus and Number Theory, either in their current or previous program. The mean proportions of topics studied as reported by the future teachers within each program group by country were used to compute a Tertiary Level Mathematics OTL scale, with values ranging from 0 to 1. NIE FPMT reported the lowest mean of 0.38 among the countries in the respective program groups, and this was due to the fact that tertiary level mathematics, as covered in the NIE AS courses, was optional for most of these FPMT. These tertiary level mathematics topics may constitute the "mathematics on the horizon" (Hill et al. 2008) to enrich the mathematical experiences of the future teachers. However, there is also the risk of "vertical disconnect" (Cuoco 2001) between what these future teachers need to know and what they have to teach.

In a similar way, the School Level Mathematics OTL scale was created based on responses to seven major topics: numbers, measurement, geometry, functions and relations, data representation, probability and statistics, and validation, structuring, and abstracting. NIE FPMT reported a moderate mean proportion of 0.62, which was lower than the means of most of the countries in the respective program groups. Most of these topics were covered in the NIE SK courses. The exception was the topics of validation, structuring, and abstracting; indeed, only 10 % of NIE FPMT reported having the opportunities to learn these topics.

The opportunities to learn mathematics pedagogy were evaluated in two ways. The first way, similar to the two scales above, was to create the Mathematics Pedagogy OTL scale by computing the mean proportions of how many of the eight mathematics education topics were studied. These eight topics were: foundations of mathematics, context of mathematics education, development of mathematical ability and thinking, mathematics instruction, development of teaching plans, mathematics teaching, mathematics standards and curriculum, and affective issues in mathematics. NIE FPMT reported a moderate proportion of 0.70 of coverage of these topics in their Mathematics CS courses. This suggests that the NIE programs were quite adequate in covering the mathematics pedagogy topics listed in the TEDS-M survey.

The second way to measure mathematics pedagogy OTL was to ask future teachers to indicate on a 4-point scale (never = 1, rarely = 2, occasionally = 3, often = 4) how frequently they were engaged in each of the 15 listed activities during the NIE Mathematics CS courses. On the basis of the mean scores of individual items, NIE FPMT reported that they frequently listened to lectures (3.57), worked in groups (with highest mean of 3.70), participated in whole class discussion (3.32), and solved mathematics problems using multiple strategies (3.23). The findings about these four activities show that the NIE Mathematics CS courses had provided these FPMT with learning experiences that covered both direct instruction and the constructivist approaches. At the other end of the scale, NIE FPMT reported that they rarely wrote mathematical proofs (with lowest mean of 1.88) and read about research on mathematics (2.17). Whether or not *primary* mathematics teachers need to know how to write proofs and to understand mathematics research is open for further discussion since they hardly encounter these two situations in their normal teaching.

Finally, 94 % of NIE FPMT rated their training program as highly effective or effective, compared to only 74 % internationally. Thus, NIE FPMT were more positive about their training than the international future teachers about their programs.

We will explore the relationships of these OTL measures with performance in MCK and MPCK in Sect. 6 below.

5 Performance in TEDS-M Primary MCK and MPCK Tests

The TEDS-M MCK framework covers four content knowledge domains (Number, Geometry, Algebra, and Data) and three cognitive domains (Knowing, Applying, and Reasoning). The MPCK framework covers three aspects: Mathematical curricular knowledge, Knowledge of planning for mathematics teaching and learning (pre-active), and Enacting mathematics for teaching and learning (interactive) (Tatto et al. 2008). Five different booklets were created to cover these test items, and each FPMT took only one of these booklets. This arrangement ensured that, across any sufficiently large group of respondents, the full range of content was tested.

5.1 Overall Performance in MCK and MPCK

The overall performance of NIE FPMT in the TEDS-M tests is given in Table 3. As a country, Singapore ranked first or second in MCK and MPCK in each program group, but individual NIE programs had different performance levels. The BSc (Ed) group had the best performance in both MCK and MPCK because some of them had the opportunity to study tertiary level mathematics. The Dip Ed (C) group had the lowest performance, and a plausible reason was that these student teachers, who did not qualify for a degree program, had to learn to teach three teaching subjects within two years. This may be quite challenging for some of them.

5.2 Relationship Between MCK and MPCK

The Pearson product-moment correlation coefficients between the MCK and MPCK scores, also given in Table 3, range from 0.32 (Singapore) to 0.54 (Poland). These values suggest that both types of tests had measured some common trait, likely to be mathematics in this case.

The average of the six correlations for countries in the Generalist group was 0.39, lower than the average of the six correlations for countries in the Specialist group (0.48). This may be because future teachers in the Specialist group tend to spend more time and effort working on MCK and MPCK for only one or two subjects, and this more focused experience is likely to mutually reinforce their development of MCK and MPCK. Although correlations do not necessarily imply causation, the fact that MPCK is built on mathematics suggests that a sufficient level of mathematics content knowledge is necessary for the development of sound pedagogical content knowledge.

In the case of Singapore, the correlations were moderated by the types of programs, from the lowest of 0.17 (PGDE, option C, generalist) to the highest of 0.42

Table 3 MCK and MPCK mean scores and correlations of future primary mathematics teachers

Program groups	Country/NIE programs	MCK Mean	MPCK Mean	Pearson *r* between MCK and MPCK
Primary Generalist (Grade 6 maximum)	**Singapore: BSc (Ed)**	**625**	**626**	**0.28**
	Chinese Taipei	623	592	0.43
	Singapore: PGDE(P) (C)	**593**	**596**	**0.17**
	Singapore (All)	**586**	**588**	**0.32**
	Singapore: BA (Ed)	**586**	**587**	**0.30**
	Singapore: Dip Ed (C)	**567**	**568**	**0.35**
	Switzerland	547	539	0.38
	USA	517	543	0.48
	Spain	481	492	0.41
	Philippines	439	457	0.34
Primary Mathematics Specialist	Poland	614	574	0.54
	Singapore: PGDE(P) (A)	**600**	**601**	**0.42**
	Singapore (All)	**599**	**603**	**0.40**
	Singapore: Dip Ed (A)	**598**	**607**	**0.39**
	Germany	555	552	0.52
	Thailand	528	506	0.50
	USA	519	544	0.47
	Malaysia	488	503	0.44

Notes. International mean: 500, standard deviation: 100. Special annotations about primary future teachers in various countries: (a) Poland: Combined participation rate between 60 % and 75 %; institutions with consecutive programs only were not covered. (b) Switzerland: Only institutions where German is the primary language of use and instruction were covered. (c) USA: Only public institutions were covered; combined participation rate between 60 % and 75 %. Caution about comparing findings across these countries. Singapore option (A) covers two teaching subjects and option (C) covers three teaching subjects

(PGDE, option A, specialist). These two extreme values are consistent with the above observation about the generalist-specialist divide and the different opportunities to learn how to teach two or three subjects required by these two different programs.

In the following sections, we have selected, from the items released by TEDS-M, two MCK and two MPCK items for discussion because they illuminate certain *mathematical* ideas that will become apparent later on. The overall results for these four items are given in Table 4.

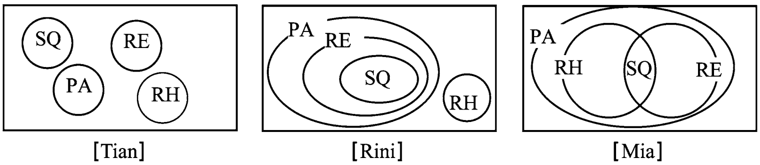
Table 4 Results of Singapore and international performance on two MCK and two MPCK items (Primary) (in percentages to 1 decimal place; correct answers in bold)

		A	B	C	D
MFC204	Singapore	3.9	30.3	**65.7**	
	International	11.2	25.1	**63.7**	
MFC412A	Singapore	**82.2**	4.0	3.3	10.5
	International	**60.0**	6.7	16.2	17.1
MFC412B	Singapore	13.7	**77.7**	8.0	0.7
	International	22.9	**54.8**	18.1	4.2
MFC108	Singapore	27.8	31.2	**32.9**	8.1
	International	30.6	28.5	**30.2**	10.7

		Correct	Partially Correct	Incorrect
MFC208A	Singapore	53.8	13.5	32.7
	International	25.6	15.8	60.4
MFC208B	Singapore	39.2	35.9	24.9
	International	23.0	23.3	58.3

5.3 MFC204: MCK: Geometry, Knowing

Three students have drawn the following Venn diagrams showing the relationships between four quadrilaterals: Rectangles (RE), Parallelograms (PA), Rhombuses (RH), and Squares (SQ).



Which student's diagram is correct? (A) [Tian] (B)[Rini] C [Mia]

This geometry knowledge item is an example of the specialized content knowledge for teaching (Graeber and Tirosh 2008; Hill et al. 2008), and similar Venn diagrams are found in many mathematics textbooks. About 66 % of NIE FPMT had chosen the correct option C. Although the 30 % choosing option B might not know the properties of rhombuses, they seemed to know the relationships among parallelograms, rectangles, and squares. Given that similar geometric relationships have been covered in the NIE SK courses and that this item is likely to be at only Level 3 (Abstraction) of the five levels of the van Hiele theory of geometry thinking (van Hiele 1986), we expect NIE FPMT to perform better in this task than the result reported here.

5.4 *MFC412A and MFC412B: MCK: Algebra, Knowing*

[Sam] wanted to find three consecutive EVEN numbers that add up to 84.
He wrote the equation $k + (k + 2) + (k + 4) = 84$.

(a) What does the letter k represent?

(A) The least of the three even numbers.
(B) The middle even number.
(C) The greatest of the three even numbers.
(D) The average of the three even numbers.

This is a straightforward test about interpreting the meaning of a letter used in a given equation. Hence, it is not surprising that 82 % of NIE FPMT had chosen the correct option. The small percentage of NIE FPMT (10 %) who had chosen option D may have been prompted by the surface feature of adding numbers together as part of the procedure to find an average.

Part (b) of the above item deals with *odd* numbers.

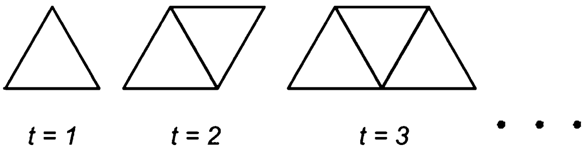
(b) Which of the following expressions could represent the sum of three consecutive ODD numbers?

(A) $m + (m + 1) + (m + 3)$
(B) $m + (m + 2) + (m + 4)$
(C) $m + (m + 3) + (m + 5)$
(D) $m + (m + 4) + (m + 6)$

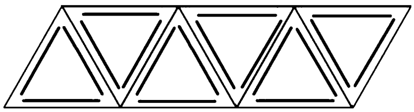
This item may be "tricky" because the correct option involves *even* rather than *odd* numbers. It challenges the future teachers to resolve this mathematical conflict, for example, by identifying the correct option even though it contains no odd numbers! Apparently about 22 % of NIE FPMT were affected by the distraction of seeing "ODD" in the stem and odd numbers in the options (A and C), although 78 % had chosen the correct option. The responses to both items taken together suggest the need to distinguish between surface and deep features of the symbols used in algebra.

5.5 *MFC108: MPCK: Enacting, Advanced Level*

[Amy] is building a sequence of geometric figures with toothpicks by following the pattern shown below. Each new figure has one extra triangle. Variable t denotes the position of a figure in the sequence.



In finding a mathematical description of the pattern, [Amy] explains her thinking by saying: I use three sticks for each triangle.



Then I see that I am counting one stick twice for each triangle, except the last one, so I have to remove those. Variable n represents the total number of toothpicks used in a figure. Which of the equations below best represent [Amy's] statement in algebraic notation?

(A) $n = 2t + 1$ (B) $n = 2(t + 1) - 1$ (C) $n = 3t - (t - 1)$ (D) $n = 3t + 1 - t$

Equivalent algebraic expressions can take different forms depending on how they are derived, especially when manipulatives are used in teaching. This is another form of specialized content knowledge for teaching.

Only 33 % of NIE FPMT could relate Amy's action to the correct expression (C). The other two popular options were B (31 %) and A (28 %); these two answers are mathematically correct, but the $2t$ term in these answers is not *directly* related to Amy's action of beginning with three sticks. As teachers are encouraged to use manipulatives to teach mathematics, it becomes imperative that they can link the kinesthetic actions of using the manipulatives to the underlying mathematics; otherwise, hands-on activities can degenerate into interesting "busy" work devoid of significant mathematics contents. The poor results above (both Singapore and international) suggest the need to find ways to help student teachers establish this critical mathematical-pedagogical link.

5.6 *MFC208A and MFC208B: MPCK: (A) Enacting, Intermediate Level; (B) Enacting, Advanced Level*

A crucial aspect of MPCK is the ability to recognize the mathematical nature of pupils' misconceptions and then to design appropriate remediation to help them

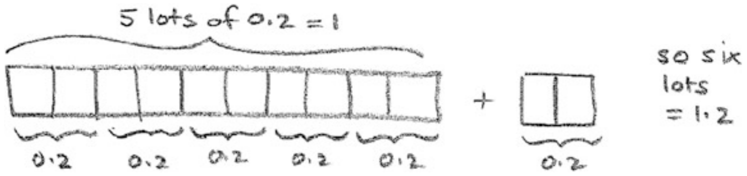


Fig. 2 A correct visual representation for 0.2×6

to overcome these misconceptions. At the primary level, visual representation is a powerful technique to help pupils learn, and this should be part of the repertoire of competent mathematics teachers. The following two items deal with the widely cited misconceptions of "multiplication makes bigger" and "division makes smaller" and how to deal with them.

> [Jeremy] notices that when he enters 0.2×6 into a calculator his answer is smaller than 6, and when he enters $6 \div 0.2$ he gets a number greater than 6. He is puzzled by this, and asks his teacher for a new calculator!
> (a) What is [Jeremy's] most likely misconception?

Slightly more than half (54 %) the NIE FPMT could state both misconceptions, for example, "He thinks that when you multiply the answer should be larger and when you divide the answer should be smaller". About 14 % gave only one misconception, probably thinking that the word "is" in the item requires only one answer. What was truly surprising to us was that about one third of NIE FPMT could not recognize these misconceptions, giving irrelevant responses, such as "Jeremy did not know how to use calculator" or "did not understand decimals". These two misconceptions are covered in the NIE CS courses, and this poor result is of particular concern to us.

Part (b) tests whether future teachers could provide a visual representation to help Jeremy come to grips with one of the misconceptions.

> (b) Draw a visual representation that the teacher could use to model 0.2×6 to help [Jeremy] understand WHY the answer is what it is?

A correct representation given in the TEDS-M marking scheme is shown in Fig. 2.

Partially correct responses include a correct diagram but not showing how 1.2 is obtained. About 40 % of NIE FPMT could give a correct response, and only 23 % gave the correct responses to both parts (a) and (b). These poor results are not satisfactory.

It is worthwhile at this stage to compare the above findings with two Singapore studies that investigated the multiplication misconception in a more direct way

(Cheang et al. 2007; Lim-Teo et al. 2007). The following item was used. It was scored on a 0–4 point scale.

> A pupil tells you that when you multiply two numbers together, the product is always larger than either of the two numbers. How do you respond to the pupil?

In the first study, a cohort of 80 Dip Ed student teachers responded to the item as a pretest in 2003 at the beginning of their program and two years later, 67 of them took it in a posttest at the end of their training. The mean increased significantly from 2.09 to 3.25. In the second study, a cohort of 113 PGDE (Primary) student teachers also performed significantly better between July 2005 and February 2006, with the mean increasing from 2.27 to 3.08. These two findings show that the NIE CS courses had been effective because the student teachers "showed more awareness in the post-test by quoting counter examples of multiplying by one or by zero" (Lim-Teo et al. 2007, p. 252). The TEDS-M items and this local item illustrate that the same misconception can be assessed differently.

A limitation about studying the performance of future teachers using the TEDS-M tests should be noted here. Doing well in these tests may indicate quality program outcomes, but future teachers who score highly in paper-and-pencil tests may or may not become competent mathematics teachers after graduation. This link between pre-service performance and enacted performance in the future is worthy of further investigation.

6 Relationships between OTL and Performance

This section explores the relationships between the OTL measures reported in Sect. 4.2 and the performance of FPMT in MCK and MPCK. Pearson product-moment correlation coefficients were computed between these two sets of variables, and the results are given in Table 5. The weak correlations suggest that further analysis is not warranted.

Performance in MCK was associated with Tertiary Level Mathematics OTL. This is not unexpected because more opportunities to learn higher level mathematics are likely to equip future teachers with stronger content knowledge, which was assessed by the MCK items. A similar positive relationship between MPCK and Tertiary Level Mathematics OTL may arise because the scoring of some of the MPCK items required correct mathematics in the answers. Thus, it appears that Tertiary Level Mathematics OTL could be a factor that led to the significant correlations between MCK and MPCK as reported in Table 3.

A plausible hypothesis is that MPCK is related to Mathematics Pedagogy OTL. However, the near-zero correlations between MPCK and the two measures of mathematics pedagogy OTL did not support this hypothesis. Furthermore, perceptions of effectiveness of the training program were not linked to performance in MCK and MPCK. These two issues require further investigation.

Table 5 Correlations between performance in MCK and MPCK and OTL scales (NIE FPMT)

	TM	SM	MP	Activity	Effective
Mean	0.38	0.62	0.70	2.81	3.14
MCK	0.160	−0.004	0.006	−0.035	−0.019
MPCK	0.153	0.016	−0.005	0.013	0.072

Notes. TM: Tertiary Level Mathematics OTL (0 = Not studied; 1 = Studied); SM: School Level Mathematics OTL (0 = Not studied; 1 = Studied); MP: Mathematics Pedagogy OTL (0 = Not studied; 1 = Studied); Activity: Activities engaged in during mathematics pedagogy course (1 = Never; 4 = Often); Effective: Perceptions of effectiveness of training program (1 = Very ineffective; 4 = Very effective)

7 Discussions and Conclusion

The search for effective strategies to place enough well-qualified mathematics teachers into schools is an ongoing challenge in many countries. Comparative findings about teacher education may suggest "good" practices that are taken for granted in one country but could stimulate other countries to re-examine their current practices leading to possible adaptations. We now offer several suggestions based on our interpretations of the Singapore practices and the TEDS-M findings reported in the earlier sections. These suggestions are discussed under the two broad areas of recruitment and training.

7.1 Recruitment Matters

The international community has stressed the importance of recruiting suitably qualified applicants with strong academic qualifications, communication skills, and appropriate motivations into the teaching profession. We wish to reiterate the point made by Auguste et al. (2010) that the top performing school systems recruit "the top third of the academic cohort" (p. 5) into the teaching profession. If the applicants were weak in subject matter knowledge, then "initial teacher preparation programmes are forced to teach mathematics that could have been learnt in secondary school" (Schwille and Dembélé 2007, p. 61), and this would not be productive use of the limited resources allocated to teacher training in some countries. One Singapore policy worthy of mention is that all primary future teachers must pass O-Level Mathematics, regardless of whether or not they are trained to teach it. This ensures minimum competency of mathematics among all Singapore teachers.

As noted earlier, Singapore has actively recruited qualified professionals who wish to make a mid-career switch to teaching because of their avowed altruistic reasons for becoming teachers, for example, to make a difference to the lives of children. This policy has opened up the education profession to many different types of applicants, and this will help to alleviate any shortage of teacher recruits from the traditional groups, namely fresh graduates from high schools, polytechnics, or universities. However, training these mid-career future teachers presents different challenges than training first-career future teachers. Some mid-career future teachers at the NIE require additional assistance to recall their mathematical content knowledge and skills. Nevertheless, they are more "mature" in understanding the complex contexts of teaching, being able to compare and contrast their previous professional experiences with the values, knowledge, and skills required for teaching. Teacher education institutes that intend to recruit these future teachers should be well prepared to offer different opportunities to learn for these adults, for example, incorporating techniques of andragogy (Knowles et al. 2005) into their course delivery.

A notable recruitment policy in Singapore is that all student teachers selected to attend the NIE draw full salaries for the first two years of training, with tuition fees paid by the MOE. They are able to devote full attention to the training without having to worry about living expenses and fees, as is the case of future teachers in countries that do not provide such generous financial support. This policy is costly, but the Singapore experience shows that it is worthwhile investment as it has produced the required number of qualified teachers for the country.

7.2 Training Counts

Well-qualified recruits require proper training to realize their potentials. Section 3 documents that the NIE, like many teacher education institutes around the world, has designed programs that vary in duration and curriculum structures to train different groups of future teachers. NIE programs have been revised regularly to ensure that they are responsive to both external changes such as recruitment numbers and education initiatives launched by the MOE and within-institution research and feedback from the student teachers about their training. Although such feedback has been gathered in the NIE on a regular basis, TEDS-M provides the first opportunity for the perceptions about the NIE programs to be compared across countries. Differences between NIE and international practices, some of which are briefly reported in Sect. 4, are appropriate starting points for future review. Further insights could be obtained from secondary analyses of the international dataset after it has been released in the public domain.

We mention in Sect. 3.4 that NIE student teachers are exposed to effective pedagogy that is a blend of global "best" practices with local experiences. This is illustrated by the training resources for Mathematics CS courses produced by NIE mathematics educators. This approach could have contributed to the overall positive perception of NIE FPMT to the effectiveness of their programs. On the other hand, the weak correlations of performance in mathematics pedagogical content knowledge with this effectiveness perception and opportunities to learn mathematics pedagogy as noted in Sect. 6 suggest that helping future teachers to develop effective pedagogy is more complex than merely providing opportunities to learn in specific courses conducted on campus.

Training that counts should include activities specially targeted for the specific subjects. For Mathematics, NIE student teachers are trained to strengthen their ability to solve and design challenging mathematics problems using the "model method" in the SK courses. The relatively high scores of NIE FPMT on the TEDS-M tests suggest that they have mastered much of the MCK and MPCK domains tested, but the detailed analyses of the four items given in Sect. 5 have highlighted some gaps in their mastery of the more advanced teaching scenarios. For example, it was noted above that as many as one third of NIE future teachers may need more opportunities to learn about ways to deal with pupils' misconceptions in mathematics. Further analyses of the remaining items in the TEDS-M tests will inform mathematics educators of different teaching scenarios that they can discuss in their courses.

The released MCK and MPCK items from the TEDS-M study can be used as another training resource. The NIE TEDS-M team is preparing a book consisting of these released items, the scoring guides, the Singapore results against international benchmark, and samples of constructed responses from NIE FPMT. NIE lecturers can use these materials in their lessons with future cohorts of student teachers in a number of ways: explore strategies to remedy misconceptions, design classroom activities that mirror the scenarios described in the TEDS-M items, linking assessment items with the TEDS-M framework, and so forth. Thus, although the TEDS-M items were originally created as "assessment *of* teacher training" (summative), it can be used as "assessment *for* teacher training" (formative). Other countries might wish to develop similar materials from the rich data from the TEDS-M study.

At the NIE, mathematicians teach the SK and AS courses and mathematics educators teach the CS courses, but they belong to the same Academic Group (equivalent to department). Under this organization, there are many opportunities for mathematicians and mathematics educators to work in committees and projects that draw on their separate expertise to achieve the common goal of training competent mathematics teachers. They can also share information about the same student teachers who have taken these different types of courses. Furthermore, all NIE mathematicians learn to supervise practicum of student teachers at secondary schools through a process of informal mentoring, and this requirement provides an important opportunity for them to observe first-hand school mathematics teaching and to share their views as a subject specialist with the student teachers. Getting mathematicians actively involved within a well-defined structure in the training of future mathematics teachers is still not common in traditional teacher education institutes in many countries, but this could be a promising area to strengthen the discipline-pedagogy link as a factor to make pre-service training really count.

Although the TEDS-M study did not address in-service teacher education, it is necessary to consider pre-service and in-service teacher education along a continuum of life-long learning (Musset 2010). As the sole teacher education institute in Singapore, the NIE is in the prime position to establish this continuity by balancing the contents and delivery of both types of training, for example, to decide whether to include proofs and validation topics in initial teacher preparation or in-service professional development. In countries where these two types of training are offered by different institutes or where there is little collaboration among the ministry of education, schools, and teacher education institutes, providing this coherent transition can be challenging but this needs to be properly addressed.

To conclude, we hope to learn much more from the international reports of the TEDS-M study and comparative analyses of the TEDS-M data. Through participating in fruitful dialogues across different systems, teacher educators from around the world can work together to create quality teacher education programs so that well-qualified teachers are available to educate their pupils to lead meaningful lives in the 21st century.

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Teacher Education Effectiveness: Quality and Equity of Future Primary and Future Lower Secondary Teachers' General Pedagogical Knowledge

J. König, S. Blömeke, L. Paine, W.H. Schmidt, and F.-J. Hsieh

Abstract For more than two decades, three components of teacher knowledge have been discussed, namely, content knowledge (CK), pedagogical content knowledge (PCK), and general pedagogical knowledge (GPK). Although there is a growing body of analytic clarification and empirical testing with regard to CK and PCK, especially with a focus on mathematics teachers, hardly any attempt has been made to learn more about teachers' GPK. In the context of the Teacher Education and Development Study in Mathematics (TEDS-M), Germany, Taiwan, and the United States worked on closing this research gap by conceptualizing a theoretical framework and developing a standardized test of GPK, which was taken by representative samples of future elementary and middle school teachers in these countries. Four task-based subdimensions of GPK and three cognitive subdimensions of GPK were distinguished in this test. TEDS-M data are used (a) to test the hypothesis that GPK is not homogenous but multidimensional and (b) to compare the achievement of future elementary and middle school teachers in Germany, Taiwan and the US. The data revealed that US future teachers were outperformed by both the other groups. They showed a relative strength in one of the cognitive subdimensions, generating

Expanded version of König et al. (2011).

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S. Blömeke et al. (eds.), *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn*, Advances in Mathematics Education,
DOI 10.1007/978-94-007-6437-8_9,

strategies to perform in the classroom, indicating that in particular they had acquired procedural GPK during teacher education.

Keywords Adaptivity · Assessment · Classroom management · Cognitive process · General pedagogy · General pedagogical knowledge · GPK · Generating · Instruction · Motivation · Recall · Structure · Teaching methods

Researchers identify and distinguish among three domains of teacher knowledge (Baumert and Kunter 2006; Bromme 1997; Grossmann and Richert 1988; Shulman 1986, 1987): content knowledge (CK), pedagogical content knowledge (PCK), and general pedagogical knowledge (GPK). Regarding the latter domain, one can state that although over the past few years the body of research on teacher knowledge has been growing, it still remains an open question what exactly is meant by the term GPK and what this knowledge domain incorporates. In a time of globalization when the discourse on teacher education and the definition of what pre-service and in-service teachers have to know and be able to do are no longer limited to institutional, regional or national boundaries, the fact that the term itself is not used in all countries or at least not in the same way will inevitably come to the front and increasingly lead to the need for clarification.

Discussions about the reform of teacher education are often dominated more by normative than evidence-based statements (Ball et al. 2008; König and Blömeke 2013, in press). Especially with regard to general pedagogy as a component of teacher education programs, broad claims about its "uselessness" as well as about what future teachers need to know at the end of their training have been made and linked with requests either to eliminate this component or to structure it in a new way (Grossman 1992; Kagan 1992). Even if such discussions and assumptions may provide promising hypotheses, without empirical testing they have their limits in the process of improving teacher education (Larcher and Oelkers 2004).

The growing body of research in the field of (future) teachers' knowledge has a special focus on subject-related issues—mostly exemplified by mathematics teachers in prominent research studies like *Learning Mathematics for Teaching* (LMT; e.g., Ball et al. 2008; Hill et al. 2004), *Mathematics Teaching in the 21st Century* (MT21; Schmidt et al. 2013) or *Professional Competence of Teachers, Cognitively Activating Instruction, and Development of Students' Mathematical Literacy* (COACTIV; Baumert et al. 2010; Krauss et al. 2008). These studies mainly focused on CK and PCK in mathematics.

Also in the "*Teacher Education and Development Study: Learning to Teach Mathematics* (TEDS-M)", the first comparative study on tertiary education with representative samples and direct testing of teacher knowledge, the common international questionnaire focused on the future teachers' mathematics content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK) in their last year of training. Three participating countries—the USA, Germany, and Taiwan—decided therefore to develop a national option measuring future teachers' GPK. The option was developed under the leadership of the German TEDS-M team (König and Blömeke 2009, 2010a, 2010b, 2010c; Blömeke and König 2010a, 2010b).

In this chapter, we report first how the general pedagogical knowledge test was conceptualized. It specifies the elements of GPK which future teachers have to acquire in teacher education in order to progress from the stage of teacher "novices" to "advanced beginners" (Berliner 2001, 2004). Based on data from future teachers in the USA, Germany, and Taiwan, the structure of GPK as well as specific strengths and weaknesses of future primary and future lower secondary teachers will be examined in a second step. Third, we will draw conclusions for next research steps and the reform of teacher education.

1 Defining General Pedagogical Knowledge of Future Teachers

Seen from an international perspective, it is a great challenge to determine what is meant by the term GPK and what this knowledge domain incorporates. In the USA, two broad labels—"educational foundations" and "teaching methods"—are needed to cover what may be labelled as "general pedagogy" in another country. In yet another country, the theoretical underpinnings of education may be provided by educational psychology, sociology of education or history of education. The opportunities to learn (OTL) implemented in these components of teacher education may be very diverse, too, not only across countries but also within one country.

The shape of general pedagogy is probably influenced by cultural perspectives on the objectives of schooling and on the role of teachers (Hopmann and Riquarts 1995). But at the same time evidence exists that there is some communality in the OTL due to the nature of teaching (Blömeke 2012; Blömeke and Kaiser 2012). A literature review reveals that two tasks of teachers are regarded as core tasks in almost all countries: instruction and classroom management. Generic theories and methods of instruction and learning as well as of classroom management can therefore be defined as essential parts of GPK. Less agreement exists as to what extent and what kind of knowledge about counselling or nurturing students' social and moral development or knowledge about school management should also be included in the area of general pedagogy.

According to Shulman (1987, p. 8) general pedagogical knowledge involves "broad principles and strategies of classroom management and organization that appear to transcend subject matter" as well as knowledge about learners and learning, assessment, and educational contexts and purposes. Similarly, and extending this definition, Grossman and Richert (1988, p. 54) stated that GPK "includes knowledge of theories and learning and general principles of instruction, an understanding of the various philosophies of education, general knowledge about learners, and knowledge of the principles and techniques of classroom management." Future teachers need to draw on this range of knowledge and weave it into coherent understandings and skills if they are to become competent to deal with what McDonald (1992) called the "wild triangle" that connects learner, subject matter and teacher in the classroom.

Since there was a lack of empirical studies on (future) teachers' GPK (Wilson and Berne 1999) when TEDS-M started, many key questions were at that time unanswered. There were virtually no studies showing how to fill these relatively broad domains of GPK so that one could develop items and actually test teachers (Baumert and Kunter 2006). Another open question was how to discriminate GPK from MPCK. In the USA, Germany and Switzerland, some first attempts existed to measure the GPK of future or practicing teachers (Baer et al. 2007; Grossman 1992; Schulte 2007) but these studies were restricted to specific institutions, languages or regions. Other studies had tried to capture GPK with self-reports of future teachers (e.g., Oser and Oelkers 2001) but these did not include objective tests.

Against the background of this research gap, the authors of this chapter aimed at developing a theoretical framework of future teachers' GPK that could be tested empirically across countries in the context of TEDS-M. Due to the complexity of GPK, the audience of an international survey, the target population of future teachers (and not practicing teachers) and with regard to standardized test procedures on a large scale, it was necessary to make certain restrictions in the definition of general pedagogy.

Following the concept of "competence" (see in general Weinert 2001; specified for the teaching profession by Bromme 1992, 1997, 2001), the study's framework focused on the mastering of professional tasks and reaching important objectives of teaching. This meant that the theoretical framework of GPK was structured in a task-based way and explicitly not according to the formal structure of general pedagogy as an academic discipline. Since it is widely accepted that instruction represents the core activity of teachers (Baumert and Kunter 2006; Berliner 2001, 2004; Blömeke et al. 2008; Bromme 1997), the central demands placed on teachers are related to student learning. While other teacher tasks like counselling or nurturing students' social and moral development were regarded equally important, they could not be included within the framework of TEDS-M. The cultural differences not only across the three participating countries but also within these did not suggest a common sense of "correct" or "incorrect" performance strategies necessary for objective testing. Clearly, these restrictions leave space for future research that follows a broader understanding of GPK.

2 Subareas of General Pedagogical Knowledge

Our focus on instruction served as a heuristic to select different topics and cognitive demands of general pedagogical knowledge. In order to operationalize it, we referred to the extensive research on instruction. Instructional research provides various models of school learning (e.g., Carroll 1963; Bloom 1976). Such models contain elements that are directly under the control of the teacher (e.g., the effectiveness with which a lesson is actually delivered) and elements that are characteristics of the students which are difficult to change (e.g., the students' general abilities to learn). Since our perspective focused on elements teachers can influence, we decided to

Content dimensions	Topics covered by the test items
structure	- structuring of learning objectives - lesson planning and structuring the lesson process - lesson evaluation
motivation/ classroom management	- achievement motivation - strategies to motivate single students/ the whole group - strategies to prevent and counteract interferences - effective use of allocated time/ routines
adaptivity	- strategies of differentiation - use of a wide range of teaching methods
assessment	- assessment types and functions - evaluation criteria - teacher expectation effects

Fig. 1 Content dimensions and topics covered in the TEDS-M test of GPK

take the QAIT model by Slavin (1994) as a basis to describe teacher tasks in more detail.

The QAIT model is a model of effective instruction which focuses on four elements:

- The first element, "Quality of instruction" (Q), refers to activities of teaching that make sense to students, for instance, presenting information in an organized way or noting transitions to new topics.
- "Appropriate Levels of instruction" (A) is an element that refers to dealing with a heterogeneous class. For teachers, it is challenging to adapt instruction to students' diverse needs. adaptivity deals, for instance, with the level of instruction (that is appropriate when a lesson is neither too difficult nor too easy for students) or with the different methods of within-class ability grouping.
- The third element, called "Incentives" (I), deals with the motivation of students to pay attention, to study, and to perform the tasks assigned. For a teacher, this means, for instance, relating topics to students' experiences.
- "Time" (T) is the fourth element of the model. It refers to the quantitative aspect of instruction and learning, e.g., strategies of classroom management enabling students to spend a high amount of time on tasks.

According to Slavin (1994), the four elements are linked to each other and instruction is only then effective if they are all applied. The QAIT elements correspond to elements of other models and listings of effective teaching (Helmke 2003; Baumert et al. 2004; Good and Brophy 2007). So, the four elements can in fact be regarded as basic dimensions of teaching quality (Brophy 1999).

However, to identify potential shortcoming of this framework, we compared the basic dimensions of teaching quality with didactical points of view (cf. Klafki 1985; Tulodziecki et al. 2004; Good and Brophy 2007). In these, diagnosing and assessing student achievement was in fact more strongly focused. In the QAIT model it is more regarded a precondition of adaptivity. However, we specifically added it because assessing students is an essential teacher task (Good and Brophy 2007).

The approach of combining findings from instructional research and didactics led us to conceptualize GPK for teaching as is shown in Fig. 1. Teacher education was regarded as effective if future teachers in their last year of their training had acquired general pedagogical knowledge allowing them to prepare, structure, and evaluate lessons (“structure”), to motivate and support students as well as manage the classroom (“motivation/classroom management”), to deal with heterogeneous learning groups in the classroom (“adaptivity”), and to diagnose and assess student achievement (“assessment”). Three of these content dimensions corresponded to the QAIT model. Assessment was added due to its didactical relevance.

Apart from the task-based content dimensions of GPK, we defined dimensions of cognitive processes describing the cognitive demands on future teachers when they respond to test items. Following Anderson’s and Krathwohl’s elaborate and well-known model (Anderson and Krathwohl 2001), we distinguished three cognitive processes which summarized the original six processes: recalling, understanding/analyzing, and generating. Future teachers had to retrieve information from long-term memory in order to respond to a test item. They had to understand or to analyze a concept, a specific term or a phenomenon outlined by a specific test item. And they were asked to generate concrete strategies on how they would solve a typical classroom situation problem which includes evaluating this situation. Our hypothesis was that future teacher performance on test items varies according to these cognitive processes.

Distinguishing between declarative and procedural knowledge as another cognitive approach is very common in teacher research (besides Anderson and Krathwohl 2001 see e.g., Fenstermacher 1994; Bromme 2001). In our instrument, test items requiring future teachers to recall information predominantly measured declarative knowledge (“knowing that . . . ”) including factual and conceptual knowledge while test items requiring future teachers to generate strategies not only measured declarative but also procedural knowledge (“knowing how . . . ”). Procedural knowledge is of a situated nature (Putnam and Borko 2000).

3 Test Instrument

Content dimensions of GPK and cognitive demands made up a matrix which served as a heuristic for item development (see Fig. 2). For each cell, a subset of items was developed. Several expert reviews in the USA, Germany, and Taiwan as well as two large pilot studies were carried out. All experts who participated in the first item review which aimed at selecting items for the first pilot study testing a large pool of items were teacher educators in the field of general pedagogy. Their research had to be related to the topic of teacher knowledge and they had to be at least PhD candidates. Experts that participated in the second and following reviews which aimed at selecting items for the final test instrument according to specific criteria or that aimed at validating the test instrument, respectively, had to endow a university chair with a specialization on research about teacher knowledge. Based on these review

	recall	Understand/ analyze	Generate
structure			
motivation/classroom management			
adaptivity			
assessment			

Fig. 2 Test design matrix

processes and empirical findings from the two pilot studies (e.g., item parameter estimates) as well as on conceptual considerations with respect to the framework, the final item set was selected (König and Blömeke 2009, 2010a, 2010c; Blömeke and König 2010a).

The TEDS-M test measuring GPK of future primary teachers consisted of 85 test items. The TEDS-M test measuring GPK of future lower secondary teachers consisted of 77 test items. In both cases, we used dichotomous and partial-credit items, open-response (about half of the items) and multiple-choice items. Items were fairly equally distributed across the four content dimensions and the three cognitive dimensions.

Following the TEDS-M test design for MCK and MPCK (for details see Tatto et al. 2008), a balanced incomplete block-design (Adams and Wu 2002; von Davier et al. 2006) with five booklets for the future primary teacher survey and three booklets for the future secondary teacher survey was used so that each person had to respond to only 60–65 % of all test items. With permission of the TEDS-M International Study Center, the GPK test was added at the end of the original TEDS-M future teacher questionnaire.

Germany and the USA used an identical booklet design in their primary and lower secondary survey instruments allowing future teachers 30 minutes to respond to the GPK test items. In contrast, Taiwan selected five complex test items due to limited survey time, covering each cell of our test design matrix and corresponding to criteria such as difficulty level, estimated response time and item discrimination. In addition, Taiwan decided to implement the GPK in the lower secondary future teacher survey only.

Item-Response-Theory (IRT) scaling methods were used to estimate scores across the different booklets for the primary and the lower secondary future teacher survey, respectively. With the methods implemented in a software package like *Conquest* (Wu et al. 1997), it is possible to create reliable achievement scores even if a person has only responded to a selection of test items if this selection was done rigorously according to a range of specific criteria. Technical issues of the test instrument such as the booklet design were reviewed by experts from each country as well. These had do have at least a PhD in psychometrics, most of them had university chairs.

Which of the following cases represents an example of intrinsic motivation, and which represents an example of extrinsic motivation?

Check one box in each row.

A student learns before a test in mathematics, because he/she ...		**intrinsic motivation**	**extrinsic motivation**
A.	expects a reward for a good grade.	□ 1	□ 2
B.	wants to avoid the consequences of a bad grade.	□ 1	□ 2
C.	is interested in problems of mathematics.	□ 1	□ 2
D.	does not want to disappoint his/her parents.	□ 1	□ 2
E.	wants to maintain his/her relative rank in the class.	□ 1	□ 2

Fig. 3 Item example for GPK about “motivation” and “analyze”

Imagine you are helping a future teacher to evaluate her lesson because she has never done this before. To help her adequately analyze her lesson, what question would you ask? Formulate ten essential questions and write them down.

Fig. 4 Item example for GPK about “structure” and “generate”

Two item examples (see Figs. 3 and 4) illustrate the GPK test.[1] The first item measured knowledge about “motivating” students. Future teachers had to recall basic terminology of achievement motivation (“intrinsic motivation” and “extrinsic motivation”) and they were asked to analyze five statements against the background of this distinction. Statement C represented an example of “intrinsic motivation” whereas A, B, D, and E were examples for “extrinsic motivation”.

The second item example (see Fig. 4) was an open-response item. Here, future teachers were asked to support another future teacher and evaluate her lesson. This is a typical challenge during a peer-led teacher education practicum, but practicing teachers are also regularly required to analyze and reflect on their own as well as their colleagues’ lessons. The item measured knowledge of “structuring” lessons. The predominant cognitive process was to “generate” fruitful questions.

For the open-response items, coding rubrics were developed and reviewed by experts on teacher education in the USA, Germany, and Taiwan to avoid culturally biased coding and scoring. The coding instructions were developed in an extensive interplay of deductive (from our theoretical framework) and inductive approaches (from empirical teacher responses). In a pilot phase, codes from several independent

[1] Since we plan to use the test instrument in future studies, we cannot include more item examples in this article. If other researchers are interested in doing research in this field, we will be pleased to provide a complete documentation of a shorter version of the instrument with half of the test item pool, including various materials such as coding rubrics, scoring instruction, empirically based information on item parameters, and test booklets. This documentation (König and Blömeke 2010c) allows to use the GPK test independently from the authors who developed it. Please contact the initial author of this article for further information.

1) *Do your students have prior knowledge about the subject?*
2) *What are your objectives?*
3) *Are the students working individually or in groups?*
…
10) *Have your students gained the knowledge from the lesson?*

Fig. 5 A US future teacher's response to the item presented in Fig. 4

raters were discussed and coding instructions were revised and expanded. The result was then reviewed by experts. Thus, the coding manual is theoretically based as well as data-based. The codes were intended to be low-inferent, i.e. every response was coded with the least possible amount of inferences by the raters.

All questionnaires were coded by two raters independently of each other on the basis of the coding manual. As a measure of consensus and internal consistency, Cohen's Kappa was estimated (Jonsson and Svingby 2007). It ranges from 0.80 to 0.99 with an average of M = 0.91 (SD = 0.07). This can be regarded as a good result. If the raters did not agree, agreement was achieved in joint discussions, calling on a third rater if necessary.

After having established reliable and culturally unbiased coding schemes, a scoring strategy for complex open-response items was developed to decide which codes should be rewarded and which could not because they were not appropriate. Again, experts from the three countries had to agree which codes would appropriately reflect outcomes of their teacher education systems. Illustrating this strategy with the test item shown in Fig. 4, codes were scored as appropriate if they addressed the "context" of the lesson (e.g., prior knowledge of students), the "input" (e.g., objectives of the lesson), the "process" (e.g., teaching methods used), or the "output" of the lesson (e.g. student achievement). The original answer given by a future teacher from the USA is an example for the scoring strategy (see Fig. 5).

4 Research Questions and Data Aalyses

Two main research questions lead the present article: How is the general pedagogical knowledge of future primary and lower secondary teachers structured? Which level of achievement did future teachers from the USA, Germany and Taiwan show?

Our theoretical framework (see Fig. 2) had outlined four content and three cognitive dimensions of GPK. This means we assumed that GPK is multidimensional. An alternative hypothesis would be that GPK is homogeneous or one-dimensional. Technically spoken, the latter would imply an IRT scaling model in which only one latent variable was specified by all test items. Model 1 in Fig. 6 shows a graphical representation of this idea. Certain psychometric indicators (which are described below in detail) can be used as criteria to evaluate the alternative models.

Multi-dimensional IRT scaling models can be applied to models 2 and 3 in Fig. 6 which hypothesize that it is possible to distinguish four content dimensions

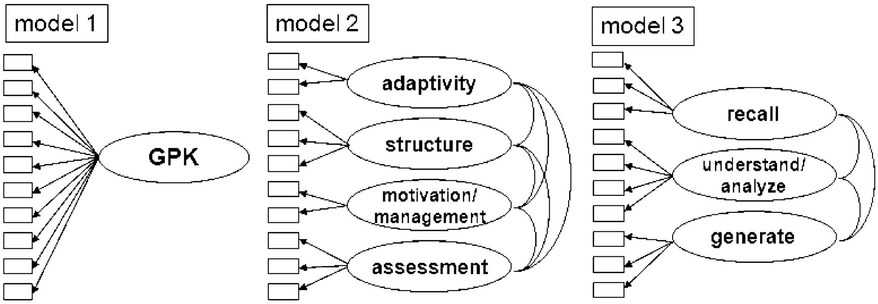

Fig. 6 One-dimensional and multi-dimensional modelling of GPK

(structure, motivation/management, adaptivity, assessment) and three cognitive dimensions (recalling, understanding/analyzing, generatinig). We specified a four-dimensional model with four latent variables and a three-dimensional model with three latent variables. All analyses were done with the IRT software *Conquest* (Wu et al. 1997; Wu 1997).

Findings on this research question provide information as to how we should report GPK results when the achievement future teachers from different countries is compared: as one overall score or as separate subscores. If reliable sub-dimensions of GPK can be modelled, we would be able to compare the outcomes of teacher education in the USA, Germany, and Taiwan in more detail because strengths and weaknesses of GPK could be described along content domains and cognitive processes. Findings of this kind would provide insight into the effectiveness of teacher education systems across countries. In a time of globalization and international competition, this is becoming increasingly important when discussing reforms of teacher education.

5 Empirical Findings on the Structure of GPK

Figure 7 shows an item-person map from the uni-dimensional IRT analysis (future lower secondary teacher survey). On the left side, abilities of future teachers are represented (one "X" represents eight persons), while on the right side the distribution of test items is shown (each of the 77 test items has a number). If the location of an item and a person match, the person has a probability of 0.5 to succeed on that item. The higher a person is above an item on the scale, the more likely the person will succeed on the item. The lower a person is below an item on the scale, the more likely the person will be unsuccessful on the item.

The map reveals that the GPK test covered the TEDS-M sample of future lower secondary teachers from the USA, Germany and Taiwan quite well as the range of person abilities (left side) was well covered by item difficulties (right side). The one-

Fig. 7 Item-person map of one-dimensional Rasch-scaling

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----------------------------------------------------
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  3             |                                  |
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                |                                  |
  2            X|43 77                             |
               X|                                  |
               X|33 34                             |
               X|63                                |
             XXX|72                                |
              XX|76                                |
            XXXX|36                                |
           XXXXX|59 62                             |
          XXXXXX|19 23 71                          |
  1    XXXXXXXXX|74                                |
         XXXXXXX|25                                |
         XXXXXXX|35 69                             |
        XXXXXXXX|56 57 70                          |
       XXXXXXXXX|49                                |
       XXXXXXXXX|12                                |
      XXXXXXXXXX|5 6 47 48 66                      |
       XXXXXXXXX|9 52 67 75                        |
       XXXXXXXXX|                                  |
  0     XXXXXXXX|4 15 61                           |
       XXXXXXXXX|13 14 17 24 32 50                 |
      XXXXXXXXXX|8 65 73                           |
       XXXXXXXXX|18 68                             |
        XXXXXXXX|20                                |
        XXXXXXXX|10 45 54 60                       |
            XXXX|3 16                              |
           XXXXX|11 28 51                          |
           XXXXX|41 55                             |
 -1          XXX|30 31 42 46                       |
              XX|1 2 22                            |
              XX|26 37 39                          |
               X|                                  |
               X|27 38                             |
               X|29                                |
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dimensional model and its results showed that it was possible to create an overall GPK test score. The reliability was good (EAP reliability 0.78).[2]

[2]Another psychometric indicator is the item fit statistics. The weighted mean squares mainly ranged from 0.80 to 1.20 with a few exceptions, which is a good result (Adams 2002; Wright

Model 2	Future primary teachers			Future lower secondary teachers		
	(1)	(2)	(3)	(1)	(2)	(3)
(1) adaptivity						
(2) structure	.72			.60		
(3) motivation/management	.77	.77		.63	.57	
(4) assessment	.81	.74	.81	.69	.65	.80

Fig. 8 Intercorrelations of GPK content domains

In contrast to a model in which all items measure one latent ability (see model 1 in Fig. 7), in a multi-dimensional IRT model test items were scaled as documented in our conceptual framework according to their content (structure, motivation/classroom management, adaptivity, assessment; see model 2 in Fig. 6) or, alternatively, according to the cognitive processes requested (recalling, understanding/analyzing, generatinig; see model 3 in Fig. 6).[3] The reliability estimates of the content domains were lower than the reliability of the overall GPK score but mostly acceptable (future primary teachers: 0.79 for structure, 0.78 for adaptivity, 0.74 for motivation/classroom management, 0.72 for assessment; future lower secondary teachers: 0.70, 0.72, 0.65, 0.64).

The intercorrelations of the four domains were high but they did not indicate homogeneity taking into account that they did not include measurement error (see Fig. 8). This result represents an indicator to assume the hypothesized multi-dimensionality rather than uni-dimensionality of future teachers' GPK.[4] Interestingly, "assessment" and "motivation/classroom management" showed the highest intercorrelation. This pattern might mirror coherence of corresponding opportunities to learn in teacher education in contrast to "structure" and "adaptivity" which seems to be more distant to the area of assessment.

The cognitive dimensions were also modelled as hypothesized. The reliability of each of these three subscales was acceptable for "recalling" and "understanding/analyzing". The reliability of "generatinig" was rather low, however, indicating that this dimension was difficult to measure. Again, intercorrelations between the different cognitive domains were partly relatively low showing that GPK is a heterogeneous construct (see Fig. 9).

Recalling and understanding/analyzing seem to be well interconnected cognitive processes (almost 0.80). By contrast, they both were only loosely connected with the third cognitive demand of generatinig (less than 0.50). Although its lower reliability

et al. 1994). Exceptions occurred when an item could not be excluded for theoretical reasons. Then we accepted weighted mean squares ranging from 0.75 to 0.79 and from 1.21 to 1.25.

[3]To investigate the internal consistency of the four dimensions we used "Expected a posteriori estimation (EAP)" as these parameters deliver an unbiased estimation of the population and take into account the multidimensional structure of the model (Wu 1997).

[4]The item fit indices generally showed a slight improvement in the four-dimensional model compared to the one-dimensional model. The deviance of the two models revealed that the four-dimensional model fits the data significantly better than the one-dimensional model. Thus, there were in fact several indications suggesting the hypothesized multidimensionality of future middle school teachers' GPK.

Model 3	Future primary teachers		Future lower secondary teachers	
	(1)	(2)	(1)	(2)
(1) recall				
(2) Understand/analyze	.76		.79	
(3) Generate	.69	.83	.49	.46

Fig. 9 Intercorrelations of cognitive domains of GPK

Country	**Future primary teachers**			**Future lower secondary teachers**		
	M	**SE**	**SD**	**M**	**SE**	**SD**
Germany	601	3.7	95	576	4.9	85
Taiwan	-	-	-	572	3.2	52
International	500	0.7	100	500	2.2	100
USA	462	2.7	72	440	3.0	66

Fig. 10 Overall GPK test score

has to be taken into account, we can hypothesize that GPK seems to consist of the ability to generate strategies as a response to typical classroom situation vignettes on the one hand and of declarative knowledge measured by items labelled as "recalling" or "understanding/analyzing" on the other hand.

6 International Comparison of Future Teachers' GPK

Our effort to measure GPK as an element of the professional knowledge of future teachers aimed at an international comparison. First, we present results on the overall GPK test score. To facilitate the reading, the mean was transformed to 500 test points with a standard deviation of 100 test points for each the primary and the lower secondary survey. Figure 10 shows the means, standard errors of the means, and the standard deviation for each country.

The data revealed that future teachers from Germany and Taiwan significantly outperformed their counterparts from the USA. The achievement of US future teachers was more than one and a half standard deviation lower than the achievement of German and, in the case of the future lower secondary teacher survey, Taiwanese future teachers. This is a difference which is of high practical relevance. There was no statistically significant difference between teacher achievement in Germany and Taiwan.

The next step in describing the GPK of future teachers is related to the subdimensions of this knowledge area. Because of the very large country mean differences on the overall GPK test score, we used ipsative measures to depict strengths and weaknesses of each country's performance on the subdomains. Ipsative measures describe the *relative* achievement of future teachers in subdimensions; they

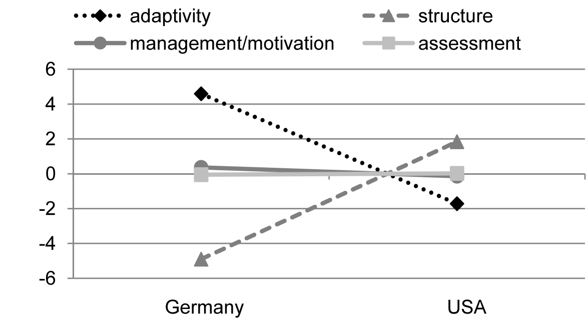

Fig. 11 Relative strengths and weaknesses for content domains of GPK (future primary teachers)

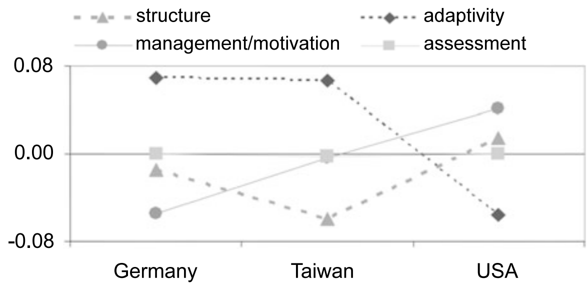

Fig. 12 Relative strengths and weaknesses for content domains of GPK (future lower secondary teachers)

are standardized differences of each subdimension compared to the overall mean (the unit is one standard deviation).[5]

With regard to future lower secondary teachers, Fig. 12 reveals that US future teachers showed relatively equal performances on the three content domains structure, classroom management/motivation, and assessment (these scores were not significantly different from zero) whereas they showed a relatively weak performance on adaptivity. In contrast, future teachers in Taiwan and Germany showed a relatively high performance on this subdimension, leading to the assumption that they acquired general pedagogical knowledge to a particularly large extent about the use of a wide range of teaching methods in order to deal with heterogeneity in classroom situations. For the future primary teachers, adaptivity is in Germany similarly mirrored as strength (Fig. 11).

[5]Fischer (2004) once explained ipsative measures by using the following analogy: "Let us consider the example of a mouse and an elephant. Assume someone measured the extremities of both animals and used within-subject (within-animal) standardization [i.e. ipsative measures]. If the researcher would now proceed to compare the length of, let us say, the legs, probably no significant differences would be found. This is despite the fact that the legs of an elephant and a mouse are surely different. This is because all the measures are related to the size of the whole animal. [...] If we compare the tail of the mouse and the elephant using ipsative measures, we would probably conclude that the mouse's tail is significantly longer than the tail of the elephant. It is important to note that this comparison makes sense only if we consider the length of the tail relative to the overall size of both animals. Obviously, relative to the overall size of the mouse and the elephant, the mouse's tail is longer than the tail of the elephant."

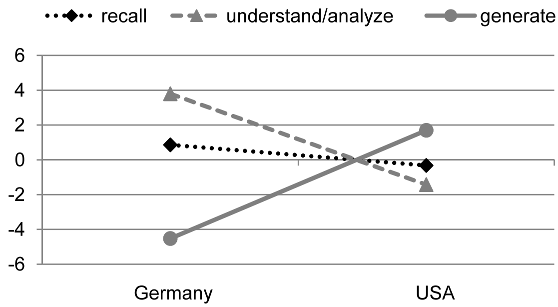

Fig. 13 Relative strengths and weaknesses for cognitive processes (future primary teachers)

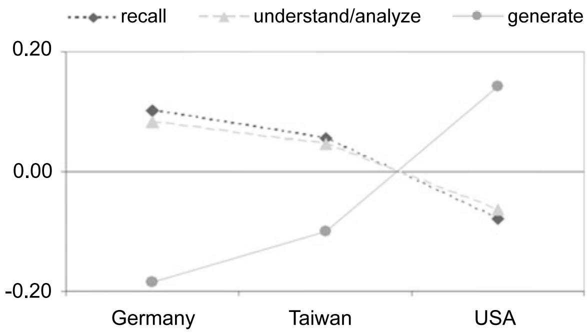

Fig. 14 Relative strengths and weaknesses for cognitive processes (future lower secondary teachers)

Figures 13 and 14 depict the relative country differences with respect to cognitive processes. Future teachers in Germany showed a relatively weak performance in generatinig strategies. Compared with their performances in recalling GPK, understanding or analyzing a concept of general pedagogy related to teaching, they seemed to struggle when asked to evaluate classroom situations and to create adequate and multiple strategies to solve typical problems.

Taiwanese future teachers showed a balanced performance on these three cognitive processes (i.e. there were no statistically significant differences from zero). In contrast to Germany, US future teachers showed a relative strength in generatinig strategies related to typical classroom situations. Although their overall GPK performance was low compared with the performance of future teachers in Taiwan and Germany, the US GPK profile pointed to a suitable strength which would be very helpful in class. So, their declarative general pedagogical knowledge was below expectations whereas their procedural knowledge was above—however, the low mean has still to be taken into account.

7 Summary and Conclusions

General pedagogical knowledge (GPK) is a central component of teacher knowledge. Teacher education programs in many countries therefore provide corresponding opportunities to learn although they are sometimes labelled differently ("educational foundations", "teaching methods", "general pedagogy", "educational psychology" etc.). In this paper we outlined a way to define and conceptualize general pedagogical knowledge for teaching in order to develop a test for large-scale assessments and international comparisons. Researchers from three countries—the USA, Germany and Taiwan—worked together in order to achieve validity with respect to teacher tasks and to avoid cultural bias.

According to our findings and in contrast to our hypothesis, it is legitimate to regard GPK as a homogenous construct. However, several indicators revealed that it is at the same time appropriate to distinguish between four content domains (structure, adaptivity, classroom management/motivation, assessment) and three cognitive processes (recalling, understanding/analyzing, generatinig) as we had assumed when we had developed our theoretical framework. Technically speaking, multi-dimensional IRT models fit the data significantly better than a one-dimensional model. Conceptually speaking, knowledge in one of these subdimensions does not necessarily mean an equal amount of knowledge in another subdimension. Since using the multi-dimensional estimates, future teachers' performance can be described in more detail, we reported both approaches.

US future teachers were significantly outperformed by future teacher in Germany and Taiwan with regard to the overall GPK test score. The difference of more than 1.5 standard deviations was very large. It meant that there was almost no overlap between US teachers on the one side and Germany and Taiwanese teachers on the other side. Most of the worst achieving teachers from the latter two groups did still better than most of the best achieving teachers from the USA. Neglecting this large mean difference, country-specific profiles revealed that US future teachers had a relative GPK strength in generatinig classroom strategies but a weakness in recalling knowledge and analyzing problems. Future teachers from Germany showed a contrary profile whereas that from Taiwan was balanced.

Such results provide important information about teacher education in the USA compared to teacher education in Germany and Taiwan. The data indicated that there were probably more opportunities in the latter two countries to acquire systematic (declarative) knowledge whereas the focus in US teacher education seemed to be on acquiring teaching skills and, by this mean, developing procedural knowledge. One may conclude that even if teaching skills is what finally matters in the classroom, there is some evidence that procedural knowledge and skills should built on extensive and systematic factual and conceptual knowledge. The low mean achievement of US future teachers may be caused by their limits in systematic knowledge. Thus, if it were possible to increase the mean GPK level of US teachers but to keep their strength in generatinig classroom-based solutions, the advantages of both approaches would be realized. Such a reform would require a careful look at the curriculum of teacher education in the USA compared to other countries.

Our results revealed that a comparative approach enables us to move beyond the familiar, and to see teacher education of different countries with a kind of "peripheral vision" (Bateson 1994). Strengths and weaknesses become visible. The structure and the content of teacher education may depend on a deeper rationale which may be a result of cultural boundaries (see specifically with respect to teacher education in the US and Germany Blömeke and Paine 2008). Stigler and Hiebert (1999) argue that teaching reflects "cultural scripts". And like the fish who is not aware of the water in which it swims, cultural givens are too often invisible to our consideration as we debate research designs, in this case, for research about teacher education. As a consequence, there is an increasing demand for comparative information about teacher education programs (König and Blömeke 2013, in press).

However, from a methodological point of view comparative studies are challenging—not least in an ill-defined area like general pedagogy. Cross-country validity is a request hard to achieve. Evaluation designs and measurement instruments have to be developed, for example, in order to precisely examine the outcomes of teacher education on the basis of educational standards. A central deficit of teacher education research has for a long time been a too narrow focus on single institutions or even single classes (Cochran-Smith and Fries 2005; Risko et al. 2008). Broader perspectives provide more orientation.

Grossman and McDonald (2008), for example, make a persuasive argument for the need for teacher education research to move beyond its "adolescence" (185) by identifying common factors that allow shared and more precise language. They suggest the "progress … will require researchers … to reach outside their immediate communities, to look over their backyards to see and learn from what their neighbors are doing" (199). A measurement instrument that is valid across three culturally very different countries as we have presented in this chapter is therefore a very important step on the way to encounter this research deficit in a time of globalization.

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Part III
Beliefs at the End of Teacher Education: International Perspectives

The Cultural Dimension of Beliefs: An Investigation of Future Primary Teachers' Epistemological Beliefs Concerning the Nature of Mathematics in 15 Countries

Anja Felbrich, Gabriele Kaiser, and Christiane Schmotz

Abstract Beliefs constitute a central part of a person's professional competencies and are crucial to the perception of situations as they influence our choice of actions. This paper focuses on epistemological beliefs about the nature of mathematics of future primary teachers from an international perspective. The data reported are part of a larger sample originating from the TEDS-M study which compares primary mathematics teacher education in 15 countries. In this paper we examine the pattern of beliefs of future teachers aiming to teach mathematics at primary level. We explore whether and to what extent beliefs concerning the nature of mathematics are influenced by cultural factors, in our case the extent to which a country's culture can be characterized by an individualistic versus collectivistic orientation according to Hofstede's terminology. In the first part of the paper, the literature on epistemological beliefs is reviewed and the role of culture and individualism/collectivism on the formation of beliefs concerning the nature of mathematics will be discussed. In the empirical part, means and distributions of beliefs rating will be reported. Finally, multilevel analyses explore how much of the variation of belief preferences between countries can be explained by the individualistic orientation of a country.

Keywords Epistemological beliefs · Mathematics education · Teacher education · Multilevel analysis · Primary teacher · Individualism · Collectivism

Corrected version of a paper originally published in ZDM—The International Journal on Mathematics Education, 2012, 44(3), 355–366.

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S. Blömeke et al. (eds.), *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn*, Advances in Mathematics Education,
DOI 10.1007/978-94-007-6437-8_10,

1 Theoretical Framework of the Study

Beliefs concerning the nature of mathematics, which are the focus of this paper, are a crucial part of the professional competence of mathematics teachers. In line with this view almost all theoretical classifications of teachers' professional competence include beliefs or attitudes besides knowledge (e.g., Baumert and Kunter 2006; Calderhead 1996; Richardson 1996; Thompson 1992). As beliefs are thought to guide a person's perception and actions, they are regarded as crucial for the application of professional knowledge in classroom situations (Leder et al. 2002; Thompson 1992). Therefore beliefs play an important role in the teaching of primary teachers, as they can be conceptualised as a bridge between knowledge and actual teaching. Results from several empirical studies are supportive of this link (Dubberke et al. 2008; Peterson et al. 1989; Staub and Stern 2002; Stipek et al. 2001; Voss et al. 2011). Furthermore, some studies even substantiate the claim that teachers' beliefs are relevant for the outcomes of teaching in terms of student achievement in mathematics (Dubberke et al. 2008; Peterson et al. 1989; Staub and Stern 2002).

Despite the extensive debate on beliefs, a precise definition of the belief construct, as well as clear-cut differentiations from other concepts such as attitudes, perceptions or conceptions, have not yet been established (Goldin et al. 2009; Hofer and Pintrich 2002; Pajares 1992; Törner 2000). Whereas Richardson (1996) developed a rather broad definition, where beliefs are seen as "psychologically held understandings, premises, or propositions about the world, that are felt to be true" (p. 103), the experiential and context-bound nature of beliefs is stressed in the work of Schoenfeld (1998).

With respect to the domain of mathematics teachers, epistemological beliefs can be separated into beliefs concerning the nature of mathematics and beliefs concerning the acquisition of mathematical knowledge (Baumert and Kunter 2006; Blömeke et al. 2008b; Ernest 1989; Goldin et al. 2009; Op 't Eynde et al. 2002; Pajares 1992; Philipp 2007; Sullivan and Wood 2008). In this chapter we focus on beliefs concerning the nature of mathematics.

1.1 Beliefs on the Nature of Mathematics

Several conceptualisations of the structure of beliefs regarding the nature of mathematics have been established, which more or less correspond to each other (Liljedahl et al. 2007). Ernest (1989) differentiates between three fundamental views of mathematics: the instrumentalist, the Platonist, and the problem-solving view. Alternatively, an earlier conception of Dionne (1984) distinguishes between a traditional, a formalist and a constructivist perspective on mathematics. Yet another conceptualisation is that of Grigutsch et al. (1998), who originally proposed four fundamental views on mathematics:

– the *formalism*-related view, resembling Ernest's Platonist view as well as Dionne's formalist perspective, where mathematics is viewed as an exact science that has an axiomatic basis and is developed by deduction;

- the *scheme*-related view corresponding to the instrumentalist and traditional view, where mathematics is regarded as a collection of terms, rules and formulae;
- the *process*-related view (or problem-solving view in the terms of Ernest, or constructivist as coined by Dionne), where mathematics can be understood as a science which mainly consists of problem-solving processes and discovery of structure and regularities; and
- the *application*-related view, where mathematics can be seen as a science which is relevant for society and life.

The empirical work of Törner and Grigutsch (1994; also Grigutsch et al. 1998) showed that these four views can be subsumed under two overarching perspectives on mathematics: the formalism-related and the scheme-related view characterise mathematics as *static* science; whereas the process-related and the application-related view conceptualise mathematics as a *dynamic* process. Furthermore, they showed that these two perspectives are not mutually exclusive and that university mathematicians in particular emphasise all four aspects of mathematics (Roesken and Törner 2010).

Empirical investigations of epistemological beliefs regarding the nature of mathematics have primarily focused on students (Grigutsch 1996; Leder et al. 2002) and on practising teachers of primary and secondary schools (Dubberke et al. 2008; Grigutsch et al. 1998; Peterson et al. 1989). With respect to future primary teachers of mathematics, Blömeke et al. (2008a) showed that they assign the highest importance to dynamic aspects followed by both static aspects. The reduction of the four views on mathematics to only two overarching perspectives has proven efficient and useful in quantitative large-scale research, where testing time is limited. It should, however, be kept in mind that this reduction may also obscure interesting cultural differences.

1.2 The Impact of Culture on the Formation of Beliefs

In line with Schoenfeld (1998), who argued that beliefs have an experiential and context-bound nature, beliefs can be understood as socially and culturally shaped mental constructs, which are acquired in educational settings with different historical traditions that vary significantly between countries. In this paper we define culture as the "shared motives, values, beliefs, identities, and interpretations or meanings of significant events that result from common experiences of members of collectives that are transmitted across generations" (House et al. 2004, p. 15). It is hypothesized that through socialisation processes a country's culture has an impact on the preferred modes of learning (Hofstede 1986).

In anthropology as well as cross-cultural psychology, several conceptualisations exist with which different dimensions of cultural differences can be described. In this paper we focus on one of the four dimensions originally described by Hofstede (1986) which seems useful in explaining differences in belief on the nature of mathematics, namely collectivism versus individualism. The collectivism–individualism dimension refers to the extent to which the individuals of a society are perceived as autonomous. In individualistic countries, learners are perceived as autonomous subjects acquiring knowledge mainly independently on their own (Triandis 1995). Lack of success in learning is often attributed to a misfit between the conditions of learning and the individual learner, i.e. in terms of composition of groups of learners or too demanding tasks, rather than to individual characteristics of the learner.

In contrast, in collectivistic countries the role of social relationships for the acquisition of knowledge is more prominent. Learners engage in learning processes because of an obligation towards their teachers, their families and other societal entities, which in turn are seen as obliged to grant the learner the necessary support. School failure in these countries is attributed to a lack of effort by the learner. Hofstede also assumed that specific differences exist in both teacher–student and student–student interactions between individualistic and collectivistic countries. In individualistically oriented societies students expect to learn how to learn and to think, whereas in collectivistic societies they expect to learn how to do something. Whereas in the latter diploma certificates are of utmost importance, they have little symbolic value in individualistic countries (Hofstede 1986). Based on these ideas it can be assumed that students in collectivistic societies more strongly endorse a schematic view on mathematics, because teachers and final examinations expect them to be proficient in the application of rules and formulae. In contrast, students from individualistic societies should feel more comfortable in engaging in mathematical investigations on their own and should therefore prefer a dynamic view of mathematics.

The claim that beliefs on the nature of mathematics are culturally imprinted can further be substantiated by drawing on research from cross-cultural psychology on preferred learning styles and works from the field of history of mathematics. Investigations from a cross-cultural perspective indicate that culture may indeed have an impact on how people learn. Several cross-culturally comparative studies have found systematic variation in learning styles depending on cultural background (for an overview see Yamazaki 2005; see also Joy and Kolb 2009). With respect to the individualism–collectivism dimension, several studies involving different Western and Asian countries show that learners from individualistic societies show a preference for abstract conceptualisation and for active experimentation in learning (Auyeung and Sands 1996; Barmeyer 2004). With respect to mathematics these learners should also prefer a process and application oriented and thus dynamic view, where mathematics is thought of as a process and used as a tool for problem solving. Learners from collectivistic societies on the other hand show a preference for concrete experiences in learning, as well as for reflective observation. This may predispose them to take a scheme and formalism related, and thus static, view of mathematics.

Additionally, based on historical studies, Siu (2009a, 2009b) elaborates that practical-algorithmic views on mathematics prevailed in Asian countries, in contrast to Western countries where dialectic-theoretical views on mathematics were dominant. However, he also points to the fact that due to the westernisation and opening up of Asian societies to Western influences, mathematics education has also incorporated Western ideas about mathematics. Consequently, nowadays both the dynamic and the static view of mathematics should be displayed by Asian teachers. This corresponds with empirical findings by Leung (2006), who was able to show that teachers in Beijing more often agreed to the static-algorithmic character of mathematics than teachers in London, who more often held a dynamic-heuristic view concerning mathematics. In contrast, views of teachers from Hong Kong, who are influenced by both Eastern and Western perspectives, were located in between the two groups.

Empirical research does not yet exist on the impact of such cultural expectations on the formation of beliefs related to the teaching and learning of mathematics in individualistic and collectivistic countries. While research on differences in beliefs about the nature of mathematics so far has usually involved a small sample of countries (e.g. Andrews 2007; Graumann and Pehkonen 1993; Pehkonen and Lepmann 1994; Pepin 1999a), the *MT21* Study (*Mathematics Teaching in the 21st Century*; Blömeke et al. 2008b; Schmidt et al. 2011) was the first study to compare future primary teachers' beliefs in six countries, namely Bulgaria, Germany, USA, Mexico, Chinese Taipei and South Korea (Blömeke et al. 2008b). The results clearly showed that country-specific differences in beliefs on the nature of mathematics do exist. In Chinese Taipei, South Korea and Bulgaria future teachers agreed with both the dynamic and the static aspect, whereas German, Mexican and US future teachers agreed more strongly with dynamic statements than with static ones (Schmidt et al. 2007, 2011).

The results of the *TALIS* Study (*Teaching and Learning International Survey*, OECD 2009) which refer to teachers' epistemological beliefs on teaching and learning of mathematics, point in the same direction as *MT21*; In this study cultural patterns concerning beliefs were identified for the first time. The study showed that in individualistically oriented societies, for example Australia and Northwest European countries, constructivist beliefs on teaching and learning are more prevalent. In contrast, in collectivistically oriented nations, such as Malaysia and South American states, transmission views have more strongly been articulated by teachers. Beliefs of Eastern European teachers and Korean teachers are situated between both groups of countries (Klieme and Vieluf 2009; Vieluf and Klieme 2011).

In summary, research shows that beliefs on the nature of mathematics entail both dynamic and static aspects and the few existing internationally comparative studies point to the fact that these beliefs differ between countries and may be influenced by cultural factors. In this paper we first explore the level and patterns of beliefs concerning the nature of mathematics of future primary teachers of mathematics from an international perspective using data of the TEDS-M study. Secondly, the data set is used to investigate the question whether these two aspects of beliefs are meaningfully related to the collectivistic or individualistic orientation of a country. It is hypothesised that teachers in countries with a more collectivistic orientation emphasise the static nature of mathematics, whereas teachers from more individualistically oriented countries show a higher preference for the dynamic aspects of mathematics.

2 Research Methods

2.1 Design of the Study and Sample

For the present investigation the TEDS-M sample of future primary teachers of mathematics is used. The population of future teachers within the TEDS-M framework has been defined as "future teachers who are in their final year of training, before they are eligible to become practicing teachers of mathematics in primary schools (either as generalist teachers or as mathematics specialists)" (Tatto et al. 2008, p. 39). The sample contained future primary teachers trained in consecutive as well as concurrent teacher education programmes (for a more detailed definition of programmes see Senk et al. 2014).

Future primary teachers participating in the survey were sampled following a two-step sampling design. First, a national probability sample of institutions preparing future primary teachers of mathematics was drawn for each of the participating countries. If a teacher preparation institution was selected for participation, all programmes of primary teacher education were included in the study. In a second step, a random sample of future teachers was selected from these institutions and their teacher education programmes. In order to meet the IEA sampling standards an effective sample of 400 teachers was aimed at for each participating country. This entailed that in smaller countries all teacher education institutions were selected, or in some countries all future teachers of the sampled institutions took part in the survey. Participation rates were computed on the level of institutions, as well as of individuals sampled. If the country's combined participation rate did not meet the IEA sampling standards of at least 60 percent of returned questionnaires, the national sample was excluded from the study. If response rates varied between 60 and 75 percent, results of the country were reported but annotated.

In the TEDS-M primary sample a total of 13,871 future primary teachers of mathematics from 15 countries was surveyed (for more detailed information on key features of teacher education programmes in participating countries see Senk et al. 2014). Complex analyses of the influence of several factors, such as opportunities to learn and entry selectivity, on the effectiveness of teacher education were carried out and are published in this book and elsewhere (cf. Blömeke et al. 2011, 2012).

2.2 Instruments

In TEDS-M, future primary teachers' beliefs on the nature of mathematics were assessed with 12 items adapted from the instrument of Grigutsch et al. (1998), which had already been used in the internationally comparative research of the *MT21* study (Blömeke et al. 2008b). The two-dimensional structure of the items representing the static and dynamic perspectives on mathematics was tested using explorative and confirmatory factor analysis, confirming that both perspectives on mathematics can be assessed with a homogeneous set of items. Future teachers' agreement with the items was assessed using a six-point Likert scale (1 = strongly disagree, 6 = strongly agree). In a second step the data were scaled using a Rasch model to arrive at more efficient scales (interval scales), which are of special importance for more complex methods of analysis (for details see the TEDS-M technical report; Blömeke et al. 2010). For ease of interpretation of the Rasch scores, individual scores were transformed to a scale with a mean value of 10, which represents a neutral perspective and thus neither agreement nor disagreement. The scale *dynamic perspective* (*Mathematics as a process of enquiry*) comprised six items emphasising the process-related and application-related character of mathematics, e.g., "In mathematics many things can be discovered and tried out by oneself" or "Many aspects of mathematics have practical relevance". The scale *static perspective* (*Mathematics as a set of rules and procedures*) contained six items stressing the importance of definitions, formulae and mathematical facts and procedures, e.g., "Mathematics is a collection of rules and procedures that prescribe how to solve a problem" and "Fundamental to mathematics is its logical rigor and preciseness".

In order to describe the cultural orientation of the participating countries the *Individualism scale* (IDV) of Hofstede was used. This continuous scale ranging from 0 to 100 is a well recognised and approved instrument to describe the respective orientations and has been validated by various large-scale research projects (Hofstede 2001). High values indicate that a country is strongly individualistically oriented, whereas low values denote countries with a collectivistic orientation. Drawing on Hofstede's research, each of the participating TEDS-M countries in the study has been assigned its score on the individualism scale, with the exception of Botswana and Georgia, where the scale has not yet been used and therefore no IDV values exist. Consequently, both countries have been excluded from the dataset in analyses involving the IDV scale. TEDS-M countries scoring high on the IDV, indicating an individualistic orientation, are the United States (91), Norway (69), Switzerland (68) and Germany (67), followed by Poland (60) and Spain (51) with somewhat smaller values. In contrast, scores pointing to a collectivistic orientation have been assigned to the Philippines (32), Malaysia (26), Singapore (20), Thailand (20) and Chinese Taipei (17), and also to Chile (23) and Russia (39).

2.3 Multilevel Analysis

As our data set has a multilevel structure, where future primary teachers (individual level) are nested within countries (aggregated level), multilevel analysis with the software package MPlus 5.0 (Muthén and Muthén 1998–2007) was used in order to obtain correct parameter estimates. Using regression analyses we tested the hypothesis that the degree to which a country can be classified as either collectivistic or individualistic can explain a significant amount of variance in future primary teachers' beliefs at the country level. We were also interested in whether high levels on the IDV scale are associated with a preference for dynamic over static beliefs. Three regression models were specified. The first model (M1) explored the predictive power of the IDV scale for the preference of dynamic beliefs on the country level. In a second model (M2), control variables on the individual level were introduced. Explorative analyses showed that future teachers' level of mathematics knowledge (MCK) as measured with the TEDS-M achievement test (see Senk et al. 2014) as well as average self-reported school achievement were positively correlated with dynamic beliefs on the nature of mathematics. *Average school achievement* was assessed with the question: "In secondary school, what was the usual level of grades that you received?" Answers could be checked on a 5-point scale ranging from 1 ("Always at the top of my year level") to 5 ("Generally below average for my year level"). For ease of interpretation the polarity of the scale was reversed with high values now indicating higher self-reported achievement. It is hypothesized that high levels of mathematics knowledge and school achievement are positively related to a preference for dynamic beliefs over static beliefs. In a third model (M3), knowledge of mathematics has been introduced as a predictor on the aggregated level in order to obtain an idea of the size of the cultural effect. It could be argued that besides the influence of mathematical knowledge at the individual level, beliefs regarding the nature of mathematics are also influenced by the absolute level on which mathematical discourse future teachers' experience in teacher education is situated. Thus, it can be assumed that high levels of mathematical knowledge at the country level are indicative of a high level of mathematical discourse in teacher education classes, which is presumably associated with a more formalistic and deductive approach to mathematics. In this respect, mathematics knowledge on the country level can be conceptualised as a compositional effect describing a particular country-specific learning environment which has an effect on the formation of mathematics-related beliefs.

3 Results

The following section is organised into three parts. First, the levels and variation of beliefs regarding the dynamic and static view on mathematics for the future teachers in the 15 countries are reported. This is followed by an examination of the country-specific patterns of beliefs in the 15 countries. Finally, using multilevel analysis it is explored how much of the variation in the belief patterns between countries can be attributed to the individualistic (versus collectivistic) orientation of countries.

3.1 International Comparison of the Beliefs on the Nature of Mathematics

Future primary teachers' beliefs concerning the static nature of mathematics vary considerably with respect to the mean level of agreement. In Table 1 means, standard

Table 1 Beliefs concerning the nature of mathematics: static perspective (mean values, standard errors and standard deviations)

	Mean	Standard error	Standard deviation
Philippines	*12.64*	*0.13*	*1.41*
Botswana	*11.96*	*0.15*	*1.36*
Thailand	*11.86*	*0.05*	*1.31*
Malaysia	*11.74*	*0.07*	*1.55*
Singapore	11.05	0.06	1.12
International	**11.03**	**0.02**	
United States[c,d,e]	11.01	0.08	1.24
Georgia	11.00	0.09	2.28
Poland[b,d]	*10.91*	*0.04*	*1.31*
Chile[d]	*10.88*	*0.04*	*1.30*
Chinese Taipei	*10.75*	*0.04*	*0.92*
Spain	*10.75*	*0.05*	*1.01*
Russian Federation	*10.75*	*0.05*	*0.94*
Norway[d,f]	*10.19*	*0.04*	*0.86*
Germany	*10.02*	*0.05*	*0.90*
Switzerland[a]	*9.99*	*0.02*	*0.71*

[a]Colleges of education in German-speaking regions

[b]Institutions with concurrent programs

[c]Public universities

[d]Combined participation rate <75 %

[e]Substantial proportion of missing values

[f]Results for Norway are reported by combining the two data sets available in order to cover the entire population of future primary teachers

errors and standard deviations of future teachers' agreement in each of the 15 countries are shown. It is apparent that future primary teachers in the Asian countries of the Philippines, Thailand and Malaysia, as well as in Botswana, highly stress a static perspective on mathematics. In general, the acceptance of static statements in these south-east Asian countries is significantly higher than the international mean[1] and also higher than the theoretical scale mean of 10, which represents a neutral perspective, i.e. neither agreement nor disagreement with statements concerning the static nature of mathematics.

[1]In Table 1, countries in which the mean significantly differs from the international mean are italicized. Individual t-tests were used for comparison. The international mean represents the unweighted mean of the 15 country means. In order to equally represent countries with small samples, each country is given the same weight, so that countries with larger samples are not overrepresented in the international mean. All country means were computed using country-specific weights.

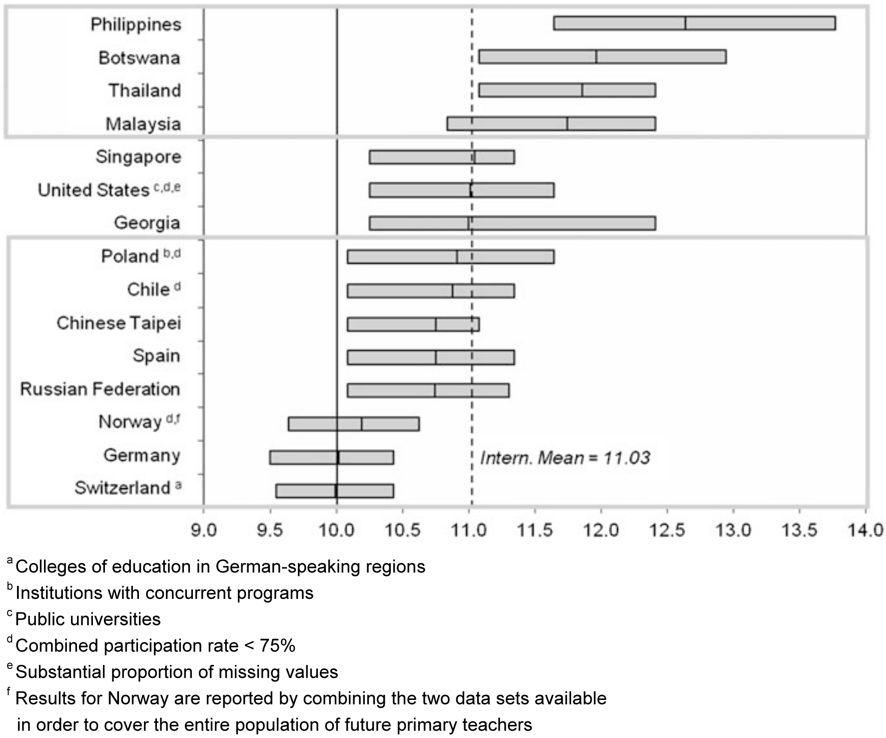


Fig. 1 Future primary teachers' beliefs concerning the nature of mathematics: static perspective (means and 25th and 75th percentiles)

In all Western and Eastern European TEDS-M-countries, as well as in Chile and Chinese Taipei, the average agreement with statements representing a *static belief* is significantly lower than the international mean. Chinese Taipei is the only Asian country in this group. It might be speculated that this is due to the intensive efforts taken in teacher education to initiate a societal change by integrating Western ideas into the curriculum (Lin and Li 2009; Hsieh et al. 2012). However, all country means except those of German and Swiss primary teachers are higher than the theoretical scale mean, indicating slight agreement with the static perspective on mathematics. In contrast, future primary teachers' agreement to static statements in Germany as well as Switzerland is the least pronounced, representing a more neutral perspective. No deviation from the international mean value can be identified for the USA, Singapore and Georgia.

With respect to the distribution of values around country means, differences between countries emerge (see Fig. 1). As an indicator of distribution we used the interquartile range. Boxes in Fig. 1 represent the 25th and 75th percentile of the distribution of values. The international mean is depicted by the dotted line, whereas the solid line indicates a neutral perspective (scale mean). The width of the percentile range thus shows the variation in individual values for those 50 percent of future teachers in a country who score higher than the lowest 25 percent and show smaller values than the 25 percent highest scoring future teachers.

An inspection of the interquartile range shows that future primary teachers' beliefs concerning a static perspective on mathematics vary to a large extent in most of the TEDS-M countries. Rather large interquartile ranges can be found for Georgia and the Philippines, indicating very heterogeneous answers of future teachers. A closer look at the distribution of Georgia in fact reveals a bimodal distribution. While one group of primary teachers highly agrees with static statements, a somewhat smaller group of future teachers rejects the static perspective on mathematics. The large difference between both groups is possibly due to the different programmes leading to a teaching certificate for primary schools in Georgia (for details on the different programmes in each country see Blömeke et al. 2010). Particularly small interquartile ranges can be found for the Western European countries of Norway, Germany and Switzerland, pointing to rather homogeneous answers of future teachers in these countries.

The descriptive analysis regarding the *dynamic perspective* on mathematics reveals that the future primary teachers from all participating countries, with the exception of Georgia, on average strongly agree with this perspective (see Table 2). However, even for the Georgian future teachers, the mean ratings suggest a tendency to agree with the dynamic belief statements.

The belief that mathematics is a dynamic science is most pronounced for future primary teachers from the Philippines, Botswana, Malaysia and Thailand, as well as future teachers from Chile and the United States. Means of these countries significantly differ from the international mean. Future teachers from the Asian countries and Botswana also showed the highest agreement for statements representing a static perspective on mathematics, indicating that in these countries both perspectives on mathematics are of similar importance. Although significantly below the international mean, future primary teachers from Georgia, Russia, Poland, Germany, Switzerland and Norway also agree to dynamic aspects of mathematics. No difference in agreement from the international mean could be detected for future teachers from Singapore, Chinese Taipei and Spain.

With respect to the spread of ratings for the dynamic perspective on mathematics, huge differences in the within-country variation can be observed (Fig. 2). Here the largest spread of answers was found in Chile and Malaysia, while the smallest range is apparent for future teachers in Russia and Switzerland. In summary, the distributions for each country show that the future teachers from different countries not only differ with respect to the level of their agreement but also with respect to the homogeneity of their answers.

3.2 The Relation of the Static and the Dynamic Perspective on Mathematics

In a second step of analyses, the relation between future teachers' agreement to the static and dynamic perspective on mathematics is examined. As outlined above we

Table 2 Beliefs concerning the nature of mathematics: dynamic perspective (mean values, standard errors and standard deviations)

	Mean	Standard error	Standard deviation
Philippines	*13.25*	*0.18*	*1.53*
Botswana	*13.09*	*0.19*	*1.62*
Malaysia	*12.63*	*0.09*	*1.87*
Thailand	*12.48*	*0.06*	*1.42*
Chile[d]	*12.43*	*0.05*	*1.75*
United States[c,d,e]	*12.18*	*0.06*	*1.59*
Singapore	11.99	0.06	1.43
International	**11.94**	**0.02**	
Chinese Taipei	11.94	0.04	1.42
Spain	11.91	0.07	1.48
Norway[d,f]	*11.82*	*0.05*	*1.42*
Switzerland[a]	*11.32*	*0.04*	*1.24*
Germany	*11.28*	*0.07*	*1.57*
Poland[b,d]	*11.26*	*0.04*	*1.46*
Russian Federation	*11.20*	*0.07*	*1.11*
Georgia	*10.25*	*0.07*	*1.56*

[a]Colleges of education in German-speaking regions

[b]Institutions with concurrent programs

[c]Public universities

[d]Combined participation rate <75 %

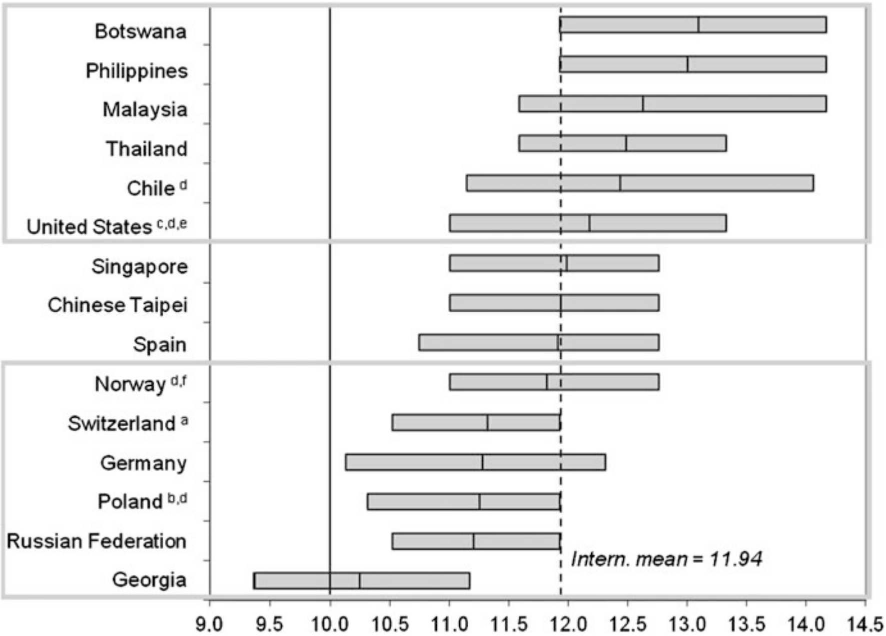
[e]Substantial proportion of missing values

[f]Results for Norway are reported by combining the two data sets available in order to cover the entire population of future primary teachers

assume that these relations differ between countries and that the country-specific profiles of beliefs can be explained using a cultural frame of reference. In particular, the difference in the social relevance and the role of mathematics between countries with a collectivistic versus an individualistic orientation should influence the formation of beliefs concerning the nature of mathematics.

In order to identify country-specific profiles of beliefs, ipsative values have been used, which allow identification of the relative approval (or rejection) of statements for both perspectives on mathematics. Although measurement invariance of the instruments used in TEDS-M has already been shown (Blömeke et al. 2008b), on an international level cultural invariance of the theoretical scale means cannot simply be assumed as these values are most likely influenced by culture-specific response tendencies (Johnson et al. 2005).

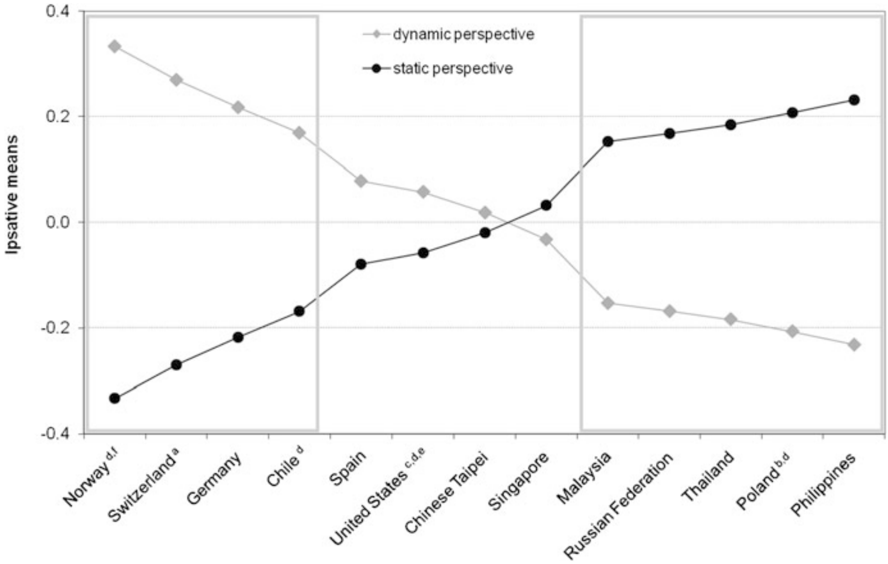
The use of ipsative values minimises this bias in future teachers' answers by levelling out response tendencies (Fischer 2004). Ipsative scores for the dynamic



[a] Colleges of education in German-speaking regions
[b] Institutions with concurrent programs
[c] Public universities
[d] Combined participation rate < 75%
[e] Substantial proportion of missing values
[f] Results for Norway are reported by combining the two data sets available in order to cover the entire population of future primary teachers

Fig. 2 Future primary teachers' beliefs concerning the nature of mathematics: dynamic perspective (means and 25th and 75th percentiles)

belief scale were calculated for each future teacher by subtracting the mean of all the 12 items measuring beliefs on the nature of mathematics from the average of the six items measuring only dynamic beliefs. Thus, a future teacher's mean score of agreement to dynamic aspects is corrected for his/her overall tendency to agree to any of the beliefs statements (Klieme and Vieluf 2009). Ipsative values for the static belief scale were computed accordingly, subtracting the mean agreement to all items from future teachers' average agreement to static belief items. Note that the average of both ipsative scales is zero for each future teacher and also for each country. The ipsative scores of the dynamic belief scale describe the relative endorsement of dynamic belief statements compared with the agreement on the static scale and vice versa. Thus, positive ipsative values of the dynamic belief scale indicate that the respective dynamic belief statements have been supported more strongly than static belief statements.



[a] Colleges of education in German-speaking regions
[b] Institutions with concurrent programs
[c] Public universities
[d] Combined participation rate < 75%
[e] Substantial proportion of missing values
[f] Results for Norway are reported by combining the two data sets available in order to cover the entire population of future primary teachers

Fig. 3 Country-specific profiles of beliefs concerning the nature of mathematics

Figure 3 illustrates country-specific profiles[2] of ipsative values representing beliefs concerning the nature of mathematics. The TEDS-M countries can be separated into three groups based on whether a significant deviation of ipsative values from zero exists. The first group comprises countries in which future teachers more strongly approve of the dynamic perspective than the static perspective (left-hand frame in Fig. 3). Countries in this group are highly individualistically oriented, Western countries, namely Norway, Switzerland, and Germany, as well as the rather collectivistically oriented country of Chile, which is the only Latin-American country in the study. This pattern is reversed in Malaysia, Russia, Thailand, Poland and the Philippines, where future primary teachers more strongly agree to the static orientation than to the dynamic perspective on mathematics. According to the Hofstede Index all these countries can be identified as collectivistically oriented, with the exception of Poland.

[2]In Botswana and Georgia the Hofstede Index has not yet been applied, so that both countries have been excluded from the following analyses.

A balanced profile of beliefs can be found for the third group of future primary teachers from Spain, the USA, Chinese Taipei and Singapore, which show no preference for either of the two orientations towards mathematics.

Overall, Fig. 3 shows that, with the exception of Chile and Poland, future primary teachers from individualistic and collectivistic countries can be differentiated based on their belief profiles. However, in interpreting the results of those countries which cannot be allocated to one of the extreme groups, it has to be taken into account that the IDV index represents a continuum. In these countries ipsative means do not differ significantly from zero. Thus, Spain has been classified as a country with an individualistic orientation on the IDV index but shows belief values between both perspectives. The reverse is true for Chinese Taipei and Singapore, which are collectivistically oriented but show rather low ipsative means indicating no preference of future teachers for either of the two belief dimensions.

3.3 Variation Between Countries Due to Cultural Orientation

The previous analyses indicate that the cultural orientation of a country and the supposed role of mathematics in society indeed have an effect on the formation of beliefs concerning the nature of mathematics. For the following multilevel regression analysis the ipsative values of the dynamic belief scale were used.[3] A decomposition of variance showed that 11 % of the variance in the ipsative dynamic beliefs of future teachers is located at the country level, indicating that the assumption of independence of individual measurements is violated and multilevel analysis should be used instead.

The results of model 1 (Table 3) show that the individualistic orientation of a country is a relevant predictor of the preference for dynamic over static beliefs, in the sense that future teachers in individualistic countries more strongly agree with statements representing a dynamic perspective on mathematics in relation to static belief statements than future teachers in collectivistic countries. This factor can account for almost one fourth of the variance in beliefs at the country level. Furthermore, the strength of the relationship is only slightly diminished, controlling for predictors on the individual level (model 2). As hypothesised, there is also a significant relationship between knowledge of mathematics and dynamic beliefs at the individual level, with future teachers scoring higher on the mathematics knowledge test also endorsing dynamic beliefs more strongly than static beliefs. Additionally, future teachers who report higher average school achievement tend to prefer a dynamic view on mathematics. Finally, model 3 shows that the level of the average mathematics knowledge of future primary teachers is also a significant predictor of dynamic beliefs. As the negative sign of the coefficient indicates, future teachers in

[3]Note that an analysis based on the static belief scale would have yielded the same results, as both scales have identical absolute values with opposite signs.

Table 3 Model results of multi-level analyses predicting dynamic beliefs (ipsative) with standardised coefficients

	M1	M2	M3
Individual level			
Math. Knowledge (MCK)		0.318^{**}	0.318^{**}
Average school achievement		0.091^{**}	0.091^{**}
R^2 *within*		12.2^{**}	12.2^{**}
Aggregated level			
Math. Knowledge (MCK)			-0.424^{*}
Individualism (IDV)	0.492^{**}	0.48^{+}	0.439^{+}
R^2 *between*	24.2	23.2	40.9^{**}

$^{*}p < 0.05$, $^{**}p < 0.01$, $^{+}p < 0.10$

countries with high average achievement on the TEDS-M mathematics test significantly prefer static aspects of mathematics over dynamic ones. The magnitude of the regression coefficient for IDV with dynamic beliefs preference is only slightly diminished when controlling for average mathematics knowledge at the country level. In fact both indicators are largely uncorrelated ($r = -0.09$) and can be assumed to describe different aspects of mathematics learning environments. With the average mathematics knowledge and the individualistic orientation as predictors at the country level, almost 41 percent of the variance in the beliefs between countries can be explained.

4 Discussion and Conclusions

The results of this study show that the beliefs concerning the nature of mathematics held by future primary teachers vary strongly within but also between countries participating in TEDS-M. Future primary teachers from the collectivistic societies of the Philippines, Thailand, Malaysia and Botswana show a distinctive agreement both with a static and dynamic perspective on mathematics. In contrast, future teachers from Germany, Switzerland and Norway are neutral towards static aspects and their average agreement to static beliefs is below the international mean, which is also the case for Chilean and Taiwanese future teachers. Furthermore, future primary teachers from these three European countries, as well as from the former communist countries of Georgia, Russia and Poland, also agree to dynamic aspects of mathematics, but score significantly below the international mean.

We further showed that different belief patterns emerge if individual response tendencies are controlled for using ipsative values. Our analyses of belief profiles based on ipsative values do indeed point to culturally imprinted beliefs. Future primary teachers from highly collectivistically oriented countries—such as Malaysia, Russia, Thailand, and the Philippines—agree more strongly to static aspects of mathematics in relation to dynamic aspects. In contrast, future teachers from highly individualistically oriented countries—namely Norway, Switzerland, and Germany—more strongly stress the dynamic nature of mathematics relative to the static nature. Additionally, future teachers in countries that cannot clearly be characterised as individualistic or collectivistic, such as Spain, Chinese Taipei, and Singapore, emphasise both aspects of mathematics to the same extent.

Thus, on the level of countries the distinctive patterns of beliefs concerning the nature of mathematics seem to correspond with the individualistic–collectivistic orientation of a country. This observation has been substantiated by a multilevel regression analysis of dynamic belief preferences, indicating that a large part of the between-country variation is associated to the individualistic–collectivistic dimension. Future teachers in individualistically oriented nations show a preference for dynamic over static beliefs, whereas teachers in collectivistic countries stress a static view on mathematics more strongly. Furthermore, the magnitude of this effect is not substantially reduced by controlling for mathematical knowledge (MCK) at the country level. Thus, it seems that both indicators represent different aspects of the cultural variation.

One limitation of the present study is the somewhat simplistic operationalisation of culture using only one cultural dimension. Studies from the field of cross-cultural psychology clearly show that several descriptive dimensions are needed in order to describe cultural differences in socially shared values, motives and orientations which have been defined as a country's culture (e.g. Joy and Kolb 2009). Our results indeed point to the fact that the individualism dimension alone is not sufficient to fully explain the variation in beliefs on the nature of mathematics between the TEDS-M countries. Further analyses should characterise a nation's culture on more than just one dimension. Several cross-cultural comparative studies indicate that, besides individualism, also additional cultural dimensions established by Hofstede—namely power distance and uncertainty avoidance—are relevant for the description and explanation of cultural differences in outcomes of mathematics education (e.g. Chiu and Klassen 2010).

Furthermore, countries in our study that seem to deviate from the overall pattern also point to the relevance of historical traditions in mathematics education (Pepin 1999a, 1999b). This is especially apparent in the somewhat unexpected results of the future teachers from the highly collectivistic Chinese Taipei, who show no preference for either belief aspect. We hypothesise that this particular belief pattern of Taiwanese future teachers might result from the intensive efforts taken to initiate societal change by integrating Western ideas into the curriculum (see Hsieh et al., 2012). It might also be speculated that the similar patterns of preference of future primary teachers in Singapore and in the United States mirror the strong influence of research in mathematics education from the English-speaking countries on the disciplinary discourse in Singapore (Kaur and Yeap 2009). A similar argument can be made to explain the results from Chile and Poland. For both countries the characterisation of the social structures using the individualism index does not seem sufficient to explain the belief structure of future primary teachers. Although Chile has been classified as a collectivist country, an analysis of future teacher characteristics in TEDS-M showed that Chilean primary teachers are more similar in their individual characteristics to teachers from the individualistic countries in our sample (Blömeke et al. 2010). Here it may be that specific educational traditions outplay social factors, i.e. Western European in the case of Chile or the Eastern European tradition with its high emphasis on mathematics and formal structures in the case of Poland (Yeager 1991; Schwartzman 1993; see also Pepin 1999b). With respect to the outcomes of teacher education, such studies can help to explain unexpected deviations from the overall pattern. In the case of the TEDS-M beliefs data on the nature of mathematics, both cultural psychology and the history of education and mathematics seem to be complementary approaches for explaining the present results.

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The Cultural Notion of Teacher Education: Future Lower Secondary Teachers' Beliefs on the Nature of Mathematics, the Learning of Mathematics and Mathematics Achievement

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Abstract The paper aims to highlight the cultural notion of mathematics-teaching related beliefs of future lower secondary teachers at the end of their training. The analysis is based on the data of TEDS-M and the belief scores produced by the authors through employing a Rasch partial credit model. Six belief scales were created: beliefs that the nature of mathematics is open and creative or conservative and rigorous, beliefs that the learning of mathematics should be guided by student initiative, teacher instruction (TI) or utilitarianism in Teaching (UT), and beliefs that mathematical ability is natural and fixed (NF). UT is a new scale created for this paper and different from the original TEDS-M scales. Cultural country groups adapted from Blömeke and Kaiser (ZDM, Int. J. Math. Educ. 44(3):249–264, 2012) are used for comparing the outcomes of our study. A common pattern among all groups exhibits the belief that the nature of mathematics is open and creative on the one hand and conservative and rigorous on the other hand. Furthermore, it is stressed that mathematics is best learned through considering student initiatives such as figuring out own solutions regardless of the time consumed. The least approved belief among all country groups is an utilitarianism in teaching such as learning works best through memorization or other non-time-consuming ways. The cultural groups may be further classified into two classes using TI and NF as dividers. The first class holding negative views on both TI and NF contains Developed Europe (including Germany, Norway and Switzerland), Confucian Asia (including Singapore and Taiwan), and the American group (including Chile and the USA). The other class holding positive views on both TI and NF contains Developing Asia (including Malaysia, the Philippines and Thailand) and East Europe (including Poland, Russia, and Georgia). Breaks within cultural groups may also be seen, which prompts a need for further studies on cultural classification.

Parts of this paper were adapted from two Chinese papers written by the authors Tang and Hsieh (2012a, 2012b).

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S. Blömeke et al. (eds.), *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn*, Advances in Mathematics Education,
DOI 10.1007/978-94-007-6437-8_11,

Keywords Cultural · Mathematics teaching · Belief · Nature of mathematics · Student achievement · Teacher belief

1 Introduction

Many researches have revealed that teachers' beliefs plays an important role in mathematics teaching (Ernest 1991; Nespor 1987; Op't Eynde et al. 2002; Pajares 1992; Philipp 2007; Thompson 1992; Törner 1997; Wilson and Cooney 2002). When we talk about teacher performance in class, beliefs represent dispositions toward action and a bridge between teacher knowledge and actual teaching (Felbrich et al. 2012; Rokeach 1960; Philipp 2007). Therefore, future teachers' beliefs may be crucial to their perception of classroom situations and their decision on what kind of knowledge to draw on or how to react. Beliefs are generally harder to change than so-called "attitudes" though (Philipp 2007; Raymond 1997; Skott 2001; Stipek et al. 2001; Thompson 1984). This is an important point for those who care about teacher education.

"Beliefs" can be defined as "psychologically held understandings, premises, or propositions about the world that are thought (or felt) to be true" (Philipp 2007; Richardson 1996). In spite of a lack of an agreed definition of beliefs, the definition above reflects a rather broad approach. According to this definition, the belief construct relates to one's cognition, knowledge, or affect, and additionally, it can be regarded as having an experiential and context-bound nature based on the social context in which one's beliefs developed (Beswick 2005; Felbrich et al. 2012; Hoyles 1992; Op't Eynde et al. 2002; Philipp 2007; Schoenfeld 1998). This also means that beliefs might be a culturally shaped mental construct. This idea is necessary for understanding the deeper meaning of beliefs especially when comparing beliefs of teachers, including future teachers, from many countries with different historical traditions or educational identities that vary significantly among countries (Felbrich et al. 2012).

The "Mathematics Teaching in the 21st Century" study (MT21) was a pioneer study of the "Teacher Education and Development Study in Mathematics" (TEDS-M). It compared future secondary teachers' beliefs in six countries, namely Bulgaria, Germany, Mexico, Taiwan, South Korea and the USA. The results have shown cross-country patterns and country-specific differences in the beliefs of future teachers (Schmidt et al. 2011a). For example, one of its findings was that "it is remarkable how homogeneously future teachers from South Korea and Taiwan view most of the issues", and this indicates that "some shared values about mathematics, teaching, and learning exist in South Korea and Taiwan, which probably reflects a relatively high level of social and cultural homogeneity in these two countries as compared with other countries" (Schmidt et al. 2011a, p. 187).

Furthermore, the MT21 study identified significant country differences in addition to the cultural split between East Asian and Western countries. It noted substantial differences between South Korea and Taiwan: the future teachers in South Korea showed "only a neutral view on the design of school life as a fundamental goal of teachers" but "the Taiwanese strongly supported this goal (Schmidt et al. 2011a, p. 187)." These findings remind us that the cultural homogeneity and heterogeneity in teaching-related beliefs of future teachers at the secondary-school level is subtle.

Culture is basically a collection of traits defining particular groups of people or a category specified by geographical boundaries, so that the members of this group or category can be distinguished from others (Hofstede 2001; Markus and Hamedani 2007). Properly speaking, culture consists of explicit and implicit patterns of historically derived or selected ideas including belief and value systems, and their embodiment in institutions or practices. These cultural patterns might be simultaneously considered to be products of action as well as conditioning elements of future action (Markus and Hamedani 2007).

On the other hand, culture involves shared understandings that serve as a medium through which individual human minds interact in communication with one another (Stenhouse 1967). This is why some people said culture can shape "the way things are done" and our understanding of reasons how things should be. The beliefs, as a kind of culturally shaped mental constructs, may serve as a means for our understanding of the way things are done and should be. Revealing beliefs may thus reflect not only the understanding and dispositions of the belief holders but also the influence caused by culture. This study takes the chance of the national-representative large-scale samples in TEDS-M to explore the patterns of mathematics-related beliefs across countries. The patterns of beliefs may help to reveal meaningful cultural images within or between countries.

2 Study Design

2.1 The Basic Aim

This paper is based on earlier studies of the authors (Tang and Hsieh 2012a, 2012b) and aims at highlighting the cultural notion of mathematics teacher education by analyzing beliefs of future lower secondary teachers.[1] The major questions of this study are as follows: (1) What patterns of beliefs about the nature of mathematics and about teaching and learning mathematics do future lower secondary teachers hold at the end of their preparation? (2) What cultural meanings do the above patterns of beliefs imply?

2.2 Theoretical Framework

Because many studies have suggested that beliefs about mathematics and mathematics learning that beginning teachers hold might influence how they teach and

[1]The data set used in this paper is TEDS_MS_DRAFT_IDB_20101103_v31.

subsequently how their students learn, TEDS-M gathered data about the following three aspects of mathematics future teachers' teaching-related beliefs: (1) beliefs about the nature of mathematics; (2) beliefs about the learning mathematics; and (3) beliefs about mathematics achievement (Tatto et al. 2012).

Generally speaking, there are several major views of mathematics, mathematics learning, and mathematics ability. In terms of the nature of mathematics, the dichotomy between "dynamic" and "static" has often been discussed. The former perceives mathematics as being continually undergoing change and revision, or a creative and generative process. But the latter views mathematics as a static and immutable unified entity, maybe including the accumulation of facts, rules, and skills (Ernest 1988; Stipek et al. 2001).

In terms of learning mathematics, the "transmission/traditional" view and "learner-centredness/constructivist" view are two common and important categories (Barkatsas and Malone 2005; Cross 2009; Perry et al. 1999). The first perspective tends to allot the role of transmitting mathematics to a teacher, and the role of carefully obeying teachers' instruction to a student. The second perspective tends to emphasize the initiative of a student and that teachers should establish a learner-focused environment for students as well as providing them opportunities to construct their own meanings.

When mentioning mathematics achievement in teaching or learning, the "ability" construct plays a critical role. Theorists usually make a distinction between the "entity" and the "incremental" views (Stipek et al. 2001). The former considers "ability" to be stable and not very amenable to change. Hence, any efforts to enhance the achievement are usually limited. The latter is opposite to the former.

Beliefs have been viewed as a component in culture that identifies people as belonging to the same or to different collectives (Hofstede 1984; Tylor 1889). How to distinguish countries by cultural identities is still an open question. It is an appropriate beginning to distinguish mostly by geographical region or historical traditions (Blömeke and Kaiser 2012; Leung 2006). This paper adopts the schemes of Blömeke and Kaiser, and Leung's in distinguishing countries into different cultural groups for probing probable cultural meaning relative to mathematics teacher education at the secondary school level. The classification of cultural groups will be delineated in the data analysis section below.

2.3 The Sample

The sampling plan of TEDS-M followed a stratified multistage probability sampling design (Tatto et al. 2009). The target populations were the future lower secondary teachers in their last year of training to teach mathematics. Although the International Association for the Evaluation of Educational Achievement (IEA) set a minimum requirement of a 75 % combined participation rate as a threshold, samples having a participation rate of 60–75 % were also suitable for use, according to the IEA's criterion, with an annotation of low participation rates.[2] Our analyses included data from fifteen participating countries covering Botswana, Chile, Georgia, Germany, Malaysia, Norway, Oman, the Philippines, Poland (concurrent type of institution only), the Russian Federation (Russia), Singapore, Switzerland (German-speaking cantons), Taiwan, Thailand, and the United States of America (the USA, public institutions only). 8207 future lower secondary teachers participated in total.

2.4 Instrument

TEDS-M surveyed the future teachers' beliefs about the nature of mathematics, about learning mathematics, and about mathematics achievement (Tatto et al. 2012). These teaching-related topics are considered as three indicators of the cultural meanings behind the future teachers' mathematics-teaching views in this paper. Mathematics teaching is one kind of common and important cultural activity in modern human society. How to view mathematics and its learning/transitive process is crucial in this context.

There were 12, 14, and 8 items capturing the beliefs about the nature of mathematics, about learning mathematics, and about mathematics achievement respectively in the TEDS-M questionnaires. The future teachers were asked to choose one from six response alternatives in each item: 1—strongly disagree; 2—disagree; 3—slightly disagree; 4—slightly agree; 5—agree; 6—strongly agree (Tatto et al. 2012).

2.5 Data Analysis

Originally TEDS-M analyzed the date from primary and secondary future teachers as a whole. Because of the differences in mathematical materials and some pedagogical goals between primary and secondary levels, our study decided to develop new scales only for future lower secondary teachers by factor analysis and Rasch modeling (employing IRT). The study conducted factor analyses to examine the factor structure of the future teachers' responses on each topic with the software SPSS, and continued to use the partial credit model (Masters 1982), one model out of the family of Rasch models, to complete the Rasch scaling on each factor with the software Winsteps.

Every scale was treated as one indicator to depict the future teachers' teaching-related beliefs. When calibrating the items for each of the scales, those cases with

[2]The combined participation rates of Chile and Poland were between 60 % and 75 %. Of Norway, it was 58 % (Tatto et al. 2012). The other participating countries like the USA and Germany suggested to make an exception and to include the national data of Norway (e.g. Blömeke et al. 2011; Schmidt et al. 2011b).

50 % or more missing responses on the items for the scale were excluded from the analysis. In addition, the sampling weights were transformed linearly so that each country contributed equally to the calibration and analysis scale by scale. During each analysis, this study set the Rasch score of 10 as neutral belief, i.e. neither positive nor negative. A value more than 10 implied a positive belief or endorsement, and on the contrary, a value less than 10 implied a negative belief or disagreement.

This study also classified the TEDS-M countries roughly into several groups with specific educational cultures or backgrounds according to the scholarly distinction (Blömeke and Kaiser 2012; Leung 2006). Notwithstanding that each of them may have its own complicated educational culture, it should be an appropriate beginning to distinguish mostly by geographical region or historical traditions: (1) Confucian Asia: including the two Asian countries, Singapore and Taiwan, having a Confucian tradition. (2) Developing Asia: including the three countries, Malaysia, the Philippines and Thailand, belonging to the Association of Southeast Asian Nations. (3) East Europe: including three countries, Poland and Russia of East Europe and Georgia of Central Asia, coming from the former Eastern European block led by the Soviet Union. (4) Developed Europe: including Germany, Norway and Switzerland, belonging to the traditional developed group. (5) American group: including Chile and the USA. (6) The others: including Botswana of Africa and Oman of West Asia. The last group will not be discussed as a whole in this paper due to not being able to identify their similarity by geographical region or historical tradition. However, their individual results will still be reported. The other groups are called "cultural groups".

In some cases, the percentages of approval or disapproval toward a belief item are used in this paper. The approval includes responses checking 4—slightly agree, 5—agree, and 6—strongly agree. The disapproval includes responses checking 1-strongly disagree, 2—disagree, and 3—slightly disagree. The percentages were calculated by applying total sample weights of the future teachers. A percentage of a belief for a country was the average over all individuals in that country. The international percentage of a belief was obtained by averaging over the percentages of all countries so that every country was weighted equally in this paper. This approach of weighting also applied to calculating the averages of any group of countries. The percentages of approval are denoted by *perA* and the percentages of disapproval are denoted by *perD*.

3 Results

3.1 Overall Factors

With respect to the nature of mathematics, the analysis yielded two principal factors explaining a total of 52.1 % of the variance in the belief items (Table 1). The first factor was labeled "Open and Creative nature (OC)" due to the high loadings

Table 1 Factor loadings of the belief items about the nature of mathematics

Item	Factor Loadings		Communality
	OC	CR	
I. Many aspects of mathematics have practical relevance	0.750	–	0.573
D. In mathematics many things can be discovered and tried out by oneself	0.724	–	0.530
F. If you engage in mathematical tasks, you can discover new things (e.g., connections, rules, concepts)	0.708	–	0.531
C. Mathematics involves creativity and new ideas	0.695	–	0.502
J. Mathematics helps solve everyday problems and tasks	0.691	–	0.495
H. Mathematical problems can be solved correctly in many ways	0.686	–	0.479
E. When solving mathematical tasks you need to know the correct procedure else you would be lost	–	0.760	0.577
A. Mathematics is a collection of rules and procedures that prescribe how to solve a problem	–	0.757	0.575
B. Mathematics involves the remembering and application of definitions, formulas, mathematical facts and procedures	–	0.743	0.565
L. Mathematics means learning, remembering and applying	–	0.737	0.557
K. To do mathematics requires much practice, correct application of routines, and problem solving strategies	–	0.685	0.546
G. Fundamental to mathematics is its logical rigor and preciseness	–	0.517	0.316
Eigenvalue	4.135	2.112	6.247
% of total variance	34.46 %	17.60 %	
Total variance	34.46 %	52.06 %	

Note. Only the loadings with the absolute value over 0.400 were shown in this table. OC = Open & Creative nature; CR = Conservative & Rigorous nature. Source: "Beliefs of Future Secondary Mathematics Teachers", (in Chinese) by S.-J. Tang & F.-J. Hsieh, 2012, in F.-J. Hsieh (Ed.), *Taiwan TEDS-M 2008: Teacher Education and Development Study in Mathematics* (in Chinese; pp. 221–252). Department of Mathematics, National Taiwan Normal University. Adapted with permission

of the items about openness and creativity as characteristics of the nature of mathematics. The next factor was labeled "Conservative and Rigorous nature (CR)". This was due to the high loadings of the items about accepted procedures or rules as characteristics of the nature of mathematics. The OC factor explains 34.5 % of the variance. It is about twice as much as the variance explained by the CR factor.

Table 2 Factor loadings of the belief items about learning mathematics

Item	Factor Loadings			Communality
	SI	TI	UT	
N. It is helpful for pupils to discuss different ways to solve particular problems	0.737	–	–	0.571
M. Teachers should encourage pupils to find their own solutions to mathematical problems even if they are inefficient	0.726	–	–	0.605
H. Teachers should allow pupils to figure out their own ways to solve mathematical problems	0.724	–	–	0.560
L. Pupils can figure out a way to solve mathematical problems without a teacher's help	0.655	–	–	0.498
G. In addition to getting a right answer in mathematics, it is important to understand why the answer is correct	0.555	–	−0.422	0.567
K. Time used to investigate why a solution to a mathematical problem works is time well spent	0.555	–	–	0.415
B. Pupils need to be taught exact procedures for solving mathematical problems	–	0.756	–	0.579
E. Pupils learn mathematics best by attending to the teacher's explanations	–	0.695	–	0.553
A. The best way to do well in mathematics is to memorize all the formulas	–	0.655	–	0.484
D. To be good in mathematics you must be able to solve problems quickly	–	0.556	0.416	0.482
J. Hands-on mathematics experiences aren't worth the time and expense	–	–	0.708	0.544
C. It doesn't really matter if you understand a mathematical problem, if you can get the right answer	–	–	0.661	0.501
F. When pupils are working on mathematical problems, more emphasis should be put on getting the correct answer than on the process followed	–	–	0.644	0.538
I. Non-standard procedures should be discouraged because they can interfere with learning the correct procedure	–	0.423	0.468	0.399
Eigenvalue	3.506	2.666	1.125	7.296
% of total variance	25.04 %	19.04 %	8.03 %	
Total variance	25.04 %	44.08 %	52.12 %	

Note. Only the loadings with the absolute value over 0.400 were shown in this table. SI = learning through Student Initiative; TI = learning by following Teacher Instruction; UT = Utilitarianism in Teaching. Source: "Beliefs of Future Secondary Mathematics Teachers" (in Chinese) by S.-J. Tang and F.-J. Hsieh, 2012, in F.-J. Hsieh (Ed.), *Taiwan TEDS-M 2008: Teacher Education and Development Study in Mathematics* (in Chinese; pp. 221–252). Department of Mathematics, National Taiwan Normal University. Adapted with permission

Table 3 Factor loadings of the belief items about mathematics achievement

Item	Factor loadings	Communality
	NF	
C. Mathematics is a subject in which natural ability matters a lot more than effort	0.752	0.565
B. To be good at mathematics you need to have a kind of "mathematical mind"	0.731	0.535
D. Only the more able pupils can participate in multi-step problem solving activities	0.712	0.506
F. Mathematical ability is something that remains relatively fixed throughout a person's life	0.632	0.400
H. Some ethnic groups are better at mathematics than others	0.620	0.385
E. In general, boys tend to be naturally better at mathematics than girls	0.618	0.381
G. Some people are good at mathematics and some aren't	0.607	0.369
A. Since older pupils can reason abstractly, the use of hands-on models and other visual aids becomes less necessary	0.533	0.284
Eigenvalue	3.425	3.425
% of total variance	42.82 %	
Total variance	42.82 %	

Note. Only the loadings with the absolute value over 0.400 were shown in this table. NF = Natural & Fixed mathematical ability. Source: "Beliefs of Future Secondary Mathematics Teachers", (in Chinese) by S.-J. Tang and F.-J. Hsieh, 2012, in F.-J. Hsieh (Ed.), *Taiwan TEDS-M 2008: Teacher Education and Development Study in Mathematics* (in Chinese; pp. 221–252). Department of Mathematics, National Taiwan Normal University. Adapted with permission

For the beliefs on the learning of mathematics, the analysis yielded three factors explaining a total of 52.1 % of the variance in the items (Table 2). The first principal factor was labeled "Learning through Student Initiative (SI)" because most of the items with high loading for this factor emphasized the students' priority and initiative in learning processes. The next factor was labeled "Learning by following Teacher Instruction (TI)" due to the high loadings of the corresponding items. More than 40 % of the variance can be explained by these two factors: the SI factor explains 25.0 % and the TI explains 19.0 %. The third factor was labeled "Utilitarianism in Teaching (UT)" and explains 8.0 % of the variance. Those items with higher loadings for the UT factor usually contained utilitarian calculations about learning or teaching.

With respect to the beliefs on mathematics achievement, the analysis yielded only one principal factor for the set of items. This factor explains a total of 42.8 % of variance (Table 3). It was labeled "Natural and Fixed mathematical ability (NF)" because the items with high loading for this factor involve how to interpret the ability behind mathematics achievement.

3.2 Descriptive Results

The majority of future teachers of the TEDS-M sample (about 8,000 lower-secondary mathematics teachers in their final year of teacher education from 15 countries) tended to agree with the belief items of the OC factor about the nature of mathematics and the SI factor about learning mathematics. All approval percentages of the belief items in the OC factor were high and exceeded 88 %. The most popular belief items were as follows (*perA* > 90 %, *perD* < 10 %):

- *Mathematical problems can be solved correctly in many ways* (*perA* = 93.7 %, *perD* = 3.5 %).
- *Many aspects of mathematics have practical relevance* (*perA* = 91.6 %, *perD* = 5.3 %).
- *If you engage in mathematical tasks, you can discover new things* (*e.g.*, *connections, rules, concepts*) (*perA* = 91.5 %, *perD* = 5.7 %).

The high *perA* of the second belief item means that for most future teachers mathematics is not abstract, rather it has practical relevance. The high *perA* of the third belief is a rather surprising result, especially in the Confucian Asia countries where the difficulty level of their mathematics curricula barely endorses a discovery of new rules or concepts. In contrast to the OC factor, all the approval percentages of the belief items in the CR factor were smaller than 85 % except item K (*perA* = 88.7 %, *perD* = 8.5 %), which emphasizes the heavy practice and correct application of routines in mathematics. The fact that all but one belief item in the OC factor received more percentages of approval than that of item K implies that most of the future teachers also held strong beliefs of some "temperament" of mathematics beyond the beliefs of practicing or applying routines in mathematics. More than about a quarter of future teachers disapproved the following items in the CR factor:

- *When solving mathematical tasks you need to know the correct procedure else you would be lost* (*perA* = 64.3 %, *perD* = 33.0 %).
- *Mathematics is a collection of rules and procedures that prescribe how to solve a problem* (*perA* = 73.2 %, *perD* = 24.2 %).

Overall, all belief items about the nature of mathematics were approved by more than 60 % of the future teachers. This suggests that future lower-secondary mathematics teachers perceive mathematics as a multifaceted entity.

In terms of the beliefs about learning mathematics, all the percentages of the items in the SI factor were above 80 %. These items earned at least a 20 % higher approval than the rest of the items in terms of the beliefs about learning mathematics. The most popular belief statements emphasizing an initiative of learners were as follows (*perA* > 90 %, *perD* < 10 %):

- *It is helpful for pupils to discuss different ways to solve particular problems* (*perA* = 93.4 %, *perD* = 3.6 %).
- *Teachers should allow pupils to figure out their own ways to solve mathematical problems* (*perA* = 92.5 %, *perD* = 4.6 %).

- *In addition to getting a right answer in mathematics, it is important to understand why the answer is correct* (*perA* = 92.3 %, *perD* = 5.0 %).

The results also showed the future teachers' tendency to disapprove the items in the factors TI and UT. Almost all the approval percentages were below 60 % and the percentages of disapproval were above one third except the item B which emphasized exact procedures for solving mathematical problems (*perA* = 61.3 %, *perD* = 35.7 %). Most of the approval percentages for utilitarian items were especially low such that the most unpopular statements of all items fell in the UT factor, which were as follows (*perA* < 33 %, *perD* > 70 %):

- *When pupils are working on mathematical problems, more emphasis should be put on getting the correct answer than on the process followed* (*perA* = 25.4 %, *perD* = 71.6 %).
- *Hands-on mathematics experiences aren't worth the time and expense* (*perA* = 21.0 %, *perD* = 75.4 %).
- *It doesn't really matter if you understand a mathematical problem, if you can get the right answer* (*perA* = 17.4 %, *perD* = 79.5 %).

For the belief items in the TI factor, the approval percentages were between 35 % and 61 %. The beliefs of the future teachers were split in two rather equal groups. To summarize, most of the future teachers consistently emphasized the students' priority and initiative, and tended to reject utilitarian schemes in learning mathematics, whereas the consensus on whether to stress teachers' instruction was relatively low.

Regarding the beliefs about mathematics achievement, the future teachers seemed to have no consensus on seven of the eight belief items. The percentages of approval for these seven items spread from 33.5 % to 59.0 %. Only three items obtained approval of more than 50 %:

- *Some people are good at mathematics and some aren't* (*perA* = 73.4 %, *perD* = 22.7 %).
- *To be good at mathematics you need to have a kind of "mathematical mind"* (*perA* = 59.0 %, *perD* = 37.7 %).
- *Mathematical ability is something that remains relatively fixed throughout a person's life* (*perA* = 54.8 %, *perD* = 41.1 %).

Each percentage of disapproval with respect to the other items was over 60 % except the item H referring to ethnic issues (*perA* = 36.7 %, *perD* = 59.2 %).[3] In general, most future teachers believed that natural mathematical ability or mind was something relatively fixed such that some people were good at mathematics or not. But when gender,[4] ethnicity,[5] and equality or opportunities to learn[6] became an

[3] The item H is "*Some ethnic groups are better at mathematics than others*".

[4] As the item E: "*In general, boys tend to be naturally better at mathematics than girls*".

[5] As the item H.

[6] As the item A: "*Since older pupils can reason abstractly, the use of hands-on models and other visual aids becomes less necessary*", and the item D: "*Only the more able pupils can participate in multi-step problem solving activities*".

issue of a statement, their responses were affected. This result may be helpful for our understanding of the "differentiation" of beliefs of future secondary teachers (Rokeach 1960).

3.3 *Factors and Distribution*

The central tendency of the future teachers' beliefs from the different countries and the distribution of their beliefs about the nature of mathematics, learning mathematics, and mathematics achievement can be seen at a glance in the following tables (see Tables 4, 5 and 6) and figures (see Figs. 1 to 5).

The Belief of the Nature of Mathematics With respect to the nature of mathematics, the means of each country were significantly greater than the neutral value 10 on both the OC and CR scales (Table 4). This means that future teachers in all

Table 4 Means of beliefs about the nature of mathematics

Country	OC		Country	CR	
	M	(SE)		M	(SE)
Philippines	13.34[a,b]	(0.13)	Philippines	12.76[a,b]	(0.11)
Oman	13.20[a,b]	(0.09)	Thailand	12.06[a,b]	(0.04)
US-public	12.85[a,b]	(0.11)	Malaysia	11.79[a,b]	(0.07)
Thailand	12.83[a,b]	(0.05)	Botswana	11.74[a,b]	(0.16)
Botswana	12.74[b]	(0.15)	Oman	11.62[a,b]	(0.05)
Chile	12.69[a,b]	(0.08)	Georgia	11.54[b]	(0.15)
Malaysia	12.45[b]	(0.09)	Chile	11.32[a,b]	(0.04)
Taiwan	12.43[b]	(0.07)	US-public	11.30[a,b]	(0.16)
IA	12.43		IA	11.24	
Germany	12.33[a,b]	(0.08)	Singapore	11.19[b]	(0.06)
Poland	12.24[a,b]	(0.09)	Taiwan	11.09[a,b]	(0.05)
Singapore	12.10[a,b]	(0.06)	Russia	10.81[a,b]	(0.04)
Switzerland	12.08[a,b]	(0.10)	Poland	10.63[a,b]	(0.06)
Norway	12.03[a,b]	(0.06)	Norway	10.57[a,b]	(0.03)
Russia	11.77[a,b]	(0.06)	Switzerland	10.20[a,b]	(0.05)
Georgia	11.32[a,b]	(0.15)	Germany	10.04[a,b]	(0.03)

Note. IA = international average; OC = Open & Creative nature; CR = Conservative & Rigorous nature. Source. "Beliefs of Future Secondary Mathematics Teachers" (in Chinese) by S.-J. Tang and F.-J. Hsieh, 2012, in F.-J. Hsieh (Ed.), *Taiwan TEDS-M 2008: Teacher Education and Development Study in Mathematics* (in Chinese; pp. 221–252). Department of Mathematics, National Taiwan Normal University. Adapted with permission

[a]Mean ≠ IA, significant ($p < 0.05$)

[b]Mean ≠ 10, significant ($p < 0.05$)

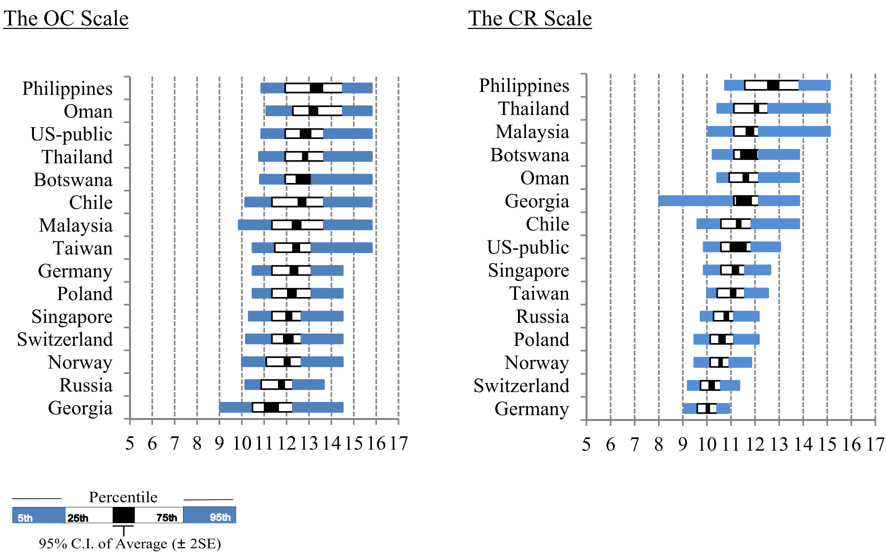

Fig. 1 Distribution of belief scores about the nature of mathematics (*two scales*). From "Taiwanese Future Secondary Mathematics Teachers' Beliefs under the Perspective of International Comparison", (in Chinese) by S.-J. Tang and F.-J. Hsieh, 2012, in F.-J. Hsieh (Ed.), *Taiwan TEDS-M 2008 Secondary Analysis* (in Chinese). Department of Mathematics, National Taiwan Normal University. Adapted with permission

countries approved that mathematics is an open, practical and creative field on the one hand, as well as rigorous and requiring fixed procedures on the other hand. However, the OC beliefs were significantly stronger than the CR beliefs for each country. An interesting result is shown in Fig. 1: those countries that scored higher tended to have wider spreads of scores for both the OC and CR scales.

Regarding the cultural notions, we can see from Table 4 that Developing Asia and the American groups always have larger means than the international average and the other cultural groups, so they tend to hold stronger beliefs on the nature of mathematics than the other groups. Table 4 and Fig. 1 also show that some cultural groups have their own characteristics or specific foci on the nature of mathematics. For the countries of the Confucian Asia group, all the means are not significantly different from the international averages. It seems to exist a "Doctrine of the Mean" (middle path) of Confucianism for mathematics. On the other hand, the ranges of belief scores of Developed Europe and East Europe countries (excluding Georgia) are the narrowest on the CR scale and their means are lower than the other cultural groups. This result may reflect the general situation that the future secondary teachers' cognition on conservative and rigorous nature of mathematics was quite alike in any one of these countries.

The Belief of Leaning of Mathematics The international averages in Table 5 show that the central tendencies of the participating countries on the three scales

Table 5 Means of belief scores in the scale about learning mathematics

Country	SI		Country	TI		Country	UT	
	M	(SE)		M	(SE)		M	(SE)
Chile	12.55[a,b]	(0.08)	Philippines	11.13[a,b]	(0.08)	Malaysia	10.24[a,b]	(0.04)
Switzerland	12.39[a,b]	(0.12)	Georgia	10.79[a,b]	(0.12)	Philippines	10.04[a,b]	(0.08)
Taiwan	12.26[a,b]	(0.05)	Malaysia	10.70[a,b]	(0.06)	Georgia	9.61[a,b]	(0.11)
Germany	12.23[a,b]	(0.06)	Oman	10.54[a,b]	(0.05)	Oman	9.51[a,b]	(0.04)
US-public	12.11[a,b]	(0.09)	Botswana	10.18	(0.15)	Botswana	9.42[a,b]	(0.12)
Poland	12.08[a,b]	(0.11)	Russia	9.99[a]	(0.04)	Chile	9.30[a,b]	(0.04)
Oman	11.95[b]	(0.07)	Singapore	9.99	(0.05)	IA	9.32	
IA	11.92		Chile	9.95[b]	(0.04)	Thailand	8.96[b]	(0.05)
Philippines	11.85[a,b]	(0.13)	IA	9.92		Russia	8.94[a,b]	(0.04)
Thailand	11.85[a,b]	(0.05)	US-public	9.62[a,b]	(0.12)	Singapore	8.87[a,b]	(0.06)
Russia	11.79[a,b]	(0.06)	Poland	9.49[a,b]	(0.06)	Poland	8.63[a,b]	(0.08)
Botswana	11.78[b]	(0.14)	Thailand	9.48[a,b]	(0.05)	Taiwan	8.39[a,b]	(0.06)
Norway	11.73[a,b]	(0.05)	Taiwan	9.37[a,b]	(0.05)	Germany	8.38[a,b]	(0.08)
Singapore	11.49[a,b]	(0.05)	Norway	9.23[a,b]	(0.03)	Norway	8.38[a,b]	(0.05)
Georgia	11.44[a,b]	(0.19)	Switzerland	9.19[a,b]	(0.08)	Switzerland	8.38[a,b]	(0.10)
Malaysia	11.31[a,b]	(0.06)	Germany	9.13[a,b]	(0.06)	US-public	8.38[a,b]	(0.14)

Note. IA= international average; SI = learning through Student Initiative; TI = learning by following Teacher Instruction; UT = Utilitarianism in Teaching. From "Beliefs of Future Secondary Mathematics Teachers", (in Chinese) by S.-J. Tang and F.-J. Hsieh, 2012, in F.-J. Hsieh (Ed.), *Taiwan TEDS-M 2008: Teacher Education and Development Study in Mathematics* (in Chinese; pp. 221–252). Department of Mathematics, National Taiwan Normal University. Adapted with permission

[a]Mean ≠ IA, significant ($p < 0.05$)

[b]Mean ≠ 10, significant ($p < 0.05$)

are different about the learning of mathematics. A tendency of endorsing students' initiative and ignoring teacher instruction and utilitarian ideas in teaching can be seen. All means are significantly greater than 10 for belief items in the SI scale, but only so for a few means on the other scales. On the scale of TI and UT, the ranges of mean scores among countries are wider than the range on SI. The UT means though spread out but almost entirely located under 10 (Table 5), which may reveal a general idealism.

Regarding the distribution of the belief scores in each country, Fig. 2 shows that in general SI scores spread out wider than TI and UT scores. It means that though SI gains higher approval from future teachers, the beliefs among future teachers in a country are more varied. Further, for the SI scale, Fig. 2 shows that those countries scored higher tended to have wider spreads of scores about the belief for SI, whereas narrower about the belief for TI.

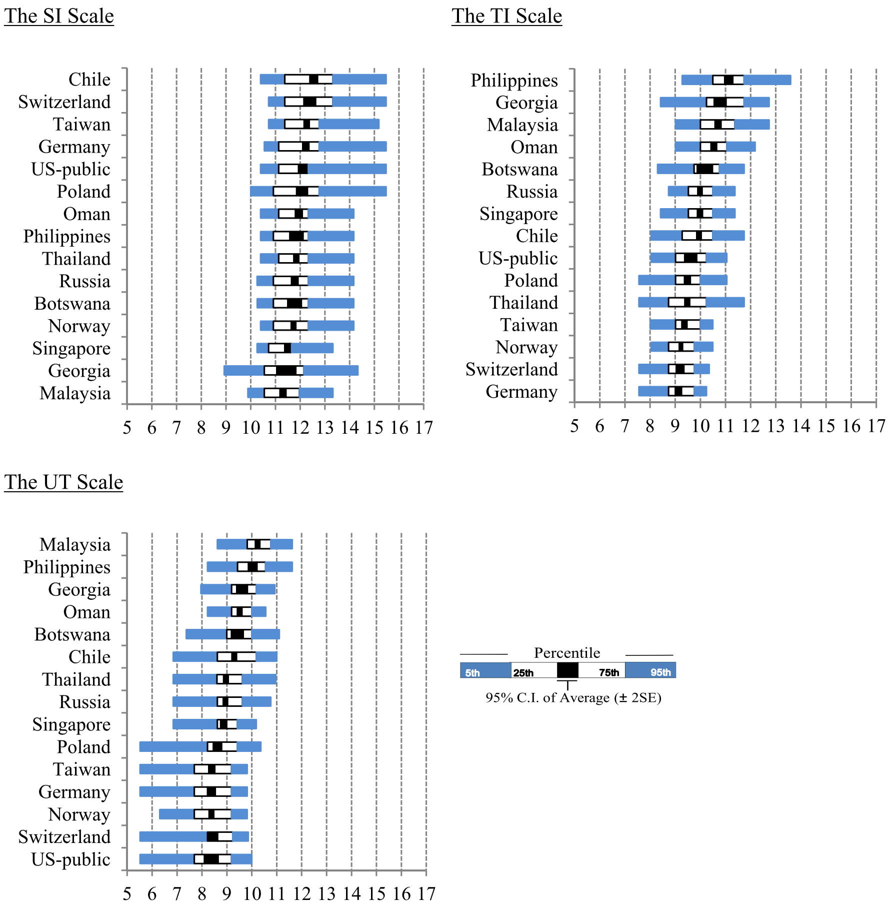

Fig. 2 Distribution of belief scores about learning mathematics (*three scales*). From "Taiwanese Future Secondary Mathematics Teachers' Beliefs under the Perspective of International Comparison", (in Chinese) by S.-J. Tang and F.-J. Hsieh, 2012, in F.-J. Hsieh (Ed.), *Taiwan TEDS-M 2008 Secondary Analysis* (in Chinese). Department of Mathematics, National Taiwan Normal University. Adapted with permission

Regarding the cultural notion, Fig. 3 shows that for some of the cultural groups, beliefs of some scales among countries are similar but are not similar in other scales or to other cultural groups. The most consistent group is the Developed Europe, where TI and UT are almost identical among countries. American group and Confucian Asia are similar in terms of the high SI and below-neutral TI and UT. Countries of Developing Asia express a larger variation and so do those of East Europe. Countries in Developing Asia generally have narrower ranges than the other groups. If we concentrate on the spots of scores for all scales of every country, we can see that the USA, Taiwan, Switzerland, and Germany have quite similar distributions. This is a break of cultural group and needs further study.

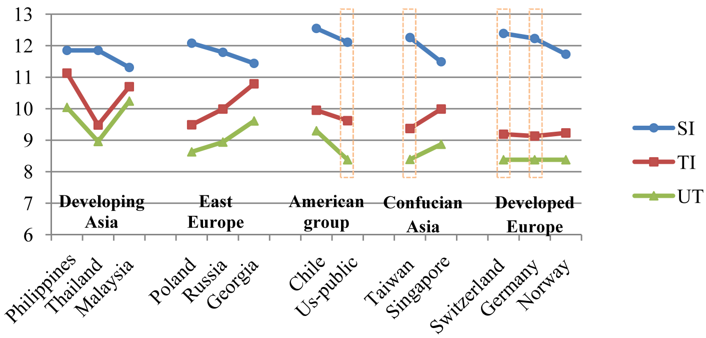

Fig. 3 Plots of mean logit scores on the scales of the beliefs about learning mathematics by cultural groups

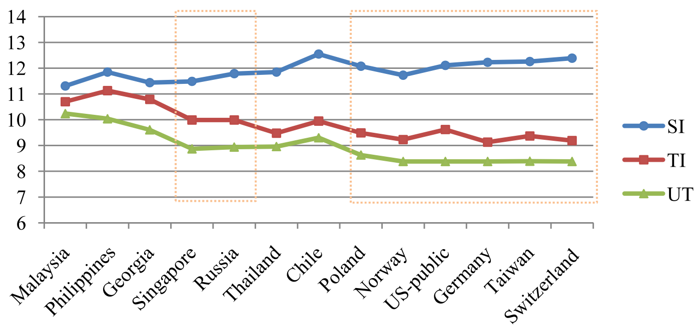

Fig. 4 Plots of mean logit scores on the scales of the beliefs about learning mathematics according to the size of ranges of all three scales

The variance among countries in some cultural groups prompted this study to check other combinations of countries in terms of beliefs about leaning mathematics. Figure 4 shows that the USA, Poland, and Taiwan join the Developed Europe group if we consider the threshold that none of the differences between any two countries on any scale were bigger than 0.5 points (except the 0.66 for Norway and Switzerland on the SI scale). For both the USA and Poland a relation with Europe can be identified in terms of geographical region or historical tradition. The USA has a root from England, a developed European country. Poland is located in central Europe. It seems reasonable if they share similar patterns of learning beliefs with the Developed Europe. However, in the case of Taiwan as an East Asian country embedded in the Confucian ideology, its consistency with Developed Europe is a mystery that needs further study.

The Belief of Natural and Fixed Mathematical Ability According to the international average in Table 6, the central tendency of the belief scores of all participating countries is below the value of 10 on the NF scale. In spite of the tendency of disapproval, Mean of most countries of the Developing Asia and East Europe

Table 6 Means of belief scores in the scale about mathematics achievement

Country	NF	
	M	(SE)
Malaysia	10.58[a,b]	(0.05)
Philippines	10.53[a,b]	(0.07)
Georgia	10.37[a,b]	(0.09)
Thailand	10.33[a,b]	(0.02)
Botswana	10.13[a]	(0.10)
Oman	10.09[a,b]	(0.05)
Russia	10.06[a,b]	(0.02)
Poland	9.89[a,b]	(0.04)
IA	9.84	
Taiwan	9.83[b]	(0.04)
Singapore	9.73[a,b]	(0.04)
Chile	9.35[a,b]	(0.04)
Norway	9.34[a,b]	(0.03)
Switzerland	9.22[a,b]	(0.06)
Germany	9.13[a,b]	(0.04)
US-public	9.02[a,b]	(0.16)

Note. IA = international average; NF = Natural & Fixed mathematical ability. From "Beliefs of Future Secondary Mathematics Teachers" (in Chinese) by S.-J. Tang and F.-J. Hsieh, 2012, in F.-J. Hsieh (Ed.), *Taiwan TEDS-M 2008: Teacher Education and Development Study in Mathematics* (in Chinese; pp. 221–252). Department of Mathematics, National Taiwan Normal University. Adapted with permission

[a]Mean ≠ IA, significant ($p < 0.05$)

[b]Mean ≠ 10, significant ($p < 0.05$)

(Poland excluded) reveals that they tend to agree with the statements included in this scale. Contrarily, the American group and Developed Europe tend to disapprove the idea of natural and stable nature of mathematical ability. The range of the USA is the widest (see Fig. 5).

3.4 *Cultural Notion Across Scales*

(1) *Common Ground*. The results showed that there are common trends of mathematics-related teaching beliefs internationally. Over 80 % of future secondary teachers (82.9–93.7 %) approved (including slightly agree, agree, and strongly agree) three quarters of the belief items[7] about the nature of mathematics, and all belief items in the OC (open and creative nature) factor were included therein. This represents that the open and creative nature of mathematics is a prevalent belief across countries.

Regarding the items about the belief of learning mathematics, all, and only these, belief items of the SI (about student initiative in learning) factor received

[7]The *perA* of the items about the nature of mathematics are as follows: H (94 %), I (92 %), F (92 %), J (89 %), D (89 %), K (89 %), C (88 %), G (85 %), and B (83 %). See Table 1 for the content.

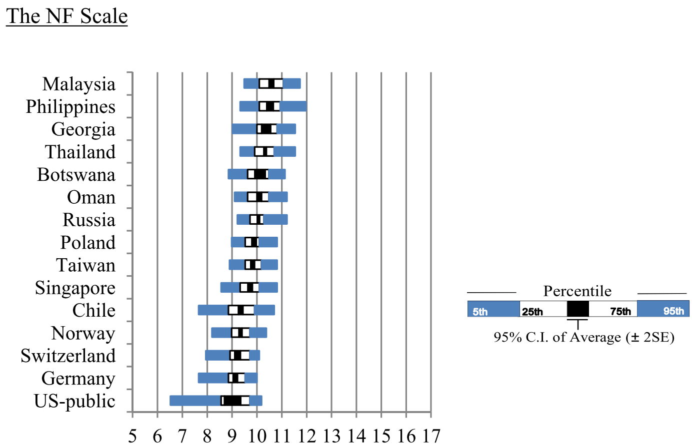

Fig. 5 Distribution of belief scores about mathematics achievement (*one scale*). From "Taiwanese future secondary mathematics teachers' beliefs under the perspective of international comparison", (in Chinese) by S.-J. Tang and F.-J. Hsieh, 2012, in F.-J. Hsieh (Ed.), *Taiwan TEDS-M 2008 Secondary Analysis* (in Chinese). Department of Mathematics, National Taiwan Normal University. Adapted with permission

approval from over 80 % of future teachers (81.2–93.4 %),[8] which accounted for 43 % of the total items about learning mathematics. On the other hand, although all the individual disapproval percentages do not exceed 80 %, some of them are either close to or not far from it. Many future teachers disapproved that understanding a mathematical problem didn't really matter if you could get the right answer ($perD = 79.5$ %), hands-on mathematics experiences were not worth the time and expense ($perD = 75.4$ %), and getting the correct answer deserved more emphasis than the working process ($perD = 71.6$ %).[9] Generally speaking, it seemed that most of the future secondary teachers tended to reject utilitarianism and hold the ideas of openness and "learning first" in mathematics teaching even though the nature of content involved logical rigor and preciseness, the remembering and correct application of formulas, and mathematical routines and procedures. The existence of common beliefs suggests that a part of the beliefs is shaped by some matters; a possible one is the wave of globalization, but it is still a hypothesis to test.

(2) *Cultural Patterns*. The concept of logit scores allows a comparison of means across different scales. Figure 6 shows the mean logit scores of all scales of mathematics-teaching related beliefs by cultural groups. Among all the scales, SI receives the smallest dispersion which means that the belief about students' initiative gains strongest agreement among the cultural groups. On the contrary, CR

[8]The *perA* of the items about the learning of mathematics are as follows: N (93 %), H (93 %), G (92 %), K (87 %), M (84 %), and L (81 %). See Table 2 for the content.

[9]These items were on the UT scale.

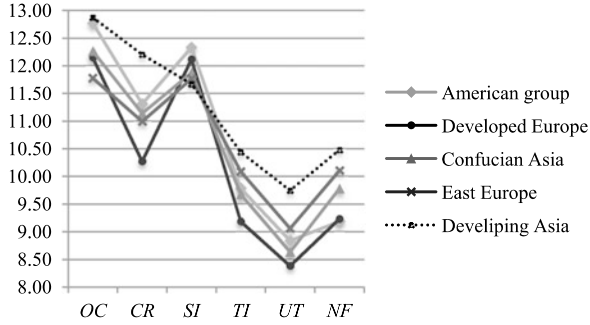

Fig. 6 Plots of mean logit scores of all scales of mathematics-teaching related beliefs by cultural groups. OC = Open & Creative nature; CR = Conservative & Rigorous nature; SI = learning through Student Initiative; TI = learning by following Teacher Instruction; UT = Utilitarianism in Teaching; NF = Natural & Fixed mathematical ability

receives the largest dispersion which means that the cultural groups has little consensus on the conservation and rigor nature of mathematics. Figure 6 also shows that the two groups, East Europe and Confucian Asia exhibit the most closed patterns on all the scales, while Developing Asia has the most distinct pattern than other groups. A rough pattern among all groups except Developing Asia is that they have the strongest beliefs on OC and SI, the second on CR, the third on TI and NF, and the last on UT.

If we consider the neutrality of the beliefs among cultural groups, we found that all groups approved both scales of the nature of mathematics and the SI of mathematics learning, and also disapproved the UT of mathematics learning. But on TI and NF scales, the cultural groups split into two classes. The first class includes Developed Europe, Confucian Asia, and the American group which has negative views on both TI and NF. The other class includes Developing Asia and East Europe that hold positive beliefs on both TI and NF (see Table 7).

The analysis of neutrality provides a comparison of cultural groups with an absolute measure, while an analysis of above or below international averages provides the comparison with a relative measure. The results of the latter analysis are shown in Table 7.

In general, the two classes probably reflect a kind of classification by the relative ideology of the future teachers. The key dividers of these two classes include the scales about whether to approve (relatively) on the natural and fixed mathematical ability (NF) and emphasize (relatively) the role of teacher instruction (TI) and whether to focus (relatively) on getting right answers in a non-time-consuming way (UT). For the first class, a relative negative view on the scales of key dividers can be seen, while the second class has a relative positive view. The views about nature of mathematics and initiative of students are the advanced dividers within classes.

Table 7 Patterns of relative positive or negative views of two classes of cultural groups

Classification	Nature of mathematics		Learning mathematics			Mathematics achievement
	OC	CR	SI	TI	UT	NF
Class 1						
American group	+	+	+	−	−	−
Developed Europe	−	−	+	−	−	−
Confucian Asia	−	−	−	−	−	−
Class 2						
East Europe	−	−	−	+	+	+
Developing Asia	+	+	−	+	+	+

Note. OC = Open & Creative nature; CR = Conservative & Rigorous nature; SI = learning through Student Initiative; TI = learning by following Teacher Instruction; UT = Utilitarianism in Teaching; NF = Natural & Fixed mathematical ability. "+" = the group average is higher than international average; "−" = the group average is lower than international average

4 Conclusion

The future lower-secondary teachers' beliefs about the nature of mathematics, learning mathematics, and mathematics achievement were analyzed in this paper. The analysis was based on six scales gained by factor analyses and by running a procedure employing a Rasch partial credit model (Masters 1982) for all the lower-secondary samples of TEDS-M. In comparison with the original TEDS-M factors, a factor "Utilitarianism in Teaching" (UT) was extracted from the "teacher direction" factor in TEDS-M. The remaining items were labeled as teacher instruction (TI).

Regarding cultural notions, this paper adapted the classification of Blömeke and Kaiser (2012) to divide the participating TEDS-M countries into five cultural groups. The results reveal that there are common patterns as well as distinct patterns across cultural groups. A common one is that all groups believe that the nature of mathematics is open and creative (OC) on the one hand and conservative and rigorous (CR) on the other hand; this kind of mathematics is best learned through considering student initiatives (SI) such as figuring out own solutions regardless of the time consumed. The existence of common beliefs suggests that a part of the beliefs is shaped by an underlying tendency; a possible one is the wave of globalization, but this is still a hypothesis to test.

Comparing the patterns of all cultural groups across all scales about mathematics-teaching related beliefs, this study found that all groups but one (the group of Developing Asia) shared a rough pattern. This pattern has the strongest beliefs on an open and creative nature of mathematics and student initiatives of learning mathematics, the second strongest beliefs on the conservative and rigorous nature of mathematics, and the third on the importance of teacher instruction and explanation to students (NF). In this pattern, the least approved belief is the utilitarianism in teaching such as learning is best through memorization or other non-time-consuming ways.

When considering neutrality, all cultural groups gain consistent positions in terms of positive or negative views on the OC, CR, and SI scales but not on the TI and NF scales. When the latter two scales are used as dividers, the cultural groups can be divided into two classes: one with negative views on both TI and NF, including Developed Europe, Confucian Asia, and the American group, the other with positive views on both TI and NF, including Developing Asia and East Europe.

Though this study analyzed cultural patterns according to the aforementioned cultural groups, breaks within groups may also be seen. For the beliefs on learning mathematics, the USA, Poland, and Taiwan join the Developed Europe pattern. This situation prompts a need for further studies of cultural group classifications such as whether the USA should join the Developed Europe group due to historical roots or whether Poland should join this group for its geographical location. Besides, why Taiwan abandons Singapore to join the Developed Europe pattern remains an open question that requires further study.

The most valuable cultural meaning of the belief patterns may be the presentation of diversity in mathematics teaching and the beauty of regular patterns. In fact, all the so-called "high-achieving" countries, the means of which were higher than the international average of 500 test points in the MCK or MPCK surveys of TEDS-M, come from almost every culture group in this paper. It is therefore difficult to say which belief pattern or culture of mathematics teaching should be best for teacher education. Examining and realizing where we are is always the first policy in terms of belief or culture.

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The Cultural Notion of Teacher Education: Comparison of Lower-Secondary Future Teachers' and Teacher Educators' Beliefs

Ting-Ying Wang and Feng-Jui Hsieh

Abstract This chapter describes a study whose aim is to highlight the cultural notion of beliefs related to mathematics teaching with respect to future mathematics teachers and teacher educators at the lower-secondary level. Through conducting multi-sample latent profile analysis, this study identified the belief profiles for the teachers and the educators. By associating the profiles with countries and employing hierarchical cluster analysis (HCA), cultural features influencing beliefs were revealed.

Our results show that the beliefs of future teachers and teacher educators in the same country are homogeneous. This suggests that the *country* is an important factor for shaping beliefs. We also discovered that the beliefs are homogenous in countries that share the same cultural features: geographical regions, historical traditions, levels of human development, or knowledge achievement. HCA grouped all Western countries with a Greek/Latin/Christian tradition together, and divided the East Asian countries into two clusters—whether or not having Confucian heritage. All countries with very high human development indices (HDI) were grouped in one cluster. The countries in the other cluster had a lower HDI. All higher-achieving countries were in the same cluster as well. Our results also indicate that the process-of-inquiry view on the nature of mathematics and the active-learning view on teaching and learning mathematics dominated in all countries with respect to future teachers as well as to teacher educators.

Keywords TEDS-M · Mathematics belief · Mathematics teaching belief · Mathematics learning belief · Achievement belief · Future teacher · Teacher educator · International comparison · Culture

1 Introduction

How mathematics teachers' beliefs are shaped is an important issue because the literature has documented the influence of teachers' beliefs on their instructional

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S. Blömeke et al. (eds.), *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn*, Advances in Mathematics Education,
DOI 10.1007/978-94-007-6437-8_12,

practice (Cooney 1999; Stipek et al. 2001). Several studies point out that how beliefs are shaped is closely related to the experiences during their teacher preparation (Brown and Borko 1992; Nespor 1987; Raymond 1997). In this experience, teacher educators play an important role because they are those who have the authority to design and develop the structure and contents of teacher preparation, and also are those who directly execute instruction to future teachers. Teacher educators' beliefs may thus be crucial, as teachers of future teachers, when it comes to influencing their instructional practice. To understand teacher education, not only future teachers' beliefs are worth investigation but also educators' beliefs and a comparison between the beliefs of these two groups.

A universally accepted definition for beliefs does not exist. However, as Pajares (1992) indicated, adopting only one specific definition would not work in the educational research community. Various definitions from different perspectives provide us with opportunities to understand multiple facets of beliefs. One concept commonly mentioned by researchers is that beliefs are regarded to be *true* by an individual. For example, Richardson (1996) defined belief as "psychologically held understandings, premises, or propositions about the world that are felt to be true." Sigel's (1985) definition was from the perspective of cognitive psychology: "mental constructions of experience—often condensed and integrated into schemata or concepts." These definitions reveal that beliefs are related to one's cognition, knowledge, and affect, and that the development of beliefs involves mental constructions within social and cultural contexts in which the individual gains experiences. Therefore, beliefs can be regarded as bounded within social and cultural contexts (Perry et al. 2006).

This potential nesting within contexts is an important notion when comparing beliefs of teachers, including future teachers and teacher educators, from various countries in which the educational cultures may be different. Some research has explored the beliefs of future teachers in various countries with respect to cultural dimensions, and these studies provide evidence that country-specific differences or a cultural split in beliefs may exist (e.g., Felbrich et al. 2012; Schmidt et al. 2011).

Tylor (1889) indicated culture as a "complex whole which includes knowledge, belief, art, law, morals, custom, and any other capabilities and habits acquired by man as a member of society." Culture also identifies the members in a mind collective and distinguishes them from members in other groups (Hofstede 1984). Beliefs, as a component of culture, also serve to identify people with certain common perspectives. Therefore, the following questions are intriguing: Do people from different countries or from different groups within a country have heterogeneous or homogenous beliefs? If people from different groups within a country interact, do they influence each other regarding beliefs and thus possess homogeneous beliefs?

This study uses the data of TEDS-M, the first IEA-sponsored international study of mathematics teacher education with national representative samples, to examine the beliefs of future lower-secondary teachers and teacher educators related to the teaching of mathematics. The aim of this study is to identify the belief profiles for future teachers and teacher educators in the countries participating in TEDS-M. We also examine similarities and disparities of the belief profiles of the two cohorts, and the homogeneity and heterogeneity of the beliefs across or within countries. Another focus of our study is to reveal the cultural meanings in the above findings.

2 Study Design

2.1 Theoretical Framework

This study used the TEDS-M structure of beliefs related to mathematics teaching. The structure was around three aspects: the nature of mathematics, teaching and learning mathematics, and mathematics abilities. These aspects have been seen as crucial aspects in scholarly discussion of mathematics teachers' beliefs (Barkastas and Malone 2005; Stipek et al. 2001).

Regarding beliefs about the nature of mathematics, various constructs emerged. We focus on two major perspectives. One views mathematics as dynamic, expanding human creations, involving a series of inquiring and problem-solving processes. The other views mathematics as a static, immutable unified body of knowledge, including the accumulation of facts, rules, and skills (Barkastas and Malone 2005; Ernest 1988).

There were also two major categories that could be extracted from various structures of beliefs concerning mathematics teaching and learning. One category could be characterized by "child-centeredness" and "constructivist". In this perspective, teachers should establish a learner-focused environment for students and provide them with opportunities to construct their own meanings. The other category is characterized by "transmission" and "instrumentalist". In this perspective, the teachers' role is to transmit mathematical knowledge, and the students' role is to carefully listen, receive, and follow rules and procedures explained by teachers (Barkastas and Malone 2005; Cross 2009; Perry et al. 1999).

Regarding beliefs about mathematics abilities, an incremental view and an entity view were distinguished. In the former, abilities can be developed through making efforts and learning; they are amenable to change. In the latter, abilities are stable and not very amenable to change; the effectiveness of efforts to enhance them is limited (Stipek et al. 2001).

Future teachers and teacher educators have many interactions during the process of teacher preparation, and so possibly have shared visions. Simultaneously, these two cohorts have different positions and experiences in teacher preparation (Nespor 1987), which may cause them to have some different viewpoints. This study starts with identifying the profiles of future teachers' and teacher educators' mathematics-teaching-related beliefs, and analyzes the similarities and disparities of the belief profiles. Then we make comparisons between countries according to the prevalence of these belief profiles. Because people from different countries with their own historical traditions and educational identities possess several different beliefs (Felbrich et al. 2012; Leung 2006; Schmidt et al. 2011), we expect to find some between-country heterogeneity in beliefs. Additionally, some countries are categorized as one group for having common characteristics relative to culture (e.g., Blömeke and Kaiser 2012; Felbrich et al. 2012; Krainer et al. 2012). We also expect to discover some homogeneity in beliefs among countries sharing characteristics, such as geographical regions, historical traditions, or levels of human development.

2.2 Participants

The target population of this study was future lower-secondary teachers in their last year of teacher education who would receive a license to teach mathematics, and teacher educators (also referred to as "educators") who had regular and repeated responsibilities to instruct the future teachers in the fields of mathematics, mathematics pedagogy, or general pedagogy.[1]

The TEDS-M sampling plan used a stratified multi-stage probability sampling design. The future teachers and educators were randomly selected or censused from teacher preparation institutions that had also been randomly selected. According to the criterion of IEA, a 75 % combined participation rate was the minimum requirement for all target populations to carry out international comparisons. Samples with a participation rate between 60 % and 75 % were suitable for data analysis but should be annotated as having a low participation rate. However, for the educator samples, we used a threshold rate of 50 % to overcome the difficulty of obtaining adequate response rates for surveying this kind of adult (Hsieh et al. 2011) and to include more information. Although a sample with a participation rate between 30 % and 60 % was advised to be reported separately by IEA, our decision of 50 % threshold was close to 60 % and contained more than half of the sample. The participating countries analyzed by this study are shown in Table 1.

In the international TEDS-M data set, teacher educators were not distinguished by levels. In order to make appropriate comparisons with future lower-secondary teachers, this study therefore further categorized educators into primary level and lower-secondary level according to the teacher preparation units they served.[2] Only lower-secondary level educators, labeled *educators*, were analyzed by this study. Our study included 8,207 future secondary teachers from 15 countries and 3,288 secondary-level educators from 13 countries.

[1]For more information on the samples, see the TEDS-M international report (Tatto et al. 2012).

[2]An educator was counted in each level if he or she served at both primary and secondary levels. German data sets did not provide enough information to categorize its educators by teacher preparation units. Following the German national representative coordinator's suggestion, its educators were distinguished into two levels through one question in the Teacher Educator Questionnaire. The question asked educators for how many years they have prepared each of future primary and secondary teachers. If an educator teaches future secondary teachers for more than zero years, he or she would be categorized into the data set of secondary level educators. We did not apply this method to all countries because substantial proportions of missing values were found or the question was not administered in some countries.

Table 1 The countries participating in TEDS-M at the lower-secondary level

Botswana	Chile[a,d]	Germany[d]	Georgia[a]	Malaysia[d]
Norway[b,e]	Oman	Philippines	Poland[a,c,f]	Russia
Singapore	Switzerland[d,g]	Taiwan	Thailand	United States[a,e,h]

[a]The combined participation rate of future teachers was between 60 % and 75 %

[b]The combined participation rate of future teachers was 58 %, which was just slightly below the threshold set by the IEA for direct comparisons with other countries and was therefore included in this study. Data sets of four Norwegian program types are available, which were combined in an attempt to accurately represent the situation in Norway

[c]The combined participation rate of educators was between 60 % and 75 %

[d]The combined participation rate of educators was between 50 % and 60 %

[e]The combined participation rate of educators was deemed unacceptable and the data were not processed by IEA Data Processing and Research Center. Therefore, no educator data were presented in this study

[f]Institutions providing consecutive programs only were not covered

[g]Only German-speaking regions were covered

[h]Only public universities were covered

2.3 Instruments

Future teachers' and educators' beliefs about the nature of mathematics, teaching and learning mathematics, and mathematics abilities were captured by three questions including 12, 14, and 8 items respectively. Six-point Likert scales were used to capture agreement for each item: "strongly disagree" to "strongly agree" were associated with 1 to 6 points respectively.

TEDS-M employed factor analyses on the future teachers' data to ensure the validity of each scale across groups and the selection of homogeneous items in each index. A partial-credit model (Masters 1982) from the Rasch family was used to scale the data with the Rasch logit score of 10 as a neutral position. Scores higher than 10 indicated the propensity to agree with the indices and scores lower than 10 indicated the propensity to disagree with the indices. The scale scores of educators were estimated by using the item parameters calibrated for the future teachers. Therefore, the scores of the two samples were placed on the same scale and thus comparisons were facilitated (Tatto et al. 2012).

Regarding the beliefs about the nature of mathematics, the two indices developed by TEDS-M were *mathematics as a process of inquiry* and *mathematics as a set of rules and procedures*, which were consistent with two major perspectives in the literature—dynamic and static. Two indices for the beliefs about mathematics teaching and learning were *learning mathematics through active involvement* and *learning mathematics through following teacher direction*, which were consistent with a constructivist perspective and a transmission perspective in the literature. Only one index was developed for the beliefs about mathematics abilities—*fixed abilities*, reflecting the view that abilities are not amenable to change (entity view) and the opposite view (incremental view) in the literature.

2.4 Data Analysis

The study employed a heterogeneous unrestricted T-class model of multi-sample latent profile analysis (LPA; Clogg and Goodman 1985) to analyze the three aspects of mathematics-teaching-related beliefs (nature of mathematics, teaching and learning mathematics, and mathematics abilities). Multi-sample LPA, a model-based approach (Muthén 2001), allows for simultaneously studying the interrelationships between observed continuous variables from several samples. In this study, the agreement of future teachers and educators with every belief item was used as variables.

A heterogeneous unrestricted T-class model was chosen by this study because (1) it fixed the number of latent classes for two samples to facilitate comparisons between their beliefs; and (2) for any belief item, it accepted different means for different samples in the same latent class. For each aspect of beliefs, the probabilities of a future teacher/educator being assigned into each latent class were obtained, which were summed to 1 across the latent classes. The average probability (weighted) of all samples for any specific latent class was calculated as the overall relative size of the class (model-based size rather than actual size).

The overall relative sizes of distinct latent classes provided quantitative information about the prevalence of the profiles specified by the latent classes for all samples; the bigger the relative class size, the more prevalent the profiles. The profiles tied with the belief items used in the LPA and the relative strengths of the beliefs on these items provided qualitative information about the beliefs.

For each aspect of beliefs, each future teacher/educator was classified into only one latent class, which he or she had the highest probability of being in. In each country and for each cohort, this study calculated the size of the latent classes by country, that is, the percentage of members belonging to the latent class of the cohort in the country. The size of the latent class by country described the prevalence of the profile specified by the class in that country. The bigger the size of a class of a country, the more prevalent the profile specified by the class in that country.

To determine the optimal number of classes, this study employed three fit statistics: adjusted Bayesian Information Criterion (BIC), difference in BIC, and entropy (Muthén 2001). The adjusted BIC was used as goodness-of-fit criterion where a smaller value indicated a better fitting model. Differences in the BIC were used to assess the improvement of model parsimony by comparing the n-class model with the $n - 1$-class model. Entropy indicated the precision with which the model classified future teachers and educators. Reaching 1.0, 0.8, 0.6, and 0.4 represented perfect, high, medium, and low levels of classification (Clark and Muthén 2009). Furthermore, when comparing the alternative competing models, in addition to fit statistics it is also recommended to consider their connections to previous results and related theoretical issues (Bollen 1989; Marsh et al. 2004). We took this into account, too.

All data analyses used the final sampling weights of future teachers and teacher educators provided by TEDS-M. The sums of the weights for future teachers and for educators in each country were both adjusted to be the same to prevent the dominancy of countries or any cohorts with larger sample sizes.

Some statistical analyses and procedures used in this study are explained in the following sections as they become applicable.

3 Results

In the following sections we report firstly on the results of the LPA regarding the number of classes. Then, the results on the beliefs profiles with respect to the nature of mathematics, teaching and learning mathematics, and mathematics abilities are reported separately. We include the profiles of future teachers' and educators' beliefs, similarities and disparities of the belief profiles among future teachers, among educators, and across them, and the homogeneity and heterogeneity of beliefs within and across countries. In the final part of this section, we simultaneously consider the three aspects of mathematics-teaching-related beliefs to see homogeneity and heterogeneity of beliefs across countries from a holistic perspective.

The model-fit statistics to decide on the optimal classification for the patterns of future teachers' and educators' beliefs are shown in Table 2. For all three aspects of mathematics-teaching-related beliefs, the entropies of models with 2, 3, or 4 classes were above 0.8 and all models reached a high level of precision of classification (Clark and Muthén 2009). BIC criteria indicated the preference of models with more classes. However, differences in BIC gradually diminished as the number of classes increased, indicating that improvements in model parsimony shrank. The improvement was the largest when two classes were distinguished.

In light of the fact that the PDA model we used allows disparities between belief profiles of different cohorts in a class, and since we have three aspects of beliefs to compare, this study decided to choose the 2-class model for all the three aspects. This choice may avoid blurring the characteristics of belief profiles due to an overwhelming amount of latent classes.

3.1 Nature of Mathematics

3.1.1 Belief Profiles

The belief profiles of future teachers and educators about the nature of mathematics in the 2-class model are shown in Fig. 1. This study grouped the items according to the indices developed by TEDS-M: *mathematics as a process of inquiry* (briefly, *process-of-inquiry view*) and *mathematics as a set of rules and procedures* (briefly, *rules-and-procedures view*). The process-of-inquiry view, including 6 items, emphasizes the creativity and application features of mathematics. The rules-and-procedures view, including 6 items, emphasizes the importance of definitions, rules, procedures, and rigor and precision in mathematics.

Both future teachers and educators in class 1 endorsed the statements supporting the process-of-inquiry view, and they endorsed the statements supporting the

Table 2 Fit statistics for the latent profile analysis of mathematics-teaching-related beliefs ($n =$ 11,495)

Number of classes	Adjusted BIC	Difference in BIC	Entropy
The nature of mathematics			
1	419664.058	–	–
2	401133.071	18530.99	0.898
3	389772.052	11361.02	0.907
4	383043.994	6728.06	0.899
Teaching and learning mathematics			
1	508486.248	–	–
2	491290.734	17195.51	0.900
3	483303.567	7987.17	0.898
4	477984.538	5319.03	0.893
Mathematics abilities			
1	329305.118	–	–
2	310858.582	18446.54	0.894
3	305429.043	5429.54	0.871
4	303981.086	1447.96	0.852

rules-and-procedures view simultaneously. The belief profiles in class 1 were, therefore, labeled "comprehensive". For the process-of-inquiry view, the means of future teachers and educators in this class were 12.62 and 13.25 respectively, and for the rules-and-procedures view, the means were 11.67 and 11.78 respectively. All four means were significantly higher than the score representing a neutral position of 10 ($p < 0.01$).

Both future teachers and educators in class 2 endorsed the statements supporting the process-of-inquiry view, but the level of their endorsement of statements supporting the rules-and-procedures view was lower. This class agreed with viewing mathematics as a process of inquiry (future teachers: $M = 11.17$, educators: $M = 12.09$; both significantly higher than 10, $p < 0.01$), but disagreed with viewing mathematics as a set of rules and procedures (future teachers: $M = 9.84$, educators: $M = 9.42$; both significantly lower than 10, $p < 0.01$). The label "inquiry-preferred" was suitable to describe the profiles in this class.

Endorsement of the process-of-inquiry view was the common feature among the two classes, but the two classes were dissimilar regarding the levels of endorsement of rules-and-procedures.

There was also heterogeneity between the belief profiles of the two cohorts (see Fig. 1). This study employed t-test and Cohen's d to identify the statements on which future teachers and educators differed. The difference between the average scores of the two cohorts had to reach (1) statistical significance and (2) Cohen's (1992) operational definition of a medium effect size (ES), 0.5, before heterogeneity

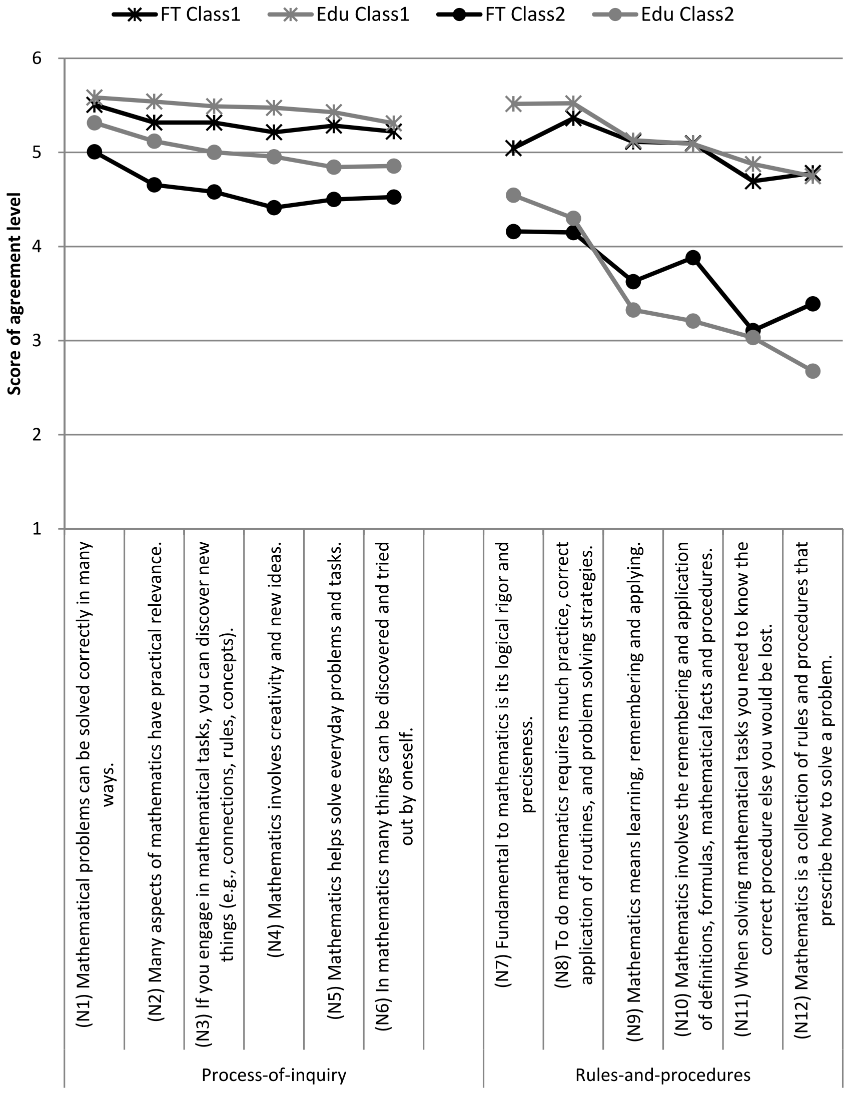

Fig. 1 Profiles for beliefs about the nature of mathematics. FT = future teachers; Edu = educators

was assumed. Such rigorous criteria were chosen because the profiles were deemed as being homogeneous in the LPA.

In class 2, the educators' scores of N2 and N4 were higher than future teachers', indicating educators' stronger agreement on the practicality-related characteristics of mathematics. By contrast, the future teachers' scores of N10 and N12 were higher than educators', indicating future teachers' stronger agreement on the rule-orientation characteristics of mathematics (see Fig. 1). Compared with class 2, future teachers and educators in class 1 were more homogeneous in their beliefs.

Table 3 The proportions of classes for the belief about the nature of mathematics in each country

Country	Future teachers		Educators	
	Class 1	Class 2	Class 1	Class 2
Germany	18.2 (2.6)	81.8 (2.6)	17.4 (2.8)	82.6 (2.8)
Switzerland	22.9 (3.7)	77.1 (3.7)	23.1 (5.0)	76.9 (5.0)
Poland	43.7 (2.8)	56.3 (2.8)	32.3 (2.4)	67.7 (2.4)
Russia	42.5 (2.4)	57.5 (2.4)	43.6 (2.8)	56.4 (2.8)
Georgia	65.0 (5.1)	35.0 (5.1)	88.6 (5.8)	11.4 (5.8)
Oman	90.2 (1.6)	9.8 (1.6)	67.4 (5.4)	32.6 (5.4)
Malaysia	81.1 (2.0)	18.9 (2.0)	74.3 (6.1)	25.7 (6.1)
Philippines	93.6 (2.4)	6.4 (2.4)	93.0 (2.2)	7.0 (2.2)
Thailand	87.8 (1.3)	12.2 (1.3)	76.6 (2.7)	23.4 (2.7)
Taiwan	60.1 (2.8)	39.9 (2.8)	58.8 (12.9)	41.2 (12.9)
Singapore	63.2 (1.9)	36.8 (1.9)	47.1 (5.6)	52.9 (5.6)
Botswana[a]	88.3 (4.6)	11.7 (4.6)	52.8 (12.1)	47.2 (12.1)
Chile	69.3 (1.9)	30.7 (1.9)	39.6 (2.8)	60.4 (2.8)
US-Public	77.4 (3.8)	22.6 (3.8)		
Norway	35.2 (2.1)	64.8 (2.1)		

The numbers in the parentheses indicate *SE*

[a]The sample of educators did not reach 30 so that it might not be appropriate to use the data for firm conclusions

3.1.2 Homogeneity and Heterogeneity of Beliefs Across Countries

The overall relative sizes of class 1 and class 2 were 59 % and 41 % respectively. The belief profiles characterized by "comprehensive" dominated more strongly among future teachers and educators internationally.

In 10 of the 13 countries with both groups, the dominant belief profiles of future teachers and educators belonged to the same class, showing homogeneity across the two cohorts (see Table 3). Among them, four countries were dominated by the inquiry-preferred belief profiles. These were all Western countries, sharing the Greek/Latin/Christian tradition (Leung 2006), in Europe—Germany, Switzerland, Poland, and Russia.[3] The other six countries were dominated by the comprehensive belief profiles. All East Asian countries, except Singapore, were of this kind. In the remaining three countries, the prevalent belief profiles of future teachers and those of educators belonged to different classes, showing heterogeneities between future teachers and educators.

[3]Similar to these four Western countries in Europe, Norwegian future teachers also belonged to the inquiry-preferred belief profile.

In Taiwan and Singapore, a higher proportion of future teachers and educators belonged to the inquiry-preferred belief profiles, compared with their East Asian counterparts Malaysia, Philippines, and Thailand. In addition to the possible influence caused by westernization of mathematics education in East Asia (Siu 2009), the common Chinese/Confucian cultural background might also be an influence (Leung 2006). In this cultural tradition, mathematics curriculum content is often demanding and students are expected to learn hard and to excel (Silver 1998; Birenbaum et al. 2005). Therefore, some future teachers and educators in Taiwan and Singapore might have the opportunities to experience what their mathematicians experience—inquiry, creation, and discovery. It is possible that this experience caused them to agree more strongly with the process-of-inquiry view than the rules-and-procedures view.

3.2 Teaching and Learning Mathematics

3.2.1 Belief Profiles

The belief profiles of future teachers and educators about teaching and learning mathematics in the 2-class model are shown in Fig. 2. This study grouped the items according to the indices developed by TEDS-M: *learning mathematics through active involvement* (briefly, *active-learning view*) and *learning mathematics through following teacher direction* (briefly, *teacher-direction view*). The active-learning view supports that students must be actively involved in learning mathematics, such as conducting their own enquiries and their own methods for problem solving. A total of six belief items were included. The teacher-direction view supports that students learn mathematics through following explanations, rules, and procedures transmitted by teachers. Eight belief items were included.

Future teachers and educators in class 1 endorsed both the active-learning view and the teacher-direction view. The future teachers' and educators' logit scores for the active-learning view were 11.48 and 11.67 respectively and for the teacher-direction view were 10.38 and 10.29 respectively. Both of these scores were significantly higher than the neutral 10 ($p < 0.01$). Figure 2 also reveals that this class agreed with all statements with only a few exceptions, such as L13, in which both cohorts had average scores significantly lower than 3.5 (the midpoint of 1 to 6, representing neutral position). The profiles of this class were, therefore, entitled "comprehensive".

Future teachers and educators in class 2 strongly endorsed the statements supporting the active-learning view (see Fig. 2). Their logit scores were 12.29 and 12.68 respectively, even higher than those scores in class 1 to a significant level ($p < 0.01$). This class did not endorse the teacher-direction view. Future teachers' and educators' logit scores were 9.03 and 8.79, significantly lower than 10. Furthermore, for almost all the statements supporting this view, the average scores of the two cohorts were significantly lower than 3.5. Therefore, this study labeled class 2 as "active-learning-preferred".

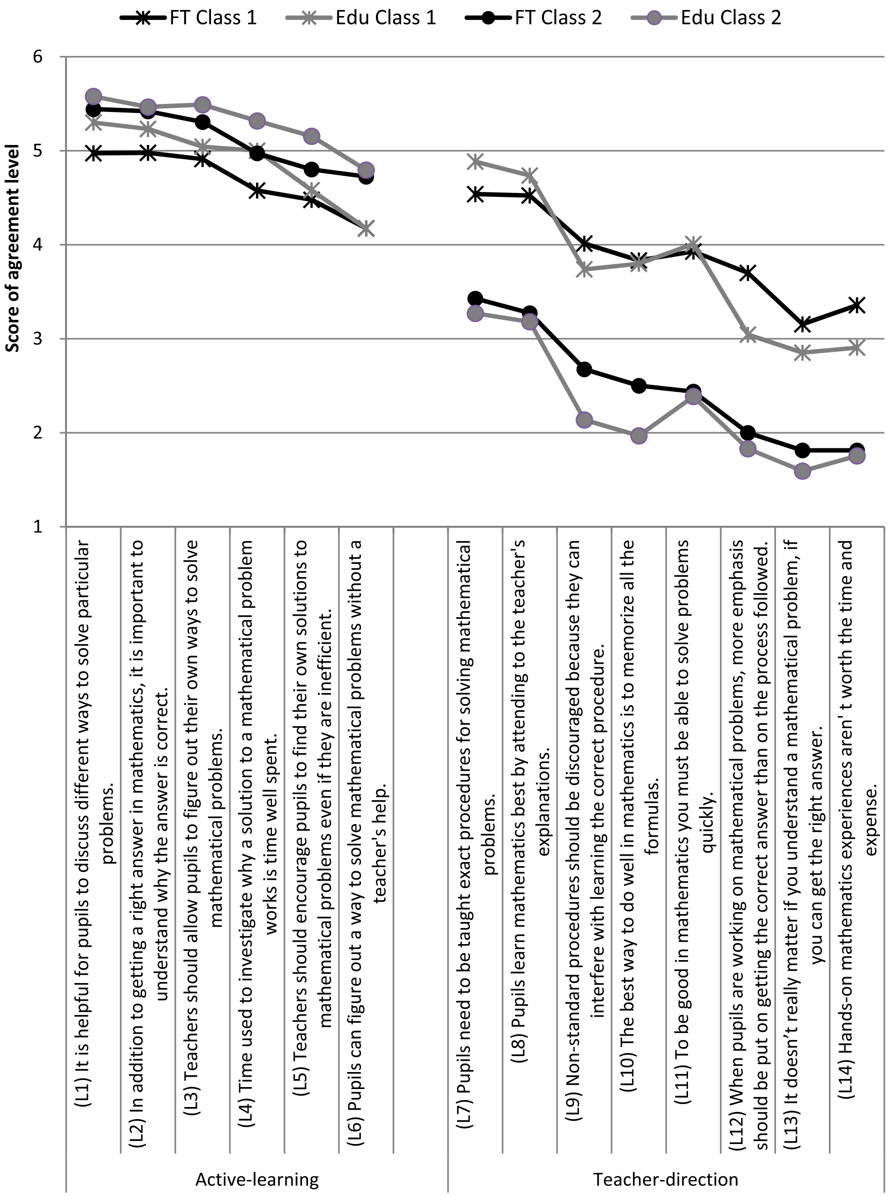

Fig. 2 Profiles for beliefs about teaching and learning mathematics. FT = future teachers; Edu = educators

Both class 1 and class 2 agreed with the active-learning view, showing international homogeneity. By contrast, the two classes were heterogeneous regarding the teacher-direction view. However, one statement supporting this view—L13—was disagreed by future teachers and educators in both classes (see Fig. 2). It is a prevailing belief in mathematics education at the secondary level that only caring about getting correct answers without seeking understanding is unacceptable for mathematics learning.

Dissimilar to the situation in the beliefs about the nature of mathematics, future teachers' and educators' belief profiles of learning and teaching mathematics were more homogeneous. Only future teachers and educators in class 1 had a different agreement level on one item—L12 (see Fig. 2). The difference between the average scores of the two cohorts reached a significant level and a medium ES. The educators did not agree with this statement while the future teachers did. In these future teachers' beliefs, it is important for students to get correct answers (L12), but students should get the correct answers combining this with their understanding (L2).

3.2.2 Homogeneity and Heterogeneity of Beliefs Across Countries

The overall relative sizes of class 1 and class 2 were 35 % and 65 % respectively. The active-learning-preferred belief profiles were more prevalent among future teachers and educators internationally.

In 10 of the 13 countries with both kinds of research subjects, the prevalent belief profiles of future teachers and of educators belonged to the same class, showing the relatively high homogeneity across the two cohorts in these 10 countries (see Table 4). Among them, eight countries were dominated by the same belief profiles, namely active-learning-preferred. The large number of countries sharing the same profiles indicated more cross-country homogeneity in the beliefs about teaching and learning mathematics than in the beliefs about the nature of mathematics. In the other two countries, the comprehensive belief profiles were more prevalent.

Among these 10 countries, all five TEDS-M Western countries (those that studied educators) except Georgia were dominated by active-learning-preferred belief profiles (see Table 4), which coincided with the findings about the West in the literature (e.g., Perry et al. 2006).[4] The literature shows that textbooks are not such an important determinant of mathematics content to be taught in Western countries as in Asian countries (Leung 2006). Therefore, teachers in the West could deal with the content more flexibly and were more likely to teach using constructivist approaches (Perry et al. 2006). Subjects in these five countries might develop their beliefs as the active-learning-preferred profiles because of learning experiences in this kind of environment. Comparing European countries, the proportions of future teachers of this belief profile were different: the proportions in Germany and Switzerland, which were among the 10 highest human development indices (HDI) in Europe (United Nations Development Programme 2011), were higher than those

[4]Future teachers in Norway and the United States, as in these five Western countries, were also of the active-learning-preferred belief profile.

Table 4 The proportions of classes for the belief about teaching and learning mathematics in each country

Country	Future teachers		Educators	
	Class 1	Class 2	Class 1	Class 2
Germany	9.9 (2.4)	90.1 (2.4)	5.7 (1.8)	94.3 (1.8)
Switzerland	4.3 (1.5)	95.7 (1.5)	8.9 (2.4)	91.1 (2.4)
Poland	20.5 (1.9)	79.5 (1.9)	12.6 (2.3)	87.4 (2.3)
Russia	35.3 (2.2)	64.7 (2.2)	16.4 (1.5)	83.6 (1.5)
Chile	41.2 (2.0)	58.8 (2.0)	22.7 (2.0)	77.3 (2.0)
Taiwan	9.9 (1.4)	90.1 (1.4)	14.9 (7.6)	85.1 (7.6)
Singapore	33.5 (2.0)	66.5 (2.0)	21.5 (4.2)	78.5 (4.2)
Thailand	27.5 (1.5)	72.5 (1.5)	22.5 (2.1)	77.5 (2.1)
Malaysia	83.3 (1.6)	16.7 (1.6)	62.6 (4.7)	37.4 (4.7)
Philippine	79.1 (3.4)	20.9 (3.4)	69.8 (2.7)	30.2 (2.7)
Georgia	80.7 (4.1)	19.3 (4.1)	54.3 (7.0)	45.7 (7.0)
Oman	61.6 (3.0)	38.4 (3.0)	53.0 (5.8)	47.0 (5.8)
Botswana[a]	43.7 (6.5)	56.3 (6.5)	55.3 (12.8)	44.7 (12.8)
Norway	7.9 (1.4)	92.1 (1.4)		
US-Public	19.0 (3.7)	81.0 (3.7)		

The numbers in the parentheses indicate *SE*

[a]The sample of educators did not reach 30 so that it might not be appropriate to use the data for firm conclusions

in Poland and Russia, which were formerly Soviet-led (in the Soviet Union or its client states).

Comparing the two Confucian heritage countries, the proportion of future teachers of the active-learning-preferred belief profile in Taiwan was much higher than that in Singapore. The explanation might relate to the different expert level of future teachers in these two countries. Singaporean future teachers were prepared to teach two subjects while Taiwanese future teachers only focused on their preparation for teaching mathematics. In addition, the latter actually outperformed the former on MCK and MPCK in TEDS-M (Hsieh 2012; Hsieh and Wang 2012). Therefore, Taiwanese future teachers might feel more confident of their teaching so that they could agree with moving beyond keeping to the textbooks and giving students some power to control their learning. Furthermore, the Taiwanese high proportion could result from a recently prevalent idea that active learning was helpful to facilitate students' understanding and to cultivate their mathematical abilities which was emphasized in Taiwan's mathematics curriculum (Ministry of Education 2009).

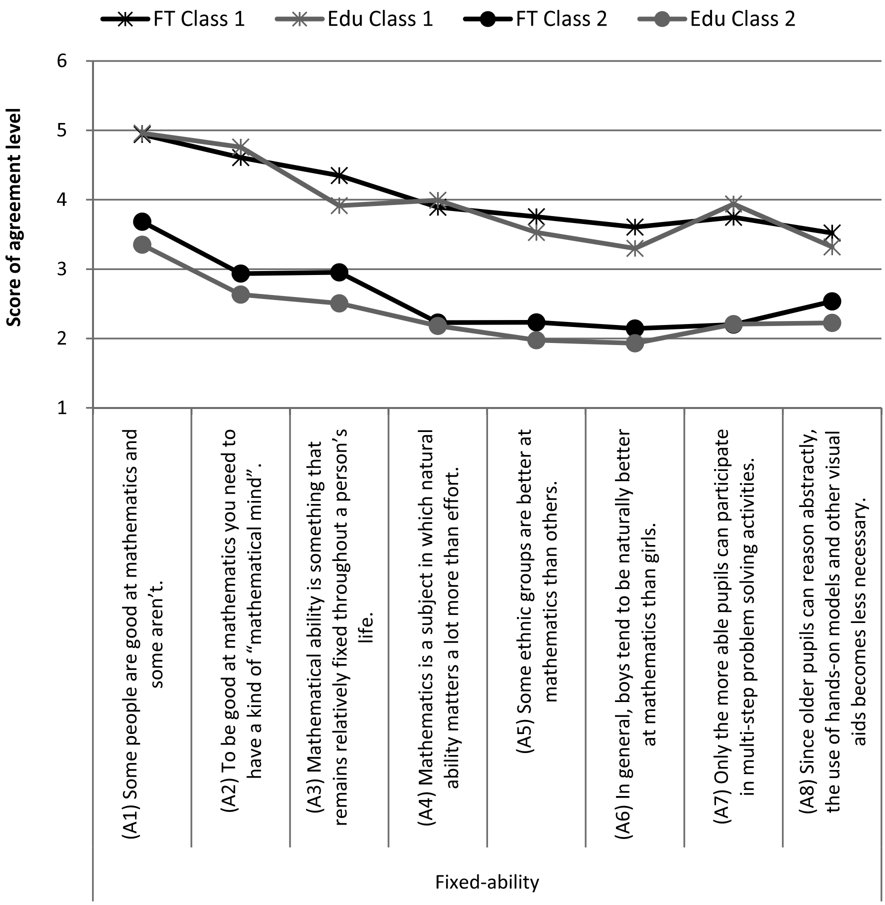

Fig. 3 Profiles for beliefs about mathematics abilities. FT = future teachers; Edu = educators

3.3 Mathematics Abilities

3.3.1 Belief Profiles

The belief profiles of future teachers' and educators' beliefs about mathematics abilities in the 2-class model are shown in Fig. 3. The index developed by TEDS-M was *fixed abilities*, which included all 8 items.

As shown in Fig. 3, both future teachers and educators in class 1 endorsed the statements supporting mathematics as fixed abilities. They believed that students' mathematics abilities were natural talents and categorical differences existed. The belief profiles in class 1, therefore, were labeled "entity-view-endorsed". The characteristics of the belief profiles could also be evidenced by the Rasch logit scores. Future teachers' and educators' scores in this class were 10.54 and 10.53 respectively, which were both significantly higher than the score representing the neutral position 10 ($p < 0.01$).

Future teachers and educators in class 2, contrary to class 1, disagreed with the statements supporting mathematics as fixed abilities. For all the statements, except A1, the average scores of the two cohorts were significantly lower than 3.5 (see Fig. 3). Furthermore, the Rasch logit scores for future teachers and educators were 9.15 and 8.79 respectively, which were both significantly lower than 10 ($p < 0.01$). This class believed that mathematics abilities were amenable to change and did not agree with the existence of categorical differences. The label "incremental-view-endorsed" was suitable to describe the profiles of this class.

Future teachers' and educators' belief profiles of mathematics abilities were highly homogeneous no matter whether in class 1 or class 2. No differences between average scores of the two cohorts in either group reached both significant level and a medium ES.

3.3.2 Homogeneity and Heterogeneity of Beliefs Across Countries

The overall relative sizes of class 1 and class 2 were 51 % and 49 % respectively. The entity-view-endorsed belief profiles and the incremental-view-endorsed belief profiles were each supported by approximately half of the future teachers and the educators, showing the belief structures were dominated by variability internationally.

In 12 of the 13 countries with both kinds of research subjects, the dominant belief profiles of future teachers and of educators belonged to the same class, showing the relatively high homogeneity across the two cohorts in these 12 countries (see Table 5). The incremental-view-endorsed and the entity-view-endorsed belief profiles were more prevalent in seven and five countries respectively.

Among the Western participating countries, countries among the 10 highest HDI in Europe, namely Germany and Switzerland, and an American country, namely Chile, were dominated by the incremental-view-endorsed belief profiles, while in the formerly Soviet-led countries, namely Russia, Georgia, and Poland, more subjects were of the entity-view-endorsed belief profiles.[5] Considering the East Asian countries, in the two Confucian heritage countries, namely Taiwan and Singapore, the incremental-view-endorsed belief profiles were more prevalent, but in the other three countries, namely Malaysia, the Philippines, and Thailand, the entity-view-endorsed belief profiles were more prevalent. The explanation might relate to the fact that, in Confucian culture, students were expected to study hard to excel academically, even by themselves (Tan and Yates 2011). They possibly experienced how the effort they put in determined their success or failure in mathematics.

[5] Future teachers in Norway and the United States were also of the incremental-view-endorsed belief profile.

Table 5 The proportions of classes for the belief about mathematics abilities in each country

Country	Future teachers		Educators	
	Class 1	Class 2	Class 1	Class 2
Germany	11.2 (1.7)	88.8 (1.7)	13.9 (2.1)	86.1 (2.1)
Switzerland	12.2 (2.5)	87.8 (2.5)	13.2 (4.7)	86.8 (4.7)
Chile	25.0 (1.6)	75.0 (1.6)	14.3 (1.8)	85.7 (1.8)
Taiwan	44.2 (2.9)	55.8 (2.9)	41.3 (3.6)	58.7 (3.6)
Singapore	45.6 (2.5)	54.4 (2.5)	39.2 (5.6)	60.8 (5.6)
Malaysia	86.5 (2.1)	13.5 (2.1)	75.1 (7.8)	24.9 (7.8)
Philippine	83.8 (2.4)	16.2 (2.4)	69.7 (4.8)	30.3 (4.8)
Thailand	71.2 (1.7)	28.8 (1.7)	75.4 (2.4)	24.6 (2.4)
Botswana[a]	73.1 (6.4)	26.9 (6.4)	57.9 (13.7)	42.1 (13.7)
Georgia	84.1 (3.7)	15.9 (3.7)	80.0 (7.4)	20.0 (7.4)
Oman	63.5 (3.4)	36.5 (3.4)	80.4 (5.6)	19.6 (5.6)
Russia	65.4 (1.8)	34.6 (1.8)	57.6 (2.2)	42.4 (2.2)
Poland	52.0 (3.5)	48.0 (3.5)	70.4 (2.8)	29.6 (2.8)
Norway	15.4 (2.0)	84.6 (2.0)		
US-Public	17.7 (3.6)	82.3 (3.6)		

The numbers in the parentheses indicate *SE*

[a]The sample of educators did not reach 30 so that it might not be appropriate to use the data for firm conclusions

3.4 Homogeneity and Heterogeneity of Beliefs Across Countries from a Holistic Perspective

This study employed hierarchical cluster analysis (HCA) to identify the homogeneous cohorts of participating countries based on their belief structures, referring to the constituent proportions of belief profiles, of all three aspects of mathematics-teaching-related beliefs. The Average Linkage method was used to determine the distances (Euclidean distance) between clusters. Through performing HCA with *SPSS*, the distances were rescaled to 0–25 units and the final merging step clustered all cohorts of all countries into one cluster at the distance of 25. A smaller distance indicated a higher level of homogeneity.

Figure 4 shows the structures of future teachers' and educators' mathematics teaching-related beliefs for all the participating countries. It can be seen that the two large clusters formed next to the final merging step grouped all cohorts in the same countries into the same cluster except Botswana. The case of Botswana was unique, as its educator cohort first grouped with the Russian future-teacher cohort and then with many other cohorts in other countries before it linked with its future-teacher cohort. Considering the small sample size of Botswana educators, further study is

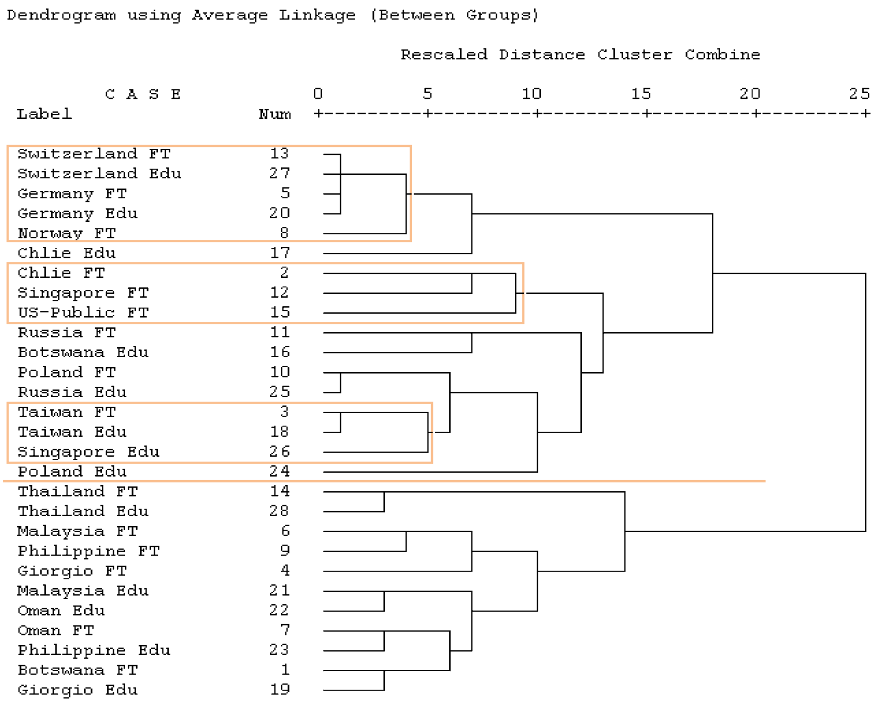

Fig. 4 Hierarchical cluster analyses based on belief structures. FT = future teachers; Edu = educators

required before any firm conclusions can be made. For the following discussion, we will put Botswana aside.[6]

Basically, the first large cluster identified through HCA is shown in the top half of Fig. 4. This linked the country cohorts at a distance of 18, and the other cluster linked the rest at 14. The first cluster included almost all Western countries, which might relate to their sharing of the Greek/Latin/Christian tradition. Those countries were the three countries among the 10 highest HDI in Europe (Switzerland, Germany, and Norway), the two formerly Soviet-led European countries (Russia and Poland), and the two American countries (Chile and the United States). In addition, two Confucian heritage countries, Taiwan and Singapore, also joined this cluster, showing their heterogeneity from other Asian participating countries and homogeneity with Western countries in mathematics-teaching-related beliefs. The second cluster included all the remaining Asian countries, those without Confucian traditions (Thailand, Malaysia, and the Philippines), the country in the Arab world (Oman), and the remaining formerly Soviet-led country (Georgia).

Observation also reveals many interesting distinctions between these two large clusters. If we put Russia aside, we can see that all countries with very high HDI

[6]The number of secondary level educators from Botswana in the study is 18.

fall in the first cluster and all others with high HDI or median HDI fall in the second cluster. Since the HDI is a composite of educational attainment, long and healthy life, and standard of living, our HCA results may reflect that countries with homogenous human well-being share homogenous mathematics-teaching-related beliefs.

Another perspective from which to view these two large clusters relates to the geographical areas. All the countries in the second cluster are located in Asia, either Southeast Asia, Western Asia, or Southwest Asia. On the other hand, all the European countries and American countries fall in the first cluster. This result may indicate a geographical effect on shaping mathematics-teaching-related belief. However, the two Asian countries, Singapore and Taiwan, falling in the first cluster prompts a need for scrutinizing other influential factors.

The knowledge achievement serves as another lens through which to compare the two clusters. All higher-achieving countries (Hsieh 2012), meaning those with the means in MCK and MPCK surveys of TEDS-M higher than the international average of 500, fall into the first cluster. This result suggests that people experiencing similar difficulty level of mathematics or putting considerable efforts and perseverance into studying may share certain homogenous beliefs.

The smaller clusters formed through the HCA also reveal certain information. Germany, Switzerland, and Norway are linked at a distance smaller than 4, which indicates that the belief structures of these countries are quite similar. These three countries share several cultural features, including geographical area, the Greek/Latin/Christian tradition, and the level of human development, which makes the linkage reasonable. By adding another common feature, high achieving in knowledge achievement, Germany and Switzerland link at the smallest value 1.

Future teachers of the two American countries and Singapore link together at a distance of 9 before they link with other countries. The linkage of the two American countries may be associated with their geographical locations. The reasons for Singapore's inclusion require more scrutiny. As pointed out by Sim (1991), Singapore, once being a British colony, continued to look towards the West to learn the models for implementing teacher education after attaining independence from Britain (Sim 1991). The United States, having a root from Europe and being also once partially colonized by Britain, has a historical tradition relating to Britain; these historical backgrounds may explain why the two countries' future teachers linked early in HCA. Before the two Singaporean cohorts linked together, Singaporean educators and the two Taiwanese cohorts linked at a distance of 5. Although further study is needed, an explanation related to historical roots and future tendency is that Singaporean educators belong to the older generation with stronger influence by Confucian ideology, while the future teachers belong to a younger generation among whom traditional Confucian values are found to be increasingly fading (Wang et al. 2005).

4 Conclusion and Discussion

Using the data sets from the TEDS-M study, this study identified several belief profiles across three crucial aspects of mathematics-teaching-related beliefs: the beliefs about the nature of mathematics, about teaching and learning mathematics, and about mathematics abilities. The data of two cohorts, future teachers and teacher educators from the lower-secondary level TEDS-M study, were available to analyze in 15 and 13 countries respectively.

For each of the three aspects of mathematics-teaching-related beliefs, future teachers' and educators' profiles identified through multi-sample LPA were similar, indicating the international homogeneity of the beliefs of the two cohorts. Furthermore, in most countries, the prevalent belief profiles of future teachers and educators were the same, showing within-country homogeneity of beliefs between the two cohorts. One of the examples for these results is the consistent endorsement of the process-of-inquiry view and the active-learning view across the cohorts and the countries.

A hierarchical cluster analysis (HCA) was performed to group countries with relatively homogeneous beliefs together. The results show that all future teachers and educators are classified into the same large clusters (at a distance of 18, except Botswana). The HCA classifies all seven Western participating countries which share the Greek/Latin/Christian tradition in the same large cluster mentioned. The two Asian countries having Chinese/Confucian cultural background are relatively more homogeneous in belief. They are grouped first (at a distance of 13) compared with the other East Asian countries without Confucian heritage. Besides geographical location and historical traditions, this study also found that human development index (HDI) and knowledge achievement are two possible cultural features influencing the classification. Among these cultural features, which is more influential than another or how they intertwine with each other for shaping the belief clusters needs further study. However, the fact that, through simultaneously examining all the three aspects of beliefs, in most countries future teachers and educators in the same country are grouped early may indicate the homogeneity of belief structures between these two cohorts. This suggests that *country* may be the most crucial factor for shaping mathematics-teaching-related beliefs.

Although all seven higher-achieving countries were in the former cluster, they deviated regarding the dominant profiles of the beliefs about the nature of mathematics and mathematics abilities. However, regarding the beliefs about teaching and learning mathematics, all of these countries were dominated by the active-learning-preferred profiles. The prevalence of these belief profiles might relate to the future teachers' knowledge performance. Another intriguing phenomenon is that the educators of the three countries ranked the top three on MCK and MPCK, which were Taiwan, Russia, and Singapore, link together at a distance of 6. The distance is small when compared with the distance 12 that links their future teachers. That is, the belief structures of these educators were more homogeneous than their future teachers'. Whether and how the educators' beliefs influence future teachers' knowledge achievement, or vice versa, requires further study.

This chapter has pointed out many open questions relating to the beliefs about mathematics teaching; the homogeneity and heterogeneity around cultural tradition is the key notion after all. The study would not have been possible without the international comparison study TEDS-M. In the wave of globalization, future international comparison studies are vital to examine the future influence of cultural changes on belief profile changes and the underlying educational meanings.

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An Examination of Future Primary Teachers Attitudes About the Teaching of Mathematics: An International Perspective

Nathan Burroughs and William Schmidt

Abstract In this paper we explore the dynamics of teacher beliefs about mathematics with a special focus on future primary teachers. After reviewing earlier research about teacher beliefs, with special attention to the MT21 study and other work based on TEDS data, we examine the relationship among the different dimensions of teacher beliefs and the extent to which these beliefs are associated with teacher knowledge. We find considerable average variation in teacher beliefs about teaching and learning mathematics across countries, but find that most of the variation in beliefs is at the individual level. By contrast, teacher preparation programs appear to play little role in shaping beliefs. Employing multi-level modeling, we also find that teacher beliefs have a statistically significant and substantively important association with future primary teachers' knowledge of mathematics. Finally, our results raise questions about the cross-national validity of a sharp constructivist-traditionalist dichotomy.

Keywords Primary teacher · Attitude · Mathematics · TEDS-M · Belief · Teacher knowledge · Nature of mathematics · Content knowledge · National culture

Teachers are a key component of any educational reform. Most efforts to improve instruction have acknowledged the importance of teacher skills, organization, and support, but teacher *attitudes* also serve a critical role in student learning. Teacher attitudes influence the outcome of policies in relatively direct ways, of course. As principle stakeholders in schools and the ones most responsible for implementing curricular changes, their reactions can make or break a policy. A teaching force that is hostile to a policy, or accepts it only grudgingly, can spell disaster.

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S. Blömeke et al. (eds.), *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn*, Advances in Mathematics Education,
DOI 10.1007/978-94-007-6437-8_13,

The attitudes of teachers about teaching itself can have major consequences too. What teachers believe about the content of what they teach, the best way of teaching it, and what students are capable of learning—all have powerful effects on what occurs in the classroom. Teachers are not passive instruments that neutrally convey information, but active participants in the process of educating students, and so their predispositions condition the success or failure of all educational reforms.

In this paper we explore the dynamics of teacher beliefs about mathematics with a special focus on future primary teachers. Secondary school mathematics instructors tend to be specialists in their fields. Given their more intensive exposure to mathematics, their attitudes about mathematics instruction may be quite different from those of primary school teachers, who are responsible for giving basic instruction in many subjects. As generalists with what may be only a smattering of math courses during their preparation to become teachers, the cultural background of primary school teachers could play an especially large role in shaping their beliefs. In addition, as their first exposure to formal mathematics, students' attitudes about math may be powerfully influenced by the beliefs of their elementary school teachers.

Our analysis is based on data drawn from the TEDS study, which contains a large international sample of future primary teachers and includes a series of questions about teacher beliefs. After reviewing earlier research about teacher beliefs, with special attention to the MT21 study and other work based on TEDS data, we examine the relationship among the different dimensions of teacher beliefs and the extent to which these beliefs are associated with teacher knowledge. Of major interest is evaluating how national culture might shape these relationships.

1 Previous Research on Teacher Beliefs

The study of teacher beliefs is fraught with difficulties. In some respects this is because studying beliefs of any sort (rather than behaviors) is an inherently tricky exercise. Beliefs are internal characteristics that people possess, and therefore very hard to measure validly. Asking people to describe their beliefs relies on the honesty, clarity, and self-knowledge of the respondent, while having people respond to previously defined categories risks having them mangle their (actual) beliefs in order to fit the framework.

Sometimes the expressed beliefs of teachers may seem contradictory or ill-formed, but as Leatham (2006) argues, we should accept this ambiguity and treat these beliefs as "sensible" if not necessarily coherent. Understanding the beliefs of teachers is a particular problem, not least of which because scholars have not always been clear what they mean by the term "belief" (Philipp 2007). As noted by Pajeres (1992) and Philipp (2007), there is not a clear consensus on how to define teacher beliefs. Philipp (2007) attempts to untangle the differences between the many similar terms (affects, beliefs, conceptions, knowledge, value, etc.), and in this paper we will try to follow his working definition:

> Psychologically held understandings, premises, or propositions about the world that are thought to be true. Beliefs are more cognitive, are felt less

intensely, and are harder to change than attitudes. Beliefs might be thought of as lenses that affect one's view of some aspect of the world or as dispositions toward action. Beliefs, unlike knowledge, may be held with varying degrees of conviction and are not consensual. Beliefs are more cognitive than emotions and attitudes.

Given the difficulties in defining "beliefs," it should be no surprise that there are a host of different means of conceptualizing beliefs about mathematics (Thompson 1992) and scales for measuring them (Philipp 2007). Ernest (1989) has developed an influential categorization of different sorts of beliefs: beliefs about the nature of mathematics, beliefs about the teaching of mathematics, and beliefs about the learning of mathematics. Beliefs about the nature of mathematics have been broken up into several distinct conceptions by Grigutsch et al. (1998) that reduce to four basic types. Quoting Schmidt et al. (2011)'s, mathematics is viewed as:

A *creative* science that consists of discovery and problem-solving
A *useful* science that can be applied to society and life
A *formal* and logical science that has an axiomatic basis and develops by deduction
An *algorithmic* science that represents a collection of terms, formulas, and rules

These four perspectives can be further collapsed into two broader conceptions: mathematics as a static perspective, characterized by a view of mathematics as a formal, exact science bound by set rules and procedures; and a dynamic perspective that sees mathematics as a process of problem-solving that can be readily applied in daily life (see for more details Chap. "The Cultural Dimension of Beliefs: An Investigation of Future Primary Teachers' Epistemological Beliefs Concerning the Nature of Mathematics in 15 Countries" by Felbrich et al. in this book).

Beliefs about the best means for teaching and learning mathematics involve a rich literature of mathematical pedagogy, which we will only brush upon here. In a review of the literature at the time, Kuhs and Ball (1986) laid out four basic approaches to the teaching of mathematics: learner-focused, content-focused with an emphasis on concepts, content-focused with an emphasis on performance, and classroom-focused literature. Much of the research on the pedagogical beliefs of mathematics teachers emphasizes a learner-focused approach, or what Peterson et al. (1989) dubs a "cognitively based perspective." This student-centered approach is closely related to the long-running debate about traditional/transmission vs. constructivist education (Barkatas and Malone 2005; Howard et al. 1997; Handal 2003; Raymond 1997), or the similar conceptual/calculational dichotomy of Thompson et al. (1994).This cognitive approach to studying beliefs is quite natural, given that the study of beliefs itself has a psychological orientation.

A presumption in studying teacher beliefs is that orientations towards the nature, teaching, and learning of mathematics may be related to one another or to educational practice. There is a plausible connection between a dynamic view of mathematics and a constructivist approach to teaching, for example. Empirical research suggests that there may be some link between what teachers believe about the nature of mathematics and what they believe about the teaching of mathematics

(Beswick 2005; Stipek et al. 2001; Barkatas and Malone 2005, but see Cross 2009, Yates 2006), although the only study focusing on future primary school teachers failed to find such a link (Yates 2006). However, beliefs about mathematics teaching is more directly connected to actual day-to-day instruction (Raymond 1997). Further, some researchers have found students with teachers adopting constructivist orientation may experience greater learning gains (Staub and Stern 2002; Peterson et al. 1989).

Assuming for the moment that the beliefs of teachers are related to their performance and ultimately to how their students learn, an important question is how malleable these beliefs are, and in particular whether teacher preparation programs can help foster the "right" beliefs about mathematics. Here the research suggests both good news and bad news. On a positive note, some research suggests that interventions can modify teacher attitudes. Hart (2002), Kajander (2007) and Gill et al. (2004) found that classroom-based interventions could move teacher beliefs in a more constructivist direction, while Akiba (2011) found that pre-service coursework could improve multicultural awareness. Field experiences may also encourage learner-centered perspectives (Ambrose 2004).

Despite these encouraging results, other scholars caution against expecting too much of pre-service interventions. Pajeres (1992) notes that attitudes about teaching and learning are formed early and are quite durable. Van Zoest et al. (1994) and Handal (2003) also highlight the contextual influences on teacher attitudes. Once teachers enter the workforce, their attitudes may revert to more traditional ones due to environmental pressure, or may find it difficult to translate their beliefs into practice. Notably, nearly all of these studies focused on future primary teachers.

On the whole the empirical literature on teacher beliefs about mathematics is rather thin. Much of it is based on relatively small sample sizes with limited geographic scope. The most important exception to these limitations are the Mathematics Teaching in the 21st Century (MT21) and the Teacher Education Study in Mathematics (TEDS-M). As described in Schmidt et al. (2007, 2011), MT21 surveyed approximately 2600 future teachers at 34 institutions across 6 countries. MT21 surveyed primary, middle, and lower secondary future teachers.

Along with testing teachers' content and pedagogical knowledge and asking about course-taking, the MT21 study also included a number of items regarding teachers' beliefs about the nature, teaching, and learning of mathematics. The study followed the 4-fold Grigutsch typology about the nature of mathematics, inquiring about teachers' orientation was algorithmic, useful, creative, or formal. Generally speaking, across countries future teachers inclined towards the "dynamic" (useful & creative) perspective, but found significant differences across countries. Taiwan, Korea, and Bulgaria future teachers adhered to all four conceptions at once, with the Taiwanese the most in favor of an algorithmic view of mathematics and German future teachers the least in favor. The US was something of an outlier, being the only country with more support for the algorithmic than formal conception. In addition, US elementary and middle school future teachers were more supportive of an algorithmic conception than were secondary future teachers.

With respect to beliefs about learning mathematics, the MT21 study asked questions tapping into five basic notions: the use of standard procedures, focusing on the right answer, mastery of skills, gaining understanding, and independent work by students. There was general agreement that students should try to gain understanding and work independently. Taiwanese future teachers had the strongest support for using standard procedures. The MT21 study also probed ideas about whether all students were capable of learning math. In most countries there was resistance to the idea that mathematics was based on natural ability (particularly in Germany the United States), and to the importance of gender and race (especially in Germany), with the exception of Taiwanese future teachers.

Building on MT21, the TEDS included a much larger sample of nearly 23,000 future teachers in 498 institutions across 16 countries (Tatto et al. 2012; Schmidt et al. 2013). The TEDS reduced the number of items related to teacher beliefs, compressing the number of dimensions into five: beliefs about the nature of mathematics (as a set of rules and procedures or as a process of inquiry), about learning mathematics (through teacher direction or through active involvement), and about mathematics achievement (whether mathematics is a fixed ability).

The TEDS found substantial variation across countries in beliefs about the nature of mathematics, and with primary, middle, and lower secondary teachers generally evincing similar beliefs within the same country. All countries' future teachers embraced the notion of mathematics as a process of inquiry and should be learned actively, and opposed the idea that mathematics is a fixed ability. There was much more variation across countries about whether mathematics is a set of rules and procedures and whether learning is best when directed by teachers. Using a simple correlational analysis, countries whose future teachers had more conceptual beliefs about mathematics (active learning, process of inquiry) generally had higher mean scores on the mathematical content knowledge (MCK) and mathematics pedagogical knowledge (PCK) tests than those with calculational perspectives (a set of rules and procedures, teacher direction).

Felbrich et al. (see Chap. "The Cultural Dimension of Beliefs: An Investigation of Future Primary Teachers' Epistemological Beliefs Concerning the Nature of Mathematics in 15 Countries" in this book) delved deeper into the TEDS data on future primary teachers' beliefs, with a specific emphasis on the static/dynamic dichotomy about the nature of mathematics. They noted a broad range of opinion within countries (as measured by standard deviations). After combining "math as a process of inquiry" and "math as a process of rules and procedures" into a single scale using ipsative values, the authors conducted a two-level analysis (country and individual) examining the dependence of teacher beliefs on mathematical content knowledge, previous school achievement, and the individualism of each country's culture. Higher-performing future teachers were found to have more dynamic attitudes about mathematics, while country individualism had a marginal effect (controlling for other factors).

2 Empirical Examination of Future Primary Teacher Beliefs

We add to these results by making use of the TEDS data set to explore the structure and impact of future primary teacher beliefs in a detailed way. As a preliminary step we describe the TEDS data to evaluate two different scales of beliefs and present descriptive data about these indices. Two main questions serve to structure our analysis. First, what is the relationship among the different dimensions of teacher beliefs? Second, what is the relationship between teacher beliefs and teacher knowledge? An important theme underlying both questions is the degree to which these relationships vary between and within countries.

2.1 Using TEDS Data to Examine Teacher Beliefs

A prerequisite to addressing all of these questions is resolving the problem of how to conceptualize teacher beliefs. The TEDS represents a considerable advance on earlier efforts given its large sample size and international character, but the design of the survey imposes certain limitations. The TEDS allows us to compare within and between country beliefs with a fairly high degree of precision. However, in designing the survey the authors of TEDS selected a smaller pool of items than existed within the previous MT21 study. The TEDS survey comprises the same three basic categories of beliefs as MT21 (nature of mathematics, learning of mathematics, beliefs about mathematics achievement), but reduced the number of distinct dimensions from twelve to five, and had only 33 belief items rather than the original 44 (a 25 % reduction).

In our analysis, we re-constructed the original MT21 scales using those items that remained in the TEDS. These scales are only rough estimates of the indexes as they would have manifested if the entire bank of MT21 belief items had been included. Some dimensions are at greater risk than others. For example, both of the mathematics achievement indices were essentially intact, but the formalism index (within the "Nature of Mathematics") had only one item as opposed to the original 5. It should therefore be no surprise that the reliabilities of some of the MT21 indices are lower than we would like. While the "nature of mathematics" indices (except for formalism, which had only one surviving item) and the "natural ability" element of mathematics achievement have Cronabch's alphas of about 0.8, the reliabilities of the "learning mathematics" beliefs are only about 0.6. The TEDS scales have a higher reliability (between 0.7 and 0.8), and also performed fairly well when re-created using MT21 data, but as simplified expressions of teacher beliefs may be missing more nuanced elements. While illuminating, both indices are only approximations of the structure of teacher beliefs.

A second restriction present in both the MT21 and TEDS studies is that they do not map perfectly onto the three types of beliefs as developed by Ernest and heavily employed in the literature. While TEDS and MT21 include the "nature of mathematics" category, the teaching of mathematics and learning of mathematics concepts have been partly combined into one group, while a component of learning mathematics has been separated into a different area related to beliefs about whether all students are capable of learning mathematics. While there it is certainly logical to posit a close connection between what a teacher believes about teaching and what he or she believes about learning, it is open to question whether this relationship is as tight as one might suppose. Any direct application to other empirical literature should therefore be treated with caution.

Mean values of both sets of indices are presented for all nations and by country in Table 1. Each index is the mean of responses to the items within each dimension, weighted by respondent. Each item posed a question rated on a 1 to 6 scale, with higher scores indicating greater agreement. We reproduced the TEDS scales using means rather than IRT scaling because of data limitations, but there was a very high correlation between the two (over 0.9). The mean scores for the TEDS scales are of course quite similar to that presented in the TEDS report, with more agreement with the concept of mathematics as a process of inquiry acquired through active learning. Math as a fixed ability and learning through teacher direction received much less support. Interestingly, this method of aggregating responses found nearly as much endorsement of math as a system of rules as it did for a process of inquiry.

The MT21 report sampled all three populations of future teachers and the report presented pooled results across grades, so it is difficult to make precise comparisons between the MT21 and TEDS samples for only future primary teachers. A few countries (Taiwan, Germany, and the United States) participated in both studies, and the mean responses using the smaller-item indexes in the TEDS sample are fairly close, despite the fact that it compares primary teachers in one sample to all teachers in the other. For the reproduced MT21 scales, we found considerable support for all four conceptions of the nature of mathematics (global means ranging from 4.4 to 4.9), the importance using different approaches (4.7) and student understanding (5.1). There was much less support for other beliefs. For both sets of scales there was appreciable variation in average beliefs by country, with a range in mean responses between 1.1 and 1.8.

Country-level averages reinforce the point that primary future teacher beliefs are partly conditioned by cultural context (see Chap. “The Cultural Dimension of Beliefs: An Investigation of Future Primary Teachers’ Epistemological Beliefs Concerning the Nature of Mathematics in 15 Countries” by Felbrich et al. in this book). There is considerable within-country variation in future teacher beliefs as well. The variation in beliefs within countries was about one standard deviation for the pooled sample (and a cross-country mean standard deviation of 1.5). The US was a clear outlier at around 4 standard deviations, but this might be due in part to its much larger sample size. Differences in teacher preparation programs, either through selection effects or a different approach for training future teachers, could also account for the variation in teacher beliefs. In an attempt to sort out how much variation is due to country and institution-level effects, we performed a three-level variance decomposition analysis for the 5 TEDS and 11 MT21 belief scales.

The results presented in Table 2 suggest that although country-level effects have a substantial influence on the variation in future primary teacher beliefs (explaining

Table 1 Mean beliefs about mathematics

Beliefs dimension	Scales	All countries	Botswana	Chile	Georgia	Germany	Malaysia	Norway	Philippines	Poland	Russsia	Singapore	Spain	Switzerland	Taiwan	Thailand	USA
Nature of mathematics	Algorithmic	4.4	5.2	4.5	4.4	3.8	5.0	3.9	5.5	4.4	4.3	4.6	4.4	3.9	4.4	5.1	4.6
	Usefulness	4.9	5.5	5.4	3.9	4.5	5.3	5.0	5.4	4.7	4.7	5.2	5.2	4.5	4.8	5.2	5.3
	Creative	4.8	5.3	5.1	3.9	4.6	5.2	4.9	5.5	4.5	4.6	5.0	4.9	4.8	5.1	5.3	4.9
	Formalism	4.6	4.5	4.5	4.3	3.8	4.8	4.2	5.2	4.8	5.2	4.9	4.7	3.7	5.0	4.9	4.3
Teaching mathematics	Products	2.4	2.3	2.7	3.4	2.0	3.6	1.9	3.7	2.3	2.5	2.0	2.0	1.9	2.2	2.2	2.2
	Different approaches	4.7	4.9	5.1	4.0	5.0	4.3	4.7	4.7	4.8	4.8	4.5	4.5	5.1	5.0	4.6	4.5
	Algorithms	3.2	3.4	3.0	4.0	2.8	4.1	2.7	4.4	3.4	3.6	3.3	2.8	2.6	2.8	2.9	3.1
	Standard procedures	2.6	2.8	2.9	3.5	2.2	3.9	2.0	3.8	2.7	2.6	2.3	2.3	2.1	2.2	2.6	2.0
	Understanding	5.1	5.1	5.3	4.3	5.1	4.8	5.3	5.3	5.1	5.0	5.3	5.2	5.2	5.1	5.3	5.4
Nature of ability	Categorical differences	2.8	2.9	2.3	3.8	2.2	3.6	2.5	3.7	3.0	2.9	2.4	2.0	2.2	3.4	3.6	2.3
	Natural ability	3.6	3.8	3.3	4.2	3.2	4.4	2.9	4.7	4.1	4.0	3.2	3.3	2.9	3.5	4.1	2.8
Learning mathematics	Enquiry	4.9	5.5	5.2	4.0	4.7	5.2	5.0	5.5	4.7	4.7	5.1	5.1	4.8	5.0	5.3	5.1
	Rules	4.5	5.1	4.5	4.4	3.8	5.0	4.0	5.4	4.5	4.5	4.6	4.4	3.8	4.5	5.1	4.6
	Active	4.9	5.0	5.2	4.1	5.1	4.6	5.0	5.0	5.0	4.9	4.9	4.9	5.2	5.1	4.9	5.0
	Directions	2.9	3.0	3.1	3.8	2.4	4.0	2.3	4.1	3.0	3.1	2.7	2.6	2.3	2.6	2.6	2.6
	Fixed	3.2	3.4	2.8	4.0	2.8	4.1	2.7	4.1	3.6	3.6	2.9	2.7	2.6	3.2	3.7	2.5
	N	14633	86	657	506	1032	576	551	592	2112	2266	380	1093	936	923	660	2263

Table 2 Variance decomposition of teacher beliefs

	Institution	Country	Individual
Enquiry	4.1 %	21.5 %	74.4 %
Rules	6.5 %	27.7 %	65.8 %
Active	4.8 %	14.6 %	80.6 %
Directions	7.4 %	38.1 %	54.5 %
Fixed	4.5 %	35.2 %	60.3 %
Algorithmic	7.2 %	28.0 %	64.8 %
Usefulness	3.3 %	19.1 %	77.6 %
Creative	4.6 %	20.3 %	75.1 %
Formalism	2.3 %	16.0 %	81.7 %
Products	5.0 %	24.0 %	71.0 %
Different approaches	6.0 %	13.8 %	80.2 %
Algorithms	8.4 %	27.4 %	64.2 %
Standard procedures	4.9 %	27.2 %	67.9 %
Understanding	3.1 %	13.5 %	83.5 %
Categorical differences	4.9 %	21.6 %	73.5 %
Natural ability	4.0 %	30.4 %	65.6 %

from 14 % to 38 % of the total variance), most of the variation was in fact attributable to student-level differences (55 % to 84 % of total variance). The impact of teacher preparation institutions was slight (2 % to 7 %). One salient finding is that the "constructivist" beliefs tended to have far more of the variation explained at the individual level, whereas "traditional" beliefs tended to have a greater proportion of variance explained by country-level influences.

2.2 The Relationship Among Beliefs

The relationship among dimensions of primary teacher beliefs includes two different considerations. First, there is the methodological concern about the extent to which the MT21 and TEDS indexes tap into the same phenomena—in short, whether the simplified TEDS typology adequately captures the range of teacher beliefs. Second, and more substantively important, the connection of different categories of beliefs to each other touches upon one of the most contested issues in mathematics education research, as well as the validity of a great deal of research related to teacher beliefs: the distinction between a more constructivist or more traditional approach to mathematics education.

Some relationship between the MT21 and TEDS beliefs scales is to be expected, given that the TEDS indices were based on the MT21 approach and include many of the same items, but also maintain the integrity of MT21 concepts: although the number of items were slimmed down, MT21 belief dimensions were not broken up across TEDS categories. For the nature of math indices, the algorithmic and formalist views were combined into the math as rules and procedures concept, while math as a creative science and math as a useful activity were combined into the concept math as a process for inquiry. Among the two "beliefs about learning math" indexes, "teacher direction" included elements of the "algorithms," "focus on products," and "standard procedures" scales; "active learning" comprised "different approaches" and "understanding" items. Finally, the "math as fixed ability" index in TEDS incorporated questions from the "categorical differences" and "natural ability" scales.

Correlation analysis indicated that most of the MT21 scales are strongly related to the relevant TEDS scale (see Table 3). "Math as a Process of Inquiry" was strongly correlated to the Usefulness and Creative scales (0.84, 0.88), as was math as active learning to the understanding (0.78) and different approaches (0.87). The association of fixed beliefs about mathematics with categorical differences (0.77) and natural ability (0.92), and of a directive orientation with standard procedures (0.73), products (0.77), and algorithms (0.84) was also quite strong. Finally, there was almost perfect collinearity between a rule-based outlook and an algorithmic perspective (0.98), but less overlap with formalism (0.62). However, because the MT21 items were truncated, the index scores could be somewhat biased towards alignment with TEDS scales.

As should be evident from the literature review, the study of teacher beliefs has been closely connected to the debate over whether a broadly constructivist or traditional approach to mathematics instruction is to be preferred. A plurality of the researchers studying teacher beliefs appears to support the idea of a more active, learner-centered, cognitive pedagogical strategy. Underlying this debate is the assumption that dynamic attitudes about the nature of mathematics and the belief that math is best learned through a process of active learning exists at the opposite end of a continuum from beliefs that mathematics is a static discipline that should be taught under the direction of teachers. The presumption therefore is that individuals (or countries) that generally support one sort of belief will oppose the other.

However, the relationship between the two different beliefs about the nature of math (math as rules and math as inquiry) do not appear to be contrary, at least according to TEDS data. In fact, there was virtually no relationship between the two dimensions, with a (very weakly) positive correlation of 0.06. When responses for the entire TEDS sample of 16 countries were pooled together, static and dynamic conceptions appeared to be orthogonal to each other, rather than inversely related to each other.

The weak relationship between static and dynamic conceptions of mathematics was replicated using MT21 scales: formalistic and algorithmic beliefs about the nature of mathematics had the same very low correlation with the usefulness (0.01, 0.11) and creative (0.04, 0.02) dimensions. Further, the relationship between the concepts of mathematics learning and the nature of mathematics are broadly "conceptual"—the belief that mathematics requires active learning and that it is a process of inquiry, were only moderately related (0.45). The correlation between the "calculational beliefs"—math as rules and learning through teacher direction—was identical (0.45). These results suggest a link between beliefs about the nature

Table 3 Correlations among teacher beliefs

	Algo-rithmic	Usefulness	Creative	Formal-ism	Products	Different approaches	Algorithms	Standard procedures	Under-standing	Categorical differences	Natural ability	Enquiry	Rules	Active	Direc-tions	Fixed
Algorithmic	1.00	0.11	0.02	0.45	0.19	−0.17	0.53	0.21	0.09	0.21	0.28	0.06	0.98	−0.06	0.43	0.26
Usefulness	0.11	1.00	0.55	0.01	−0.13	0.20	−0.12	−0.18	0.40	−0.14	−0.26	0.84	0.10	0.34	−0.16	−0.25
Creative	0.02	0.55	1.00	0.04	−0.10	0.34	−0.16	−0.14	0.39	−0.09	−0.17	0.88	0.03	0.44	−0.16	−0.16
Formalism	0.45	0.01	0.04	1.00	0.11	−0.03	0.34	0.19	0.04	0.19	0.33	0.04	0.62	0.01	0.30	0.31
Products	0.19	−0.13	−0.10	0.11	1.00	−0.09	0.47	0.46	−0.26	0.30	0.38	−0.14	0.19	−0.20	0.77	0.43
Different approaches	−0.17	0.20	0.34	−0.03	−0.09	1.00	−0.16	−0.14	0.37	−0.04	−0.03	0.32	−0.15	0.87	−0.17	−0.04
Algorithms	0.53	−0.12	−0.16	0.34	0.47	−0.16	1.00	0.42	−0.12	0.32	0.47	−0.17	0.54	−0.17	0.84	0.49
Standard procedures	0.21	−0.18	−0.14	0.19	0.46	−0.14	0.42	1.00	−0.29	0.34	0.44	−0.20	0.23	−0.25	0.73	0.50
Understanding	0.09	0.40	0.39	0.04	−0.26	0.37	−0.12	−0.29	1.00	−0.17	−0.20	0.45	0.09	0.78	−0.25	−0.25
Categorical differences	0.21	−0.14	−0.09	0.19	0.30	−0.04	0.32	0.34	−0.17	1.00	0.56	−0.13	0.23	−0.11	0.41	0.77
Natural ability	0.28	−0.26	−0.17	0.33	0.38	−0.03	0.47	0.44	−0.20	0.56	1.00	−0.24	0.32	−0.13	0.55	0.92
Enquiry	0.06	0.84	0.88	0.04	−0.14	0.32	−0.17	−0.20	0.45	−0.13	−0.24	1.00	0.06	0.45	−0.19	−0.24
Rules	0.98	0.10	0.03	0.62	0.19	−0.15	0.54	0.23	0.09	0.23	0.32	0.06	1.00	−0.05	0.45	0.30
Active	−0.06	0.34	0.44	0.01	−0.20	0.87	−0.17	−0.25	0.78	−0.11	−0.13	0.45	−0.05	1.00	−0.25	−0.16
Directions	0.43	−0.16	−0.16	0.30	0.77	−0.17	0.84	0.73	−0.25	0.41	0.55	−0.19	0.45	−0.25	1.00	0.60
Fixed	0.26	−0.25	−0.16	0.31	0.43	−0.04	0.49	0.50	−0.25	0.77	0.92	−0.24	0.30	−0.16	0.60	1.00

Table 4 Correlation between static and dynamic beliefs by country

Country	Correlation
Botswana	0.19
Chile	0.08
Georgia	0.80
Germany	−0.23
Malaysia	0.78
Norway	−0.25
Philippines	0.47
Poland	0.09
Russian Fed.	0.35
Spain	−0.10
Switzerland	−0.30
Taiwan	0.16
Thailand	0.40
USA-All	−0.12
Singapore	0.19
All	0.06

of mathematics and beliefs about teaching and learning mathematics, with a more conceptual and more calculational approach as distinct families of beliefs, but ones that are not in opposition to one another.

Within-country correlations indicate that the relationship between beliefs about the nature of mathematics varied dramatically across national units (see Table 4). In some countries the relationship between these two beliefs reflected the international average, with very low correlations between math as a process of inquiry and math as a set of rules: Botswana, Taiwan, the US, Singapore, Poland, Chile, and Spain all posted correlations of 0.2 or less. However, there were a few countries—in particular Georgia—that suggested a strong positive correlation between a static and dynamic view of mathematics. The two kinds of beliefs about the nature of mathematics were modestly negatively correlated in a few countries; Switzerland, Germany, and Norway all saw negative correlations of between 0.2 and 0.3.

2.3 The Relationship Between Beliefs and Knowledge

As a study directed strictly at mathematics teacher preparation, TEDS data did not include K-12 student data, and therefore cannot be used to directly measure the impact of teacher beliefs on K-12 student performance. However, given the strong link between the mathematics knowledge of primary teachers and student learning gains (Hill et al. 2005), there may be an indirect effect of teacher beliefs on student learning via teacher knowledge. The relationship between beliefs on the one hand

and mathematical content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK) assessment results on the other hand could give a hint as to the ultimate impact of beliefs in the classroom. The connection between teacher beliefs and teacher knowledge is of particular concern at the primary school level, as elementary school teachers are rarely mathematics specialists.

We conducted a statistical analysis using a two-level model using PROC MIXED, measuring future teacher characteristics at the individual level and controlling for country-level clustering in the second level. We experimented with two outcome variables (MCK and PCK), but the high degree of correlation between these two measures led to substantively similar results and we therefore present only MCK results. We ran a series of regressions, with each belief index (both MT21 and TEDS) as the main independent variable for each regression. Treating teacher beliefs as a fixed or random effect had substantively identical results, with minimal change to parameter estimates and significance levels.

Our analysis builds on several previous studies. First, the TEDS report presented the country-level correlations between MCK and PCK on the one hand and teacher beliefs on the others. Our work therefore adds an additional level of sophistication to this analysis by incorporating country-level and individual effects into one model and incorporating a number of control variables. These control variables included a number of student-level measures. Following Felbrich et al. (Chapter “The Cultural Dimension of Beliefs: An Investigation of Future Primary Teachers’ Epistemological Beliefs Concerning the Nature of Mathematics in 15 Countries” in this book), we included the student’s self-reported typical class ranking in secondary school as a proxy for a student’s mathematical knowledge before he or she entered a teacher preparation program. As an additional background characteristic, we included the average number of books the student reported in the home, standardized within each country to adjust for differences in wealth across countries. Finally, following Schmidt et al. (2011), we controlled for the effects of program coursework by including the percentage of mathematics and general pedagogy courses taken by the student.

The results of our analysis are presented in Table 5. Our results suggest that teacher beliefs have a statistically significant and substantively important association with future primary teachers’ knowledge of mathematics. Only one of the indices (formalism) failed to register a significant effect, which may be due to it being limited to a single item in the TEDS study. Consistent with the TEDS report, the more “conceptually” oriented beliefs (math as inquiry, active learning) were associated with higher mathematics knowledge scores, and the “calculational” oriented beliefs with lower MCK scores (math as rules, directive learning), as is belief in fixed abilities.

The MT21 scales produced similar results, although with slightly smaller coefficients. Beliefs in mathematics as algorithmic, focused on the right answer, using standard procedures, and ability as categorical or natural were negatively related with the mathematics knowledge of future primary teachers. By contrast, the belief that mathematics is useful, creative, requires different approaches, and student understanding were associated with higher MCK scores. Again, these findings are

Table 5 Multilevel model of beliefs' relationship to MCK

Effect	Estimate	StdErr	Probt
Enquiry	16.15	0.92	0.000
Rules	−15.88	0.83	0.000
Active	15.26	1.05	0.000
Directive	−18.94	0.89	0.000
Fixed	−14.60	0.81	0.000
Algorithmic	−17.00	0.79	0.000
Usefulness	12.49	0.77	0.000
Creative	12.24	0.83	0.000
Formalism	−1.10	0.61	0.072
Products	−10.11	0.64	0.000
Different approaches	10.33	0.79	0.000
Algorithms	−11.42	0.70	0.000
Standard procedures	−12.75	0.68	0.000
Understanding	11.61	1.02	0.000
Categorical differences	−8.91	0.54	0.000
Natural ability	−12.46	0.68	0.000

unchanged if one uses PCK rather than MCK as an outcome, or if the relationship of beliefs to MCK is permitted to vary by country.

Estimates and significance levels for the control variables are not shown, but were statistically significant, quite consistent across models, and replicated previous results. The number of books in the home and previous performance in school were associated with higher MCK scores. The estimates for opportunity-to-learn (OTL) measures in our multi-country model were virtually identical to the US-only analysis by Schmidt et al. (2011), with each additional percentage of mathematics courses resulting in an extra half-point in MCK scores, and each percentage of general pedagogy associated with a 0.9 point decline in performance. These results strengthen the findings of the Schmidt et al. (2011) piece and suggest that OTL has a similar effect across different educational systems.

Once again we found that beliefs have disparate dynamics in different countries. In Table 6 we present the results of within-country regression analyses employing the same set of independent variables as used in the previous analysis. Although the direction of the relationship between beliefs and mathematical knowledge was consistent for those countries in which it is statistically significant, the size of the coefficients varied quite a bit across countries. In addition, there were some instances (in particular Botswana) where beliefs appeared to have little relationship with MCK scores. There was certainly a link between the beliefs and mathematical knowledge of future primary teachers, but the strength of this association was apparently conditioned by national cultures and institutions.

Table 6 Regression of the relationship between beliefs and MCK, by country

Country	Enquiry	Rules	Active	Directive	Fixed	Algo-rithmic	Useful	Creative	Forma-lism	Products	Different approaches	Algo-rithms	Standard procedures	Under-standing	Cat. diff.	Nat. ability
Botswana	7.56	−8.06	11.34	−0.01	−10.2	−6.6	0.3	7.6	−6.2	1.8*	4.5	−0.9	−0.8	9.5	−2.2	−15.7*
Chile	12.32*	−9.29*	10.59*	−15.47*	−8.5*	−9.9*	9.1*	10.3*	−1.3	−5.0*	3.5	−8.7*	−9.6*	13.7*	−5.9*	−5.8*
Georgia	2.46	3.57	4.15	2.02	0.2	3.7	1.4	3.1	1.4	−2.4	3.7	1.2	4.4	3.5	1.2	−0.4
Germany	20.06*	−9.93*	18.82*	−16.53*	−3.3	−11.5*	14.6*	13.3*	1.3	−8.1*	14.9	−7.2*	−12.1*	15.0*	−6.1*	−1.1
Malaysia	6.47*	0.76	5.96*	−5.89*	1.2	0.6	6.3*	5.6	0.8	−4.3*	2.2	−0.6	−6.1*	7.4*	0.8	0.0
Norway	26.84*	−28.77*	21.50*	−21.70*	−11.7*	−30.1*	14.4*	22.3*	−2.1	−9.2*	19.1	−12.1*	−17.8*	10.8*	−6.6*	−10.1*
Philippines	19.90*	−0.90	9.42*	−14.64*	−8.7*	−2.2	11.2*	18.0*	1.8	−7.1*	0.5	−7.5*	−10.1*	14.2*	−3.2	−7.1*
Poland	14.45*	−26.12*	16.93*	−32.18*	−18.4*	−27.6*	8.4*	13.8*	−0.8	−16.1*	15.6	−23.6*	−16.0*	10.7*	−10.2*	−14.1*
Russian Fed.	15.67*	−4.73	16.81*	−17.27*	−14.8*	−8.0*	14.0*	11.6*	14.2*	−12.5*	12.1	−8.9*	−14.4*	12.5*	−5.7*	−12.9*
Singapore	−0.59	−16.38*	−4.17	−17.36*	−9.7*	−16.6*	1.9	−3.1	−2.2	−8.9*	−6.1	−12.9*	−7.5	3.7	−8.1*	−5.9
Spain	9.89*	−13.26*	9.18*	−12.30*	−8.0*	−13.3*	6.9*	5.7*	−1.5	−6.8*	5.3	−5.4*	−7.9*	8.8*	−3.9*	−7.4*
Switzerland	9.77*	−13.27*	−4.84	−5.43	−9.9*	−13.8*	3.5	11.6*	−1.2	−7.7*	−7.1	4.7	−8.5*	−0.2	−6.1*	−6.8*
Taiwan	17.82*	−8.89*	18.98*	−24.64*	−12.2*	−8.9*	10.2*	14.8*	0.4	−14.4*	16.7	−14.1*	−14.0*	11.9*	−7.0*	−7.8*
Thailand	12.74*	−20.88*	8.46	−37.40*	−23.3*	−21.7*	5.2	6.4	−4.9	−22.9*	4.8	−28.0*	−19.8*	7.8	−14.0*	−16.2*
USA	15.27*	−17.62*	15.57*	−15.81*	−15.1*	−18.6*	13.0*	11.2*	−3.5*	−7.9*	10.0	−9.6*	−11.9*	11.3*	−10.1*	−13.7*

3 Discussion

Beliefs do not exist in a vacuum. They are both shaped by and interact with individual and social context. But given the considerable autonomy possessed by teachers in the classroom, how teachers conceive of mathematics—what mathematics is, how it is best taught and learned, and who is capable of learning it—could have a substantial influence on how their students ultimately approach mathematics. The earlier these beliefs are instilled, the greater the potential long-term effects, and hence the critical importance of understanding the beliefs of elementary school mathematics instructors.

Our analysis has yielded two principal insights. First, properly modeling beliefs is a devilishly tricky task. Drawing the proper conceptual boundaries around ideas so that they are mutually exclusive and exhaustive is a difficult enterprise in any field, but are doubly so when the meanings of these ideas vary so much across cultural contexts. The dynamic-static dichotomy about the nature of math, and the constructivist-traditional dichotomy about mathematics pedagogy, were developed in very specific cultural milieus. Our results raise questions about whether these categories are quite so distinct. It is important to remember that simply because two beliefs may be logically opposed doesn't not mean that people aren't fully capable of subscribing to both simultaneously. What might be seen from one perspective as a battle between "good" and "bad" conceptions of mathematics may in fact simply be an example of a fruitful tension between two different, worthwhile approaches.

Secondly, differences in national culture and teacher preparation programs to shaping teacher beliefs should not be overdrawn. Most variation in the beliefs of future primary teachers lies not at the national or program level, but with individuals. Further, although the link between different dimensions of belief differs across societies, there are relatively stable connections between beliefs about mathematics and mathematical knowledge. In virtually every country those future teachers who see mathematics as an engaging discipline and emphasized student understanding tended to know more about mathematics and mathematics pedagogy. Those future teachers who adopted a more rigid, didactic approach to mathematics and hewed to an essentialist view of human characteristics tended to know less. In developing interventions to improve mathematics instruction and teacher preparation, the idea that math is a living discipline rather than a collection of facts is most common among the brightest teachers, something that policymakers and researchers should probably keep in mind.

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Part IV
Does Teacher Education Matter?

Homogeneity or Heterogeneity? Profiles of Opportunities to Learn in Primary Teacher Education and Their Relationship to Cultural Context and Outcomes

Sigrid Blömeke and Gabriele Kaiser

Abstract The curriculum of teacher education has been described as heterogeneous across countries and influenced by the context in which it is implemented. The present study investigates this potential heterogeneity by conducting latent class analysis of opportunities to learn mathematics, mathematics pedagogy and general pedagogy in primary teacher education as indicated by future teachers from 15 countries at the end of their training. The aim was to identify *curricula profiles* based on data from the "Teacher Education and Development Study: Learning to Teach Mathematics (TEDS-M)". In each teacher education component, three groups of primary teachers were identified which differed quantitatively and qualitatively. Associations between these profiles and countries revealed broader cultural influences on OTL and thus shared (within a culture) and at the same time distinct (between cultures) visions of what primary teachers should know. Within countries, associations between curricula profiles and teacher education programs pointed to shared (within a program) and at the same time distinct (between programs) visions of what primary teachers should know at the end of their training. The OTL profiles were significantly related to outcomes of primary teacher education in terms of mathematics content knowledge, mathematics pedagogical content knowledge and general pedagogical knowledge.

Keywords TEDS-M · Comparative study · Latent class analysis · Mathematics · Curriculum · Mathematics pedagogy · General pedagogy · Culture · Content knowledge · Pedagogical content knowledge

Originally published in ZDM—The International Journal on Mathematics Education.

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S. Blömeke et al. (eds.), *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn*, Advances in Mathematics Education,
DOI 10.1007/978-94-007-6437-8_14,

1 Introduction

The "Teacher Education and Development Study: Learning to Teach Mathematics (TEDS-M)" was the first study to examine primary teachers' professional competence with nationally representative samples across countries. It was carried out under the supervision of the "International Association for the Evaluation of Educational Achievement (IEA)".[1] More than 13,000 teachers at the end of their training from 15 countries in Africa, America, Asia, and Europe were tested on their knowledge of mathematics and mathematics pedagogy as well as (in two countries) on their knowledge of general pedagogy. The ranking of countries and teacher education programs according to these outcomes provided benchmarks to evaluate the quality and effectiveness of primary teacher education in the participating countries and to initiate evidence-based reforms.

The knowledge-related outcomes can be summarized as follows: Taiwan and Singapore achieved the best result of all TEDS-M countries, for both mathematics content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK) of primary teachers (see Table 1). The difference to the international mean of 500 test points was large in both cases—about one standard deviation which is according to Cohen (1988) a large effect given the small overlap of the populations' distribution. Norway, Switzerland and the USA achieved results significantly above the international mean as well. In contrast, the MCK as wells as the MPCK achievement of primary teachers from Poland, Spain, the Philippines, Botswana, Chile and Georgia was significantly below the international mean. In the latter two cases, the difference was very large (for further details see, e.g., Blömeke et al. 2011, 2012).

TEDS-M was also the first comparative study which addressed the assessment of general pedagogical knowledge with nationally representative samples. Germany and the USA assessed their primary teachers' knowledge about lesson planning, classroom management and motivation, dealing with heterogeneity, and assessment. The main result was that German teachers significantly outperformed US teachers (for further details see, e.g., König et al. 2011). The difference was about one standard deviation and represents again a large effect (Cohen 1988).

To move toward an evidence-based reform of teacher education, it is important to examine the kinds of opportunities to learn (OTL) future teachers had had during their training, what kinds of cultural or institutional influences might have driven the development of these OTL in the participating countries and then to relate OTL to outcomes.

The curriculum of primary teacher education is assumed to be heterogeneous across countries and influenced by the context in which it is implemented—with

[1]The international costs of TEDS-M were funded by the IEA, the National Science Foundation (REC 0514431), and the participating countries. In Germany, the German Research Foundation funded TEDS-M (DFG, BL 548/3-1). The instruments are copyrighted by the TEDS-M International Study Center at MSU (ISC). The views expressed in this paper are those of the authors and do not necessarily reflect the views of the IEA, the ISC, the participating countries or the funding agencies.

Table 1 Mathematics content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK) of future primary teachers

MCK		MPCK	
Country	Mean (SE)	Country	Mean (SE)
Taiwan	623 (4.2)	Singapore	593 (3.4)
Singapore	590 (3.1)	Taiwan	592 (2.3)
Switzerland[a]	543 (1.9)	Norway[d,f]	545 (2.4)
Russia	535 (9.9)	USA[c,d,e]	544 (2.5)
Thailand	528 (2.3)	Switzerland[a]	537 (1.6)
Norway[f]	519 (2.6)	Russia	512 (8.1)
USA[c,d,e]	518 (4.1)	Thailand	506 (2.3)
Germany	510 (2.7)	Malaysia	503 (3.1)
International	500 (1.2)	Germany	502 (4.0)
Poland[b,d]	490 (2.2)	International	500 (1.3)
Malaysia	488 (1.8)	Spain	492 (2.2)
Spain	481 (2.6)	Poland[b,d]	478 (1.8)
Botswana	441 (5.9)	Philippines	457 (9.7)
Philippines	440 (7.7)	Botswana	448 (8.8)
Chile[d]	413 (2.1)	Chile[d]	425 (3.7)
Georgia	345 (3.9)	Georgia	345 (4.9)

[a]Colleges of education in German speaking regions
[b]Institutions with concurrent programs
[c]Public universities
[d]Combined participation rate <75 %
[e]High proportion of missing values
[f]Results for Norway are reported by combining the data sets available in order to present a proxy of the country mean

many subtypes, each supposed to generate teacher competence (see especially the 13th and the 15th ICMI studies, Leung et al. 2006, and Even and Ball 2009). The aims of this paper are to examine this curriculum heterogeneity and to identify *profiles* of OTL. If they exist, these profiles will be related to countries, to teacher education programs within countries and to outcomes of teacher education in order to explore potential explanations for the shape of the profiles.

2 Theoretical Framework and Hypotheses

TEDS-M followed the tradition of the IEA connecting educational opportunity and educational achievement. The idea is to describe important aspects of the education process and to determine whether cross-national differences in teachers' professional competence at the end of their training are related to differences in their opportunities to learn mathematics, mathematics pedagogy and general pedagogy (McDonnell 1995).

OTL in teacher education can be regarded as intentionally developed by educational policy makers and teacher education institutions (Stark and Lattuca 1997). National and program specifications of OTL reflect particular visions of what primary teachers are supposed to know before they enter the classroom. In all TEDS-M countries except Thailand and Malaysia, the teaching of mathematics represents an important part of primary teachers' responsibilities because they work as class teachers and teach most subjects including mathematics. In Thailand and Malaysia, primary teachers of mathematics are even trained as subject specialists and they teach mainly mathematics.

The current state of research on OTL in teacher education has to be regarded as weak. It suffers from the fact that only crude data exists about the components of teacher education which led to inconsistent results (Cochran-Smith and Zeichner 2005). In many studies, only the type of license or the number of courses taken was used to define OTL. These quantitative measures reflect the amount of content coverage without taking into account *which* content was offered, thereby ignoring qualitative similarities or differences between countries or teacher education programs. Data from a small comparative study on the lower secondary mathematics teacher education in six countries (Schmidt et al. 2011) had revealed that specific OTL profiles may exist and that these may be influenced by context characteristics: in five countries, the multiple institutions where teacher education took place tended to cluster together with respect to the OTL offered, suggesting within-country agreement and between-country heterogeneity reflecting cultural effects (Schmidt et al. 2008).

The first aim of this paper is, therefore, to identify groups of future primary teachers who had similar opportunities to learn mathematics, mathematics pedagogy and general pedagogy during primary teacher education. Given the state of research that points to curricula heterogeneity, we expect to find many different groups of future teachers across the 15 TEDS-M countries. Their profiles are supposed to reveal *quantitative* OTL differences (e.g., high, medium, and low coverage of mathematics pedagogy topics) as well as *qualitative* differences (e.g., coverage of specific mathematics pedagogy topics whereas others were neglected so that a pattern emerges).

Second, the relationship of these OTL profiles to cultural contexts will be examined. We assume that the *country*—or more precisely: its educational philosophy—is an important characteristic that shapes OTL and thus indicates between-country heterogeneity, stemming from shared national visions about teaching which differ between countries. Prior work on mathematics student achievement in TIMSS had revealed that in the students' OTL and their test results such cultural differences could be discovered (Klieme and Baumert 2001). The TEDS-M countries can be distinguished into major educational backgrounds: From Asia, two East Asian countries with a Confucian heritage took part in the study (Taiwan and Singapore) and three South-East Asian countries without a Confucian background (Malaysia, the Philippines and Thailand). From the former Eastern European block led by the Soviet Union, three countries took part (Russia, Georgia and Poland). Western countries in the study were the four European countries (Germany, Norway, Spain and Switzerland) and the two American countries (Chile and the USA). One African country (Botswana) took part as well. Given what we know about education in the "East" and the "West" (Leung et al. 2006; Schmidt et al. 2011), we expect to discover different OTL profiles along these groups of countries.

Third, we assume that within each country *program* requirements, e.g. the difference between the preparation as a class teacher or a subject specialist, influence the profiles. The future teachers reported which topics they had studied during their training. We assume that they had to follow to some extent institutional guidelines which reflect shared (within a program) but at the same time distinct (between programs) visions of what primary teachers should know before they start teaching. If so, in each country heterogeneity between programs should appear as a systematic correlation of the future teachers' OTL to the type of program they are in.

Finally, we hypothesize that the OTL profiles are related to differences in outcomes of teacher education in the sense that broader coverage of mathematics, mathematics pedagogy and general pedagogy leads to higher achievement in the respective fields.

3 Study Design

3.1 Instruments

TEDS-M intended to describe opportunities to learn during primary teacher education across countries. The topics listed in the survey were generated so as to be exhaustive of the content exposures in mathematics, mathematics pedagogy and general pedagogy in the participating countries. The future teachers indicated whether they had "studied" or "not studied" these topics. Their responses were prompted by three initial requests "Consider the following topics in university level mathematics (or mathematics pedagogy or general pedagogy respectively). Please indicate whether you have studied each topic."

Nineteen topics in mathematics were included as well as eight topics in mathematics pedagogy and eight topics in general pedagogy. For mathematics, these topics included categories such as "linear algebra", "abstract algebra", "analytic geometry" or "probability". In consultation with mathematicians in each of the countries and through a series of pilot and field studies, these categories were found to have the same meaning across countries. Mathematics pedagogy included categories such as "mathematics standards and curriculum", "development of mathematics ability and thinking", or "developing teaching plans". The history, philosophy and sociology of education were included under general pedagogy as were topics related to assessment, teaching and the theory of schooling. National expert reviews and pilot studies ascertained the cultural validity of these items in all participating countries.

The mathematics-related outcome assessments consisted of five booklets with 104 items in total; 72 mathematics and 32 mathematics pedagogy items. Items were assigned to booklets following a balanced-incomplete-block design. The mathematics items covered the content areas "number" (as that part of arithmetic most relevant for primary teachers), "algebra," and "geometry," with each set of items having about equal weight, as well as a small number of items about "data" (as a hypernym for that part of probability and statistics most relevant for primary teachers). The mathematics pedagogy items included aspects of "curricular and planning knowledge" and "knowledge about how to enact mathematics in the classroom." These two sets of items were of about equal weight. The majority of items were complex multiple-choice items. Some of the items were partial-credit items (for more details see Tatto et al. 2012).

The instrument measuring general pedagogical knowledge consisted of 85 test items. These included dichotomous and partial-credit items as well as open-response (about half of the test items) and multiple-choice items. The items were fairly equally distributed across different teacher tasks like lesson planning, dealing with heterogeneity, motivation, classroom management and assessment. Following the MCK and MPCK test design, five booklets in a balanced-incomplete-block design were used (for more details see König et al. 2011).

3.2 Data Analysis

Latent class analysis was used to examine the potential heterogeneity of opportunities to learn mathematics, mathematics pedagogy and general pedagogy in primary teacher education as indicated by the future teachers at the end of their training. LCA is a model-based exploratory method to classify similar objects—in our case primary teachers—into homogenous groups where the number of classes as well as their properties are unknown and inferred from the data (McLachlan and Peel 2000). At the same time, measurement error is taken into account. The class membership describes the relationships among the opportunities to learn during primary teacher education; each class is characterized by a specific OTL pattern, called "profile". The results describe the conditional probabilities of having studied the content topics given the class membership. Teachers were assigned to the class, for which their observed response pattern is most probable. Latent class analysis is therefore not a deterministic but a probabilistic clustering approach.

We used weights which adjusted the collected data for stratification and unequal probabilities of selection as well as for non-response in order to obtain unbiased estimates of the population parameters. In addition, we adjusted the sample size per country upwards or downwards to $n = 500$ in order to avoid that properties of countries with large sample sizes dominated over properties of countries with small sample sizes. Within countries, teacher education programs are represented proportional to their size in the target population in order to represent the national curricular structure which is the core unit of analysis in this paper.

Part of a LCA is a decision about the number of classes necessary to explain the dependencies among the OTL items. We made the decision by comparing several models which differed with respect to the number of classes. The classification quality of the models was evaluated based on an aggregated classification uncertainty measure called "entropy" (Ramaswamy et al. 1993). Entropy is a standardized measure of how accurately primary teachers were classified and is implemented in the software package MPlus (Muthén and Muthén 2008) which we used for this paper. Entropy values above 0.7 can be regarded as indicating a sufficient separation; values above 0.8 indicate good separation.

The estimate of the average posterior probability for class membership was taken into account as well. It should be above 0.7 if sufficient and above 0.8 if high classification quality is intended. Finally, we evaluated relative criteria (Nylund et al.

2007). The adjusted Bayesian Information Criterion (Schwarz 1978) is a measure of the goodness of fit of a latent class model that considers the number of parameters and the number of observations; the log likelihood is a function of the observed responses for each teacher and the model parameters. Lower values indicate a better fitting model.

The final decision about the number of classes was in each case also based on substantive considerations. Based on a literature review, Marsh et al. (2004)warned against the practice of using goodness-of-fit indexes as “golden rules” only. Instead they recommended selecting a model that balances fit statistics and sense in relation to theory and previous research as well as to a convincing interpretation of the results. We followed this recommendation.

The between-country heterogeneity of OTL in primary teacher education was examined based on the frequency distribution. If in fact culture had shaped the OTL profiles, large proportions of a country’s future teachers should be classified into one OTL class only and thus pointing to shared national visions. However, precisely into *which* class the future teachers were classified should *differ between* countries and this along the lines of educational traditions stated above (Eastern and Western countries, countries with Confucian and non-Confucian heritage and so on) and thus pointing to curricula heterogeneity between cultural contexts.

The *within*-country heterogeneity was examined by relating class membership to teacher education *programs* through Chi-square testing finally leading to Cramer’s V as estimate. With this approach we capture OTL heterogeneity between programs for each country using a single coefficient. Cramer’s V is adjusted for sample size and table size, and the test statistics was created against the null hypothesis that all programs have the same distribution of future teachers into the country’s different OTL classes. In this case of within-country homogeneity, the Chi-square deviance and Cramer’s V would be zero. In contrast, the larger V (i.e., the closer to 1), the stronger our hypothesis of within-country heterogeneity would be supported by the data. Institutions were treated as offering different programs because of the freedom most of them have in designing their curriculum. In the context of TEDS-M this unit of analysis (programs per institution) is called “teacher preparation unit (TPU)”.

The relationship of class membership to MCK, MPCK and GPK was estimated through general linear models with the class membership as factor after first having estimated the deviance of individual outcomes from the country mean in order to avoid a cultural bias and to get pure class effects (Cronbach 1976). This approach corresponds with centering the outcomes around their country means.

3.3 Sample

The target population of the present study was defined as future teachers in their final year of teacher education who would receive a license to teach mathematics in primary schools (Tatto et al. 2008). A teacher education program was identified as preparing primary teachers if the license would include one of the grades 1 through

4 as the common denominator in the "International Standard Classification of Education" (primary or basic education, cycle 1; UNESCO 1997).

In a two-stage process, random samples were drawn from this target population in each participating country. The samples were stratified according to important teacher education features like "route" (consecutive vs. concurrent programs), "type" of program (grade span the license includes, e.g., grades 1 through 4 vs. 1 through 10), "focus" of opportunities to learn (with or without extensive mathematics) or "region" (e.g., federal states) in order to reflect accurately the distribution of future primary teachers' characteristics at the end of their training.

In 2008, more than 13,000 future primary teachers from 15 countries were surveyed. All countries had to meet IEA's quality requirements as known from TIMSS or PIRLS. These included controlling of translation processes, monitoring of test situations, and meeting participation rates. If a country missed the participation benchmark only slightly, its results are reported in an annotated way ("Combined Participation Rate <75 %").

In most countries, TEDS-M covered the full target population. Only Switzerland (German speaking regions), Poland (institutions with concurrent programs) and the USA (public universities) had to limit their participation for economic reasons. Particularly complex is the composition of the Norwegian sample. Two sets of primary data are available for this country, each one biased: either to a focus on mathematics or without such a focus. The present paper combines them in order to present the most accurate picture possible (for a detailed discussion of this decision see Blömeke et al. 2010).

4 Results

First, we will identify profiles of opportunities to learn across the 15 TEDS-M countries. These profiles are then described with respect to the underlying educational philosophy. In a second step, the OTL profiles are related to the countries, teacher education programs and outcomes.

4.1 OTL in Mathematics

4.1.1 Profiles of OTL in Mathematics

Based on an evaluation of all fit indices, a distinction of two or three groups of future primary teachers turned out to be the best solutions with respect to their OTL in mathematics (see Table 2). We can interpret this basic result as an indicator that more classes exist than only one and thus as a first indicator of (some) cross-country OTL heterogeneity. The log likelihood and the adjusted BIC improved the most when *two* classes were distinguished. In contrast, the entropy measure improved and was well above the threshold of 0.8 when *three* classes were distinguished. The average posterior probabilities for class membership for each class was above 0.9

Table 2 Fit indices of different latent class solutions for OTL in mathematics ($n = 13{,}809$)

# Classes	Entropy	Log likelihood	Adjusted BIC	Diff(LL)	Diff(BIC)
1	–	−154,415.40	308,951.56	–	–
2	0.815	−137,169.66	274,587.18	−17,245.74	−34,364.38
3	0.831	−132,210.75	264,796.45	−4,958.92	−9,790.73
4	0.793	−130,629.10	261,760.25	−1,581.65	−3,036.20
5	0.769	−129,393.70	259,416.57	−1,235.40	−2,343.69
6	0.748	−128,667.04	258,090.35	−726.66	−1,326.21

Table 3 Classification table for OTL in mathematics

Most likely class	Mean posterior probabilities		
	Class 1	Class 2	Class 3
Class 1	0.929	0.071	0.000
Class 2	0.042	0.925	0.033
Class 3	0.000	0.099	0.901

in the two-class solution as well as in the three-class solution (see Table 3 for the latter). Since the three-class solution provided more substantive information compared to the two-class solution and in addition balanced parsimony and differences in the OTL profiles, we decided to use this solution. Our decision was supported by the fact that the entropy measures of the three-class solution and the mean posterior probabilities for class membership were satisfactory *within* the participating countries as well.

Across countries, a primary teacher was classified into class 1 with a probability of only one quarter (25.6 %) whereas the chance that she was classified into class 2 was much higher (55.2 %). The probability of a classification into class 3 was the lowest (19.3 %). Thus, across countries the second OTL profile represented the dominating idea to structure opportunities to learn mathematics in primary teacher education and thus to a large extent of cross-country OTL *homogeneity*.

With a probability of about 90 % (see Fig. 1), teachers in this second class indicated that they had had the opportunity to learn number theory and probability—the two topics closest related to the teaching of mathematics in primary school as it has been proclaimed, for example, by professional organizations like the National Council of Teachers of Mathematics (NCTM 2000), by mathematics educators like Niss (1999) or by policy makers like The Standing Conference of the Ministers of Education and Cultural Affairs of the Länder in the Federal Republic of Germany (KMK 2005). Furthermore, with a probability of at least 60 %, they had also had the opportunity to learn linear algebra, beginning calculus, the foundations of geometry, and statistics—topics very typical for the first, introductory sequence of university mathematics in each area but still related to the teaching of mathematics in primary schools and, therefore, often regarded as important background knowledge. The 60 % probability level points to a lower level of cross-country standardization

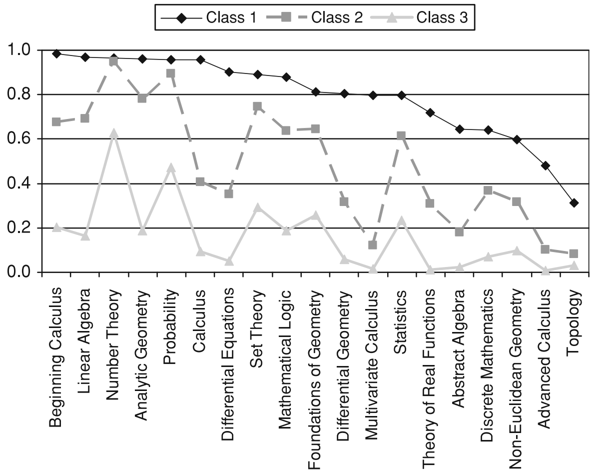

Fig. 1 Probability of OTL in mathematics in primary teacher education by class

with respect to these topics though. More advanced topics like multivariate calculus or differential geometry had probably not been part of these teachers' training. This second class is therefore well characterized by the concept of "basic university mathematics".

In class 1, those teachers were grouped who had the most OTL in mathematics during teacher education. Their OTL profile comes close to students who major in university mathematics and it can be labeled as "advanced university mathematics". With the exception of a few specialized topics like abstract algebra, the primary teachers of this class had studied mathematics topics like number theory, linear algebra, or analytic geometry with a probability of at least 80 %. Topology was the only topic taken with a probability as low as one-third—but this is typically an elective topic for mathematics majors as well.

The OTL profile of the third class followed essentially the same pattern as the second class but the probability that a topic was taken was lower each time. Number theory (63.0 %) and probability (47.2 %) were the only two topics of university mathematics taken with a substantial probability by the primary teachers in this class. The other topics had probably not been part of their teacher education. This group of teachers seems to be supposed to teach mathematics mainly based on their content knowledge from school. Even basic university topics like beginning calculus, or foundations of geometry were taken with a probability lower than one quarter. The label "school mathematics" fits therefore the best to describe this idea of primary teacher education.

The commonalities and differences in the opportunities to learn mathematics during primary teacher education documented in Fig. 1 reveal that a single OTL profile accepted in all TEDS-M countries does not exist. None of the mathematics topics was taken by almost all future primary teachers surveyed, no matter which class they belonged to. Number theory and probability came closest to such an idea. With

Table 4 Observed proportions (standard error) of OTL classes in mathematics by country

	Advanced university mathematics	Basic university mathematics	School mathematics
Thailand	91.3 (0.7)	8.7 (0.7)	0.0 (0.0)
Malaysia	64.5 (2.4)	29.5 (1.9)	6.0 (1.2)
Taiwan	7.6 (0.9)	78.1 (1.2)	14.2 (0.8)
Norway	11.9 (1.3)	71.2 (1.7)	16.9 (1.3)
Russia	21.4 (2.9)	70.3 (3.5)	8.3 (1.7)
Georgia	18.1 (1.4)	68.8 (1.6)	13.1 (1.2)
Philippines	26.4 (4.1)	67.4 (5.8)	6.2 (3.4)
Chile	6.1 (0.9)	66.0 (1.8)	27.9 (1.7)
Botswana	11.2 (3.3)	65.2 (4.8)	23.6 (5.2)
Switzerland	29.3 (1.5)	64.5 (1.7)	6.2 (0.9)
Spain	24.9 (1.8)	60.8 (2.3)	14.3 (2.2)
Poland	27.6 (0.6)	57.3 (1.0)	15.1 (0.9)
United States	10.8 (1.6)	55.2 (1.6)	34.0 (1.9)
Germany	5.5 (1.3)	36.0 (1.8)	58.5 (1.9)
Singapore	18.0 (1.9)	35.9 (2.9)	46.1 (2.6)

a probability close to 1 for future teachers in classes 1 and 2 and a probability of about 0.5 for teachers belonging to class 3, the two topics may represent the core of mathematics in primary teacher education. Teaching basic numeracy skills has in fact a long tradition in primary schools around the world and calls for teacher knowledge in number theory while an increasing emphasis on applying mathematical knowledge calls for knowledge about probability. In contrast, even though geometry may be regarded a standard component of mathematics, an emphasis on this area depends more on cultural traditions. In many English-speaking countries, for example, geometry had never played an important role because primary mathematics was centered around number (Fielker 1986).

4.1.2 Relationship of the OTL Profiles to Cultural Contexts

In all countries except Singapore, future primary teachers were classified into one specific class with a probability above 50 %, whereas the classification probability into another class was at most about one-third and into the third class 15 % only (see Table 4). This result points to relative homogeneity within countries.

Only in Thailand and Malaysia substantial proportions of future teachers belonged to class 1. This result can be related to the fact that these countries had decided to train mathematics teachers for primary schools as specialists and not as classroom teachers with responsibility for many subjects. Advanced university mathematics seems to represent the common OTL standard in this teacher education model.

In 11 countries, the majority of future primary teachers indicated an OTL profile of basic university mathematics. This result replicates the overall result that such a profile represents a widespread standard how primary teacher education is organized. Obviously, a substantial knowledge of mathematics content beyond school mathematics is regarded necessary across countries if class teachers are to be trained—no matter which educational tradition a country belongs to. Thus, this results points to more cross-country homogeneity than expected.

Germany is the only country where more than half of the future teachers indicated an OTL profile that relied mainly on content knowledge from school mathematics. Singapore is another country with a substantial proportion of future teachers in this class. Further analyses reveal that this result may be related to the countries' decision to have a highly stratified teacher education system. In both countries, primary teachers can be trained in different types of programs of which some offered only few or even no opportunities to learn university mathematics although the future teachers would receive a license to teach mathematics in primary schools.

With respect to Singapore, this result is astonishing, given the overall achievement of the country's future primary teachers (see Table 1). Although further analysis is needed, our hypothesis is that the teachers may have received their mathematics training elsewhere. We know from country reports (see Wong, Boey, Lim-Teo & Dindyal in this book; Lim-Theo 2009) that teacher education in Singapore is more and more relying on mid-career teachers who had, e.g., a training as engineers before. Mathematics represents an important part of such programs.

4.1.3 Relationship of the OTL Profiles to Teacher Education Programs

The dominating profiles point to more cross-country and within-country homogeneity in the mathematics teacher education curriculum than initially expected. However, the data revealed at the same time some remaining within-country heterogeneity, indicated by up to one-third of a country's future primary teachers classified into a second and up to 15 % classified into a third class. Country by country, the relationship of the OTL profiles to teacher education programs was examined in order to get an estimate of the between-program heterogeneity, probably caused by differences in the institutional guidelines.

In fact, as Table 5 reveals, in most countries teacher education programs did not have the same distribution of future teachers into the three OTL classes. In contrast, in some countries a rather strong relationship between OTL profiles and programs existed, especially if they had a large number of institutions and several programs that provided primary teacher education. This applied, for example, to Poland and Thailand. In contrast, in countries like Singapore or Taiwan where the number of institutions or programs was small and in addition national guidelines existed, TPU differences were small, thus indicating within-country homogeneity.

Table 5 Within-country relationship of OTL profiles in mathematics to teacher education units

Country	Teacher education programs	Number of institu-tions	Teacher preparation units	Cramer's V
Poland	8	78	126	0.62
Thailand	2	45	53	0.61
Philippines	1	33	33	0.48
Germany	4	14[a]	24	0.46
Russia	1	49	49	0.44
Spain	1	45	45	0.36
USA	4	51	79	0.34
Norway	2	26	26	0.30
Botswana	1	4	4	0.26
Malaysia	3	23	24	0.25
Chile	1	31	31	0.24
Georgia	2	9	10	0.22
Switzerland	2	14	23	0.21
Taiwan	1	11	11	0.14
Singapore	6	1	6	0.13

[a]Federal states

4.1.4 Relationship of the OTL Profiles to the Outcomes of Teacher Education

As a last step, the OTL profiles are related to outcomes of primary teacher education. If the country effect is partialed out as explained in Sect. 2.2 in order to get pure class effects, the different types of OTL in mathematics turn out to be closely related to teachers' mathematics content knowledge (see Fig. 2; $F = 28.68$, $p < 0.001$). The mean MCK differences between the three classes were each highly significant. On average, teachers belonging to class 1 achieved higher mathematics scores than teachers from class 2 (mean difference = 6.3 test points) and these again achieved higher mathematics scores than teachers from class 3 (mean difference = 13.3 test points). The overall difference between class 1 and 3 was thus about 20 test points which is one-fifth of a standard deviation. This is a small but relevant effect (Cohen 1988). Higher investments into OTL—if defined as coverage of the topics listed—seem to correspond with higher outcomes.

4.2 OTL in Mathematics Pedagogy

As we did with the OTL in mathematics, we will firstly identify profiles of OTL in mathematics pedagogy across the 15 TEDS-M countries. These profiles are then described with respect to the underlying philosophy. In a second step, the OTL profiles are related to countries, teacher education programs and outcomes in terms of MPCK.

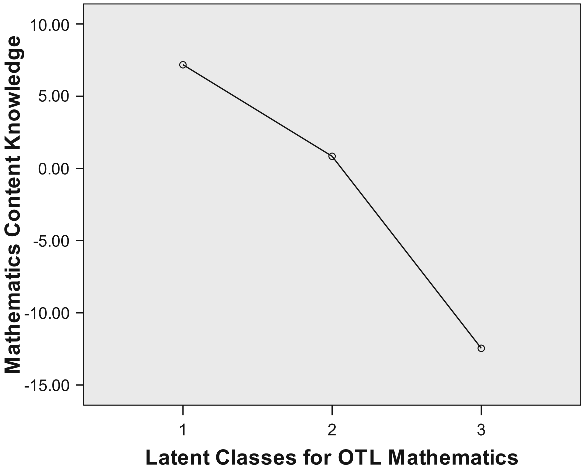

Fig. 2 Relationship of OTL profiles in mathematics to outcomes of teacher education (*1* advanced university mathematics, *2* basic university mathematics, *3* school mathematics)

Table 6 Fit indices of different latent class solutions for OTL in mathematics pedagogy ($n = 13{,}499$)

# Classes	Entropy	Log likelihood	Adjusted BIC	Diff(LL)	Diff(BIC)
1	–	−62,314.65	124,679.95	–	–
2	0.708	−56,888.05	113,883.76	−5,426.59	−10,796.19
3	0.663	−55,615.73	111,396.11	−1,272.32	−2,487.65
4	0.662	−54,954.10	110,131.64	−660.73	−1,264.48
5	0.659	−54,773.75	109,826.13	−181.25	−305.51
6	0.647	−54,715.52	109,766.65	−58.23	−59.47

4.2.1 Profiles of OTL in Mathematics Pedagogy

The fit indices of the class solutions for OTL in mathematics pedagogy were not as good as they were for mathematics but they still allowed for a satisfying model. The evaluation criteria supported decisions for two, three or four classes (see Tables 6 and 7) and pointed thus to the existence of several OTL profiles and some cross-country heterogeneity. Based on a balance of statistical and substantive considerations and taking parsimony into account, a distinction of three groups of future primary teaches represented the optimal solution for our purpose: although the entropy measure dropped slightly below the desired value of 0.7, the average latent class probabilities for the most likely latent class membership were still between 0.8 and 0.9. Also, the relative measure log likelihood and the adjusted BIC improved considerably in the three-class compared to the two-class solution. However, the most important reason for our decision was a substantive one. The additional information gained was very valuable (see below).

Across the 15 TEDS-M countries, a future primary teacher was classified into class 1 with a probability higher than two-fifth (43.1 %), whereas the chance that

Table 7 Classification table for OTL in mathematics pedagogy

Most likely class	Mean posterior probabilities		
	Class 1	Class 2	Class 3
Class 1	0.859	0.117	0.024
Class 2	0.100	0.811	0.088
Class 3	0.039	0.074	0.887

she was classified into the second group was about one-third (34.6 %) and only one-fifth with respect to class 3 (22.3 %). Thus, the first OTL profile represented the most important idea across countries to structure opportunities to learn mathematics pedagogy in primary teacher education.

In class 1 those teachers were grouped who reported coverage of almost all OTL in mathematics pedagogy listed (see Fig. 2). The probability that a topic was taken was each time at least 80 %—no matter whether the content was related to instructional aspects like teaching methods or to broader issues of mathematics instruction like context conditions. This concept can, therefore, be labeled as a "broad mathematics pedagogy curriculum". The high probabilities point to teacher education guidelines which had to be followed by the future teachers. Thus, they point to a certain degree of homogeneity and shared visions of what future primary teachers are supposed to know before they enter the classroom.

The second class of future primary teachers is best characterized by the concept of a "functional mathematics pedagogy curriculum". The OTL most probably taken in this class showed a narrow focus on mathematics instruction. Teachers in this class had the opportunity to develop teaching plans, to learn about teaching methods and the school curriculum as well as to observe and analyze mathematics instruction with a probability of more than 80 %. In contrast, broader issues like the context conditions of mathematics education or affective aspects of mathematics learning had probably not been part of their OTL.

Learning about teaching methods was the only mathematics pedagogy topic taken by primary teachers in the third class with a substantial probability (i.e., more than 60 %). These teachers did probably neither learn about further instructional topics like the development of mathematics ability nor about broader context issues. The label "teaching methods" fits, therefore, best to describe this class. The probabilities of most opportunities to learn mathematics pedagogy were so low that they even might not have been electives.

The OTL profiles discovered in the field of mathematics pedagogy are systematically related to the OTL profiles in mathematics ($r_c = 0.22^{***}$). Those future primary teachers who reported the broadest coverage of mathematics topics (class 1) also reported more often a broad coverage of mathematics pedagogy than future teachers from the other classes. Vice versa, primary teachers with especially low OTL in mathematics (class 3) fell short with respect to mathematics pedagogy as well. This pattern does not only apply across countries but also within most countries. The latter can be interpreted as an indicator that the correlation does not result

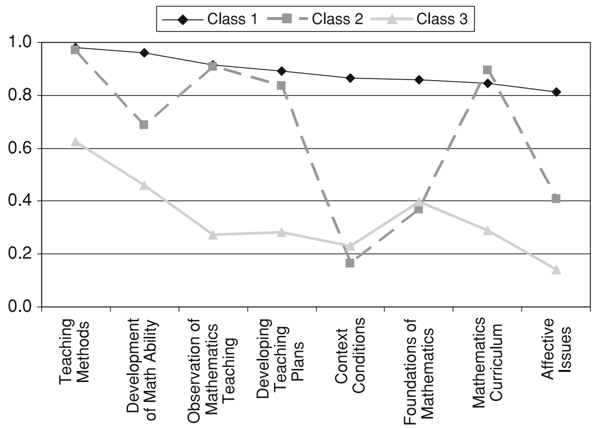

Fig. 3 Probability of OTL in mathematics pedagogy in primary teacher education by class

from response tendencies (e.g., that those who react strongly to one statement react strongly to others as well or vice versa) but reflects an existing relationship.

The commonalities and differences in the opportunities to learn mathematics pedagogy during primary teacher education documented in Fig. 3 revealed that, on the class level, the heterogeneity was comparable to that of OTL in mathematics. There was no mathematics pedagogy topic studied with a probability close to 1 in all classes. However, a few classes were sufficient to describe the OTL appropriately across countries and one topic came close to the idea of a core curriculum in the TEDS-M countries. "Teaching methods" was studied by the teachers in classes 1 and 2 with a probability of more than 90 % as well as by the teachers in class 3 with a probability of more than 60 %. Certainly, teaching methods represent a core element of teachers' professional knowledge.

4.2.2 Relationship of the OTL Profiles to Cultural Contexts

In 11 countries (see Table 8), at least half of the future primary teachers were classified into one specific mathematics pedagogy profile. This result points to relative homogeneity within countries. Between countries the underlying structure of the curriculum profiles is less clear and in the remaining four countries, variability is the dominating pattern.

In Taiwan, Singapore and Poland the majority of primary teachers indicated a functional OTL profile in mathematics pedagogy (in Russia and Georgia this applied to about 40 %). This result may reflect a specific educational philosophy in the "East" (in this case in East Asia and Eastern Europe). In both educational traditions, the cognitive learning of mathematics is highly valued and mathematics education plays a privileged role in the school curriculum (Leung et al. 2006; Schmidt et al. 2011).

Table 8 Observed proportions (standard error) of OTL classes in mathematics pedagogy by country

	Broad curriculum	Functional curriculum	Teaching methods
Botswana	80.6 (4.2)	4.5 (2.3)	14.9 (3.6)
Malaysia	76.6 (1.7)	17.3 (1.6)	6.1 (1.0)
Philippines	64.5 (3.2)	15.2 (2.3)	20.4 (1.9)
Switzerland	58.4 (1.8)	28.3 (1.4)	13.2 (1.0)
Thailand	55.4 (1.7)	37.7 (2.0)	6.9 (0.9)
Russia	52.6 (3.5)	40.7 (3.3)	6.7 (1.3)
Chile	50.3 (2.1)	24.5 (1.8)	25.2 (1.3)
Taiwan	14.9 (1.3)	63.7 (1.6)	21.4 (1.3)
Singapore	34.2 (2.3)	56.1 (2.5)	9.8 (1.4)
Poland	24.7 (1.2)	54.5 (1.7)	20.8 (1.0)
Germany	12.0 (1.0)	33.8 (2.3)	54.2 (2.0)
United States	48.7 (2.7)	40.2 (2.4)	11.1 (1.5)
Norway	47.7 (1.8)	30.7 (1.8)	21.7 (1.8)
Spain	31.8 (2.1)	27.6 (2.0)	40.6 (2.5)
Georgia	28.3 (2.4)	39.7 (2.1)	32.1 (2.0)

Variability was the overall characteristic of OTL in mathematics pedagogy in the United States, Norway and Spain, thus in three Western countries. A low level of standardization is a typical criticism in these countries (NCATE 2001; NOKUT 2006). It is a bit surprising that Germany and Switzerland were not part of this group. The assumption would have been that they would be dominated by variability as well because teacher education has been discussed in this way (Criblez 1999; Terhart 2000).

In Germany, by far the highest proportion of future primary teachers—more than half of them—was classified into an OTL profile focused on teaching methods. This result may reflect that German teacher education consists of two training steps: a first academic phase at university and a following second practical training phase at state institutions closely related to schools. During this second phase, teaching methods are crucial because the future teachers have to teach several classes with full responsibility.

The broadest perspective on mathematics pedagogy is taken in seven countries, including the three non-Confucian South-East Asian countries, Malaysia, Thailand and the Philippines, as well as the only African country Botswana. It seems as if these countries have a special interest in the teaching and learning of mathematics although it is difficult to find a common denominator for these cultural contexts.

Table 9 Within-country relationship of OTL profiles in mathematics pedagogy to teacher preparation units

Country	Cramer's V
Thailand	0.45
Germany	0.44
Spain	0.40
Poland	0.37
Russia	0.37
USA	0.37
Switzerland	0.33
Norway	0.30
Chile	0.30
Philippines	0.26
Malaysia	0.27
Botswana	0.24
Taiwan	0.24
Georgia	0.19
Singapore	0.18

4.2.3 Relationship of the OTL Profiles to Teacher Education Programs

Although focused on the country level and an identification of between-country heterogeneity, the previous examination has already revealed a large amount of heterogeneity *within* the TEDS-M countries. The question is whether it stems from systematic between between-program differences as in the case of mathematics and thus reflecting shared but distinct visions of teacher knowledge.

Again, it turns out that programs differ with respect to the distribution of class membership (see Table 9). In all countries, a systematic relationship of OTL profiles in mathematics pedagogy to TPUs exists which provides evidence for within-country heterogeneity. Even though culturally shaped visions seem to exist on the country level, within countries programs may have different versions of these if they belong to different institutions or if they prepare different types of primary teachers.

4.2.4 Relationship of the OTL Profiles to Outcomes of Teacher Education

The OTL profiles were related to the primary teachers' MPCK test results (see Fig. 4; $F = 13.64$, $p < 0.001$). The differences between future teachers who had taken either a broad or a functional curriculum and future teachers who had taken OTL focused on teaching methods were highly significant. On average, a tendency existed as well that teachers reporting a functional curriculum achieved higher mathematics pedagogy scores than teachers reporting a broad curriculum (mean difference = 4.2 test points). The latter in turn had significantly higher math pedagogy scores than teachers in the methods class (mean difference = 9.5 test points).

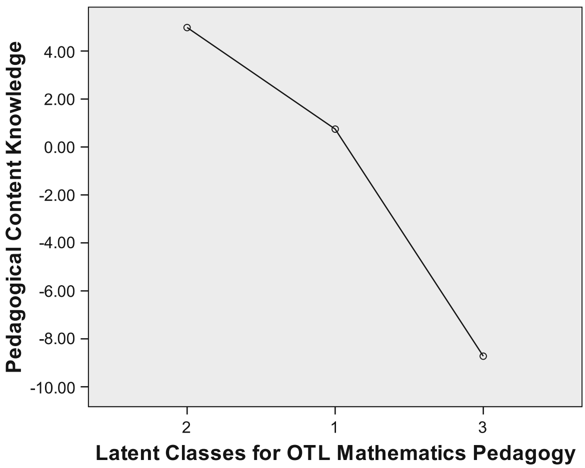

Fig. 4 Relationship of OTL profiles in mathematics pedagogy to outcomes of mathematics teacher education (*1* broad curriculum, *2* functional curriculum, *3* teaching methods)

The overall difference between the functional and the methods classes was about 14 test points, which was not as high a difference than in the case of MCK, but still a small effect. This result indicates once more that larger investments into OTL pay off. A teacher education concept that is based on OTL focused on teaching methods only is associated with weaker MPCK.

4.3 OTL in General Pedagogy

4.3.1 Profiles of OTL in General Pedagogy

The situation in general pedagogy was clearer than in mathematics pedagogy. The different criteria would allow for a two-or a three-class solution. Based on an evaluation of all fit indices and including substantive considerations, again a distinction of three groups of future primary teachers represented the optimal solution for our purpose (see Tables 10 and 11). First, the entropy measure was as desired around 0.7 and it did not drop compared to a two-class solution. Second, the average latent class probabilities for class membership were above 0.9 for the classification into class 1 and well above 0.8 for classification into class 3. Only the classification into class 2 dropped slightly. Third, the substantive value added by distinguishing between three instead of only two classes was regarded as important (see below).

With a probability higher than two-thirds was a future primary teacher classified into class 1 (70.0 %), whereas the classification probability was only about one-sixth for the second (17.3 %) and the third classes (16.7 %). Interestingly, this distribution reveals more homogeneity across countries with respect to OTL in general pedagogy than in the other components—especially compared to mathematics pedagogy where the same number of topics was used to describe the OTL. One topic was taken

Table 10 Fit indices of different latent class solutions for OTL in general pedagogy ($n = 13{,}250$)

# Classes	Entropy	Log likelihood	Adjusted BIC	Diff(LL)	Diff(BIC)
1	–	−46,905.13	93,860.76	–	–
2	0.700	−42,713.50	85,534.34	−4,191.62	−8,326.42
3	0.695	−42,304.56	84,773.27	−408.95	−761.07
4	0.669	−41,948.36	84,117.70	−356.20	−655.57
5	0.687	−41,875.74	84,029.29	−72.62	−88.41
6	0.687	−41,830.33	83,995.29	−45.41	−34.00

Table 11 Classification table for OTL in general pedagogy

Most likely class	Mean posterior probabilities		
	Class 1	Class 2	Class 3
Class 1	0.913	0.055	0.032
Class 2	0.132	0.730	0.139
Class 3	0.045	0.124	0.831

in each class with a probability of at least 80 %. This topic was educational psychology, which reflects the high importance of aspects such as learning theories or child development in primary education. Further topics were taken in all classes with a probability of at least 50 %: theories of schooling and the knowledge of teaching. Methods of educational research represented the topic which distinguished the best between the three classes.

In class 1 those teachers were grouped who had the most OTL in general pedagogy (see Fig. 5). All topics were covered with a probability of at least 80 %, no matter whether the content was related to theory or practice, to instruction, context or research. Analogous to the mathematics pedagogy profiles, this concept can be labeled as a "broad general pedagogy curriculum". Given the size of this class, it

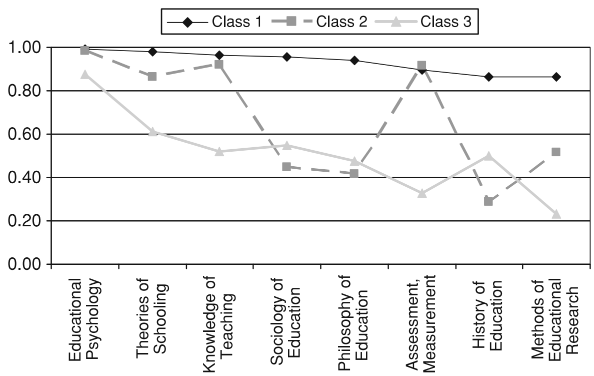

Fig. 5 Probability of OTL in general pedagogy in primary teacher education by class

reflects the dominating profile across countries and at the same time, given the high probabilities of each OTL, it reflects some type of guideline that future teachers were required to follow.

The second class of future primary teachers is best characterized by the concept of a "functional general pedagogy curriculum". The OTL taken showed a strong focus on topics closely related to teaching and learning. Across countries, teachers grouped into this class had had the opportunity to learn about theories of schooling and educational psychology as well as about knowledge of teaching and assessment with a probability of at least 80 %.

A distinct underrepresentation of assessment and educational research characterizes the OTL in class 3 whereas—compared to class 2—the topics sociology, philosophy and history of education were relatively more stressed. This concept can, therefore, be labeled as "context-related general pedagogy". However, generally speaking future teachers from this class had had the least opportunities to learn general pedagogy.

As with mathematics pedagogy and mathematics, the general pedagogy OTL profiles are systematically related to the profiles in the other components—not unexpectedly more pronounced with respect to mathematics pedagogy ($r_c = 0.29^{***}$) than to mathematics ($r_c = 0.15^{***}$). On the class level, future teachers with broad OTL in general pedagogy had probably also had more OTL in mathematics pedagogy and in mathematics and vice versa. Again, this pattern applies across countries and within most countries which means that it is a function of true relationships instead of response tendencies.

4.3.2 Relationship of the OTL Profiles to Cultural Contexts

In all countries except Singapore, Taiwan, Germany and Georgia, at least two-thirds of the teachers were classified into class 1 (see Table 12). This result replicates the overall profile of a broad curriculum as the dominating way to organize OTL in general pedagogy in primary teacher education across countries. Thus, the result is in this respect a strong indicator of between-country *homogeneity*.

In Singapore, the highest proportion of teachers belonged to the class "functional curriculum" (42 %) but at least a quarter of the primary teachers in Taiwan, Germany and Georgia was classified into this OTL profile as well. Whereas the first two countries would be expected in this group given their educational tradition (see, e.g., Blömeke et al. 2008), the classification of Germany is once again surprising given the traditionally strong influence of the concept of general education, so-called "Allgemeinbildung". The distinction of two phases of teacher education with a marginal role of general pedagogy in the first academic phase and a second practical phase at schools may again play an important role.

None of the countries had a majority of teachers indicating a context-related OTL profile. Countries with at least about a quarter of primary teachers in this class were Taiwan, Singapore, Germany, Georgia and Spain. This results points to relatively low agreement about the arrangement of OTL in general pedagogy in the first four

Table 12 Observed proportions (standard error) of OTL classes in general pedagogy by country

	Broad curriculum	Functional curriculum	Context-related curriculum
Philippines	94.3 (1.3)	3.2 (0.6)	2.5 (1.3)
Russia	91.8 (1.3)	4.3 (1.3)	3.9 (0.8)
Switzerland	91.4 (1.0)	6.2 (0.8)	2.4 (0.6)
Chile	83.9 (1.5)	8.4 (0.9)	7.8 (1.0)
Thailand	83.4 (1.3)	14.6 (1.1)	2.0 (0.6)
Malaysia	80.4 (1.6)	9.2 (1.1)	10.4 (1.2)
Poland	79.0 (1.4)	1.7 (0.3)	19.3 (1.3)
United States	74.7 (2.6)	12.9 (2.3)	12.4 (1.3)
Norway	69.1 (1.7)	19.6 (1.5)	11.3 (1.1)
Botswana	64.7 (5.2)	21.7 (4.5)	13.6 (3.2)
Spain	63.3 (2.4)	10.4 (1.1)	26.3 (2.2)
Georgia	49.5 (2.6)	26.5 (2.5)	24.0 (1.8)
Taiwan	42.4 (2.0)	29.9 (1.7)	27.6 (2.3)
Germany	39.2 (2.3)	32.0 (2.1)	28.8 (1.9)
Singapore	28.1 (2.6)	42.1 (2.8)	29.9 (2.4)

countries given that their teachers dominate class 2 and that they have substantial proportions in class 1 as well.

4.3.3 Relationship of the OTL Profiles to Teacher Education Programs

Such within-country heterogeneity is revealed in the relationship of the general pedagogy OTL profiles to teacher education programs. The participating TPUs may follow distinct visions in this component but at the same time offering shared OTL to their students. In all countries, such a systematic relationship of OTL profiles to programs exists (see Table 13). However, the strength of the relationship varies. Again heterogeneity is largest in those countries in which more institutions and programs exist than in countries with only a few TPUs.

4.3.4 Relationship of the OTL Profiles to Outcomes of Teacher Education

Very similar to the result in mathematics pedagogy, the different types of OTL in general pedagogy were related to outcomes of primary teacher education in terms of general pedagogical content knowledge if country differences are controlled ($F = 3.33$, $p < 0.05$). Only Germany and the USA took part in this component of TEDS-M though. On average, primary teachers belonging to classes 1 and 2 achieved significantly higher general pedagogy scores than teachers from class 3

Table 13 Within-country relationship of OTL profiles in general pedagogy to teacher preparation units

Country	Cramer's V
Poland	0.51
USA	0.37
Thailand	0.29
Spain	0.28
Malaysia	0.27
Botswana	0.26
Russia	0.26
Germany	0.25
Chile	0.24
Norway	0.24
Taiwan	0.22
Philippines	0.22
Switzerland	0.21
Singapore	0.18
Georgia	0.17

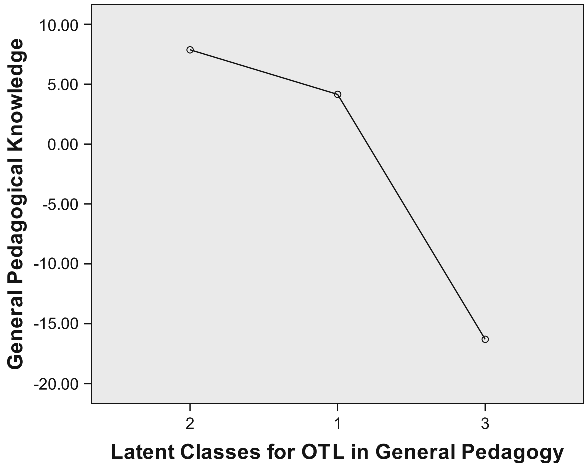

Fig. 6 Relationship of OTL profiles in general pedagogy to outcomes of primary teacher education (*1* broad curriculum, *2* functional curriculum, *3* context-related curriculum)

(see Fig. 6). In addition, a tendency existed that teachers belonging to class 2 outperformed teachers from class 1 (mean difference = 3.6 test points). This is an unexpected but similar result to mathematics pedagogy and may reflect the functional orientation of the test instrument. The difference between classes 2 and 3 was almost 25 test points which means a small but relevant effect.

5 Discussion

The aim of this paper was to identify profiles of the mathematics teacher education curriculum in 15 countries as reported by future primary teachers in their final year of training. We can summarize that a comprehensive core curriculum accepted in all TEDS-M countries neither existed in mathematics pedagogy and general pedagogy nor in mathematics. The latter result is more surprising than the first. Based on research about student achievement (Mullis et al. 2008), more homogeneity of OTL in this component of teacher education could have been expected.

In contrast, the state of research on mathematics pedagogy and general pedagogy had indicated large cultural diversity (Bishop 2004; Blömeke and Paine 2008). Based on our evidence we are able to conclude though that the heterogeneity of primary teacher education may be less pronounced than usually discussed. In mathematics as well as in mathematics pedagogy and general pedagogy, topics existed which were taken by most teachers. Furthermore, it was sufficient to distinguish between a few profiles of OTL in each component to describe appropriately the mathematics, mathematics pedagogy and general pedagogy curriculum. Even though the fit criteria did not always match, they pointed in all cases to more than one class but a maximum of four classes. The common topics and the low number of profiles reveal that some homogeneity exists in primary teacher education across countries. It may reflect shared visions of what primary teachers are supposed to know before they enter the teaching profession.

In mathematics, number theory and probability represented common topics. In mathematics pedagogy, teaching methods, and in general pedagogy, educational psychology represented such topics. They can, in fact, be regarded as important preconditions for teaching mathematics in primary schools.

Three profiles were identified in each teacher education component. In mathematics, they are best described as "advanced university mathematics", "basic university mathematics" and a restriction to "school mathematics". The dominating philosophy across the TEDS-M countries was to provide OTL in basic university mathematics. In mathematics pedagogy and in general pedagogy, two of the three profiles were similar to each other: a "broad (mathematics or general) pedagogy curriculum" and a "functional (mathematics or general) pedagogy curriculum". The third profiles were "teaching methods" (in mathematics pedagogy) or "context-related conditions" (in general pedagogy). In both components of primary teacher education, the dominating philosophy across the 15 TEDSM countries was to provide a broad curriculum.

Countries were not evenly distributed in these classes. A distinct classification was visible which may represent cultural influences. In mathematics, future teachers had then more OTL if they were trained as subject specialists and supposed to teach mainly mathematics. This profile applied to Thailand and Malaysia. It explains especially well the good results of Thailand in the overall rankings of TEDS-M compared to what one would expect based on their students' achievement in TIMSS (Mullis et al. 2008). Regarding the pedagogical OTL profiles, primary teachers from the "East" (Taiwan and Singapore but also Poland and partly Russia) reflected functional OTL in mathematics pedagogy and those from Singapore in addition in general pedagogy. Some indications exist that the OTL profiles in other countries were influenced by educational philosophies as well.

Besides the identification of dominating profiles and cultural influences on these, which may reflect shared but distinct visions of what primary teachers should know *between* countries, *within*-country variation existed in all three teacher education components. Programs differed systematically according to the distribution of class membership. This result indicates between-program heterogeneity, especially in those countries with large numbers of institutions and programs. It seems as even though culturally shaped visions of teacher knowledge on the country level exist, within the countries programs may have implemented different versions of these visions.

In all teacher education components higher investments—if defined as extensive coverage of the topics listed—corresponded with higher outcomes. Future primary teachers with a chance to learn advanced or basic university mathematics, broad or functional mathematics pedagogy as well as general pedagogy did significantly better on our achievement tests than future teachers who had only the chance to rely on school mathematics, teaching methods or context-related topics. However, it is important to point out that the nature of the two pedagogy assessments has to be taken into account which played out in favor of functional topics.

We presented a model of OTL in primary teacher education across the 15 TEDS-M countries. It represents well those topics with latent class probabilities close to 0 or 1. If the probability is around 0.5, it is equally likely that a teacher in that class had been exposed to the topic or not. If the future teachers had studied under the same conditions, the interpretation would be that this topic is an elective. However, we examined individuals from different countries, institutions, teacher education programs and license levels. So, cultural or program specific priorities could exist although teachers were classified into the same class. Yet, it is interesting to note that only a few topics were taken with a probability of around 0.5 and this in only some classes.

A limit of our analysis is the nature of our OTL data (future teachers' self reports). It was felt that since the respondents were still students when asked to respond to the survey that they would be able to remember accurately the content they studied. This, however, may be a weak assumption. Furthermore, broad terms like "linear algebra" or "educational psychology" leave not only room for considerable variation but, as in all comparative studies (van de Vijver and Leung 1997), a risk exists that responders differ in their willingness to mark such a topic as "studied" or "not studied".

However, our results provide first insights into the profiles of different kinds of curricula in primary teacher education and how they were shaped. The details are worthwhile to be examined in more detail in the future. We developed a first approach that leads to the conclusion that more homogeneity may exist in mathematics pedagogy and general pedagogy, but less in mathematics as usually discussed. The specific philosophies underlying the curriculum profiles are probably deeply rooted either in educational cultures and/or in the nature of the teaching profession.

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Family Background, Entry Selectivity and Opportunities to Learn: What Matters in Primary Teacher Education? An International Comparison of Fifteen Countries

Sigrid Blömeke, Ute Suhl, Gabriele Kaiser, and Martina Döhrmann

Abstract First findings of IEA's "Teacher Education and Development Study in Mathematics (TEDS-M)" had revealed differences in the demographic background, prior knowledge, opportunities to learn (OTL), and outcomes of primary teacher education between future teachers from different countries. In this chapter, two hypotheses are examined: (1) OTL and teacher background are significant predictors of mathematics content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK) as teacher education outcomes. (2) OTL effects are partly mediated by differential student teacher intake. Data from multilevel models reveal that effects on MCK are in general larger than on MPCK. Gender, prior knowledge and OTL in mathematics are significantly related to both types of outcomes whereas other background characteristics affect MPCK only. Motivation mediates the effects of prior knowledge and the OTL effects are partly mediated by teacher intake. Consequences for educational policy are discussed based on these results. Policymakers have on the one hand to be aware of the continuing problem of societal inequalities. Providing OTL in mathematics as well as increasing entrance selectivity may, on the other hand, have positive consequences for the outcomes of primary teacher education.

Keywords Effectiveness (of teacher education) · Teacher knowledge · MCK · MPCK · Opportunities to learn · OTL · Value added · Comparative study · Pre-service teacher education

Adjusted version of Blömeke et al. (2012).

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S. Blömeke et al. (eds.), *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn*, Advances in Mathematics Education,
DOI 10.1007/978-94-007-6437-8_15,

1 Introduction

With the publication of comparative studies on K-12 *student* achievement, the competencies of their *teachers* have become areas of considerable interest. This interest is reflected by the "Teacher Education and Development Study: Learning to Teach Mathematics (TEDS-M)" which examined the competencies of mathematics teachers in fifteen countries at the end of their training (Tatto et al. 2008, 2013).[1] Mathematics teachers have a central role in the preparation of future generations of K-12 students. Mathematics not only belongs to the core academic subjects worldwide (Mullis et al. 2008) but is also essential for meeting everyday occupational requirements (Freudenthal 1983).

An examination of mathematics teachers' competencies and to ascertain whether and how teacher education contributes to their development is therefore one of the most important parameters of school quality. Efforts to fill corresponding research gaps have already been made since the 1990s (Cochran-Smith and Zeichner 2005; Darling-Hammond 2000). Most of the research, however, focused on future teacher *beliefs* as one subdomain of teacher competencies (see e.g. Bramald et al. 1995; Calderhead 1991; Tamir 1988). Large-scale assessments or studies including direct measures of teacher *knowledge* as another subdomain of teacher competencies are still widely lacking (Brouwer 2010; Wilson et al. 2002).

TEDS-M offers a unique chance to examine the relationship of teacher education and future teachers' knowledge in detail. It was the first comparative large-scale assessment of higher education in which graduates from fifteen countries were tested. The first descriptive results revealed significant mean differences in the future teachers' background, their opportunities to learn (OTL) during teacher education, and outcomes in terms of mathematics content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK) between countries (Babcock et al. 2010; Blömeke et al. 2011, 2012; Hsieh et al. 2010; Oser et al. 2010). It had to remain an open question though to what extent teacher background and OTL influenced the outcomes. This relationship, examined with respect to future primary teachers, is the focus of the present chapter.

In examining the effects of program characteristics on teacher education outcomes, the chapter contributes to effectiveness research on the level of the teacher education system. It transfers an approach frequently used in K-12 research where effectiveness is defined as "the degree to which schools achieve their goals, in comparison with other schools that are 'equalized', in terms of student-intake" (Scheerens 2000, p. 20). The advantage of an approach on the system level is a high precision of estimates due to large sample sizes. At the same time our study is a first approximation of a value-added model because the effects of OTL in teacher

[1]The international costs of TEDS-M were funded by the IEA, the US National Science Foundation (REC 0514431) and the participating countries. The instruments are copyrighted by the TEDS-M International Study Center at MSU (ISC). This publication was made possible by grant number BL 548/3-1 from the German Research Foundation (DFG). Any views expressed in this paper are those of the authors and do not necessarily reflect the views of the funders or the ISC.

education are not distorted by teacher background (McCaffrey et al. 2003). The advantage of such an approach is that it filters out characteristics which are not under the control of teacher education.

2 Theoretical Framework

2.1 Opportunities to Learn

TEDS-M followed the tradition of the IEA in connecting educational *opportunity* and educational *achievement*. As it was done in TIMSS, OTL were framed as content coverage, specifically as "the content of what is being taught, the relative importance given to various aspects of mathematics and the student achievement relative to these priorities and content" (Travers and Westbury 1989). OTL were in this sense defined as future primary teachers' encountering occasions to learn about particular topics during teacher education. Since subject matter specificity is the defining element of an educational opportunity (Schmidt et al. 1997), in the case of TEDS-M as a study about "learning to teach *mathematics*" the particular topics reflected the areas of mathematics and mathematics pedagogy. Teaching mathematics represents a small but important part of primary teachers' responsibilities since they usually work as class teachers and teach most subjects.

OTL in teacher education can be regarded as having been intentionally developed by educational policymakers and teacher education institutions (Schmidt et al. 2008). They give characteristic shape and direction to instruction. Every choice provides some OTL at the expense of others. National program choices in this sense reflect particular visions of what primary teachers are supposed to know and be able to do in class and how teacher education should be organized in order to provide the knowledge and skills necessary for successful accomplishment of their professional tasks.

In expansion of TIMSS, TEDS-M also examined the *quality* of OTL, e.g. the teaching methods experienced during teacher education (McDonnell 1995). The idea of teacher education as a model for future teaching in class has always played an important role in pedagogical discourse; see e.g. the theory of "signature pedagogies" developed by Shulman (2005). In this paper we take OTL quality into account by including research-based learning approaches.

2.2 Teacher Background

In studies of school effectiveness, not only OTL but also K-12 student background is almost always a powerful predictor of achievement. Specifically with respect to mathematics, gender, socio-economic status and language background as well as generic and domain-specific prior knowledge play an important role (Scheerens and

Bosker 1997). Equity with respect to these characteristics is rarely accomplished. It is reasonable to assume that the same applies to teacher education.

Mathematics has been regarded as a male-dominated subject for a long time (Burton 2001). Longitudinal and trend studies reveal that even though differential mathematics achievement by *gender* has decreased over the past decades, females still show lower achievement compared with their male counterparts in mathematics tests in higher school grades and college (Fan et al. 1997; Hyde et al. 2008). The reasons for such inequity mainly point to socio-psychological aspects: the females had received less support and encouragement from teachers and parents and they had had fewer opportunities to learn mathematics (Henrion 1997). One of the few studies on gender effects in teacher education, the comparative study "Mathematics Teaching in the 21st Century (MT21)", carried out with future lower-secondary mathematics teachers (Schmidt et al. 2011), provides evidence that gender-related achievement differences in mathematics also apply to teacher education. Male lower secondary teachers from Germany significantly outperformed their female counterparts in mathematics tests (Blömeke and Kaiser 2010).

Differences in *language background* are a characteristic known to be associated with K-12 student achievement as well. In many countries, those students whose OTL occur in their second language perform significantly worse than first-language learners (Walter and Taskinen 2007). Classroom discourse plays a major role in this context as Schütte and Kaiser (2011) show with respect to German primary students. The magnitude of language disadvantages is usually increased by the difference between language skills sufficient for communication at home or with peers and the language proficiency necessary for school success (Council of Chief State School Officers 1990; Cummins 1983). Correspondingly, Thomas and Collier (1997) found cumulative effects in the sense that language effects increase in higher grades of K-12 schooling.

Students' *socio-economic status* is generally significantly associated with achievement as well (Coleman et al. 1966). The higher the SES, the better students perform in tests. SES represents in this context access to resources important for learning like wealth or education (Mueller and Parcel 1981). These resources are actively used, or implicitly play out, as support for student progress.

Prior *generic* and *domain-specific knowledge* has to be included in a study about teacher education effects not only because it has frequently proven to be associated with K-12 student achievement in a strongly positive way (Simmons 1995) but also because not correcting for it could result in an overestimation of other background or institutional effects (Goldhaber and Brewer 1997; Thomas and Mortimore 1996). Prior knowledge has to be regarded as probably having been affected by these characteristics in the past.

Motivation is often positively related to learning outcomes, especially if the learning tasks are complex (Benware and Deci 1984; Grolnick and Ryan 1987) and if motivation is modeled as intrinsic motivation (Singh et al. 2002). With respect to teachers, intrinsic reasons to decide on this profession can be distinguished into *altruistic-pedagogical* and *subject-related motives* (Brookhart and Freeman 1992; Watt and Richardson 2007). How these affect cognitive outcomes of primary teacher education is an open question. The effects of *extrinsic motivation* on achievement—with respect to teachers it is related to job security and job benefits (Brookhart and Freeman 1992)—are generally mixed (Ryan and Deci 2000). A controversy exists about the extent to which motivation can be regarded as a background characteristic at all. Some researchers argue that including motivation in effectiveness studies would require the inclusion of a variable that may mediate real background effects like socio-economic status and therefore represents an explanation of how these effects play out. Thus, motives will only be included stepwise in the present study about primary teacher education.

2.3 Outcomes of Teacher Education

TEDS-M is based on the notion of professional competencies as they are defined in general by Weinert (2001) and specifically with regard to teaching by Taconis et al. (2004). Competencies in this tradition mean the cognitive and affective-motivational wherewithal to solve job-related problems successfully. In the case of TEDS-M, cognitive abilities have been categorized into three facets which are frequently discussed in the literature: mathematics content knowledge (MCK), mathematics pedagogical content knowledge (MPCK), and—due to feasibility reasons in only three countries: Germany, Taiwan, and the US—general pedagogical knowledge (GPK) (Blömeke 2002; Shulman 1985). The job-related problems to be dealt with by future primary teachers were defined according to existing standards (see e.g. NCTM 1991).

In the present study, MCK and MPCK were used as indicators of the outcomes of primary teacher education. Since we had GPK data from only three countries, we had to leave out this component of teacher competencies. But by including two subdimensions of teachers' professional competencies we lowered the risk of a "mono-operation bias" (De Maeyer et al. 2010): Evidence exists that teachers need to draw on MCK *and* MPCK in order to foster student achievement in mathematics (Baumert et al. 2010). If we used only one of these as outcome indicator, we would miss the breadth of teacher competencies. Although school effectiveness research has established a certain degree of consistency across cognitive outcome measures (Scheerens and Bosker 1997; Thomas et al. 1997), we do not have the same kind of information in teacher education research.

3 Hypotheses

In line with the results from school effectiveness research, we hypothesize that OTL matter for teacher education outcomes (H1). More specifically, we expect that across the fifteen TEDS-M countries OTL in mathematics and mathematics pedagogy as well as research-based learning during primary teacher education significantly predict outcomes in terms of MCK and MPCK. The strengths of the relationships may vary though. OTL in mathematics should have a stronger impact on MCK than on MPCK, and OTL in mathematics pedagogy should have a stronger impact on MPCK than on MCK because the respective predictors and outcomes correspond more closely to each other. Still, we expect cross-effects, especially an influence of OTL in mathematics on MPCK, because MPCK requires by definition MCK and the two latent traits correlate. Research-based learning should have a stronger influence on MPCK because—in the way it was defined in TEDS-M (including videos of mathematics instruction, for example)—it is much more prominent in the field of mathematics pedagogy than in mathematics.

At the same time, and again in line with the results from school effectiveness research, we suppose that background matters for teacher education outcomes (H2). In particular, we hypothesize significant effects of gender (in favor of males), socioeconomic status (in favor of higher SES) and language background (in favor of first-language learners), prior generic and domain-specific knowledge (in favor of those primary teachers with higher perceived high-school achievement), and motivation (in favor of those with higher altruistic-pedagogical and subject-related motives and lower extrinsic motives) on the acquisition of MCK and MPCK.

Finally, we hypothesize that OTL effects are partly mediated by differential teacher intake (H3). The first descriptive results of TEDS-M had revealed that the composition of future teachers differed in many countries by teacher education program (Tatto et al. 2013). This applied especially to prior knowledge in the sense that teachers who reported better high-school achievement were more often selected—either formally by the institutions or by self-selection—for teacher education programs with more OTL in mathematics and mathematics pedagogy.

4 Study Design

4.1 Sample

The target population of the present study was defined as future teachers in their final year of teacher education who would receive a license to teach mathematics in primary schools (Tatto et al. 2008). This definition included primary teachers who would work as class teachers. A teacher education program was identified as preparing primary teachers if the license covered one of the grades 1 through 4 as the common denominator of education level 1 in the "International Standard Classification of Education" (primary or basic education, cycle 1; UNESCO 1997).

In a two-stage process, random samples were drawn from this target population in each participating country. The samples were stratified according to important teacher education features like "route" (consecutive vs. concurrent programs), "type" of program (grade span the license included, e.g. grades 1 through 4 vs. 1

Table 1 Participating countries in the TEDS-M primary study

Botswana	Chile[e]	Germany	Georgia
Malaysia	Norway[d,e]	Philippines	Poland[b,e]
Russia	Spain	Switzerland[a]	Singapore
Taiwan	Thailand	USA[c,e,f]	

[a]Colleges of Education in German-speaking regions

[b]Institutions with concurrent programs

[c]Public Universities

[d]Results for Norway are reported by combining the two data sets available to cover the entire population of primary future teachers

[e]Combined Participation Rate <75 %

[f]Substantial proportion of missing values

through 10) or "focus" of opportunities to learn (with or without extensive opportunities to learn mathematics) in order to reflect accurately the distribution of primary teachers' characteristics at the end of their training.

In 2008, about 14 000 future primary teachers from more than 500 teacher education programs in fifteen countries (see Table 1) were tested on their MCK and MPCK in a standardized paper-and-pencil assessment. All countries had to meet the quality requirements of the "International Association for the Evaluation of Educational Achievement (IEA)" as known from studies like the "Third International Mathematics and Science Study (TIMSS)". These included controlling of translation processes, monitoring of test situations, and meeting participation rates. If a country missed the participation benchmark only slightly, its results are reported briefly ("Combined Participation Rate <75 %"). This applies to Chile, Norway, Poland, and the US. In the US, about a quarter of the primary sample had to use a shortened version of the survey instrument for administrative reasons. Therefore, the basic proportion of missing values is higher than in other countries.

In most countries, TEDS-M covered the full target population. Only Switzerland, Poland and the US had to limit their study for economic or other reasons. In Poland, due to difficulties identifying the target population, it was not feasible to include about 10 % of the teacher education institutions where teachers were trained in consecutive programs only. In the US, it was not feasible immediately to include private universities where about one third of the teachers in the target population were trained. They were examined in a separate step; the results did not differ systematically from those at public universities. In Switzerland, only the German speaking regions agreed to participate in the study. Particularly complex is the composition of the Norwegian sample. Data from two different primary programs are available for this country. Although these sub-populations are not completely disjunct because students had the chance to change to the other programs, the present chapter combines them in order to cover the entire population of primary future teachers in Norway.

4.2 Instruments

The *gender* variable was dichotomous with two values (0: female, 1: male). Across the fifteen TEDS-M countries, on average 81 % of the primary teachers in their final year of training were female (range: 59 % in Botswana through 100 % in Georgia).

The *language* spoken at home in contrast to the official language of instruction in teacher education was captured with a four-point Likert scale (0: "never" through 3: "always"). A distinct difference between two groups of countries existed. In Botswana, Malaysia, and the Philippines, future teachers were tested in English although this was the language always or almost always spoken at home by less than 13 %. In Singapore, Thailand and Taiwan, between 30 and 40 % of the teachers always or almost always spoke a different language at home. In the other nine countries, between 86 and 99 % of the future teachers always or almost always spoke the official language of instruction at home.

Measuring *socio-economic status* (SES) is complex. Owing to its multidimensionality, SES can be indicated by different aspects or be a composite of parental education, home resources, parental occupation, and/or parental income (Sirin 2005; van Ewijk and Sleegers 2010). These subdimensions are commonly associated with each other but represent different aspects of societal inequality. Based on their meta-analysis, van Ewijk and Sleegers (2010) recommend either the use of a composite or the use of one single indicator as continuous variables. Dichotomies have to be regarded as unreliable measures of the underlying continuous construct. Including several SES indicators may lead to ambiguity in the interpretation and the true effect would probably be underestimated. Therefore, in the present study *parent education* was used as an indicator of SES. It was separately measured for future teachers' fathers and mothers on scales covering the seven most important ISCED levels (1 = "primary" through 7 = "beyond ISCED 5A"). One variable was created to represent the parents' highest education level. On average, almost 40 % of the primary teachers had parents with a university degree (range: 12 % in Botswana through 52 % in Norway).

Perceived high-school achievement was used as a proxy for *generic prior knowledge*. It was measured across school subjects with a five-point Likert scale representing the perceived high-school achievement compared with a future teacher's age cohort (1: "generally below average" through 5: "always at the top"). Across the TEDS-M countries, about 38 % of the primary teachers reported high-school achievement at or near the top (range: 14 % in Germany through 58 % in Malaysia).

Domain-specific prior knowledge was surveyed through the number of mathematics classes taken during K-12 schooling as a proxy (five-point Likert scale from 1: "below year 10" through 5: "year 12 (advanced level)"). Across the fifteen countries, 68 % of the primary teachers reported at least twelve years of mathematics at school with a minimum of 0 % in Russia where high school ends after grade 11 and a maximum of 100 % in Taiwan and Poland where twelve years of mathematics are mandatory.

The *motives* to become a teacher were captured in three subdimensions: *altruistic-pedagogical, subject-related* and *extrinsic motivation*. Four, two or three statements respectively had to be rated on four-point Likert scales (1: "not a reason" through 4: "a major reason"). An indicator of altruistic-pedagogical motives was e.g. "I like working with young people." An indicator of subject-related motives was "I love mathematics" and an indicator of extrinsic motives was "I seek the long-term security associated with being a teacher." On average, altruistic-pedagogical motives dominated the decision to become a primary teacher much more (M = 3.18, SD = 0.65) than extrinsic motives (M = 2.05, SD = 0.69) but also more than subject-related motives (M = 2.04, SD = 0.79). In an international context, the reliability of the pedagogical scale was sufficient (Cronbach's $\alpha = 0.73$) whereas the reliability of the other two scales was only at or slightly above the critical limit ($\alpha = 0.50$ or 0.60 respectively). Thus, the number of items turned out to be too low. If we had used more items, we still would have achieved a higher reliability though. In any case, we have to be wary of drawing conclusions in the context of this study if we do not find significant correlations.

The OTL index for *mathematics* was based on the future primary teachers' responses to what extent content was covered in 15 domains across three key areas: (1) continuity and functions, e.g. beginning calculus or multivariate calculus, (2) discrete structures and logic, e.g. linear algebra or number theory, and (3) geometry, e.g. axiomatic geometry or differential geometry. Opportunities to learn probability and statistics were ignored in this paper because the corresponding knowledge is only poorly represented in the mathematics test. The index represents a regression score (M = 0, SD = 1) with a minimum of −0.75 in Germany (SD = 0.94) and a maximum of 1.56 in Thailand (SD = 0.46) from a factor analysis with the three OTL indices which explained 68 % of the variance.

The OTL index for *mathematics pedagogy* was based on eight domains, including foundations like the development of mathematics ability and thinking, and instructional applications like developing teaching plans. The index once again represents a regression score (M = 0, SD = 1) based on a factor analysis with the two counts which explains 71 % of the variance. The minimum was −1.05 in Germany (SD = 1.11) and the maximum was 0.75 in Malaysia (SD = 0.73).

In TEDS-M, *teaching methods* were captured in several subdomains. For the purpose of this paper, the scale "research-based learning" was chosen which was the only one that corresponds with subject-specific OTL and points to their academic nature of teacher education. Its reliability was good ($\alpha = 0.83$). Four statements covered the reading of research papers as well as active research strategies like analyzing videos. They had to be rated on four-point Likert scales (1: "never" through 4: "often"). Across the fifteen countries, primary teachers reported a medium level of research-based learning during teacher education (M = 2.36, SD = 0.81) with the lowest level in Germany (M = 1.65, SD = 0.67) and the highest in Russia (M = 2.76, SD = 0.70).

TEDS-M sought to measure future teachers' *MCK* and *MPCK* as outcomes at the end of primary teacher education. For this purpose, a 60-minute paper-and-pencil assessment had to be completed during a standardized and monitored test session. The items were supposed to depict classroom performance of mathematics teachers in grades 1 through 4 as closely as possible. A matrix design with five test booklets of the type "Balanced Incomplete Block Design" was applied. Scaled scores were created separately for MCK and MPCK in 1-dimensional models using item response theory. For dichotomous items, the standard Rasch model and for polychotomous items the partial credit model were used (see Tatto et al. 2013). Both item types were analyzed simultaneously with ACER Conquest software (Wu et al. 2007). The resulting achievement estimates were transformed into a scale with an international mean of 500 and a standard deviation of 100 test points.

The 74 items of the mathematics test covered number (25 items), algebra (23) and geometry (21) but only to a small extent data (5). Three cognitive dimensions were covered: knowing (33), applying (29) and reasoning (12). About a quarter of the TEDS-M items have been released by the IEA and are available at: teds@msu.edu.

The 32 items of the mathematics pedagogy test covered two subdimensions: preactive curricular and planning knowledge (16 items) which is necessary before a teacher enters the classroom (e.g. establishing appropriate learning goals, knowing different assessment formats or linking pedagogical methods and instructional designs, identifying different approaches for solving mathematical problems) and interactive knowledge about how to enact mathematics for teaching and learning (16 items; e.g. diagnosing typical students' responses including misconceptions, explaining or representing mathematical concepts or procedures, providing appropriate feedback).

4.3 Validity of the TEDS-M Measures

As an IEA study, TEDS-M had to meet the benchmarks set by prior large-scale assessments like TIMSS in order to prove validity of its instruments. First of all, the item development had to follow a conceptual framework (Tatto et al. 2008) and it had to be connected to previous research. These precautions provided strong validity-related evidence regarding the content of the scales as well as their meaningfulness and appropriateness. To avoid cultural bias, items had to be sent in from all participating countries. The item pool was reviewed by large groups of experts, and this on the international level and within the participating countries. Translation processes had to follow strict rules and they were controlled by the IEA headquarter. All national research coordinators had to approve the final version of the different instruments in order to satisfy ethical aspects of the research.

In addition to this conceptual validity, measures were taken to ensure high psychometric quality, including the provision of internal-consistency evidence, score reliability evidence, and particularly evidence of measurement invariance (see Tatto et al. 2013). Based on data from an extensive pilot study, initial exploratory factor analyses were carried out. These were followed by confirmatory factor analyses based on data from the main study and referring to the conceptual framework in order to assess the fit of each scale to the data. The structure of the scales was similar to the pilot findings and there was strong consistency between the primary future teacher and secondary future teacher studies. These results again provided validity-related evidence regarding the construct definitions.

To assess the degree to which these factor structures were invariant across countries, Multiple Group Confirmatory Factor Analysis (MCFA) was used. The results provided evidence of the fit of the given factor structure in each country—an important test to defend the meaningfulness of each scale within and across countries.

OTL was measured by asking the future teachers what they perceived had been covered. Such self-reported data always includes certain kinds of risks. Therefore, evidence was collected to prove the validity of these data by correlating the future teacher data to curriculum data (Blömeke and Kaiser 2010).

4.4 Data Analysis

The analyses took the multi-level structure of the TEDS-M data into account. The international sampling plan used a stratified multi-stage probability sampling design (Tatto et al. 2013). The future teachers (individual level) were randomly selected from a list of future teachers for each of the randomly selected teacher education institutions in a country. Teachers from all teacher education programs (level 2) offered by an institution selected were considered in scope if the license formally allowed for the teaching of mathematics in one of the grades 1 through 4 (including in a class teacher's role) and if they were in their final year of teacher education. Countries represented the third level in our multi-level analyses.

Explicitly modeling the cluster structure has several advantages. First, we obtain statistically efficient estimates of regression coefficients and correct standard errors (Hox 2002). Second, and this is important in the context of this paper, we can use covariates at any level of the hierarchy which enables us to examine the extent to which differences in achievement are accountable for by OTL or teacher background. One measure in this context is to adjust for intake differences.

The influence of individual level characteristics (teacher background) on MCK and MPCK was examined first. The background variables were introduced by group centering in order to separate level-1 effects from higher-level effects accurately. When level-2 effects were examined (OTL and teacher intake), the individual-level variables were controlled and therefore introduced by grand mean centering them. In order to determine the mediating effect of teacher intake, it is not only important to state separate significant effects of the predictors as well as of the mediator on outcomes of teacher education but also a significant relationship between the predictors and the mediator (Baron and Kenny 1986). Therefore, an additional multi-level model will be estimated in which this relationship is examined.

In order to check how justified it was to aggregate the OTL and teacher-intake data (self-reported high-school achievement) which were collected on the individual level, we estimated the $ICC(K)$ and the $r_{wg(J)}$ indices as indicators of reliability across our clusters and agreement within these (McGraw and Wong 1996; James et al. 1993). Overall, the results indicated that it was justified to aggregate these measures (see Table 2). Based on the $ICC(K)$ index, we can conclude that all four

Table 2 Indices of reliability and agreement of future primary teachers with respect to self-reported OTL and high-school achievement

	$ICC(K)$	$ICC(K)$ range	$r_{wg(J)}$	$r_{wg(J)}$ range
OTL in mathematics	0.85	(0.53–1.00)	0.90	(0.83–0.97)
OTL in mathematics pedagogy	0.88	(0.55–0.99)	0.84	(0.77–0.90)
Research-based learning	0.88	(0.63–0.99)	0.67	(0.50–0.78)
Teacher intake	0.83	(0.50–0.97)	0.59	(0.38–0.79)

measures were stable enough across programs in the 15 TEDS-M countries to use them as composites (LeBreton and Senter 2008). The average reliability was very good and none of the scales dropped below 0.50 in any of the countries.

Based on the $r_{wg(J)}$ index, the within-group agreement was sufficient with respect to both OTL measures and research-based learning as well. The perceived high-school achievement showed only moderate agreement though (ibid.). However, the average reliability across teacher education programs was high. This result pointed to a lack of consensus *within* programs—may be because of an insufficient supply of applicants so that institutions had to fill their slots with a wide range of future teachers—but still to a relative high consistency across programs which is the more important feature in the context of our analyses.

Within countries, it can reasonably be assumed that effects of predictors play out in the same way. Thus, slopes were defined to be the same across programs in our multi-level analyses. In contrast, due to possible cultural differences between countries the strength of effects like gender could vary on this level. If the number of countries were large enough, random slopes should be estimated. However, due to the already relatively low number of countries this procedure was not feasible in our case and the strength of predictor effects was defined to be the same across countries as well.

One question was whether the model for the examination of OTL effects had to include these variables—introduced on the aggregated level—on the individual level as well. In many studies of composition effects this is a common practice and it is recommended in technical handbooks (see e.g. Snijders and Bosker 1999) because peer effects would be overestimated otherwise. In fact, we followed this recommendation when we examined the role of teacher intake. The focus was different when OTL were concerned, however. Here, we were not interested in separating individual and composition effects. The variables represented OTL offered by programs according to their specific requirements but may have been used with some variation by the future teachers. The mixture of level-1 and level-2 effects is therefore precisely what we would like to obtain.

By including two indicators of teacher education outcomes—MCK and MPCK—we increased the construct validity of our study. At the same time, however, we "bought" an increased risk of type 1 errors because our dependant variables were correlated to each other (Hox 2002). The range varied from a strong manifest correlation in Poland ($r = 0.68$) to a low correlation in Botswana ($r = 0.28$). A multivariate multi-level model would have taken care of this problem but it was not feasible. We already had three levels to consider—future teachers, teacher education programs and countries—so that adding another level would have led to unstable results and difficulties in interpreting the results. Given the obvious fact that the risk of missing important effects is negligible (De Maeyer et al. 2010), we applied two univariate three-level models.

Weights were incorporated in order to reflect non-response rates so that robust population estimates could be obtained. Teacher education programs with fewer than four future teachers in an institution were excluded from the analyses in order to insure stable estimates. This measure reduced the original data set of 13 871 primary teachers in their final year of teacher education to 13 829 (=99.7 %) nested in 527 teacher education programs and fifteen countries.

Given this large sample size, statistical significance is not sufficient to distinguish between practically relevant results and results less relevant. Therefore, each effect will be discussed with respect to its practical relevance based on its proportion of one standard deviation. All analyses were done with HLM for Windows Version 6.08.

5 Results

5.1 Variance in the Outcomes of Primary Teacher Education

The unconditioned models revealed that the country level explained a large proportion of variance in the outcomes of primary teacher education. About one-third of the MCK as well as of the MPCK variance was explained by this level (see the footnotes below Tables 4 and 5). This result reflects the huge disparity in the country means (see Table 3). Systematic variance also existed between teacher education programs within countries. The proportion of variance in the future teacher level was higher in the case of MPCK than MCK.

5.2 Effects of Background Characteristics on Teacher Education Outcomes

Our data generally supported H2 in that background matters for outcomes of primary teacher education. We have to be careful, however. There was large variation depending on whether we examined MCK or MPCK and whether we examined demographics, prior knowledge or motivation.

With respect to MCK (see Table 4), *gender* turned out to be the most important individual characteristic across the participating TEDS-M countries. On average, a difference between one-fifth—if gender was introduced separately—or even a quarter of a standard deviation—if the other background characteristics

Table 3 Means and standard errors (SE) of future primary teachers' MCK and MPCK

Mathematics content knowledge		Mathematics pedagogical content knowledge	
Country	Mean (SE)	Country	Mean (SE)
Taiwan	623 (4.2)	Singapore	593 (3.4)
Singapore	590 (3.1)	Taiwan	592 (2.3)
Switzerland[a]	543 (1.9)	Norway[d,e]	545 (2.4)
Russia	535 (9.9)	USA[c,e,f]	544 (2.5)
Thailand	528 (2.3)	Switzerland[a]	537 (1.6)
Norway[d,e]	519 (2.6)	Russia	512 (8.1)
USA[c,e,f]	518 (4.1)	Thailand	506 (2.3)
Germany	510 (2.7)	Malaysia	503 (3.1)
International	**500 (1.2)**	Germany	502 (4.0)
Poland[b,e]	490 (2.2)	**International**	**500 (1.3)**
Malaysia	488 (1.8)	Spain	492 (2.2)
Spain	481 (2.6)	Poland[b,e]	478 (1.8)
Botswana	441 (5.9)	Philippines	457 (9.7)
Philippines	440 (7.7)	Botswana	448 (8.8)
Chile[e]	413 (2.1)	Chile[e]	425 (3.7)
Georgia	345 (3.9)	Georgia	345 (4.9)

[a]Colleges of Education in German-speaking regions

[b]Institutions with concurrent programs

[c]Public Universities

[d]Results for Norway are reported by combining the two data sets available in order to approximate a country mean

[e]Combined Participation Rate <75 %

[f]High proportion of missing values

were controlled—between male and female teachers existed in favor of the males. This is a highly substantial effect. In contrast, future teachers' *language background* and their *parents' education* were influential but the effect sizes were small. The effect of language even disappeared when the teachers' motivation was controlled.

Important for the acquisition of MCK were both proxies of prior knowledge, the *perceived high-school achievement* as well as the *number of mathematics classes*. Those future primary teachers within a program who perceived themselves as good students compared to their peers and reported more years of mathematics during schooling performed better on average in our MCK test. One more year of mathematics and a one-point difference on the perceived high-school achievement scale led to a difference of about twelve test points. Once motivation was introduced, the effect sizes of perceived high-school achievement and number of math-

Table 4 Three-level modeling of future primary teachers' MCK regressed on background characteristics

	M1 *b* (SE)	M2 *b* (SE)	M3 *b* (SE)	M4 *b* (SE)	M5 *b* (SE)	M6 *b* (SE)	M7 *b* (SE)	M8 *b* (SE)	M9 *b* (SE)	M10 *b* (SE)
Individual predictors										
Gender	20.7*** (3.2)					23.7*** (2.9)				24.7*** (2.8)
Parent education		2.2** (0.8)				1.8* (0.9)				2.0* (0.9)
Language background			2.9* (1.5)			3.3** (1.2)				ns
High-school achievement				11.7*** (1.0)		12.3*** (1.0)				9.8*** (0.8)
Mathematics classes					13.8*** (1.5)	11.8*** (1.3)				9.5*** (1.4)
Pedagogical motives							ns			−4.4*** (1.2)
Subject-rel. motives								13.8*** (2.0)		13.2*** (1.6)
Extrinsic motives									−2.2** (0.8)	−6.9*** (1.6)

Variance components in the unconditioned model: Country 33.6 %, Teacher education program 18.6 %, Future teacher 47.9 %. Predictors have been centered around their group means (programs within institutions) in order to obtain pure level-1 effects. ns: not significant, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

ematics classes taken decreased slightly. This result may indicate a mediating effect.

Motivation itself had a varying influence on the acquisition of MCK—depending on which subdimension was concerned. Across the fifteen TEDS-M countries, the correlation of *subject-matter related motives* to *subject-matter knowledge* was positive and especially strong, even stronger than prior knowledge. The correlation of *extrinsic motivation* to MCK was significant as well but negative. *Altruistic-pedagogical motives* neither supported nor limited substantially the acquisition of MCK if this characteristic was introduced separately. If all background characteristics were controlled, a small negative effect emerged.

With respect to MPCK (see Table 5), fewer or less substantial effects of background characteristics existed across the fifteen TEDS-M countries. With respect to demographics, only *gender* had on average a small significant effect in favor of male primary teachers and this effect even disappeared if the other background variables were controlled. Neither which language a future teacher spoke at home nor his/her *parents' educational background* was significantly correlated to the acquisition of MPCK.

Table 5 Three-level modeling of future primary teachers' MPCK regressed on background characteristics

	M1 *b* (SE)	M2 *b* (SE)	M3 *b* (SE)	M4 *b* (SE)	M5 *b* (SE)	M6 *b* (SE)	M7 *b* (SE)	M8 *b* (SE)	M9 *b* (SE)	M10 *b* (SE)
Individual predictors										
Gender	4.8* (2.2)					ns				ns
Parent education		ns				ns				ns
Language background			ns			ns				ns
High-school achievement				10.4*** (0.8)		10.1*** (0.9)				8.8*** (0.9)
Mathematics classes					9.0*** (1.1)	7.1*** (0.7)				5.5*** (0.6)
Pedagogical motives							3.4** (1.3)			ns
Subject-rel. motives								10.9*** (1.2)		9.1*** (1.5)
Extrinsic motives									−3.0*** (0.9)	−7.2*** (1.6)

Variance components in the unconditioned model: Country 31.6 %, Teacher education program 11.5 %, Future teacher 56.9 %. Predictors have been centered around their group means (programs within institutions) in order to obtain pure level-1 effects. ns: not significant, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

In contrast, both proxies of *prior knowledge* turned out to be significantly influential in relation to the acquisition of MPCK. On average, those future primary teachers who indicated better school achievement by one point (e.g. the difference between "generally about average for my year level" and "generally above average for my year level") performed better by ten test points. One more year of mathematics at school added another seven test points. Also in this case, the effect sizes of perceived high-school achievement and the number of mathematics classes decreased slightly once motivation was introduced.

Motivation itself significantly influenced the acquisition of MPCK. Across the TEDS-M countries, the correlation of *subject-matter related motives* to this type of outcome had about the same positive effect size as perceived high-school achievement. If introduced separately, *altruistic-pedagogical motives* had a positive effect on the acquisition of MPCK as well. The effect size was small, however, and disappeared altogether if all background characteristics were controlled. It is important to note that extrinsic motives were generally significantly negatively correlated to the acquisition of MPCK. A one-point difference on the four-point Likert scale was associated with a loss of seven points in the MPCK test.

5.3 *Effects of Opportunities to Learn on Teacher Education Outcomes*

With respect to the acquisition of MCK (see Table 6), two of the program features were of high relevance across the TEDS-M countries: OTL taken in mathematics and OTL taken in mathematics pedagogy. Thus, the data strongly supported H1. Both factors led to differences in MCK of one-third or almost a quarter of a standard deviation in favor of those future teachers in a program where they had had one standard deviation more of the respective OTL during teacher education. In particular, OTL taken in mathematics explained a substantial proportion of variance in the outcomes of primary education between programs whereas the proportion was relatively low in the case of OTL taken in mathematics pedagogy. Correspondingly, the data revealed that—if the OTL were mutually controlled—a substantial proportion of the mathematics pedagogy effect on MCK was mediated by OTL in mathematics. Across the TEDS-M countries, the effect size was almost halved.

The research-based learning during primary teacher education generally did not have a significant effect. The acquisition of MCK was neither supported nor limited significantly by reading research papers or using active research strategies like analyzing videos. In this respect, H1 has to be rejected.

Some of the results for MPCK correspond to the MCK results (see Table 7). Similarly to MCK, OTL in mathematics were important for the acquisition of MPCK. In addition to background characteristics, this type of OTL explained a substantial proportion of variance between teacher education programs in the fifteen TEDS-M countries. No matter whether OTL in mathematics were introduced separately or whether other OTL characteristics were controlled, a difference of a quarter of a standard deviation in MPCK existed in favor of those future teachers whose program had offered one standard deviation more of OTL in mathematics during teacher education.

Slightly less additional variance in MPCK was explained by OTL taken in mathematics pedagogy. If introduced separately, the data revealed that future primary teachers who had taken more of these topics performed better in our test, and this by one-fifth of a standard deviation. Similarly to MCK but against our hypothesis, the relevance of OTL in mathematics pedagogy decreased if the OTL in mathematics were controlled. Across the TEDS-M countries, the effect size was more than halved then.

An interesting deviance from the MCK results was the relevance of research-based learning for the acquisition of MPCK. Even though the substantial positive effect disappeared if the other two OTL variables were controlled, the separate effect may point to an important feature of primary teacher education. The proportion of variance explained across the TEDS-M countries by using active and passive research strategies and the average gain in test points corresponded to the effect size of OTL in mathematics pedagogy.

Table 6 Three-level modeling of future primary teachers' MCK regressed on OTL in teacher education and teacher intake (controlling for background characteristics)

	M0 *b* (SE)	M1 *b* (SE)	M2 *b* (SE)	M3 *b* (SE)	M4 *b* (SE)	M5 *b* (SE)	M6 *b* (SE)
Individual predictors							
Gender	25.5*** (3.2)	25.0*** (2.8)	25.4*** (3.1)	25.4*** (3.1)	25.0*** (2.8)	25.1*** (3.0)	24.7*** (2.8)
Parent education	2.2* (0.9)	2.3* (1.0)	2.2* (0.9)	2.2*	2.3* (1.0)	2.2* (0.9)	2.2* (0.9)
Language background	ns	ns	ns	ns	ns	ns	ns
High-school achievement	10.3*** (0.9)	10.3*** (0.9)	10.2*** (0.9)	10.3*** (0.9)	10.3*** (0.9)	9.7*** (0.8)	9.8*** (0.8)
Mathematics classes	9.8*** (1.3)	9.7*** (1.2)	9.8*** (1.4)	9.8*** (1.3)	9.7*** (1.3)	9.7*** (1.4)	9.6*** (1.3)
Pedagogical motives	−4.7*** (1.0)	−4.4*** (1.2)	−4.8*** (1.0)	−4.8*** (1.0)	−4.4*** (1.2)	−4.7*** (1.0)	−4.4*** (1.2)
Subject-rel. motives	14.4*** (1.9)	13.6*** (1.5)	14.3*** (1.9)	14.3*** (1.9)	13.6*** (1.5)	14.1*** (1.8)	13.5*** (1.5)
Extrinsic motives	−7.5*** (1.8)	−7.4*** (1.7)	−7.5*** (1.8)	−7.5*** (1.8)	−7.4*** (1.7)	−7.2*** (1.7)	−7.2*** (1.6)
Program predictors							
OTL in mathematics		34.1* (14.9)			31.5* (14.6)		25.9* (11.8)
OTL in math pedagogy			23.7* (11.9)		13.7† (7.6)		ns
Research-based learning				ns	ns		ns
Teacher intake						44.7** (13.8)	31.9*** (3.7)
R^2	24.6 %	38.1 %	29.5 %	27.5 %	40.7 %	37.8 %	46.3 %

R^2: Proportion of null-model variance on the program level explained by the respective model. Individual predictors have been centered around the grand mean in order to control for level-1 effects and thus obtain pure level-2 effects. OTL predictors have been centered around their group means (countries) in order to obtain pure level-2 effects. ns: not significant, † $p < 0.10$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

5.4 The Role of Teacher Intake

Entry selection according to perceived high-school achievement seemed to play a major role in the acquisition of MCK and MPCK across the fifteen TEDS-M countries although program effects in terms of OTL in mathematics were still substantial even after controlling for teacher intake and background effects (see Tables 6 and 7, M6). The data revealed that across the TEDS-M countries a difference of about two-

Table 7 Three-level modeling of future primary teachers' MPCK regressed on OTL in teacher education and teacher intake (controlling for background characteristics)

	M0 b (SE)	M1 b (SE)	M2 b (SE)	M3 b (SE)	M4 b (SE)	M5 b (SE)	M6 b (SE)
Individual predictors							
Gender	ns	ns	ns	ns	ns	ns	ns
Parent education	ns	ns	ns	ns	ns	ns	ns
Language background	ns	ns	ns	ns	ns	ns	ns
High-school achievement	9.3*** (1.0)	9.3*** (1.0)	9.3*** (1.0)	9.3*** (1.0)	9.3*** (0.9)	8.6*** (0.8)	8.8*** (0.9)
Mathematics classes	6.0*** (0.6)	5.8*** (0.6)	6.0*** (0.6)	5.8*** (0.6)	5.8*** (0.6)	5.8*** (0.6)	5.7*** (0.6)
Pedagogical motives	ns	ns	ns	ns	ns	ns	ns
Subject-rel. motives	10.3*** (2.3)	9.4*** (1.6)	10.1*** (2.2)	10.2*** (2.3)	9.3*** (1.6)	9.9*** (2.0)	9.2*** (1.6)
Extrinsic motives	−7.8*** (1.8)	−7.7*** (1.7)	−7.8*** (1.8)	−7.8*** (1.8)	−7.7*** (1.7)	−7.4*** (1.6)	−7.4*** (1.6)
Program predictors							
OTL in mathematics		27.0* (12.8)			24.6* (12.3)		19.9* (9.9)
OTL in math pedagogy			20.5* (9.6)		9.7† (5.1)		ns
Research-based learning				22.2* (10.5)	ns		ns
Teacher intake						37.6** (12.2)	26.5*** (3.5)
R^2	20.1 %	32.2 %	25.9 %	25.3 %	36.7 %	32.9 %	41.7 %

R^2: Proportion of null-model variance on the program level explained by the respective model. Individual predictors have been centered around the grand mean in order to control for level-1 effects and thus to obtain pure level-2 effects. OTL predictors have been centered around their group means (countries) in order to obtain pure level-2 effects. ns: not significant, † $p < 0.10$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

fifths of a standard deviation in MCK as well as in MPCK existed between teacher education programs in favor of those programs where the primary teachers reported a one-point higher mean school achievement level if this indicator of teacher intake was introduced separately. These are highly substantial effects. The corresponding school achievement effect on the individual level decreased only slightly after the composite was introduced.

If the composition characteristic was introduced in addition to the OTL characteristics, the intake effect and the effects of OTL in mathematics on MCK and

Table 8 Two-level modeling of teacher intake regressed on OTL in primary teacher education

	M1 *b* (SE)	M2 *b* (SE)	M3 *b* (SE)
Predictors			
OTL in mathematics	$0.15^{\dagger}$ (0.08)		ns
OTL in mathematics pedagogy		0.19^{**} (0.07)	0.15^{*} (0.08)

Variance components in the unconditioned model: Country 46.0 %, Teacher education program 54.0 %. Predictors have been centered around their group means (countries) in order to obtain pure level-2 effects. ns: not significant, † $p < 0.10$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

MPCK decreased by 13 or 11 and by 6 or 5 test points respectively and the effects of OTL in mathematics pedagogy disappeared completely. These results pointed to a mediating effect in the sense that primary teachers with a better perceived school achievement were selected or selected themselves to a higher extent for programs with more OTL to learn mathematics and mathematics pedagogy so that the entrance differences mediated the OTL effects.

In order to support this hypothesis, it is necessary to show in addition to our previous results that OTL as predictors significantly influenced teacher intake as the assumed mediator as well (Baron and Kenny 1986). For this purpose an additional two-level model was examined with programs as level 1 and countries as level 2. This model allowed us to use composition of programs according to perceived high-school achievement as the dependant variable and OTL in mathematics and mathematics pedagogy as predictors.

In fact, if introduced separately both OTL characteristics showed a systematic relationship with the mean level of perceived high-school achievement in teacher education programs (see Table 8). In particular, the effect of OTL in mathematics pedagogy was significant which fits well to our prior results. It seems as if OTL in mathematics pedagogy are an especially important feature of primary teacher education programs that drove the (self-)selection process—and thus have an indirect effect on MCK and MPCK. If examined separately without taking teacher intake into account (Tables 6 and 7, M2 and M4), there is a significant relationship of OTL in mathematics pedagogy to MCK and MPCK. This relationship disappears if one controls for teacher intake. In contrast, OTL in mathematics does not have a significant relationship to teacher intake. So, the effects remain in M6 compared to M4 (see Tables 6 and 7).

6 Discussion

Data from the comparative TEDS-M study revealed that the mathematics content knowledge (MCK) and the mathematics pedagogical content knowledge (MPCK) of primary teachers differed significantly at the end of teacher education between the participating countries and between teacher education programs within countries. In this chapter, we examined to what extent teacher background, prior knowledge, motivation, opportunities to learn (OTL) during teacher education and teacher intake influenced the knowledge acquisition across countries on average in order to contribute to a global theory of teacher education effectiveness.

Our hypothesis that teacher background generally influenced the outcomes of teacher education (H2) was only partly supported by the data. *Gender* turned out to be an important individual characteristic but only with respect to the acquisition of MCK and not with respect to MPCK. In the first case, university training may have suffered from cumulative effects during a long history of gender inequity in K-12 schooling (Hyde et al. 2008). The acquisition of MPCK started only after that, which may have reduced the disadvantages of females.

Against our hypothesis, the *language background* of the teachers and their *parents' education* were relevant neither for MCK nor for MPCK. Given that these are important predictors on the school level (Coleman et al. 1966; Thomas and Collier 1997), this result is surprising. It seems as if the many selection processes during schooling had filtered out those students who were at a disadvantage because of their background.

In contrast, our data strongly confirmed our hypotheses that the *perceived high-school achievement* as well as the *number of mathematics classes at school* significantly correlate with MCK and MPCK. Effect sizes were large in both cases. Assuming that both predictors are appropriate to indicate prior knowledge, these results are in accordance with the general state of research (see e.g. Anderson and Lebière 1998; Simmons 1995). A possible explanation may be that higher prior knowledge facilitates the acquisition of new knowledge, e.g. by supporting the integration of new information into existing schemata, the modification of knowledge structures or the compilation and chunking of knowledge.

With respect to motivation, it is important to distinguish between subdimensions because it had either no practically relevant (*altruistic-pedagogical motives*) or contradictory effects (positive: subject-related motives, negative: extrinsic motives) on the outcomes of primary teacher education. It seems as if the persistence to overcome mathematics-related learning difficulties or to invest time and energy in the learning of mathematics decreases if somebody wants to become a teacher primarily because s/he wants the long-term security of the job but increases if s/he is interested in the subject (Wigfield and Eccles 2000). Some evidence surfaced that motivation was one of the channels through which prior knowledge played out. Further research is needed at this point but such a result would support the critical evaluation laid out at the beginning of the paper that motivation should not be regarded purely as a background characteristic.

With respect to program characteristics, the data supported our hypotheses that OTL and teacher intake are highly relevant to teacher education outcomes (H1 and H3). Both features were introduced as aggregated variables on the program level in order to increase the reliability of the measures. In fact, the $ICC(K)$ estimates revealed strong agreement within programs.

OTL in mathematics were of outstanding relevance for the outcome of primary teacher education. They had not only a strong direct influence on MCK but also on MPCK and they probably mediated the effects of OTL in mathematics pedagogy.

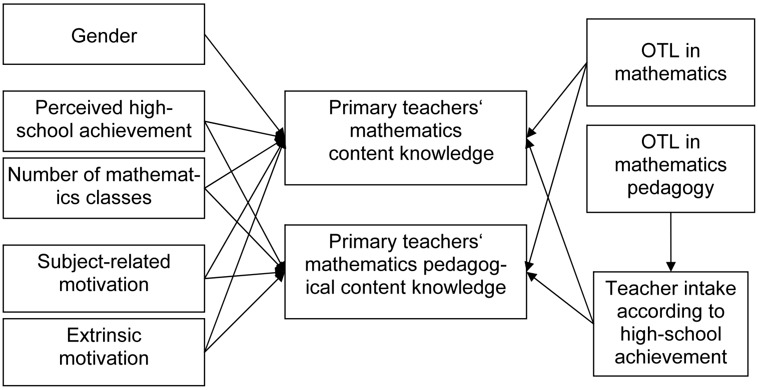

Fig. 1 Hypothetical model of the effects of teacher background, opportunities to learn and teacher intake on outcomes of primary teacher education

These in turn probably mediated the effect of research-based learning although further research is needed about the specifics of these processes.

Besides the relevance of OTL, the relevance of entry selection at the beginning of primary teacher education—either carried out officially by an institution or program or implicitly happening as self-selection by the future teachers—became apparent as well. OTL in mathematics pedagogy were an important feature here and thus had an indirect effect on MCK and MPCK. This result probably reflects the widespread nature of primary teacher education programs as trainings of generalists with broader coverage of mathematics pedagogy than of mathematics. The larger this coverage is, the more it attracts students with higher self-perceived high-school achievement who in turn show higher MCK and MPCK at the end of teacher education.

In addition, the composition effect significantly mediated the effects of opportunities to learn. It is important to note, however, that OTL in mathematics were still substantial even after controlling for teacher intake and background effects.

These results lead to a first hypothetical model of the effectiveness of primary teacher education from a global perspective, which is summarized in Fig. 1.

Before conclusions are drawn, we have to point out some methodological limitations of our study. TEDS-M was a cross-sectional study with a retrospective self-report about school achievement. Longitudinal data and a better measure of prior knowledge are needed for far-reaching conclusions. Furthermore, owing to the low number of countries we had to use a "one size fits all approach" (van Ewijk and Sleegers 2010) with parameter estimates the same for all countries. Thus, a risk exists that country-specific variation in the effects sizes of some predictors was overlooked (with respect to variation in gender and language effects by country see Blömeke et al. 2011). At least for the larger countries in the TEDS-M sample, it seems therefore worthwhile to estimate country-specific models.

In future research, in addition to MCK and MPCK as subject-specific criteria of teacher education outcomes, other cognitive criteria like general pedagogical knowledge or affective characteristics like teacher beliefs should be included in order to develop a full model. Such an approach would increase the validity of a study of teacher education effectiveness. In this context the increased risk of type 1 errors owing to correlation between different criteria should be addressed as well, e.g. by multi-level structural equation modeling.

With respect to effects of single variables, we have to point out that the SES effect may have been underestimated because a single indicator instead of a composite was used (van Ewijk and Sleegers 2010). To create a composite, in our study data about parental occupation were missing. In addition, the reliability of the scale measuring extrinsic motivation was at a critical limit. Since we discovered a significant effect in any case, we can assume that its size was underestimated as well.

7 Conclusions

If school effectiveness can be defined as "the degree to which schools achieve their goals, in comparison with other schools that are 'equalized', in terms of student-intake" (Scheerens 2000, p. 20), we examined in this chapter the effectiveness of teacher education in 527 programs from fifteen countries with respect to MCK and MPCK as cognitive outcomes after equalizing their teacher intake. Future research should continue this line of research but aim at improving some of the methodological weaknesses discussed above. Also, it seems necessary to include classroom observations of teacher performance and possibly even K-12 student achievement to examine the construct validity of our outcome measures. With respect to OTL, it may be beneficial to go into more detail instead of examining broad constructs like "OTL in mathematics" to gain more insight into the relationship between program characteristics and knowledge acquisition. Subdomains like number or algebra or indicators like types of practical experience are worth examination.

Policymakers have to be aware of the continuing problem of societal inequalities even in teacher education outcomes. Special support of female teachers when it comes to the acquisition of MCK in order to overcome cumulative disadvantages of a long history of K-12 schooling seems to be a meaningful measure in many TEDS-M countries.

For achieving an increase of teacher education effectiveness, our study points to two potential measures, each with separate effects. Providing OTL in mathematics as well as increasing entrance selectivity may have positive consequences for the outcomes of primary teacher education and thus in the long run for student achievement in mathematics. Mathematics is one of the most important school subjects and a gatekeeper to academic and professional success. Investments in the training of teachers should therefore pay off quickly. Entrance selectivity is a sensitive issue, however. Not everywhere is teaching at primary schools such a popular and rewarding job that enough applicants for teacher education are available. Higher selectivity, however, may increase the reputation of the profession in the long run so that institutions can recruit from a larger pool.

Teacher educators may want to compare the outcomes of different programs and different institutions in their country. Within almost all countries, huge between-program disparity existed. This means that within the same cultural context some institutions are doing better than others. They may represent a benchmark and provide important information about features of teacher education which can be more easily adapted than features from other countries. Especially the structure and content of the mathematics and the mathematics pedagogy curriculum should be put to the test.

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Primary Teacher Preparation in the United States: What We Have Learned

Leland Cogan, William H. Schmidt, and Richard Houang

Abstract Motivating the recently conducted Teacher Education and Development Study—Mathematics (TEDS-M) was the question of how high performing countries prepare their teachers to teach their challenging curriculum to primary and lower secondary students? The study found that countries prepared teachers in substantially different types of programs. These differences are reflected in the many different teacher preparation approaches available in the United States. Although US private institutions of higher learning attract stronger students on average than their public counterparts, performance of their future teachers on the TEDS-M mathematics knowledge and mathematics pedagogy knowledge scaled scores did not significantly differ. In addition, the balance among the three types of teacher preparation courses, i.e., formal mathematics, mathematics pedagogy, and general pedagogy, was nearly the same in the US and in the top achieving TEDS-M countries. Differences seen in international assessments at eighth grade may indicate that the pool of teacher preparation students also differs among these countries; differences which may affect what is studied and learned in teacher preparation.

Keywords Elementary · Elementary teacher · Teacher preparation · United States · Third International Mathematics and Science Study (TIMSS) · Achievement · Mathematics · Curriculum · Assessment · Future teacher · Teacher Education and

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An expanded version of this chapter was previously released as a research report by the author with the Center for Research in Math and Science Education at Michigan State University (Center for Research in Math and Science Education, 2010).

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S. Blömeke et al. (eds.), *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn*, Advances in Mathematics Education,
DOI 10.1007/978-94-007-6437-8_16,

Development Study in Mathematics (TEDS-M) · Primary · Lower-secondary · Opportunities · Mathematics content knowledge · Pedagogical content knowledge · Performance · Scale · Common Core State Standards · Course Work · Mathematics pedagogy · General pedagogy · Programme for International Student Assessment (PISA) · Formal mathematics · Private institutions · Public institutions · K-12

The Third International Mathematics and Science Study (TIMSS) K-8 curriculum study and the grade 8 achievement data revealed that countries with higher student achievement also had teachers who taught substantially different content than that found elsewhere (see Schmidt et al. 1996, 2001, 2005). Other reports have highlighted the idea that what teachers know and do in the classrooms is consequential for students' learning (National Commission on Teaching & America's Future 1996). US reform efforts, consistent with this line of thought, have introduced standards to measure teacher quality in connection with student achievement, which has led to accountability concerns regarding teacher preparation programs (INTASC 1995; Murray 2000; Leithwood et al. 1999; NCATE 2000).

Given the substantial differences in the coherence, rigor, and focus seen in the mathematics curriculum among the highest achieving countries as identified by the outstanding performance of their students on international assessments, a critical question is, how do high performing countries prepare their teachers to teach challenging curriculum to lower secondary students? This question was the motivation for the small-scale *Mathematics Teaching in the 21st Century* study that contributed to the conceptual framework and instrument development for the Teacher Education and Development Study in Mathematics (TEDS-M) project (Schmidt et al. 2007; Tatto et al. 2008).

1 Sample Design

Future Teachers near the end of their final year of teacher preparation were the focus of the study. Three sampling approaches were used to obtain nationally representative data for participating countries. A few countries such as Norway, Singapore, and Thailand obtained a census of all teacher preparation institutions in their country and a census of all future teachers fitting the TEDS-M target population definitions. Other countries such as Poland, Switzerland, and Taiwan obtained a census of teacher preparation institutions and randomly sampled from eligible future teachers. The last set of countries including the Philippines, the Russian Federation, Spain, and the United States obtained random samples of both teacher preparation institutions and eligible future teachers within each teacher preparation institution. In each case, the specific sampling plan was developed in consultation with the International Association for the Evaluation of Educational Achievement (IEA) sampling referee and deemed appropriate for representing the country's production of possible future teachers of mathematics.[1]

Two approaches for the preparation of primary teachers were found in Germany and Poland. The first prepares generalists who, as in all of the other 15 TEDS-M countries other than the US, teach mathematics along with most of the other subject areas included in a primary curriculum. These teachers are not specifically trained to teach mathematics but are prepared to teach mathematics only as one of the many topics they will be teaching. The second approach in these two countries prepares future mathematics teachers as specialists for the primary grades.

The US also has two approaches but neither one focuses on the preparation of specialists in mathematics. Actually one of the two approaches was described for lower secondary teacher preparation where future teachers are prepared to teach grades 1–8 resulting in future teachers who could teach mathematics at the primary or middle school level. The other approach focuses on preparing generalists for grades 1–5.

No attempt was made to adjust the data obtained from the groups of potential future teachers to reflect who might actually end up teaching in the classroom. The national recruitment and training contexts in each country vary considerably making any such attempt difficult at best. Therefore, the focus of TEDS-M must be understood to be directly on the preparation of potential future teachers of mathematics for either the primary or lower secondary grades and not on characterizing the teaching force in general nor necessarily those who enter the classroom for the first time.

2 Data Collection

TEDS-M sought to measure and to characterize what individuals learned in their teacher preparation programs, e.g., what learning opportunities were provided, how they are structured, and what knowledge may have been gained. This was accomplished primarily by three main surveys developed for this purpose:

1. Institution Program Questionnaire to be completed by an official familiar with the program including entry requirements, academic course requirements, and program length;
2. Educator Survey for those teaching the mathematics, mathematics pedagogy, or general pedagogy courses associated with the program. This brief survey included questions about their academic and professional background and the type of learning activities employed in the courses taught; and
3. Future Teacher Survey and Assessment. These are described in more detail below and the initial results presented in this report are derived solely from this survey.

[1] See the TEDS-M technical manual for a full description of project details including random sampling, translation, weight creation, and quality control.

Table 1 Composition of future teacher survey and assessment

Section	Focus	Time (minutes)
A	Background	5
B	Opportunity to learn (course taking)	15
C	Mathematics content and pedagogical content knowledge assessment	60
D	Beliefs about mathematics and teaching	10
E	General pedagogy knowledge assessment	30

The Future Teacher Survey and Assessment had four main parts and was completed during a standardized administration session. The focus of each part and the time allotted to completing it are shown in the Table 1.

TEDS-M employed a rotated block design in order to measure the desired breadth and depth of knowledge. There were five primary booklets that had rotated blocks of items in Part C and three lower secondary booklets that had rotated item blocks in Part C. Rasch scaling was used to create individual scaled scores for each future teacher (see Tatto et al. 2008, for details of item development, cognitive domain frameworks, and scaling). Results from Part E, which was administered only in Germany, Taiwan, and the US, are not included in this chapter.

The chapter concerns elementary school which, for most of the United States involves grades 1–5, however, for some states it also includes grades 6–8. The story centers on the future teachers who are prepared to teach those grades—who they are, what they studied, and what they know.

3 Results

The mathematics content knowledge measured in TEDS-M focused on the mathematics supporting the topics that would typically be covered in grades 1–8. The test itself, however, measured the type of advanced knowledge teachers should possess in order to teach the more elementary topics typically included in the primary grades. In other words the test itself was not about the mathematics that would be taught to the students but about the mathematics related to and supporting those topics typically taught to children in these early grades. The test itself was developed reflecting what was viewed as the international standard of mathematics knowledge that would be expected of future teachers at the primary level.

3.1 What They Know

International comparative studies present us with the temptation to focus on the ranking of the countries. However, statistically this is not desirable since the rankings are relatively unstable and the differences when characterized by rankings may well suggest differences that are very small and insignificant among pairs of coun-

Table 2 TEDS-M countries' overall performance with respect to mathematics content knowledge at the primary level

Country	Mn	(SE)
Taiwan	623	(4.2)
Singapore	590	(3.1)
Switzerland	543	(1.9)
Russian Federation	535	(9.9)
Thailand	528	(2.3)
United States-private	527	(3.6)
Norway	519	(2.6)
United States-public	518	(4.1)
Germany	510	(2.7)
Poland	490	(2.2)
Malaysia	488	(1.8)
Spain	481	(2.6)
Botswana	441	(5.9)
Philippines	440	(7.6)
Chile	413	(2.1)
Georgia	345	(3.9)
Significantly above US-public		
Not significantly different from US-public		
Significantly below US-public		

tries. For this reason, Table 2 shows the countries divided into three groups, those countries that statistically significantly outperformed the United States public colleges and universities, the group of countries who had a similar performance, and finally the group of countries that the United States public institutions statistically significantly outperformed.

Looking at the table, the United States is found somewhat near the middle of the international distribution suggesting a performance similar to that of Germany, Norway and the Russian Federation, but not at a level of performance consistent with the top achieving countries such as Taiwan, Singapore and Switzerland. This would suggest the mathematical content knowledge of future teachers in the United States is neither distinctive in terms of being particularly low, nor being particularly strong. In any case this is not where we as a nation would like the knowledge level of our primary teachers to be.

Table 3 gives the results for pedagogical content knowledge. What is measured here is also mathematical knowledge but the type of such knowledge needed to understand how the mathematics topics fit together to define the K-12 curriculum, how students learn mathematics and how it should be taught. It is a type of applied mathematics knowledge specifically related to K-5 instruction. Here the performance of the United States future teachers is somewhat stronger, outperforming a larger number of countries, but still finding themselves behind Singapore and Taiwan as was the case with the mathematics content knowledge.

Table 3 TEDS-M countries' overall performance with respect to pedagogical content knowledge at the primary level

Country	Mn	(SE)
Singapore	593	(3.4)
Taiwan	592	(2.3)
Norway	545	(2.4)
United States-private	545	(3.1)
United States-public	544	(2.5)
Switzerland	537	(1.6)
Russian Federation	512	(8.1)
Thailand	506	(2.3)
Malaysia	503	(3.1)
Germany	502	(4.0)
Spain	492	(2.2)
Poland	478	(1.8)
Philippines	457	(9.7)
Botswana	448	(8.8)
Chile	425	(3.7)
Georgia	345	(4.9)
Significantly above US-public		
Not significantly different from US-public		
Significantly below US-public		

The two assessments portrayed in Tables 2 and 3 were constructed to have an international mean of 500 and a standard deviation of 100. For mathematics content knowledge, this implies that the US performance is about one standard deviation behind that of the future teachers in Taiwan. This represents a rather large difference in content knowledge between the future teachers of those two countries. A similar large difference exists with respect to Singapore as well. Table 2 indicates the relative country positions with respect to the overall mathematics content knowledge scale.

We now examine whether the US's performance with respect to mathematics content knowledge varies depending on the sub-areas of mathematics that were measured in the TEDS-M study. The TEDS-M item design included enough items to produce three sub-scales: algebra, geometry, and number. The relative performance of the US (combining the private and public samples) across these three areas did not differ in any appreciable way from that of the overall performance (see Table 4). However, there were differences between the public and private universities and colleges in terms of their performance on both the algebra and geometry subtests. The future teachers prepared at private institutions statistically significantly outperformed the public sample on algebra but performed more poorly on the geometry test.

Table 4 TEDS-M countries' overall performance across three sub-areas—algebra, geometry and number

Algebra		
Country	Percent Correct	(SE)
Taiwan	80.5	(0.5)
Singapore	78.4	(0.7)
Russian Federation	69.7	(1.1)
United States-private	69.0	(0.6)
Switzerland	68.5	(0.6)
Thailand	68.1	(0.6)
Norway	64.3	(0.8)
Germany	64.2	(0.9)
United States-public	63.7	(0.7)
Malaysia	60.6	(0.8)
Spain	57.7	(0.6)
Poland	57.0	(0.6)
Botswana	51.7	(1.6)
Philippines	47.5	(1.1)
Chile	41.2	(0.7)
Georgia	32.8	(0.9)

Geometry		
Country	Percent Correct	(SE)
Taiwan	80.3	(0.7)
Singapore	74.2	(0.9)
Switzerland	66.4	(0.6)
Russian Federation	64.2	(1.3)
Thailand	61.7	(0.6)
United States-public	61.2	(0.8)
Germany	60.8	(1.0)
Norway	60.5	(0.9)
Malaysia	59.9	(0.7)
Poland	57.5	(0.7)
United States-private	56.2	(0.8)
Spain	54.2	(0.6)
Botswana	48.3	(1.9)
Philippines	44.9	(1.3)
Chile	40.2	(0.8)
Georgia	24.7	(0.9)

Number		
Country	Percent Correct	(SE)
Taiwan	84.3	(0.6)
Singapore	73.1	(0.7)
Switzerland	70.6	(0.6)
Thailand	68.6	(0.7)
United States-private	66.9	(0.6)
Russian Federation	66.3	(1.0)
United States-public	65.7	(0.6)
Norway	64.5	(0.7)
Germany	61.0	(1.0)
Spain	56.9	(0.7)
Poland	56.7	(0.6)
Malaysia	55.0	(0.6)
Philippines	48.7	(1.1)
Botswana	46.5	(1.5)
Chile	41.7	(0.7)
Georgia	28.9	(0.8)

Significantly above the US
Not significantly different from the US
Significantly below the US

With respect to pedagogical content knowledge, there were also three sub-scales dealing with the future teachers' knowledge of the K-8 curriculum, the pedagogical knowledge related to instructional practices in the classroom, and finally, knowledge related to the planning of instruction. Here again the future teachers performed at about the same level in all three of these sub-areas and similar to that of the overall scale. In effect, what this suggests is that the knowledge of the US future elementary teachers, both in terms of mathematics content as well as pedagogical content related to mathematics, is neither weak nor particularly strong when placed in an international context. The overall scale-scores were consistent with the sub-areas for the country as a whole suggesting that the results indicated in Tables 2 and 3 essentially characterize the country differences and the relative position of the United States with respect to that international distribution. It is clearly not where we want our teachers' knowledge level to be in order to be able to teach the more demanding curriculum put forth by the National Governors Association (NGA) and Council of Chief State School Officers (CCSSO). The standards defining this curriculum, called the Common Core State Standards for Mathematics, have been adopted by 46 states.

Perhaps, not surprisingly, the performance of the US elementary future teachers internationally is quite consistent with the performance of third and fourth graders in the TIMSS studies—mired near the international mean. The data characterized in the previous tables represent the United States as a whole. Teacher preparation at the elementary level as indicated previously can be done through at least two types of programs—elementary programs focused on grades 1–5 and secondly elementary programs allowing for certification up through grade 8. Actually the number of different types of programs is much larger, but to make the study manageable, the elementary certification programs were classified into these two broad types. The main question behind the TEDS-M research project was to understand the relationship of various teacher preparation programs with respect to the knowledge acquired during that preparation program. Ultimately in one sense the question was: does teacher education matter, at least in terms of the knowledge acquired during the preparation program?

We now look at the relationship of the two different types of elementary teacher preparation programs in terms of their relationship to knowledge of mathematics content and knowledge of mathematics pedagogy. In addition, we look more closely as to whether there is any difference between the teacher preparation programs provided by public versus private universities and colleges.

Consider first, mathematics content knowledge: the difference between the two types of programs was relatively small. In fact, for the public institutions, the difference in the two means was trivial—520 versus 518 with the higher average score associated with those programs allowing certification up through middle school. For the private institutions the difference was more substantial with a difference of 8 points (533 versus 525). Although larger, the differences are not statistically significant. A similar pattern emerges with respect to the pedagogical content knowledge, where the difference between the two program types for the public universities was again trivial, but with respect to the private institutions, the difference was substantial. The average test score of those prepared to be certified at the K-8 level was 16 points higher than was the case for those prepared to teach only at the primary level (558 versus 542).

It is interesting to note that with respect to teacher preparation at public universities and colleges there were essentially no differences between the two types of preparation programs, but this was not the case for the private institutions where the differences between the two programs for both the content knowledge and the pedagogical content knowledge favored those prepared to be able to teach at both the primary and middle school level.

The other major dimension we examined is the difference between public and private teacher preparation itself. This comparison was confounded by the fact that typically the students entering private universities have higher levels of mathematics knowledge upon entering the university. So the differences that might be noted with respect to what the future teachers knew as they left their programs, especially in mathematics content knowledge, could be influenced by the entry level knowledge of those students.

Ignoring this caveat for the moment, Table 2 indicates that with respect to mathematics content knowledge the future teachers prepared through private universities and colleges scored about 8 points higher than their counterparts at public institutions of higher learning. The difference, however, was not statistically significant. For pedagogical content knowledge, the difference between the preparation programs in the two types of universities was negligible and not significant. The latter is not surprising since one would imagine that most of the pedagogical content knowledge that students would have at the end of their programs would have come primarily through those preparation programs rather thant through course experiences in the K-12 system. However, the mathematics content knowledge could have been influenced by the level of knowledge of those students as they entered the teacher preparation program. As mentioned previously there likely would be such differences given the US admissions procedures associated with college attendance.

In fact, the data show that there are such differences on average between those students who enter teacher preparation programs at private universities and colleges versus those who enter such programs at public institutions. The data upon which that is based are of three types. We used the Barron rankings of university prestige as one such measure. Additionally we used the 25th and 75th percentiles of ACT scores for those matriculating at the universities. Finally we also have from the students themselves an indication of the highest level of mathematics they took while in high school. Using those three variables we found statistically significant differences between public and private schools with the differences in the expected direction. We then adjusted the TEDS-M scale scores for these initial differences and the small, insignificant differences were eliminated after adjusting for these entry level differences. In other words, the apparent and small differences in mathematics content knowledge of the future teachers upon finishing their teacher preparation programs was probably more the result of the differences in admission procedures between the two types of universities and colleges and not some systemic difference in the nature of teacher preparation between the public and private sector.

3.2 Characterizing Course Taking

The previous section characterized the mathematics knowledge of US future elementary teachers as being somewhat adequate as represented by their relative position in the international distribution but also suggested that the level of knowledge does not put them where we would hope they might be, that is among the top performing countries. The desire that these future elementary teachers would possess higher levels of mathematics knowledge is especially important as this nation confronts the international realities suggested by the Program for International Student Assessment (PISA) and TIMSS detailing how far behind we are. In addition, the TIMSS curriculum analysis has pointed out that our K-12 curricular expectations are not competitive internationally.

The US has recently responded calling for curriculum that are focused, coherent, and rigorous. The new Common Core State Standards for Mathematics have such high level expectations for students. As these standards are adopted by the states, this places an increasingly high level of demand that US teachers have a more sophisticated and deeper understanding of mathematics.

How can we as a nation meet that challenge? That question can be addressed by focusing on the curricular experiences that the future elementary teachers had while in their teacher preparation programs. Our hypothesis was that the differences among the countries would be related at least in part to differences in terms of the experiences and course requirements that these students had while part of those teacher preparation programs. In this section we examine that issue.

Future teachers were asked which courses they took in each of three areas—formal mathematics, mathematics pedagogy and general pedagogy. Mathematics content was defined in the traditional way and there was little difficulty internationally in specifying those topic areas. Future teachers were asked to indicate whether they had studied each of 15 different content topics associated with university level mathematics. These included such things as: calculus, differential equations, linear algebra, topology, real analysis, and probability, among others. In many of the countries including the US, these various topics would represent particular courses but in other countries, these were topics that could have been covered in multiple courses. Consequently, there is an ambiguity as to whether the sum of these represents the total number of courses taken or the total number of topics studied while in their preparation programs. Whichever the case, the indicator suggests the amount of exposure to the area of formal mathematics. In describing these results we refer to them as courses, which is essentially the way they would typically be designated here in the United States, although the reader should keep in mind the caveat that in some places these are not formal courses but simply topics covered as a part of multiple courses.

Course work in mathematics pedagogy included courses on the foundations of mathematics including the philosophical underpinnings of mathematics, the history of school mathematics, the development of mathematical ability and thinking in children. It also included methods of teaching mathematics, practical experiences with respect to developing, and forming instructional lesson plans for the teaching of mathematics, as well as practical experiences such as teaching elementary students or observing them in their classes as they are taught mathematics.

The third area was general pedagogy and included traditional courses such as the history, philosophy, and sociology of education, as well as educational psychology. Also included were courses focusing on generalized methods of teaching as well as classroom management. In both general pedagogy and mathematics pedagogy these are likely to be different topics that might be considered in one or more courses of pedagogy but, again, they represent the breadth of exposure to various areas of pedagogy and are used in that vein. Again for simplicity sake we refer to these as the number of courses.

We look first at the relative allocation of course work across the three areas as reported by the future teachers. It is our belief that the allocation of the limited amount of time—typically four years of course work—across the three areas is one of the key, if not the central, policy issue confronting teacher preparation.

Such relative allocations serve in some way as an institutional definition of what constitutes quality teacher preparation. Surely all teacher preparation institutions have as their goal to prepare a high quality future primary teacher. What the relative allocation across the three areas defines, no matter how many total hours might be required for the program, is their definition of the type of expertise future teachers should possess as they finish their teacher preparation program.

For the US the distribution across the three areas is roughly characterized as one-third, one-third, one-third. More specifically, mathematics course work constituted somewhere around 29–34 % of the teacher preparation course taking (ignoring other course work such as liberal arts, electives, etc.) with about 35 % focused on mathematics pedagogy. The remaining 32–35 % focused on general pedagogy.

US private institutions devoted more time to pedagogy (both general and mathematics pedagogy), but not by a large amount. Conversely, the public university teacher preparation programs devoted more time to course taking in mathematics. So in general, given the small differences between public and private teacher preparation programs we can approximate the time distribution as a one-third allocation across the three areas implying that students took about the same amount of course work in all three areas.

How does this compare with the other countries, especially those whose future teachers performed particularly well on the mathematics content and pedagogical content knowledge tests? The distributions were very similar. The average across the top achieving countries showed a slightly greater allocation in mathematics and correspondingly less in the general pedagogy area. The differences, however, were not substantial. For example, Taiwanese future teachers on average had a ratio that approximated a 38/34/28 percent time allocation across the three areas, thus implying slightly more mathematics course work taken as a part of the preparation program. Singapore's distribution can be summarized as a 35/40/25 percent distribution. To understand the different opportunities provided by teacher preparation programs in Taiwan and Singapore as opposed to the US we focused on specific course taking differences among countries.

One of the more distinctive differences reflects the percent of future elementary teachers in each of the countries who took a basic two-course introductory calculus sequence. In the US about one-fourth of the future elementary teachers took that sequence, while more took it in Switzerland (62 %) and in Singapore (41 %). However, a similar percentage of Taiwan's future elementary teachers took the calculus sequence—25 %. There were differences among some of the countries and the US with respect to the amount of mathematics taken, but there is no single pattern that differentiates the preparation of future elementary teachers in the top achieving countries from that of the United States. Given that result, the question that emerges is what might account for such differences in performance. On the surface, it does not appear that the difference is in the nature of the teacher preparation program at least as reflected in the relative allocations across the three areas.

This leads to an hypothesis that for primary future teachers, the differences among countries may simply go back to the fact that the pool from which future teachers are selected within each of those nations differs. In other words, from international studies, we know that the country distributions of mathematics achievement are quite different. For example, we used international TIMSS eighth grade mathematics data to define the pool from which primary future teachers would be drawn. We can then make assumptions as to where in that distribution the US typically draws its primary teachers and then compare that to the distribution for Taiwan and Singapore as representatives of the top achieving TEDS-M countries. Given that all three of the countries are on a common international scale, this makes such analyses possible.

For example, if Taiwan and Singapore were to draw their future elementary teachers from the middle of their distributions (the 50th percentile) as represented by the eighth grade TIMSS 2003 results, this would correspond to the US's having to draw its future elementary teachers from above the 75th percentile—actually closer to the 85th–90th percentile—to be comparable in their entry level knowledge of mathematics (see Fig. 1). The pool from which future elementary teachers are drawn in those three countries is radically different, and those differences may well account for the differences at the end of teacher preparation, more so than the differences among the countries in terms of their teacher preparation requirements and the actual course taking. This implies that an important issue is one of recruitment and admission policies. So, for example, even if Taiwan were drawing its pool of future elementary teachers from somewhere below its country mean, in order for the US to be comparable, the US would have to draw its pool of future elementary teachers from the 75th percentile of the distribution. From other data, this is clearly not happening in the United States.

3.3 US Implications for Policy

There are no statistically significant differences in mathematics content knowledge or mathematics pedagogy knowledge between public and private universities and

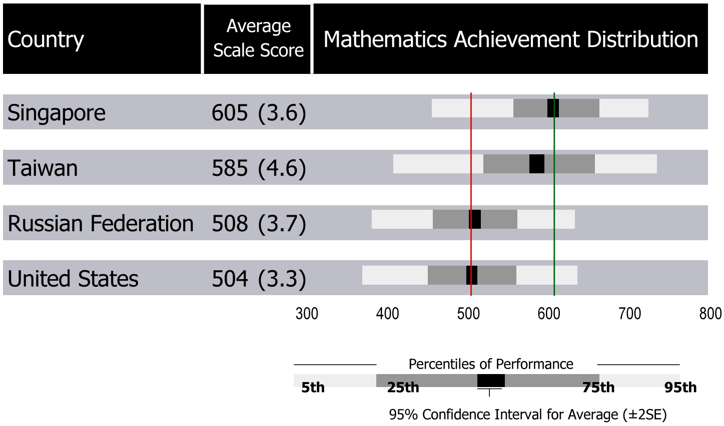

Fig. 1 TIMSS 2003 Eighth Grade Mathematics Achievement Distributions. Source. Mullis et al. (2004). TIMSS 2003 International Mathematics Report: Findings From IEA's Trends in International Mathematics and Science Study at the Fourth and Eighth Grade (p. 465). Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Lynch School of Education, Boston College

colleges. This is in spite of the fact that characteristics of the institutions do vary between private and public universities. In the sampled institutions the 25th and 75th percentile of matriculating students' ACT mathematics scores were higher in the private schools as was the Barron rating associated with the prestige of those institutions. Assuming that this would be true of the future teachers at that institution and hence the sample we drew, the selection bias associated with the private schools would suggest that their knowledge of mathematics would likely be greater at the outset, however, at the end there were no significant differences between the public and private colleges and universities. This was true in spite of the private institutions' more selective admission policies.

At the elementary level, the patterns are not as clear as the course taking for the US does not seem that dissimilar from the other countries especially those whose future teachers performed the best on the mathematics content knowledge test. This suggests that there are other differences related to the relative position of the mathematics knowledge of US future elementary teachers. One hypothesis is that the difference may have to do with the nature of the K-12 mathematics curriculum itself. We know from the TIMSS study that the K-12 curriculum is more demanding and challenging in countries such as Singapore, Taiwan and the Russian Federation, whose future teachers demonstrated greater knowledge of mathematics upon completion of their program. Those teachers came to the teacher preparation program with a stronger background enabling them to likely take more advanced mathematics, but relative to the amount of pedagogy preparation it would still be similar to the United States. In the US the one-third of the teacher preparation that is formal mathematics would perhaps need to be at a lower level of mathematics than would be the case in a country such as the Russian Federation or Taiwan since the high school curriculum in the US is weaker and does not have as high expectations as is the case in those other countries.

Coupled with this is the fact that in some of these countries, such as Taiwan, the students who enter an elementary teacher preparation program likely come from a higher percentile of the international distribution of mathematics performance as reflected in TIMSS at the eighth grade. This would imply that the US, given its relatively lower position in that international distribution, would have to draw from the very high end of the distribution in order to even be comparable to future teachers being drawn from the middle or even the lower end of the mathematics knowledge distribution in other countries.

This places US future elementary teachers at a disadvantage both in terms of their entry level knowledge as well as the substance of the mathematics they would encounter as a part of their teacher preparation program. If students enter the program with a higher level of mathematics knowledge from high school, the corresponding coursework that they would experience while at the university would be of a higher level. This was made clear from the data where, in these other countries, a larger percentage took at least one of the two calculus courses than was the case in the United States. This is in spite of the fact that the relative allocation across the three areas of preparation is constant, but the nature of the mathematics taken was different. Much additional analysis needs to be pursued in order to understand more fully the relationship of what the future teachers studied in their teacher preparation program and what knowledge they possessed as they left that program.

In the end the real question is whether these professional competencies such as the knowledge level in mathematical content, mathematical pedagogical content and general pedagogical content makes any difference as to how much mathematics the K-8 students learn and achieve.

So what might be a good approach to this problem? It would appear that the solution may well lie in some combination of recruitment and inducement to enter teaching for those who have quantitative backgrounds together with a more demanding curriculum. The other serious issue that needs to be addressed is the certification issue which states control and consequently should be looked at carefully because, as is indicated by these data, that choice has likely consequential impact on what students learn.

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Emphasis and Balance among the Components of Teacher Preparation: The Case of Lower-Secondary Mathematics Teacher Education

William H. Schmidt, Leland Cogan, and Richard Houang

Abstract This article examines the teacher preparation program learning opportunities afforded future lower secondary mathematics teachers and future elementary teachers who may teach mathematics. Data from U.S. participation in the recent international Teacher Education and Development Study in Mathematics are explored against international profiles to address a critical issue often cited in the teacher education literature: Given the finite time available, what sort of balance is provided for course work across the areas of mathematics content, mathematics pedagogy, and general pedagogy? Results demonstrated major differences for lower secondary preparation programs in both the types of topics or courses covered and the relative emphasis across the three areas in those countries statistically outperforming the United States in comparison to U.S. programs. Similar but less striking differences were noted among elementary programs. These results should provide important empirical evidence relevant to the ongoing policy dialog concerning identifying the specific content of a quality teacher preparation program.

Keywords International benchmark · TEDS-M · Middle school · Assessment · Common core state standards · Content knowledge · Mathematics · Opportunity to learn (OTL) · Mathematics pedagogy · General pedagogy · Belief · Mathematics content knowledge (MCK) · Pedagogical content knowledge (PCK)

Elements were previously published as "The Role of Opportunity to Learn in Teacher Preparation: An International Context", Journal of Teacher Education, 62 (2), pp. 138–153. Printed with permission of the Journal of Teacher Education.

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S. Blömeke et al. (eds.), *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn*, Advances in Mathematics Education,
DOI 10.1007/978-94-007-6437-8_17,

The initial results from the Teacher Education and Development Study in Mathematics (TEDS-M) revealed a somewhat disappointing level of performance by U.S. future middle school teachers on the TEDS-M mathematics knowledge scaled scores (The Center for Research in Math and Science Education 2010). Their performance, especially on the content knowledge assessment, placed them in the middle straddling the divide between those countries whose middle school students have outperformed the U.S. on international assessments and those whose middle school students have been outperformed by the U.S. (The only exception to this pattern was Malaysia).

Shortly thereafter, the National Governors Association (NGA) and the Council of Chief State School Officers (CCSSO) released the Common Core k-12 mathematics content standards that as of late 2012 were adopted by 45 out of 50 U.S. states. As a partial response to an increasing globalization in which individuals as well as corporations and governments confront competition and evaluation from an international context, these standards were designed to be more in line with those of other countries especially those from countries whose students have performed particularly well on international assessments such as the Third International Mathematics and Science Study (TIMSS) and the Programme for International Student Assessment (PISA) (TIMSS 1995, 1999, 2003, 2007; PISA 2000, 2003, 2007). The TEDS-M results call into question whether US future teachers are being prepared to teach internationally benchmarked challenging mathematics found in the recently developed and adopted Common Core State Standards (Schmidt and Houang 2012).

U.S. performance in both the TIMSS and PISA mathematics assessments consistently lags behind many other countries especially those who are our world-wide peers. Over 15 years ago, the original TIMSS (1995) examined curricular differences among the countries. The TIMSS K-8 curriculum data combined with the grade 8 achievement data revealed that countries with higher achievement also had teachers who taught substantially different content than that found elsewhere (see Schmidt et al. 1996, 2001, 2005). This result likely speaks to both curricular differences as well as to differences related to the typical level of mathematics knowledge of the countries' teachers. Country differences in teacher knowledge may also be one of the reasons that the mathematics level of the curriculum varies across countries.

Other reports have highlighted the idea that what teachers know and do in the classroom is consequential for students' learning (National Commission on Teaching and America's Future 1996). U.S. reform efforts, consistent with this line of thought, have introduced standards to measure teacher quality in connection with student achievement which has led to accountability concerns regarding teacher preparation programs (INTASC 1995; Murray 2000; Leithwood et al. 1999; NCATE 2000). Given the coherence, rigor, and focus seen in the mathematics curricula of the highest achieving countries as identified by the outstanding performance of their students on international assessments, the question arises as to how such countries prepare their teachers to teach a challenging curriculum to K-8 students? This question was one of the motivating factors for the TEDS-M study.

The other striking feature of the U.S. TEDS-M report was the large amount of variability in the level of future teacher mathematics content knowledge across the sampled colleges and universities that prepare teachers. That variation was so large that future teachers in some institutions on average performed at levels associated with institutions in the top achieving countries and some at the level of the lowest achieving countries. That within U.S. variation in mathematics content knowledge across institutions combined with similar large cross-country variation (Tatto and Senk 2011; The Center for Research in Math and Science Education 2010) suggest two main hypotheses as to what might be related to such variation: variation in who chooses to become a teacher and variation in what constitutes teacher preparation, i.e., what courses and experiences are taken by future teachers. The latter produces variation in opportunity to learn (OTL).

Teacher preparation does not take place in a vacuum but functions within the broader context of a country's k-16 (or more) education system. In the U.S., this includes states enacting policies governing what school subjects may be taught at which grade levels with what sort of academic education and professional training. The system specific nature of teacher preparation may lead some to doubt the wisdom of considering any sort of international comparison of teacher education given that it does not share common meanings across various cultural contexts (Akiba et al. 2007). Nonetheless, the OECD and the European Union have been engaged for a number of years in an effort to characterize and harmonize higher education in general with an intentional focus on teacher education across these many national systems (Eurydice 2001; OECD 2004). A recent report from the American Federation of Teachers documents the large number of teachers recruited from other countries to teach in U.S. schools (AFT 2009). Given that many are recruited to teach mathematics, the TEDS-M study may provide an important window on the preparation of teachers that end up in U.S. classrooms.

The TIMSS curriculum analysis made it clear that there were very different curricular opportunities for K-8 students to learn mathematics both across countries and across classrooms in the U.S. In a similar fashion, this chapter explores differences in the opportunities to learn (OTL) that future teachers experience in their teacher preparation programs. Put simply, does teacher preparation vary in terms of course taking in mathematics, mathematics pedagogy and general pedagogy across countries and across colleges and universities within the U.S. in ways that are related to the variation in performance on the mathematics content knowledge and pedagogical content knowledge assessments?

1 Theoretical Framework

TEDS-M owes part of its intellectual heritage to the Mathematics Teaching in the 21st Century (MT-21) study (Schmidt et al. 2008, 2011). The defining element for both studies is based on the concept of teacher competence as it is defined in general, e.g., by Spencer and Spencer (1993), Eraut (1994), and Weinert (2001), and specifically with regard to teaching by Bromme (1997) and Taconis et al. (2004). Competence is to have the knowledge and the skills at one's disposal to successfully solve core, job-related problems. Professional competence has been described to include both professional knowledge as well as beliefs about subject matter, teaching, and students (Blömeke et al. 2008, 2010, 2011). Teacher knowledge is viewed as an important competency that teachers need in order to be effective in the classroom. Observed test performance is used in this study as an indicator of teacher competence. This definition also takes into account that we are not able to observe actual classroom performance. As a result, TEDS-M was not a study of classroom effectiveness nor student learning but a study of the opportunities provided to and experienced by future teachers while engaged in teacher preparation toward developing the competencies deemed by the literature to be relevant to quality classroom instruction. This particular paper examines the role of teacher preparation through the opportunities provided and taken in developing such competencies.

Opportunity to learn is defined here in the original sense of the concept as it was developed by Carroll (1963) and further refined and developed in the IEA studies (McDonnell 1995). For an elaboration of the OTL concept and a review of the development of the idea see Schmidt and Maier (2009). Thus the focus in TEDS-M was on the content that future teachers are exposed to as a part of their teacher preparation program. The goal of teacher education undoubtedly is to prepare highly competent future teachers. To do so, universities and colleges design a set of courses and experiences which presumably help to develop the necessary knowledge and skills. Research in teacher education has suffered from the fact that only crude indicators have been used as measures of OTL. The following excerpt from Schmidt et al. (2011) related to MT21 summarizes that previous set of studies:

> Many studies use the number of courses taken or the kind of teaching license to define OTL. So, not surprisingly, findings about the effects of content in teacher education on professional competence are inconsistent (Blömeke 2004; Cochran-Smith and Zeichner 2005; Wilson et al. 2001). A continuous, positive link between future teachers' opportunities to learn and their professional knowledge and beliefs could not be identified through analysis of the courses they take. This does not necessarily mean that content features can be left out in studies of teacher education. It merely points to the need for more sophisticated measures of OTL than are presently available. Regardless of how common it is to use indicators like degrees, majors, examination results, or the number of classes taken (see, e.g., Akiba et al. 2007; Goldhaber and Brewer 2000; Monk and King 1994), this approach is at high risk of washing out any kind of relationship between opportunities to learn in teacher education and the outcomes because there is unfortunately nothing in teacher education "that share[s] a relatively common meaning across various cultural contexts" (Akiba et al. 2007).
>
> An example of this is the difference in the meaning of opportunities to learn "general pedagogy." In comparison to a broad central European understanding of general pedagogy, the understanding in English-speaking countries is rather narrow since it is mainly operationalized as classes in teaching methods or

> classroom management (Hopmann and Riquarts 1995). This methodological weakness results in a disturbing inconsistency of study results because differences due to cultural definitions of general pedagogy overlay differences between programs. In addition, because of the inconsistent findings, almost any inference can be drawn: teacher education may or may not matter; personality may or may not matter, and so on (see, e.g., Abell Foundation 2001a, 2001b vs. Darling-Hammond and Youngs 2002). Thus, there is a need to develop less aggregated measures that capture the content of teacher education in a low-inference way. (Chap. 3, pp. 88–89)

Such an undertaking was attempted in TEDS-M. Future teachers were asked which topics they studied in the courses they took as a part of their teacher education program. The lists represented an international definition of the topics in mathematics, mathematics pedagogy and general pedagogy that may be studied as a part of mathematics teacher preparation. In mathematics the list is orientated toward an international definition of university baccalaureate level courses that can be taken (for example, calculus, linear algebra, or differential geometry).

The content of mathematics was quite standardized across countries, but finding an internationally comparable and at the same time nationally accurate representation of the topics in mathematics pedagogy and general pedagogy was more challenging. The lists were vetted internationally and were made as detailed as possible so as to record the content exposure as precisely and accurately as possible.

The measurement of OTL is designed to provide an empirical basis for addressing one of the major controversies related to mathematics teacher education identified in the literature—to which extent should each of the three components: mathematics, mathematics pedagogy and general pedagogy be taught in the limited amount of time available overall in teacher education (Wilson et al. 2001). This is the central focus of this chapter.

2 The Study

The Teachers Education and Development Study in Mathematics (TEDS-M) was an international comparative study of teacher education that focused on the preparation of teachers of mathematics at the primary (elementary) and lower secondary (middle school) levels. The study was carried out under the aegis of the International Association for the Evaluation of Educational Achievement (IEA), an independent, international consortium of countries representing national research institutions and governmental research agencies—the same organization that sponsored the Third International Mathematics and Science Study (TIMSS). TEDS-M is the first international study of higher education and the first international study focusing on teacher preparation. Participating countries in addition to the U.S. included Germany, Norway, Poland, the Russian Federation, Spain, Switzerland, Taiwan, Singapore, Thailand, Malaysia, Botswana, the Philippines, Chile, Georgia, and Oman.

2.1 Sampling Design

Future Teachers near the end of their final year of teacher preparation were the focus of the study. Three sampling approaches were used to obtain nationally representative data for participating countries. A few countries such as Norway, Singapore, and Thailand, obtained a census of all teacher preparation institutions in their country and a census of all future teachers fitting the TEDS-M target population definitions. Other countries such as Poland, Switzerland, and Taiwan, obtained a census of teacher preparation institutions and randomly sampled from eligible future teachers. The last set of countries including the Philippines, the Russian Federation, Spain, and the U.S. obtained random samples of both teacher preparation institutions and eligible future teachers within each teacher preparation institution. In each case, the specific sampling plan was developed in consultation with the IEA sampling referee and deemed appropriate for representing the country's overall production of possible future teachers of mathematics. Sampling weights were used to appropriately weight the results from the different types of programs within a country so that the averages represented the typical future teacher, taking into account the proportions prepared by the different types of programs. For the U.S. a two stage cluster sample was randomly drawn with the colleges and universities serving as the clusters. Eighty-one public and private institutions were drawn which had future teacher overall response rates of around 70 %.[1]

There are three distinct approaches to preparing lower secondary teachers of mathematics in the U.S. The first prepares teachers to teach all secondary

[1]The rationale for a 2-stage cluster sample drawn with probability proportional to the size of the institutions (TEDS-M ISC, *Sample Preparation Manual*, 2007) was that with few exceptions, teachers in the participating countries were prepared by identifiable institutions such as universities, colleges, teacher colleges, normal schools, etc. The first stage was to identify and select institutions with probabilities proportional to the size of the institutions. Then a sample was drawn randomly from eligible training programs within each institution.

The desired target population was to have national coverage. For TEDS-M, the target populations included Level 1 (primary/elementary) teachers who are prepared by their teacher education programs and certified by the states to teach mathematics, and Level 2 (lower secondary/middle grades) teachers. In the U.S., Level 1 teachers are prepared by primary or elementary programs (K-5, K-6, K-8, 1–5, etc.). Level 2 teachers are prepared by programs for secondary and/or middle school mathematics. The U.S. TEDS-M sampling frame focused on the 1351 colleges and universities that have teacher preparation programs approved by the U.S. Department of Education. The sampling frame, therefore, excluded teachers prepared under "alternate routes" as these individuals are most often already teaching in classrooms and thus fell outside the definition of "future teachers" which was the population in focus for TEDS-M.

The resulting U.S. sampling frame includes 498 publicly controlled institutions and 853 privately controlled institutions. Based on the sampling frame, publicly controlled institutions represent 37 % of all institutions but they are responsible for 60 % of the total institutional production.

For both organizational and operational reasons, it was necessary to conduct the data collection in two consecutive years in 2008 and 2009. A sampling fraction of 12 % was used to draw the sample. Data collection followed strictly the guidelines and procedures provided by the ISC (*Institution Contact and Site Coordinator Manual*, 2008).

Because of the complex sampling design, standard errors for any estimators and comparisons had to be estimated using Balanced Repeated Replication (BRR) (Dumais and Meinick 2009). Es-mathematics—including the curriculum of the lower secondary (middle school) grades. The second route focuses specifically and exclusively on preparing teachers for the lower secondary/middle school grades. The third approach prepares lower secondary/middle school teachers as an extension of elementary teacher preparation. All three of these were represented among the TEDS-M participating countries. While a few countries such as Chile, Germany, and Norway combine two of these approaches to prepare all the needed teachers for lower secondary mathematics, the U.S. is unique in having programs at various institutions in different states that exemplify each of these three.

2.2 Instruments

TEDS-M designed and developed two sets of assessments—mathematics content knowledge (MCK) and pedagogical content knowledge (PCK)—for future teachers at each of the two levels of preparation. The MCK assessment was designed to measure advanced mathematics related to the appropriate school mathematics taught at that level rather than the level of knowledge associated with advanced undergraduate academic mathematics courses such as the theory of complex functions (see Blömeke et al. 2011; also Appendix B in *Breaking the Cycle* for example items). The study does not measure the classroom practices of these future teachers only what knowledge competencies they possess as they finish their preparation. The study also sought to measure and to characterize what learning opportunities were provided and how they were structured as well as to measure a set of beliefs. This was accomplished primarily by a survey of future teachers. The Future Teacher Survey and Assessment had four main parts and was completed during a standardized administration session. The focus of each part and the time allotted to completing it are shown in Table 1.

Parts A, B, and D were the same for all future teachers in both primary and lower secondary programs. Two different tests were developed to assess mathematics content knowledge and pedagogical content knowledge: one for those preparing

sentially, weights were determined according to the sampling design, adjusted for non-participation and non-response. Replicate samples were then created for computing the desired standard errors.

Finally, because data collection spanned two academic years, a second sample was collected from 8 of participating public institutions in 2009 for comparison. These 8 institutions were selected randomly after the sample of participating public institutions was stratified according to the response rates. The comparison revealed that there were no significant differences between institutional samples from the two years. The two samples were compared on a set of variables relating to the future teachers' background (high school GPA, highest course taken in mathematics in high school, SAT, and ACT scores), as well as mathematics courses taken in college. The analysis was performed controlling for differences among the institutions. There were no statistically significant differences between data collected from the 2 years.

For further details, see Appendix A of *Breaking the Cycle: An International Comparison of U.S. Mathematics Teacher Preparation* (The Center for Research in Math and Science Education 2010).

Table 1 Composition of future teacher survey and assessment

Section	Focus	Time (minutes)
A	Background	5
B	Opportunity to Learn (Course Taking)	15
C	Mathematics Content and Pedagogical Content Knowledge Assessments	60
D	Beliefs about Mathematics and Teaching	10

to teach the primary grades and another for those preparing to teach mathematics in the lower secondary (middle grades). TEDS-M employed a rotated block design in order to measure the desired breadth and depth of knowledge. There were five primary booklets that had rotated blocks of items in Part C and three lower secondary booklets that had rotated item blocks in Part C. Rasch scaling was used to create individual scaled scores for each future teacher (see Tatto et al. 2008, for details of item development, cognitive domain frameworks, and scaling.)

2.3 Measurement of Opportunity to Learn

In TIMSS, variation across countries in the implemented curriculum was found to be related to achievement. The actual content exposure that was delivered to students by teachers was found to be among the most salient features of schooling related to academic performance. Countries with higher achievement gains from one year to the next had mathematics teachers who taught substantially different content than their counterparts in less accomplished countries (Schmidt et al. 1999, 2001).

A basic hypothesis of TEDS-M is that this is the case for mathematics teacher education as well—that in countries with higher future teacher levels of knowledge, different content and different amounts (associated with specific content) are taught than is the case in other countries. Therefore, we measured the opportunities individuals had in their teacher preparation programs. We did this by surveying the students' experiences related to mathematics and pedagogy. Future teachers were asked to choose the type of content (topics) covered in the courses they had taken during their teacher preparation. Three internationally developed sets of content were included: one each for mathematics, mathematics pedagogy and general pedagogy. In most countries the topics listed usually defined specific courses given their "grain size". This was especially true for the mathematics and mathematics pedagogy topics yet our references to "courses" remains an inference although one that is supported by the curriculum work of the MT21 project (Schmidt et al. 2011). Nonetheless, in some countries there were topics that were covered in one or more courses but, by themselves, did not constitute a single course. This was more likely the case with the general pedagogy topics. To accommodate this ambiguity, we use both terms throughout the results section.

For ***mathematics*** this included content such as linear algebra, abstract algebra, calculus, theory of complex functions, differential equations and topology. The 19 distinct mathematics courses or topics can be conceptually grouped into seven broader categories of university level mathematics: linear algebra; number theory; geometry; probability and statistics; abstract algebra; basic calculus; and advanced mathematics which includes topics such as topology, differential geometry, multivariate calculus, differential equations and the theory of real functions.

Mathematics pedagogy included such content as the history and psychology of mathematics, methods of teaching mathematics, and the principles and theory of various school-level mathematics topics (e.g. arithmetic, measurement, etc.). For conceptual purposes the 11 courses or topics in the future teacher questionnaire can be grouped into four categories: mathematics pedagogy; the history of school mathematics; the principles and theories of basic school-level mathematics topics; and the principles and theories of advanced school-level mathematics topics.

Pedagogy includes not only what is specific to the teaching of mathematics, the focus of the previous paragraph, but it also deals with understanding the psychology of learning, the dynamics of the social context from which the students come as well as the micro social system of the classroom, the organization and history of schooling and the theory and principles of instruction including curriculum assessment, lesson planning, and classroom management. It is particularly in this area that there is often much debate about just how much teachers really need to know in a theoretical way as opposed to amassing a repertoire of practical tips for professional functioning in the classroom or acquiring a teacher's "bag of tricks". To address this there were eight broad topic areas included in the future teacher questionnaire: history of education, philosophy of education, sociology of education, educational psychology, theories of schooling, assessment and measurement, education research methods, and knowledge of teaching. Again, future teachers were asked to indicate which topics they had taken as a part of their preparation.

These can be grouped conceptually into those specific to the operation of the classroom including, but not limited to classroom management; topics related to general methods of teaching including lesson planning and motivating students; and courses/topics related to the theories that provide the rationale and academic background of schooling.

2.4 An International Benchmark

To facilitate the characterization of OTL in relationship to future teacher professional knowledge we borrowed a methodological approach from TIMSS (Schmidt et al. 2005). In that study the K-12 curriculum was characterized for the top achieving countries calling the result the A+ curriculum which was then used as an international benchmark by which to compare the U.S. curriculum.

Figure 1 displays country ranks on the MCK assessments. Since ranks are not stable given the presence of sampling error, the list of countries in each figure is

Primary Level

Country	Mn	(se)
Taiwan	623	(4.2)
Singapore	590	(3.1)
Switzerland	543	(1.9)
Russian Federation	535	(9.9)
Thailand	528	(2.3)
United States-Private	527	(3.6)
Norway	519	(2.6)
United States-Public	518	(4.1)
Germany	510	(2.7)
Poland	490	(2.2)
Malaysia	488	(1.8)
Spain	481	(2.6)
Botswana	441	(5.9)
Philippines	440	(7.6)
Chile	413	(2.1)
Georgia	345	(3.9)

Lower Secondary Level

Country	Mn	(se)
Taiwan	667	(3.9)
Russian Federation	594	(12.8)
Singapore	570	(2.8)
Poland	540	(3.1)
Switzerland	531	(3.7)
Germany	519	(3.6)
United States-Private	512	(16.3)
United States-Public	505	(9.7)
Malaysia	493	(2.4)
Thailand	479	(1.6)
Oman	472	(2.4)
Norway	444	(2.3)
Philippines	442	(4.6)
Botswana	441	(5.3)
Georgia	424	(8.9)
Chile	354	(2.5)

Significantly above US-Public

Not significantly different from US-Public

Significantly below US-Public

t - tests for statistical significance of mean differences adjusted for multiple comparisons at .05 level.

Fig. 1 TEDS-M countries' overall mathematics content knowledge scaled scores for primary and lower secondary future teachers

divided into three distinct groups. The first indicates the countries that statistically significantly outperformed the U.S.—this becomes the set of A+ countries for our purposes. The table is further divided into those countries which performed like the U.S. and the group of countries which the U.S. statistically significantly outperformed.[2] Taiwan, Russia, Singapore and Poland function as the A+ countries for the lower secondary future teacher sample. The A+ group of countries for the elementary future teachers a result, Taiwan, Singapore and Switzerland, are included in Fig. 1 and in some discussion to provide some comparison and contrast.

3 Results

This section is organized around the course taking of future teachers in the three broad areas: mathematics, mathematics pedagogy (including the study of school-level mathematics topics), and general pedagogy. Within each of these three areas

[2]Statistical significance for differences between country means was determined using the computed standard errors and employing Bonferroni multiple comparisons which controls the family-wise significance level at 0.05. Countries were then grouped with respect to the U.S. Public institution mean: statistically significantly greater than the U.S. Public mean; no different than the U.S. Public mean; less than the U.S. Public mean. The full distribution of scores for each country including the 95 % confidence interval for the means is included in Appendix C of the report, *Breaking the cycle: An international comparison of U.S. mathematics teacher preparation*.

Table 2 Mean number of courses (topics) in mathematics, mathematics pedagogy, and general pedagogy (including the standard errors and standard deviations) taken by the primary and lower secondary future teachers

	Mathematics			Mathematics pedagogy			General pedagogy		
	Mean	(se)	SD	Mean	(se)	SD	Mean	(se)	SD
Primary level									
A+ Country Group	9.0*	(0.07)	3.5	7.0	(0.05)	2.1	6.0*	(0.07)	1.8
U.S. Composite	*7.1*	*(0.23)*	*4.3*	*7.3*	*(0.13)*	*2.2*	*6.9*	*(0.07)*	*1.4*
Lower Secondary level									
A+ Country Group	17.1*	(0.09)	2.8	9.9*	(0.07)	1.4	6.6	(0.09)	1.7
U.S. Private	8.9	(0.94)	5.2	7.7	(0.20)	2.5	6.8	(0.05)	1.6
U.S. Public	9.9	(0.63)	4.7	7.7	(0.33)	2.3	6.6	(0.14)	1.4
U.S. Composite	*9.5*	*(0.77)*	*4.9*	*7.7*	*(0.29)*	*2.4*	*6.7*	*(0.11)*	*1.5*

*Significantly different from U.S. Composite, $p < 0.05$

we characterize the amount of course work taken by U.S. future teachers as compared to that of the future teachers in the A+ countries. Secondly, we look at the balance across the three areas, as to which receives, relatively speaking, the most preparation in terms of course work. The last section examines the variation across the 81 sampled U.S. colleges and universities toward understanding the large variation across the institutions in terms of average performance on the mathematics content knowledge assessment.

3.1 What Amount of Course Work Did Future Teachers Take?

To answer this question, as indicated in the previous section, future teachers were asked to indicate from a list of possible topics that could have been covered in their preparation programs which of them they had experienced in their particular coursework. We tabulated the total number of courses (topics) taken in each of the three areas for each future teacher. The country mean values were averaged over all future teachers at the end of their program within an institution and then a weighted average was taken over all sampled institutions. The A+ mean was the average of the means of the countries defining the international benchmark. The resulting means were rounded to a whole number (see Tables 2 and 3).

We first consider all 38 possible topics/courses defining the three areas of coursework directly related to the teaching of mathematics. This total does not include general university-related coursework such as liberal arts requirements or courses related to other areas of teaching such as additional areas needed for certification, i.e., second majors, or optional courses unrelated to mathematics or the teaching of mathematics.

Table 3 Mean percentage of the total number of courses (topics) allocated to the three areas of general pedagogy, mathematics pedagogy, and mathematics

	Mathematics			Mathematics pedagogy			General pedagogy		
	Mean	(se)	SD	Mean	(se)	SD	Mean	(se)	SD
Primary level									
A+ Country Group	39.6*	(0.3)	11.4	53.6*	(0.4)	11.6	46.4*	(0.4)	11.6
U.S. Composite	*33.3*	*(0.8)*	*16.0*	*52.1*	*(0.6)*	*9.7*	*47.9*	*(0.6)*	*9.7*
Lower Secondary level									
A+ Country Group	50.8*	(0.3)	6.6	60.5*	(0.2)	7.5	39.5*	(0.2)	7.5
U.S. Private	34.3	(2.9)	16.0	52.1	(0.6)	13.1	47.9	(0.6)	13.1
U.S. Public	42.8†	(2.0)	16.4	55.4†	(1.2)	8.9	44.6†	(1.2)	8.9
U.S. Composite	*39.4*	*(2.4)*	*16.2*	*54.1*	*(1.0)*	*10.8*	*45.9*	*(1.0)*	*10.8*

*Significantly different from U.S. Composite, $p < 0.05$

†Significantly different from U.S. Private, $p < 0.05$

Future lower secondary mathematics teachers in the top achieving countries took a total of 34 of the 38 courses—half again as many courses as were taken by their counterparts in the elementary preparation programs. The U.S. future teachers, on the other hand, took a total of only 25 teacher preparation related courses, some nine less than that taken in the A+ countries. It is also relevant to note that this average total is only four more than was the case for their counterparts in the primary preparation programs.

Table 2 describes the typical number of courses (topics) taken in each of mathematics, mathematics pedagogy and general pedagogy separately together with their standard errors and indicators of statistically significant differences.[3] For the most part, lower secondary or middle school future teachers are specialists who primarily teach mathematics and perhaps on other subject. As mathematics specialist teachers, in contracts to generalist teachers who teach mathematics along with other subject matters, one would expect them to take more mathematics courses. This in fact is the reality, even in the A+ countries future middle school teachers take on average almost twice as many courses in mathematics as compared to their counterparts in the elementary programs—17 compared to 9. This was not the case for U.S. future middle school teachers who took on average only about three more courses than those preparing to teach mathematics in elementary school. This is also some seven fewer courses than are taken in the A+ countries. There were no statistically significant differences in mathematics course taking between the public and private institutions. In addition, the mathematics pedagogy area revealed statistically signif-

[3]Statistical significance of the comparisons made with the A+ group and those between the U.S. Public and Private samples appearing in Tables 2 and 3 were computed using the Balanced Repeated Replication (BRR) estimated standard errors.

icant differences between the A+ countries and the U.S. amounting to a difference of two courses more being taken in the top achieving countries.

The last area to be compared for differences in course taking is general pedagogy. The pattern of differences changes from the other two content areas as no differences were apparent between future teachers in the U.S. and the A+ countries as both covered on average around seven topics.

Large and likely consequential differences were evident between the OTL experiences of lower secondary future teachers in the U.S. and their counterparts in the top achieving countries. These differences were particularly large in mathematics and mathematics pedagogy where in total on average U.S. middle school future teachers took nine fewer courses. Yet there was no trade off as the number of topics covered in general pedagogy was the same for future teachers in both the A+ countries and in the U.S.: future teachers in the top achieving countries simply encountered more topics/courses overall and all of these were directly related to mathematics.

3.2 How Does Course Work in the Three Areas Fit Together?

The previous section described the amount of course work taken by future teachers in each of the three key areas associated with teacher preparation. In this section we look at the relative allocation of course work so as to address the question of balance across the three areas which was identified in the literature as central to teacher education research. By examining the three areas of mathematics, mathematics pedagogy and general pedagogy in concert, we gain an understanding of teacher preparation as a whole. In focusing on such profiles we also remove some of the possible cultural bias resulting from differences in response tendencies across the countries.

We began this section with an indication of the size of the whole program—the total number of teacher education courses taken by future mathematics teachers. We now look at how that total was allocated across the three areas. To characterize the balance across areas we calculated two proportions. The first was defined as the number of mathematics courses taken relative to the total number of courses taken in all three areas, i.e., the denominator included mathematics courses and both types of pedagogy courses (mathematics education and general pedagogy).

This notion of balance can also be expressed as a ratio of the number of mathematics courses taken to the total number of pedagogy courses taken. Such ratios and proportions are descriptive indicators of the relative importance of course taking in each of the three area while removing the effect of the total number of courses taken, i.e., they are independent of the total.

The ratio and proportion capture how those courses are traded off against each other. The average value of such an indicator across the future teachers within an institution likely reflects the sense or “zeitgeist” of the institution relative to which areas of study are more important for teacher preparation. Put another way it is an indicator of how the limited finite time available for teacher preparation was divided between content knowledge and pedagogy in an institution.

The value of the ratio (not the proportion) can be less than one (indicating on average more pedagogy courses taken than mathematics courses); one (an equal allocation of course taking between the two areas); or greater than one (indicating relatively more coursework in mathematics than pedagogy). In effect, we argue that values less than one imply the conception that pedagogy is more important than content. When the ratio is greater than one, content is viewed as more important than pedagogy, and when equal to one there is an even balance between the two. On the other hand, the total number of courses taken over all three areas reflects the country's or institution's definition of how much teacher education course work overall is needed to prepare a high quality teacher of mathematics. We argue both are important.

The second proportion is defined as the number of general pedagogy courses taken relative to the total number of pedagogy courses taken including both general and content specific courses. This proportion reflects the relative importance of general pedagogy course taking to mathematics pedagogy course taking. This can also be expressed as a ratio.

As would be expected for teachers of mathematics, a larger proportion of future middle school teachers' content course work was allocated to mathematics that was the case for future primary teachers. This likely reflects their focus on a single content area excluding what might be a minor teaching area if they had one. In the top achieving countries future teachers on average took about half (51 %) of their education related course work in mathematics.

For U.S. future teachers this percentage is statistically significantly less as it almost reaches 40 % representing a difference of more than 10 %. The difference—about 9 %—between future teachers in private versus public institutions was large and also statistically significant. The average mathematics emphasis for private institutions was only slightly more than one-third, indicating a content to pedagogy ratio below one (0.6) compared to the ratio for the A+ countries that was essentially one. Projecting this 10 % difference into an American context, and assuming topics and courses are the same, this represents nearly two semesters more of mathematics—some 20 semester hours more—being taken by future teachers in the A+ countries compared to those in the U.S.

Which type of pedagogy constituted the other half of the course work in the A+ countries? By a ratio of 3 to 2 most of the course work was taken in mathematics specific pedagogy. This implies that nearly 80 % of the course work for the future middle school teachers of mathematics in the A+ countries was allocated to course work related to mathematics, i.e., courses that focused on mathematics content or combined the consideration of mathematics content and pedagogy. For U.S. future teachers this percentage was 72 %. In the U.S. the ratio of mathematics pedagogy to general pedagogy course taking was also more than one as almost 55 % of the pedagogy course taking was allocated to mathematics pedagogy.

All of this suggests clear differences between future teachers prepared in the top achieving countries and those prepared in the U.S. Given that their mathematics knowledge was higher than that of the U.S. this provides a point of dialogue regarding both the absolute and relative allocation of course work that should be required in the preparation of future teachers of mathematics.

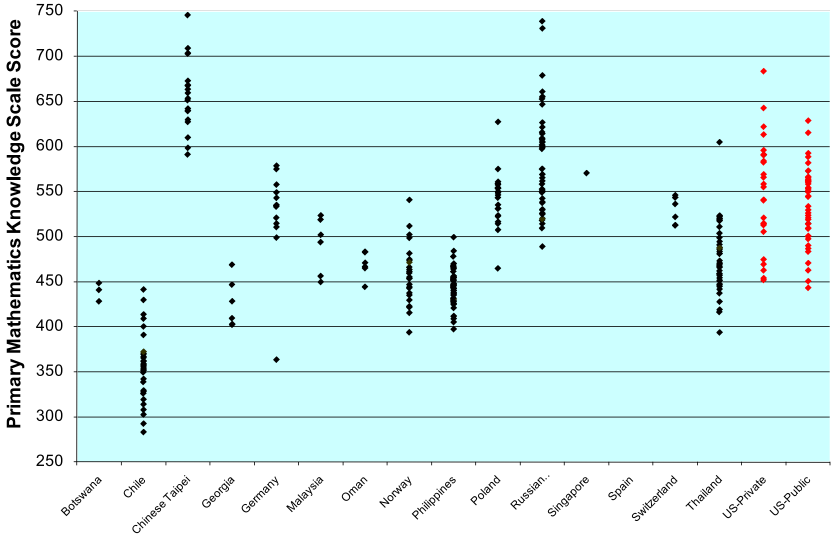

Fig. 2 Institution level mathematics knowledge scale scores by country at the lower secondary level

3.3 OTL in Relationship to U.S. Institutional Variation

The degree of within country variation in sampled institutions' average future teacher performance on the mathematics content knowledge assessment is displayed in Fig. 2. U.S. institutions vary substantially such that some of them have average performance levels commensurate with those of the A+ countries yet some perform at levels consistent with institutions in the countries with the lowest levels of performance.

In this section we examine OTL for the U.S. and the A+ institutions using internationally benchmarked indices related to the allocation of course work to the three areas by the sampled future teachers in each institution. To develop international benchmarks, we analyzed the course taking patterns (how many courses were taken in each area) of future teachers in the A+ countries in order to find the most common patterns.

The international benchmark for middle school teacher preparation programs was set at 15 or more mathematics courses, 10 or more mathematics pedagogy courses and seven or more general pedagogy courses. This pattern of course taking was taken by around 45 % of the future teachers in the A+ countries. Only 4 % of U.S. institutions met this criterion.

Another way of examining the variability in OTL across U.S. institutions is to explore the variation in the two indices considered in the previous section—the percentage of total course work allocated to mathematics and the percentage of total pedagogy course work allocated to mathematics pedagogy. Across U.S. institutions preparing middle school teachers, the percent of education related course work allo-

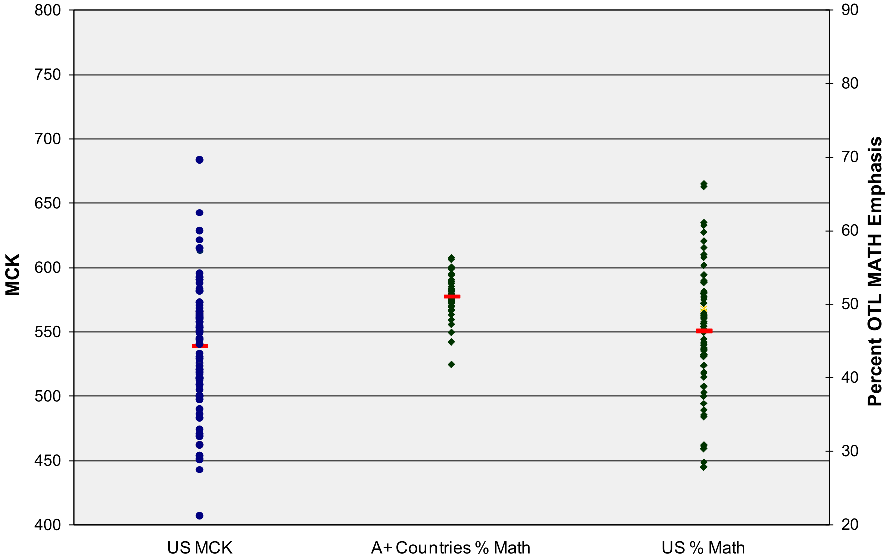

Fig. 3 Distribution of lower secondary mathematics knowledge score and pedagogy OTL percent mathematics pedagogy institution means for the U.S. and Top-Performing Countries

cated to mathematics courses ranged from 28 to 66 %. In the A+ countries the range was much smaller, ranging from 42 to 56 %—essentially all institutions hovering around the average of 50 %.

For the proportion of total pedagogy course work related to mathematics specific pedagogy, the range across U.S. institutions went from 23 to 86 %. Among A+ institutions the range was similar to that observed for mathematics ranging from 54 to 68 %. The relative amount of mathematics course work for the A+ institutions was approximately half of the education related course work. In addition, there was agreement among A+ institutional that well over half of all pedagogy was mathematics specific. Such was not the case in the U.S. It is likely that such variation might be related to the variation in the mathematics assessment results. The variation in the two indices is graphically portrayed in Fig. 3.

Using variance component analyses, we estimated the source of the variation. Such variation as described above can result from different sources: institutional variation, some of which can be attributed to the type of institution (public versus private); and individual future teacher variation, resulting from future teachers taking different patterns of courses even in the same institution. The policy implications, given the different sources of variations, are different.

Table 4 gives the estimated variance components for the two proportions—the proportion of course work related to mathematics and the proportion of pedagogy course work related to mathematics pedagogy. The patterns are almost identical for both. Variation related to differences between public and private institutions accounts for as little as 3 to 8 %. The vast majority of the variation in course taking was at the individual future teacher level. This ranged from 74 to 89 %. This likely

Table 4 Estimated variance component for percent of OTL devoted to mathematics and percent of pedagogy OTL devoted to mathematics pedagogy

Source	Percent mathematics		Percent mathematics pedagogy	
	Variance component estimate	Percent total variance	Variance component estimate	Percent total variance
Lower secondary level				
Institution (Public/Private)	8.4	3	3.5	3
Institutions	41.5	16	30.3	23
Future teacher	206.2	81	95.8	74

reflects one of two possibilities. First, in some universities and colleges there are different program types even within the broader designation of lower secondary. The other and most likely dominant source of the variation is related to individual future teacher choices related to course taking.

The last source of variation is directly attributable to institutional variation. Variation here is most likely attributable to either differences in the visions and definitions institutions have as to what constitutes quality teacher preparation or to differences in the quality of students attending the university or college. Such differences can lead to varied course choices. For example, if certain institutions recruit students more capable in mathematics than other institutions and those future teachers choose, because of their stronger backgrounds to take more mathematics, this would be reflected in the institutional variance component.

Given the partial control of such background differences introduced by accounting for the differences between public and private institutions suggests that the majority of such institutional variation might be more reflective of differences in the nature of the teacher preparation programs. This must remain a hypothesis, however, and cannot be tested within this study. Nonetheless, this hypothesis is further supported by the fact that the proportion of variation attributable to institutional differences is the largest for secondary preparation programs and the largest component within those programs is related to the pedagogy index. It is in the area of pedagogy where one could imagine more program differences. The component for mathematics course taking is also larger. This is consistent with the more detailed analyses done as a part of the MT21 Study (Schmidt et al. 2011).

These patterns were confirmed by a formal hierarchical analysis of variance which found that for lower secondary programs, the differences for both course-taking indices in terms of the contrast between public and private institutions was statistically significant ($p < 0.01$ and $p < 0.03$). These formal analyses confirm that the differences in the patterns described in the previous paragraphs are statistically significant.

The ultimate question posed by Figs. 2, and 3 is: to what is the large variation in mathematics content knowledge across institutions attributable? Clearly, one hypothesis is OTL as reflected by the institutional variation in the two indices discussed in the previous paragraphs. However, there is a competing hypothesis related

Table 5 HLM estimated effects on mathematics content knowledge and mathematics pedagogy knowledge scaled scores for lower secondary future teachers

Source	Math knowledge scale			Math pedagogy scale		
	Est	(se)	$p <$	Est	(se)	$p <$
Lower Secondary level						
Intercept	501.8	(13.2)	0.000	476.4	(15.2)	0.000
U.S. Public/Private Contrast	8.7	(10.5)	0.410	11.6	(9.6)	0.231
Percent of OTL devoted to Mathematics	1.2	(0.2)	0.000	1.3	(0.2)	0.000

to differences in the student body composition of different universities and colleges related to differences in admissions criteria. Our goal is to examine if there is a relationship between OTL and performance and not to argue causality. This is beyond the scope of this paper.

To examine this relationship, we used hierarchical regression analyses. At the lower secondary level (see Table 5) there were no statistically significant differences between the public and private institutions for the mathematics knowledge assessment ($p < 0.41$) nor for the pedagogical assessment ($p < 0.23$). For both assessments the percentage of course work related to mathematics was statistically significant ($p < 0.0001$) and in both cases the estimated effect size was around 1.25. However, for the type of pedagogy index the relationship was different depending on the assessment. For the mathematics assessment, the percentage of pedagogy course work related to mathematics pedagogy was statistically significant ($p < 0.023$). For the mathematics pedagogy assessment, however, it was not significant ($p < 0.372$).

In short, after accounting for the differences in admissions standards related to public and private institutions (at least a partial control for selection bias), OTL was related to the two competencies measured by the assessments. In all cases the proportion of education related course work devoted to mathematics was significantly related to performance with effect sizes indicating that a 10 % increase in mathematic course taking would predict a one-tenth of a standard deviation increase in the mathematics content knowledge assessment. These results indicate the presence of a relationship but do not warrant a causal inference due to the lack of control for initial differences in knowledge related to who attends which university. In the U.S. we were able to obtain SAT/ACT scores for a subset of these future teachers. Yet controlling for this in the regressions did not remove the statistically significant effects of OTL.

4 Discussion

This chapter set out to examine the issue of whether opportunity to learn was related to mathematics and mathematic pedagogy knowledge for future middle school mathematic teachers. Specifically we addressed the issue cited in the literature as in need of crucial empirical work—the balance of course work across the three areas of mathematics content, mathematics pedagogy and general pedagogy.

Using data from 81 randomly sampled U.S. public and private institutions as well as international data from the top achieving countries, we examined the nature of the differences in OTL between the U.S. and those countries whose future teachers statistically significantly outperformed the U.S.

These results showed major differences in course taking between the A+ countries and the U.S. The differences represented a 10 % tradeoff between mathematics and general pedagogy with the A+ countries opting for more mathematics content preparation. Typically and with little variation across the institutions within the A+ countries, they allocated almost half of their education related course work to mathematics and by a ratio of 3 to 2 more mathematics pedagogy than general pedagogy. By contrast the U.S. institutions not only on average allocated less course work to mathematics, but there was also much more variation in the range across the U.S. institutions both with respect to the amount of mathematics course work taken and the proportion associated with the two types of pedagogy courses which was enormous ranging in some cases between approximately 25 and 86 %.

Clearly within the U.S. there is little agreement as to what constitutes teacher preparation across the many colleges and universities that educate lower secondary or middle school teachers. Presumably, since all institutions design their programs consistent with their vision of what a highly qualified mathematics teacher should know, there is substantial disagreement within the U.S. as to what that vision is. Related to these differences in OTL there is also substantial variation in what they know upon completion of their formal schooling. That variation results in some U.S. institutions where the average performance is at a level commensurate with institutions in the top performing countries but also some where the average performance is at the level of institutions in developing countries such as Botswana. The data are not able to support a more causal inference but clearly the amount of topics encountered/courses taken as well as the relative allocations across the three areas are related to the enormous variation in the levels of mathematics and mathematics pedagogical knowledge. Further analyses using additionally available data are needed to explore these relationships. The prior mathematics knowledge of these future teachers is only adjusted for in terms of controlling for institutional differences related to differences in admissions standards for public and private institutions.

At this stage in the analyses related to the U.S. TEDS-M data, the implications are strikingly similar to what was found in TIMSS—that curricular differences in terms of content coverage are related to achievement. In the case of TEDS-M this implies that the course work offered, required and taken has a relationship to the professional competencies in terms of knowledge that future teachers have upon completion of their preparation programs. Issues related to eliminating bias relevant to variation in selection and admissions requirements across institutions must be further addressed but at this point it is hoped that this paper both stimulates and provides some empirical evidence for a dialog related to what good teacher preparation should be.

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Greater Expectations in Lower Secondary Mathematics Teacher Preparation: An Examination of Future Teachers' Opportunity to Learn Profiles

William Schmidt and Leland Cogan

Abstract The Teacher Education and Development Study in Mathematics (TEDS-M) tested students in their final year of teacher preparation on their knowledge of mathematics undergirding secondary school mathematics (MCK). Several articles have explored the relationship between students exposure to specific opportunities to learn (OTL) in their programs to their knowledge as demonstrated on the TEDS-M assessment. Here we sought to identify the courses that virtually all future teachers took in the top-achieving (A+) TEDS-M programs. Despite the fact that the top-achieving programs came from four countries on three continents, a set of nine courses that nearly every future teacher in these programs had taken was readily evident. Requirements had a strong emphasis on calculus with a majority of the nine courses, six, being university mathematics courses. This set of courses differed dramatically in number and focus from the set of empirically identified required courses among the international bottom 25 percent of programs or the set identified among the top-achieving programs in the U.S. The relatively large number of A+ requirements and electives demonstrated a greater consistent vision for teacher preparation than the few standards identified among the international bottom 25 percent of programs. This observation led to the hypothesis that excellence, at least as its measured by the TEDS-M MCK, may have very few paths leading to it but conversely many ways to arrive at much less impressive performance.

Keywords Opportunity to learn · Mathematics teacher preparation · International comparisons of education · Teacher Education Study—Mathematics · TEDS-M

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S. Blömeke et al. (eds.), *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn*, Advances in Mathematics Education,
DOI 10.1007/978-94-007-6437-8_18,

1 Introduction

The 1995 Third International Mathematics and Science Study (TIMSS), conducted by the International Association for the Evaluation of Educational Achievement (IEA) in over 40 countries, administered assessments to students in grades 4, 8, and at the end of secondary school and included an analysis of official curriculum standards and textbooks. This expanded the IEA tradition of collecting information about the curriculum taught and studied in schools, referred to as students' opportunity to learn (OTL), in contextualizing students' academic performance (Floden 2002; Schmidt and Maier 2009). Subsequent analyses demonstrated that these curricular measures were powerful predictors of student performance (Schmidt et al. 2001). The greater expectations embedded in the curricula of the highest achieving countries, called the "A+" group, appeared to be focused and coherent in contrast, for example, with that in the U.S. which was seen as being "a mile wide and an inch deep" (p. 122, Schmidt et al. 1997, 2005; Valverde and Schmidt 2000).

Both the recently conducted Teacher Education and Development Study—Mathematics (TEDS-M) sponsored by the IEA and the earlier Mathematics Teaching in the 21st Century (MT-21) project extended the OTL concept to their investigation of tertiary education (Tatto et al. 2012; Schmidt et al. 2011a). One report found significant relationships between the OTL experiences future teachers had during their teacher preparation programs and their mathematical content knowledge (MCK) (Schmidt et al. 2011a). In addition, some evidence of country differences in OTL has been reported that may reflect, at least in part, cultural perspectives on teacher preparation with varying emphases (Schmidt et al. 2007; Blömeke and Kaiser 2012). Here we wanted to examine the issue differently. Given a particular level of performance on the TEDS-M MCK score, to what extent are future teachers' OTL experiences similar or dissimilar? In other words, similar to what had been done exploring the greater expectations of the highest achieving countries in TIMSS, is there a group of courses that virtually all future teachers took in the top-achieving, i.e., "A+", TEDS-M programs? How might this top-achieving A+ OTL pattern look different from the OTL patterns found in other programs?

2 Background

More than 13,000 future teachers of mathematics in 15 countries participated in TEDS-M. A central element to both the earlier MT-21 study (Schmidt et al. 2007, 2011a) and TEDS-M was the concept of professional competence generally defined and, more specifically, with respect to teaching (Spencer and Spencer 1993; Weinert 2001; Taconis et al. 2004). The conception of competence included both professional knowledge and beliefs as these are related to the knowledge and the skills needed to successfully address core professional responsibilities. TEDS-M focused on the opportunities provided to and taken by future teachers while engaged in teacher preparation toward developing the competencies deemed relevant to quality classroom instruction. It assessed their OTL experiences, their beliefs about the teaching and learning of mathematics, as well as their knowledge about mathematics (MCK) and their knowledge related to the teaching of mathematics (PCK) (see Tatto and Senk 2011). The PCK measurement was an attempt to capture the sort of professional knowledge discussed by Shulman (1986, 1987) and developed by Ball and her colleagues (Ball and Bass 2003; Ball et al. 2005).

The OTL construct or "educational opportunity" reflects the curricular focus as originally developed by Carroll (1963) and employed in IEA international studies (McDonnell 1995; Floden 2002; Schmidt and Maier 2009). The interest in OTL has been to provide context for student achievement. Although some IEA studies have sought OTL information from primary and lower secondary students the most informative OTL reports have come from classroom teachers (Schmidt et al. 2001). This sort of classroom based OTL report is more challenging to obtain in a higher education context. Consequently, MT21 and TEDS-M collected course syllabi and asked future teachers to indicate which courses or topics they had experienced during their teacher preparation program. In most cases the "grain size" of the listed topics did correspond to a specific course yet the reference to a "course" remains somewhat of an inference although it has been supported by the syllabi analysis (Schmidt et al. 2011a). Nonetheless, there were topics that were covered in one or more courses in some institutions. The equating of topics with courses was least ambiguous in considering the university mathematics topics but is important to bear in mind. For brevity and readability, the term course is used in this paper. In an analysis of these future teacher reports summarized by the institutions they attended, institutions appeared to have different implied definitions of teacher competence as reflected in the opportunities to learn (OTLs) they provided (Schmidt et al. 2007). Aggregated to the country level these institutional patterns seemed to reflect different cultural positions or philosophies of what a "qualified" future teacher should know.

TEDS-M investigated the preparation of primary level teachers of mathematics in addition to that of lower secondary mathematics teachers. Investigating the OTL of these future primary mathematics teachers, Blömeke and Kaiser (2012) conducted a latent class analysis of OTL. The latent constructs identified exhibited significant relationships with future teachers knowledge as measured by the MCK and PCK scores. In addition, they found some evidence of cultural differences in the way the constructs were patterned across countries.

In previous analyses, both OTL and the TEDS-M MCK and PCK knowledge scores have demonstrated substantial differences across countries with some evidence for cultural patterns (Blömeke and Kaiser 2012; Schmidt et al. 2011b, 2011c). As intriguing as these cultural patterns of OTL may be we wanted to explore the relationship between OTL and the knowledge measures from the perspective of outstanding performance. Do the highest performing programs demonstrate a relatively consistent OTL pattern and is this different from other groups of programs? The goal in asking this question is to identify an international A+ benchmark for lower secondary mathematics teacher preparation similar to what had been done exploring the greater expectations of the highest achieving countries in TIMSS.

3 Methods

The goal of this analysis was to find which, if any, courses had been taken by essentially all of future teachers in the top performing, A+, programs. This required four steps: (1) identifying the top-performing teacher preparation programs; (2) identifying the courses taken by the future teachers in the top-performing programs; (3) identifying the pattern, i.e., set of courses, taken most commonly by these future teachers; and (4) comparing the OTL pattern identified from the A+ programs' future teachers with other groups of future lower secondary teachers.

3.1 Identifying International Top-Performing Programs

The A+ benchmark efforts in TIMSS were conducted at the country level. Up to this point, most of the TEDS-M OTL work also has been conducted using summaries at the country and or institutional level. However, previous analysis has shown considerable overlap across countries in the distributions of programs' mean MCK score (Center for Research in Mathematics and Science Education 2010; Schmidt et al. 2011b). In addition, in some countries, particularly with respect to lower secondary teacher preparation, institutions may have more than one type of program that prepares teachers. In Germany, for example, some teachers are prepared in a program that prepares teachers to teach mathematics across grades that range from the primary grades through grade 10. In another program, future mathematics teachers are prepared to teach through the end of secondary, i.e., the mathematics taught in pre-university secondary schools (e.g., gymnasium). Similarly, in the U.S. future lower secondary mathematics teachers are prepared in one of three types of programs: (1) as mathematics specialists who will be licensed to teach all secondary grades, e.g., grades 6 or 7 through grade 12; (2) as mathematics specialists licensed to teach in the middle grades only, e.g., grades 4 or 5 through grades 8 or 9; and (3) as mathematics specialists who will be licensed to teach mathematics in all the grades k-8 as well as being licensed to teach all subjects in the primary grades, k-5 or 6. Even though each of these programs prepares future teachers to be mathematics specialists, their focal grade levels differ. Consequently, this analysis used the combination of institution and program to examine which courses future teachers indicated they had been exposed to during their teacher preparation program. This situation occurs in only six of the 15 TEDS countries: Chili, Germany, Norway, Poland, Thailand, and the US. Table 1 reports the number of each type of program and the percent of each country's future teacher sample by program type for each of the participating TEDS-M countries.

The mean level of proficiency for each program was determined by averaging the TEDS Mathematics Content Knowledge (MCK) score for all the future teachers in that program. All programs across all countries were ordered according to their mean MCK and the top ten percent were identified. The top ten percent, referred to as the international A+ programs, included 39 programs from four countries: Poland (1), the Russian Federation (15), Taiwan (17), and the U.S. (6).

Table 1 Number of programs for each country preparing teachers to teach either at the secondary level only or at the secondary and primary level and the weighted percent of sampled future teachers in them

Country	Primary and Secondary	Secondary	Percent[a] of each country's sampled future teachers
BOTSWANA	–	3	100
CHILE	–	9	8
CHILE	28	–	92
GEORGIA	–	6	100
GERMANY	–	12	70
GERMANY	7	–	30
MALAYSIA	–	6	100
NORWAY	–	5	4
NORWAY	23	–	96
OMAN	–	7	100
PHILIPPINES	–	48	100
POLAND	23	–	100
RUSSIAN FED.	–	48	100
SINGAPORE	–	1	100
SWITZERLAND	–	6	100
TAIWAN	–	19	100
THAILAND	45	–	100
USA	–	71	42
USA	24	–	58

[a]Weighted by TEDS-M future teacher weights

3.2 OTL Benchmarks

The OTL data came from part B of the TEDS-M future teacher questionnaire. Future teachers were asked to indicate if they had "studied each topic as part of your current teacher preparation program." They were asked to do this for 19 university level mathematics topics such as differential geometry and multivariate calculus; seven secondary school mathematics topics such as numbers and calculus; and eight math education/pedagogy topics such as development of mathematics ability and thinking and mathematics standards and curriculum (see Appendix B, Tatto et al. 2008). Two math education topics—context of math education and affective issues in mathematics—were excluded from this analysis.

The quest here was to find the OTL requirements among the A+ top-achieving programs, i.e., the set of courses that virtually all future teachers in these programs had. As we did not have access to these requirements as officially defined by these programs we developed an empirical definition based on the OTL experiences reported by the future teachers in these A+ programs. One might expect that a requirement would mean that all future teachers would have had the specific OTL experience yet this criterion appeared from the data to be too rigid. Consequently,

Table 2 Criteria used to identify required and elective courses within the international A+ programs

Empirically Defined Courses Sets		Percent of Future Teachers within program who had course	Percent of Programs Exhibiting the Future Teacher Criterion
1	Requirements	80 % or more	90 % or more
2	Electives	80 % or more	at least 75 %

two criteria were employed on this quest. The first was the percent of future teachers within a program that indicated they had experienced a particular course. The benchmark that defined this criterion was 80 percent. The second was the percent of programs that exhibited the first criterion. The second criterion differentiated courses in the "requirement" group from those in the "electives" group.

Required courses demonstrated a rather broad consensus on OTL experiences with more than 90 percent of the programs meeting the 80 percent future teacher criterion. The second group of courses, referred to as electives, met the same 80 percent or more future teacher criterion but fewer programs met this criterion. Table 2 identifies the thresholds used for the two criteria in defining the requirement and elective set of OTL experiences.

4 Findings

4.1 Identifying International A+ Benchmarks

Given that the 39 A+ teacher preparation programs came from four countries, on three continents, each with their own culture and tradition with respect to education and mathematics, there was reason to doubt the success of a quest to find commonality among them, at least enough commonality among the OTL experiences to quality as requirements. Nonetheless, across all the future teachers in these A+ programs a set of nine courses were nearly universally experienced by them. These courses included six university mathematics courses (beginning calculus, calculus, linear algebra, probability, differential equations, and multivariate calculus); two math education courses (mathematics instruction and observation, analysis and reflection of mathematics teaching); and one school mathematics topic (functions, relations, and equations). Each of these courses was experienced by over 90 percent of all the future teachers in the 39 A+ programs: a low of 92 percent for the math education course, observation, analysis and reflection of mathematics teaching, and a high of 99 percent for three university mathematics courses, beginning calculus, calculus, and linear algebra.

This empirically derived required course set reflects a strong emphasis on mathematics and a concentration on calculus (4 courses). This emphasis on mathematics is again obvious in the nine electives. Between 88 percent and 92 percent of these

Table 3 Empirical requirements and electives identified within the international A+ programs

Requirements	Electives
University Mathematics	University Mathematics
Beginning Calculus	Abstract Algebra
Calculus	Analytic Geometry
Differential Equations	Axiomatic Geometry
Linear Algebra	Number Theory
Multivariate Calculus	Set Theory
Probability	
Math Education	Math Education
Math Instruction	Math Standards
Observing Math Teaching	–
School Mathematics	School Mathematics
Functions	Geometry
–	Numbers
–	Statistics

A+ future teachers had these courses but only between 80–87 percent of the A+ programs had at least 80 percent of their future teachers reporting having had the course. Table 3 summarizes the number and types of courses identified as requirements and electives for the A+ programs.

The geographical, cultural, and, likely, educational diversity represented by the 39 A+ programs[1] lends credibility and validity to identifying this set of nine required courses as an international benchmark. The benchmark's strong emphasis on university mathematics is intensified by the elective set that adds five additional university mathematics courses. It is also instructive that the other two areas, math education and school mathematics, are not neglected in this A+ benchmark.

[1]Required and elective courses stemmed from the A+ programs as identified by the TEDS-M MCK score. Using the TEDS-M PCK score to identify the top 10 % of programs yielded only slightly different results. The PCK top 10 percent of programs come from the same four countries, Poland (1), the Russian Federation (14), Taiwan (19), and the U.S. (5) with only slightly different programs within those countries. This may be explained, at least in part, by the .93 correlation between the two scores at the program level. This correlation is only .75 at the individual future teacher level (see Robinson 1950 for a discussion of the relationship between individual correlations and group correlations). The vast majority (31) of the 39 A+ MCK programs were also in the PCK top 10 percent. Eight programs appeared only in the MCK top 10 (4 from the Russian Federation and 4 from the U.S.); eight others (three from the Russian Federation, two from Taiwan, and three from the U.S.) appeared only in the PCK top 10. Requirements according to the PCK top 10 were the same as the MCK top 10 with the addition of one university mathematics topic (analytic geometry). Because the results differed so little between the MCK A+ and the PCK A+ programs, no further analyses based on PCK are reported.

Table 4 Empirical requirements and electives identified within the U.S. top ten percent programs

Requirements	Electives
University Mathematics	University Mathematics
Beginning Calculus	Abstract Algebra
Calculus	Axiomatic Geometry
Linear Algebra	Number Theory
Multivariate Calculus	Probability
–	Statistics
Math Education	Math Education
Math Instruction	–
Math Standards	–
Observing Math Teaching	–
Dev. Teaching Plans	–
School Mathematics	School Mathematics
–	Functions
–	Statistics

4.2 Identifying U.S. Top Ten Benchmarks

The previous section examined required and elective courses as defined by the international A+ group of programs. It is possible to define a top-achieving group within a single country as well. This prompts the question, what might the set of required and elective courses look like as identified by the top performing programs in the U.S.? This is particularly interesting as the U.S. had six programs in the international A+ and given the size of the U.S. sample few programs would need to be added to these six to reach the ten percent threshold. Indeed, the top ten percent of U.S. programs included an additional four programs: 10 out of the total of 95; 60 from publically supported colleges/universities and 35 from private colleges/universities.[2] Technically, 10 percent would be 9.5 programs but we applied the standard rounding algorithm to obtain ten.

The required and elective courses identified by the U.S. top ten programs are displayed in Table 4. Given that 60 percent of them, i.e., 6 out of the 10, were also in the international A+ one might expect few differences in requirements and electives. All of the U.S. top ten university mathematics course requirements are to be found in the international A+ requirements. In addition, all of the international A+

[2]For both organizational and operational reasons, public and private sample data collection was conducted in two consecutive years. The public colleges/universities sample adhered to the TEDS-M timeline and was the only sample included in official TEDS-M reports. The following year the study was conducted with the sample of private colleges/universities according to all TEDS-M sampling and study procedures. For further details, see Appendix A of *Breaking the Cycle: An International Comparison of U.S. Mathematics Teacher Preparation* (The Center for Research in Mathematics and Science Education 2010).

math education requirements are found in the U.S. requirements. Yet the U.S. required one less course overall; two fewer university mathematics courses and two additional math education courses. Although university mathematics and math education are represented by the same number of courses the overall emphasis seems weighted more towards math education as these four represent 67 percent of all the math education courses but the four university mathematics courses represent only 21 percent of all the possible university mathematics courses listed in the TEDS-M survey.

Recall that requirements are those courses for which over 80 percent of future teachers in a program reported having the course and 90 percent or more of the programs exhibited this criterion. Three U.S. requirements were as close to being universal as might be possible with 98–99 percent of all future teachers in those programs reporting having had the course and all ten programs having reached the 80 percent criterion. These three were beginning calculus, calculus, and the math education course, mathematics instruction (e.g., representation of mathematics content and concepts, etc.). The 80 percent criterion was met by nine of the programs for the other required courses. The fewest overall percent of future teachers for a required course was for the math education course, developing mathematics teaching plans (89 percent).

Elective courses were those for which the 80 percent criterion was met in seven or eight of the top ten U.S. programs. The overall percent of future teachers reporting having had these courses ranged from a low of 87 percent for two university mathematics courses, axiomatic geometry and statistics, to a high of 95 percent for the university mathematics probability course.

4.3 Comparing Benchmarks Across Four Program Groups

In the previous section we sought to identify a set of courses that were taken by virtually all the future teachers in their teacher preparation programs. Criteria were established and both required and elective course sets were identified. These required and elective courses identified among the A+ programs provide a benchmark for examining other groups of programs. The required and elective courses as identified by the U.S. top 10 programs were contrasted with this A+ benchmark. Requirements were readily identified with both of these top performing program groups. This was particularly surprising in the case of the A+ in that we found a relatively large number of courses experienced so broadly across so many programs located in four different countries. This prompts the question as to whether such broadly based experiences, i.e., required courses, can be identified with other groups of programs. If such requirements can be identified, are they the same or similar to the A+ benchmark?

To explore these questions we looked at the bottom 25 percent of all 392 international programs—a group that might be quite different in terms of requirements—similarly as to how the A+ group was defined and explored. We also looked at

Table 5 Total number of each type of course identified as requirements or electives by program group

	International A+ Programs	U.S. Top 10 Programs	Lowest Performing Programs	Lowest Performing U.S. Programs
University Mathematics	11	9	4	2
Math Education	3	4	2	5
School Mathematics	4	2	5	5

the bottom 25 percent of all U.S. programs. The international bottom 25 percent included 97 programs from nine countries: Botswana (2), Chile (37), Georgia (5), Germany (1), Norway (11), Oman (1), the Philippines (28), Thailand (8), and the U.S. (4). These programs represented varying proportions of each country's total programs that participated in TEDS-M. Twenty-three programs were in the U.S. bottom 25 percent. Four of these were also in the international bottom 25 percent. The U.S. was the only country to have programs in both the international A+ group and in the bottom 25 percent of all international programs.

Table 5 summarizes requirements and electives together by type of course for the four program groups, the international A+, the U.S. top 10, and the two bottom 25 percent groups. Both the A+ and the U.S. top 10 groups included more courses in the required and electives sets (18 and 15) than did the lowest program groups (11 and 12). In addition, the contrast in the types of courses is also rather striking: the top programs exhibited 2 to 5 times as many mathematics courses as did the two lowest programs.

Although the A+ benchmark included four school mathematics courses, the bottom program groups emphasized these to a greater extent. For both of these groups the majority of the mathematics learning occurred in school mathematics courses rather than in university mathematics courses. Both emphasized school mathematics to a greater extent than university mathematics as five school mathematics courses represent about 70 percent of all the school mathematics courses whereas the two or four university mathematics courses represent only about 10 or 20 percent respectively of the university mathematics courses listed in the TEDS-M survey.

Tables 6 and 7 detail the obvious differences summarized in Table 5 by identifying the specific required and elective courses. Requirements among the A+ reflect a heavy emphasis on calculus. Although the required courses as defined by the U.S. top ten included only one less course than the A+, the focus was decidedly different. The A+ benchmark requirements included six mathematics courses, two math education topics, and one school mathematics topic. In contrast, the U.S. top ten requirements included two fewer mathematics courses, two additional math education courses and no school mathematics courses. Nonetheless, there is greater agreement exhibited between these top groups as to what is necessary in mathematics teacher preparation, i.e., what is required, than with either of the two bottom groups.

Although all bottom 25 percent required courses did reach the 80 percent future teacher criterion, the percent of programs meeting this criterion was less than that

Table 6 Empirical requirements identified within each group of programs

	International A+ Programs	U.S. Top 10 Programs	Lowest Performing Programs	Lowest Performing U.S. Programs
University Mathematics				
Beginning Calculus	1	1	–	–
Calculus	2	2	–	–
Differential Equations	3	–	–	–
Linear Algebra	4	3	–	–
Multivariate Calculus	5	4	–	–
Number Theory	–	–	1	1
Probability	6	–	–	2
Math Education				
Math Instruction	7	5	2	–
Math Standards	–	6	–	3
Observing Math Teaching	8	7	–	–
Dev. Teaching Plans	–	8	–	–
School Mathematics				
Functions	9	–	–	–
Measurement	–	–	3	4
Numbers	–	–	4	5
Statistics	–	–	–	6

for the top performing groups. The most pronounced international bottom requirement was the school mathematics numbers course. Over 99 percent of all future teachers in these programs reported having taken this course and every one of the programs reached the 80 percent future teacher criterion. The math education course on mathematics instruction was the least popular requirement with about 87 percent of future teachers having had this course and only 80 percent of the programs meeting the 80 percent future teacher criterion.

4.4 Exploring the International A+ Benchmarks Across Program Groups and Countries

4.4.1 A+ Courses

Using the international A+ course taking as a benchmark, what sort of differences, if any, are evident in how many future teachers in each program group reported having experienced these requirements and electives? Responses to this question are graphically presented in Table 8. For all the required courses and for the vast majority of electives, well over 90 percent of A+ future teachers took these courses.

Table 7 Electives identified within each group of programs

	International A+ Programs	U.S. Top 10 Programs	Lowest Performing Programs	Lowest Performing U.S. Programs
University Mathematics				
Abstract Algebra	1	1	1	–
Analytic Geometry	2	–	–	–
Axiomatic Geometry	3	2	–	–
Number Theory	4	3	–	–
Probability	–	4	2	–
Set Theory	5	–	3	–
Statistics	–	5	–	–
Math Education				
Dev. of Math Thinking	–	–	4	1
Math Instruction	–	–	–	2
Math Standards	6	–	–	
Observing Math Teaching	–	–	–	3
Dev. Teaching Plans	–	–	–	4
School Mathematics				
Functions	–	6	5	5
Geometry	7	–	6	6
Numbers	8	–	–	–
Statistics	9	7	7	–

This set of courses was a bit less popular with those in the U.S. top ten. Other than probability, number theory, and set theory, the university mathematics courses were taken by a little more than half of those in the bottom programs. Both the math education courses and the school mathematics courses were taken more frequently by these future teachers than were the university mathematics courses.

All A+ future teachers had three of the nine required courses: beginning calculus, calculus, and linear algebra. Although well over 90 percent of these teachers took the other six, the course-taking pattern of taking all nine of these courses was exhibited by about 83 percent. The other most popular course taking patterns were to have had eight of the nine required courses. Adding the future teachers exhibiting these single course omission patterns to those having had all nine captures the OTL experience of over 96 percent of the A+ future teachers.

In spite of the fact that many of the future teachers in the U.S. top ten were also in the A+ group only about 50 percent of the top U.S. future teachers reported having taken all nine courses. However, adding those exhibiting the single course omission patterns accounted for nearly 82 percent. These OTL experience patterns were much rarer among future teachers in the two bottom program groups: about 13 percent in the international group and only about 2 percent in the U.S. Including those exhibiting the single course omission patterns only accounted for about a quarter or less of

Table 8 Percent of future teachers in each program group reporting they had experienced each international A+ required or elective course

	International A+ Programs	U.S. A+ Programs	International Bottom 25 Percent	U.S. Bottom 25 Percent
Empirical Requirements				
University Mathematics				
Beginning Calculus	■	■	⬓	⬓
Calculus	■	■	⬓	◔
Differential Equations	■	⬓	⬓	□
Linerar Algebra	■	■	⬓	⬓
Multivariate Calculus	■	■	◔	□
Probability	■	■	◕	◕
Math Education				
Math Instruction	■	■	◕	◕
Observing Math Teaching	■	■	◕	◕
School Mathematics				
Functions	■	◕	◕	◕
OTL Pattern: All Required Courses	83%	50%	13%	2%
Popular Electives				
University Mathematics				
Abstract Algebra	◕	■	◔	□
Analytic Geometry	■	⬓	⬓	⬓
Axiomatic Geometry	■	◕	⬓	⬓
Number Theory	■	■	■	◕
Set Theory	◕	◕	◕	◔
Math Education				
Math Standards	◕	■	⬓	◕
School Mathematics				
Geometry	■	◕	◕	◕
Numbers	■	⬓	■	■
Statistics	◕	◕	◕	◕
OTL Pattern: All Electives	65%	42%	12%	6%

Key:

33-50%	51-75%	76-90%	>90%
◔	⬓	◕	■

the future teachers: in the international bottom group, 27 percent; in the U.S. bottom group, 11 percent.

By definition, fewer future teachers reported having taken the elective courses. Among the A+ electives over 90 percent of the A+ future teachers reported having taken five of these nine courses with well over 80 percent for the other four. The most popular course taking pattern was to have taken all nine. Over 65 percent of the A+ future teachers exhibited this pattern which was less popular among the others: about 42 percent in the U.S. top ten, 12 percent in the international bottom group, and six percent in the U.S. bottom group. An additional 20 percent of the A+

group future teachers exhibited one of the single course omission patterns. Single course omission patterns were exhibited by an additional 28 percent of the U.S. top ten future teachers, 20 percent of those in the bottom international programs and about 11 percent of those in the bottom U.S. programs.

4.4.2 A+ Course Patterns

Fewer future teachers exhibited the pattern of taking all required (or elective) courses than the percent indicating that they had taken the least popular course in the set. Here we look at how prevalent this pattern of taking all the required or elective courses was by asking three questions: (1) how many programs have any future teachers exhibiting the course taking pattern of taking all courses in the defined set; (2) how many programs reach the criterion of having 80 percent of their future teachers exhibiting this pattern; and, (3) what is the average percent of future teachers exhibiting the given course taking pattern? Results for this last question for the four program groups were also included in Table 8.

Tables 9 and 10 respond to these three questions for the four program groups along with each TEDS-M country. Note that the pattern considered here, having taken all nine required courses, is rather stringent. Required courses were identified using two criteria: more than 80 percent of future teachers in a program took them and more than 90 percent of the programs met this 80 percent future teacher threshold. The previous section revealed that although over 90 percent of all A+ future teachers took each of the required courses, the pattern of having taken all nine courses was exhibited by only about 83 percent of these future teachers. Once these future teachers are situated in their respective programs the percent of programs demonstrating this 80 percent threshold is less than one might expect. The explanation for this somewhat counterintuitive phenomenon is that a number of relatively small programs did not reach the 80 percent criterion. This also explains the anomaly observed for some countries such as Taiwan.

Looking across countries the percent of teachers exhibiting the OTL pattern of taking all A+ required courses ranged from less than two (Chile) to nearly 87 (the Russian Federation). The Russian Federation was the only country in which this percentage exceeded that found among the A+ programs. A similar pattern across the countries is apparent in the results for the A+ electives OTL pattern of having taken all the courses (see Table 10). In both Table 9 and 10 all of the programs in many countries have at least some future teachers exhibiting the OTL pattern of having taken all the A+ requirements and electives yet very few if any of the programs reached the 80 percent criterion for this pattern. Again, this apparent discrepancy stems from a relatively large number of small programs in which the 80 percent criterion is not met.

Beyond requirements it must be acknowledged that the A+ future teachers in general had a general tendency to have had experience with nearly every course. It almost seems that that the common sense notion of the relationship between OTL and achievement holds here: those who take more learn more. Yet, one must not

Table 9 Percent of programs and future teachers with pattern of having all A+ required courses

	Percent of programs in which some future teachers took all required courses	Percent of programs in which 80 % or more of future teachers took all required courses	Percent of future teachers taking all required courses
International Top 10 % of Programs	94.9	51.3	82.7
U.S. Top 10 % of Programs	80.0		50.3
International Bottom 25 % of Programs	52.6	7.2	13.2
U.S. Bottom 25 % of Programs	16.7	–	2.0
BOTSWANA	100.0	–	25.2
CHILE	32.3	–	1.2
GEORGIA	100.0	–	42.7
GERMANY	72.9	0.4	13.5
MALAYSIA	100.0	24.0	70.7
NORWAY	72.5	–	4.6
OMAN	100.0	76.0	81.7
PHILIPPINES	91.2	2.0	33.1
POLAND	99.1	28.2	66.2
RUSSIAN FED.	100.0	79.1	86.6
SINGAPORE	100.0	–	45.7
SWITZERLAND	100.0	–	16.2
TAIWAN	100.0	47.5	75.2
THAILAND	98.4	33.4	65.6
USA	63.2	0.5	14.2

move too quickly to this conclusion. Even in the lowest programs there were *some* future teachers who took *all* the A+ required courses—yet not all the other courses A+ future teachers reported taking.

This conundrum prompts the speculation that, just as was been seen with same-named courses at the lower secondary level in the U.S., the context in which a course occurs may have a large effect on the extent and quality of the content covered (Cogan et al. 2001). In other words, the extent and depth of the content covered in a differential geometry or number theory course could differ substantially from one program to another, particularly as these are housed in different educational institutions in different countries having different education traditions. Unfortunately, the type of within classroom OTL data available from TIMSS was understandably not possible to obtain in TEDS-M to explore this issue in the same way. Nonetheless, this remains a reasonable hypothesis to explain some of the incongruous results for some of the lower performing countries, e.g., Malaysia, Oman, and Thailand.

Table 10 Percent of programs and future teachers with pattern of having all A+ elective courses

	Percent of programs in which some future teachers took all electives	Percent of programs in which 80 % or more of future teachers took all electives	Percent of future teachers taking all electives
International Top 10 % of Programs	97.4	20.5	64.6
USA Top 10 % of Programs	90.0	–	40.4
International Bottom 25 % of Programs	63.9	4.1	11.4
USA Bottom 25 % of Programs	33.3	–	5.8
BOTSWANA	100.0	–	5.6
CHILE	58.7	–	2.7
GEORGIA	62.9	–	13.2
GERMANY	85.9	–	12.6
MALAYSIA	100.0	–	20.1
NORWAY	94.4	–	10.1
OMAN	100.0	–	46.4
PHILIPPINES	88.0	0.3	19.4
POLAND	100.0	18.6	61.3
RUSSIAN FED.	100.0	25.9	68.5
SINGAPORE	100.0	–	3.0
SWITZERLAND	100.0	–	41.3
TAIWAN	100.0	1.1	49.8
THAILAND	100.0	29.2	63.8
USA	75.0	0.1	15.6

4.5 Exploring the U.S. Top Ten Benchmarks Across Program Groups and Countries

4.5.1 U.S. Top Ten Courses

The required group of courses as identified among the U.S. top 10 included one less course overall than seen in the international A+ benchmark. However, this reflected two fewer university mathematics courses and two additional math education courses resulting in a profile with more of an overall emphasis on math education. Table 11 summarizes individual future teachers OTL experiences for each of the U.S. top ten benchmark courses. Course taking for each course is reported together with the pattern of taking all required or all elective courses.

Over half, six of the nine, A+ required courses were university mathematics courses. In contrast fully half, four of the eight, U.S. top ten requirements were math education courses. Nonetheless, as can be seen in Table 11, the course taking reported by future teachers in these two top groups for the U.S. top ten required and

Table 11 Average percent of future teachers in each program group reporting they had experienced each U.S. A+ required or elective course

	International A+ Programs	U.S. A+ Programs	International Bottom 25 Percent	U.S. Bottom 25 Percent
Empirical Requirements				
University Mathematics				
Beginning Calculus	>90%	>90%	51-75%	51-75%
Calculus	>90%	>90%	51-75%	33-50%
Linear Algebra	>90%	>90%	51-75%	51-75%
Multivariate Calculus	>90%	>90%	33-50%	□
Math Education				
Math Instruction	>90%	>90%	76-90%	76-90%
Math Standards	76-90%	>90%	51-75%	76-90%
Observing Math Teaching	>90%	>90%	76-90%	76-90%
Dev. Teaching Plans	76-90%	76-90%	51-75%	76-90%
OTL Pattern: All Required Courses	79%	61%	12%	6%
Popular Electives				
University Mathematics				
Abstract Algebra	76-90%	>90%	33-50%	□
Axiomatic Geometry	>90%	76-90%	51-75%	51-75%
Number Theory	>90%	>90%	>90%	76-90%
Probability	>90%	>90%	76-90%	76-90%
Statistics	76-90%	76-90%	51-75%	51-75%
School Mathematics				
Functions	>90%	76-90%	76-90%	76-90%
Statistics	76-90%	76-90%	76-90%	76-90%
OTL Pattern: All Electives	64%	60%	14%	9%

Key:

33-50%	51-75%	76-90%	>90%
◱	⬓	◩	■

elective courses was quite similar. Somewhat surprising, perhaps, is the observation that more A+ future teachers exhibited the pattern of taking all of the U.S. top ten required courses, 79 percent, than those who were in the top U.S. programs, 61 percent. Adding in the percent of future teachers exhibiting the single course omission from the pattern of taking all required courses maintains the gap, about 93 percent of the A+ group compared to about 85 percent in the top U.S. programs.

4.5.2 U.S. Top Ten Course Patterns

Tables 12 and 13 correspond to Tables 9 and 10 but summarize results for the U.S. top ten course sets for the four programs groups and participating TEDS-M countries. As previously noted, more of the A+ future teachers had all of the U.S. top ten requirements than those in the U.S. top ten programs. Further comparing results for these two top-performing program required course sets reveals a tendency

Table 12 Percent of programs and future teachers with pattern of having all U.S. top ten required courses

	Percent of programs in which some future teachers took all required courses	Percent of programs in which 80 % or more of future teachers took all required courses	Percent of future teachers taking all required courses
International Top 10 % of Programs	100.0	41.0	79.2
USA Top 10 % of Programs	100.0	30.0	61.3
International Bottom 25 % of Programs	58.8	5.2	12.0
USA Bottom 25 % of Programs	25.0	–	5.7
BOTSWANA	100.0	–	35.0
CHILE	46.1	–	2.0
GEORGIA	100.0	–	39.8
GERMANY	85.2	0.4	21.1
MALAYSIA	100.0	12.4	68.5
NORWAY	73.2	–	4.2
OMAN	100.0	9.0	64.0
PHILIPPINES	93.0	0.3	25.8
POLAND	100.0	5.0	56.5
RUSSIAN FED.	100.0	59.1	79.3
SINGAPORE	100.0	–	46.7
SWITZERLAND	100.0	–	15.7
TAIWAN	100.0	3.7	63.1
THAILAND	97.2	40.0	70.5
USA	71.8	3.1	23.0

among the A+ group and the highest performing countries, i.e., Taiwan and the Russian Federation, to have a larger percentage of future teachers exhibiting the A+ required set than the U.S. top ten required set.

As was the case for the A+ OTL patterns, none of the countries exhibited as many future teachers with the OTL pattern of taking all U.S. top ten required courses as found among the A+ programs. The Russian Federation was the exception with essentially the same percentage as that found among the A+ programs, about 79 percent. Across all countries this ranged from two (Chile) to nearly 79 (the Russian Federation) with an average of about 41 percent.

For the U.S. top ten elective OTL pattern, the percent of future teachers with this pattern ranged from less than three percent (Singapore) to over 65 percent (Taiwan). The top of this range was about the same as the percent seen among the A+ program group. The average across all countries was about 31 percent; ten percent less than the average for the U.S. top ten required courses OTL pattern.

Table 13 Percent of programs and future teachers with pattern of having all U.S. top ten elective courses

	Percent of programs in which some future teachers took all elective courses	Percent of programs in which 80 % or more of future teachers took all elective courses	Percent of future teachers taking all elective courses
International Top 10 % of Programs	100.0	23.1	64.4
USA Top 10 % of Programs	100.0	30.0	60.0
International Bottom 25 % of Programs	70.1	3.1	14.4
USA Bottom 25 % of Programs	37.5	–	9.0
BOTSWANA	58.3	–	5.1
CHILE	76.9	–	3.5
GEORGIA	63.8	–	13.5
GERMANY	99.4	–	15.7
MALAYSIA	100.0	–	19.9
NORWAY	96.0	–	14.1
OMAN	100.0	12.8	62.7
PHILIPPINES	85.7	0.3	25.9
POLAND	100.0	18.9	61.8
RUSSIAN FED.	100.0	16.9	56.9
SINGAPORE	100.0	–	2.5
SWITZERLAND	82.5	–	38.6
TAIWAN	100.0	22.4	65.5
THAILAND	100.0	27.0	66.6
USA—ALL	80.2	1.5	20.7

5 Discussion

The purpose of this paper was to identify the courses that virtually all future teachers took in the top-achieving TEDS-M programs; to see if an empirical benchmark could be identified similar to what had been done exploring the greater expectations for the K-12 system of the highest achieving countries in TIMSS.

Despite the fact that the 39 A+ programs came from four countries on three continents, a set of courses that nearly every future teacher in these programs had taken was readily evident. Requirements had a strong emphasis on calculus with a majority of the nine courses, six, being university mathematics courses. All of the U.S. top 10 programs requirements and electives were evident in the A+ benchmarks with the exception of the math education course, developing teaching plans. Yet the U.S. benchmarks included fewer university mathematics courses and more math education courses yielding an overall emphasis that seemed to stress math education more.

Examining the lowest performing group of programs did reveal some requirements and electives in common across all these future teachers. The standards among these programs, however, were fewer in number than those observed among the top-performing programs and included much less university mathematics and more pedagogy. The bottom U.S. program standards revealed that the vast majority of the mathematics studied was school mathematics rather than university level mathematics. The rigor and focus of these school mathematics courses varied across programs and countries based on the level of secondary mathematics future teachers have had before matriculating to their teacher preparation programs. In some instances, future teachers were studying school functions or school calculus for the first time never having had a trigonometry, pre-calculus, or calculus course before enrolling at their college/university.

The relatively large number of A+ requirements and electives demonstrated a greater consistent vision for teacher preparation than the few standards identified among the international bottom 25 percent of programs. This prompts the speculation that excellence, at least as its measured by the TEDS-M MCK, may have very few paths leading to it but conversely many ways to arrive at much less impressive performance.

Whether the standard was the A+ benchmarks or those stemming from the U.S. top ten the two bottom program groups demonstrated very few future teachers having had these requirements and very few, if any, programs reaching the 80 percent future teacher criterion. The bottom U.S. programs had remarkably fewer future teachers having had all A+ requirements, two percent, than what was evidenced among the international bottom, about 13 percent. This contrast between the standards, i.e., what's required and the performance of the top-achieving programs and that those programs in the bottom performing groups prompts the hypothesis that the presence of these standards is an important part of what differentiates these two performance groups.

Looking across countries, the percent of future teachers exhibiting the A+ required course taking pattern varied greatly from less than two percent (Chile) to nearly 87 percent (the Russian Federation). Similar variation was evident in the percent of programs in which the 80 percent future teacher criterion was met. In this respect it seems quite unsurprising that two of the countries well-represented among the A+ programs, the Russian Federation and Taiwan, lead the way.

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Does School Experience Matter for Future Teachers' General Pedagogical Knowledge?

Johannes König and S. Blömeke

Abstract Researchers commonly distinguish between three domains of teacher knowledge (Baumert and Kunter in Z. Erzieh.wiss. 9(4):469–520, 2006; Bromme in Enzyklopädie der Psychologie: Psychologie des Unterrichts und der Schule, pp. 177–212, 1997; Grossman and Richert in Teach. Teach. Educ. 4(1):53–62, 1988; Shulman in Educ. Res. 15(2):4–14, 1986; Harv. Educ. Rev. 57:1–22, 1987): content knowledge (CK), pedagogical content knowledge (PCK), and general pedagogical knowledge (GPK). The common international tests of the "Teacher Education and Development Study: Learning to Teach Mathematics (TEDS-M)" covered mathematics content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK). Three participating countries—the USA, Germany, and Taiwan—decided to develop an additional test to cover future teachers' GPK as well (König and Blömeke in Z. Erzieh.wiss. 12(3):499–527, 2009; Blömeke and König in TEDS-M 2008—Professionelle Kompetenz und Lerngelegenheiten angehender Mathematiklehrkräfte im internationalen Vergleich, pp. 239–269, 2010a; pp. 270–283, 2010b; König and Blömeke in TEDS-M 2008—Professionelle Kompetenz und Lerngelegenheiten angehender Primarstufenlehrkräfte im internationalen Vergleich, pp. 253–273, 2010a; pp. 275–296, 2010b; in Pädagogisches Unterrichtswissen (PUW). Dokumentation der Kurzfassung des TEDS-M-Testinstruments zur Kompetenzmessung in der ersten Phase der Lehrerausbildung, 2010c). Chapter "Teacher education effectiveness: quality and equity of future primary and future lower secondary teachers' general pedagogical knowledge" of this book describes the conceptual framework of this GPK and the tests instruments in detail. It reports also about core mean and structural results.

The present chapter examines in addition the relationship of GPK to the opportunities to learn provided during teacher education. In detail, it analyses how practical

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Adjusted version of König and Blömeke (2012).

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S. Blömeke et al. (eds.), *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn*, Advances in Mathematics Education,
DOI 10.1007/978-94-007-6437-8_19,

in-school experience reported by future primary teachers is related to their GPK. The findings enrich the discussion on in-school OTL during teacher education.

Keywords Field experience · Latent-class analysis · General pedagogy · General pedagogical knowledge · GPK · In-school experience · In-school OTL · Practicum · School experience

Researchers commonly distinguish between three domains of teacher knowledge (Baumert and Kunter 2006; Bromme 1997; Grossman and Richert 1988; Shulman 1986, 1987): content knowledge (CK), pedagogical content knowledge (PCK), and general pedagogical knowledge (GPK). The common international tests of the "Teacher Education and Development Study: Learning to Teach Mathematics (TEDS-M)" covered mathematics content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK). Three participating countries—the USA, Germany, and Taiwan—decided to develop an additional test to cover future teachers' GPK as well (König and Blömeke 2009, 2010a, 2010b, 2010c; Blömeke and König 2010a, 2010b). Chapter "Teacher education effectiveness: quality and equity of future primary and future lower secondary teachers' general pedagogical knowledge" of this book describes the conceptual framework of this GPK and the tests instruments in detail. It reports also about core mean and structural results.

The present chapter examines in addition the relationship of GPK to the opportunities to learn provided during teacher education. In detail, it analyses how practical in-school experience reported by future primary teachers is related to their GPK. The findings enrich the discussion on in-school OTL during teacher education.

1 Research Questions

1.1 State of Research in TEDS-M

On the mean level of achievement, future in general pedagogy primary teachers from the USA were significantly outperformed by future teachers in Germany (see for details König and Blömeke 2010b). The difference of nearly 1.5 standard deviations was very large, meaning that there was almost no overlap between US teachers and German teachers—most of the worst-achieving teachers from Germany did better than most of the best-achieving teachers from the USA. Similar results were reported from the TEDS-M survey of future lower secondary teachers (see chapter "Teacher education effectiveness: quality and equity of future primary and future lower secondary teachers' general pedagogical knowledge").

Regarding the relationship between teacher education programs, opportunities to learn (OTL) and the professional knowledge acquired, Blömeke et al. (2010a, 2010b) developed a multilevel conceptual framework distinguishing between various individual and institutional components that could be considered as potential predictors. With regard to the GPK of future primary teachers in Germany, multilevel modeling in fact revealed significant positive effects of the number of topics studied in general pedagogy and mathematics pedagogy on GPK. That is, the more opportunities to learn a future primary teacher had during her training, the better she did on the TEDS-M test.

Continuing such kinds of examination, the question arises whether practical in-school experience is also a relevant OTL with respect to the GPK acquired by future primary teachers.

1.2 School Experience and General Pedagogical Knowledge

School experience during teacher education is regarded a core component providing future teachers valuable experience with instructional approaches through group tutoring, classroom observations or student teaching (Clift and Brady 2005). In the practicum setting of teacher education, learning is situated in authentic contexts (Putnam and Borko 2000). Future teachers get the chance to connect their theoretical knowledge acquired during courses in the academic setting to practical situations in the classroom. Such an approach is assumed to enhance in turn the theoretical knowledge and is therefore discussed as the ideal way towards professionalism of teachers (Dann 2000; Kolbe and Combe 2004).

From that, it can be inferred that future teachers' knowledge should correlate positively with the extent of practical experiences they had had. With regard to GPK as measured in TEDS-M (focus on instruction), such practical experiences whereby future teachers have the chance to teach students in the classroom should turn out to be particularly important. Future teachers should then be forced to reflect on tasks such as structuring lessons, dealing with heterogeneity, or motivating students, and thus to activate their GPK. Presumably, while making use of GPK in such situations, future teachers become also increasingly flexible in how to apply their knowledge (Anderson 1982; Hatano and Inagaki 1986; Berliner 2001, 2004; Gruber and Rehrl 2005; König 2010).

Large-scale studies giving insight into the relationship between OTL in schools through a practicum and the GPK of future teachers are virtually non-existent. Neither do we know precisely how different kinds of OTL are related to GPK. This is mainly due to the lack of studies measuring GPK or performance of pre-service teachers in general (König 2012; König and Seifert 2012). Another open question is, for example, whether the number of lessons pre-service teachers teach during their practicum influences the acquisition of GPK or to what extent support by supervisors or mentors in the field does so. TEDS-M is the first survey allowing international comparisons in this area.

2 Methods

2.1 Test Development

As laid out in chapter "Teacher education effectiveness: quality and equity of future primary and future lower secondary teachers' general pedagogical knowledge", teacher tasks and cognitive demands made up a matrix which served as a heuristic for the development of GPK items. For each cell, a subset of items was developed. The instrument measuring GPK of future primary teachers in the USA and Germany consisted of 85 test items.

2.2 Instruments Measuring Opportunities to Learn

Among various items and scales that allow the examination of future teachers' OTL in TEDS-M, one section of the future teacher survey asks specifically about school experiences as part of the teacher education program (see Tatto et al. 2008). Future teachers were asked how long they had spent on teaching students in relation to the total time spent in school: "For what proportion of this time were you temporarily in charge of teaching the class (as opposed to observation, assistance, individual tutoring, etc.)?" In addition, they were asked to what extent they had been supported by a mentor or supervisor: "For about how much of the time in the field experience/practicum was one of your assigned mentors/supervisors present in the same room as you?" Future teachers had to respond to these two questions by checking one out of four options ("Less than 1/4 of the time", "1/4 or more, but less than 1/2", "1/2 or more, but less than 3/4", "3/4 or more").[1]

Apart from such items asking for time-related aspects of in-school OTL, future teachers were also asked about particular activities in which they had been engaged. To examine the relationship between practicum and GPK, we focused on the questions asking future teachers how often they had the opportunity to reflect on and to improve their teaching practice. To measure these OTL two scales were used in TEDS-M: "Teaching for Reflection on Practice" (4 items, e.g. "develop strategies to reflect upon your professional knowledge") and "Teaching for Improving Practice" (8 items, e.g. "develop and test new teaching practices"). The introductory question was the following: "In your teacher preparation program, how often did you have the opportunity to learn to do the following?" The response formats were four-point Likert scales ("never", "rarely", "occasionally", "often"). IRT scaling was applied to the raw data followed by a transformation to a mean of 10 (Tatto 2009).

2.3 Latent-Class Analysis

Latent-class analysis (LCA) is used to identify groups of persons who share similar characteristics (Rost 2004; Magidson and Vermunt 2004). Compared with traditional types of cluster analysis, LCA includes a probability-based classification (persons are classified into clusters based upon membership probabilities estimated

[1] Although items asking for the absolute time spent on teaching are important as well, they were not included in the TEDS-M survey. The relative items still provide information about the type of practicum which can be seen in the results section.

directly from the Latent-Class Model). Persons grouped according to the results of a LCA are denoted as "latent classes". Educational concepts that cannot be directly measured such as milieu, lifestyle or behavior can thus be modeled and typologies can be generated using manifest indicator variables.

Using TEDS-M data for in-school OTL of primary future teachers, Latent-Class Models for ordinal data were computed for the purpose of this paper. By using LCA, we aim at identifying groups of future primary teachers in Germany and the USA who share similar in-school experience. The analyses were done with the software Mplus (Muthén and Muthén 1998–2006) using the "knownclass" option to specify countries (Germany and the USA), the option "type = complex" to integrate the teacher preparation unit (and thus to take into account the cluster structure of the sample) and the weight option to include the TEDS-M future teacher sample weight.

The decision about the number of classes was made based on the information criteria AIC (*Akaike's Information Criterion*) and the adjusted BIC (*Bayesian Information Criterion*). The smaller the numeric value, the better the model fits the data (see Rost 2004).

3 Results

3.1 Patterns of In-school Experience in Germany and the USA

Our assumption was that in-school experience can be classified into different patterns indicating (1) different proportions of time spent on teaching and (2) different amounts of time mentors/supervisors had been present. Four models were estimated, each specifying another number of latent classes (Fig. 1). Based on the information criteria, especially based on the adjusted BIC which has been proven in simulation studies to be most robust when dealing with large sample sizes (Nylund et al. 2007), the model specifying two latent classes showed the smallest numeric value followed by the model specifying three latent classes. In contrast, the AIC pointed to the opposite result, namely that the solution with three latent classes had the best model fit followed by the model specifying two classes only. In any case, the model to be chosen would be the two- or the three-class solution.

An additional examination of the step parameters showed that the item steps were sorted as expected in both models, in the German as well as in the US sample. Thus, neither of the two models includes a class which deviates from the assumed order. This is an important quality criterion (Rost and Georg 1991). Examining the entropy values and the average latent class probabilities for the most likely latent class pattern, we found that the values for the model specifying three latent classes (entropy = 0.66; average probabilities between 0.71 and 0.87) were still in an acceptable range while the relevant values for the two-class solution were only slightly better (entropy = 0.71; average probabilities between 0.74 and 0.90).

Against this background, where the two models showed little difference in their fit criteria, we decided to choose the more differentiated model specifying three

Model	Log-Likelihood	Number of Parameter	Adjusted BIC	AIC
1 Latent Class	-6409.997	13	12878.193	12845.994
2 Latent Classes	-6354.028	26	12824.454	12760.056
3 Latent Classes	-6338.349	39	12851.296	12754.698
4 Latent Classes	-6338.162	52	12909.121	12780.324

Fig. 1 Results from the latent-class analyses of in-school opportunities to learn

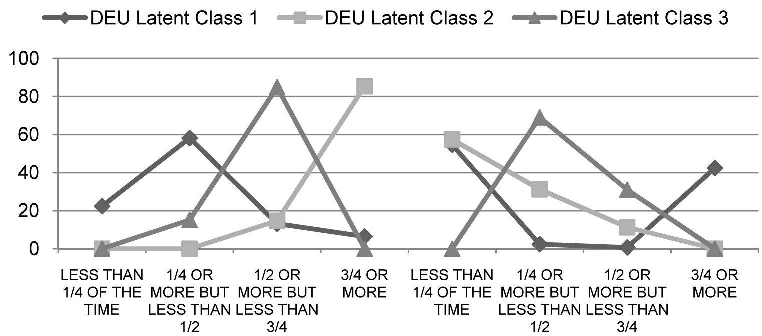

Fig. 2 Results from LCA for Germany (*left*: time spent on teaching; *right*: mentor being present)

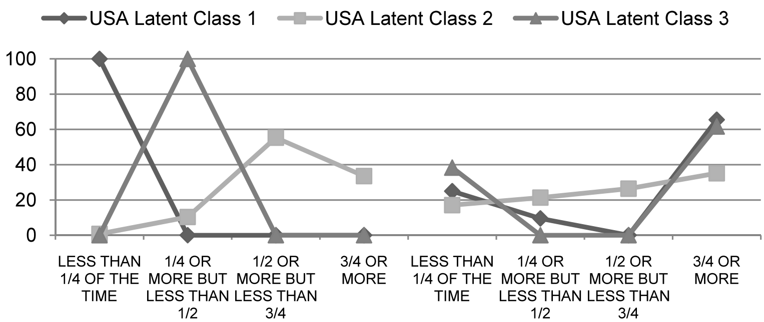

Fig. 3 Results from LCA for the US (*left*: time spent on teaching; *right*: mentor being present)

classes in order to get a more detailed picture of future teachers' in-school experience. To legitimate this decision, we considered it highly relevant to examine external validation criteria, as will be described in Sect. 3.2.

Figures 2 and 3 show the probability distribution for each class for the German and the US sample. In addition to that, Table 1 contains the precise numeric values. On the basis of these results, we can describe the three types of future primary teachers in detail.

Table 1 Results from LCA for Germany and the US

Country	Latent class	% (SE)	Variable	Less than 1/4 of the time	1/4 or more but less than 1/2	1/2 or more but less than 3/4	3/4 or more
Germany	1	10.0 (1.6)	TST	22.3	58.1	13.2	6.4
			MSP	54.6	2.4	0.6	42.4
	2	51.4 (2.3)	TST	0.0	0.0	14.7	85.3
			MSP	57.5	31.2	11.3	0.0
	3	32.4 (1.8)	TST	0.0	15.2	84.8	0.0
			MSP	0.0	69.0	31.0	0.0
	Missing	6.2 (1.5)					
USA	1	7.3 (0.9)	TST	100.0	0.0	0.0	0.0
			MSP	25.0	9.5	0.0	65.5
	2	53.2 (1.8)	TST	0.8	10.4	55.3	33.5
			MSP	17.1	21.3	26.4	35.2
	3	15.2 (1.1)	TST	0.0	100.0	0.0	0.0
			MSP	38.3	0.0	0.0	61.7
	Missing	24.4 (2.5)					

TST—Time spent on teaching, MSP—Mentor/supervisor being present

Type 1 "early beginners": In the USA and in Germany, a group of future primary teachers exists that report a relatively small amount of time on teaching during their in-school OTL. With almost all teachers reporting teaching less than one quarter of the time, the relative teaching time was particularly low in the USA. About two-thirds of the US future teachers and about 40 percent of the German future teachers of this type also report that they were accompanied most of the time by a mentor or supervisor. About 10 percent of the future primary teachers in Germany and about seven percent of the future primary teachers in the USA belong to this type, which we refer to as "early beginners".[2]

Type 2 "autonomous teachers": By contrast to type 1, another group of future primary teachers—that again can be identified both in Germany and the USA—reports they have spent a relatively large amount of time on teaching during their practicum. With almost all teachers in this group reporting teaching during more than three quarters of the time, the relative teaching time was particularly high in Germany. The future teachers' mentors or supervisors have been far less present in class when compared with type 1, especially in Germany. Relatively speaking,

[2]A subgroup of future teachers belonging to this type reported less than 1/4 of the time being accompanied by a mentor/supervisor (Germany: 54.6 percent; USA: 25.0 percent; see Table 1). However, since they also reported that they spent less than 1/4 of the time on teaching and can thus be contrasted to the other two types, we still consider the label "early beginners" to be appropriate.

Table 2 Indicator means and standard error for latent classes

Country	Class	Type description	Item	M	SE
Germany	1	Early beginners	TST	2.04	0.08
	2	Autonomous teachers		3.85	0.02
	3	Balance of autonomy and supervision		2.85	0.03
USA	1	Early beginners		1.00	0.00
	2	Autonomous teachers		3.22	0.04
	3	Balance of autonomy and supervision		2.00	0.00
Germany	1	Early beginners	MSP	2.31	0.23
	2	Autonomous teachers		1.54	0.04
	3	Balance of autonomy and supervision		2.31	0.03
USA	1	Early beginners		3.06	0.15
	2	Autonomous teachers		2.80	0.06
	3	Balance of autonomy and supervision		2.85	0.13

TST—Time spent on teaching, MSP—Mentor/supervisor being present

future teachers of this type have a much higher teaching workload than future teachers of type 1 while have to act more on their own responsibility and be given less support by a mentor or supervisor. In Germany as well as in the US, this group forms about half of each country's sample of future primary teachers.

Type 3 "balance of autonomy and supervision": This type is located between type 1 and type 2. In both countries, it shows a middling amount of time spent on teaching compared with each country's other types, while the presence of a mentor or supervisor seems to be well balanced for the German sample, though a bit heterogeneous for the US sample. In Germany, about one-third of all future primary teachers participating in the survey belong to this type, whereas in the USA it amounts to 15 percent.

Across all classes, German future teachers report that they spent more relative time on teacher than their counterparts from the USA. At the same time, US future teachers report that their supervisors or mentors are more present in the classroom than in Germany; and in contrast to Germany, they do not differ as much in this aspect of teacher education quality (see Table 2 and Fig. 4).

3.2 Validating the Typology

We had decided to choose the model specifying three latent classes although it turned out to be the second best result only on some of the fit indicators because it provided the opportunity to investigate in-school experience in a more differentiated way. Therefore, it is important to examine external validation criteria before using the typology to investigate differences in the future teachers' GPK.

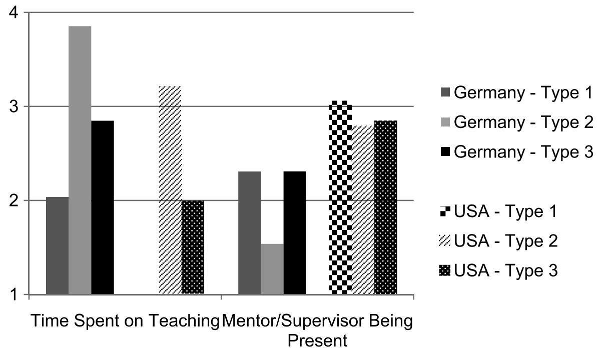

Fig. 4 Indicator means for latent classes

Table 3 Means, standard error, and standard deviation for selected TEDS-M OTL-scales

	Latent class	Teaching for reflection on practice			Teaching for improving practice		
		M	SE	SD	M	SE	SD
Germany	1	9.53	0.36	2.80	9.52	0.25	1.67
	2	10.14	0.22	3.01	10.03	0.08	1.37
	3	10.51	0.24	2.91	10.18	0.09	1.25
USA	1	13.26	0.31	2.80	11.19	0.22	1.77
	2	14.10	0.15	2.54	12.02	0.09	1.75
	3	14.14	0.13	2.45	11.77	0.09	1.57

We assume that future teachers who were both challenged to teach during the practicum and sufficiently mentored (type 3) would agree more strongly on the TEDS-M teacher education quality indices "Teaching for Reflection on Practice" (i.e. that they had more often had the opportunity to e.g. "develop strategies to reflect upon your professional knowledge") and "Teaching for Improving Practice" (i.e. that they had more often had the opportunity to e.g. "develop and test new teaching practices") than future teachers who did not teach much (type 1). As Table 3 shows, this is fairly true. In both Germany and the USA, future teachers belonging to type 3 outperform type 1 future teachers on both scales. Furthermore, type 2 future teachers show higher means than type 1 (due to the large standard error this result is not statistically significant in Germany though).

3.3 Future Teachers' General Pedagogical Knowledge by Type of Practicum

In examining future primary teachers' GPK, similar mean differences between the three types of practical experiences can be observed for the German and the US

Table 4 General pedagogical knowledge by latent class

Latent class		Germany	USA
1	M	571.5	446.2
	SE	11.4	8.1
	SD	102.3	75.2
2	M	602.7	459.3
	SE	5.4	3.9
	SD	93	70.4
3	M	613.3	479.3
	SE	6.9	3.4
	SD	93.4	71.3

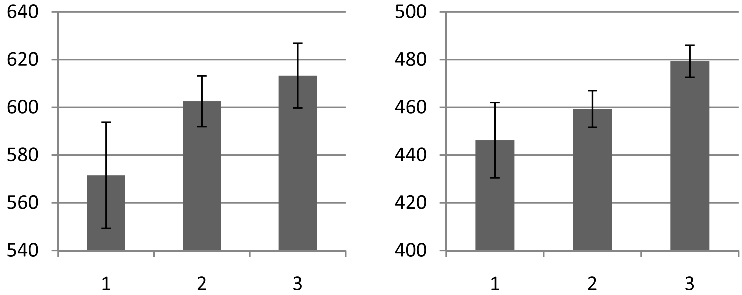

Fig. 5 General pedagogical knowledge for latent classes in Germany (*on the left*) and the US (*on the right*) with 95 %-confidence intervals

samples. In both countries, type 1 is clearly outperformed by type 3 (see Table 4 and Fig. 5; mean differences are statistically significant, $p < 0.05$). Moreover, in Germany, type 1 is also outperformed by type 2, whereas in the US, type 2 is outperformed by type 3. The results reveal that the GPK scores turn out to vary in the expected direction: the higher the quality of in-school OTL, the higher the test score in general pedagogy. It can be interpreted as another external validation criterion of the three latent classes model.

4 Summary and Discussion

General pedagogical knowledge (GPK) is a central component of teacher knowledge. Teacher education programs in many countries provide corresponding opportunities to learn (OTL), and in-school experience is regarded as a core component of OTL fostering knowledge in the area of general pedagogy. First findings from TEDS-M 2008 had revealed large country differences in GPK between future primary teachers in the USA and Germany (see Chap. “In-Depth Analyses of Different Countries’ Responses to MCR Items: A View on the Differences Within and

Between East and West"). Further multilevel modeling had then revealed significant positive effects of OTL programs provided in the field of general pedagogy (Blömeke and König 2011).

In this chapter, we investigated the relationship between practical in-school OTL of German and US future primary teachers and their GPK. On the basis of results from latent-class analysis (LCA) that included two items indicating the proportion of relative time spent on teaching and mentors or supervisors being present in the classroom, we distinguished three types of future teachers in the USA as well as in Germany:

(1) Future teachers with only a small proportion of relative time spent on teaching and a relatively large amount of time during which their mentors or supervisors were present ("early beginners"),
(2) future teachers with the other extreme ("autonomous teachers"), most time spent on teaching while mentors or supervisors were usually not present,
(3) and future teachers who reported a relatively balanced type of in-school OTL with sufficient time for teaching and mentoring ("balance of autonomy and supervision").

In both countries, type 3 future primary teachers reported that they had had more OTL to reflect on and improve their teaching than type 1 teachers. Type 3 teachers also generally achieved better GPK test results than type 1 teachers. Furthermore, there is also a tendency that type 3 future teachers show better results than type 2.

Based on these results, we hypothesize that the quality of future teachers' activities during in-school OTL matters with regard to important outcomes of teacher education, making in-school OTL an effective component of teacher education. However, we did not find evidence that the extent of practical experiences is *linearly* significant for GPK in the sense of "the more, the better". Obviously, future primary teachers grow if they are both challenged to teach to an adequate extent *and* if they are sufficiently supported by a supervisor or mentor.

By contrast, there is only limited evidence that a lot of teaching practice without support leads to the desired outcome in a straightforward way. Possibly, future teachers experiencing such OTL (type 2) are confronted with what Johnson denotes a "lost at sea" or "sink and swim" experience (Johnson 1990; Johnson and Birkeland 2003) to a larger extent than type 3 future teachers. Thus, teacher education programs that require future teachers to teach as much as possible during their practicum (type 2) may have limited value. Least effective seems to be a strategy not to challenge future teachers with teaching experiences at all during their practicum (type 1), leaving them in a stage of early beginners even when they are in their last year of teacher education.

Our research findings enrich the discussion on the relationship between theory and practice during teacher education which is relevant with regard to program requirements set by the governments (Townsend and Bates 2007). Reforms on teacher education need an empirical basis upon which policy makers decide about the structure and the content of teacher education programs. In Germany, for example, new forms of designing the first and second phase of teacher training are being introduced—especially with regard to in-school OTL. As a consequence, there is an increasing demand for comparative information about the effectiveness of teacher education programs in general (König and Blömeke 2013) and particularly when questioning the effectiveness of in-school OTL (König 2012).

From a methodological point of view, some concluding remarks have to be made—not least examining such an ill-defined area like general pedagogy. Cross-country validity is an unquestionable but hard to fulfil request. More evaluation designs and measurement instruments have to be developed, for example in order to examine the outcomes of teacher education on the basis of national educational standards. There is no longer any doubt that "scientific evidence ought to be the grounding for educational practice, policy, and resource allocation" (Cochran-Smith and Fries 2005, p. 102). A central deficit of teacher education research is too narrow a focus on a single institution or even a single college or university class (Risko et al. 2008). Broader perspectives that will provide orientation in a global context are needed. Cross-cultural patterns such as we have identified—describing the relationship between in-school OTL and GPK—are of high relevance and should stimulate further research. The restrictions taken in this paper for such a first approach leave space for future research that follows a broader understanding of GPK and a more detailed view on in-school OTL.

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Part V
Assessment Challenges with Respect to Teacher Knowledge

The Conceptualisation of Mathematics Competencies in the International Teacher Education Study TEDS-M

Martina Döhrmann, Gabriele Kaiser, and Sigrid Blömeke

Abstract The main aim of the international Teacher Education and Development Study in Mathematics (TEDS-M), carried out under the auspices of the International Association for the Evaluation of Educational Achievement (IEA), was to understand how national policies and institutional practices influence the outcomes of mathematics teacher education. This paper reports on the definition of effective mathematics teacher education in TEDS-M, distinguishing between mathematics content knowledge and mathematics pedagogical content knowledge as essential cognitive components of mathematics teachers' professional competencies. These competence facets were implemented as proficiency tests based on extensive coordination and validation processes by experts from all participating countries. International acceptance of the tests was accomplished whereas, by necessity, national specifications had to be left out, as is common in comparative large-scale assessments. In this paper, the nature of the TEDS-M tests for the primary study is analysed and commented on detail. The aims are to increase our understanding of mathematics content knowledge and mathematics pedagogical content knowledge, which are still fuzzy domains, to provide a substantive background for interpretations of the test results and to examine whether some educational traditions may be more accurately reflected in the test items than others. For this purpose, several items that have been released by the IEA are presented and elaborately analysed in order to substantiate the test design of TEDS-M. Our main conclusion is that the overall validity of the TEDS-M tests can be regarded as a given, but that readers have to be aware of limitations, amongst others from a continental European point of view.

Originally published in ZDM—The International Journal of Mathematics Education.

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S. Blömeke et al. (eds.), *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn*, Advances in Mathematics Education,
DOI 10.1007/978-94-007-6437-8_20,

Keywords TEDS-M · Mathematics content knowledge (MCK) · Pedagogical content knowledge (PCK) · Comparative study · Large-scale assessment · Validity · Test item · Fachdidaktik · Didactique

1 Introduction

The aim of the international comparative study Teacher Education and Development Study in Mathematics (TEDSM), carried out from 2006–2009 under the auspices of the International Association for the Evaluation of Educational Achievement (IEA), was to understand how national policies and institutional practices influence the outcomes of mathematics teacher education.[1] The international study was based on national representative samples of primary and lower secondary mathematics teachers from 15 countries. Due to the focus of this issue, this paper is limited to primary teacher education; thus the part of the TEDS-M study referring to secondary mathematics teacher education is not covered here.

The main research questions of TEDS-M were:

> What is the level and depth of the mathematics and related teaching knowledge attained by prospective primary and lower secondary teachers? How does this knowledge vary across countries? (Tatto et al. 2008, p. 13)

In order to measure the effectiveness of mathematics teacher education, TEDS-M developed a conceptual model of mathematics teachers' professional competencies whose promotion is the central goal of mathematics teacher education. Based on the approach by Shulman (1986), TEDS-M describes mathematics teachers' professional competencies consisting of mathematics content knowledge (MCK), mathematics pedagogical content knowledge (MPCK) and general pedagogical knowledge (GPK) as essential cognitive components complemented by personality traits and beliefs. This paper focuses on the knowledge components MCK and MPCK, which were implemented in TEDS-M as proficiency tests. The knowledge in these sub-domains of more than 13,000 future primary school teachers in their last year of teacher education was measured by a paper-and-pencil test. The theoretical background of the test is summarised in the framework of TEDS-M (Tatto et al. 2008), to which the following analyses refer.

The TEDS-M concept of teachers' mathematical competencies was the result of a long and intense discussion among the participating countries, in which international acceptance was eventually accomplished. Our comments from a German point of view do not target this basic validity of the study. We only point to some limitations important to consider when discussing the results.

[1]The international costs of TEDS-M were funded by the IEA, the National Science Foundation (REC 0514431) and the participating countries. In Germany, the German Research Foundation funded TEDS-M (DFG, BL 548/3-1). The instruments are copyrighted by the TEDS-M International Study Center at MSU (ISC). The views expressed in this paper are those of the authors and do not necessarily reflect the views of the IEA, the ISC, the participating countries or the funding agencies.

In order to achieve international acceptance, national specifications of what we understand by "mathematics content knowledge" or by "mathematics pedagogical content knowledge" had—by necessity—to be left out. Further, if one analyses the theoretical framework of TEDS-M, core decisions reveal in addition that the understanding of teaching and learning processes was slightly more connected to approaches predominantly taken in English-speaking countries and less to continental European traditions on subject-related reflections, called *Fachdidaktik* in German or *didactique* in French.

As Pepin (1999) pointed out, the emergence of research on teacher knowledge in a particular subject has in continental Europe led to the development of subject-related didactics which describe the pedagogical transformation of disciplinary content to teaching content taking into account the whole teaching-and-learning process. According to Pepin (1999), these continental traditions are based on educational philosophical, theoretical reflections, and they include normative descriptions of teaching-and-learning processes. It is revealing that the development of subject-related didactics did not start until the end of the nineteenth century, taking place within the transformation of teacher education in many European countries (Schneuwly 2011).

In English-speaking countries, these perspectives of the continental European debate on subject-related didactics can be found partly in the debate either of curriculum theory or of educational psychology (Kansanen 1999). However, reflections on the knowledge transformation, that is, its student-related simplification throughout the process to teaching knowledge, called elementarisation in Germany—central for the tradition of didactics—can hardly be found. The concept of elementarisation described by Ball and Bass (2000) appears similar, but focuses on the unpacking or decompressing of mathematical content as a main task of the teacher.

In contrast, the Anglo-American type of educational research has been from the beginning more outcome-based and thus to a large extent based on empirical studies, in order to identify and determine influential factors (as predictors) of successful teaching and learning in order to understand the relationship. Broader normative, subject-related reflections were of lower importance. As Westbury (2000) pointed out, the dominant features of the US curriculum tradition were of an organisational nature, referring to schools as institutions, where teachers were expected to be agents for an optimal school system.

Kaiser (1999, 2002) described the understanding of mathematics and mathematics teaching in English-speaking countries as more algorithm-oriented. The conceptual understanding of mathematics, an understanding of mathematical structures as well as of argumentation and proof, are of lower relevance than in the above mentioned subject-oriented *Fachdidaktik*. According to Kaiser (1999) and Kaiser et al. (2006), such subject-oriented views on mathematics and its education are indicative for continental European mathematics traditions. (For an overall discussion on European tradition concerning didactics and other central European concepts see Hudson and Meyer 2011.)

These differences in basic orientations of the countries participating in TEDS-M led to decisions about the objectives of the TEDS-M test, the considered knowledge domains and knowledge facets, and the item development, as will become apparent in our later descriptions of the TEDS-M proficiency test. The conceptual understanding of mathematics and an understanding of mathematical structures, as well as argumentation and proof and heuristic problem solving, were of slightly lower relevance in the MCK and MPCK tests than they would be in the tradition of *Fachdidaktik* (for a similar critique of the TEDS-M test from an East Asian perspective see the paper by Hsieh et al. 2012).

The following elaborations on the conceptual framework of TEDS-M and the nature of the tests focus on such differences. In this way, we intend to increase the understanding of mathematics content knowledge and mathematics pedagogical content knowledge, which are still fuzzy domains. The overall reliability, validity and credibility of the tests has already been demonstrated many times (Senk et al. 2012; Blömeke et al. 2011, 2012). Now, we can look beyond what was accomplished in order to examine further research needs.

2 Teachers' Professional Competencies as Theoretical Framework of TEDS-M

Teachers' professional tasks in everyday school life are extensive and manifold. However, teaching is the core task of teachers, and thus the development of teaching abilities internationally constitutes the main function of teacher education. Correspondingly, teaching abilities are described as the main objective by various educational documents all over the world (see the documents by the German Standing Conference of the Ministers of Culture and Education on teacher education, KMK 2004a; or the documents by the US National Council for Accreditation of Teacher Education, NCATE 2008), which are the starting point of the theoretical framework of TEDS-M. The teaching abilities—called 'professional competencies'—include cognitive as well as affective-motivational facets (Richardson 1996; Thompson 1992; Weinert 2001). According to Shulman (1986) and Bromme (1992), three domains of knowledge as main cognitive components of mathematics teachers' professional competencies can be discriminated: MCK, MPCK and GPK.

In addition, beliefs and affective traits such as motivation, and also metacognitive abilities such as self-regulation, are indispensable parts of the professional competencies of teachers, as displayed in Fig. 1.

This framing of teachers' professional competencies is visualised also in the TEDS-M framework, where cognitive and affective-motivational facets of the future teachers' competencies were measured as criteria for effective teacher education. The future teachers' MCK and PCK were assessed in every participating country of TEDS-M, as well as their subject-related beliefs and professional motivations. Germany, Taiwan and the USA also assessed the GPK in a supplementary study. Metacognitive abilities, however, were not part of the TEDS-M surveys.

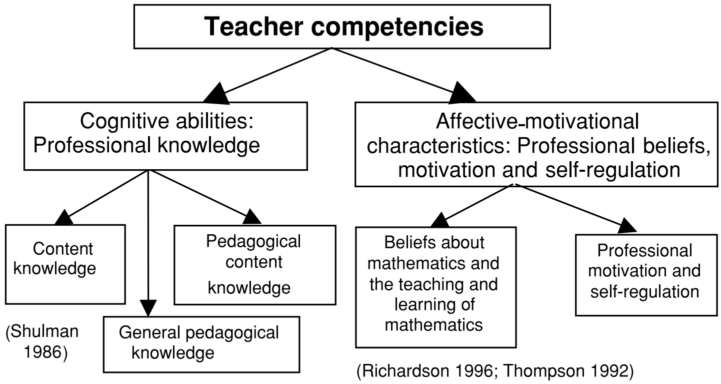


Fig. 1 Conceptual model of teachers' professional competencies

In commenting on the TEDS-M definition of professional competencies, one can point to the following benefits and limitations. Instruction is the core task of teachers all over the world; thus, their main activity is broadly covered. Furthermore, evidence in fact suggests that successful teaching depends on professional knowledge *and* teacher beliefs. Thus, the multidimensional nature of teacher competencies is taken into account.

However, successful accomplishment of all teacher tasks requires skills that go beyond merely teaching. In addition to the organisation and planning of teaching and learning processes, teachers are responsible for the social education of students, cooperation with parents, students' counselling, active participation in school development, and many other activities. Therefore, the standards on teacher education by the German Standing Conference of the Ministers of Culture and Education KMK (2004a) extend the demands on teacher education beyond teacher-related competencies. Successful teacher education should thus advance communicative skills and impart strategies in order to prevent and overcome conflicts. Furthermore, the standards demand knowledge about legal conditions of school and education, qualifications about cooperation with colleagues, and awareness of stress management methods. The limitation of the TEDS-M framework to the teachers' core task of teaching does not reduce the relevance of these responsibilities of teacher education.

2.1 Conceptualisation of MCK

As TEDS-M is the first cross-national large-scale study on teacher education the theoretical conceptualisation of MCK and MPCK as well as developing proficiency tests necessitated extensive work and an enormous amount of time before the realisation of the study. So, in 2002, representatives from the countries participating in TEDS-M met for the first time to discuss their nationally and culturally shaped conceptions on the professional knowledge of mathematics teachers. The aim was

Table 1 Analytical description of the four content domains included in TEDS-M (according to Tatto et al. 2008, p. 36)

Number	Whole numbers	Fractions and decimals
	Number sentences	Patterns and relationships
	Integers	Ratios, proportions and percentages
	Irrational numbers	Number theory
Geometry	Geometric shapes	Geometric measurement
	Location and movement	
Algebra	Patterns	Algebraic expressions
	Equations/formulas and functions	
Data	Data organisation and representation	
	Data reading and interpretation	Chance

to develop a collective cross-national core of MCK and MPCK which was approved by every participating country.

The result emerging from this process was a definition of MCK that predominantly focused on teachers' tasks rather than normative—often implicit—curricular requirements. Thus, a teacher's mathematical knowledge was expected to cover from a higher and reflective level at least the mathematical content of the grades the teacher would teach. In addition, a teacher was considered to need to be able to integrate the educational content into the overall mathematical context as well as to connect the content to higher levels of education. The mathematical proficiency test was oriented towards these aspects. All items were categorised into levels of difficulty arising from the item's curricular level. In detail, the *novice* level of difficulty indicates mathematics content that is typically taught at the grades the future teacher will teach. The *intermediate* level of difficulty indicates content that is typically taught one or two grades beyond the highest grade the future teacher will teach and, finally, the *advanced* level of difficulty indicates content that is typically taught three or more years beyond the highest grade the future teacher will teach (Tatto et al. 2008, p. 37).

The operationalisation of the content domains was guided by the TIMSS framework as described in the TIMSS assessment framework (Mullis et al. 2008). Table 1 shows the four content domains assessed in the primary study of TEDS-M.

In order to accomplish their professional activities, teachers need cognitive skills in addition to content knowledge. These cognitive skills were included in the development of the mathematics test for TEDS-M. Hence, the three cognitive domains *knowing*, *applying* and *reasoning* (according to TIMSS) were specified in addition to the content domains (Tatto et al. 2008, p. 37). Together, the cognitive and the content domains constituted a heuristic tool for the item development.

The sub-domain *knowing* includes various abilities such as recalling definitions and properties, recognising and classifying geometrical objects or sets of numbers, carrying out algorithmic procedures, retrieving information given in graphs and tables, and using measuring instruments. The sub-domain *applying* refers to abilities such as selecting efficient operations, methods or strategies for solving problems, generating and applying appropriate models for routine problems, and representing information and data in diagrams and tables. Finally, the sub-domain *reasoning* includes the abilities to prove and reason mathematically and to analyse and characterise mathematical relations (Tatto et al. 2008, p. 37). Although it is not directly mentioned in the framework, the analysis and characterisation of mathematical relations involves the ability to describe and present them.

The assessment of TEDS-M focused at the primary level on the cognitive domain *applying*, followed by the domains knowing and *reasoning*.

The nature of the content domains covered in the TEDSM test can be commented on from a critical stance as follows. The standard repertoire of mathematics education internationally includes three of these domains, namely number, algebra and geometry (see Schmidt et al. 1997; NCTM 2000; KMK 2003, 2004b). In TEDS-M, MCK is, therefore, mainly measured by items from these domains. In contrast, the topic area data and probability is strongly unequally implemented in the mathematics curricula of schools and teacher education in the participating countries and in many countries does not belong in the core curriculum (see the curricular analyses of Schmidt et al. 1997; for the state of discussion at the end of the 1990s see Li and Wisenbaker 2008). However, a growing interest in this domain has become apparent in many countries due to its relevance for applications in everyday life and sciences. For instance, the NCTM standards for teaching in the USA include "Data Analysis and Probability" throughout from kindergarten to college (NCTM 2000) and thus give this domain a prominent place. The educational standards for mathematics teaching in Germany also specify "data and probability" as a key domain for both primary school and lower secondary school examinations (KMK 2003, 2004b). As a consequence of this inconsistent state of discussion, the mathematical content in this domain was reduced in TEDS-M to basic ideas of the concept of probability and data handling. Thus, it is incorporated to only a minor extent into the definition of MCK.

Another area the test left out was the use of technology that nowadays constitutes an increasingly important part of teacher education and working life in many countries. However, it is difficult to simulate technology use in paper-and-pencil tests and so we would need other test formats. Another aspect that is demanded, for example by the German standards of teacher education but could not be displayed by the test, concerns the development of students' mathematical concepts. Understanding mathematics as a science that contains fundamental structures with regard to content and specific procedural methods constitutes an aspiring learner's concept of mathematics education. Of course, these concepts about the nature of mathematics influence the beliefs on mathematics, which were surveyed in another part of TEDS-M, but the assessment of professional competencies was not influenced by those concepts. Again, complex item formats were needed to capture this, which go beyond the limitations of paper-and-pencil tests in large-scale assessment.

Thus, of the three cognitive domains covered in the MCK part of the TEDS-M test, most emphasis was put on items addressing the domain of application. This not only provides a systematic connection to the general design of IEA studies but also connects TEDS-M to approaches of cognitive psychology. Special reference is made to the taxonomy of Anderson et al. (2001) who advanced the taxonomy of Bloom concerning cognitive processes. To be able to apply mathematical knowledge is highly significant to the future teachers' tasks whereas the mere possession of declarative knowledge may complicate practical implementations (Gruber and Renkl 2000; Anderson et al. 2001).

As part of the domain *reasoning*, argumentation and proof were included in the TEDS-M test. However, their emphasis is narrower than in European traditions on argumentation and proof, and only seldom refer to differentiations developed in educational traditions from continental Europe (see Reid and Knipping 2010).

Modelling competencies are covered by the sub-domains *reasoning* and *applying*. "Generating an appropriate model, such as an equation or diagram, for solving a routine problem" (Tatto et al. 2008, p. 37) comes into the category *applying*, while solving "problems set in mathematical or real-life contexts where future teachers are unlikely to have encountered closely similar items" (Tatto et al. 2008, p. 38) is in the category *reasoning*. However, cognitive skills of solving non-routine problems were not taken into account during test development. In particular, the validation of a result as an essential step of a modelling process is not mentioned in the description of the cognitive domains. Dealing with non-routine problems is generally rather extensive and time-consuming and is thus almost impossible to realise within an accelerated test.

2.2 Conceptualisation of MPCK

Reaching a consensus about the essential knowledge and abilities that mathematics teachers should possess in the area of MPCK proved to be an even greater challenge in TEDS-M than for mathematics content knowledge. In this regard, theories and trends are affected even more strongly by educational traditions and culture. The conceptualisation of MPCK was oriented towards the teacher's core task of teaching. For TEDS-M, two sub-domains of MPCK were differentiated according to Shulman (1986) and Fan and Cheong (2002): *Curricular knowledge and knowledge of planning for mathematics teaching and learning and knowledge of enacting mathematics for teaching and learning* (Tatto et al. 2008, p. 38).

As specified in the framework, the sub-domain *curricular knowledge and knowledge of planning for mathematics teaching and learning* not only contains knowledge about the primary school's mathematics curriculum, but also covers the ability to identify the key ideas in learning programmes, seeing connections within the curriculum, establishing appropriate learning goals and knowing different assessment formats. Furthermore, this sub-domain refers to various abilities and skills that are essential to concrete planning of mathematical lessons in primary school. This applies to the selection of an adequate approach to mathematical ideas, choosing appropriate teaching methods, identifying different approaches for solving mathematical problems and predicting typical students' responses for the purpose of choosing assessment formats. In order to interpret and evaluate students' mathematical solutions and arguments in school as well as providing appropriate feedback, it is necessary to possess the abilities of analysing and diagnosing which are assigned to the sub-domain of *enacting mathematics for teaching and learning*. In addition, this sub-domain contains the abilities to guide the classroom discourse as well as to explain or represent mathematical concepts or procedures (Tatto et al. 2008, p. 39).

This theoretical frame guided the item development in the field of mathematics pedagogy. In the domain of the curricular and planning knowledge, the tasks especially relate to identifying mathematical key ideas and conceptions in mathematical tasks and problems and to analysing a task's mathematical content with respect to the required precognition and level of difficulty. Additionally, consequences for the planning of teaching due to thematic changes of the curriculum's organisation should be identified in the process. Further, abilities are demanded in order to reveal adequate approaches for mathematical ideas and select appropriate methods to represent mathematical situations.

Furthermore, TEDS-M includes items in which tasks or problems given to students have to be analysed in terms of possible understanding difficulties and students' responses. The knowledge of enacting mathematics teaching and learning was measured by tasks predominantly referring to analysing and evaluating students' mathematical solutions or arguments. The focus is on items referring to the already mentioned limited concept of reasoning and argumentation in accordance with the requirements with regard to the content of primary school mathematics.

Analogous to the items concerning MCK, the MPCK items were categorised into the three levels of theoretical difficulty, i.e. novice, intermediate and advanced.

Regarding the nature of MPCK covered in the TEDS-M test, one can state that both domains of MPCK characterise substantial knowledge and skills required to teach mathematics effectively. They can be described as an internationally accepted common core of MPCK that is universally required by future mathematics teachers. However, even this core largely corresponds to elements of mathematics pedagogy that are limited to the conveyance of mathematical ideas. Furthermore, the two foci "curricular knowledge and knowledge of planning" and "knowledge of enacting mathematics" are primarily in accordance with an orientation towards curriculum theory and educational psychology dominating in English-speaking countries, in contrast to continental European countries.

But even in the US, current school standards for mathematics education go beyond conveying content. Thus, pupils are expected to acquire process-related competencies based on mathematical content. The standards developed by NCTM, which have influenced the discussion all over the world (see the standards compulsory in German schools since 2003), include competencies such as problem solving, reasoning and proof and suggest connections that additionally deal with applications to non-mathematical topics or the area of communication (NCTM 2000). Nowadays, in many countries, teachers are requested to teach with regard to the development of student competencies. The corresponding approach of Niss (2003) on competence-oriented mathematics education is widely accepted. However, the extent of the future teachers' capability to support students' acquisition of process-related competencies is only marginally surveyed in TEDS-M and the development of competencies such as modelling skills is completely left out of consideration.

National characteristics of MPCK from individual participating countries had, of course, to be excluded as well, because of cultural boundaries and dependences on educational traditions. For example, scaffolding measures, which support students at different ability levels or from different ethnic or linguistic backgrounds, play a prominent role in German mathematics teacher education, but are not covered by the TEDS-M test. Hsieh et al. (2012) describe several topics, important in Taiwanese mathematics education, which could not be considered in TEDS-M and which may explain the difference between the high achievements of Taiwanese students in TEDS-M in contrast to their unsatisfactory achievements in national Taiwanese tests. Likewise, neither theoretical knowledge about preschool age mathematical knowledge development nor the knowledge about research in mathematics pedagogy was an object of the TEDS-M study.

3 Test Design and Example Analysis of Items

3.1 Test Design

The instrument development in TEDS-M was guided by and based on the preparatory study "Mathematics Teaching in the 21st Century" (MT21), an independent six-country study which aimed at developing central conceptualisations of professional knowledge for prospective mathematics teachers and its measurement. MT21 produced, amongst others, items for the TEDS-M test to measure the knowledge in mathematics, mathematics pedagogy and general knowledge for the teaching of future lower secondary teachers (Schmidt et al. 2011; for a detailed description of the framework and the instruments used see Blömeke et al. 2008).

Because MT21 focused on prospective teachers for secondary level, new items had to be developed concerning the assessment of future primary school teachers with the involvement of national research teams as well as the international project management of TEDS-M. Two American studies in particular provided a basis for the item development, namely the study "Knowing Mathematics for Teaching Algebra" (KAT) of Michigan State University (Ferrini-Mundy et al. 2005) and "Learning Mathematics for Teaching" (LMT) of the University of Michigan (Hill et al. 2008).

The adapted items and the newly developed items were translated and retranslated in all participating countries. Subsequently, a review process was performed by the international project management examining whether international standards were maintained. The translated items were simultaneously tested for adequacy, correctness and clarity of wording by experts in mathematics and mathematics pedagogy in each respective country.

In 2007, the items underwent extensive field testing in eleven countries. Items were only carried into the main study if their psychometric properties proved to be suitable in regard to descriptive statistics. Explorative and confirmative factor analyses were applied as well. The time for the testing of MCK and MPCK was limited to 60 min. To enable the use of item response theory (IRT) methods and in order to report reliable measures for the sub-domains, it was decided to apply a rotated booklet design. For the primary study, five blocks were used in a balanced incomplete block design. The number of items in the booklets for the sub-domains of the MCK and MPCK were nearly uniformly distributed. The items were designed in three question formats: multiple-choice (MC), complex multiple-choice (CMC), and open constructed-response (CR) requesting a short self-dependent verbalised answer. For the scoring of the open constructed-response items, detailed coding manuals were developed to assure uniformity. Further, extensive training courses were arranged internationally in the first instance and were subsequently organised nationally in every participating country to make the TEDS-M staff familiar with the manuals. Double encoding was adopted in order to control uniform coding.

Regarding the nature of the TEDS-M test design it can be stated that MCK was measured by 73 items and MPCK by 33 items. The range of item numbers represents an imbalanced focus of the testing, which led to quite imbalanced information being derived from the test. Although several of the European countries agreed that the measurement of MPCK should be the core of the study, as it had been in MT21 (Schmidt et al. 2011), the International Study Centre decided to focus on MCK. Thus, the objective was rather to report several facets of MCK than of MPCK.

3.2 Example Item Analysis

In order to provide insight into the nature of the TEDS-M test, selected items—featuring special aspects of the items used—and their specific requirements are presented in the following detailed item analyses, which are partially based on ACER documents. The provision of background information of the items, the percentage correct frequencies[2] as indicators of the countries' range of proficiency and our analysis are intended to help the reader to reflect on the commentaries we offer. The complete set of 34 TEDS-M primary items released by the IEA together with coding guides is available on the ACER TEDS-M website (http://www.acer.edu.au/research/projects/iea-teacher-educationdevelopment-study-teds-m/).

3.2.1 MCK Item Examples

The first two tasks are both related to the MCK domain *algebra* and require cognitive skills in the category of *applying* (ACER 2011, p. 34 and p. 20). They demonstrate in particular the differences between the levels of difficulty.

The task 'pattern of matchsticks' (Fig. 2) was classified as being at *novice level of difficulty* since its mathematical content might be implemented in primary school

[2]The calculations of the percentage frequencies are based on the international TEDS-M data set version 3.0, provided by the IEA.

Matchsticks are arranged as shown in the figures.

Figure 1 Figure 2 Figure 3

If the pattern is continued, how many matchsticks would be used to make Figure 10?

Check one box.

A.	30	☐[1]
B.	33	☐[2]
C.	36	☐[3]
D.	39	☐[4]
E.	42	☐[5]

Fig. 2 TEDS-M example measuring MCK in the domain algebra (Source: ACER 2011, p. 34)

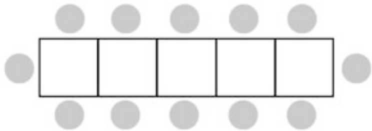
by using a hands-on or iconic approach. Pre-algebraic skills are required in order to solve the given task by identifying the regularity and continuance of the geometrical pattern. The required arithmetic operations are easy and minor calculation errors can be detected and revised owing to the predefined multiple-choice answers. On average, 75 % of the future primary school teachers marked the correct answer '33' (B). Thus, the task was relatively easy from an empirical point of view as well.

The percentage correct range of this item is remarkable, since 94 % of future teachers from Taiwan were capable of solving the task correctly, but this only applied to 12 % (!) of the future teachers from Georgia who predominantly selected option E. We assume that the participants who chose the wrong answer of 42 counted the first figure's number of matchsticks, which is 6. Given the fact that each subsequent figure pictures another additional square, they may have added 4 (instead of 3) matchsticks for each following figure ($6 + 9 \times 4 = 42$).

At first glance, the task 'pattern of seats' (Fig. 3) appears to be similar to the task 'pattern of matchsticks', but it explicitly asks for formulation of an algebraic expression for the figured numerical sequence in addition to recognising the regularity and its continuance. Furthermore, the item was given in an open response format without possible responses given. The term '$2n + 2$' was rated as the correct answer as well as each equivalent term such as '$2(n + 1)$' and '$(n \times 2) + 2$' (Australian Council for Educational Research for the TEDS-M International Study Center 2011, p. 21).

Because of its thematic reference to lower secondary school mathematics, the item was classified as *intermediate* level difficulty. For this item, 50 % of the participants succeeded in formulating an appropriate term. Again, the percentage correct frequencies of the participating countries covered the full range from 4 % in Georgia to 93 % in Singapore.

A square table can seat four people, one on each side. When 5 square tables are placed side by side, as shown below, 12 people can sit around them, 5 on each side and 2 on the ends.

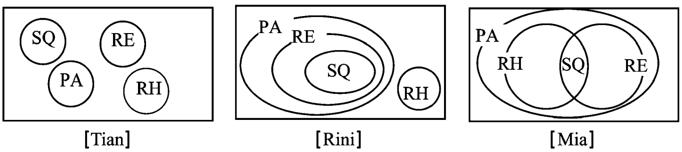


How many people can sit around *n* square tables when they are placed side by side?

Write your answer to the problem in terms of *n*.

Fig. 3 TEDS-M example measuring MCK in the domain algebra (Source: ACER 2011, p. 20)

Three students have drawn the following Venn diagrams showing the relationships between four quadrilaterals:
Rectangles (RE), Parallelograms (PA), Rhombuses (RH), and Squares (SQ).



Which student's diagram is correct?

Check one box.

A. [Tian] ☐1

B. [Rini] ☐2

C. [Mia] ☐3

Fig. 4 TEDS-M example measuring MCK in the domain geometry (Source: ACER 2011, p. 8)

Solving the task 'Venn diagrams on quadrangles' (Fig. 4) from the domain *geometry* requires *knowledge* about subset relations of types of quadrangles. Therefore, it is necessary to be familiar with squares being special rectangles and rhombuses, whereas all of these three quadrangles are parallelograms. In addition, the future primary school teachers are requested to detect the figure of this particular relation within the given Venn diagrams. The item's level of difficulty was classified as *novice* in advance, due to its reference to primary school geometry with regard to content. The correct answer is 'Mia' (C), which was on average given by 60 % of the study participants. The percentage correct frequency was lowest in Germany with 38 % and highest in Taiwan reaching 89 %. Less than half of the future teachers from Germany being able to solve this item correctly might indicate a rather low understanding of geometrical relations or uncertainty dealing with the types of figures pictured. However, another possible explanation for the displayed high dif-

Indicate for each number whether it is rational or irrational.

Check *<u>one</u>* *box in each* *<u>row</u>*.

		Rational	Irrational
A.	π	☐1	☐2
B.	2	☐1	☐2
C.	$\sqrt{49}$	☐1	☐2
D.	$-\frac{3}{2}$	☐1	☐2

Fig. 5 TEDS-M example measuring MCK in the domain number (Source: ACER 2011, p. 31)

ficulty might be the inadequate translation of the term 'rhombuses' to the German word 'Rhomben' since this term is less common than the synonymous German term 'Rauten'.

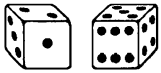
Mathematical *knowledge* in the domain *number* was required in order to solve the task 'irrational numbers' (Fig. 5), which was given in a complex multiple-choice format. A priori, the task was classified as *advanced* level of difficulty, because it requires knowledge about irrational numbers being defined as real numbers that cannot be expressed as fractions consisting of two integers. Considering the given response options, only π is an irrational number (Item A). A correct classification was given by 74 % of the study participants on average, thus indicating a rather low complexity as well as great familiarity with the number π. The percentage correct frequency in Georgia (37 %) was even below the guessing probability while 89 % of the future teachers from Taiwan solved the task correctly.

Item B was solved correctly by almost every future primary teacher (average 89 %; range 53 % in Georgia—99 % in Taiwan). The percentage correct frequency for item C was on average 69 % (range 53 % in Georgia—95 % in Singapore). Item D proved to be of greatest empirical difficulty since more than half of the study participants (59 %) were not capable of answering it correctly. The number of incorrect answers was especially high in Botswana (84 %) while it was only 24 % in Thailand.

The task "game of dice" (Fig. 6) originates from the mathematical domain *data* and the cognitive domain *applying*. The chances of profit of two teenagers who play the game have to be compared. An extremely time-consuming possibility to find the correct answer is counting every favourable and every possible dice score, whereas a faster possibility is given by plausibility considerations that necessitate a deeper understanding. Josie has greater chances of winning (B), because there are more possibilities to compile the differences 0, 1 and 2 than 3, 4 and 5 from the results of the two dice.

This task requires an application of knowledge about computations with probability and was internationally rated as *advanced* level of difficulty, although the subject matter is in some countries such as Germany generally conveyed at the be-

Two fair six-sided number cubes are thrown in a probability game and the two numbers at the top are recorded.



[Josie] wins if the difference between the two numbers is 0, 1 or 2.
[Farid] wins if the difference between the two numbers is 3, 4 or 5.

The students discuss whether the game is fair.

Which of the following statements is correct?

Check one box.

A.	Both have an equal chance of winning.	$\square_1$
B.	[Josie] has the greater chance of winning.	$\square_2$
C.	[Farid] has the greater chance of winning.	$\square_3$
D.	As the game involves number cubes, it's not possible to say who has the greater chance of winning.	$\square_4$

Fig. 6 TEDS-M example measuring MCK in the domain data (Source: ACER 2011, p. 4)

ginning of secondary school. The testing revealed a high degree of empirical difficulty as well. The correct option B was on average chosen by only 29 % of the study participants, ranging from merely 5 % in Georgia to 51 % of the future teachers from Taiwan.

On average, 33 % of the prospective teachers claimed that Josie and Farid have equal chances of winning (option A). Presumably, equal probabilities for the possible differences of two dice scores were erroneously inferred from the assumption that every elementary event is equiprobable according to Laplace or classical probability. The chances of winning are mistakenly assumed to be equal, because Josie and Farid both selected three of the possible differences.

Finally, 69 % of the participants from Georgia as well as 30 to 40 % of the prospective teachers from a number of other countries believed that it is impossible to conclude who is the person with the greater chance of winning (option D). Due to the fact that predicting the outcome of random experiments, such as throwing a die, is impossible, it is concluded that the chances of winning are unpredictable as well. The prospective teachers marking this option either held an insufficient understanding of the function and possibilities of data and probability or they misapprehended the term "chance" and thought they should predict the outcome of the dice-throwing.

Summarising the analysis of the MCK items, we can point out that the items refer to standard topics from the primary and secondary school levels. However, in this context the geometry item (Fig. 4) particularly demonstrates that a novice-level classification does not imply that primary school students are able to answer the item correctly. A comprehensive and profound knowledge is required for the solution of the item that thematically refers to primary school. The TEDS-M test for the primary school level did not include items requiring higher mathematical knowledge that is taught in university courses.

Only 5 of 73 MCK-items are assigned to the sub-domain *data* with the presented item (Fig. 6) being the most challenging. Another item requires calculating the probabilities of events of a Laplace experiment while the others require interpretation of diagrams. The empirical difficulty of all four items was rather low. This low aspiration level together with the relatively small number of items shows also the minor importance of the sub-domain data in the MCK-test. However, a further four items assigned to the sub-domain data were applied when measuring MPCK.

The task 'pattern of seats' (Fig. 3) contains characteristics of modelling competencies such as generating an appropriate model for solving a routine problem (sub-domain *applying*). There are several other items in the test that require the translation of a verbally given context into a mathematical term and accordingly the interpretation of a term according to its real-world context. Apart from these abilities, the test does not measure modelling competencies.

3.2.2 MPCK ITEM Examples

The task 'fuel consumption' (Fig. 7) demonstrates the combination of MCK and MPCK being measured in one task consisting of two items. Both items belong to the content domain of *number* and they were assigned to an *intermediate* level of difficulty previous to the testing. The first mathematical part (a) requires cognitive abilities in the field of *applying*. The correct answer to the question is '8.0', which can be developed by using proportionality arguments or the rule of three. On average, the item was solved correctly by 78 % of the study participants, empirically indicating a rather basic level of difficulty.

The second part of the question concerning MPCK requires knowledge from the field of planning instructions and is related to the sub-domain *curricular knowledge and knowledge of planning for mathematics teaching and learning*. It is expected to identify difficulties of primary school learners calculating with decimal numbers, in this case the division of 2.4 by 30 (or 3). Subsequently, the question is expected to be simplified in a constructed-response format. A possible correct answer from the scoring guide (ACER 2011, p. 11) was: 'A machine uses 3 litres of fuel for every 30 hours of operation. How many litres of fuel will the machine use in 100 hours if its fuel consumption remains constant?'

Simplifying the calculation keeping the decimal number 2.4 was also rated as being correct if the overall calculations became easier, for example: 'A machine uses 2.4 litres of fuel for every 50 h of operation. How many litres of fuel will the machine use in 100 h if its fuel consumption remains constant?' If the new problem varied contextually but still required a simplified calculation, the answer was accepted as well.

The proportion of future teachers within the 15 participating countries who solved the task correctly ranged between 18 % in Georgia and 82 % in Singapore. On average, 55 % of the future teachers' answers were rated as appropriate.

The task 'pattern of teeth' (Fig. 8) requires an interpretation as well as a comparison of the given diagrams. It was assigned to the MPCK domain *enacting math-*

(a) A machine uses 2.4 litres of fuel for every 30 hours of operation.
How many litres of fuel will the machine use in 100 hours if it continues to use fuel at the same rate?

Check one box.

A.	7.2	☐ 1
B.	8.0	☐ 2
C.	8.4	☐ 3
D.	9.6	☐ 4

(b) Create a different problem of the same type as the problem in (a) (same processes/operations) that is **EASIER** for <primary> children to solve.

Fig. 7 TEDS-M example measuring MCK in the domain number (a) and MPCK in the domain curricular knowledge and knowledge of planning for mathematics teaching and learning (Source: ACER 2011, p. 9)

ematics for teaching and learning and was classified as *intermediate* level of difficulty. The future teachers were expected to provide an answer that specified both a similarity and a significant difference of the given diagrams. Describing that both diagrams show equivalent data is an example of an accepted similarity, as is stating that both diagrams are pictograms. Declaring that Mary arranged people according to the amount of their lost teeth as opposed to Sally, who listed each person separately, exemplifies an accepted difference. The item was classified as partly correct if just one common feature or one difference was mentioned. On average, 30 % of the test persons completely solved the task while 37 % reached a partial solution. The range of complete answers was between 4 % in Georgia and 73 % in Taiwan.

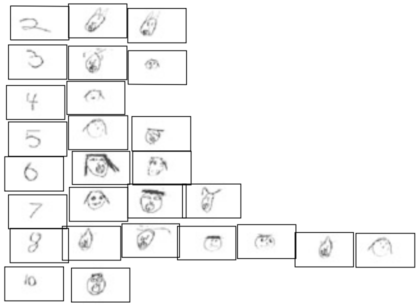
The task ‘introducing length measurement’ (Fig. 9) was used in TEDS-M in order to assess the MPCK in the domain *curricular knowledge and knowledge of planning for mathematics teaching and learning*. In advance, it was classified as *advanced* level of difficulty requiring the prospective teachers to analyse the described teaching method and to specify two reasons to justify it.

Answers were accepted as correct if the reasoning included two of the following three arguments in favour of the chosen type of introduction:

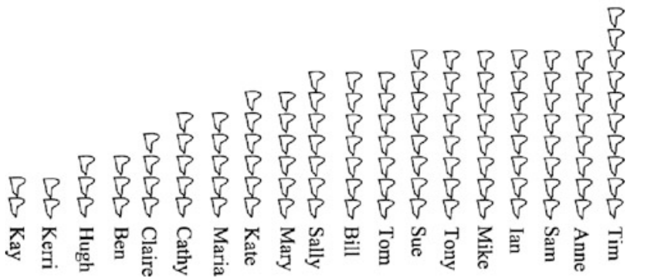
- Supporting an understanding of what ‘measurement’ actually is (using the given objects as a unit enables the understanding of the fundamental idea of measurement being a comparison of an unknown length with a well-established dimension).
- Identifying the need for standard units (i.e. the use of a non-standard unit results in differences of measured values and can show the need for a standard unit).
- Choosing the most appropriate unit (using objects of different lengths fosters an examination on the question which unit/object is the most appropriate to measure a given length).

Imagine that two <primary> students in the same class have created the following representations to show the number of teeth lost by their classmates.

[Mary] drew pictures of her classmates on cards to make this graph.



[Sally] cut out pictures of teeth to make this graph.



From a data presentation point of view, how are the representations alike and how are they different?

Fig. 8 TEDS-M example measuring MPCK in the domain enacting mathematics for teaching and learning (Source: ACER 2011, p. 24)

When teaching children about length measurement for the first time, Mrs. [Ho] prefers to begin by having the children measure the width of their book using paper clips, then again using pencils.

Give **TWO** reasons she could have for preferring to do this rather than simply teaching the children how to use a ruler?

Fig. 9 TEDS-M example measuring MPCK in the domain curricular knowledge and knowledge of planning for mathematics teaching and learning (Source: ACER 2011, p. 41)

Naming one of the given aspects was classified as partly correct while non-cognitive criteria such as motivating the students were not accepted, because of their lack of specificity. Empirically, the task appeared to be extremely challenging since merely 9 % of the future primary teachers succeeded completely in solving the task correctly by specifying two reasons (range from 2 % in Georgia to 19 % in

Singapore). Less than half of the study participants (40 %) named one of the three reasons listed above. The first reason was named most frequently (by 26 % of the future teachers) since the fundamental idea of measurement immediately suggests this reason. Germany was the only participating country where the future teachers most frequently named the second option, which is not unexpected as the need of using standard units is emphasised in the subject-related didactical literature (see Padberg 1997).

Summarising the analysis of the MPCK items, it can be seen that the three described tasks illustrate different facets of the MPCK concept of TEDS-M. Item (b) of the task 'fuel consumption' (Fig. 7) demands an analysis of the mathematical content of the posed problem with respect to the required precognition of primary children and the adaptation of the problem. Teachers' daily routine is composed of such analysing and evaluating the appropriateness of tasks as to their applicability in specific classroom situations. The same applies for analysing and interpreting students' solutions as demanded by the example 'pattern of teeth' (Fig. 8). The task 'introducing length measurement' (Fig. 9) surveys the knowledge about mathematical conceptions concerning measurement which can also be assigned to the substantial knowledge and skills required to effectively teach mathematics. The German didactical teacher education conveys those mathematical conceptions to the future teachers. This is a suitable example to clarify the distinct setting of priorities of German subject-related didactics. The subject-related didactics (*Fachdidaktik*) not only deals with mathematical conceptions but also concerns the students' corresponding learning process and its constructive support. For instance, the 'didactical gradation' which was developed by Radatz and Schipper (1983) should be mentioned in the context of the introduction of the concept of length measurement since it constitutes a relevant part of the didactical training in mathematics teacher education in many German universities. The didactic gradation proposes a model to develop the concept of quantities. In this process, an idea about quantities can be perceived based on nine stages beginning with initial experiences in playful situations. Subsequently, additional stages are passed, such as directly comparing different representations of quantities, indirectly comparing units of measurement at random, and recognising the invariance of quantities.

4 Concluding Remarks

Overall, the MCK and the MPCK of future teachers was successfully conceptualised and efficiently surveyed through the proficiency tests of TEDS-M. The items measured the knowledge in the domains as priori defined and they were suitable for identifying different proficiency levels of future primary school teachers from various countries. Therefore, we can confirm the reliability and validity of the tests from an international point of view.

A great challenge in the development and design of the test was the separation of the two domains mathematics content knowledge and mathematics pedagogical content knowledge. It is impossible to construct disjoint sub-domains, because the solution of an item in the domain MPCK generally requires MCK.

A few items, which are classified by TEDS-M as belonging to the sub-domain MPCK, at first glance rather seem to need MCK. In the following we will show the difficulty concerning the distinction of MPCK and MCK and their inseparable linkage. Due to the difficulty of separating the domains it has to be decided from case to case whether an item set within a teaching context refers to MPCK or MCK only. Two sample items will be discussed, which are both embedded into a teaching context but which measure different knowledge.

The first example item to be discussed is displayed in Fig. 6 and is taken from the mathematical domain data. The item is embedded into a classroom context and requires the evaluation of the correctness of given answers. According to the framework the requirement 'Analysing or evaluating students' mathematical solutions or arguments' (p. 39) belongs to the MPCK sub-dimension 'Enacting Mathematics for Teaching and Learning'. However, the correct solution of the item merely requires mathematical knowledge, because for example neither the pre-knowledge of the students nor their solution approaches need to be analysed or taken into account. Therefore, this item was defined to measure MCK, although it is embedded into a teaching context.

In contrast to the item described, there are items based on teaching situations which are defined to measure MPCK due to the reference of the item to classroom activities. The following example uses a released item taken out of the study for future secondary teachers, which requires the evaluation of students' answers. This item is adapted from a study by Healy and Hoyles (1998) and used with their permission in TEDS-M (Fig. 10).

This item refers to the mathematical domain *number* and the cognitive domain *reasoning* and can obviously not be solved without mathematical knowledge. From a mathematical point of view, an argument that $6 = 2 \times 3$ is a prime factorisation, which cannot be further reduced, would be expected. However, mathematical knowledge is not sufficient in order to assess whether a specific kind of proof is accepted as valid in school. The embedding of the task in a classroom context is not artificial, but has substantive consequences for the correct assessment of the proofs given. From a university mathematical point of view none of the described statements would be accepted as valid proofs—arguments concerning the prime factorisation would be expected. However, acceptable proofs at university mathematics level cannot simply be transferred into mathematics teaching. In order to justify which statement can be accepted in mathematics teaching as a valid argument or proof, mathematics didactical reflections are necessary, for example knowledge about different kinds of arguments or proofs such as formal proofs and pre-formal proofs or generic proofs (for an overview see Reid and Knipping 2010). Based on this kind of knowledge Kate's answer would be evaluated as a valid and coherent proof formulated in a pre-formal language, known as "content-related argumentation" in German didactics (Blum and Kirsch 1991; Wittmann and Müller 1988). Leon's and Maria's answers cannot be accepted as valid, but for different reasons. Leon's argument is only example-based without referring to a generalisable core

Some <lower-secondary school> students were asked to prove the following statement:

When you multiply 3 consecutive natural numbers, the product is a multiple of 6.

Below are three responses.

[Kate's] answer

A multiple of 6 must have factors of 3 and 2.
If you have three consecutive numbers, one will be a multiple of 3.
Also, at least one number will be even and all even numbers are multiples of 2.
If you multiply the three consecutive numbers together the answer must have at least one factor of 3 and one factor of 2.

[Leon's] answer

1 x 2 x 3 = 6

2 x 3 x 4 = 24 = 6 x 4

4 x 5 x 6 = 120 = 6 x 20

6 x 7 x 8 = 336 = 6 x 56

[Maria's] answer

n is any whole number

$n \times (n+1) \times (n+2) = (n^2+n) \times (n+2)$
$= n^3 + n^2 + 2n^2 + 2n$

Cancelling the *n's* gives 1 + 1 + 2 + 2 = 6

Determine whether each proof is valid. *Check one box in each row.*

	Valid	Not valid
A. [Kate's] proof	$\square_1$	$\square_2$
B. [Leon's] proof	$\square_1$	$\square_2$
C. [Maria's] proof	$\square_1$	$\square_2$

Fig. 10 TEDS-M example measuring MPCK in the domain enacting mathematics for teaching and learning (Source: ACER 2011, p. 12)

necessary for a generic proof (Mason and Pimm 1984), and resembles an empirical argumentation common in Anglo-Saxon classrooms (see the empirical study by Kaiser 1999). Maria's argument is correct at the beginning, before the error occurs. Hers is the most formal argument of the three, which might lead many test persons to think that it must be valid. To summarise, these kinds of assessments need a deep understanding in the area of MPCK, based on a sound knowledge of MCK as well.

As described in this paper, the TEDS-M items—validly capturing the international core of teachers' professional competencies—could not cover the entire spectrum of the MCK and MPCK of the future primary teachers, which vary due to cultural and traditional differences among the participating countries. In addition, the orientation of the theoretical framework and the test development towards a pragmatic conception of teaching and learning, predominant in English-speaking countries, means that other theoretical conceptions, for example those common in continental Europe, were taken into account to a slightly lesser extent. Items referring to the continental European tradition of didactics with a focus on subject-related reflections would refer more strongly to the topic area of argumentation and proof, which is not of high importance within the TEDS-M test. Continuing this strand of the debate, these missing items would refer to the different functions of proof within teaching-and-learning processes and would tackle various classification systems of proofs in the European mathematics didactics such as pre-formal and formal proof, taking into account the different level of formality and content-related reflections. From our national view in Germany, the role of 'Anschauung', insufficiently translated as visualisation or imagination, which plays a special role in the German didactics on proof—in contrast, for example, to the French debate (see Knipping 2008)—would allow interesting open items on argumentation and proof, in which the role and function of imagination or visualisation could be reflected by the future teachers. However, such national specifics are difficult to cover in comparative large-scale assessments.

Another important kind of item missing in the TEDS-M test is items on concept development and concept introduction, which are at the heart of the German subject-related didactics (so-called *Stoffdidaktik*), but have links to the other European tradition of didactics. Possible items could deal with different basic ideas of central mathematical concepts such as number, percentages or fractions and different ways to introduce them. It is an interesting side remark that the basic idea of percentage and its different constituents are labelled in German with different notions, not known in English and not translatable. Such limitations are a strong plea for additional national studies on teaching and teacher education—anchored on the international TEDS-M scales in order to keep these as benchmarks but designed to reflect national peculiarities.

Furthermore, ideas for possible items refer to the diagnosis of students' errors based on a detailed subject-related analysis of cognitive barriers, which would overcome the more organisational orientation of the American curriculum debate as described at the beginning of this paper (for possible examples see Schwarz et al. 2008). More open items would allow per se more insight into the professional knowledge of future teachers and would give the chance to display the richness of various cultural traditions from all over the world.

The ambitious work carried out by TEDS-M is to be continued with further studies on the conceptualisation and testing of the professional knowledge of future teachers. A few studies are planned or already under way, with varying aims. For example, a group of US, German and Taiwanese researchers (to which the authors of this paper belong) follow up the tested cohort in their first year of teaching, evaluating the development of their professional knowledge. Based on a theoretical model describing the development of expertise of teachers (Blömeke 2002), the study evaluates their professional competence in activity-oriented settings using short (3–5 min) video-clips, in which classroom situations are shown. After observing the classroom situation, the participants are requested to answer several questions containing different levels of requirement. On the one hand, Likert rating scales are applied to assess the teachers' perceptive and observational qualities. On the other hand, open question formats are used in order to assess the teachers' analytical abilities and their abilities to anticipate possible learning progressions a and suggest teaching alternatives. A special question format focuses on areas such as the teachers' adequate continuance of a given teaching situation, formulating subsequent homework or providing cognitively stimulating exercises.

For example, the following question originates from one of the videos showing a third grade class of a German primary school. The video focuses on a boy called Tim with suspected dyscalculia. During the video sequences, Tim is working on a weekly schedule in mathematics containing several subtraction tasks when the teacher is offering individual help with the aid of Dienes blocks. The scene ends with Tim solving subtraction tasks algorithmically. Thus, the video sequence offers several indicators arguing for and against the diagnosis of dyscalculia. Therefore, the teachers are asked to answer the following question:

> Tim is suspected to have dyscalculia. But this diagnosis is still arguable and has to be based on indicators that are predominantly ambiguous.
>
> Describe three clearly identifiable indicators from the video that support this diagnosis.
>
> However, the situation is often not clear-cut. Describe two situations which support the assumption that Tim does not have dyscalculia.

These questions require the participants to recollect and reflect on Tim's approaches and solutions to the mathematical tasks. Initially, indicators have to be perceived as relevant information about Tim's abilities and misconceptions. Some clearly identifiable indicators that can be observed in the video and might support the diagnosis of dyscalculia include, for example, counting calculation methods or using fingers for calculations, inverting left and right, and the non-usage of material structures. By contrast, there are several incidents where Tim's approaches show age-appropriate mathematical abilities, including unbundling the Dienes rods while subtracting and bundling the remaining blocks in the process of counting (we thank Jessica Benthien for this idea for a video clip).

With this approach we aim to evaluate the professional knowledge of teachers in a realistic setting which allows the participants to show their teaching competence and competence of promoting learning processes based on careful observations of classroom situations and judging their adequacy.

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A Conceptualization of Indicators for Mathematics Teacher Education Quality for International Studies

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Abstract The Teacher Education and Development Study in Mathematics, sponsored by the International Association for the Evaluation of Educational Achievement, is the first data-based study about mathematics teacher education with large-scale samples; this article is based on its data but develops a stand-alone conceptual framework to investigate the quality of teacher education among various countries. The framework includes five indicators: future teacher achievement, instructor effectiveness, coherence between universities and schools, courses/content arrangement, and overall effectiveness of teacher education programs. One of the findings provides indications that it is necessary to combine theoretical knowledge with practical teaching into teacher education; another finding is that for all countries involved, future teachers are less approving of the courses/content arrangement of teacher education programs than are program educators, thus perhaps lowering educators' motivation to improve the arrangement. The data also indicate that there is a high degree of synchronization and organization in teacher education programs in the United States; however, these programs still require further development and promotion of their future teachers' knowledge achievements.

Keywords Teacher education · Teacher development · Mathematics education · International education · International study · School effectiveness · Teacher effectiveness

This paper is an adjusted version of Hsieh et al. (2011).

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S. Blömeke et al. (eds.), *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn*, Advances in Mathematics Education,
DOI 10.1007/978-94-007-6437-8_21, © Springer Science+Business Media Dordrecht 2014

1 Introduction

The world is flat! Countries around the world affect one another, no matter their economic structures, thoughts, beliefs, or values. Therefore, we know that no country or person can be independent. Acknowledging the situations and trends of other countries is thus essential. According to this view, the international Trends in International Mathematics and Science Study (TIMSS) and the Programme for International Student Assessment (PISA) enable different nations to understand one another's educational situations.

Results from the TIMSS and PISA studies reveal that there are significant differences among countries in mathematics competency (National Center for Education Statistics [NCES] 2009; Organisation for Economic Co-operation and Development [OECD] 2007). These include mathematics knowledge and its application, analysis and problem solving, and model utilization and augmentation, among other proficiencies (OECD 2007). In addition, these studies indicate that different countries have different curricula, a fact that might constitute the reason for various students' significant levels of difference. However, these indications are the total of our current interpretations; we still do not know how and in what way this difference of curricula affects our students' vast differences in knowledge.

To understand the factors that can affect students' levels of achievements, the TIMSS 1999 video study (NCES 2003) focused on three aspects of mathematics teaching: the way lessons are organized, the nature of content implemented in lessons, and instructional practices. The study found that there were detectable differences in the relative emphasis or arrangement by mathematics teachers in different countries. It further suggested that teaching methods should align with what teachers want their students to learn and that one cannot say which teaching method may be best to implement in a given country.

In these three aspects, a common latent factor is noticed—the quality of teachers. Many studies have shown that teacher quality is the most important school-related factor influencing student achievement (Goe 2007; Kaplan and Owings 2001; Rice 2003). Some also have found that the methods and content used by teachers have a definite influence on their students' learning (Abell Foundation 2001; Fetler 1999; Goldhaber and Brewer 2000).

Various nations have therefore established teacher certification to control the quality of teachers (e.g., NCES 1999; see also Goldhaber and Anthony 2004). Different certifications thus exist in order to assess candidates' different types of knowledge for mathematics teaching. Among these different types of knowledge are subject matter knowledge, subject specific knowledge for teaching, and pedagogical knowledge, all of which enjoy considerable favor in modern certifications, such as the Praxis Series (Hill et al. 2007; NCES 1999). These types of knowledge have also gained attention in academic circles (Hill et al. 2007). Many researchers further claim that a given teacher's knowledge of mathematics and knowledge of how to translate mathematics into a form that can be understood by students play the most important role in effective teaching (Ferrini-Mundy et al. 2005). These two types of knowledge are indeed consistent with, if not identical to, the two of Shulman's categories for teachers' knowledge that are applicable to mathematics, namely, mathematics content knowledge and mathematics pedagogical content knowledge (Shulman 1987).

How a country can guarantee that its teacher quality is high has been a seriously considered issue. One straightforward inference is to guarantee the quality of the basic learning environment in which we train and equip our future teachers, that is, teacher education programs (TEPs). The different features and practices involved in TEPs are therefore worth investigating. A recent study conducted by the National Research Council (2010) in the United States addressed various issues about teacher preparation, including faculty and staff qualifications, the requirements for subject matter knowledge, general pedagogy and professional knowledge, and field experience. However, this study provided results only from the United States. Therefore, we realized that it is vitally important to globalize the study of the knowledge of future teachers and the features as well as the practices of TEPs and to be able to compare these results among various countries.

To reflect on the demands of globalization, the international Teacher Education and Development Study in Mathematics (TEDS-M) was launched to study and compare the policies, practices, and outcomes of teacher preparation programs among different countries. TEDS-M, sponsored by the International Association for the Evaluation of Educational Achievement (IEA), is the first cross-national study about mathematics teacher education with large-scale samples. The TEDS-M study team developed a thorough analytical framework and completed a process of data collection (Tatto et al. 2008). An important issue for TEDS-M was to describe and compare teacher education quality among diverse countries. However, the TEDS-M data analysis is still in its initial stage; therefore, the available resources are limited in scope. This article consequently is based on a stand-alone study that we conducted and uses the data collected by TEDS-M, while also referring to the earlier results of Taiwan's TEDS-M national report (Hsieh et al. 2010). The main purpose of this study is to depict the phenomena, patterns, and comparisons of the participating countries' TEPs in terms of effectiveness.

2 Framework

Based on the purpose of this study, we face the following problem: What features of TEPs can be treated as indicators of effectiveness?

Darling-Hammond (2000) claims that policies regarding teacher education, licensing, hiring, and professional development might make an important difference in the qualifications and capacities that teachers bring to their work. Wang et al. (2010) agree with this point and propose that teacher education should prepare and retain sufficient numbers of high-quality teachers who can work effectively with students in order to establish a credible public image of what they do. Many researchers have attempted to figure out how best to evaluate teacher effectiveness and which criteria should be included in such an evaluation. Among all types of teacher knowledge, there are essentially two types of mathematics teacher knowledge: mathematics content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK). Some researchers have noted that MCK is necessary for mathematics teachers to be effective (Allen 2003; Goldhaber and Brewer 1997), whereas others have posited that MPCK is an important element to effectiveness (Hill et al. 2007; Ingvarson et al. 2007; Shulman 1987). Accordingly, as both MCK and MPCK are regarded as essential ingredients in *future teacher achievement*, both types of knowledge make up the first indicator of the effectiveness of TEPs in our conceptual framework. This indicator concerns the issue of outcome, an essential part of the effectiveness of TEPs, particularly relating to the quality of people being cultivated.

Many studies have shown that there is a strong correlation between students' achievement and the quality of their instructors' teaching (Ferguson 1998; Goe 2007; Kaplan and Owings 2001; Rice 2003). Others have paid particular attention to levels of quality regarding how well instructors can teach (Clark 1992; Ducharme and Ducharme 1999; Howey 1995). From these points, there is no doubt that the quality of instructors' teaching is an important factor in determining the quality of the TEP. Regarding mathematics TEPs, instructors in mathematics-related courses (MR-instructors), who play a crucial role in helping future teachers learn to teach mathematics (Tatto et al. 2008), and school-based supervising teachers (SB-supervisors), who have the important responsibility of mentoring future teachers' learning during field-based experiences (Putnam and Borko 2000), cannot be ignored when evaluating the effectiveness of TEPs. Therefore, the second indicator of the effectiveness of TEPs in our conceptual framework is the *effectiveness of instructors*, composed of the effectiveness of both MR-instructors and SB-supervisors.

On the one hand, MR-instructors provide future teachers with theoretical concepts about teaching ideas, principles, and standards as well as demonstrating models, evaluations, and reflections in college. On the other hand, SB-supervisors offer practical knowledge in field experiences and teaching methods and an understanding of pupils at school sites. Thus, future teachers should apply these teaching theories in real classroom teaching and advance their field experiences in developing knowledge (Bates et al. 2009; Wang et al. 2010; Zeichner 2010). For a TEP to be effective, it is mandatory to evaluate whether or not those ideas and principles taught in colleges, or the standards provided by them, are coherent with the experiences needed in schools. The *teaching coherence* between teacher education universities and schools is therefore integrated into the framework of this study as the third indicator of the effectiveness of TEPs.

Although the coherence of teaching between a university and a school is important and contributes to successful teaching for future teachers, course arrangement is also an important characteristic of TEPs (Tatto et al. 2008) and one that is usually expected to meet the main needs as an effective teacher (Florida State Department of Education 1983). In light of this, the effectiveness of *courses/content arrangement* in TEPs is considered and incorporated into the framework of this study as the fourth indicator of the effectiveness of TEPs. In this article, future teachers are

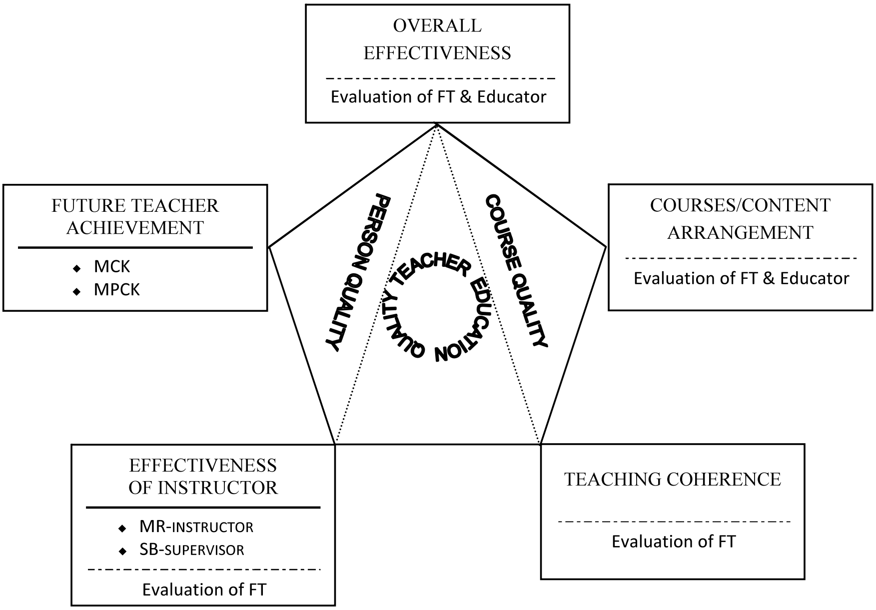

Fig. 1 Conceptual framework utilized in this study. *FT* = future teacher; MCK = mathematics content knowledge; *MPCK* = mathematics pedagogical content knowledge; *MR-instructor* = instructor in mathematics-related courses; *SB-supervisor* = school-based supervising teacher

assumed to provide the pragmatic view, and the educators, as the planners and executors, are assumed to represent the advanced view. Their evaluations together may depict the effectiveness of the courses/content arrangement.

Each of these four indicators, while referring to some part of the quality of teacher education, nevertheless paints an incomplete picture. This study fills this gap with a final indicator—the *overall effectiveness* of TEPs, the inclusive nature of which lends itself to evaluation by both future teachers and educators. Together, these five indicators depict the effectiveness of TEPs from different perspectives: the one being educated, the educator, and the circumstances under which this education takes place. This article is based on the proposed framework shown in Fig. 1 and seeks to investigate teacher education quality among various countries from an effectiveness point of view. This framework consists of five indicators that fall into two major categories: person quality and course quality.

Based on this conceptual framework, we therefore addressed three research questions to guide the analysis and discussion of this study:

1. What are the phenomena or patterns regarding effectiveness for each of the five indicators among the participating countries?
2. What are the levels of effectiveness for each of the five indicators for each country?
3. What are the correlations among these five indicators and the possible concomitant interpretations?

3 Research Method

The target populations in this study included future primary and lower secondary teachers in their last year of training to teach mathematics and teacher educators who instructed these future teachers in the fields of (a) mathematics and mathematics pedagogy and (b) general pedagogy. All persons with regular and repeated responsibilities in teaching future primary and lower secondary mathematics teachers were classified as teacher educators in this study.

The sampling plan contained in this study followed a stratified multistage probability sampling design (Tatto et al. 2009), and samples of teacher preparation institutions were randomly selected with probability proportional to size within explicit strata according to the specific context of each country. For each selected teacher preparation institution, individuals including educators and future teachers were randomly selected or a census was used. The sampling designs and processes for all the countries were developed in consultation with IEA sampling referees and the regulations of the IEA-developed sampling guide.

The future teacher samples of this study either met the IEA's threshold (at least 75 %) or met the criterion to use with an annotation (60 %–75 %; for a more detailed description see Chap. "Framing the Enterprise: Benefits and Challenges of International Studies on Teacher Knowledge and Teacher Beliefs—Modeling Missing Links" of this book). For the educator samples, this study used a threshold rate of 50 %. This rate takes into account previous research that shows that it is difficult to get satisfactory response rates when surveying adults, and it was chosen to ensure the inclusion of more information. Some studies have accepted participation rates much lower than 50 % for adult samples (e.g., Archambault and Crippen 2009; Enochsson 2010). IEA advises that a sample with a participation rate of 30 %–60 % is to be reported separately. However, our choice of a rate above or equal to 50 % is not far from 60 % and also equates to more than half of the sample. All participation rates were calculated and are reported in Table 1.

The international TEDS-M data set did not distinguish teacher educators by levels. The entire group is thus designated as *all educators* in this article. To match the levels with future teachers, these educators were further recategorized by this study into primary level or lower secondary level based on the levels of the teacher preparation units in which these educators served. If an educator served at both levels, he or she would be counted in each level.[1] For the purpose of this article, a distinction is made between *teacher* and *educator*, where *educator* refers to an educator of future teachers. The groups of educators that serve in preparing lower secondary and primary level future teachers are named *lower secondary educators* and *primary educators*, respectively, in this article.[2]

[1]Levels are indistinguishable from Germany's data. Thus, when comparing data of educators with the levels distinguished, the data of all educators are used for Germany.

[2]This study uses the newest release of data sets TEDS_MS_NRC-USE_IDB_20091209_v30 for national research coordinators from the international Teacher Education and Development Study in Mathematics.

Table 1 Participation numbers and participation rates of each level

Country	Educator		Lower secondary future teacher		Primary future teacher	
	n	PR	*n*	PR	*n*	PR
Botswana	43	x	53	x	86	x
Chile	392	50–60	746	60–75	657	60–75
Georgia	62	x	78	60–75	506	x
Germany	482	50–60	771	x	1032	x
Malaysia	255	50–60	389	60–75	576	x
Norway[a]			550		551	
Oman	84	x	268	x	–	–
Philippines	589	x	733	x	592	x
Poland	734	60–75	298	60–75	2112	60–75
Russia	1212	x	2141	x	2266	x
Singapore	77	x	393	x	380	x
Spain	533	x	–	–	1093	x
Switzerland	220	50–60	141	x	936	x
Taiwan	195	x	365	x	923	x
Thailand	312	x	652	x	660	x
US-Public[b]			607	60–75	1501	x
Total	5190		8185		13871	

Note. n = number of unweighted participants; PR = range of the participation rate. An "x" indicates the participation rates that meet International Association for the Evaluation of Educational Achievement's threshold. Blanks indicate the data were collected but not yet processed. Dashes indicate the country did not participate in the denoted level

[a]PR cannot be confirmed yet for Norway

[b]The data from the United States include only public institutions

For future teachers and educators, TEDS-M developed three instruments: a future primary teacher questionnaire, a future secondary teacher questionnaire, and an educator questionnaire. The future teacher questionnaires included both tests and Likert-type scale items, whereas the educator questionnaire included only the latter. Using a self-report method to study and measure the effectiveness of TEPs has its limitations, as respondents' self-impressions may be different from reality. Although other methods may overcome some of these limitations, self-report questionnaires are economical and simple to administer to large numbers of respondents, especially for a cross-national study involving different cultures and languages. Moreover, direct evaluation of effectiveness by future teachers constitutes a pragmatic benefit that is similar to customer evaluation. Therefore, we adopt the data obtained by TEDS-M using both testing and a self-report method of data collection in this study.

According to our proposed conceptual framework and research questions, several variables from those instruments were adopted in this study. The variables MCK and MPCK were used as the indicator of future teacher achievement. The rest of the variables all came from Likert-type scale items: Effectiveness of instructors consisted of MR-instructors' and SB-supervisors' effectiveness in teaching, teaching coherence was concerned with the connection between the teaching of universities and the teaching of schools, courses/content arrangement dealt with the consistency of courses and/or content within a university itself, and overall effectiveness treated the TEP as a whole. Both future teachers and educators were involved in the last two indicators. For the indicators measured based on Likert-type scale items, factor analyses were done to put the items together. Except for overall effectiveness, all variables for the indicators in this study were estimated by using the partial-credit Rasch model with a center at the value 10 as an essentially neutral position. In other words, a logit score of 10 represents a neutral rating toward the rated index. According to the attributes of logit scores, a higher score therefore means a higher index. The data collection period for participating countries varied from late 2007 to early 2009.

In addition to the statistics used by TEDS-M, this study further utilized a variety of statistical analyses and statistical procedures, which will be delineated as they become applicable.

4 Research Findings

International comparisons are widely used to indicate the degree of success of a nation's education system and also the levels of performance to which a given country should aspire. To some degree, cross-national comparisons of education can serve as indicators of a country's educational qualities and have thus constituted a powerful impetus for educational reforms. Thus, from an international perspective, we propose to focus our concentration and begin our discussions on the phenomena, patterns, and comparisons of those indicators within the conceptual framework that are relevant to the effectiveness of TEPs.

4.1 Future Teacher Knowledge Achievements

The results of international analyses show that the mean differences between the highest and lowest rated countries were strikingly large in terms of the standard deviation of 100. The least divergence ($SD = 2.48$) appears in MPCK at the primary level, whereas the most ($SD = 3.13$) appears in MCK at the lower secondary level. The dispersions of the means reported here were in comparison with those of the fourth and eighth graders' achievements in TIMSS, and it appeared that the variability of future teachers' knowledge among these countries was bigger than that of school-level students. This may mean that in these countries the differences in future teachers' MCK scores have a more serious impact than the achievement scores of school students. One point worth mentioning is that the primary-level MCK items included only those at the school level. In this case, the significant differences between countries should be a cause of concern to the teacher education field, as it seems that in some countries primary-level teachers lack some of the basic mathematics knowledge that is commonplace among future primary teachers in other countries. In contrast, the lower-secondary-level mathematics tests included items from the primary level to the college level. Though this may cause a greater difference in achievement among countries, it also demonstrate that some countries emphasize mathematics up to the college level, whereas others do not.

Two more interesting phenomena emerge if we investigate the data further by school levels and knowledge types. The range for MCK was larger than that for MPCK at both school levels, and the lower-secondary-level MCK means were spread out much more widely than those of the primary. It is difficult to reach a sure interpretation concerning these phenomena because each country presents a separate contextual element. However, one possible conclusion is that between the countries there is a greater difference in the emphasis on MCK than there is in the emphasis on MPCK. The wider spread of lower-secondary-level MCK scores may further confirm that the inclusion of tests from the primary level to the college level may yield a big achievement difference between countries with a narrow range in mathematics and those with a wide range including college mathematics.

In terms of all countries' means, the ranks varied case by case, with some relatively more stable than others. By taking a look at only six countries that have achieved levels beyond the international mean of 500 on all four measures, primary and lower secondary MCK and primary and lower secondary MPCK, we found that Singapore and Taiwan ranked consistently within the top-three highest achieving countries, and Germany and the United States almost always remained in the middle, with means a little higher than the international mean (see Fig. 2).[3] Figure 2 also shows that the Russian Federation exhibited a trend of means similar to but lower than that of Taiwan, whereas Singapore exhibited a trend of means similar to but higher than that of the United States, and Switzerland had a trend of means similar to but higher than that of Germany.

Because the future teachers of Singapore, Taiwan, Germany, and the United States scored higher than the international mean in the primary vital indicator of knowledge achievements, they demonstrate an evenly kept balance among different school levels as well as in the types of knowledge in their teacher education policies. They also exemplify both the high-achieving countries, Singapore and Taiwan, and the mid-achieving countries, Germany and the United States, in terms of knowledge. For this reason, we anticipated that it would be informative to explore their strengths and limitations; therefore, these four countries are used as examples,

[3]None of the countries persistently stayed in the middle range, but Germany and the United States stayed frequently in the middle range, each having three means in between ± 0.25 standard deviations from the international means.

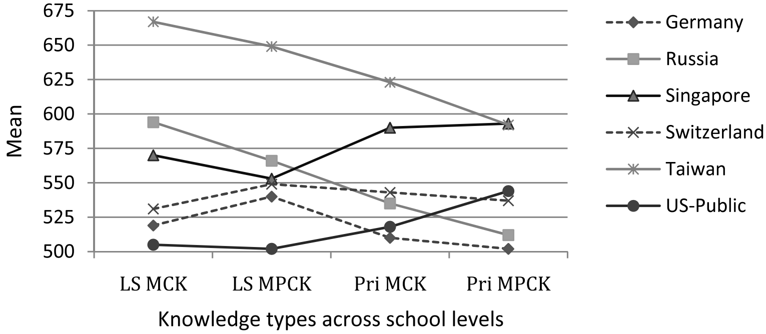

Fig. 2 The within-country trends of knowledge types across school levels for the six countries having all means above the international mean of 500. *LS* = lower secondary; *Pri* = primary; *MCK* = mathematics content knowledge; *MPCK* = mathematics pedagogical content knowledge

and the concomitant analyses use their performances in other indicators whenever possible. In some cases, when the Russian Federation and Switzerland demonstrate unique features, they are also included in discussion.

4.2 Effectiveness of Instructor

As discussed earlier in the conceptual framework, two kinds of instructors were evaluated in this study with respect to their effectiveness: MR-instructors (mathematics-related instructors) and SB-supervisors (school-based supervisors). Based on TEDS-M future teacher questionnaires, the effectiveness of MR-instructors was determined by demonstrating good models in their teaching, evaluations, and reflections; drawing on and using research that is relevant to the content of their courses; and valuing future teachers' learning and experience. The effectiveness of SB-supervisors was measured by whether their feedback could help future teachers improve their understanding of pupils, curricula, teaching methods, and knowledge of mathematics content. The ratings for both types of these instructors were obtained through a set of Likert-type scale items.

In Table 2, we present the means of MR-instructors' and SB-supervisors' scores for each of the participating countries at both the lower secondary and primary levels.

From Table 2, one can notice that all means go beyond the neutral rating of 10, which means that every participating country had positive ratings regarding the effectiveness of their MR-instructors and SB-supervisors. This indicates that future teachers can benefit from both academic and school-based instructors, and it also shows the necessity and appropriateness of integrating theoretical knowledge and practical teaching in teacher education. By going into more detail concerning the ranks of countries across all categories, we see that all Eastern and Southeastern Asian participating countries, other than Malaysia, ranked in the upper half, whereas

Table 2 Country means of future teachers' logit scores of MR-instructor and SB-supervisor

Lower secondary FT				Primary FT			
MR-instructor		SB-supervisor		MR-instructor		SB-supervisor	
Country	M	Country	M	Country	M	Country	M
Philippines	14.00	Philippines	14.49	Philippines	14.17	Philippines	14.71
Thailand	13.81	Taiwan	14.12	Thailand	13.64	Thailand	14.26
Singapore	13.04	Thailand	14.02	US-Public	13.35	Taiwan	13.95
Malaysia	12.82	Singapore	13.77	Russia	13.28	Singapore	13.89
US-Public	12.81	Botswana	13.55	Singapore	13.25	Russia	13.68
Taiwan	12.62	US-Public	13.52	Malaysia	13.21	US-Public	13.46
Russia	12.48	Oman	13.39	Taiwan	12.15	Botswana	13.33
Chile	11.86	Russia	13.05	Botswana	12.10	Malaysia	12.84
Oman	11.85	Malaysia	12.87	Chile	11.99	Poland	12.02
Georgia	11.81	Chile	12.19	Georgia	11.78	Norway	11.99
Botswana	11.79	Norway	12.09	Poland	11.62	Chile	11.88
Poland	11.33	Poland	12.08	Switzerland	11.25	Spain	11.86
Germany	11.19	Switzerland	10.97	Spain	10.83	Georgia	11.42
Switzerland	10.73	Germany	10.68	Norway	10.68	Switzerland	10.80
Norway	10.47	Georgia	10.56	Germany	10.57	Germany	10.54

Note. MR-instructor = instructor in mathematics-related courses; SB-supervisor = school-based supervising teacher

the United States and the Russian Federation were the only other two countries that also ranked in the upper half. Although one cannot say that the MR-instructors and SB-supervisors of those countries ranked in the upper half provided more professional help to facilitate their students in becoming well-trained teachers, we can say that they earned a stronger endorsement from their students, namely, the future teachers.

Based on the homogeneity and heterogeneity of the duties of MR-instructors and SB-supervisors, it would be fascinating to know what these future teachers regard as important in order to be more functionally effective. Thus, we started our inspection by focusing on the mean differences within countries, and an interesting pattern emerged. More than two thirds of the countries gave evidence that the SB-supervisors are more effective than MR-instructors in helping future teachers become well trained.

This study further examined whether the effectiveness of these two groups correlated with each other, and the Spearman's correlation coefficients at the lower secondary and primary levels were $r_s = 0.76$, $p < 0.01$, and $r_s = 0.88$, $p < 0.01$, respectively.[4] This means that the rankings of the effectiveness of MR-instructors

[4] Spearman's rank correlation analyses were chosen because of the small number of countries sampled in this study.

were highly associated with those of the SB-supervisors. This result indicates why the placements of countries are so consistent in these rankings.

With these strong correlations, we sought to investigate whether the effectiveness of these two groups of instructors influenced future teachers' MCK and MPCK achievements. We employed Pearson's correlation analyses to examine each country's situation and showed that of these countries the results were either without significance or with small significant coefficients ($-0.2 < r < 0.2$), with the exception of Georgia.

These findings revealed that the effectiveness of MR-instructors and SB-supervisors did not have a noticeable influence on future teachers' knowledge achievement in any country except Georgia. The future teachers' high ratings for effectiveness of instructors thus did not guarantee high knowledge achievement and vice versa. For example, Germany generally placed midrank in achievement scores, but the effectiveness of Germany's instructors is ranked near the bottom. Operating on a premise that students learn from their instructors, why are there no significant correlations between the future teachers' knowledge and the effectiveness of their instructors? By reviewing the items examining the effectiveness of instructors in the future teacher questionnaires, we noticed that the content of these items is highly related to real teaching instead of knowledge accomplishment. This probably explains why the correlations are not significant.

4.3 Teaching Coherence Between Universities and Schools

As an indicator, teaching coherence reveals the effectiveness of the education future teachers receive at their universities in relation to their future needs as teachers. Being experienced teachers, the SB-supervisors not only play the role of mentors in TEPs but are also in the position of inspecting whether the content or approaches of courses taken by future teachers in their universities are consistent with the needs of teaching in schools. Because both the SB-supervisors and the future teachers possess firsthand observations, experiences, and a sense of the learning consistency, summaries that include a probe of SB-supervisors' views will be more informative. In TEDS-M, the evaluation of teaching coherence was obtained by the future teachers' ratings on five Likert-type four-point scale items. These items took into account the extent to which SB-supervisors appreciated the teaching ideas, approaches, and standards employed in their teacher education universities in terms of applicability to the real classroom settings.

Table 3 shows the means of teaching coherence for each of the participating countries at both the lower secondary and primary levels.

Similar to the effectiveness of instructors, all means go beyond the neutral rating of 10, which indicates that every participating country had positive ratings regarding the coherence of the content taught in their teacher education universities compared to what future teachers should know in real classroom teaching. A noticeable phenomenon emerged: The United States, a mid-achieving country, compiled the

Table 3 Country means of future teachers' logit scores of teaching coherence

Lower secondary FT		Primary FT	
Country	M	Country	M
US-Public	13.00	US-Public	13.13
Philippines	12.91	Russia	13.04
Russia	12.61	Philippines	13.02
Botswana	12.56	Thailand	12.52
Thailand	12.52	Singapore	12.46
Singapore	12.45	Chile	12.18
Chile	12.29	Poland	11.77
Oman	12.09	Switzerland	11.63
Poland	11.81	Botswana	11.51
Malaysia	11.79	Norway	11.41
Switzerland	11.59	Spain	11.36
Norway	11.39	Malaysia	11.35
Taiwan	10.98	Georgia	11.27
Georgia	10.95	Taiwan	11.06
Germany	10.60	Germany	10.80

Note. FT = future teacher

highest average scores in teaching coherence at both school levels, but Taiwan, a high-achieving country, remarkably descended toward a bottom position.

This phenomenon told us that there was most likely no correlation between the indicators of teaching coherence and future teachers' knowledge achievement. Therefore, Pearson's correlation analyses were employed to determine whether these correlations existed. Although some countries' correlation coefficients reached the 0.05 level of statistical significance, all of these coefficients were small ($-0.2 < r < 0.2$). On the other hand, Spearman's rank correlation analyses showed no significant correlations between the countries' means of teaching coherence and MCK or MPCK achievements. These results revealed that the degree to which teaching coherence between universities and schools related to the teaching ideas, principles, and standards was not statistically associated with future teachers' performance on MCK or MPCK.

This result does not seem to be predictable. A common concept is that learning will be motivated and promoted if we can reinforce it. A teacher education system with a high rating of teaching coherence seems to be reinforced at the second learning location: the school. Why is there no statistical relationship between teaching coherence and knowledge achievement? One possible explanation for this is that teaching coherence as employed in this study evaluates the degree of the coherence between universities and schools in the dimension of real teaching, not the dimension of knowledge achievement; therefore, their correlation is not significant.

At the country level, Spearman's rank correlation analyses showed that the ranking of teaching coherence was significantly correlated with the ranking of

MR-instructors' ratings ($r_s = 0.58$ and $r_s = 0.69$, respectively, for the lower secondary and primary level) and the ranking of SB-supervisors' ratings ($r_s = 0.63$ and $r_s = 0.55$, respectively, for the lower secondary and primary level). These analyses further indicate that, for both kinds of instructors, the more effectively the instructors were ranked, the higher the TEPs' coherence was also rated. Although these indicators have significant correlations, country evaluation differences still exist, as exemplified by Singapore, whose future teachers rated teaching coherence around 12.5, about 0.5 to 1.5 logits less than their ratings for effectiveness of instructors. Nevertheless, we still found that some countries' means of teaching coherence were closer to those of effectiveness of instructors, such as the United States.

It is not easy to change a person's attitude toward something in a short period of time, for example, in a few classes. Therefore, if the ratings of teaching coherence and effectiveness of instructors are nearly the same and at a high level, it should be perceived that the teaching ideas, principles, and standards taught by university instructors; the teaching models, evaluations, and reflections demonstrated by MR-instructors; and the field experiences, teaching methods, and understanding of pupils induced by SB-supervisors are tightly integrated. We perceive the teaching of these types of programs as being synchronized. By taking the sum of the rating scores for teaching coherence and effectiveness of instructors, with each of the MR-instructors and SB-supervisors weighted a half, as a score for the degree of synchronization, we found that the United States and Singapore presented the most synchronized programs at the lower secondary level and that the United States and the Russian Federation shared first place at the primary level. For the six countries included here, the United States demonstrated the most synchronized teaching in TEPs at both the lower secondary and the primary levels.[5]

4.4 Courses/Content Arrangement

Not only is the connection of teachings between a university and a school important but also the connection of teachings within a university itself. The evaluation of the courses/content arrangement can be an indicator of the effectiveness of courses and the practicality of the materials being taught. Both future teachers and educators are involved in this indicator. On the one hand, the future teachers, who are those persons being directly exposed to the courses/content arranged by their TEP, can evaluate from a practical standpoint. On the other hand, with higher academic backgrounds and richer experiences involving researching or teaching, educators can represent a more advanced standpoint. In fact, educators usually play the most important role in developing and executing the content or even planning courses for their

[5]For the six countries in concern, at the primary level, the top-three countries, in order, were the United States, Singapore, and the Russian Federation. For the lower secondary level, the United States remained the first, but the second and third countries exchanged positions. For both levels, Taiwan, Switzerland, and Germany were sequentially ranked. The rating scores of the United States and those of the countries ranked third or lower at both levels were significantly different.

Table 4 Country means of future teachers' and educators' logit scores of courses/content arrangement and the differences in the means

Lower secondary level				Primary level			
Country	FT	Edor	DM	Country	FT	Edor	DM
	M	M			M	M	
Philippines	13.76	14.88	−1.11**	Philippines	13.98	15.23	−1.24**
US-Public	13.50			Russia	13.50	13.61	−0.12
Russia	12.96	13.13	−0.16	US-Public	13.35		
Thailand	12.94	14.17	−1.23**	Malaysia	13.10	14.03	−0.93**
Botswana	12.70	14.06	−1.35	Thailand	13.06	14.17	−1.11**
Malaysia	12.70	14.24	−1.54**	Singapore	12.68	13.43	−0.75**
Oman	12.28	12.92	−0.64*	Botswana	12.57	13.58	−1.01
Singapore	12.01	13.53	−1.52**	Chile	11.88	13.63	−1.75**
Taiwan	11.96	13.29	−1.34**	Georgia	11.62	14.07	−2.45**
Chile	11.79	13.50	−1.71**	Taiwan	11.47	12.24	−0.77**
Poland	11.43	12.20	−0.77**	Poland	11.26	12.43	−1.18**
Georgia	10.83	14.31	−3.49**	Spain	10.30	11.15	−0.86**
Switzerland	10.42	11.70	−1.28**	Norway	10.22		
Norway	9.96			Switzerland	10.20	11.79	−1.59**
Germany	9.22			Germany	9.07		

Note. FT = future teacher; Edor = educator; DM = difference of means, which were obtained by subtracting future teachers' mean of logit scores by educators' for each country. The German mean for all educators, which is not distinguishable by levels, is 11.17. It is much higher than the average of their future teachers' means, 9.15, of both levels. $^{*}p < 0.05$, $^{**}p < 0.01$

students. To determine the effectiveness of courses/content arrangement, TEDS-M focused on the organization of the sequences, the links of the courses/content, and whether the courses/content met the needs of future teachers. Sets of six Likert-type scale items were included in both the future teachers' and educators' questionnaires in order to obtain their ratings.

Table 4 presents the means of courses/content arrangement of both the future teachers and educators for each of the participating countries at both the lower secondary and primary levels.

Although almost all countries' future teachers, regardless of their levels, approved of the courses/content arrangement, some were below 10, as shown in Table 4. Because teaching involves the use of the different kinds of knowledge taught in universities and any effective teaching method is subject to different kinds of learners or situations, it therefore requires teachers to incorporate a large block of ideas and skills simultaneously. However, in most TEPs, subject matter knowledge and didactical methods often are separated, letting future teachers integrate related concepts and skills by themselves. This situation made the sequences and links of the courses/content, a part of courses/content arrangement, vitally important. Consequently, if a TEP has a high rating of effectiveness from future teachers for its courses/content arrangement, we call this program's curriculum well organized.

At both of these levels, the Russian Federation and the United States ranked in the top three, meaning that in comparison to other countries, their courses/content arrangements were appropriate and, from the view of future teachers, met their needs. These two countries demonstrate good examples of programs with well-organized curricula. In contrast to the United States, Germany showed a lack of organization in its curriculum. Because the arrangement of courses and teaching content for future teachers should always consider the targeted levels of instruction, we conducted a comparison between the teaching grade spans and specializations among these three countries.

The Russian Federation prepared generalists at the primary level (up to the fourth grade) and specialists in mathematics at the upper primary and lower secondary levels. The United States was similar to the Russian Federation, the only exception being that there was a mix of generalists and specialists at the Grade 4–5 levels. These models of program organization are probably better in terms of the courses and teaching content arrangement. Germany, on the other hand, had complicated program types. Not only are there some mixes of the four types of future teachers—generalists with mathematics, generalists without mathematics, specialists in two subjects with mathematics as one of these two, and specialists in two subjects without mathematics—but also there are programs that prepare future teachers to teach grades with wide spans, such as 1–10 and 5–13. What kinds of courses or content can a program offer for a future teacher to be eligible to teach from Grade 1 to Grades 5–8 and further into Grades 9 and 10? It seems reasonable to conclude that the German TEP's model of specialization and teaching grade spans does not produce positive results with respect to the courses/content arrangement. Another possible reason for Germany's situation may be the fact that its TEP has struggled with different forms of revisions and reforms since the 1970s and that its state (*Länder*) ministries are formally in charge of the structure, course content, and methods of teacher education, causing considerable differences among the 16 states. Consequently, this may lead Germany's future teachers to feel at a loss as to what to do (Foraker 1999).

The correlation of this indicator with knowledge achievement was again calculated. The results show that the degree of well-organized curriculum from future teachers' views and their MCK and MPCK achievements are not statistically correlated. These results suggest that the future teachers' MCK and MPCK achievements and the organization or arrangement of courses they received in TEPs are not necessarily related. One possible reason for this result is that there might be other factors relating to courses, such as the amount and difficulty level of the course content, that influence knowledge achievement.

With regard to the educators' viewpoints, the data in Table 4 primarily indicate that no matter what types of educators exist within a given country, the means of the logit scores exceeded the neutral score of 10, which implies that educators in every country approved of their courses/content arrangements on average. Thus, an

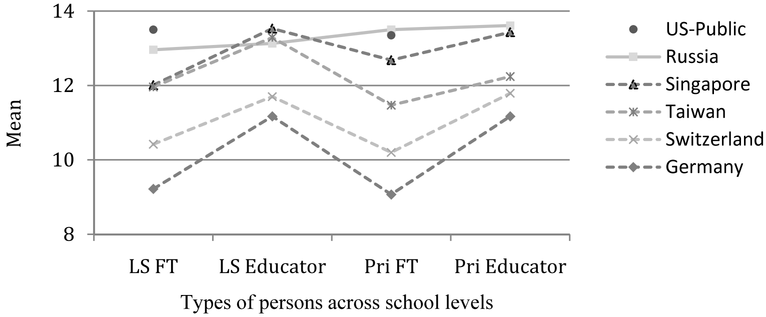

Fig. 3 The within-country trends of courses/content arrangement across school levels for the six countries that have all means of knowledge achievement above the international mean of 500. *LS* = lower secondary; *Pri* = primary; *FT* = future teachers. The lines of US-Public are not drawn since its data of educators were not processed

inconsistency in the evaluations among future teachers and educators on the effectiveness of courses/content arrangement appears, prompting us to start our inspection by focusing on the mean differences of future teachers and educators within countries. A common pattern emerged in that educators at both levels gave statistically significantly higher ratings than future teachers did for all applicable countries, with the exception of two, the Russian Federation being one. This phenomenon tells us that educators had more confidence in courses/content arrangements in TEPs than did future teachers. However, educators are often the planners and executors of their curricula; therefore, the higher ratings they provided may translate into a lack of motivation to improve. These results in issues worth considering: For example, does this idea mean that educators are unfamiliar with the lower secondary and/or primary level, or are they just more optimistic? What kinds of courses/content do future teachers desire or need?

Undeniably, in terms of courses/content arrangement, each TEP possesses a different degree of focus on either the advanced or the pragmatic standpoints. Based on our scales, when the arrangement of courses/content are rated to the same degree from both the educators' advanced standpoints as well as the future teachers' pragmatic standpoints, then the TEP shows an arrangement that possesses equilibrium. From the six countries that all have achievement means above the scale mean of 500, we discovered two different patterns (see Fig. 3). Germany, by using the type of all educators as an estimation of both the lower secondary and primary educators, shares the same pattern as Switzerland, where the arrangements are much more valued from the advanced view at both school levels. Taiwan and Singapore also are similar to Germany and Switzerland but with a slight difference at the primary level in that the degree of equilibrium between both views is slightly better.

The Russian Federation, however, presented a totally different pattern in that its TEP is in perfect equilibrium. The United States, like the Russian Federation, having almost the highest ratings from future teachers, unfortunately did not have the educators' data, and this fact hindered an investigation on whether they would fall in the same pattern as the Russian Federation in terms of the degree of equilibrium between both views in courses/content arrangement.

4.5 Overall Effectiveness

The overall effectiveness of TEPs in educating future teachers on mathematics teaching is taken as the last indicator of teacher education quality with respect to the ratings of persons inside the TEPs. Again, both the pragmatic view and the advanced view are valuable in this indicator. The international TEDS-M included a question at the end of both the future teachers' and the educators' questionnaires inquiring about this topic. Four levels of ranks (*very ineffective, ineffective, effective*, and *very effective*) were provided as the levels of satisfaction in association with effectiveness, ranging from one to four points, respectively.

Table 5 presents the means of overall effectiveness for each of the participating countries in both the lower secondary and primary levels.

One observation from the lower secondary future teachers' data was that the countries whose overall effectiveness ranked in the top six in terms of the means of the effectiveness points were all among the eight countries that prepared only specialists teaching in one subject. The Russian Federation and Taiwan fell into this category. For those countries that prepared some generalists, namely, Chile, Norway, and Switzerland, the effectiveness means were low. An important inference made from these preliminary results is that effectiveness is influenced by the degree of specialization for the lower secondary level.

With regard to primary future teachers, the means of the effectiveness points of the nine countries preparing only generalists were spread across the rating scale. The only two countries that prepared only specialists—Thailand (in one subject) and Malaysia (in two subjects)—ranked high, at the third and fourth positions. From the means of the effective points, we can see that some of the countries' programs had reached the effectiveness threshold of three points from both the future teachers, who represent a pragmatic standpoint, and the educators, who represent an advanced standpoint. These countries include Taiwan at the lower secondary level and the Russian Federation at the primary level. Unfortunately, at both levels, Germany and Switzerland, as well as the United States at the primary level, did not meet the threshold of three points.

Another interesting issue relates to the differences in ratings from the future teachers' pragmatic standpoint and the educators' advanced standpoint. We started our inspection by focusing on the mean differences of future teachers and educators within countries. The differences were very small for each country (absolute values less than 0.3), indicating near equilibrium of the two standpoints in a system. These results were different from those for the courses/content arrangement indicators. For the Russian Federation, the degrees of equilibrium concerning advanced and pragmatic views were high for both the courses/content arrangement and overall effectiveness in preparing future teachers to teach mathematics at both

Table 5 Country means of effectiveness points of future teachers and educators rating their teacher education programs and the differences in the means

Lower secondary level				Primary level			
Country	FT	Edor	DM	Country	FT	Edor	DM
	M	M			M	M	
Botswana	3.27	3.26	0.01	Russia	3.24	3.02	0.22**
Russia	3.24	2.97	0.26**	Philippines	3.19	3.24	−0.05
Philippines	3.20	3.14	0.06	*Thailand*	3.15	3.04	0.11**
Oman	3.19	3.15	0.04	Malaysia	3.14	3.00	0.14**
Taiwan	3.14	3.28	−0.14**	Singapore	3.14	2.93	0.21*
Thailand	3.11	3.04	0.07*	Botswana	3.13	2.85	0.28
Malaysia	3.04	3.02	0.02	Georgia	3.08	3.08	0.00
Singapore	3.04	2.95	0.08	US-Public	3.03		
US-Public	2.98			Taiwan	2.83	2.88	−0.04
Georgia	2.94	3.15	−0.21	Switzerland	2.81	2.73	0.08
Poland	2.94	2.99	−0.05	Poland	2.77	2.91	−0.14**
Switzerland	2.84	2.80	0.04	Chile	2.66	2.82	−0.16**
Germany	2.75			Norway	2.59		
Chile	2.53	2.82	−0.29**	Spain	2.51	2.66	−0.15*
Norway	2.48			Germany	2.40		

Note. FT = future teacher; Edor = educator; DM = difference of means, which was obtained by subtracting future teachers' means of the ratings of program effectiveness by educators' means for each country. The significances of the differences of means were tested by t tests. The italicized countries are those that prepare only specialists teaching in one subject. The German mean for all educators, which is not distinguishable by levels, is 2.75. It is higher than the average of their future teachers' means, 2.58, for both levels. $^{*}p < 0.05$, $^{**}p < 0.01$

levels. This point means that the TEP of the Russian Federation showed balance between advanced and pragmatic views. On the other hand, the educators of Germany, Singapore, Switzerland, and Taiwan gave higher ratings for courses/content arrangement than their future teachers did; however, ratings for overall effectiveness were balanced among educators and future teachers. This development may be a result of the educators' confidence in the curricula that they have formed, thus causing their ratings for courses/content arrangement to be higher. However, for the educators of these countries, with the incorporation of other considerations into the overall evaluation of the effectiveness of a country in preparing future teachers to teach mathematics, such as the courses taken and the level of knowledge of their students, their overall ratings became lower. In contrast, the future teachers did not find the courses/content arrangements effective, but their ratings for the other factors, such as the effectiveness of instructors or the satisfaction of their knowledge achievement, managed to bring their overall ratings close to that of the educators.

But were future teachers' ratings of the overall effectiveness of their TEPs associated with their own MCK and MPCK achievements? Spearman's rank correlation analyses showed that in all countries, except the United States and Germany, at both school levels the correlation coefficients (significant) were comparatively low ($-0.2 < r < 0.2$). Germany and the United States, however, reached the 0.05 level of statistical significance.[6] In addition, conducting Spearman's rank correlation analyses to examine whether or not the correlation exists between countries' means of overall effectiveness and their MCK and MPCK scores also returned a negative result.

Finally, this study tried to examine what kinds of effectiveness indicators influenced future teachers or educators in evaluating the overall effectiveness of their TEPs. Spearman's rank correlation coefficient analyses were used at the individual level for every country because all the scales of effectiveness were ordinal. Results showed that, regardless of the school level, the correlations between the overall effectiveness and other indicators of effectiveness ranked in the following order: MR-instructors, courses/content arrangement, teaching coherence, and SB-supervisors (see Table 6). At the lower secondary level, there were two major groups of five and six countries categorized in terms of which indicators influenced the countries' overall effectiveness the most. Germany, the Russian Federation, Taiwan, and the United States all fell into the first group, where their overall TEP effectiveness in preparing future teachers to teach mathematics was most influenced by the effectiveness of the MR-instructors. Singapore and Switzerland fell into the second group, with their overall effectiveness most influenced by the effectiveness of both MR-instructors and courses/content arrangement. At the primary level, the dominating pattern was the second group, which consisted of eight countries. The United States did not fall into this group because the indicator of teaching coherence between the teaching of universities and that of schools further influenced its overall effectiveness. The results indicated that there are more factors involved in making primary future teachers feel that their TEPs are capable of preparing them for mathematics teaching than in the lower secondary level. The US primary-level programs especially need to take care of at least three indicators in order for future teachers to feel satisfied with its overall TEP effectiveness.

From the data shown in Table 6, we are aware that there are some countries in which none of the four indicators has an effect on the overall effectiveness ratings. The reason for this phenomenon is still unknown and thus needs further study.

[6]At the lower secondary level, only the United States had significant correlation coefficients for mathematics content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK), 0.22 and 0.23, respectively. At the primary level, Germany had significant correlation coefficients for MCK and MPCK, 0.37 and 0.33, respectively.

Table 6 Noticeable significant correlations between future teachers' ratings of overall effectiveness and other indicators of the effectiveness of TEP

Lower secondary level					Primary level				
Country	MR-I	CA	TC	SB-S	Country	MR-I	CA	TC	SB-S
Oman	•	•	•	•	Botswana	•	•	•	•
Norway	•	•			US-Public	•	•	•	
Poland	•	•			Taiwan	•	•		
Switzerland	•	•			Spain	•	•		
Chile	•	•			Poland	•	•		
Singapore	•	•			Thailand	•	•		
Thailand	•	•			Norway	•	•		
US-Public	•				Chile	•	•		
Russia	•				Switzerland	•	•		
Taiwan	•				Singapore	•	•		
Botswana	•				Russia	•			
Germany	•				Germany	•			
George					Malaysia				
Malaysia					Philippines				
Philippines					Georgia				

Note. MR-I = instructor in mathematics-related courses; CA = *courses/content arrangement*; TC = *teaching coherence*; SB-S = school-based supervising teacher. A dot (•) indicates $r_s \geq 0.3$ at $p < 0.01$, with the exception of Botswana at $p < 0.05$. The significances for $r_s < 0.3$ are not shown in the table

By considering the comparisons across countries, some factors influential to overall effectiveness became apparent. One factor is future teachers' knowledge achievement. Taking the United States and Taiwan at the lower secondary level as examples, with Taiwan's ratings in overall effectiveness higher than those of the United States, we see that the United States, having all other indicators of effectiveness rated highly with only knowledge achievement placing at the middle, did not receive higher ratings in overall effectiveness compared with Taiwan, which had effectiveness indicators that usually did not reach the levels of the United States but which had extremely high achievement scores. In the same way, Singapore and Taiwan, being high-achieving countries, together with Germany, a mid-achieving country, and Switzerland, an upper-half-achieving country, did not reach the same ranks in their overall effectiveness as their knowledge achievement, given that their other indicators of effectiveness were not as positive as their achievement levels. The Russian Federation, the only country that ranked high for all indicators of effectiveness and achievements, always remained within the top two ranks in overall effectiveness. Perhaps this point is exemplary of a country's TEP that can make its future teachers feel that they are being aptly prepared to teach mathematics, therefore proving that all indicators of effectiveness and achievements are necessary.

5 Conclusion

Through international comparison, countries around the world acquire opportunities to learn from each other and reflect on themselves for the future. This initial study looks at the picture of mathematics teacher education quality in terms of effectiveness across countries by constructing a number of indicators and using various TEDS-M collected or scaled data. These indicators not only consider TEPs in terms of the outcome of knowledge future teachers possess but also the persons involved at the other end of TEPs—academic and school-based instructors; effectiveness is evaluated both from the perspectives of these persons and from the circumstances. Certain types of these data had never been collected prior to this study, such as statistics relating to educators. Several reflections and implications can be drawn from this study.

5.1 Effectiveness of Instructors and Teaching Coherence

Whether practical teaching and theoretical knowledge should both be included in the TEP, and how much of each is necessary, has always been a topic of discussion. This study shows that future teachers report that they benefit from both academic and school-based instructors in every participating country, and this result supports the necessity and appropriateness of integrating theoretical knowledge and practical teaching into teacher education. Based on the fact that future teachers in more than two thirds of the countries gave evidence that the effectiveness of SB-supervisors is higher than that of MR-instructors in helping them become well-trained teachers, it seems reasonable for countries to raise the following question: Should we reorganize our TEPs in order to allow future teachers more time in practicum?

Regardless of from whom the future teachers have benefitted the most, all countries' future teachers rated both positions of instructors as effective in providing professional help to facilitate them in becoming well-trained teachers. However, the effectiveness of instructors does not produce any noticeable influences on future teachers' knowledge achievement. Given that the future teachers feel their instructors are effective in educating them, they must have learned something and been influenced by their instructors. So what aspects of the future teachers' experience did the educators influence—their future classroom teaching or their knowledge achievement? This question is worth investigating and reminds us that a paper-and-pencil measure may not provide the whole picture for the achievements of future teachers.

The MR-instructors usually provide a theoretical foundation, and the SB-supervisors can use the future teachers' qualifications and what they have learned in the universities to strengthen their real classroom teaching. This study produces a concept of synchronization by joining the three indicators—teaching coherence, the effectiveness of MR-instructors, and the effectiveness of SB-supervisors. When the three indicators are highly rated, this reveals that university and school instructors are effective and make use of tightly integrated teachings. A program with this characteristic is regarded as synchronized.

The United States demonstrated an effective model in terms of having the most synchronized teaching in TEPs at both the lower secondary and primary levels. This means that we can expect that US teachers will be good at real classroom teaching in terms of building their instructional frameworks together with theoretical support. Other countries, for example, Taiwan, should reflect on what they could learn from the features of the US TEP. Further study regarding this issue is needed.

5.2 Courses/Content Arrangement and Overall Effectiveness

A high-quality TEP not only is synchronized in its teaching but also needs to attend to the organization of the sequences and links of the courses/content to meet the needs of future teachers. For this reason, the indicator courses/content arrangement emerges. This indicator serves as a criterion to determine whether a program has a well-organized curriculum in educating future teachers. Ideally, a program is well organized if it is perceived as being equipped with a well-organized curriculum from both the advanced and pragmatic points of view. However, given that some of the countries involved did not provide educator samples, the rating from an advanced view could not be obtained; therefore, this study employs only the pragmatic view of future teachers.

The United States and Russian Federation are good examples of well-organized programs. In contrast to the United States, Germany shows a lack of organization in its curriculum. Further analysis of these countries' cases shows that the complicated mixes of specializations and teaching grade spans may influence the organization of curricula in TEPs. It is easy to recognize the difficulty of building a curriculum that encourages competence in preparing a future teacher to be eligible to teach from the primary to senior high grades.

A complicated mixture of different specializations and teaching grade spans also shows a negative influence on overall effectiveness, the last and most comprehensive indicator of the quality of TEPs. The six countries chosen for further investigation in this study provide evidence showing the tendency that TEPs preparing only specialists at the lower secondary level and TEPs preparing generalists at the primary level are better in terms of overall effectiveness; however, the mixture of specializations and grade spans is not the only influence involved. Other factors, such as the effectiveness of MR-instructors and/or the courses/content arrangement, also show moderate influences on future teachers' ratings. Among the six countries we chose to further investigate, we found that in order for a TEP to make its future teachers feel that they are being aptly prepared to teach mathematics, all indicators of effectiveness and achievement are necessary and influential to overall effectiveness.

Last but not least, focus should be put on the equilibrium of a TEP between both the advanced and pragmatic views. For most of the countries involved in this research the overall effectiveness is in equilibrium; however, the indicator courses/content arrangement is not balanced. This phenomenon produces an issue worthy of consideration by the mathematics education community. Another fact is that among all participating countries, all levels of educators, being the planners and executors of courses/content arrangements, rated their arrangements in providing suitable courses/content for their students much higher than their students did. The higher ratings they provided may translate into a lack of motivation to improve. To us, as teachers of teachers, this is not only a heavy blow but also a wake-up call raising issues worth considering: Does this mean that teacher educators are too unfamiliar with the situations at the lower secondary and/or primary levels, or are they just being more optimistic? What kinds of courses/content do future teachers desire or need? These are issues that the academic community should immediately pursue.

This study discovered that many effectiveness indicators do not correlate with the knowledge indicator, which is regarded as the most important indicator by some people. Somewhat based on the results from the section titled Overall Effectiveness, we may put forth a hypothesis that other factors exist that may be combined with our indicators to guarantee the knowledge of future teachers. For example, the mathematics knowledge of future teachers at the entry point of the TEP or the amount and depth of the courses taken in the TEP may be other factors that influence future teachers' knowledge at the exit point. Further research is suggested.

Research concerning teacher education has always been highly valued; yet, how many national studies expose this reality? Although some studies (e.g., Judge et al. 1994) criticize US teacher education as a "non-system" in that it is not under national control but has a great deal of autonomy for teacher educators, it is worth noting that, as observed from this international comparison study, the US TEP is synchronized and well organized from the pragmatic views of its future teachers. One thing to which the United States should pay more attention is the elevation of its future teachers' MCK and MPCK, which may be the reason why the overall effectiveness of the US TEP does not stand out in the international ranks.

From the abundance of information this research has obtained, it is reasonable to say that this international comparison study provides new information to many countries. The insufficient aspect of this study was the small number of participating countries; it therefore lacked complete international representation. Furthermore, some countries, like the United States, provided insufficient data concerning their educators; this caused certain pieces of information to remain unresolved, and therefore they could not be presented. Nevertheless, the results of this initial analysis show that teacher education matters and that international teacher education studies are valuable. This should not mark the end of teacher education studies; instead, this is the beginning.

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Diagnosing Teacher Knowledge by Applying Multidimensional Item Response Theory and Multiple-Group Models

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Abstract Researchers are still struggling to define a concept of pedagogical content knowledge that separates this dimension from content knowledge. Based on data from TEDS-M, an IEA study of mathematics teacher education in 16 countries, this paper aims to contribute to this discourse by using different multidimensional approaches to modeling teacher knowledge. Another question of cross-cultural research is whether the characteristics of the latent traits examined and their interplay are homogeneous across countries (measurement invariance) or if it is necessary to treat the countries as separate groups. Our basic hypothesis is that more sophisticated multidimensional and multiple-group IRT models lead to valuable additional information that gives diagnostic insight into the composition of teacher knowledge. This is demonstrated using the TEDS-M data.

Keywords TEDS-M · Comparative study · Validity · Mathematics content knowledge (MCK) · Pedagogical content knowledge (PCK) · Measurement invariance · Culture · Between-item multidimensionality · Within-item multidimensionality · 2-parameter logistic model

1 Introduction

TEDS-M, a multinational study of mathematics teacher education in 16 countries, surveyed future primary and lower secondary teachers in their final year of teacher

This chapter is an adjusted version of Blömeke, S., Houang, R. & Suhl, U. (2011). TEDS-M: Diagnosing teacher knowledge by applying multidimensional item response theory and multi-group models. *IERI Monograph Series: Issues and Methodologies in Large-Scale Assessments, 4*, 109–126.

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S. Blömeke et al. (eds.), *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn*, Advances in Mathematics Education,
DOI 10.1007/978-94-007-6437-8_22,

training.[1] In addition to gathering data on the teacher trainees' backgrounds, the courses they were taking, and their beliefs about teaching, the study assessed their content knowledge and their pedagogical content knowledge, that is, the knowledge they would need to be successful in the classroom.[2] In this chapter, we use the data from TEDS-M (for a first description of the results see Blömeke et al. 2010a, 2010b) to examine different approaches to defining and subsequently scaling teacher knowledge. We also examine if such approaches are invariant across countries.

2 Dimensionality of Teacher Knowledge

Latent traits such as reading literacy or mathematics literacy, typically found in PIRLS or TIMSS, are relatively well defined. They serve different purposes and are usually applied in different contexts. Despite their measures having strong correlation, it is convincing to treat them as being conceptually different and therefore to scale them separately in unidimensional item response theory (IRT) models. This conceptual clarity does not exist with respect to teacher knowledge. Researchers are still struggling to define this latent trait and to identify its subdimensions (Graeber and Tirosh 2008).

Teacher knowledge includes several cognitive abilities (Bromme 1992; Shulman 1985). Based on Shulman's initial work, two subject-related subdimensions of teacher knowledge can be distinguished:

- Content knowledge, which in the case of TEDS-M as a study on mathematics teacher education is *mathematics content knowledge* (MCK). MCK includes the fundamental definitions, concepts, and procedures of mathematics.
- Pedagogical content knowledge, which in the case of TEDS-M is *mathematics pedagogical content knowledge* (MPCK). This form of knowledge includes knowledge about how to present fundamental mathematical concepts to students, some of whom may have learning difficulties (for further details, see Tatto et al. 2008).

Both subdimensions of teacher knowledge deal with mathematics but from different perspectives. Studies by Schilling et al. (2007) and Krauss et al. (2008) demonstrate that while it is possible to distinguish between MCK and MPCK, the two are highly correlated. The challenge is to determine the appropriate model that defines

[1]TEDS-M was funded by the IEA, the US National Science Foundation (NSF; REC 0514431) and each participating country. In Germany, the study was funded by the German Research Foundation (DFG; BL 548/3-1). In the US, the study was funded by the GE Foundation, the Boeing Company, the Carnegie Corporation, and the Bill and Melinda Gates Foundation. The instruments are copyrighted by the TEDS-M International Study Center at MSU (ISC). Any views expressed in this paper are those of the authors and do not necessarily reflect the views of the IEA, the ISC or the funders.

[2]For the first results from this study, see Blömeke et al. (2011, 2013) and Tatto et al. (2012).

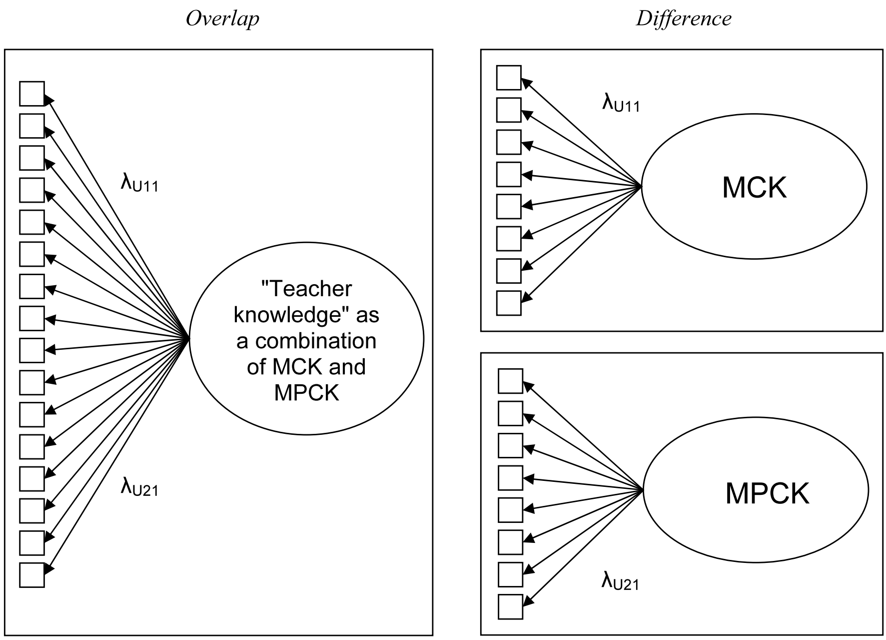


Fig. 1 Unidimensional approaches to scale MCK and MPCK (with respect to the notation cf. Hartig and Höhler 2008)

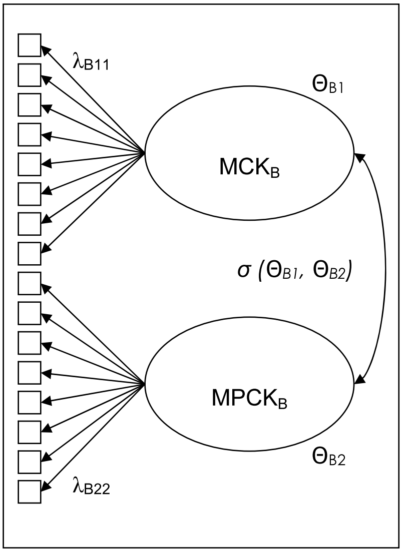
the relationship between the two latent traits. One choice is between unidimensional and multidimensional IRT models.

Unidimensional models can stress the conceptual *overlap* of MCK and MPCK, in which case teacher knowledge is regarded as a single dimension and all items are scaled together. Or the model can stress the conceptual *difference* between MCK and MPCK, which means these two forms of knowledge are regarded as separate dimensions and the mathematics and mathematics pedagogy items are scaled separately. This approach was used in TEDS-M. Figure 1 illustrates the two unidimensional models. It shows how the two types of items link to the respective latent variables.

Multidimensional approaches, in contrast, can take the conceptual overlaps and differences into account at the same time. Multidimensional item response theory or MIRT (Reckase 2009) is a relatively new but growing methodology for modeling the relationship of examinees to sets of test items as well as the relationship of the underlying latent traits using the matrix of their responses (see, for example, Finkelman et al. 2010; Wang et al. 2004; Yao and Boughton 2007). In the case of TEDS-M, two MIRT approaches are possible.

The first approach could be a two-dimensional scaling of MCK and MPCK, where each latent variable is treated as unidimensional ("between-item multidimensionality," Adams et al. 1997; "factorial simple," McDonald 2000). The conceptual overlap of MCK and MPCK is then expressed by a positive latent correlation of the two variables (see Fig. 2).

Fig. 2 Model of between-item multidimensionality



The second approach could be a two-dimensional scaling of MCK and MPCK with a general and a nested factor ("within-item multidimensionality," Adams et al. 1997; "factorial complex," McDonald 2000). This model would represent the idea that the nested factor MPCK is a mixture of different abilities and that mathematics pedagogy items measure this mix. According to this idea, solving mathematics pedagogy items requires not only MCK (as a general ability) but also specific MPCK (see Fig. 3). In order to separate the latter from the former, the two latent variables are constrained to be uncorrelated.

The different approaches to modeling the interplay of MCK and MPCK produce different scale scores, potentially leading to different interpretations. The within-item multidimensionality model depicted in Fig. 3 allows for double loadings and therefore represents an elaborated model of the interaction between teachers and items. Hartig and Höhler (2008) demonstrated (with respect to the English literacy of German students) the value of such an approach, namely that it provides more information about the nested factor. Following their reasoning, we expect that only in such a *within model* can the strength of teachers on the nested factor (in the case of TEDS-M, MPCK) be revealed for countries where mathematics pedagogy but not mathematics is stressed.

In contrast, in IRT models, where the two types of items are restricted to load only on one dimension, future teachers' achievement in MCK would obscure this strength. However, the advantage would be that we would essentially provide operational definitions of the two latent traits via the items themselves. In other words, we are relying on the face or content validity to provide meaning for the scaled scores. In this sense, the unidimensional model depicted on the right-hand side of Fig. 1 and the between-item multidimensionality model depicted in Fig. 2 are conceptually the same, except that all the items in the latter model are fitted together to yield a single statement of model fit.

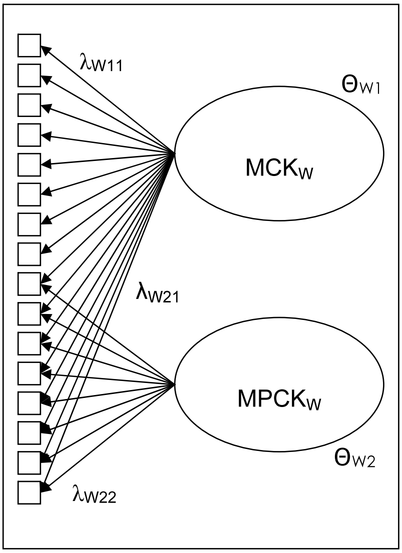


Fig. 3 Model of within-item multidimensionality

In this paper, we examine the two latent traits MCK and MPCK and their relationships to the two types of items. We therefore restrict our attention to models where all the items are fitted simultaneously. This means that we examine the fit and the measurement properties of the two multidimensional approaches and of the unidimensional model with a single latent variable, "teacher knowledge" (see Fig. 1 on the left-hand side). Because our focus is on contrasting the different models, the factor loadings of the items on their corresponding latent traits are constrained to be identical. This restriction simplifies the measurement models and limits the number of parameters to be fitted.

3 Cultural Invariance

Another question we need to ask when modeling the subdimensions of teacher knowledge is whether the interplay of this dimension is homogeneous across countries (measurement invariance) or if we need to treat the countries as separate groups. A recurring controversy in the comparative education literature centers on whether one should try to establish a universal model of educational outcomes across countries or whether the differences among countries are of such importance that they should be modeled: see, for example, Heyneman and Loxley (1982) versus Comber and Keeves (1973) and the application of these two approaches to the TIMSS 2003 study by Ilie and Lietz (2010).

Consideration of this controversy with respect to our study meant that, irrespective of the scaling approach taken, we would need to model the participating TEDS-M countries as one homogeneous group or, more precisely, as multiple groups from the same population. In the first case, we would need to treat model fit, loading patterns, variance explained, and latent correlations between MCK and MPCK as identical in all countries. The variances explained by the latent traits would then be the same in all countries. In the second case, we would need to allow cultural differences to manifest in differences in factor loadings, proportions of variance explained, and/or the latent correlations.

Moreover, even in the well-established field of studies on student achievement, the measurement quality is often slightly higher in English-speaking countries (Grisay et al. 2007; Schulz 2009; Thorndike 1973). An important reason for such non-equivalence is that, in a comparative study, most of the work such as item development or item review is done in English. In addition, Grisay et al. (2009) suggest the following further potential sources for non-equivalence:

- Language problems, in that the mother tongue and the test language are not the same in some countries. This was the case, with respect to TEDS-M, in Botswana and the Philippines.
- Differences in educational traditions among Asian and Western countries or differences in the developmental state of participating countries, which may, in turn appear, for example, with respect to our study, as differences in teacher education curricula.

Although TEDS-M was a highly collaborative effort and although the field data were subject to many checks with respect to differential item functioning, differences might still exist in how well the models measure MCK and MPCK in different countries. This situation may manifest in how well the item variances are explained country by country.

4 Research Questions

To summarize, based on our assumption of teacher knowledge being multidimensional in nature, we expect that, across the TEDS-M countries, multidimensional models are more likely than a unidimensional model of providing a better fit to the data. The between and the within models depicted in Figs. 2 and 3 would fit the data equally well, of course, because they are mathematically equivalent.[3] Taking into account the multidimensional nature of teacher knowledge should be particularly

[3]Note that this equivalence holds only if the factor loadings for each set of items—the mathematics and the mathematics pedagogy items, respectively—on their corresponding factors are constrained to be equal (see Rose et al. 2010, especially Appendix A; von Davier et al. 2011). We discuss the equivalence mathematically in detail in Blömeke and Houang (2009; available on request from the authors). The *between model* then conceptually corresponds to two simultaneously estimated Rasch models—one for each construct—allowing for a correlation between the constructs. The *within model* is a reparameterization of the between model.

Because the mathematics items have the same loadings whereas the mathematics pedagogy items have a different one for the latent variable MCK—and thus satisfy the 2PL definition of having multiple slopes—the *within model* is a simple case of the 2PL IRT model. Because the main aim of our paper is to demonstrate the implications—especially the potential value of the within favorable for the measurement of MPCK. Therefore, we expect that across countries the loadings of the mathematics pedagogy items on the underlying trait(s) will, in contrast to the loadings in the unidimensional model, vary and improve in the two-dimensional between and within models. We expect this pattern even though the loadings of the mathematics items on the underlying latent trait will be the same in all models.

In addition, and based on controversies and experiences from studies on student achievement, we expect that factor loadings, variances explained, and latent correlations between MCK and MPCK will differ from country to country. We will likely find that the countries where the test language does not match the language spoken at home are set at a disadvantage when the future teachers work on the items so that factor loadings, variances explained and latent correlations will be lower.

With respect to descriptive results, we expect that countries will show very different performance in MPCK as compared to their performance in MCK on the two-dimensional within model. The differences will vary according to the emphasis on mathematics pedagogical education in the teacher preparation programs of the respective countries. In particular, we expect the differences to be specifically apparent in countries such as Norway and the United States where mathematics pedagogy—but not mathematics—is stressed. For the two-dimensional between model, future teachers' achievement in MCK would obscure such differences.

5 Data Sources

We used the international dataset from the TEDS-M assessment of future primary school teachers in their final year of teacher education for this paper. The total sample size was 13,400. The primary assessments consisted of five booklets with 104 items in total; 72 mathematics and 32 mathematics pedagogy items. Items were assigned to booklets following a balanced-incomplete-block design. The mathematics items covered the content areas "number" (as that part of arithmetic most relevant for primary teachers), "algebra," and "geometry," with each set of items having about equal weight, as well as a small number of items about "data" (as a hypernym for that part of probability and statistics most relevant for primary teachers). The mathematics pedagogy items included aspects of "curricular and planning knowledge" and "knowledge about how to enact mathematics in the classroom." These two sets of items were of about equal weight. The majority of items were complex multiple-choice items. Some of the items were partial-credit items.

Because primary school teachers are responsible for teaching multiple subjects, including mathematics, we examined in all TEDS-M countries except Thailand[4] a

modeling approach—of these different parameterizations for the interpretation of the TEDS-M data, we restricted ourselves to this kind of measurement model, which was also close to the scaling approach used by the TEDS-M International Study Center (see Tatto et al. 2012).

[4] In Thailand, the future teachers surveyed were primary mathematics specialists.

broad range of primary teacher education programs. Although 16 countries took part in the TEDS-M primary study, Canada was excluded because it did not meet the response rate requirements. Therefore, our sample consisted of 15 countries.

The sampling process for Norway was difficult, and the final country sample consisted of two subsamples that were likely to partly overlap. While information about the seriousness of this problem is not available, we realized that using only one subsample would lead to strongly biased country estimates. Combining both subsamples would lead to imprecise standard errors (for more details, see Tatto et al. 2012). After an extensive research of the Norwegian literature about teacher education, combining TEDS-M data with publicly availably evaluation data from Norway (NOKUT 2006), and recourse to expert reviews, we decided to combine the two subsamples in order to represent the future teachers' knowledge as appropriately as possible. However, the results should be regarded as a rough approximation only.

Finally, we used sampling weights in all the analyses so that all the countries were weighted equally. For each country, we adjusted the final sampling weights upwards or downwards so that the sum of weights for each country was equal to 500 cases.

6 Method

We applied unidimensional and two-dimensional scaling models to the 104 items. We carried out calibration by applying, to the TEDS-M data, the IRT 2-parameter logistic model implemented in MPlus 5.2 (Muthén and Muthén 2008), using maximum likelihood estimation with robust standard errors (MLR). The estimation procedure took the multiple-groups and multiple-forms structure of the data into account (MLR is the MPlus default estimator when dealing with complex data structures). We used Samejima's (1969) graded-response model to model the partial credit items.

Because our focus was on comparing the different models, we constrained the factor loadings to be the same within each dimension. This constraint resulted in an identical estimate for the loadings of the same type of items, that is, mathematics versus mathematics pedagogy items, an outcome that facilitated comparison of the models.[5] Variances of the latent variables were fixed to 1. In the within-multidimensional model, the correlation between the two latent variables was restricted to 0. This meant that the specific MPCK factor was defined to be uncorrelated with the general MCK factor, which allowed us to use IRT as a "diagnostic aid"

[5] As we pointed out in the previous footnote, this is not a standard 2PL IRT model, in the sense that slopes can vary across items. In contrast, the model, because of its restrictions, comes close to a 1PL (or Rasch) model. However, due to the double loadings of the mathematics pedagogy items or the different loadings of the mathematics items and the mathematics pedagogy items on the underlying MCK trait, respectively, we consider it is still justifiable to label the model as a (constrained) 2PL model.

(Walker and Beretvas 2003). Our evaluation of model fit was based on the log likelihood, which required us to take into account the number of parameters (adjusted Bayesian information criterion; see Schwartz 1978).

When carrying out the multiple-group analyses, we used the mixture modeling procedure of MPlus, with countries as known classes. This procedure is the approach that Muthén and Muthén (2008) use when addressing this question. In the case of our study, it meant that all loading parameters and the correlation between MCK and MPCK (in the case of the between model) were estimated separately for each country.[6] For the single-group configuration, however, the parameters were restricted to be the same for all countries. Differences in the model fit between the multiple-group and the single-group configurations would point to differences among the countries.

After completing the calibration, we used the item-parameter estimates to estimate achievement for each respondent. We used, as individual-ability estimates, "expected a posteriori" (EAP), thereby assuming a standard normal distribution of the ability scores. In accordance with the practice in TEDS-M, we scored "not reached" responses, which were scored as "missing" in the calibration, as "incorrect" when estimating scores for individuals. Although Rose et al. (2010) demonstrated in a simulation study that this scoring procedure may result in bias, especially under the condition of a high proportion of not reached responses, the proportion of such responses in the TEDS-M primary study was very small compared with the proportions in the simulation settings (MCK, 0.79 %; MPCK, 1.14 %). As a consequence, the correlations between the EAP estimates used in this paper and the EAP estimates obtained when scoring the not reached as missing were very high (single-factor model, 0.97; two-dimensional models, >0.99). We standardized the EAP estimates (in logits) to a mean of 500 and a standard deviation of 100.

7 Results

7.1 Measurement Properties of the Different Calibration Models

First, we examined the fit of the calibration models with data from all of the countries together (single-group configuration). The models contained 150 or 165 estimated parameters (i.e., the item-difficulties or threshold parameters, factor loadings or item discrimination, class means, and, in the between-multidimensional model, the latent correlations), respectively, for the unidimensional and two-dimensional models. As expected, the two-dimensional between and within models showed a significantly better model fit than the unidimensional model (see Table 1, chi-squared

[6]In this sense, the procedure is actually the same as that used in the multiple-group IRT model (Bock and Zimowski 1997). The only difference is that it is labeled differently by Muthén and Muthén (2008) and may therefore cause confusion.

Table 1 Model fit for the different models under the single-group configuration

Model	Log likelihood	Scaling correction factor	Number of parameters	$BIC_{adj.}$	Latent correlation
One-dimensional model ("teacher knowledge")	−365,822.06	2.11	150	732,592.88	–
Two-dimensional between model	−365,462.40	2.10	165	731,968.44	0.85 (0.02)
Two-dimensional within model	−365,462.40	2.10	165	731,968.44	0.00 (0.00)

Note: $BIC_{adj.}$ = adjusted Bayesian information criterion

Table 2 Standardized factor loadings and variance explained for the different models

Model	Factor loadings mathematics items	Factor loadings mathematics pedagogy items		R^2	
				MCK	*MPCK*
One-dimensional model ("teacher knowledge")	0.34 (0.00)***	0.28 (0.01)***		0.11 (0.00)	0.08 (0.00)
Two-dimensional between model	0.34 (0.00)***	0.30 (0.01)***		0.12 (0.00)	0.09 (0.00)
Two-dimensional within model	0.34 (0.00)***	0.25 (0.00)*** MCK	0.16 (0.01)*** MPCK	0.12 (0.00)	0.09 (0.00)

Note: *** $p < 0.001$

difference test TRd = $chi^2_{(15)} = 359.66$, $p < 0.0001$). Both two-dimensional models produced the same log likelihood statistics because they were mathematically equivalent. This result supported our expectation of a multidimensional structure of teacher knowledge. The latent correlation between MCK and MPCK was high (0.85).

Second, we examined the loading patterns and the variance explained by the models in the single-group configuration. As we expected, the loadings of the mathematics items on the underlying MCK dimension were the same in all three models, whereas the loadings of the mathematics pedagogy items varied (see Table 2). The loadings of the mathematics pedagogy items on the underlying trait(s) were slightly higher in the two-dimensional models. But, more importantly, only the within model revealed the specific loading composition. Although the specific loadings of the mathematics pedagogy items on the MPCK trait were lower in the within model, they showed substantial additional loadings on MCK. All loadings were significant. This result points to the relevance of each dimension in this model.

Note that the loading for the mathematics pedagogy items for the between model is a composite of the loadings of these items for the within model. Thus, the square of the value of 0.30 in the between model is the sum of the squares of 0.25 and 0.16

Table 3 Model fit of the two-dimensional between model under the single-group versus the multiple-group configuration

Model	Log likelihood	Scaling correction factor	Number of parameters	$BIC_{adj.}$
Single-group configuration	−365,462.40	2.10	165	731,968.44
Multiple-group configuration	−364,924.00	2.12	207	731,157.29

Notes: $BIC_{adj.}$ = adjusted Bayesian information criterion. The fit for the two-dimensional within model is identical to the fit of the between model documented here

in the within model. In other words, as we pointed out above, the two models are mathematically equivalent.

The variance explained per item by the latent variables was higher for the mathematics items. This could be due to the smaller number of items and to a less well-defined MPCK trait, for which it is more difficult to construct items to measure it reliably.

Third, we examined if these results for the measurement properties of the calibration models applied to all countries (single-group configuration) or if there were differences among countries (multiple-group configuration). The comparison revealed a significantly better model fit of the two-dimensional multiple-group configuration (see Table 3, chi-squared difference test $\text{TRd} = \text{chi}^2_{(42)} = 489.90$, $p < 0.0001$).

Table 4 shows the country variation in the measurement properties. The language use (match of test language versus language used at home) seemed to have a systematic relationship to how well the items were associated with the latent variables. The correlations at the country level between language use and factor loadings ranged from −0.44 to −0.74. In Botswana, Malaysia, and the Philippines, almost all future teachers spoke a language at home (mainly Setswana, Bahasa Melayu, or Filipino, respectively) different from the language they were tested in (English). In particular, the mathematics items showed smaller factor loadings for these three countries than for the other countries.

The language used at home seemed to have a stronger relationship to the mathematics items than to the mathematics pedagogy items, and this was evident in both the between model and the within model. This result is somewhat surprising given that—by nature—pedagogy could be regarded as more closely associated with verbal representations than mathematics. That said, the latent correlations between MCK and MPCK were consistently high in all countries and uncorrelated to language use at home ($r = 0.06$).

As we again expected, the strength of the factor loadings and the amount of variance explained by the latent traits were significantly correlated with the developmental state of a country. We used the United Nations Human Development Index (HDI) as an indicator of the latter. However, the data revealed a relationship between measurement properties and country background for mathematics items but not for mathematics pedagogy items. The correlations between HDI and mathematics items were 0.36 and 0.26 for loadings and for variance explained, respectively,

Table 4 Standardized factor loadings, variance explained, and latent correlations for the two-dimensional multiple-group models and parameter estimates correlations with HDI and language use

Country	HDI	Language use	Between model								Within model					
			MCK math items	SE	R^2	MPCK math ped. items	SE	R^2	Corr.	SE	MCK math items	SE	MCK math ped. items	SE	MPCK math ped. items	SE
Botswana	0.664	90.30	0.19	0.03	0.04	0.22	0.05	0.05	0.97	0.20	0.19	0.03	0.21	0.07	0.04	0.35
Chile	0.874	0.61	0.30	0.01	0.09	0.32	0.02	0.10	0.83	0.05	0.30	0.01	0.27	0.02	0.18	0.03
Georgia	0.763	3.25	0.37	0.02	0.14	0.34	0.03	0.11	0.65	0.07	0.37	0.02	0.22	0.03	0.25	0.03
Germany	0.940	2.20	0.39	0.02	0.16	0.40	0.02	0.16	0.83	0.04	0.39	0.02	0.33	0.02	0.22	0.02
Malaysia	0.823	87.18	0.21	0.01	0.04	0.27	0.02	0.07	0.85	0.08	0.21	0.01	0.23	0.03	0.14	0.04
Norway	0.968	1.59	0.37	0.02	0.14	0.27	0.02	0.07	0.92	0.06	0.37	0.02	0.25	0.02	0.10	0.04
Philippines	0.745	94.99	0.24	0.02	0.06	0.20	0.03	0.04	0.77	0.15	0.24	0.02	0.16	0.03	0.13	0.05
Poland	0.875	0.83	0.47	0.01	0.22	0.44	0.01	0.19	0.94	0.02	0.47	0.01	0.41	0.01	0.15	0.02
Russia	0.806	6.99	0.46	0.01	0.22	0.38	0.01	0.14	0.87	0.03	0.46	0.01	0.33	0.01	0.19	0.02
Singapore	0.918	42.80	0.34	0.02	0.11	0.29	0.02	0.08	0.75	0.08	0.34	0.02	0.21	0.03	0.19	0.03
Spain	0.949	13.85	0.27	0.01	0.07	0.21	0.02	0.05	0.90	0.07	0.27	0.01	0.19	0.02	0.09	0.04
Switzerland	0.955	6.14	0.33	0.01	0.11	0.25	0.02	0.06	0.77	0.06	0.33	0.01	0.19	0.02	0.16	0.02
Taiwan	0.932	29.59	0.38	0.02	0.15	0.27	0.02	0.07	0.95	0.05	0.38	0.02	0.25	0.02	0.09	0.04
Thailand	0.786	38.89	0.37	0.01	0.14	0.26	0.02	0.07	0.91	0.05	0.37	0.01	0.24	0.02	0.11	0.04
United States	0.950	1.78	0.34	0.01	0.12	0.27	0.02	0.07	0.88	0.05	0.34	0.01	0.23	0.02	0.13	0.03
Correlation[a] with HDI			**0.36**		**0.26**	0.13		0.11	0.06		**0.36**		0.14		0.11	
Correlation[a] with language use			**−0.74**		**−0.68**	**−0.57**		**−0.53**	0.07		**−0.74**		**−0.49**		**−0.44**	

Notes: *HDI*: Human Development Index of the United Nations

Language use at home: Proportion of future teachers with a mother tongue other than the test language (i.e., the official language of teacher education)

Between model: Mathematics items are loaded on MCK only, while mathematics pedagogy items are loaded on MPCK only

Within model: Mathematics items are loaded on MCK only, while mathematics pedagogy items are loaded on both MCK and MPCK

[a]These correlations were computed at the country level. Due to the small number of countries included and, in the case of language use, the extreme values, these are potentially subject to changes if the observations change

Table 5 Means, standard errors, and standard deviations for the two-dimensional models

MCK—between/within models				MPCK—between model				MPCK—within model			
	Mean	SE	SD		Mean	SE	SD		Mean	SE	SD
Taiwan	622	3.4	70	Taiwan	619	3.0	69	United States	544	2.3	97
Singapore	598	2.9	67	Singapore	601	3.0	66	Singapore	544	4.4	97
Switzerland	543	1.9	66	Switzerland	543	1.8	64	Norway	542	4.5	93
Russia	529	10.5	92	USA	529	3.8	71	Taiwan	520	2.8	87
Thailand	522	2.2	75	Norway	529	2.5	75	Malaysia	512	4.1	100
Norway	522	2.6	76	Russia	525	10.3	92	Switzerland	512	2.7	99
United States	522	4.1	72	Thailand	518	2.2	74	Spain	506	2.5	94
Germany	505	3.0	88	Germany	504	3.3	90	Philippines	499	7.4	95
Malaysia	485	2.2	58	Malaysia	488	2.6	61	Germany	494	4.3	107
Poland	480	2.1	102	Spain	478	2.8	61	Russia	489	8.0	102
Spain	476	2.9	61	Poland	477	2.0	103	Poland	484	2.7	98
Philippines	429	8.9	55	Philippines	432	9.5	55	Thailand	481	3.7	95
Botswana	428	6.4	53	Botswana	427	6.7	55	Chile	481	3.9	99
Chile	397	2.4	68	Chile	399	2.7	71	Botswana	477	11.1	94
Georgia	327	3.4	74	Georgia	326	3.3	73	Georgia	450	3.9	90

but the corresponding correlations ranged from only 0.06 to 0.14 for the mathematics pedagogy items.

Generally, the loadings of the mathematics items on the latent trait MCK were relatively high for the European countries. While regional differences between Asian and Western countries did not exist, the loadings were particularly high for the two Eastern Europe countries (Poland and Russia). They were 0.47 and 0.46, respectively. In contrast, the loadings for the other countries ranged from 0.19 to 0.39. The results were similar for MPCK loading but not as pronounced.

7.2 Descriptive Summaries of Country Performance on MCK and MPCK

Table 5 shows the country descriptive summaries from the between and within models. Note that the two models produced identical scores for MCK; only one set is therefore included in the table. The country means for MPCK differed widely in the different models, however. In the between model, the rank order of countries according to MPCK was very similar to MCK, with all 15 countries having the same rank (nine countries), within one or two ranks (five countries), or within three ranks (one country) on the scales. Primary teachers from Taiwan and Singapore ranked 1 and 2 on both scales, respectively.

If we remove general mathematics ability from the latent trait MPCK as was done in the within model, the picture changes. Only three countries now have the same rank according to MCK and MPCK, while the rank order shows differences of up to six ranks for the other countries. The result from the within model now shows future primary teachers from the United States with first place ranking in MPCK, tied with the future primary teachers from Singapore. Likewise, Norway, Malaysia, Spain, and the Philippines also rank higher for their MPCK performance than for MCK. In contrast, Russia and Thailand end up below the international MPCK mean.

8 Discussion

The two-dimensional between and within models provided significantly better fit estimates than the unidimensional model that assumed a single latent construct, "teacher knowledge."

This result supports our contention that the nature of teacher knowledge is multidimensional. In accord with Hartig and Höhler (2008), we can state that the between-multidimensional model describes the performance of future primary teachers on our mathematics and mathematics pedagogy items in a straightforward way. In contrast, the within model represents a more elaborated model of the interaction between teachers and items. Thus, the between model yields similar achievement information for MCK and MPCK, as revealed in the relative country ranks, whereas the within model yields distinctive profiles that are particularly evident in the case of MPCK.

Note that our summary relied on the kind of measurement models we used to define MCK and MPCK. Because our focus was on contrasting the different approaches to modeling multidimensionality and their implications for the interpretation of the TEDS-M results, we decided to keep the measurement models as simple as possible and as close to the scaling approach applied in TEDS-M as possible. It is most likely that a more complex measurement model, such as a 2PL IRT model without constraints on the factor loadings, would fit the data better or at least as well as our models, if only due to the larger number of free parameters. However, a more complex measurement model would not only make it more difficult to contrast the within and the between models, but it would also be more difficult to interpret and thus obscure the parameterization benefits.

The main feature that, in our case, distinguishes the two two-dimensional models is that the within model attempts to isolate the specific MPCK trait from MCK. If we were to follow the descriptive results from the conditioned within model, they would suggest not only a special strength in mathematics pedagogy among the future primary teachers from the United States but also among those from Norway, Malaysia, Spain, and the Philippines. These countries moved visibly up in the rank order of countries from the within model. In contrast, with this model future primary teachers from Taiwan and Singapore no longer outperformed the teachers from all the other countries, while the performance of teachers from Russia and Thailand moved below the international mean.

The relative importance of the within model as an appropriate representation of the strengths and weaknesses of the countries' respective mathematics teacher education provision becomes evident when we examine the correlation of MPCK with opportunities to learn (OTL) in teacher education. OTL were framed as content coverage in TEDS-M, specifically as "the content of what is being taught, the relative importance given to various aspects of mathematics and the student achievement relative to these priorities and content" (Travers and Westbury 1989, p. 5, quoting Wilson). OTL is, in this sense, defined in terms of future primary teachers encountering occasions to learn about particular topics during their teacher education. Because subject matter specificity is the defining element of an educational opportunity (Schmidt et al. 1997) and because TEDS-M is a study about "learning to teach mathematics," the particular topics reflected the areas of mathematics and mathematics pedagogy.[7]

The correlation between the ipsative OTL mean for mathematics pedagogy and the MPCK measure from the between model was almost zero ($r = -0.02$). But the correlation with MPCK from the within model was $r = 0.30$. Thus, under the within model, the more a country had focused on mathematics pedagogy in relation to mathematics during primary teacher education, the more likely it would be to have a high MPCK mean.

The conclusions drawn from the results of the unconditioned-between versus the conditioned-within model would be different (see also Hartig and Höhler 2008, with respect to English literacy). A potential explanation for this difference is the focus of primary teacher education. Coverage of mathematics content is highly relevant during teacher education in Taiwan, Singapore, Russia, and especially in Thailand, where, as we mentioned earlier, mathematics specialists are trained at the primary level. This focus is accurately expressed in these countries' MCK means.

In contrast, mathematics pedagogy is a very important focus of teacher education in Norway, Spain, and the United States, even at the cost of training in mathematics content. With the high conceptual and empirical overlap of MCK and MPCK (evident in the latent correlation), the low level of mathematics content knowledge superimposes on the relative strength in mathematics pedagogy. Its specialties are evident only in the within model that distinguishes between MCK influence on the solution of mathematics pedagogy items and specific MPCK influence. For those

[7] In order to avoid cultural bias of self-reported data, which is a well-known problem in comparative studies (Triandis and Triandis 1962; Van de Vijver and Leung 1997), and which in our case would represent differences in the willingness to check a topic as studied or not studied in teacher education, ipsative measures were developed (see, for example, Cunningham et al. 1977; Fischer 2004):

- (OTL_Number + OTL_Algebra + OTL_Geometry + OTL_Data)/4 = OTL_Mathematics
- (OTL_Foundations + OTL_Applications)/2 = OTL_MathPedagogy
- (OTL_Mathematics + OTL_MathPedagogy)/2 = OTL_Subject
- OTL_Mathematics_ipsative = OTL_Mathematics − OTL_Subject
- OTL_MathPedagogy_ipsative = OTL_MathPedagogy − OTL_Subject

readers wanting to learn about MPCK in detail, the within model provides this diagnostic information.

With this conception, however, the MPCK results from the within model do not correspond to test performance on the mathematics pedagogy items, given that performance on mathematics pedagogy items is a function of both underlying traits. Performance requires a mix of mathematics and mathematics pedagogical abilities. Only the between model accurately reflects this reality.

We therefore have to point out that both models have their uses and limitations and that it would not be appropriate to substitute one for the other. Note that the latent correlation of 0.85 is high, which means that the multidimensionality observed is modest in size, even though it does appear to exist. An interesting follow-up research question in this context would be: what kind of relationship exists between the conditioned MPCK and general pedagogical knowledge? Since extraction of MPCK is purposely uncorrelated with MCK, the former may be more strongly correlated to GPK for the within measure than for the between measure.

Evidence from our study also suggests that the MCK and MPCK assessments may not have been completely equivalent in all TEDS-M countries. Although rigorous quality control took place (as it always does in IEA studies), language and cultural differences might have been related to how well these traits were measured in the 15 countries. The differences by country complicate the development of a universal model of teacher knowledge.

To our surprise, the language problems seem to have been larger with respect to MCK than to MPCK. We attribute this result to a long history of schooling in the case of mathematics content knowledge. Its acquisition had probably already suffered from language disadvantages during primary and secondary school. In this sense, our study could raise the awareness of this problem.

A cultural influence on the measurement properties in TEDS-M may exist as well. The factor loadings were surprisingly high in the two Eastern European countries Poland and Russia. Although these countries were not specifically strongly involved in the test development, it seems as if the two TEDS-M tests were more closely connected to mathematics and mathematics pedagogy traditions in these two countries. However, this conclusion can be only a very tentative one; the relationship needs to be examined in more detail.

What do these results on measurement invariance mean for the quality of the TEDS-M results? In reality, this question cannot be answered because it has to remain an open one. The number of countries in our study was only 15, with even smaller numbers of country groups from similar educational traditions (in order to determine a potential cultural bias) or with substantial proportions of teachers using a different language at home than they were tested in (in order to determine a potential language bias). In addition, there is no commonly agreed upon threshold, above which a lack of measurement invariance would invalidate results from cross-country comparisons. Moreover, it would be naive to expect perfect test equivalence in comparative research.

Future research should examine in more detail the question of measurement invariance in TEDS-M. Hierarchical IRT and multiple-group confirmatory factor analysis provide the tools to determine important properties such as configural invariance, metric invariance, and scalar invariance (Fox 2005; Vandenberg and Lance 2000). Even if full invariance—which is rarely accomplished in cross-cultural research—cannot be determined in TEDS-M, such studies would reveal the extent to which partial invariance is supported. Approaches could then be taken to appropriately deal with such problems. Using hierarchical IRT, for example, de Jong et al. (2007) were even able to relax all invariance requirements across groups while retaining the possibility to make substantive comparisons. Such studies would be of relevance not only with respect to the TEDS-M assessment data but also, and may be more importantly, with respect to the OTL and beliefs data, given the fact that self-reported data may be even more vulnerable to bias (Blömeke in press).

Acknowledgements We thank Neelam Keer for her helpful comments on the final draft of this paper and the reviewers for their productive questions about the measurement models used, but we take responsibility for whatever errors we may have made.

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Are College Rankings an Indicator of Quality Education? Comparing Barron's and TEDS-M

William Schmidt, Nathan Burroughs, Lee Cogan, and Richard Houang

Abstract Although students at more selective schools generally demonstrate greater academic performance, it is unclear whether the gains from attending an elite postsecondary institution are due to the quality of educational services provided, or merely from peer and/or selection effects. Employing data drawn from the US-TEDS study, we assess the relationship between college selectivity and the mathematics learning of future teachers controlling for previous SAT scores using two different models. In an institution-level analysis, gains in student knowledge are measured by the difference between standardized SAT scores and standardized mathematical content knowledge (MCK) scores. In a multi-level model institutional and student-level data are used to examine the effects of selectivity on MCK scores, including measures of course-taking and prior achievement. In both analyses we find that college selectivity has little relationship with added mathematical knowledge.

This chapter was previously published, as Schmidt, William, Nathan Burroughs, Leland Cogan, and Richard Houang. (2011) "Are College Rankings An Indicator of Quality Education?" Forum on Public Policy: A Journal of the Oxford Roundtable. Vol. 2011, no. 3. http://forumonpublicpolicy.com/vol2011no3/archive/schmidt.pdf. The article is being re-printed with the permission of the Forum on Public Policy, Oxford Round Table.

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S. Blömeke et al. (eds.), *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn*, Advances in Mathematics Education,
DOI 10.1007/978-94-007-6437-8_23,

Keywords Mathematics content knowledge (MCK) · SAT · Program quality · Selectivity · Lower secondary · Competitive · Course hours · Mathematics · Course indicators

1 Background

A handful of colleges and universities sit as the uncrowned princes of the U.S. system of higher education. With enormous endowments, renowned faculty, and international prestige, there is no question that a comparative handful of institutions—most of them private—stand above the rest. These colleges are highly selective, such as the prestigious Ivy League schools, which have an average acceptance rate of under ten percent.[1] They attract of preponderance of the highest-achieving high school graduates, as evidenced by the average SAT scores of incoming freshman (Hoxby 2009). And, as many of the top schools are private, they tend to be much more expensive to attend: the total charges (including tuition and fees) for in-state four year public colleges in 2010 was $16,000 per year, less than half of that of private non-profits at $37,000 (Baum and Ma 2010).

What is less clear is what these bright students and their families are getting for their money. At first blush this might seem a rather odd question. After all, students at elite institutions of higher education tend to graduate at a higher rate, are more likely to pursue advanced degrees, have more prestigious careers, and earn higher salaries than students at other universities (Carnevale and Rose 2003). The disproportionate rewards accruing to students at top colleges have aroused a great deal of concern due to their underrepresentation of women, minorities, and those from families of modest means.

However, simply because graduates of the best schools do rather well in life tells us little about the quality of education they have received. First, it should be remembered that education is in part a positional good (Hollis 1982): one need not learn a great deal, only more than one's competitors. Higher incomes associated with having gone to a Harvard or Yale are not necessarily an indicator of having received an objectively excellent education, only that their graduates' educations are deemed superior to that obtained by others elsewhere. Second, the very privileged status of those entering the most prestigious schools raises doubts about the rigor of the instruction provided.

The success of those graduating from elite colleges could simply be an instance of selection bias: they attract the best students in part through reputation and price signaling, so it should not be surprising that their alumni do rather well. These postsecondary institutions provide superior social networking advantages and peer effects, real benefits to be sure, but hardly indicators of a strong curriculum. There is every possibility that students at the most selective postsecondary institutions would do just as well having gone somewhere else. These considerations raise the question: is the selectivity of a college necessarily an indicator of the quality of education received?

[1] Calculated from data presented in Dell (2011).

The difficulty to date in judging the quality of higher educational institutions is that the most common metrics are selectivity measures like acceptance rates, input measures such as SAT scores, and outcome measures such as wage and career data. What has been lacking thus far are measures of the *value added* by these schools. In other words, we need a way of determining how much additional knowledge has been gained by students in a given educational program in comparison with other programs.[2]

From the U.S. Teacher Education Study in Mathematics (TEDS), we now have such information. The TEDS yields empirical data on the educational experiences and content knowledge gains of a nationally representative sample of future teachers prepared at a wide variety of institutions. By combining data from the TEDS study with college selectivity measures—specifically the respected Barron's college rankings—it is now possible to estimate the degree to which selective schools provide a superior education, at least in one academic discipline. Other studies, such as that of Arum et al. (2011), examined learning gains across disciplines using generic measures of cognitive ability such as critical thinking that may or may not be appropriate to specific majors. The virtue of the TEDS study is that it focuses on only one—those preparing to become teachers of primary and early secondary mathematics—which permits specific assessments of content acquisition. In this paper we focus on future lower secondary teachers.

2 Study Design & Preliminary Analysis

The data for this research were gathered from two main sources: the Barron's College Admissions Selector Rating, generously provided by Barron's, and the results of the U.S.-TEDS study. By examining the relationship between student academic achievement and content knowledge with college selectivity, we can examine the value added by attending a given institution of higher education. We perform this analysis both at the institutional and student level. If "selectivity" truly does connote a higher quality education, students who attend elite schools should perform much better than students at other schools, *controlling for the students' prior academic achievement*.

Probably the most commonly-used measure of college selectivity is the Barron's index, published every year in the Barron's Guide to the Most Competitive Colleges (College Division of Barron's Education Series 2006). The index has been used by many researchers studying college selectivity. Barron's has even partnered with the U.S. National Center on Educational Statistics to offer a publicly available longitudinal database. The Barron's College Admissions Selector Rating classifies colleges

[2] The use of the term "value added" is not to be confused with the quite different "value-added models" that are in widespread use as measures of school and teacher quality in U.S. K-12 education.

Table 1 The College Admissions Selector Rating indicates the degree of competitiveness of admission to the college, with examples[a]

Barron's Rating	Examples
Most Competitive (10)	Harvard, Northwestern University, University of North Carolina-Chapel Hill
Highly Competitive^{+} (9)	University of California, Berkeley, University of Illinois
Highly Competitive (8)	Pennsylvania State University, University of Wisconsin
Very Competitive^{+} (7)	Rochester Institute of Technology, University of Maryland
Very Competitive (6)	Michigan State University, Indiana University, Purdue
Competitive^{+} (5)	Illinois State University, University of Colorado—Boulder
Competitive (4)	San Diego State University
Less Competitive (3)	California State University
Non-Competitive (2)	University of Toledo
Special (1)	New England Conservatory of Music

[a]College Division of Barron's Education Series (Ed.). Barron's Profiles of American Colleges: 2007 (27th ed.). Barron's Educational Series, Inc., p. 252, Hauppauge, NY (2006)

into ten categories, with the most selective colleges receiving a rating of 10 and non-competitive schools a rating of 2, with specialty institutions assigned a coding of 1. Table 1 presents the ten categories, with examples.

Sponsored by the International Association for the Evaluation of Educational Achievement (IEA), the same organization that conducts the TIMSS study, the TEDS was conducted as a follow-up to the "Mathematics Teaching in the 21st Century" (MT21) study in multiple countries, including the United States (Schmidt et al. 2011). In the U.S., nine hundred future lower secondary teachers in their final year of preparation at eighty-one postsecondary schools completed a questionnaire about their personal backgrounds, pre-collegiate educational experiences (including SAT math scores[3]), the types of coursework and field experience they received at their preparatory institution, and their beliefs about and attitudes towards teaching mathematics. They also took the lower secondary version of the Future Teacher Mathematical Content Knowledge (MCK) test assessing their knowledge of mathematical topics on key domains. Additional surveys were directed towards institutional administrators and faculty to glean information about the course curriculum and minimum requirements, among other items.

Although the precise metric employed in the Barron's Selector Rating is privileged information, it is quite likely that the SAT scores of incoming freshmen are an important component. In Fig. 1 we present a scatterplot with the standardized SAT score on the x-axis and the 2 through 10 Barron's rating on the y-axis ("special" schools coded 1 are excluded from our analysis because they do not prepare teachers), with each plot representing the mean of each institution.

[3]Some institutions reported only ACT scores, which were transformed into equivalent SAT scores.

Fig. 1 Barron's selector rating vs. SAT

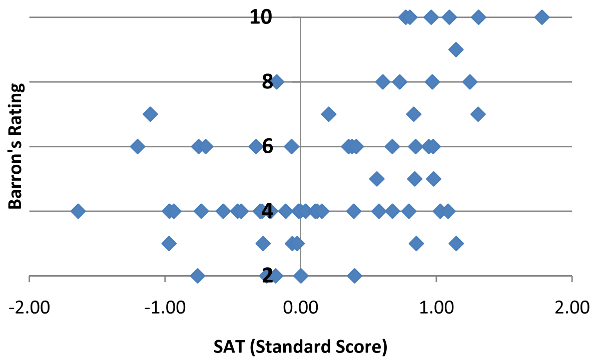
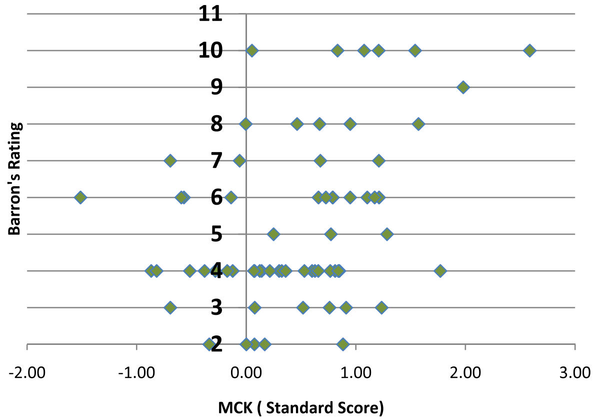


Fig. 2 Barron's selector rating vs. MCK



This spatial representation confirms that there is a relationship between the selectivity of the school according to the Barron's ranking and the quality of the student population, at least among those who intend to become lower secondary mathematics teachers. The relationship is a modest one, however, with a bivariate correlation of 0.48, which could be because the Barron's ranking presumably considers the quality of the entire student body, not just future teachers. It is somewhat surprising that the relationship between institutional ranking and incoming SAT scores isn't higher, given how fierce the competition is for slots at elite colleges.[4]

There is also a relationship between the Barron's ranking and the quality of the student body exiting postsecondary institutions, as measured by the mathematical content knowledge (MCK) of future teachers who are close to graduating from lower secondary teacher preparation programs. This relationship is presented in Fig. 2, with a modest correlation between institutional ranking and performance on the MCK test of 0.40. On the surface these results might lead one to believe that more

[4]One possibility is that those entering teacher preparation programs in less competitive schools have higher SAT scores relative to their institution, while those preparing to become teachers at competitive schools have lower relative SAT scores.

selective schools do in fact produce better-trained future teachers. However, because there is every reason to expect that students who perform well on the SAT would also receive a good score on the MCK, the apparently superior performance of students from prestigious schools might therefore be a case of selection bias.

3 Selectivity and Program Quality

A more accurate measure of the educational rigor of a given program can be found by comparing the performance of an institution's students on the SAT and the TEDS-M MCK. SAT mathematics scores serve as the measure of a student's preparation in mathematics before they enter a teacher preparation program, while the MCK score is a measure of a student's knowledge near the completion of that program. If an institution's students perform relatively better on the MCK than on the SAT, then we can infer that the teacher preparation program added a something to its future teachers' store of mathematical knowledge. However, if student inputs (SAT) and outputs (MCK) are equivalent, then we should question how much added value that program provides.

Figure 3 relates student performance on the MCK to scores on the math portion of the SAT, with both measures standardized so that they are on the same scale. The plots themselves are displayed in nine different shapes representing the nine different levels on the Barron's selectivity scale. Open shapes indicate more selective schools and closed shapes less selective schools. Since both the SAT and MCK are measures of mathematical knowledge, there is a strong correlation between the two (0.78)—students who do well on one test tend to do well on the other test. The forty-five degree line in the center of the figure represents an exact correspondence in the performance of an institution's students on the standardized SAT and MCK—in other words, that on average students demonstrated no *relative* gains in mathematical knowledge during their teacher preparation program. The further above the line an institution is, the greater the relative average learning gains for its students.

As is clear from the graph, the average student at most of the eighty-one institutions in the sample registered learning gains in MCK during their college years.[5] However, there is no clear relationship between the selectivity of the institution and relative improvement on MCK assessments. There are a substantial number of institutions ranked low on the Barron's index which nevertheless are located above the line, and some of the more selective institutions fall below the line, indicating that their students actually performed less well relative to comparable students at other schools after four years of college education.

[5]Part of this increase may be due to differing sample populations. The SAT is normed on the U.S. population, while the MCK is normed internationally. As the U.S. average on the MCK is slightly higher than the average SAT math score, part of the difference in performance between the two exams is based on that higher average. This likely contributes only to a modest proportion of the overall score gain, however.

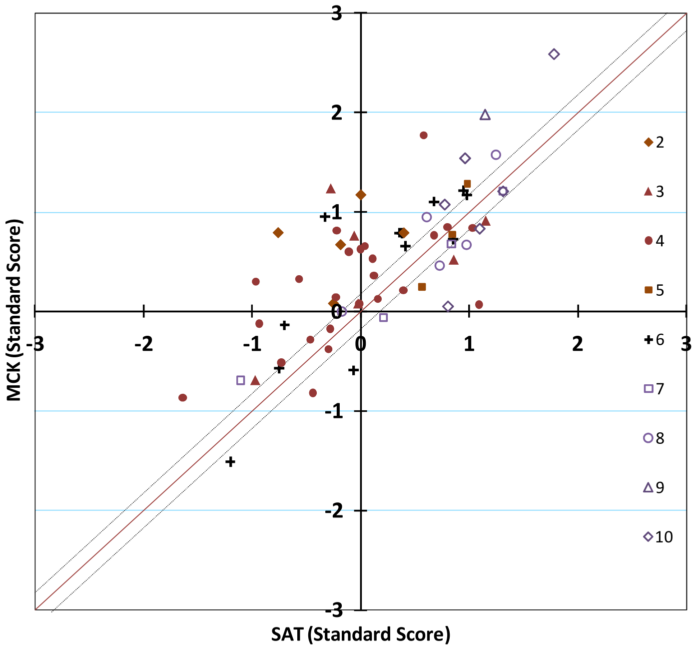


Fig. 3 SAT vs. MCK by Barron's ranking

The quadrant in which the institution is plotted also reveals information about the quality of the students attending each type of college or university. Those institutions in the top right quadrant recruit high-quality students who also do well on the MCK, with those above the line posting increases in relative performance. Although the highest-performing institutions are relatively selective, there are nearly as many lower-ranked schools that also recruit bright students who exhibiting greater relative performance on the MCK.

Given their selectivity, why aren't the average SAT scores at selective schools not all grouped in the upper right quadrant? And why isn't there a stronger relationship between selectivity and SAT scores? Although we can only speculate, it is possible that, of those interested in becoming teachers, the "best and brightest" students as measured by SAT scores are not attracted to more selective schools. Given their great expense, students instead choose to attend less selective state public schools. State schools are designed with state teaching credentials in mind, which might not be the case for private schools.

Additionally, the salary for teachers is low compared with that in other professions that have comparable educations. As a consequence it might make little sense from a strict cost-benefit perspective to obtain a very expensive degree from a private institution. As this study is focused only on future lower secondary mathematics teachers, the results should not be analogized to other disciplines, or to any class of universities as a whole. There is every possibility that the Barron's rankings are more

Table 2 Quality indicator by simplified Barron's selector categories

	Level 1: Less Selective	Level 2: Selective	Level 3: Very Selective	Level 4: Most Selective
Above Quality Baseline	16	41	32	11
On/Below Quality Baseline	19	39	32	10

(Cells indicate number of institutions in each category)

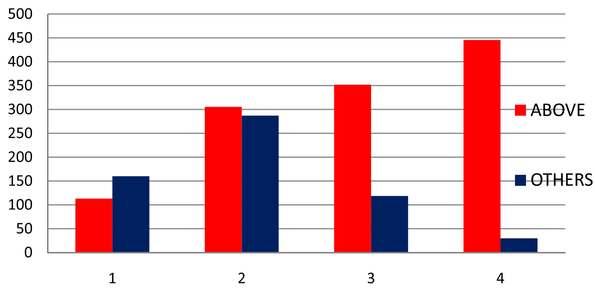


Fig. 4 Required course hours in mathematics by Barron's selector categories

strongly correlated to SAT scores for the whole student body, or that different programs at elite schools might demonstrate larger increases in relative performance.

Because of the small number of institutions in the sample and to reduce the number of interaction terms, we collapsed the competitiveness ratings into four broader categories: Level 4 Most Selective (Most Competitive and Highly Competitive +), Level 3 Very Selective (Highly Competitive, Very Competitive +, and Very Competitive), Level 2 Selective (Competitive + and Competitive), and Level 1 Less Selective (Less Competitive and Non-Competitive). Table 2 presents a simplified representation of Fig. 3 indicating the percentage of institutions in each category, with "above" referring to institutions whose average performance is greater than the baseline, which suggests that students attending those institutions have increased their relative knowledge of mathematics, and "on/below" referring to those institutions whose students have experienced no increase or have even fallen behind their peers at other institutions.

If we define a "high-quality" teacher preparation program as one whose students score more highly than their entrance examination scores would predict (and "low-quality" as those whose relative performance is lower), then it becomes clear that there is *very little relationship between the college selectivity and program quality*, at least in the field of lower secondary mathematics education. At each level of selectivity, a roughly equal proportion of institutions are above the baseline as on or below it.

An analysis of institutional data drawn from the US-TEDS study suggests that there are important differences between "high-quality" and "low-quality" teacher education programs, independent from institutional selectivity. In previous work (Schmidt et al. 2002, 2011), we have argued that a rigorous mathematics curriculum is essential for learning mathematics for both students *and* future teachers. As indicated in Fig. 4, high-quality teacher education programs—those whose stu-

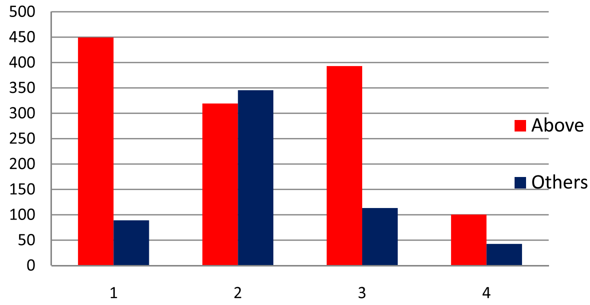


Fig. 5 Required course hours in general studies by Barron's selector categories

dents MCK scores are much greater than their SAT math scores—share a common emphasis on mathematics coursework. Although the proportion of required math courses increases with college selectivity, the difference between high-quality and low-quality selective institutions in the number of math courses is particularly pronounced. The main dividing line in program quality is *not* school selectivity. There also appears to be a tendency for higher-quality but low selectivity institutions to focus more on general studies course requirements, while high-quality and high-selectivity institutions emphasize mathematics classes (see Fig. 5).

The last stage of our examination of the relationship between college selectivity and college quality in lower secondary mathematics teacher preparation programs is a more rigorous statistical analysis using multi-level modeling techniques. The data is drawn from the US-TEDS institutional and student survey and includes both individual level variables measuring previous academic coursework (high school mathematics coursework and score on the SAT) and postsecondary training, as well as program-level indicators of course requirements, selectivity, and mean SAT scores (see Table 3). The dependent variable is the student's score on the MCK. The interaction terms are formed from two categorical variables: the simplified Barron's ranking of selectivity and whether the teacher preparation program was classified above or on/below the line of equal input and output.

High-selectivity institutions whose students perform above the baseline serve as the reference category. If selectivity is a proxy for quality, then we should expect school selectivity to relate to individual-level achievement (although it is clear from Fig. 3 that not all selective schools are "high-quality" in terms of improved relative performance). Alternatively, a strong impact from specific institutional features (such as coursework requirements) or average school outcomes (defined as being above or below the baseline) would imply that the design of the teacher program is related to student knowledge gains, i.e. that curriculum matters. Finally, stronger effects due to student characteristics (the average SAT score) would suggest that students who know relatively more entering teacher preparation programs gain relatively more knowledge by the time they graduate.

The results of the multi-level model are presented in Table 3. Predictably, individual student background is strongly associated with performance on the MCK, with students taking more advanced high school mathematics courses and with higher SAT scores earning better MCK results. This is an individual level relationship

Table 3 Predicting MCK by selection & course indicators

Source	Est	(se)	$p<$
Intercept	193.54	57.28	0.001
Future Teacher Level[a]			
Highest Mathematics Course Taken in High School	11.87	1.97	<0.0001
College Entrance Mathematics Score	0.14	0.02	<0.0001
Advanced Mathematics OTL Exposure	5.31	1.27	<0.0001
Program Level			
College Entrance Mathematics Score	0.53	0.09	<0.0001
Number of Required Math Course Hours	0.04	0.01	0.0119
Advanced Mathematics OTL Exposure	12.63	4.82	0.0101
Less Selective & At/Below Baseline	−41.60	17.54	0.0196
Less Selective & Above Baseline	−10.32	18.15	0.5711
Selective & At/Below Baseline	−43.83	15.48	0.0056
Selective & Above Baseline	−21.61	16.94	0.205
Very Selective & At/Below Baseline	−49.27	14.72	0.0011
Very Selective & Above Baseline	−30.15	15.47	0.054
Most Selective & At/Below Baseline	−47.73	19.5	0.0161
Most Selective & Above Baseline	Reference category		

[a]Variables centered on program means

which holds across all institutions. Most relevant for the present study, the only variables that fail to achieve statistical significance are the interactive terms for selective, high quality institutions (those that are on average above the baseline). Students attending institutions whose future teachers on average experienced relative knowledge gains in mathematics did no better if they went to a selective school, controlling for other factors. Similarly, future teachers at low-quality institutions did about the same no matter how selective their institution, performing forty to fifty points worse on the MCK than those at high-quality, high-selectivity schools, again controlling for other factors.

Each of the other institutional characteristics had a statistically significant and positive relation with student MCK scores. Students at institutions with higher average SAT scores tended to receive a higher MCK score. The curriculum of the teacher preparation program was also associated with better outcomes on the MCK, with both required number of course hours in mathematics and the average hours in advanced mathematics courses taken by future teachers demonstrating a statistically significant relationship. Students with more coursework also tended to have higher scores.

One key outcome of our analysis is that institutional and program-level factors appear to play a greater role than individual-level characteristics. Although they are statistically significant, the coefficients for student-level SAT performance and number of advanced math courses are substantially smaller than the coefficients representing the institutional mean for SAT or advanced math coursework. The association with MCK scores was more than twice as great for the institutional average in the number of advanced courses than individual student behavior (12.63 vs. 5.31), while coefficients for mean SAT scores were more than triple that of individual student scores on the math section of the SAT (0.54 vs. 0.14). We can only speculate precisely how institutional average SAT scores influence individual MCK scores, for example through peer effects or the ability to devote more time and resources to advanced preparation. More clear is that curriculum and institutional design is strongly related to how well future teachers are grounded in mathematics.

4 Conclusions

The top schools in the U.S. hold their status for a reason, not least of which is that students and faculty compete fiercely to become part of them. However, rankings such as those generated by U.S. News & World Report or Barron's tell us more about the reputations of those schools than about their ability to deliver a high-quality education. Any easy attributions about the worth of a postsecondary institution are confounded by the problem of selection bias.

In the instance of teacher education this problem is particularly severe, with consequences both for prospective teachers and for the educational system as a whole. Starting teachers in the United States make approximately 80 % of per capita income, much less than similarly educated professions. Attending an expensive private institution makes little sense unless it comes with a commensurate reward. Our study suggests that in many cases those interested in teaching mathematics might be better served by attending less expensive, less exclusive public institutions. More importantly, the greater importance placed upon teacher quality by policymakers makes it imperative to identify the best models of teacher preparation, most especially in mathematics. Our analysis makes it clear that we will not be able to find those models simply by examining the course requirements at Ivy League institutions. We will need to do the careful work of examining a broad range of institutions, with close attention paid not to what sort of students attend a program but what sort of teachers they are when they graduate.

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Part VI
Conclusions: What We Have Learned and Future Challenges

Learning from the Eastern and the Western Debate—The Case of Mathematics Teacher Education

Gabriele Kaiser and Sigrid Blömeke

Abstract Comparative studies have gained significant influence in the last decades, and school systems of many countries have been revised referring to better results of other countries in international large-scale assessments. Authors of such studies commonly link their interpretations of the results to distinctions between "Eastern" and "Western" cultures, in particular with respect to the consistent and continuing outstanding performance of East Asian learners compared with their Western counterparts. One question is whether the same achievement pattern holds for future teachers and whether similar cultural difference may cause it. IEA's "Teacher Education and Development Study in Mathematics" (TEDS-M) was the first comparative study that focused on the outcomes of teacher education with standardised testing. In this paper—based on the TEDS-M results—commonalities and differences in the achievement of future teachers from Eastern and Western countries are explored and related to a cultural perspective. Cultural differences between Eastern and Western approaches concerning mathematics, mathematics education and mathematics teachers are analysed with respect to the achievement pattern. The paper closes with reflections on possible consequences concerning the development of teachers' knowledge and teachers' expertise in mathematics education.

Keywords Culture in education · Cultural differences · Mathematics learning · Mathematics teachers · Effectiveness of teacher education · International study · Comparative study · Teacher expertise · East Asia

Reprinted version of a paper published in ZDM—The International Journal of Mathematics Education, 2013, 45(1), 7–19.

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S. Blömeke et al. (eds.), *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn*, Advances in Mathematics Education,
DOI 10.1007/978-94-007-6437-8_24,

1 Introduction: Cultural Differences and Their Influence on Education

Comparative studies have gained significant influence in the last decades and have influenced reforms of educational systems around the world. School systems of many countries have been revised referring to better results of other countries in international large-scale assessments. Authors of such studies commonly link their interpretations of the results to distinctions between "Eastern" and "Western" cultures, in particular with respect to the consistent and continuing outstanding performance of East Asian learners compared to their Western counterparts. According to this work, mathematics education in "Eastern" and "Western" cultures can be characterised by sharp distinctions, amongst others the acquisition of basic knowledge (East) versus creativity (West).

One question is whether, in fact, there exists such a joint characterisation of students from East Asian countries, who are assumed to represent what is called the "Eastern" culture, as distinct from a joint characterisation of students from European or English-speaking countries, assumed to represent what is called the "Western" culture.

Leung (2001) described in his "search of an East Asian identity in mathematics education" important differences between the East Asian and the Western traditions in mathematics education using strong dichotomies. Firstly, he distinguished between "product (content) versus process". According to him, in East Asian mathematics classrooms mathematics content and procedures or skills are emphasised putting basic knowledge and basic skills in the foreground, whereas Western education in the last decades has tended to focus more on the process of doing mathematics. Secondly, Leung (2001) distinguished between rote learning versus meaningful learning, with rote learning and memorisation as a legitimate and necessary way of learning, contributing to a better understanding, as seen in East Asian countries. In contrast, Western cultures emphasised the necessity of understanding the phenomenon before it can be memorised and internalised. Studying hard versus pleasurable learning was presented as the third dichotomy. It refers to traditional views in East Asian countries that studying is a serious endeavour relying on hard work and perseverance, in contrast to many Western views, which put the child in the middle of the learning process, and who has to enjoy the meaningful learning process.

The fourth dichotomy presented by Leung (2001), "extrinsic versus intrinsic motivation", described that on the motivational level Western educators value intrinsic motivation in learning mathematics more than extrinsic motivation. In contrast, their Eastern counterparts emphasise the necessity of extrinsic motivation as complementary to intrinsic motivation, reflecting the high relevance of high-stake tests. The fifth dichotomy corresponded to a different understanding of the nature and the role of the teacher, which is based on social orientations in East Asian countries. Whole-class teaching with the teacher as the role model is regarded as highly important in East Asian countries, in contrast to the stronger focus on individualised learning in Western countries, stressing the independence and individualism within learning. The sixth dichotomy developed by Leung (2001) referred again to a different understanding of the role of the teacher, namely as a scholar with profound subject-matter knowledge in East Asian countries compared to the teacher as a facilitator with profound pedagogical competencies in the West.

Summarising, Leung (2001) sees the core differences between East Asian and Western views on mathematics education in different views on "who or what the centre in the teaching and learning process should be" (p. 47)—student-centred education in Western approaches, in contrast to a tripartite emphasis on the student, the teacher and the subject matter in East Asian cultures. Leung (2001) hypothesized that this tripartite description of teaching and learning might be the essence of an East Asian identity, which is in line with other approaches, for example with the concept of learning in the Confucian Heritage Culture (CHC). Wong (2004) described as a central feature of learning based on the CHC its orientation towards social or collective achievement, in contrast to an orientation towards individual achievement in Western cultures—including an emphasis on diligence, an attribution of success to effort, a competitive spirit, and a high relevance of practice (Wong 2004).

Although Confucianism seems to have a strong influence on education, it has to be taken into account that there is no direct connection or causal relationship between schools of thought and social phenomena such as high achievements of students in mathematics, as Wong et al. (2012) pointed out. Nevertheless, there seems to be some consensus that a kind of joint identity of East Asian learning traditions exists. Whether there is a common core of Western educational traditions seems to be more controversial.

The ICMI Study "Mathematics Education in Different Cultural Traditions—A Comparative Study of East Asia and the West" (Leung et al. 2006) contrasted the "Chinese/Confucian tradition" with the "Greek/Latin/Christian tradition". Recent studies on European traditions in pedagogy emphasise its diversity. In particular, a difference between approaches coming from the United Kingdom on the one hand and from the Scandinavian countries and Continental Europe, including Belgium and the Netherlands, on the other hand is stressed, with the first approach characterised by pragmatically oriented ways of teaching and learning and the second approach sharing a joint didactics tradition (Hudson and Meyer 2011).

Such a didactics tradition is virtually unknown in the English-speaking world and is distinct from curriculum traditions prominent there (Blömeke and Paine 2008). As Hudson and Meyer (2011) pointed out, the historical origin of present-day didactics dates back to Jan Amos Comenius and his work *Didactica Magna* (Great didactic), developed in the 17th century within the framework of the Age of Enlightenment, in which he claimed to teach everything thoroughly to everybody, emphasising the necessity of carefully laid out teaching sequences based on general principles.

Referring specifically to mathematics education, several studies have pointed out that there are relevant differences in the teaching and learning traditions between Anglo-Saxon and Continental European traditions concerning the kind of mathematical knowledge to be acquired, the role of argumentation and proof and the kind of interactional activities (Kaiser 2002; Kaiser et al. 2006; Pepin 1999). However, focusing on the differences between East Asian cultures and Western cultures as elaborated above, the Western approaches seem to have more in common with each other than with the Eastern approach, such as in putting the individual in the foreground. This means that, in fact, we can assume a joint identity of students from East Asian countries distinct from a joint identity of students from European or English-speaking countries.

This distinction can be explained with theories from cultural psychology or sociology, amongst others the famous cultural-psychological framework developed by Hofstede (1986, 2001). Departing from a definition of culture as the "shared motives, values, beliefs, identities, and interpretations or meanings of significant events that result from common experiences of members of collectives that are transmitted across generations" (House et al. 2004, p. 15), Hofstede (1986) concluded that through socialisation processes a country's culture has an impact on the preferred modes of learning. Within the framework of Hofstede (1986), the collectivism-individualism dimension refers to the extent to which the individuals of a society are perceived as autonomous. This criterion seems to be particularly relevant in explaining differences between East Asian and Western teaching and learning processes. We will elaborate details of this framework within the interpretation of TEDS-M results in Sect. 3.3.

Turning to the achievement level, it is striking that all international comparative studies of the last decades, such as TIMSS, originally implemented by the International Association for the Evaluation of Educational Achievement (IEA) in 1995, or the OECD PISA studies, carried out since 2000 in three-year cycles, reveal a similar picture, namely the students from the five East Asian countries outperforming all students from Western countries with a substantial gap in average mathematics achievement between these five Asian countries and the next group of countries.

For example, in TIMSS 2011, Singapore, South Korea and Hong Kong, followed by Chinese Taipei and Japan, were the top-performing countries at fourth grade followed after a great gap by Northern Ireland and then after another gap Belgium, Finland, England, the Russian Federation, the USA and the Netherlands. Similarly, at eighth grade, South Korea, Singapore and Chinese Taipei outperformed all countries, followed by Hong Kong and Japan; after a huge gap Russia was listed followed by Israel, Finland and the USA (Mullis et al. 2012). PISA 2009, in which 65 countries participated, showed similar results with the students from Shanghai-China, Singapore, Hong Kong, South Korea and Chinese Taipei achieving the best results in mathematics followed by students from Finland, Switzerland, Japan, Canada, the Netherlands, New Zealand and Australia. German students performed above the international mean and the USA students below (OECD 2010, 130ff).

This well-documented outstanding mathematics achievement of East Asian students compared to their Western counterparts has been studied in detail with case studies and complementary studies accompanying TIMSS in 1995. Based on data from these studies, Kawanaka et al. (1999) stated: "Although there probably are many ideas in the Japanese videos that could prove useful in the classrooms in other countries, systems of teaching are not easily transported from one culture into another" (p. 103). Following this statement, the question emerges: What, in fact, can we learn from international comparative studies then? In particular: Can the Eastern and the Western traditions of mathematics education learn from each other? In order to answer these questions we analyse in the following the intentions and the development of comparative studies in the past. Afterwards we will use the "Teacher Education and Development Study in Mathematics" (TEDS-M) as an example of an international comparative study, in which we discuss the differences between Eastern and Western traditions from a cultural perspective.

2 Intentions and Historical Development of Comparative Education

An overall consensus exists that international comparative studies provide insight into other educational systems and support a better understanding of one's own educational system (Blömeke and Paine 2008). Stigler and Perry (1988) emphasised such a potential of comparative studies: "Cross cultural comparison also leads researchers and educators to a more explicit understanding of their own implicit theories about how children learn mathematics. Without comparison, we tend not to question our own traditional teaching practices and we may not even be aware of the choices we have made in constructing the educational process" (p. 199).

However, it is difficult to compare educational systems based on different cultures, different philosophical traditions, different values and other different characteristics. This difficulty is reflected in two famous characterisations of comparative education. Thut and Adams (1964) described comparative education as indispensable: "To study education well is to study it comparatively" (Back Cover). Husén—a founding member of the IEA and chair of the First International Mathematics Study (FIMS)—specified in contrast the limitations of comparative education: "Comparing the outcomes of learning in different countries is in several respects an exercise in comparing the incomparable" (1983, p. 455).

Bringing these problems together, Postlethwaite (1988) defined in his seminal work in the *Encyclopedia of Comparative Education and National Systems of Education* what comparative education actually means: "Strictly speaking, to 'compare' means to examine two or more entities by putting them side by side and looking for similarities and differences among them. In the field of education, this can apply both to comparisons between and comparisons within systems of education" (p. xvii). Comparative education in Postlethwaite's perspective would have to focus on similarities *and* differences between *and* within educational systems, to seek for patterns in the differences or similarities, which then allow deeper insights into the various systems.

Postlethwaite pointed out that such an understanding of comparative studies on education has a long history in Europe, going back as far as the ancient times of Greeks or Romans, and in medieval times to Marco Polo's travel to China or Alexis de Tocqueville's work. For Europe the studies by Sir Michael Sadler are of special importance. Sadler visited the Prussian folk school system at the beginning of the 20th century together with a British expert commission and compared it with the British educational system. In his ground-breaking article "How Far Can We Learn Anything of Practical Value from the Study of Foreign Systems of Education" (Sadler 1964, originally published 1900), Sadler analysed the gap between the educational systems of continental Europe and England. He described the high achievement of the German educational system, a decisive factor in which was, for Sadler, the strong national interest in education within Germany. He therefore proposed to send future teachers at the end of their study to Germany in order "to study ... its methods of teaching and system of education" (Sadler 1964, p. 310).

However, Sadler was also sceptical of some aspects of the German school system and to what extent its characteristics could be transferred to England. He stated that it was a common misbelief "that all other nations have better systems of education than we have. It is a great mistake to think, or imply, that one kind of education suits every nation alike" (Sadler 1964, p. 312). He therefore recommended enhancing the English school system by accepting the good aspects of the English system and by learning from the continental European school systems. He formulated the following caveat against simply transferring single components or measures of foreign educational systems into one's own system, often still quoted today:

> In studying foreign systems of Education we should not forget that the things outside the schools matter even more than the things inside the schools, and govern and interpret the things inside. We cannot wander at pleasure among the educational systems of the world, like a child strolling through a garden and pick off a flower from one bush and some leaves from another, and then expect that if we stick what we have gathered into the soil at home, we shall have a living plant. A national system of education is a living thing, the outcome of forgotten struggles and 'of battles long ago'. It has in it some of the secret workings of national life. (p. 310)

In the aftermath, researchers attempted to identify those factors influencing the development of school systems using methods from social sciences. The limitations of comparative education however lie, as Hilker (1962) pointed out, in the missing normative potential of these studies, which cannot create the norms of education and an educational philosophy out of itself. Which actions to take based on the *tertium comparationis*, which is needed as a benchmark for the initial objective, can only be decided outside comparative education.

Summarising these different issues it becomes clear that, on the one hand, comparative education is looking for general patterns and mutual understanding of various educational systems. On the other hand it is obvious that comparisons cannot result in far-reaching recommendations for the change of educational systems. The cultural dependency of comparative education presents simultaneously an opportunity and a problem, though as Alexander (1999) phrases it:

> I argue that the educational activity which we call pedagogy—the purposive mix of educational values and principles in action of planning, content, strategy and technique, of learning, and assessment, and of relationships both instrumental and affective—is a window on the culture of which it is a part,

> and on that culture's underlying tensions and contradictions as well as its publicly-declared educational policies and purposes. Second, like many others these days I argue that the comparative perspective is an important and necessary part of the quest to understand and improve the science, art or craft of teaching, and to enable us to distinguish those aspects of teaching which are generic and cross international boundaries from those which are culture-specific. (p. 149)

Returning to the current international debate on mathematics education, we argue that despite the rich database created in the many international comparative studies, the core questions of these studies' relevance and potential consequences still remain unresolved. Alexander (1999, p. 158) called this the "the 'so what?' problem in educational research", calling for cultural sensitive studies with practical insight.

We intend to offer at least some first answers to these questions. Based on TEDS-M, we analyse cultural influences on the teacher's role and function, especially with respect to teacher knowledge. Is the gap in K-12 student achievement valid for future or practising teachers as well? Does, in fact, a vicious cycle of competent students and competent teachers exist, as Leung and Park (2002) describe it on the basis of case studies? We discuss then what the East and the West potentially can learn from each other.

3 TEDS-M: An International Comparative Study on Teacher Education

Criticism about the inefficiency of teacher education has long been voiced in many Western countries. Teacher education has been described as a weak intervention compared to one's own school experience and later professional socialisation (Richardson 1996). Particularly referring to mathematics teacher education, Klein (1905) criticised more than 100 years ago in his famous metaphor of a "double discontinuity" the lack of impact of university education on teaching practice in school. Such criticisms of teacher education stimulated the implementation of a study about the effectiveness of teacher education carried out under the auspices of the IEA, TEDS-M, whose results were released in 2010 (see in particular Blömeke et al. 2011; Tatto et al. 2012; various papers in *ZDM* 2012, 44(3) and all papers in Blömeke et al. 2014).

3.1 Background and Theoretical Framework of TEDS-M

TEDS-M comprised a primary study and a lower-secondary study with 15 countries participating in each study, covering Eastern and Western countries. The focus of TEDS-M was future teachers in their final year of teacher education who would receive a licence to teach mathematics in either grades 1 through 4 (primary study) or grade 8 (lower-secondary study). The two studies were based on nationally representative samples and had to follow the rigorous IEA quality control mechanisms of sampling, data collection, coding and data analysis. The research questions of TEDS-M were multi-layered, namely:

1. What are the professional competencies of future mathematics teachers?
2. How distinctive are the institutional conditions of mathematics teacher education?
3. What are the national conditions of mathematics teacher education?

In this paper we concentrate on the first research question, the professional competencies of future teachers. According to Weinert (2001), professional competencies can be divided into cognitive facets (in our context, teachers' professional knowledge) and affective-motivational facets (in our context, professional beliefs).

The professional knowledge of teachers can again be divided into several facets. Referring to the seminal work by Shulman (1986), the following facets were distinguished in TEDS-M: mathematics content knowledge (MCK), mathematics pedagogical content knowledge (MPCK) including curricular knowledge, and general pedagogical knowledge (GPK). These facets of professional knowledge were further differentiated: MCK covered the main mathematical areas relevant for future teachers, while MPCK covered curricular knowledge, knowledge of lesson planning and interactive knowledge applied to teaching situations (see Fig. 1). The framework has similarities to models of professional knowledge developed in other studies (see Blömeke and Delaney 2012 for a detailed overview).

TEDS-M also examined the professional beliefs held by the future teachers, due to the fact that beliefs are crucial for the perception of classroom situations and for decisions on how to act, as Schoenfeld (2011) pointed out. Based on Richardson (1996), beliefs can be defined as stable, psychologically held propositions of the world around us, which are accepted to be true. In TEDS-M, several belief facets were distinguished, in particular epistemological beliefs about the nature of mathematics and beliefs about the teaching and learning of mathematics (Thompson 1992).

TEDS-M examined mathematics teacher education using a broad range of instruments. Due to our focus on future teachers, we restrict ourselves to the survey that covered the background of the future teachers, their opportunities to learn in teacher education, their knowledge of mathematics, mathematics pedagogy and general pedagogy, their beliefs on mathematics, the teaching and learning of mathematics, and schooling.

3.2 Selected Results from TEDS-M

Due to space restrictions, we present only selected results of the primary study (for details of the lower-secondary study see Blömeke et al. 2010; and in particular Blömeke et al. 2014). The evaluation of the future primary teachers' achievement

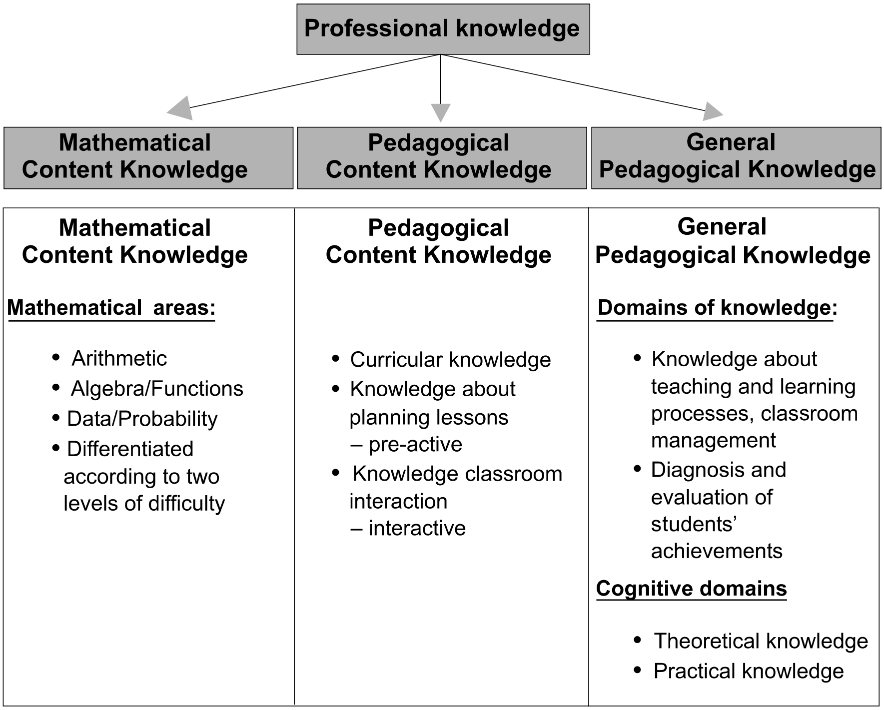


Fig. 1 TEDS-M model of professional knowledge (Tatto et al. 2008)

revealed huge differences between the participating countries concerning both MCK and MPCK. The participants from Taiwan and Singapore showed the highest performance, which was significantly distinct from the performance of the other participating countries. The future teachers from the USA and Germany were hardly above the international mean of 500 points, the difference of these from the achievement of future teachers from Taiwan and Singapore being about one standard deviation. The achievement of future teachers from the USA and Germany was not only lower than that of the future East Asian teachers, it was also significantly lower than the achievement of future teachers from Switzerland. Relating the achievement to the Human Development Index, we can point out that future teachers from Russia and Thailand showed a surprisingly good performance (Table 1).

Concerning MPCK, the performance pattern was quite similar: the future primary teachers from Singapore and Taiwan achieved much higher test results than the future teachers from the other countries. The German students' attainments were around the international mean, with their difference from Singapore and Taiwan again being about one standard deviation. In addition, the MPCK results from the German students were significantly lower than those from Switzerland, the USA and Norway.

A comparison of the relative strengths and weaknesses in MPCK and MCK highlights interesting differences. Country-specific profiles emerge which may reflect

Table 1 MCK and MPCK of future primary teachers by country

Mathematics Content Knowledge of Future Primary Teachers		Mathematics Pedagogical Content Knowledge of Future Primary Teachers	
Country	Mean (S.E.)	Country	Mean (S.E.)
Taiwan	623 (4.2)	Singapore	593 (3.4)
Singapore	590 (3.1)	Taiwan	592 (2.3)
Switzerland[a]	543 (1.9)	Norway[d,f]	545 (2.4)
Russia	535 (9.9)	USA[c,d,e]	544 (2.5)
Thailand	528 (2.3)	Switzerland[a]	537 (1.6)
Norway[d]	519 (2.6)	Russia	512 (8.1)
USA[c,d,e]	518 (4.1)	Thailand	506 (2.3)
Germany	510 (2.7)	Malaysia	503 (3.1)
International	500 (1.2)	Germany	502 (4.0)
Poland[b,d]	490 (2.2)	International	500 (1.3)
Malaysia	488 (1.8)	Spain	492 (2.2)
Spain	481 (2.6)	Poland[b,d]	478 (1.8)
Botswana	441 (5.9)	Philippines	457 (9.7)
Philippines	440 (7.7)	Botswana	448 (8.8)
Chile[d]	413 (2.1)	Chile[d]	425 (3.7)
Georgia	345 (3.9)	Georgia	345 (4.9)

[a]Colleges of Education in German speaking regions

[b]Institutions with concurrent programs

[c]Public Universities

[d]Combined Participation Rate <75 %

[e]High proportion of missing values

[f]Results for Norway are reported by combining the data sets available in order to present a proxy of the country mean

the orientation and cultural traditions of teacher education in general and mathematics teacher education in particular. The following analyses of these orientations with mathematics teacher education will allow us to develop our argumentation of what the Eastern and the Western debate can enable us to learn from each other. Based on Fig. 2, we can identify three groups:

- Higher achievement in MCK than in MPCK—from Asia, the future teachers from Taiwan and Thailand belong to this group; from Eastern and Central Europe the future teachers from Russia, Poland, Germany and Switzerland also belong to this group.
- Higher achievement in MPCK than in MCK—several Eastern and Western countries belong to this group, namely the future teachers from Norway, the USA, Spain, Chile, Malaysia and the Philippines.

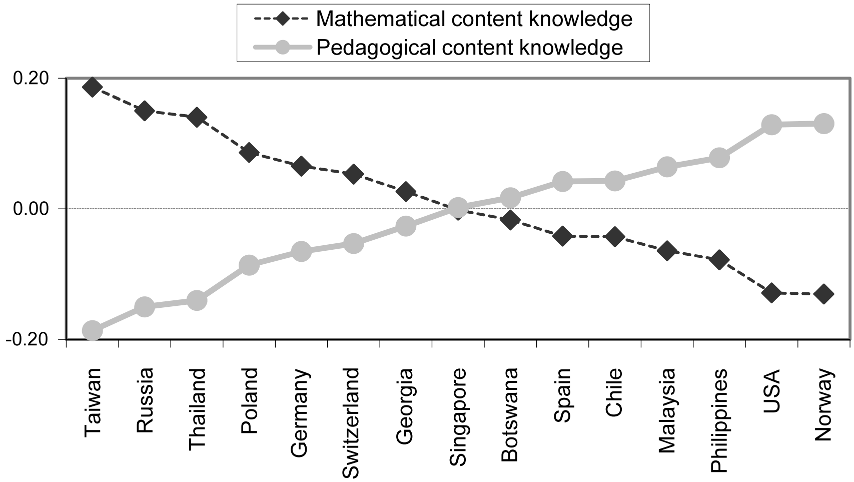


Fig. 2 Country-specific profiles of MCK and MPCK of future primary teachers

- Nearly equal level—one East Asian country, namely Singapore, and one country from the former Soviet Union, namely Georgia, belong to this group as well as the only participating African country, Botswana.

The profiles of the three groups are independent of the absolute level of achievement. Countries with higher-achieving future teachers can be found in both groups, e.g. Taiwan or Norway. The same holds for lower-achieving countries. Even the two East Asian countries belong to different groups with Singapore playing an intermediate role. The results show that apparently no "silver bullet" to high achievement in both domains exists. Neither a strong emphasis on MCK nor on MPCK seems to promote the overall achievement of the future teachers of a country.

However, a deeper analysis of the different profiles points to an influence of cultural traditions, which may have shaped the profiles. As cultural explanations, the following lines of discussion are brought forward. Influenced by the CHC tradition, the teacher is seen as an expert in many East Asian countries. The teacher is the "scholar-teacher" (Leung 2006; Li and Huang 2013) who inherits the content knowledge students need to acquire. Such a tradition leads, amongst other things, to a high importance of content knowledge in teacher education.

But content knowledge does not only play a significant role in mathematics teacher education in East Asia. In Continental Europe, content-related approaches place traditionally high priority on knowledge in the already mentioned tradition of didactics. With respect to the field of mathematics, this approach includes didactical reflections on the teaching of mathematics based on a sound and deep understanding of mathematics content as background knowledge. In the German tradition, several researchers have brought forward this position (Griesel 1974) and influenced the European tradition on didactics. The broad notion of "pedagogical content knowledge" in Continental Europe compared to English-speaking countries strongly emphasises theory-guided knowledge closely connected to the content. Didactics in this sense (Pepin 1999) is characterised by its inseparable connection of content knowledge and teaching knowledge.

Finally, Eastern European countries have historical roots linked to the German educational systems, including teacher education (Alexander 2000). Content knowledge and content-related didactics are therefore important, too. These traditions may have supported the achievement patterns displayed in Fig. 2, namely a relatively high level of MCK compared to MPCK of the future teachers from East Asian and Eastern European countries and Germany and Switzerland, which may reflect the high emphasis of content in teacher education.

The situation is quite different in Scandinavian countries, North and South America and in countries shaped by a US influence such as the Philippines (Nebres 2006). In this tradition, a so-called "progressive education" with child-centred approaches characterises K-12 education and teacher education. The child is in the foreground, whereas the content is assigned a background role. In addition, the English-speaking countries share as already mentioned a tradition of pragmatism, assigning content knowledge less importance than pragmatic reflections (Kaiser 2002). These traditions may have in turn supported the achievement patterns displayed in Fig. 2, namely a relatively high level of MPCK compared to MCK of the future teachers from Scandinavian, American and South-East Asian countries.

Such cultural influences on the results of TEDS-M cannot only be seen at the achievement level, but also in the area of the future teachers' beliefs. TEDS-M has evaluated in detail epistemological beliefs on the nature of mathematics and on the genesis of mathematical knowledge, i.e. the nature of mathematics teaching and learning (for details see Felbrich et al. 2012). Four fundamental views on mathematics were distinguished (Grigutsch et al. 1998), which can be grouped into two overarching perspectives on mathematics: a formalism-related and a scheme-related view characterise mathematics as a static science; whereas a process-related and an application-related view conceptualise mathematics as a dynamic process.

Based on relative analyses—in which the mean of all items measuring beliefs on the nature of mathematics were subtracted from the agreement to each single belief item ("ipsative score", that is, the agreement is corrected for the overall tendency of a future teacher to agree; OECD 2009)—three groups of countries can be distinguished (see Fig. 3). In the first group of countries, future teachers followed relatively strongly a dynamic orientation in their view on mathematics. These were mainly European countries, including Germany, Switzerland and Norway. Another group followed relatively strongly a static orientation. These future teachers came from South-East Asian and East European countries including Russia and Thailand. A balanced view was held by future teachers from East Asian countries including Taiwan and Singapore and Western countries including Spain and the USA.

Similar tendencies concerning beliefs on the genesis of mathematical knowledge can be identified, namely a particularly strong dominance of constructivist approaches, which were hold mainly by future teachers from Western European countries and Taiwan. In contrast, a relative dominance of transmission-oriented approaches was put forward by future teachers from Eastern European countries and

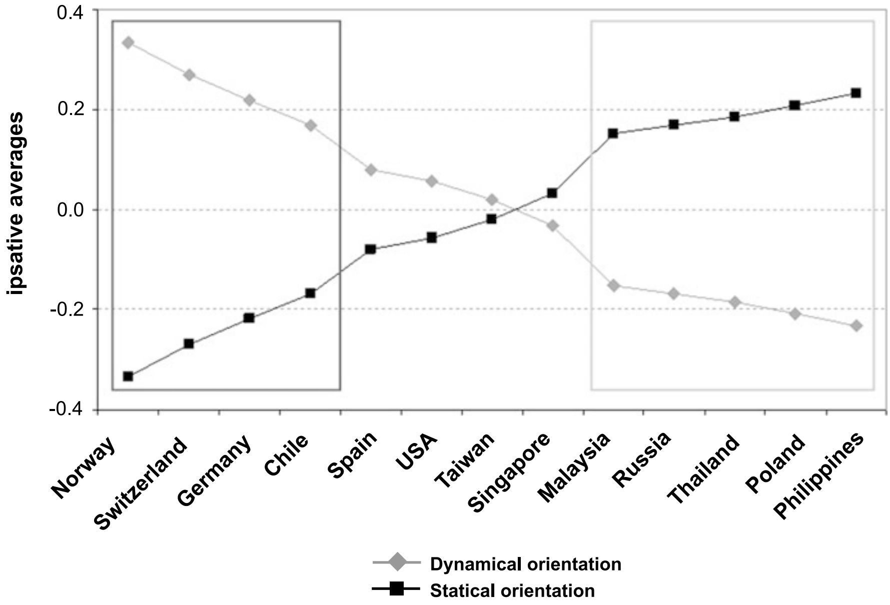


Fig. 3 Country-specific profiles of future primary teachers' beliefs on the nature of mathematics

South-East Asian countries such as Malaysia (see Fig. 4). Again, a middle group existed with countries including the USA and Spain.

3.3 First Interpretations of These TEDS-M Results

Not all of these TEDS-M results were in line with our expectations. Indeed, some of them came rather unexpectedly. The following first interpretations refer to cultural-psychological and mathematics historical approaches.

During the debate on beliefs and their influence on teaching and learning, Schoenfeld (1998) pointed out that beliefs can be understood as socially and culturally shaped mental constructs, which are acquired in educational settings with different historical traditions that vary significantly between countries. The already mentioned cultural-psychological framework of Hofstede (1986) seems to be adequate to offer explanations for the differences between countries in the beliefs on the nature of mathematics, namely the distinction between collectivism versus individualism.

The collectivism-individualism dimension refers to the extent to which the individuals of a society are seen as autonomous, and societal action is consequently seen as a result of freely negotiated contracts. Transferred to education—which is done in the work by Triandis (1995)—this means that in individualistic-oriented countries students are seen as autonomous subjects, who are not obliged to learn

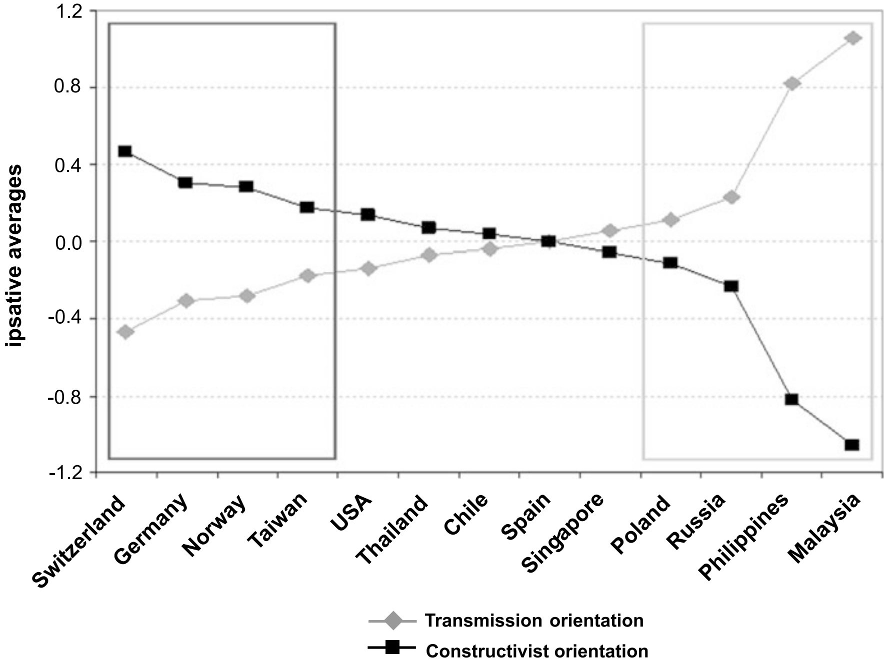


Fig. 4 Country-specific profiles of future primary teachers' beliefs on the genesis of mathematical knowledge

for familial or societal reasons. Failure in achievement is consequently attributed to context conditions inadequately addressing the individual student's needs, such as too difficult tasks, poor lessons or poor explanations by the teachers, rather than to characteristics of the learner such as lack of effort or talent. The required measures to change the situation refer to the context conditions such as improving the quality of the lessons or the school system, and seldom to obliging the individual student to put more effort into his or her learning.

In contrast, in collectivistic countries the role of social relationships in general and for the acquisition of knowledge is more prominent. In collectivistic countries, societal actions are seen as commitment towards social networks. Transferred into education, this means that student learning is seen as a commitment towards their teachers, their families and the society. Failing in school is attributed to a lack of effort, learners not engaging enough in learning processes. The required action to take is to put more effort into school because of an inner obligation.

In Hofstede's approach, additional distinctions in the teacher-student as well as in the student-student interaction exist between individualistic and collectivistic countries, which again may explain the distinction in the future teachers' beliefs. In individualistic societies, students would expect to learn how to learn and to think, whereas in collectivistic societies the focus is on passing the many high-stakes examinations. This latter approach may be connected to a more schematic view on mathematics whereas students from individualistic societies may have more chances to work on mathematical investigations.

Studies on learning styles from cross-cultural psychology substantiate these first interpretations. Several comparative studies found systematic variation in learning styles depending on the cultural background (for an overview see Yamazaki 2005). Learners from individualistic societies showed a preference for abstract conceptualisation as well as for active experimentation in learning. With respect to mathematics these learners should then also prefer a process-oriented and application-oriented and thus dynamic view, where mathematics is thought of as a process and used as a tool for problem solving. Learners from collectivistic-oriented societies on the other hand showed a preference for concrete experiences in learning as well as for reflective observation. This may predispose them to take a scheme and formalism related and thus static view of mathematics.

Finally, referring to the distinction between algorithmic mathematics as a tool for the solution of problems and dialectic mathematics as a logical science focused on the examination of the truth of statements, introduced by Henrici (1974) and based on historical studies, Siu (2009a, 2009b) describes different views on mathematics in Western and Eastern countries. He elaborates that practical-algorithmic views on mathematics prevailed in Asian countries due to their relation to the old Egyptian, Chinese and Indian mathematics; in contrast to Western countries, where dialectic-theoretical views on mathematics were dominant, influenced by their origin in the classical Greek mathematics. These differences correspond with the high importance of astronomy in old China, which has influenced the development of mathematics significantly (Martzloff 2000).

However, Siu (2009a, 2009b) also points to the fact that due to the westernisation and opening up of Asian societies to Western influences, mathematics education has also incorporated Western ideas about mathematics. Consequently, nowadays both the dynamic and the static views on mathematics, or in Henrici's terminology dialectic and algorithmic views on mathematics, are incorporated into their beliefs by Asian teachers. This corresponds with empirical findings by Leung (2006), who was able to show that teachers in Beijing more often agree with the static-algorithmic character of mathematics than teachers from London, who more often held a dynamic-heuristic view concerning mathematics. In contrast, views of teachers from Hong Kong, who are influenced by both Eastern and Western perspectives, were located in between the two groups.

To summarise, these descriptions point to an important characteristic of mathematics, namely the so-called Janus-faced or dual character of mathematics, incorporating complementarily both dynamic and static aspects. This characteristic is reflected amongst others in the theoretical approach by Sfard (1991) describing the interplay of operational and structural phases in concept development as of crucial importance. Based on these approaches, different dichotomies currently important in Eastern and Western views on mathematics, such as procedural versus conceptual knowledge and process versus object, can be integrated into a more comprehensive framework.

Large-scale research on the impact of such cultural expectations on the formation of beliefs related to the teaching and learning of mathematics in individualistic and collectivistic countries has only been developed in the last decade. The results of TALIS, the "Teaching and Learning International Survey" (OECD 2009) which examined practising teachers' epistemological beliefs on the teaching and learning of mathematics, pointed in the same direction as TEDS-M. In TALIS, the cultural patterns in beliefs were identified for the first time (OECD 2009; Vieluf and Klieme 2011). Similar results were revealed by the MT21 Study "Mathematics Teaching in the 21st Century" (Schmidt et al. 2011).

These results can be used to interpret the results of TEDS-M concerning the beliefs of future mathematics teachers on the genesis of mathematical knowledge. In individualistic countries such as Germany or Norway, constructivist principles of teaching and learning are dominant, which put the individual student into the foreground. In collectivistic countries such as Russia or the Philippines, transmission aspects are prevalent, with teachers being seen to be responsible for the transfer of knowledge to students.

4 Cultural Lenses on Teacher Education and Teacher Expertise

To answer our opening question, what the Eastern and the Western debate enables us to learn from each other, it can be hypothesized that our thinking about teachers and their knowledge is influenced by the cultural differences between Eastern and Western countries identified above. In their international overview on expertise in mathematics education, Li and Kaiser (2011) pointed out that many commonalities in the conceptions of teacher expertise exist in Eastern and Western countries. However, significant differences between Eastern and Western approaches on expertise could also be identified.

Eastern approaches put teachers' instructional practices in the foreground and specify teachers' knowledge as part of their expertise in a more holistic way. Consequently, teacher knowledge is described within the frame of teachers' instructional practices, and is not taken as a stand-alone component. Rather, it is taken as an integrated aspect of what teachers are capable of doing. Therefore there is no uniform position within the Eastern debate: different approaches are common in the various East Asian countries such as joint lesson studies, joint textbook analysis, apprenticeship practices and public lessons within the context of contests or master teachers who serve as role models (see Li and Huang 2013).

In contrast, contributions from the West likely examine and analyse teacher knowledge as an important yet stand-alone aspect of teacher expertise. Teacher expertise is regarded in an analytical way as containing different components, including knowledge, beliefs and teaching performance, which becomes obvious within the TEDS-M framework. Such differences in describing teacher expertise point to the influence of cultural orientations, distinguishing the Eastern from the Western debate.

This difference in conceptualising teacher expertise may also be linked to the unspoken difficulty of identifying expert teachers. This difficulty may pose a bigger challenge to researchers in the West than in the East, as teaching is regarded as a private practice in the West but not in the East (Kaiser and Vollstedt 2007). Thus, researchers in the West take a more theoretically driven approach to conceptualising teacher expertise, in contrast to the East, where it is possible to first identify those teachers and then to analyse their characteristics.

Other notable differences between East and West may be manifested in the various roles assigned to expert teachers. Russ et al. (2011) developed four metaphors of expertise:

- role of teachers as *diagnosticians*, describing the teachers' ability to interpret student thinking and students' problem solving strategies;
- role of teachers as *conductors*, leading the classroom discourse and establishing classroom norms for the communication about mathematical ideas;
- role of teachers as *architects*, identifying cognitively demanding tasks;
- role of teachers as *river guides*, deciding how to unfold the lesson as it progresses based on intuition and improvisation'.

This description of expertise clearly focuses on the learning process and the individual student teacher's organisation of learning processes in order to promote student learning.

A comparison with the different aspects of expertise from an Eastern perspective shows remarkable differences. Yang (2014) in his study on expert teachers in China identifies multiple roles which have to be played by an expert teacher:

- role as a *teacher* means structuring good teaching processes;
- role as a *researcher* means carrying out research on teaching and publishing papers in professional and academic journals;
- role as a *teacher educator* means serving as a mentor of non-expert teachers and promoting non-expert teachers' professional development;
- role as a *scholar* means disposing of profound knowledge in mathematics and other areas;
- role as an *examiner* means possessing the ability to pose high-quality examination problems;
- role as a *model* means to serve as an example for students and colleagues.

Similar descriptions were developed by Li et al. (2011) in their study on expert teachers. They point out that expert teachers are expected to serve as moral role models who stand for culturally valued moral characteristics and expertise for others to follow. In addition, they emphasise the necessity for an expert teacher to act as a researcher and to regularly write books and scientific papers in teacher journals published by many East Asian universities focusing on teacher education.

Taking into account the "closed-door policy" of many Western schools, the request to act within public and exemplary or teaching contests up to the national level makes the Eastern approach to defining expertise quite different from Western conceptions. In addition, the teacher promotion system, commonly practised in several East Asian countries, provides a platform for teachers to value and pursue mathematics classroom instruction excellence. Yang (2014) emphasises that in contrast

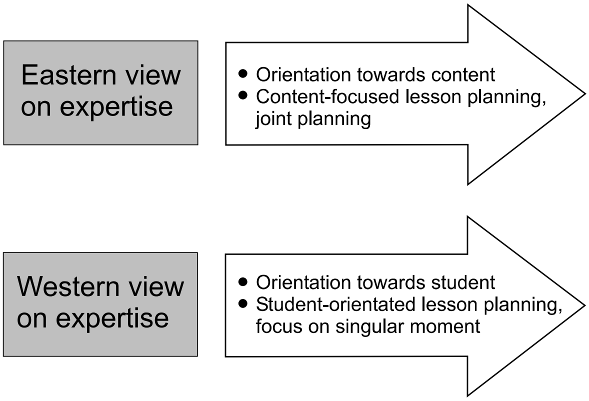


Fig. 5 Cultural perspective on teacher expertise

to Western culture, where the closed-door policy is followed, teaching in China is open for colleagues' observations, studies and discussions, mainly based in teaching research groups.

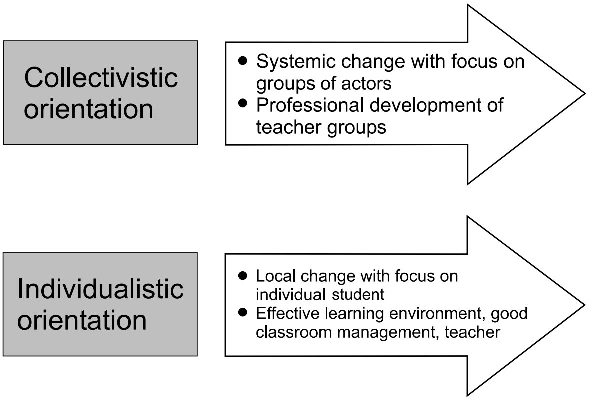
Kaiser and Li (2011) describe therefore the "Eastern perspective on teacher expertise as more holistic, aiming for a systemic change of the teaching-and-learning processes in school by strengthening teachers as researchers and developing expertise in scientific work" (p. 349). They emphasise that this holistic view is embedded in a public value of expert teachers who work not only on their teaching but also on curriculum development. According to Kaiser and Li (2011), the Western perspective is in contrast "clearly focused on the teaching-and-learning process within the classroom, where experienced teachers shall display their expertise especially in interactions with the students. Characteristic for the Western approach to expertise is the focus on the individual student, who is put into the centre of reflections and actions; the promotion of learning processes of individual students is a major goal of the classroom activities" (p. 349).

The differences in the cultural perspectives on the understanding of teacher expertise between Eastern and Western approaches are summarised in Fig. 5.

The already introduced cultural-psychological distinction of collectivistic and individualistic countries may once again allow explaining, at least partly, the differences in the understanding of teacher expertise. The strong holistic orientation on expertise in Eastern countries may be based on the collectivistic orientation of the countries, which may result in a professional development that refers to whole teacher groups, including curricular work aiming for systemic change. The individualistic orientation of Western cultures, in contrast, expects the single teacher to provide effective learning environments and good classroom management and may therefore describe expertise as a focus on individual student learning.

Cai et al. (2009) confirmed such an interpretation from the teachers' perspective on effective mathematics teaching. Based on their research work, they described Asian teachers as mathematics content-oriented, and they emphasized that an effective teacher should understand the content thoroughly and organise teaching in well-structured lessons. In contrast, teachers from the USA and Europe tended to

Fig. 6 Cultural perspectives on educational change



be more child-oriented, emphasising that an effective teacher should be passionate about mathematics, and leave enough room and time for students to develop an understanding of mathematics on their own.

These different perspectives on teacher knowledge and teacher expertise based on cultural differences of collectivism and individualism have significant consequences when it comes to possible indicators of change. Within the Eastern tradition, the focus of the indicators is at the systemic level, on groups of actors and their professional development. In contrast, indicators in the West refer to changes on the local level, putting the individual teacher and his/her ability to develop effective learning environments and manage the classroom effectively in the foreground. This characterisation is, for example, reflected in Hattie's (2012) recommendations for effective teaching based on the results of his synthesis of meta-analyses mainly from English-speaking countries.

To sum up, cultural differences concerning the description of expertise in mathematics education are visible in the different ways of implementing expertise and in the ways of attempting change in mathematics education (see Fig. 6).

We would like to close with a tentative conclusion coming back to Michael Sadler's original question posed in 1900: "How far can we learn anything of practical value from the study of foreign systems of education?" If one takes into account the cultural dependency of educational processes and the thinking of major players within these systems, who are not only teachers, but also students, parents and policy makers, it becomes obvious that it is not appropriate simply to take isolated measures from another educational system such as special teaching materials, teaching methods or teaching contests without questioning the context of these measures. Thus, the transfer of a single measure will not bring change. The full paradigm of teacher knowledge connected to the spirit of the underlying educational philosophy needs to be taken into account to accomplish any sustainable educational change.

From the perspective of the accelerating internationalisation of education at all levels, and the globalisation of nearly all societal processes, the Eastern and Western debate enables both to learn from each other. Both traditions need to pay attention to the individual student and his or her learning processes, to the content of mathematics as the base of learning, and to a combination of local and systemic measures for changing teacher education and teacher effectiveness. Effective teaching-and-learning environments may have different shapes in different countries but valuing teachers, supporting student learning and putting education in the foreground for a comprehensively educated human being will be the key for good mathematics education in all parts of the world (Zhao 2005).

International comparative studies have the potential to reveal an unbalanced view of one's own culture on mathematics (cf. dialectic or algorithmic), show collectivistic and individualistic ways of organising teaching-and-learning processes, and define adequate teacher and student behaviour. At the end of the day, they can provide insight into the possibilities to complement and enrich our own view on education. Individualism may be a benefit when it comes to creativity, as we know from studies on business organisations (Goncalo and Staw 2006). Why Western, in particular US, companies are so innovative is explained this way (Kanter 1988). East Asian countries may want to learn from such approaches without losing their strong cognitive base. In contrast, Western countries may want to learn from the East Asian motivation to strive hard for educational success without losing their creative base.

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Assessment of Teacher Knowledge Across Countries: A Review of the State of Research

Sigrid Blömeke and Séan Delaney

Abstract This review presents an overview of research on the assessment of mathematics teachers' knowledge as one of the most important parameters of the quality of mathematics teaching in school. Its focus is on comparative and international studies that allow for analyzing the cultural dimensions of teacher knowledge. First, important conceptual frameworks underlying comparative studies of mathematics teachers' knowledge are summarized. Then, key instruments designed to assess the content knowledge and pedagogical content knowledge of future and practicing mathematics teachers in different countries are described. Core results from comparative and international studies are documented, including what we know about factors influencing the development of teacher knowledge and how the knowledge is related to teacher performance and student achievement. Finally, we discuss the challenges connected to cross-country assessments of teacher knowledge and we point to future research prospects.

Keywords Mathematics teacher education · Comparative study · Mathematics content knowledge (MCK) · Mathematics pedagogical content knowledge (MPCK) · Large-scale assessment

This chapter on the "Assessment of teacher knowledge across countries" brings together two research fields that have developed largely independently from each other: comparative studies on prospective mathematics teachers' content knowledge and pedagogical content knowledge at the end of teacher preparation (Blömeke and Kaiser), and research on the mathematical knowledge of practicing teachers in several countries. The combination of these two research fields provides the reader with

Originally published in ZDM—The International Journal of Mathematics Education (adjusted version for the purpose of this book).

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S. Blömeke et al. (eds.), *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn*, Advances in Mathematics Education,
DOI 10.1007/978-94-007-6437-8_25,

an overview of what is going on in mathematics education research across countries in relation to teachers' knowledge.

In the first review of this topic for this journal, the state of research was summarized as follows: "Teacher-education research lacks a common theoretical basis, which prevents a convincing development of instruments and makes it difficult to connect studies to each other" (Blömeke et al. 2008a). Since then, research on prospective and practicing mathematics teachers' knowledge has continued to develop. Two research groups have been particularly productive in assessing teacher knowledge with direct measures: one from Michigan State University in the context of "Mathematics Teaching in the 21st Century" (MT21; see, e.g., Schmidt et al. 2011) and the "Teacher Education and Development Study: Learning to Teach Mathematics" (TEDS-M; see, e.g., Blömeke et al. 2012a; Tatto et al. 2012); the second one from the University of Michigan in the context of "Learning Mathematics for Teaching" (LMT; see, e.g., Delaney et al. 2008; Hill et al. 2008). This pioneering work has paved the way for a 2012 ZDM special issue (see, e.g., the papers by Cole 2012; Kwon et al. 2012).

Our chapter summarizes what we have learned in editing this issue, and presents an overview of research on the assessment of mathematics teachers' knowledge going on in other research groups.[1] Our focus is primarily on comparative and international studies because these are innovative areas covering long-lasting research gaps.[2] During the past two decades, the interest in international comparative studies on mathematics teachers has increased (Cochran-Smith and Zeichner 2005; Darling-Hammond 2000). Mathematics teachers will play a central role in the preparation of future generations' K-12 students. An examination of mathematics teachers' knowledge is therefore an important parameter of school quality. It is important to ascertain whether and how teacher training contributes to the development of teacher knowledge.

At the same time, international comparisons allow for analyzing cultural dimensions of teacher knowledge. By developing international studies, many matters are questioned which may remain unquestioned in national studies. The structure and the content of mathematics teacher training depend on a deeper rationale which is a result of factors which may be at least partly cultural. Like the water in the fish's tank, such cultural givens are too often invisible (Blömeke and Paine 2008)—and international comparisons provide the chance to move beyond the familiar, and to see with a kind of "peripheral vision" (Bateson 1994).

The results of comparative studies also provide benchmarks of what level and quality of teacher knowledge can be achieved and point to country-specific strengths and weaknesses. In many countries, the results of such studies on K-12 student achievement have led to fundamental reforms of the school system. The publication of the PISA 2000 results in Germany, for example (Baumert et al. 2001), one of the

[1] We highly appreciate the support of the reviewers who pointed to work relevant for the purpose of our paper. Only due to their efforts were we able to include conceptual frameworks and research from all continents.

[2] We confined our review to articles that were written in the English language.

first international studies the country took part in, and the realization that Germany performed at only a mediocre level—in contrast to the country's self-image—came as a shock. Heated debates and soul-searching among policymakers, researchers, and lay people finally resulted in changes. Similarly, the USA implemented significant reforms in its mathematics school curricula after the so-called "Sputnik shock" and the country's weak performance was confirmed in comparative studies such as SIMS (Pelgrum et al. 1986) and TIMSS (Mullis et al. 1997). Thus, comparative studies of student knowledge provided the chance to understand educational phenomena in a new way. We hope that research on teaching and teacher training across countries will produce similar effects.

Warnings that the proficiency level of mathematics teachers may not be strong enough, given the marginalized role mathematics had been playing in teacher education in many Western countries, had been put forward long ago. Mathematics educators (see, e.g., Schoenfeld 1994; Kilpatrick et al. 2001) and mathematicians (see, e.g., Cuoco 2001; Wu 1999) have repeatedly pointed to the risks of weak training in mathematics: teachers' limited understanding of what mathematics actually is, a fragmented conception with vertical and horizontal disconnects, less than enjoyable teaching routines, and an inability to implement the modern ideas in new school curricula. However, systematic evidence supporting these claims with respect to teachers has been missing for a long time.

Efforts to fill existing research gaps have been made since the late 1990s. Several comparative small-scale studies on mathematics teachers and mathematics teacher training are available (e.g., An et al. 2004; Ma 1999; Burghes 2008). An important step was also the work of a Topic Study Group on mathematics knowledge for teaching at ICME-11 in Mexico (Adler and Ball 2009). About 50 colleagues from a broad range of countries presented their approaches to measuring teachers' or future teachers' mathematical knowledge (see, e.g., Kristjánsdóttir 2008 in several Nordic countries; Naik 2008 in India; West 2008 in Japan). Much of the teacher research, however, neglected the content domain, focused on other subdomains of mathematics teachers' competencies such as beliefs (Bramald et al. 1995; Calderhead 1996), or intended to capture knowledge by self-reports. Studies including direct measures of teacher knowledge and cross-country studies are still needed (Brouwer 2010; Wilson et al. 2001).

Our paper reports about such measures and summarizes their results. It is organized as follows. First, we summarize important conceptual models underlying comparative and international studies of mathematics teachers' knowledge. Second, we develop a model of factors assumed to influence the development of teacher knowledge during teacher training and the relationship between teacher knowledge and student achievement. Third, we describe the study design and key instruments developed to assess the content knowledge and pedagogical content knowledge of future and practicing mathematics teachers. Fourth, core results are documented from comparative and international studies from several countries on the structure and the level of this knowledge and how it is associated with teacher training, teacher performance, and student achievement. Fifth, we discuss the challenges connected to cross-country assessments of teacher knowledge and identify research needed to address these. Each section is subdivided into research on future teachers and practicing teachers.

1 Modeling Mathematics Teacher Knowledge: Conceptual Frameworks

Concepts of teacher knowledge and how it is gained have changed over the past 30 years. A first important model that characterized pre-service teacher education can be labeled as "teacher learning". This included approaches including learning by observation in a kind of apprenticeship (Zeichner 1980), learning by planning, application, and reflection (Schön 1983), and teacher learning as a craft (Brown and McIntyre 1983). The concept's starting point for modeling teachers' competencies was teachers' existing classroom practices.

Similar to this concept was a second one, prominent in the 1990s, in which the cognitive basis of teachers' pedagogical practices started to emerge. The first small-scale comparative studies based on this concept were carried out in the field of mathematics teaching (see especially Pepin 1999; Kaiser 1995).

More recently, teacher education research and research on practicing teachers has focused even more strongly on the knowledge base of teachers' classroom practice. Besides the studies already mentioned in our introduction and the Topic Study Group at ICME-11, this new research paradigm included, for example, studies by Rowland et al. (2005), Chick et al. (2006), and the many chapters from different contexts in Rowland and Ruthven (2010).

Similar to this approach, but more analytical in the sense of defining and distinguishing between different knowledge facets functional for teaching and stressing the importance of mathematics *content* knowledge, is the most recent approach which underpinned not only the PISA study on student achievement but also the two most prominent international teacher studies TEDS-M and LMT. This approach is characterized by a notion of teacher competencies elaborated with respect to the field of mathematics by, for example, Niss (2002) and proficiency in teaching mathematics (see, e.g., Schoenfeld and Kilpatrick 2008).

Elaborating this latter approach, Schoenfeld (2010, p. 187), in his recent book *How we think*, describes in-the-moment decision making as follows: "People's decision making in well practiced, knowledge-intensive domains can be fully characterized as a function of their orientations, resources, and goals." Teaching is such a knowledge-intensive domain, and Schoenfeld points out that mathematics knowledge is the most important resource of mathematics teachers. He regards it as an important enterprise (p. 203) to develop analytical distinctions of knowledge facets and to clarify why particular knowledge facets are accessed in some classroom situations by teachers and others are not. Affective-motivational facets such as orientations and goals or self-regulation are supposed to be decisive in these processes as well. Only together can the full range of teacher competencies underlying classroom performance be understood.

In the following, we present two conceptual frameworks that model the knowledge of mathematics teachers in such a way: the comparative and international studies TEDS-M and LMT.

1.1 Conceptual Framework of TEDS-M as a Comparative Study on Prospective Teachers

In 2008, a comparative study was carried out that examined the knowledge and the beliefs of mathematics teachers at the end of their training: the "Teacher Education and Development Study: Learning to Teach Mathematics" (TEDS-M). Nationally representative samples of primary and lower secondary mathematics teachers in their final year of teacher training from 16 countries in Africa, the Americas, Asia, and Europe were examined, as well as representative samples of teacher educators and training institutions (Tatto et al. 2008). TEDS-M was carried out under the supervision of the International Association for the Evaluation of Educational Achievement (IEA).[3] The study looked at how teachers of mathematics were trained and what kinds of knowledge and beliefs they had at the end of their training with standardized testing. More than 23,000 prospective teachers were surveyed.

The professional knowledge of teachers can be divided into several facets: content knowledge, pedagogical content knowledge, curricular knowledge, and generic pedagogical knowledge (Shulman 1985). In the context of TEDS-M as a study about prospective mathematics teachers, the content knowledge was the knowledge of mathematics. Pedagogical content knowledge referred to knowledge about the teaching and learning of mathematics. In the TEDS-M framework, it included curricular knowledge, too. Pedagogical knowledge, finally, was the knowledge typically acquired in a teacher training program that is not subject-matter related (Blömeke and Paine 2008).

Mathematics content knowledge (MCK), in this framework, includes not only basic factual knowledge of mathematics but also the conceptual knowledge of structuring and organizing principles of mathematics as a discipline (Shulman 1987): why a specific approach is important and where it is placed in the universe of approaches to mathematics. *Mathematics pedagogical content knowledge* (MPCK) includes the subject-related knowledge for teaching. Shulman (1987, p. 9) characterizes it as an "amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding". A mathematics teacher has to know about typical preconditions of students and how to represent a topic in the best possible way. Curricular knowledge is part of it and includes teaching materials and curricula.

[3]TEDS-M was funded by the IEA, the National Science Foundation (REC 0514431), and the participating countries. In Germany, the German Research Foundation funded TEDS-M (DFG, BL 548/3-1). The views expressed in this paper are those of the authors and do not necessarily reflect the views of the IEA, the ISC, the participating countries, or the funding agencies.

Table 1 Core situations which mathematics teachers are expected to manage (Tatto et al. 2008)

Mathematical curricular knowledge	Establishing appropriate learning goals
	Knowing about different assessment formats
	Selecting possible pathways and seeing connections within the curriculum
	Identifying the key ideas in learning programs
	Knowledge of the mathematics curriculum
Knowledge of planning for mathematics teaching and learning [pre-active]	Planning or selecting appropriate activities
	Choosing assessment formats
	Predicting typical student responses, including misconceptions
	Planning appropriate methods for representing mathematical ideas
	Linking didactical methods and instructional designs
	Identifying different approaches for solving mathematical problems
	Planning mathematics lessons
Enacting mathematics for teaching and learning [interactive]	Analyzing or evaluating students' mathematical solutions or arguments
	Analyzing the content of students' questions
	Diagnosing typical student responses, including misconceptions
	Explaining or representing mathematical concepts or procedures
	Generating fruitful questions
	Responding to unexpected mathematical issues
	Providing appropriate feedback

An important implication of the TEDS-M framework that modeled teacher knowledge as a facet of teacher "competencies" (Weinert 2001) was its link to classroom situations. Since competencies were assumed to represent a latent trait that underlies performance, teacher knowledge was regarded as situated and applied by nature (Blumer 1969). The classroom situations a teacher has to deal with have to be set by constitutive features of the teaching profession. To determine which features were to be regarded as constitutive, TEDS-M referred to existing standards for the national teacher training programs in its participating countries (e.g., KMK 2004; NCTM 2000). Table 1 documents the problems which mathematics teachers were expected to solve in TEDS-M according to the study's conceptual framework (see Tatto et al. 2008), based on these standards.

In this applied perspective, it is important to distinguish between different types of MPCK (Shulman 1986): knowledge in teaching, that is *propositional knowledge* of, for example, student errors or misconceptions without being related to a specific classroom context; *case-based knowledge* that includes prototypes, borderline cases, and analogies based on individual experiences; and *strategic knowledge* or "practical wisdom" for situations when a teacher is overwhelmed by the multidimensionality and speed of what is going on in the classroom. As far as is possible

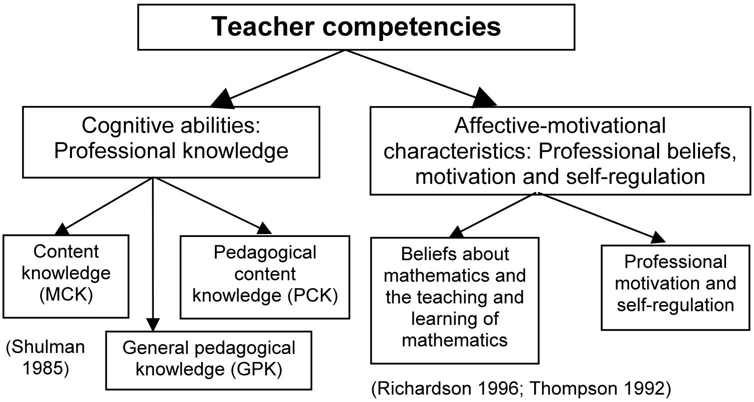

Fig. 1 Conceptual framework of teacher competencies

in a paper-and-pencil test, TEDS-M tried to cover the first two types of knowledge, the propositional and the case-based knowledge of mathematics teachers.

TEDS-M looked also at the professional beliefs held by the future mathematics teachers. Beliefs were defined by Richardson (1996, p. 103) as "psychologically held understandings, premises, or propositions about the world that are felt to be true". As Schoenfeld (2010) pointed out, teacher beliefs are crucial for the perception of classroom situations and for decisions on how to act (Leder et al. 2002; Leinhardt and Greeno 1986). Therefore, they connect knowledge and action. In this sense, they are also an indicator of the type of instruction that mathematics teachers may use in their future teaching (Brown and Rose 1995). If beliefs are operationalized specifically to both the content being taught and the challenges a specific classroom situation presents, empirical evidence exists for a link between teacher beliefs and student achievement (Bromme 1994). In TEDS-M, several beliefs facets were distinguished, in particular epistemological beliefs about the nature of mathematics and beliefs about the teaching and learning of mathematics (Thompson 1992).

By distinguishing between knowledge and beliefs and thus to include cognitive and affective-motivational teacher dispositions and by stressing the situative and applied nature of teacher knowledge, the TEDS-M framework can be connected to a measurement tradition prominent in educational psychology: the measurement of competencies. Competencies as defined in general by Weinert (2001) and specifically with regard to teaching by Bromme (1997) mean having the cognitive ability to develop effective solutions for job-related problems and, in addition, having the motivational, volitional and social willingness to successfully and responsibly apply these solutions in various situations (see Fig. 1). The purpose of such a multidimensional approach is to come as closely as possible to real behavior in the classroom that is supposed to be guided by both types of dispositions. Measuring them with separate instruments allows for examining the relationship and clustering teachers according to their beliefs-knowledge profile (Blömeke et al. 2012a).

Besides beliefs as one facet of the affective-motivational dimension, a teacher's professional motivation and self-regulation are important. A teacher who regulates her behavior is able to define her professional objectives, to decide on appropriate strategies in order to achieve her objectives, and to apply them in various situations. Furthermore, she monitors and evaluates her behavior systematically guided by metacognition (Butler and Winne 1995; Perry et al. 2006; Boekaerts and Corno 2005). Teaching strengths and weaknesses are identified and—if necessary—either behavior is adjusted or professional development activities are taken. Self-regulation capacities are a necessary precondition in order to be successful as a teacher in the long run.

Such a multidimensional notion of teacher competencies that underlie classroom performance led not only the item development in TEDS-M but also comparative studies such as MT21 and the follow-up studies to TEDS-M (TEDS-FU) carried out in Germany, Taiwan, and the USA. TEDS-FU currently looks into the transition of the samples tested in TEDS-M from teacher training into the job (see the forthcoming special issue of the *International Journal of Science and Mathematics Education*, *IJMA*, edited by the three national TEDS-M coordinators in these countries, Blömeke, Schmidt, and Hsieh).

However, such an analytical approach has also been criticized. It has its benefits with respect to the purpose of reliable measurements, but from a didactical perspective that focuses on the practical side of a teacher's work it is argued that the separate dimensions have to come together in the classroom (Huillet 2009; Bednarz and Proulx 2009). This is certainly a valid observation. We will come back to it at the end of our paper when we discuss issues of validity in our efforts to measure teacher knowledge.

1.2 Conceptual Framework of Studies on Practicing Teachers

The studies on practicing teachers that are considered here differ from the TEDS-M studies in at least two ways. First, the studies focus mainly on elementary teachers. Although measures of secondary teachers' mathematical knowledge which can be used at scale have been developed by the German COACTIV (Cognitive Activation in the Classroom) research group (e.g., Krauss et al. 2008a), none of the studies reported here have adapted these measures for use in other countries. Instead, the measures used were developed for use with elementary teachers by the US LMT (Learning Mathematics for Teaching) research group (e.g., Hill and Ball 2004). Although the LMT group has subsequently developed measures for studying teachers of middle school students, the team's original focus was on knowledge held by teachers of elementary students.

Second, unlike the conceptual framework which was developed for the purposes of comparing prospective teachers participating in the TEDS-M study, the conceptual framework for mathematical knowledge for teaching (MKT) was developed by researchers at the University of Michigan (Ball and Bass 2003, p. 399) in order to better understand teacher knowledge in the United States. The conceptual framework of MKT was inspired by Shulman's (1986) idea of pedagogical content knowledge and seeks to categorize the domains of knowledge needed to do the work of teaching mathematics (Ball et al. 2008). It consists of two broad categories—subject matter knowledge and pedagogical content knowledge—and does not include affective-motivational characteristics. Each broad category has been further subdivided into the domains of common content knowledge, specialized content knowledge, and horizon content knowledge on one hand; and knowledge of content and students, knowledge of content and teaching, and knowledge of content and curriculum on the other. Each domain refers to a hypothesized type of mathematical knowledge that is needed by teachers.

Common content knowledge (CCK) refers to mathematical knowledge "used in settings other than teaching" (Ball et al. 2008, p. 399) and an example would be recognizing and naming a two-dimensional shape such as a rectangle or a pentagon. Specialized content knowledge (SCK) is mathematical knowledge and skill that is "not typically needed for purposes other than teaching" (Ball et al. 2008, p. 400), such as knowing a range of definitions of shapes that are both comprehensible to students of different age levels, and mathematically accurate and complete. Knowledge of content and students (KCS) "combines knowing about students and knowing about mathematics" (Ball et al. 2008, p. 401) and would involve knowing for example that a square remains a square even if it is rotated 45 degrees. Knowledge of content and teaching (KCT) refers to knowledge of mathematics combined with knowledge of teaching and would include knowing how to select a poster to support the teaching of shapes by using non-examples and non-stereotypical examples. A provisional domain is horizon content knowledge (HCK), which is an "awareness of how mathematical topics are related over the span of mathematics included in the curriculum" (Ball et al. 2008, p. 403). HCK includes knowledge of the wider discipline of mathematics insofar as its content and practices can inform the work of teaching and, as a domain, requires further specification (Jakobsen et al. 2012). Knowledge of content and curriculum (KCC) is another provisional category. Empirical evidence for these domains has been sought but findings to date have been inconsistent (Schilling et al. 2007). Measures based on the framework have been developed for all domains except HCK and KCC and it is these measures which have appealed to researchers outside the United States.

An important distinction to notice between the TEDS-M study and studies using measures based on MKT is that the TEDS-M study was conducted in conjunction with the IEA, whereas the theory of MKT and the measures based on the theory were developed by the US-based Learning Mathematics for Teaching (LMT) project. Although both frameworks are related to classroom situations, the teaching which informed the development of MKT was specifically US teaching (Ball et al. 2008). Consequently, when measures based on the theory are used outside the United States, researchers need to evaluate the suitability of using US measures to study the mathematical knowledge held by teachers in other countries. This is because the theory is based on the practice of teaching, and if the practice of teaching is a cultural activity as some would argue (e.g. Stigler and Hiebert 1999) then the theory of MKT may also be culture-specific. Nevertheless, the availability of high quality measures of MKT that could be used at scale (Hill et al. 2004) has made it worthwhile to evaluate their suitability for use in several countries, despite the challenges involved in doing so.

One challenge is that a coordinated study of practicing teachers' mathematical knowledge across countries has not yet been funded. This situation contrasts with the substantial funding of several large-scale comparative studies—TIMSS and PISA—of students' mathematical knowledge. Studies which have used the MKT framework to study teacher knowledge, including those published here, tend to be conducted locally, are small-scale in nature, and are situated in specific countries. However, despite being small in scale, the lessons learned in such studies can inform comparative studies of teacher knowledge, should they take place in the future. The range of settings for the studies—involving countries in Europe (Ireland and Norway), Africa (Ghana), and Asia (Indonesia and Korea)—provides a robust and diverse test for the US measures. Such diversity is likely to ensure that the studies are well placed to evaluate as well as to contribute to the conceptual framework of MKT.

In addition to the studies published in the 2012 ZDM special issue (see, e.g., the papers by Cole 2012; Kwon et al. 2012), the MKT approach and framework (but not the measures) have been used as the basis of research into teacher knowledge by scholars in settings as diverse as New Zealand (Burgess 2009) and South Africa (Adler and Davis 2006; Kazima and Adler 2006). In addition to studies of practicing teachers, the framework has been used in studies of mathematical knowledge among Australian and Canadian pre-service teachers (Butterfield and Chinnappan 2010, 2011). Similarly, Forrester and Chinnappan (2011) studied 224 pre-service teachers in a year 1 subject in Bachelor of Education Primary drawing on the work of Ball and Bass (2003).

Alternative frameworks for studying teachers' mathematical knowledge have also been proposed. Chick and her colleagues developed a framework that includes categories that are "clearly PCK", "content knowledge in a pedagogical context", and "pedagogical knowledge in a content context" (e.g., Chick et al. 2006). Also in Australia, Chinnappan and Lawson (2005) developed a framework for studying knowledge of geometry for teaching and applied it to studying the knowledge exhibited by two experienced teachers in taped interviews. One study that did not look at teachers but at the history of mathematics, research studies, and textbooks is the work in Brazil of Ribeiro (2008) who elaborates on how understanding six specified meanings of equations can contribute to teachers' knowledge for teaching.

At the secondary level, with the exception of the COACTIV work, frameworks for studying mathematical knowledge for teaching are a more recent development. However, researchers at Penn State University have analyzed the mathematics needed by teachers of secondary students from three perspectives: a curriculum-based mathematical thinking approach, a situations approach, and a mathematical process approach (Heid 2008). A series of studies have been designed by Penn State faculty and graduate students around these perspectives to characterize prospective teachers' mathematical knowledge, to characterize their knowledge and use of mathematical processes, and (in conjunction with faculty and graduate students from the

University of Georgia) to develop a practice-based framework of MKT at the secondary level (Heid 2008).

Like most studies of teachers' mathematical knowledge, the examples we have found tend to be conducted locally and are small in scale. We found more studies of primary teachers than of secondary teachers. Many studies looked at pre-service teachers, probably because as subjects they are much more accessible to university-based researchers. Because primary teachers tend not to have subject-specific preparation, primary teachers have long been perceived by many to be lacking in MKT. However, studies of prospective and practicing secondary teachers are appearing more frequently than before as researchers find that specialized subject preparation does not necessarily equip teachers with the knowledge they need for teaching.

2 Modeling the Link from Mathematics Teacher Education to Student Achievement

The topic of this book and our review, the knowledge of prospective and practicing mathematics teachers, is one[4] crucial link between mathematics teacher education and student achievement in mathematics. In the first perspective, teacher knowledge represents a core criterion for effective teacher training and is thus a dependent variable. In the second perspective, teacher knowledge represents an important predictor of student achievement and is thus an independent variable. Only both perspectives together provide an appropriate view on the relationship of mathematics teacher training and what is accomplished in schools—multiply mediated by complex context factors.

Before we start to model this link based on the frameworks used in TEDS-M and LMT, we have to point out that the enterprise represents a huge—and maybe at present too huge—step. We have currently only data from studies examining parts of the connection between mathematics teacher education, mathematics teacher knowledge, and student achievement. Thus, we cannot claim that the overall model stands on strong grounds. Further, the studies are relational only and not longitudinal, thus causal claims cannot be made, either. This section is therefore meant to build a heuristic rather than to give a definite answer.

2.1 Mathematics Teacher Education and Teachers' Professional Knowledge

In order to examine which factors may influence the development of teacher knowledge during teacher training, potentially influential factors were divided into three categories in TEDS-M:

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[4]From here on, we focus on the state of research on teacher knowledge that is related to mathematics. This topic is already highly complex in a comparative context and covering pre-service and in-service teachers. General pedagogical knowledge, beliefs, motivation, and self-regulation have thus to be neglected although they are also highly important for teaching (for results on the future teacher's belief and their relationship to knowledge see, e.g., Blömeke et al. 2012b).

(1) the individual characteristics of future teachers
(2) the institutional characteristics of teacher training, and
(3) the national country context.

In studies of school effectiveness, K-12 students' background is almost always a powerful predictor of achievement. Specifically with respect to mathematics, gender (Hyde et al. 2008), socio-economic status (Mueller and Parcel 1981), and language background (Walter and Taskinen 2008) as well as prior knowledge (Simmons 1995) and motivation (Singh et al. 2002) play an important role (Scheerens and Bosker 1997). Such a network of individual predictors may apply to the knowledge acquisition during mathematics teacher training as well.

With respect to institutional characteristics, TEDS-M followed the tradition of the IEA in connecting educational opportunity and educational achievement. As was done in the "Third International Mathematics and Science Study" (TIMSS), opportunities to learn (OTL) were framed as content coverage, specifically as "the content of what is being taught, the relative importance given to various aspects of mathematics and the student achievement relative to these priorities and content" (Travers and Westbury 1989). OTL were in this sense defined as future mathematics teachers' encountering occasions to learn about particular topics during teacher training, including the characteristics of their educators such as background, teaching objectives, and teaching methods used. Since subject matter specificity is the defining element of an educational opportunity (Schmidt et al. 1997), the particular topics reflected the areas of mathematics and mathematics pedagogy.

OTL in teacher training can be regarded as having been intentionally developed by educational policy makers and teacher training institutions (Stark and Lattuca 1997; Schmidt et al. 2008). They give characteristic shape and direction to instruction. Every choice provides some OTL at the expense of others. National program choices in this sense reflect particular visions of what mathematics teachers are supposed to know and be able to do in class and how teacher training should be organized in order to provide the knowledge and skills necessary for successful accomplishment of their professional tasks.

Thus, teacher knowledge at the end of teacher training is assumed to depend on individual background characteristics of the prospective mathematics teachers, their differential learning experiences during teacher training, opportunities to learn provided by their training institutions, and the social, schooling, and policy context they are living in (see Fig. 2). However, as a study on the effectiveness of teacher education, TEDS-M did not look into the relationship between teachers' professional knowledge and student achievement.

2.2 *Mathematics Teachers' Professional Knowledge and Student Achievement*

While the characteristics of prospective mathematics teachers depend on their entering characteristics, the learning experiences received from interactions with teacher

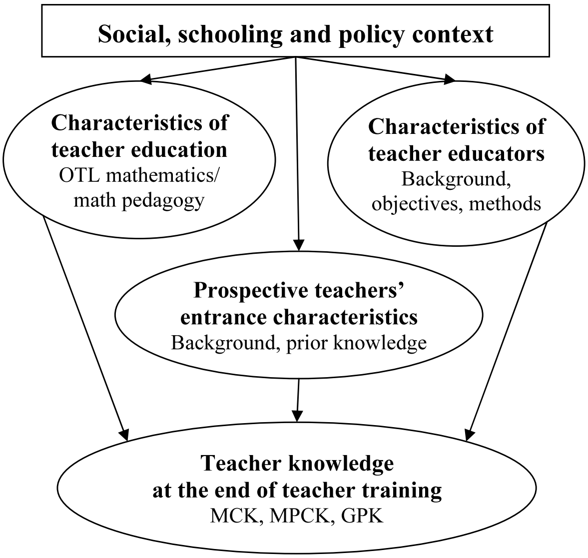


Fig. 2 Hypothetical model of characteristics of teacher training influencing teacher knowledge mediated by the prospective teachers' entrance characteristics (based on Tatto et al. 2008)

educators and their teacher education program, and from their social, schooling, and policy context, additional factors such as experience (e.g. Hill 2007) and professional development (Bell et al. 2010) may influence the knowledge held by practicing teachers. Our interest, however, in teachers' knowledge is not ultimately an end in itself but as a means of improving student achievement.

For many years researchers have attempted to study the link between teacher mathematical knowledge and student achievement in what are often referred to as educational production function studies (Monk 1989). Begle conducted an early educational production function study in 1972 (Begle 1972; Eisenberg 1977). He subsequently used his own research and studies by others to conclude that, beyond a certain level, mathematical knowledge matters little for student achievement (Begle 1979). In the studies referred to by Begle, mathematical knowledge was measured by tests of general mathematical knowledge administered to teachers (e.g. Begle 1972). Begle's studies and the studies he reviewed provided evidence that proxy measures of teacher knowledge (e.g., math courses studied) and performance on generic mathematics test items are not good predictors of student learning, suggesting that more sophisticated means of studying teachers' mathematical knowledge were needed.

More recent studies that attempt to link teacher knowledge and student achievement have used "pencil-and-paper" tests. Their goal is to test teachers' performance on mathematics problems that occur in teaching rather than on general mathematical knowledge tasks. They focus on specific mathematical domains, and have been developed by teams with expertise in mathematics, mathematics education, and psychometrics (Hill et al. 2007b). An example of such a study is one by Hill et al. (2005)

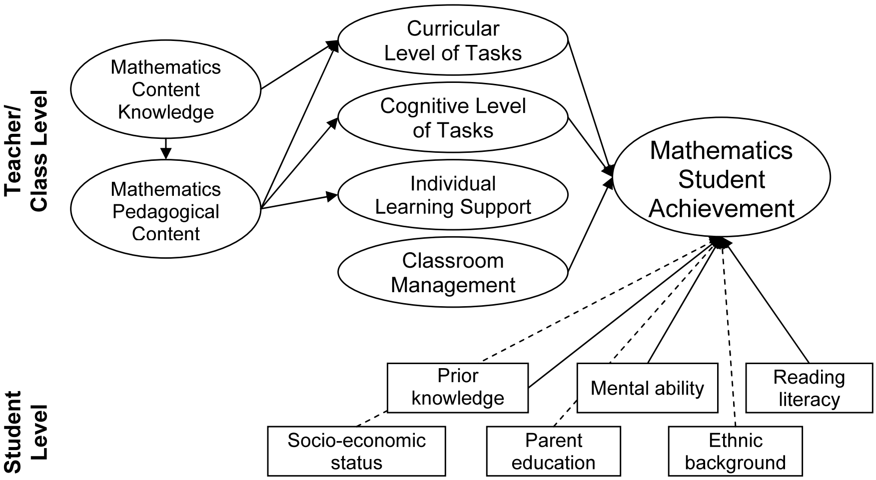


Fig. 3 Hypothetical model of the effects of teachers' mathematics content knowledge and mathematics pedagogical content knowledge on student achievement in mathematics mediated by instructional quality (based on Baumert et al. 2010)

where it was found that every standard deviation of difference in teacher knowledge was worth the equivalent of two to three weeks additional instruction time in predicting the gains made by first and third grade students on standardized math test scores. In third grade the effect size of teacher knowledge "rivaled that of SES and students' ethnicity and gender, while in the first grade models the effect size was not far off" (p. 396). Although such findings in a single study need to be treated with caution, they confirm the importance of pursuing teacher knowledge as a key variable in student achievement.

A longitudinal study conducted by Baumert and colleagues (2010) in Germany studied the effects of one domain of mathematical knowledge, pedagogical content knowledge (PCK), on student achievement at the end of grade ten (see Fig. 3). The study found a substantial effect size of PCK on student achievement, an effect which is distinguishable from the effect of general mathematical knowledge and other factors such as track membership (i.e. membership in one of the different school types existing parallel to each other in Germany). Although this finding too is promising, it is limited to studying teacher knowledge in a single country.

Comparative studies of teacher knowledge have tended to be on a smaller scale and consequently have not established "direct and statistical relationships between teaching-related factors and student mathematics performance" (Wang and Lin 2005). Although studies such as Ma's (1999) demonstrated that some Chinese teachers have deeper mathematical knowledge than some US teachers, another study found that whether US or Chinese students perform better on mathematics problems varies by problem type (Cai 2000). More importantly for our interest, factors other than teacher knowledge, such as classroom instruction or placement of topics on the school curriculum (Cai 2000; Wang and Lin 2005), may account for differences in student performance between the two countries. Studying the relationship between teacher knowledge and student achievement across several countries could yield important insights in mathematics education. Consequently, insights from country-specific studies of the teacher knowledge/student achievement relationship and from studies of teacher knowledge across countries will inform how challenges inherent in such research can be addressed.

3 Study Design and Instruments to Assess Mathematics Teachers' Knowledge

Most comparative studies have to deal with the same methodological challenges. In order to reduce the complexity, this section focuses again on selected studies as examples. The largest study on prospective mathematics teachers was TEDS-M. Its study design and instruments are therefore described and evaluated. An instrument for the assessment of practicing teachers' mathematical knowledge applied in many countries is the LMT test, which is also documented.

3.1 Design of TEDS-M as a Study on Prospective Teachers

3.1.1 Sampling

The target population of TEDS-M consisted of students in their final year of teacher training who were on track to receive a license to teach mathematics in primary or lower secondary schools (Tatto et al. 2008). Since the definition of "primary" and "lower secondary" varies across countries, an agreement on which programs to include in which of the two studies was difficult to accomplish. The complexity was increased by the substantial number of programs covering both school levels. Finally, a teacher training program was included if it prepared primary teachers for one of the grades 1 through 4 as the common denominator of level 1 education in the "International Standard Classification of Education" (primary or basic education, cycle 1; UNESCO 1997), or if it prepared lower secondary teachers for grade 8 as the common denominator of level 2 education (lower secondary or basic education, cycle 2). Programs covering both levels were included in both studies.

In a two-stage process, random samples were drawn from the target populations in each participating country. The samples were stratified according to important teacher training features such as "route" (consecutive vs. concurrent programs), "type" of program (grade span the license includes, e.g., grades 5 through 9 vs. 5 through 12), "focus" of opportunities to learn (with or without extensive mathematics), and "region" (e.g., federal state) in order to reflect accurately the distribution of prospective primary and lower secondary teachers' characteristics at the end of training.

In 2008, approximately 14,000 prospective primary and 9,000 lower secondary mathematics teachers from altogether 16 countries (15 countries in each of the two

Table 2 Countries participating in TEDS-M

Botswana	Chile	Germany	Georgia
Malaysia	Norway	Oman (Lower secondary study only)	Philippines
Poland	Russia	Switzerland	Singapore
Spain (Primary study only)	Taiwan	Thailand	USA

studies; see Table 2) were tested on their knowledge of mathematics and mathematics pedagogy by a standardized paper-and-pencil assessment. Rigorous quality requirements, as set out in TIMSS or PIRLS by the IEA, included controlling the translation processes, monitoring test situations, and meeting the required participation rates. The aim of these standards was to ensure that bias resulting from non-response was kept within acceptable limits. However, in contrast to studies on student achievement where ministries of education often can pledge school and teachers to participate, TEDS-M had to deal with universities which enjoy a large degree of autonomy. Furthermore, future teachers are adults who can decide about participation by themselves. Correspondingly, it was much harder to meet IEA's requirements than it is on the school level.[5]

3.1.2 Instruments

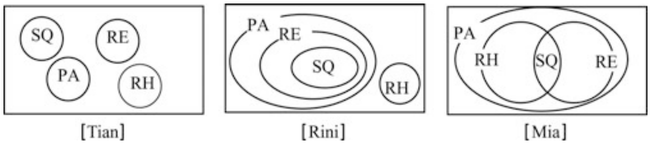
TEDS-M sought to measure prospective teachers' MCK and MPCK at the end of their training. For this purpose, a 60-minute paper-and-pencil assessment was completed during a standardized and monitored test session. The MCK test covered number, algebra, and geometry with approximately equal weight and, to a lesser extent, data (Tatto et al. 2008). In addition, three cognitive dimensions were covered by the items: knowing, applying, and reasoning. Sample items are given in Figs. 4 and 5 (the full sets of released items are available from tedsm@msu.edu).

The MPCK test covered two facets: knowledge of curricula and planning, which is necessary before a teacher enters the classroom, and interactive knowledge about how to enact mathematics for teaching and learning. In line with the MCK test, four content areas were distinguished. An example is given in Fig. 6.

The item development was mainly informed by the MT21 study (Schmidt et al. 2011), as well as the two Michigan studies entitled "Knowing Mathematics for Teaching Algebra" (KAT; Ferrini-Mundy et al. 2005) and "Learning Mathematics for Teaching" (LMT; Hill et al. 2008). Three item formats were used: multiple choice, complex multiple choice, and open constructed response.

[5]The participation rates in four countries on the primary level (Chile, Norway, Poland, and the USA) and five countries on the lower secondary level (Chile, Georgia, Norway, Poland, and the USA) did not fully meet the required benchmarks. Their results are therefore reported in an annotated way. In Poland, Switzerland, and the USA the coverage of the target population was reduced and in Norway the sample composition did not fully meet the TEDS-M definition of the target population. In the USA, a substantial proportion of missing values was observed. The results of these countries are reported in an annotated way as well. Canada had to be excluded from the study because the country missed the benchmarks to a serious extent.

Three students have drawn the following Venn diagrams showing the relationships between four quadrilaterals: rectangles (RE), parallelograms (PA), rhombuses (RH) and squares (SQ).



Which student's diagram is correct? *Check <u>one</u> box.*

A. [Tian] o_1

B. [Rini] o_2

C. [Mia] o_3

Fig. 4 Sample item from the TEDS-M primary test of MCK

Fig. 5 Sample item from the TEDS-M lower secondary test of MCK

Prove the following statement:
If the graphs of linear functions

$f(x) = ax + b$

and

$g(x) = cx + d$

intersect at a point *P* on the *x*-axis, the graph of their sum function

$(f + g)(x)$ must also go through *P*.

Fig. 6 Sample item from the TEDS-M primary test of MPCK

When teaching children about length measurement for the first time, Mrs. [Ho] prefers to begin by having the children measure the width of their book using paper clips, and then again using pencils.

Give **TWO** reasons she could have for preferring to do this rather than simply teaching the children how to use a ruler.

3.1.3 On the Nature of the TEDS-M Knowledge Tests

If one is to evaluate the nature of the TEDS-M tests as done by Döhrmann, Kaiser, and Blömeke (this book), one can summarize that the MCK and MPCK of prospective teachers was successfully conceptualized and efficiently surveyed through the TEDS-M assessments. The authors confirm the overall reliability and validity of the tests from an international point of view. However, they also point to limits of the assessment. Due to cultural differences between the participating countries, the items did not cover the entire range of the knowledge teachers should acquire during teacher training. Some teacher tasks are relevant in certain countries but not in others. Thus, the corresponding knowledge was not assessed.

In addition, Döhrmann, Kaiser, and Blömeke (this book) characterize the orientation of the conceptual framework and the item pool as slightly biased towards a pragmatic conception of teaching and learning, predominantly in place in English-speaking countries. Facets common in continental Europe were taken into account to some extent only. In this European tradition, the mathematics and the mathematics pedagogy tests would have had to include more argumentation and proof items, for example. Also, fundamental ideas of central mathematical concepts such as number or percentage and different ways to introduce them in class would have had to have a higher priority.

Hsieh, Lin, and Wang (this book) argue along the same line. Confirming the overall validity of the TEDS-M tests, they point out that the tests have more items testing MCK than MPCK, which may demonstrate an unbalanced focus of interest. Based on conceptual considerations or empirical evidence, many colleagues would make a plea for MPCK as the crucial facet of teacher knowledge (see, e.g., Baumert et al. 2010). Furthermore, Hsieh, Lin, and Wang (this book) would prefer an assessment more closely in line with Niss (2002), who emphasized the fundamental characteristics of mathematical thinking across the content domains of mathematics rather than assessment of their knowledge in these specific domains (see also Hsieh 2010; Hsieh et al. 2010). This perspective is also supported by An and Wu (2011).

An open question is to what extent the TEDS-M tests were measurement invariant across countries. The number of countries was only 15, with even smaller numbers of country groups from similar educational traditions or with substantial proportions of teachers using different languages within the countries. Thus, a potential cultural bias or a potential language bias could only be examined to a limited extent. First evidence suggests that the MCK and MPCK assessments may not have been *completely* equivalent in all TEDS-M countries (Blömeke et al. 2011a). Although rigorous quality control had taken place, language and cultural differences between and within countries seem to be related to how well these traits were measured in the TEDS-M countries. The language problem seemed to be larger with respect to MCK than to MPCK. Blömeke et al. (2011a) attributed this result to a long history of schooling in a different language from that used at home in the case of MCK. Its acquisition had probably already suffered from language disadvantages before the prospective teachers entered university.

3.1.4 Challenges of Scaling the TEDS-M Data

Scaled scores in TEDS-M were created separately for MCK and MPCK in one-dimensional models using item response theory. Such models stress the conceptual difference between the two knowledge facets (see Tatto et al. 2012). An alternative way of scaling the data was used by Blömeke et al. (2011a). They applied two-dimensional models that can take the conceptual overlap between MCK and MPCK into account (Reckase 2009). Whereas traits such as reading literacy or mathematics literacy, typically found in PIRLS or TIMSS, are relatively well-defined, such conceptual clarity does not exist with respect to teacher knowledge. Researchers are still struggling to separate its facets (Graeber and Tirosh 2008). Studies by Schilling et al. (2007) or Krauss et al. (2008b) demonstrated that MCK and MPCK were highly correlated.

Therefore, Blömeke et al. (2011a) used a multidimensional IRT approach in which MCK and MPCK were represented with a general and a nested factor ("within-item multidimensionality", Adams et al. 1997). The model represented Shulman's idea that the nested factor MPCK was a mixture of different abilities and that mathematics pedagogy items measured this mix. According to this idea, solving the mathematics pedagogy items required MCK as a general ability but also specific MPCK. The results supported, in fact, the contention that the nature of teacher knowledge is multidimensional (Blömeke et al. 2011a). Only in this model was the importance of opportunities to learn mathematics pedagogy during teacher education revealed, too. The more a country had focused on mathematics pedagogy in relation to mathematics during teacher training, the more likely it would be to have a high MPCK mean.

3.1.5 Challenges of Reporting the TEDS-M Results

The TEDS-M results gave a sound picture of the professional knowledge of prospective mathematics teachers who in 2008 were in their final year of teacher training. However, there was an important challenge in how to report the results. The data had to be analysed on two levels of aggregation because it was important to distinguish between an evaluation of the national teacher training systems and an evaluation of specific teacher training programs within countries. Both approaches have their benefits and their limitations:

(1) Due to the traditional policy orientation of IEA's large-scale assessments, TEDS-M focused on the national level. This approach stressed the overall educational effectiveness of a nation, regardless of the structure of its education system. In this perspective, with regard to international competitiveness, it considered what a nation accomplishes as a whole—and differences in the structure of teacher education systems between countries represent a function of differences in their educational policy.

(2) Additional information was gained by looking into program types. Thus, it was possible to learn about pathways to success within countries, that is, without confounding variables like cultural or societal features. Note that the relatively small sample sizes in the case of teachers (compared with students) became even smaller when types of programs were examined and that the precision of estimates was probably lower because the sampling target was mainly on the national level. This approach has therefore to be used with caution.

Ms. Chambreaux's students are working on the following problem:

Is 371 a prime number?

As she walks around the room looking at their papers, she sees many different ways to solve this problem. Which solution method is correct? (Mark ONE answer.)

a) Check to see whether 371 is divisible by 2, 3, 4, 5, 6, 7, 8, or 9.
b) Break 371 into 3 and 71; they are both prime, so 371 must also be prime.
c) Check to see whether 371 is divisible by any prime number less than 20.
d) Break 371 into 37 and 1; they are both prime, so 371 must also be prime.

Fig. 7 Sample CCK item from the LMT database of MKT multiple choice items. Released items are available from http://sitemaker.umich.edu/lmt/files/LMT_sample_items.pdf

3.2 Use of MKT Measures to Study Practicing Teachers

Because the studies that used the MKT measures to study teachers' knowledge are country-specific, they differ from each other in terms of how samples of respondents were selected, in terms of the specific measures used, and in terms of how data were analyzed. This is because studies conducted to date have not been coordinated centrally and the purpose and the resources available differed from one setting to another. What they have in common is that they all accessed the database of multiple choice measures of MKT that were developed at the University of Michigan. The measures available related primarily to three domains of MKT: CCK, SCK, and KCS.

Figure 7 is an example of an MKT item which is considered to tap into teachers' common content knowledge (CCK). A teacher responding to the item needs to consider various strategies that were used to check if a number is prime, where only one strategy is valid. Answer (a) is incorrect because it is possible that 371 could be divisible by two numbers higher than 9 and not by a number less than 9. The number 19 is the square root of 371 making 19 a factor (although in this case 7, one of the numbers to be checked, is also a factor). Response option (b) is incorrect because it is invalid to test for primes by breaking down the number additively into hundreds and tens and units. In this specific case, for example, the 3 represents 300 which is not a prime number. Response (c) is correct because 20 is greater than the square root of 371 and if 371 has a prime factor greater than 20, it must have one that is less than 20 as well. Response (d) is incorrect for reasons similar to response (b). This item taps into general mathematical knowledge of prime numbers and although it is information a teacher would use, it is not specific to the work of teaching and it requires no knowledge of students or teaching.

Figure 8 is an example of an MKT measure that taps into specialized knowledge of mathematics. A respondent is expected to analyze four different potential repre-

At a professional development workshop, teachers were learning about different ways to represent multiplication of fractions problems. The leader also helped them to become aware of examples that do not represent multiplication of fractions appropriately.

Which model below cannot be used to show that $1\frac{1}{2} \times \frac{2}{3} = 1$? (Mark ONE answer.)

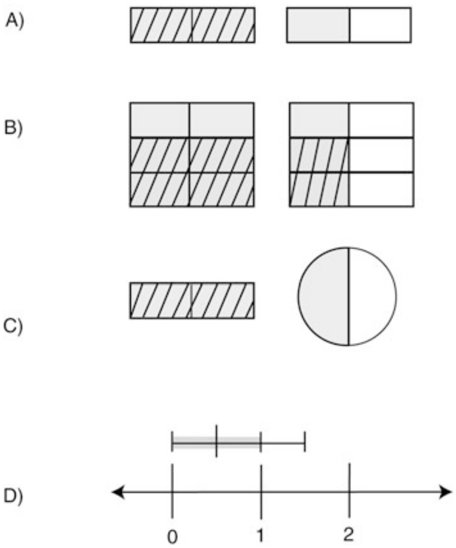
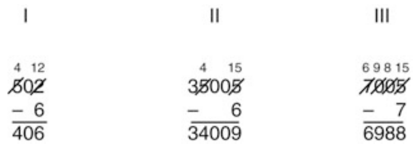


Fig. 8 Sample SCK item from the LMT database of MKT multiple choice items. Released items are available from http://sitemaker.umich.edu/lmt/files/LMT_sample_items.pdf

sentations of a calculation involving multiplication of fractions. In response (a) two unit area models are shown, with 1½ shaded in grey and 2/3 of the 1½ indicated with oblique lines. Response (b) is similar but the units are partitioned into sixths rather than halves. The third model is considered to be unsuitable because one unit is a rectangle and the second is a circle. Response (d) uses a number line model to show a line measuring 1½ partitioned in three equal parts, each measuring 1/2. Two of these parts (or 2/3 of the line measuring 1½ units) are shaded indicating the product of 1½ and 2/3. The knowledge required here is purely mathematical but it is mathematical knowledge that is specific to teaching.

Figure 9 is a sample KCS item and draws on the teacher's knowledge of mathematics and students combined. In three samples of work, students have subtracted incorrectly. However, the errors made differ. In sample I, the student exchanged one hundred for ten units and in sample II the student exchanged one thousand for ten units. A teacher who has knowledge of students and mathematics will recognize that both problems relate to a misunderstanding of renaming a number using principles of place value. In sample III the student exchanged one thousand for nine hundreds, eight tens and ten units, which is also incorrect but the error made is more sophis-

Mrs. Jackson is planning mini-lessons for students around particular difficulties that they are having with subtracting from large whole numbers. To target her instruction more effectively, she wants to work with groups of students who are making the same kind of error, so she looks at a recent quiz to see what they tend to do. She sees the following three student mistakes:



Which have the same kind of error? (Mark ONE answer.)

a) I and II

b) I and III

c) II and III

d) I, II, and III

Fig. 9 Sample KCS item from the LMT database of MKT multiple choice items. Released items are available from http://sitemaker.umich.edu/lmt/files/LMT_sample_items.pdf

ticated than a direct swap of hundreds or thousands for ten units as is the case in I or II, so the correct answer is (a).

A substantial database of MKT items has been developed and many of these have appeared on various survey forms used in studies by the research group. The LMT research team has set a standard for using such items and for analyzing responses. The purposes for which measures may be used and how results are presented are restricted and users of the measures are required to participate in a training workshop before using the measures. They may not be used for high stakes purposes such as decisions related to appointments or tenure and raw frequencies cannot be discussed publicly or compared with other groups (http://sitemaker.umich.edu/lmt/terms). The measures are suitable for measuring growth in teacher knowledge, studying how teacher knowledge relates to student achievement, studying the mathematical knowledge that teachers need, and for looking at how that knowledge is organized (Hill et al. 2004).

Factor analyses of responses to the items can be compared to the hypothesized domains of teacher knowledge (Hill et al. 2004). Item response theory (IRT) models can be used to score the teachers' responses in standard deviations where the mean is 0 (Hill 2007). The difficulty of the items and their ability to discriminate among teachers are also estimated using IRT models. Studies were conducted in the United States to evaluate the validity of using the MKT measures. Although more work remains to be done on validation, and in understanding MKT and its measurement, even within the United States, Hill et al. (2007a) found that teachers' scores on the measures could predict mathematical features of the teachers' instruction and student achievement.

One early study by the group looked at how the MKT measures might be adapted for use outside the United States (Delaney et al. 2008). This study made four categories of changes to items: to the general cultural context, to the school cultural context, to the mathematical substance, and other changes. As other researchers have used the measures in various countries, including non-English speaking countries, they have proposed additional or alternative categories of changes to be considered (see Mosvold et al. 2009; and Kwon et al. 2012) when measures developed for use in one country are adapted for use elsewhere.

4 Core Results on the Professional Knowledge of Mathematics Teachers

4.1 Prospective Teachers

4.1.1 MCK and MPCK by Countries

Prospective primary teachers from Taiwan achieved the most favourable MCK result of all of the countries participating in the TEDS-M primary study (Blömeke et al. 2011b). The difference from the international mean was large—more than one standard deviation, which is according to Cohen (1988) a highly relevant difference. The achievement of primary teachers from the US was slightly above the international mean and roughly on the same level as the achievement of teachers in Germany and Norway. Their difference from the international mean was significant but of low practical relevance. These groups of teachers also reached significantly lower performance levels than Swiss and Thai teachers. If we take into account the Human Development Index used by the UN in order to indicate the social, economic, and educational developmental state of a country, the high performance of teachers from Russia and Thailand was striking.

Regarding MPCK, the achievement of prospective primary teachers from the US was roughly on the same level as the achievement of teachers in Norway, which was significantly above the international mean. In this case, the difference from the international mean was of practical relevance. Teachers from Singapore and Taiwan outperformed the US teachers. Whereas Singapore was behind Taiwan in the case of MCK, these countries were on the same level in the case of MPCK. Regarding MPCK, Norway and the US were only half of a standard deviation behind the two East Asian countries, whereas this difference reached one standard deviation regarding MCK.

Prospective lower secondary teachers from Taiwan, Russia, Singapore, Poland, and Switzerland significantly outperformed teachers from the other countries regarding MCK (Blömeke et al. 2012a). If we take into account the Human Development Index, the performance of lower secondary mathematics teachers from Russia and Poland was remarkable. Regarding MPCK, the achievement of Taiwanese and Russian teachers was outstanding. The achievement of teachers from Singapore, Switzerland, and Russia was also well above the international mean.

The ranking of countries in TEDS-M was very similar to the ranking of countries in TIMSS (Mullis et al. 2008), which allows a preliminary tentative conclusion that we are talking about a cyclic relationship—with the option to improve student achievement by increasing mathematics teachers' professional knowledge.

Hsieh, Lin, and Wang (this book) examined the TEDS-M data based on the notion of thought-oriented mathematical knowledge (Niss 2002). They found that although Taiwan outperformed Singapore, these two East Asian countries shared the same structural pattern in their responses (see also Hsieh et al. 2011). The pattern suggested that, when compared with other countries, Taiwanese and Singaporean teachers performed relatively better with respect to mathematical language, including representing mathematical entities and handling mathematical symbols or formalisms, than with respect to modeling and reasoning.

As another relative weakness of prospective teachers in Taiwan, the authors discovered that—when compared with other countries—Taiwan performed worse in diagnosing student achievement. This result confirmed findings of domestic studies in Taiwan that teachers demonstrated an incomplete understanding of student learning. Overall, these results suggest a specific cultural pattern in teacher performance in Taiwan as discussed in the literature (Leung 2001; Leung et al. 2006) and probably related to basic cultural features of the society (Hofstede 1983, 1993).

Senk, Tatto, Reckase, Rowley, Peck, and Bankov (this book) point out large structural variations across countries in how teachers were trained to teach mathematics. The authors group teacher training programs into four groups. Primary teachers trained as mathematics specialists tend to have higher MCK and MPCK than those trained as generalists. However, within each group of teacher training programs differences of about one to two standard deviations in MCK and MPCK occur between the highest and the lowest achieving countries. The authors infer from these results that the relative performance within countries may vary greatly, especially if more than one teacher training program exists.

MCK and MPCK are strongly correlated in most countries that took part in TEDS-M (Blömeke et al. 2010a, 2010b). However, the strength varies by country and level, which may be an indicator of cultural differences on the one hand and differences in the nature of primary and lower secondary teachers' knowledge on the other. With respect to primary teachers, the weakest correlation exists in Botswana, the Philippines, and Singapore (around $r = 0.30$) whereas it is strongest in Poland, Russia, and Germany (around $r = 0.60$). With respect to lower secondary teachers, the correlations are mostly stronger (around $r = 0.70$ in Germany, Russia, and Poland; around $r = 0.40$ in the Philippines, Switzerland, Oman, and Taiwan).[6]

[6] Botswana is an exception here with $r = 0.18$.

4.1.2 Factors Related to Prospective Mathematics Teachers' Professional Knowledge

According to the TEDS-M results, countries differed with respect to the demographic background of their prospective teachers and the opportunities to learn they received during teacher training (Tatto et al. 2012). Both the individual and the institutional characteristics levels influenced the acquisition of teachers' professional knowledge (Blömeke et al. 2012a; Schmidt, Houang, and Cogan, this book; Wong, Boey, Lim-Teo, and Dindyal, this book).

Influence of Individual Characteristics on Teacher Knowledge

Gender Effects With respect to prospective primary teachers from the 15 countries that participated in TEDS-M, we have to note significant achievement differences in favor of male compared with female teachers in most countries (Blömeke et al. 2011b). Pronounced gender gaps existed at the end of teacher training in particular with respect to MCK. However, the gender effect did not apply to the same extent to MPCK; in Malaysia, female teachers even outperformed male teachers. The MCK differences between male and female teachers were the largest in Poland, whereas only in a few countries (Malaysia, the Philippines, Botswana, and Germany) did no significant differences occur. The comparative study MT21 provided first evidence that gender-related achievement differences in MCK might apply to lower secondary teachers as well (Blömeke and Kaiser 2010).

The MCK gender effect may be cumulative. MCK has been built over a long period of schooling and teacher training, whereas MPCK was taught only at university. The well-known K-12 disadvantages in the mathematics achievement of girls compared with boys in Western countries (Hyde et al. 2008) may result in differences in prior knowledge as well as in motivational differences and thus play out in teachers' MCK.

That several countries were able to avoid gender inequalities indicates that teacher achievement may reflect cultural patterns. With respect to MPCK, its pedagogical nature may reduce women's disadvantages, too. Evidence exists that female future teachers tend to support pedagogical motives more strongly than male future teachers do, specifically in comparison to subject-specific motives (Eberle and Pollak 2006).

Language Effects Another background characteristic associated with prospective teachers' professional knowledge in some countries was their *language* background (Blömeke et al. 2011b). In Germany, the United States, and Thailand, differences of high practical relevance occurred in MCK as well as in MPCK. The differences were always in favor of those teachers whose first language matched the official language of instruction in teacher training. Thus, students with a minority background, speaking a different language at home from the language of instruction, were at a disadvantage.

This difference may result from selection effects during schooling. The language background is an important predictor of K-12 achievement (Coleman et al. 1966; Thomas and Collier 1997). Students with a different language background from the one used in instruction may have been filtered out during schooling or at the beginning of teacher training. An indicator for this interpretation is that the proportion of teachers with a different language background is lower in the prospective teacher force than on average in the K-12 student population. Again, several countries (e.g., Spain) were successful in avoiding differential language effects. It would be worthwhile to examine in detail how these countries accomplished language equity.

Prior Knowledge Characteristics strongly associated with prospective teachers' MCK and MPCK, not only in a few countries but more or less universally, were the perceived high-school achievement as well as the number of mathematics classes at school (Blömeke et al. 2012a). Effect sizes were large in both cases. Assuming that both predictors were appropriate to indicate prior knowledge, these results are in accordance with the general state of research (see, e.g., Anderson and Lebière 1998; Simmons 1995). Higher prior knowledge facilitates the acquisition of new knowledge, for example by supporting the integration of new information into existing schemata, the modification of knowledge structures, or the compilation and chunking of knowledge.

Motivation Effects A final set of individual characteristics associated with MCK and MPCK was motivation. Subject-related motives were positively related whereas extrinsic motives were negatively related to teacher training outcomes if other individual predictors were controlled (Blömeke et al. 2012a). It seems as if the persistence to overcome mathematics-related learning difficulties or to invest time and energy in the learning of mathematics decreases if somebody wants to become a teacher primarily because she wants the long-term security of the job but increases if she is interested in the subject (Wigfield and Eccles 2000).

Several countries concluded from these kinds of results that an active recruitment policy is necessary in order to ensure a high-quality pool of applicants from which teacher training can draw. This, however, raises questions of feasibility with respect to salary, prestige, and motivation as the teaching profession does not always compare favorably with other professions available to highly mathematically literate college graduates. In Singapore, all students selected into teacher training receive full salaries as if they were already practicing. Tuition fees are paid by the state (Wong, Boey, Lim-Teo, and Dindyal, this book). Similarly, South Korea and Taiwan recruit their future teachers from the best high school graduates. In contrast, top students in many Western countries choose law or medicine as study fields. Schmidt, Houang, and Cogan (this book) describe therefore with respect to the US a different measure to accomplish the same objective. If the US were to successfully implement the new federal mathematics curriculum, the achievement of students in eighth grade might look different and resemble more that of high-scoring countries. This might then result in better entrance characteristics even if future teachers were drawn from the same part of the distribution as is currently done. In any case, it is necessary to have in mind that teacher knowledge at the end of teacher training is partly an amalgam of the teachers' educational background and cultural and social characteristics.

Institutional Characteristics Prior to TEDS-M, the state of research had indicated large cultural diversity in the curricula of teacher training across countries (Bishop 1988). However, a study by Adler et al. (2005) analyzed 160 papers about mathematics teacher training and pointed out that most studies were restricted to small-scale qualitative research and English-speaking countries. In quantitative studies, only the type of license or the number of courses taken was used to define OTL. These measures reflected the amount of content coverage without taking into account which content was offered.

TEDS-M was the first study that provided in-depth information about OTL. Blömeke and Kaiser (this book) summarize with respect to primary teacher training that, in fact, a comprehensive core curriculum accepted in all TEDS-M countries neither existed in mathematics pedagogy nor in mathematics. At the same time, the authors were able to conclude that the heterogeneity may be less pronounced than usually discussed. In mathematics (number theory and probability) and mathematics pedagogy (teaching methods), topics existed which were taken by most prospective primary teachers. Blömeke (2013) reached a similar conclusion with respect to prospective lower-secondary teachers.

Furthermore, it was sufficient to distinguish between a few profiles of OTL in mathematics ("advanced university mathematics", "basic university mathematics", and a restriction to "school mathematics") as well as in mathematics pedagogy (a "broad mathematics pedagogy curriculum", a "functional mathematics pedagogy curriculum", and "teaching methods") to describe appropriately the mathematics and mathematics pedagogy curriculum of primary teacher training across the participating countries. The dominating philosophy across the TEDS-M countries was to provide OTL in basic university mathematics and a broad mathematics pedagogy curriculum. A recent study of Blömeke (2013) on lower-secondary teacher education revealed a similar pattern.

The common topics and the low number of profiles may reflect shared visions of what primary and lower-secondary teachers are supposed to know before they enter the profession. This result confirmed a prior result from a comparative study on lower-secondary mathematics teacher training in six countries (Blömeke et al. 2008b; Schmidt et al. 2011). Its data indicated that specific OTL profiles may exist and that these may have been influenced by culture: in five countries, the multiple institutions where teacher training took place tended to cluster together with respect to the OTL offered, suggesting country-level agreement reflecting a cultural effect (Schmidt et al. 2008).

However, a closer examination of the OTL also revealed patterns of high-achieving vs. low-achieving TEDS-M countries. The level of mathematics and the emphasis of mathematics teaching practice were higher in the first cases (Hsieh et al. 2013). Schmidt, Houang, and Cogan (this book) confirmed these aggregated country-level results through multi-level modeling. They found within-country evidence of a significant relationship between teacher training and both MCK and

MPCK. OTL in mathematics and mathematics pedagogy predicted an increase in MCK of about half of a standard deviation, which is a substantial effect size. The largest single effect was produced by OTL closely related to experiences with mathematics instruction. OTL in mathematics had not only a strong direct influence on MCK, though, but also on MPCK, and they probably mediated the effects of OTL in mathematics pedagogy (Blömeke et al. 2012a).

Wong, Boey, Lim-Teo, and Dindyal (this book) summarized in this sense the evidence for Singapore that teacher training counts. Even well-qualified recruits require proper training to realize their potential. Therefore, programs are regularly revised in Singapore to ensure that they are responsive to both external changes such as recruitment numbers and education initiatives launched by the state and within-institution research and feedback from the student teachers about their training.

The TEDS-M results are in line with recent evidence from large national longitudinal studies. These had revealed that pure structural features, such as program or degree type, do not have significant effects on short-term outcomes of teacher training, such as teacher knowledge, or long-term outcomes, such as teacher retention or student achievement (Goldhaber and Liddle 2011). In contrast, especially in the case of mathematics teachers, the evidence increasingly suggests that the quality of programs does have an impact on teacher outcomes (Boyd et al. 2009; Constantine et al. 2009). Content courses in mathematics are an important part of these quality features as they provide the background knowledge and the conceptual and factual knowledge necessary to present mathematics topics to learners in a meaningful way and to connect the topics to one another as well as to the learner's prior knowledge and future learning objectives (Cochran-Smith and Zeichner 2005; Wilson et al. 2001). In this context, the curriculum sequence and delivery seems to have an important influence on graduates' subject matter knowledge (e.g., Tatto et al. 2010).

Knowing the content, however, provides only a foundation for mathematics teaching. Student achievement is higher if a strong subject-matter background is combined with strong educational credentials (Clotfelter et al. 2006). The importance of professional preparation, specifically the understanding of how learners acquire mathematical knowledge, how to teach racially, ethnically, and linguistically diverse students, and using a wide array of instructional strategies, represents another robust finding of teacher education research across various studies (Constantine et al. 2009; NRC 2003). A third robust finding on the impact of OTL on the outcomes of teacher education is the quality of the teaching methods experienced—in particular, the opportunity to engage in actual teaching practices, such as planning a lesson or analyzing student work, rather than only listening to lectures (Boyd et al. 2009).

4.2 Practicing Mathematics Teachers

MKT is a practice-based theory in the sense that MKT is identified with reference to the practice of teaching. Because the practice of teaching can vary from country to country, the content of MKT could vary accordingly. Furthermore, the MKT measures differ from other instruments for measuring teacher knowledge because they are not criterion referenced (Hill et al. 2007b). Criteria stating what mathematical knowledge teachers should hold have not been specified. Instead the measures were designed so that teachers could be ordered "relative to one another and to the underlying trait being assessed" (p. 131). Therefore the studies of the MKT held by practicing teachers in various countries could not be compared as the knowledge of prospective teachers could be in the TEDS-M study. In some cases, the researchers' goal was not so much to measure teachers' knowledge at present but to develop guidelines to enable such study in the future.

For example, the study by Kwon et al. (2012) considers changes made to the multiple-choice items in order to use them with Korean teachers. Despite initially proposing a more elaborate scheme of potential changes to items and despite making various changes, such as replacing terminology relating to base ten materials with an alternative term and subsequently with a diagram, the authors conclude by advocating a conservative approach to adaptation in order to maintain the integrity of the items. They identify potential risks that are inherent in adaptation, such as reducing or increasing the mathematical demand of the items or losing the validity of an item. Such an argument is helpful for others who seek to adapt the items for use outside the United States.

The study by Fauskanger et al. (2012) describes an iterative process that can be used to determine how well items—adapted or not—work when administered to a group of teachers outside the United States. Point biserial correlations (how teachers' performances on a given item correlate with their performances on all other items) and the relative ordering of item difficulties (measured in standard deviations where an item that a teacher with average ability has a 0.5 chance of responding correctly has a difficulty of 0) for US and Norwegian teachers was studied in order to identify items which could be probed further in focus group interviews. Although different causes of the problems were hypothesized and the precise problems ultimately remain somewhat uncertain, possible reasons include differences in defining mathematical objects, the use of unfamiliar teaching contexts, and problematic aspects of translation. When combined with the approach of Kwon and her colleagues (2012), the findings of the Norwegian study provide a means of identifying problematic items; but mindful of the risks inherent in any adaptation, changes should only be made if a compelling reason exists to do so.

Ng (2012) studied Indonesian teachers' performances on geometry items. Having studied the point biserial correlations, the item difficulties and the reliability of the measures and compared them to similar statistics among US teachers, he concludes that the geometry measures "may not be a good set of assessments to evaluate Indonesian teachers' mathematical knowledge for teaching geometry". He believes that a reason for this is attributable to national differences between the United States and Indonesia with regard to differences in how shapes are classified. Nevertheless, he believes that the process of studying the performance outside the United States of measures based on MKT can contribute to the overall development of our understanding of MKT.

Cole (2012) and Delaney (2012) evaluate the validity of using the MKT measures in Ghana and Ireland respectively. Cole looks at the consistency between three teachers' scores on individual items and the mathematical reasoning for choosing the responses they chose. Cole finds that most items could be used validly in Ghana despite evidence of "cultural incongruence". Sources for the incongruence came from possible differences in teaching practices, a specific question format, the length of time taken to do the test, and taking the test in English where despite English being the language of schooling, none of the teachers in the study were native English speakers.

Delaney (2012) attempted to validate the use of the measures to study Irish teachers' MKT. Like Cole (2012), he found that, in general, teachers' thinking was consistent with their responses to the measures. The factors found among teachers' responses to the items in Ireland are similar to the factors found among US teachers' responses. However, the organization of the factors in both countries differs from the hypothesized domains. Finally, Delaney (2012) found that only in the case of five of ten teachers did their MKT score predict the mathematical quality of instruction to be found in their teaching. This contrasted with findings in a similar study in the United States (Hill et al. 2007a). He concludes by identifying challenges to validating the use of the MKT measures in settings outside the United States.

Small-scale comparative studies of practicing teachers have been conducted using measures and frameworks other than MKT. An et al. (2004) compared teachers' PCK between Chinese and US groups, focused on fractions, ratio, and proportion. Based on Shulman's work, they conceptualized pedagogical content knowledge as an amalgam of content and pedagogical knowledge. As Ma (1999) also found, US teachers performed more poorly than Chinese teachers on the PCK measure. Further comparisons of opportunities to learn PCK in China and the US revealed that the Chinese had gained much of their knowledge through school-based pre-service training led by nationally well-known expert teachers and continuous professional development activities, especially by observing each other's lessons and jointly discussing them (An et al. 2004; Paine 1997; Paine and Ma 1993).

In more recent work, An (2009) studied 385 elementary school teachers at 37 schools in six cities/regions in four provinces in China. The association between CK and PK in three areas of content and six areas of pedagogy in multi-digit division was medium (Cramer's $V = 0.33$). The validation findings revealed that Chinese teachers' pedagogical knowledge was related to grade levels taught and to the majors they had chosen in college. These results provide evidence that their instruments in fact are able to measure meaningful differences between CK and PK in multi-digit division, but An concludes that further studies are needed to test PCK in other content areas.

5 Challenges of Comparative Large-Scale Assessments of Teacher Knowledge

It is a methodological challenge to assess teacher knowledge from a comparative perspective. Research perspectives have to be adjusted across borders and educational traditions. Furthermore, it is a methodological challenge to assess the development of knowledge among prospective teachers in the context of a differentiated tertiary education system. Not only do a variety of institutions, teacher training programs, and job requirements exist, but also the outcome is hard to define and even harder to measure.

5.1 Generalizability

In most of the studies reported above, a definition of teacher knowledge as a context-specific disposition was applied. The knowledge can be acquired and it is needed to cope successfully with domain-specific classroom situations and tasks. However, several controversies are unsolved. What is the role of attitudes and beliefs in this context? In future research, in addition to MCK and MPCK as subject-specific facets of mathematics teachers' knowledge, other cognitive criteria like general pedagogical knowledge or affective characteristics like teacher beliefs should be included (as was done in TEDS-M) in order to develop a full model. Such an approach would increase the validity of studies on teacher knowledge.

With respect to theory development and generalizability, it seems important not to focus solely on analytical approaches as used in TEDS-M and LMT. Shavelson (2012) unpacks competency as a complex ability construct closely related to real-life situation performance. He exemplifies how to make it amenable to measurement in a holistic way by research from business, military, and education sectors. The generalizability theory, a statistical theory for modeling and evaluating the dependability of competency scores, is applied to several of these examples. The paper then pulls the threads together into a general competency measurement model.

Shavelson (2012) points out that there are limitations to measuring competency in terms of resources, costs, and time on various levels. Performance assessment is an issue that has long been discussed (Kane 1992). It is difficult to generalize results from one situation to another, that is, there are problems with reliability (Brennan and Johnson 1995), and it is difficult to validate the measures (Kane et al. 1999). How representative are, for example, the situations to be worked on in a measurement?

We have presented the TEDS-M framework in some detail (see Table 1). Although it looks convincing, a comparison with the way that California evaluates its pre-service mathematics teachers' knowledge (Wu 2010) reveals that different approaches can be taken. The Teacher Performance Assessment (TPA) depicts classroom situations in a standardized way and according to the state's standards, "Teaching Performance Expectations". Four tasks have to be dealt with: connecting instructional planning to student characteristics, assessment, lesson design, and reflection. These have to be applied to (only) two groups of learners which are not present in the TEDS-M framework: English language learners and special education students. The difference between the TEDS-M and the Californian approaches reveals very different visions of what mathematics teachers are supposed to know and be able to do.

It is worthwhile to examine the phenomenon of context dependability and generalizability in more detail—also with new instruments beyond the limitations of paper-and-pencil tests. Approaches in this respect exist in several countries. Wu and Li (2008), for example, examine "the power to perform a teaching task" as the criterion of applying knowledge in the classroom. They videotaped 119 lessons from nine K-8 teachers, interviewed them, and assessed student achievement of these teachers. Observation instruments were designed based on components of the PCK (An et al. 2004). They discovered distinct patterns in teaching performance with differences in their relationship to student achievement. The TIMSS and TIMSS-R video studies had provided the first comparative evidence in this direction (though without testing teacher knowledge).

In this context, the slight difference in spelling between "competency" and "competence" in the English language may be relevant for evaluating the generalizability of results. Sadler (2012) points out that a conceptual distinction can be made between the two terms, which in turn leads to distinct measurement approaches. A "competency" often means an identifiable practice. "Competence", in contrast, often consists of a large number of discrete competencies—as is assumed in TEDS-M—which could be tested independently. Competence involves being able to select from and then orchestrate a set of competencies to achieve a particular end within a particular context. The competent person makes multi-criterion judgments that are consistently appropriate and situation-sensitive. What is more, the range of situations faced by many professional practitioners is potentially infinite. Decomposing competence into manageable components in order to facilitate judgments has value in certain contexts, but the act of decomposition can obscure how a practitioner would juggle the various bits together to form a coherent whole. It may be worthwhile to follow this assumption with research projects that compare the results of more integrative and holistic judgments with the results of more analytic approaches.

Also, it is necessary to consider the knowledge development of prospective and practicing teachers along a continuum of lifelong learning (Musset 2010). Such an approach would also allow including classroom observations of teacher performance and possibly even student achievement in order to examine the construct validity of measures.

5.2 *Benefits and Limits of Comparative Research*

Like everyone else, researchers are embedded in their own culture, and so they often overlook matters of culture. This is particularly the case for teacher training, given the unique way in which it incorporates or touches upon many different levels of education and stands at the intersection of education and other social, economic, and political forces (Blömeke and Paine 2008). This embedded character of the system of teacher training in any one country makes looking beyond that country's experience mandatory in order to recognize the assumptions which drive it, which are all too often taken for granted. The investigation of another teacher training system in a foreign country, for example, and the discovery that it is possible to organize the training differently, sheds new light on domestic systems. The recognition of this cultural boundedness of teacher training is an argument for approaching a comparative study in ways that maximize opportunities for cross-cultural communication and the direct examination of concepts (LeTendre 1999).

As such, language problems become important in comparative studies and are far more demanding to resolve than a "simple" translation of instruments or responses (NRC 2003). Of course, at one level, this is a common, familiar and well-studied aspect of cross-cultural studies, for which there are now widely-used conventions of translation, back translation and so on (Hambleton 2002). In teacher training, however, more language-related challenges exist that require attention. They are a problem of cultural boundaries. In some countries, Ghana for instance, the language of schooling may vary from the language of the home for many students. Many terms from native languages cannot be translated because adequate English terms are missing and vice versa. It is even difficult to name the process by which future teachers learn their profession: is it teacher education, is it teacher training, or is it perhaps teacher preparation?

These questions relate to deeper and often tacit assumptions about schooling, teaching, and learning to teach. As these terms connect to broadly shared cultural beliefs, the uniqueness of their meaning often is not explicit and can easily escape scrutiny unless outsiders to the cultural community stumble over them and begin to enquire about them (Blömeke and Paine 2008). Behind the apparently simple choice of whether to refer to the practice as teacher education, teacher training, teacher preparation, or something else, lie other aspects of history, policy, social values, and cultural norms. These are worth examining in detail.

These conceptual challenges of comparative research are extended by methodological challenges. Owing to the low number of countries, in the TEDS-M analyses a "one size fits all approach" (van Ewijk and Sleegers 2010) has to be used, though with parameter estimates the same for all countries in multi-level analyses (see, for example, Blömeke et al. 2012a). Thus, a risk exists that country-specific variation in the effect sizes of some predictors is overlooked. At least for the larger countries in the TEDS-M sample, it seems therefore worthwhile to estimate country-specific models in addition to cross-country models.

Future research should also examine in more detail the question of measurement invariance in TEDS-M (Blömeke et al. 2011a). Van de Vijver (1998, p. 43) points to a serious threat: "An instrument is biased if its scores do not have the same psychological meaning across the cultural groups involved; more precisely, an instrument is biased if statements about (similarities and differences of) its scores do not apply in the psychological domain of the scores." Equivalence is thus the objective to be achieved. It consists of several dimensions (Vandenberg and Lance 2000): conceptual equivalence of the latent trait in each group, equivalent associations between operationalizations in each group, and the extent to which they are influenced to the same degree by the same factors. Despite agreement on the importance of achieving equivalence, however, many terms have been used to describe aspects of equivalence. Johnson (1998) has identified over fifty of them, several of which overlap and many of which have not been well defined.

Hierarchical IRT and multiple-group confirmatory factor analysis provide the tools to determine important properties such as configural invariance, metric invariance, and scalar invariance (Fox 2005; Vandenberg and Lance 2000). Even if full invariance—which is rarely accomplished in cross-cultural research—cannot be determined for TEDS-M, such studies would reveal the extent to which partial invariance is supported. Approaches could then be taken to deal appropriately with the problems. Using hierarchical IRT, for example, De Jong et al. (2007) were able to relax all invariance requirements across groups while retaining the possibility of making substantive comparisons. Such studies would be relevant not only with respect to the TEDS-M assessment data but also, and perhaps more importantly, with respect to the OTL and beliefs data, given the likelihood of self-reported data being even more vulnerable to bias (Blömeke et al. 2010a, 2010b).

5.3 Research on Teacher Knowledge as a Tool to Improve Teacher Training

Wong, Boey, Lim-Teo, and Dindyal (this book) discuss the value of studies such as TEDS-M for teacher training. They point out that the released MCK and MPCK items can be used as a training resource. In fact, the Singaporean TEDS-M team is preparing a book consisting of these released items, the scoring guides, the Singapore results against international benchmarks, and samples of constructed responses. Teacher educators can then use these materials in their lessons with future cohorts of prospective teachers by, for example, exploring strategies to remedy misconceptions, designing classroom activities that mirror the scenarios described in the TEDS-M items, and linking the assessment items to the TEDS-M framework and thus analyze conceptions of teacher knowledge. Thus, although the TEDS-M items were originally created as a summative assessment of teacher knowledge at the end of teacher training, they can be used as a formative assessment of teacher knowledge.

Teacher educators may also want to compare the outcomes of different programs and different institutions in their country. Within almost all countries, huge between-program disparity existed. This means that within the same cultural context some institutions are doing better than others. They may represent a benchmark and provide important information about features of teacher education which can be more easily adapted than features from other countries. In particular, the structure and content of the mathematics and the mathematics pedagogy curriculum should be put to the test.

From those countries achieving high scores in TEDS-M, we may want to learn about promising ideas on how to organize teacher training programs. Again the Singapore example may serve as a role model (Wong, Boey, Lim-Teo, and Dindyal, this book). Here, mathematicians teach the content courses and mathematics educators teach the pedagogical content courses, but they belong to the same department. Under this organization, there are many opportunities for mathematicians and mathematics educators to work in committees and projects that draw on their separate expertise to achieve the common goal of training competent mathematics teachers. They can also share information about the prospective teachers. Furthermore, all mathematicians learn to supervise practica of prospective teachers at secondary schools through a process of mentoring, and this requirement provides an important opportunity for them to observe first-hand school mathematics teaching and to share their views as a subject specialist with the prospective teachers.

For achieving an increase of teacher education effectiveness, the TEDS-M study points to two further measures, each with separate effects. Providing OTL in mathematics as well as increasing entrance selectivity may have positive consequences for the outcomes of teacher training and thus in the long run for student achievement in mathematics. Mathematics is one of the most important school subjects and a gatekeeper to academic and professional success. Investments in the training of teachers should therefore pay off quickly. Entrance selectivity is a sensitive issue, however. Not everywhere is teaching such a popular and rewarding job that enough applicants for teacher education are available. Higher selectivity, however, may increase the reputation of the profession in the long run so that institutions can recruit from a larger pool.

In addition to such reforms, policymakers have to be aware of the continuing problem of societal inequalities in teacher education outcomes. Special support of female teachers when it comes to the acquisition of MCK in order to overcome cumulative disadvantages of a long history of K-12 schooling seems to be a meaningful measure in many TEDS-M countries.

5.4 Adaptation

Because MKT is a practice-based theory (Ball and Bass 2003), and teaching practices may be cultural in nature (Stigler and Hiebert 1999), differences may exist in relation to the form MKT takes in different countries. In order that teachers in a given country can respond to the items without being distracted by names or contexts that would be unusual or non-existent for them, some adaptation is necessary. Indeed, all of the studies of MKT adapted the measures they used in their research. However, such adaptations must avoid distorting the mathematical content or the mathematical demands of the measures. Guidelines for adaptation exist for studies such as TIMSS (Johansone and Malak 2008) and PISA, and similar guidelines will be needed for measures of MKT if the measures are to be used in diverse countries around the world. Recommendations from the studies in the 2012 ZDM special issue (see, e.g., the papers by Cole 2012; Kwon et al. 2012) could be useful in developing such guidelines.

5.5 Validation

Despite the challenges found by Cole (2012) and Delaney (2012), and despite the general absence of validity analyses in educational assessments (Hill et al. 2007b), validation of the use of the measures needs to be an important part of studies of MKT in any countries in which the measures are used. Attending to this will help to clarify the organization of the subdomains of MKT. It will also ensure that the items are tapping knowledge that is needed to teach mathematics. Above all, it will ensure that the measures are valid for the uses to which they will be put.

6 Concluding Remarks

This review has presented an overview of research on the assessment of mathematics teachers' knowledge as one of the most important parameters of school quality. Its focus has been on comparative and international studies that allow for analyzing the cultural dimensions of teacher knowledge. We presented in detail the conceptual frameworks underlying TEDS-M and MKT/LMT, the instruments designed to assess the content knowledge and pedagogical content knowledge of future and practicing mathematics teachers in different countries, and core results of its level and structure.

Although cross-national and comparative surveys of student knowledge have a longer track record and attract more sustained funding and attention than studies of teacher knowledge, the current book acknowledges the importance of learning about teacher knowledge by studying it beyond individual countries. More needs to be learned about adapting and validating measures for use in other countries. However, studying across countries has the potential to offer insights into the original frameworks and contribute to a better and clearer conception of the frameworks underlying TEDS-M and MKT/LMT.

It appears as if—not only in the context of TEDS-M and MKT/LMT—the research on mathematics teacher knowledge has made important progress. Shulman's model of teacher knowledge leads many studies so that the traits examined intend to represent the same. Our summary in Sect. 1 demonstrates this intention with respect to TEDS-M and LMT. The similarities in turn make it easier to compare the instruments and to connect the studies' results to each other than was the case in prior research.

Still, many challenges exist. Cross-country equivalence of meaning and predictive validity are the most important ones. The many studies connected to LMT had their focus on this perspective. LMT started as a one-nation enterprise but is expanding to many countries all over the world. In contrast, TEDS-M started as a comparative study but is now complemented by many national studies that go into more detail. We are starting to get ideas about how teacher knowledge develops and how it is connected to teacher education and student achievement. It turns out, as assumed, that teacher knowledge is the crucial link between mathematics teacher education and student achievement in mathematics. How much, and in what quality, opportunities to learn are provided significantly influences the knowledge achieved during teacher training.

In turn, teacher knowledge represents an important predictor of student achievement because a mathematics teacher's decision making in class is a function, among others, of her mathematical knowledge (Schoenfeld 2010). Thus, the two perspectives together—research on prospective and practicing teachers—provide an appropriate view on the relationship between mathematics teacher education and what is accomplished in schools, although multiply mediated by complex context factors. Here, much further research is needed so that we will be able to understand the nature of teacher competencies underlying classroom performance.

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S. Blömeke et al. (eds.), *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn*, Advances in Mathematics Education,
DOI 10.1007/978-94-007-6437-8, © Springer Science+Business Media Dordrecht 2014