Periodic Solution and Strange Attractor in Impulsive Hopfield Networks with Time-Varying Delays

Yanxia Cheng, Yan Yan and Zhanji Gui

Abstract By constructing suitable Lyapunov functions, we study the existence, uniqueness and global exponential stability of periodic solution for impulsive Hopfield neural networks with time-varying delays. Our condition extends and generalizes a known condition for the global exponential periodicity of continuous Hopfield neural networks with time-varying delays. Further the numerical simulation shows that our system can occur many forms of complexities including gui strange attractor and periodic solution.

Keywords Hopfield neural network · Lyapunov functions · Pulse · Time-varying delay · Periodic solution · Strange attractor

1 Introduction

In recent years, stability of different classes of neural networks with time delay, such as Hopfield neural networks, cellular neural networks, bidirectional associative neural networks, Lotka-Volterra neural networks, has been extensively studied and various stability conditions have been obtained for these models of neural

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networks. A citation will look like this, [1, 3, 6]. Here are some more citations [5, 10, 13, 16, 17].

Stability and convergence properties are generally regarded as important effects of delays. Both in biological and man-made neural systems, integration and communication delays are ubiquitous, and often become sources of instability. The delays in electronic neural networks are usually time varying, and sometimes vary violently with time due to the finite switching speed of amplifiers and faults in the electrical circuit. They slow down the transmission rate and tend to introduce some degree of instability in circuits. Therefore, fast response must be required in practical electronic neural-network designs. The technique to achieve fast response troubles many circuit designers. So, it is important to investigate the delay independent stability and decay estimates of the states of analog neural networks.

However, in implementation of networks, time delays are inevitably encountered because of the finite switching speed of amplifiers, see [2, 4, 7, 11, 12]. On the other hand, impulsive effect likewise exists in a wide variety of evolutionary processes in which states are changed abruptly at certain moments of time, involving such fields as medicine and biology, economics, mechanics, electronics and telecommunications, etc. Many interesting results on impulsive effect have been gained. Here are some more citations [2, 4, 7–9, 11, 12, 15, 18]. As artificial electronic systems, neural networks such as Hopfield neural networks, bidirectional neural networks and recurrent neural networks often are subject to impulsive perturbations which can affect dynamical behaviors of the systems just as time delays. Therefore, it is necessary to consider both impulsive effect and delay effect on the stability of neural networks.

In this chapter, we consider the following impulsive Hopfield neural networks with time-varying delays:

$$\begin{cases} \dot{x}_i(t) = -a_i x_i(t) + \sum_{j=1}^{n} a_{ij} g_j(x_j(t)) \\ \qquad + \sum_{j=1}^{n} b_{ij} g_j(x_j(t - \tau_{ij}(t))) + I_i(t), \quad t \neq t_k, \\ \Delta x_i(t_k) = \gamma_{ik} x_i(t_k), \quad i = 1, 2, \ldots, n, \quad k = 1, 2, \ldots, \end{cases} \tag{1}$$

where n is the number of neurons in the network, $x_i(t)$ is the state of the ith neuron at time t, a_{ij} is the rate at which the ith neuron resets the state when isolated from the system, b_{ij} is the connection strength from the jth neuron to the ith neuron. $g(x) = (g_1(x_1), g_2(x_2), \ldots, g_n(x_n))^T : \mathbb{R}^n \to \mathbb{R}^n$ is the output of the ith neuron at time t, $I(t) = (I_1(t), I_2(t), \ldots, I_n(t))^T \in \mathbb{R}^n$ is the ω-periodic external input to the ith neuron.

Throughout this chapter, we assume that

(**H**$_1$) For $j \in \{1, \ldots, n\}$, $g_j(u)$ $(j = 1, 2, \ldots, n)$ is globally Lipschitz-continuous with the Lipschitz constant $L_j > 0$. That is,

$$|g_j(u_1) - g_j(u_2)| \leqslant L_j |u_1 - u_2|,$$

for all $u_1,\ u_2 \in \mathbb{R} = (-\infty, \infty)$.

($\mathbf{H}_2$) There exists a positive integer p such that, $t_{k+p} = t_k + \omega, \gamma_{i(k+p)} = \gamma_{ik},\quad k > 0,\ k = 1, 2, \ldots$

($\mathbf{H}_3$) $\tau_{ij}(t)(i, j = 1, 2, \ldots, n)$ are continuously differentiable ω-periodic functions defined on $\mathbb{R}^+$, $\tau = \sup_{0 \leqslant t \leqslant \omega} \tau_{ij}(t)$ and $\inf_{t\in\mathbb{R}^+}\{1 - \dot{\tau}_i(t)\} > 0$.

In order to describe the initial condition accompanying Eq. (1), we introduce the following notations.

Definition 1 A function $\phi : [-\tau, 0] \to \mathbb{R}$ is said to be a C^*-function if the following two conditions are satisfied:

(a) ϕ is piecewise continuous with first kind discontinuity at the points t_k. Moreover, ϕ is left-continuous at each discontinuity point.
(b) For all $i \in \{1, \ldots, n\}$ and $k \in \{1, \ldots, p\}$, $\phi_i(t_k + 0) = \phi_i(t_k) + \gamma_{ik}\phi_i(t_k)$.

Let C^* denote the set of all the C^*-functions. Obviously, $(C^*, \mathbb{R}, +, \cdot)$ forms a vector space on $\mathbb{R}$. Now consider $(C^*, \mathbb{R}, +, \cdot)$ endowed with the norm defined by

$$\|\phi\|_\infty = \sup_{-\tau\leqslant\theta\leqslant0} \|\phi(\theta)\| = \sup_{-\tau\leqslant\theta\leqslant0} \max_{1\leqslant i\leqslant n} |\phi_i(\theta)|.$$

Definition 2 A function $x : [-\tau, \infty] \to \mathbb{R}$ is said to be the special solution of Eq. (1) with initial condition $\phi \in C^*$ if the following two conditions are satisfied:

(c) x is piecewise continuous with first kind discontinuity at the points t_k, $k \in \{1, \ldots, p\}$.
(d) x satisfies Eq. (1) for $t \geqslant 0$, and $x(\theta) = \phi(\theta)$ for $\theta \in [-\tau, 0]$.

Henceforth, we let $x(t, \phi)$ denote the special solution of Eq. (1) with initial condition $\phi \in C^*$

Definition 3 Equation (1) is said to be globally exponentially periodic if it possesses a periodic solution $x(t, \phi^*)$, and $x(t, \phi^*)$ is globally exponentially stable. That is, there exist positive constants ε and M such that every solution of Eq. (1) satisfies

$$\|x(t, \phi) - x(t, \phi^*)\|_\infty \leqslant M\|\phi - \phi^*\|e^{-\varepsilon t},\ forall\ t \geqslant 0.$$

2 Main Result

Now we define $\psi(t) = t - \tau_i(t)$, then $\psi^{-1}(t)$ has inverse function ν. Set

$$\delta_i = \max\left\{\frac{1}{1 - \dot{\tau}_i(\psi_i^{-1}(t))} : t \in \mathbb{R}\right\}, i = 1, 2, \ldots, n.$$

Theorem 4 *Equation* (1) *is globally exponentially periodic if the following two conditions are satisfied:*

(H₄) $|1+\gamma_{ik}| \leqslant 1$, *for all* $i \in \{1,\dots,n\}$, *and* $k \in \{1,\dots,p\}$,
(H₅) *There exist positive numbers* $\alpha_1, \alpha_2, \dots, \alpha_n$, *such that*

$$\alpha_i a_i > L_i \sum_{j=1}^{n} \alpha_j \big(|b_{ji}| + \delta_j |c_{ji}|\big), i = 1, 2 \dots, n.$$

In order to prove Theorem 4, we need the following Lemma.

Lemma 5 *Let* $x(t,\phi)$, $x(t,\varphi)$*be a pair of solutions of Eq.* (1)*. If the two conditions given in Theorem 4 are satisfied, then there is a positive number* ε *such that,*

$$\|x(t,\phi) - x(t,\varphi)\|_\infty \leqslant M(\varepsilon)\|\phi - \varphi\|_\infty e^{-\varepsilon t}, \quad \textit{for all} \ \ t \geqslant 0.$$

where

$$M(\varepsilon) = \frac{1}{\min\limits_{1\leqslant j\leqslant n} \alpha_j} \sum_{i=1}^{n} \alpha_i \left[1 + \frac{1}{\varepsilon} L_j \delta_j |c_{ij}| \big(e^{\varepsilon\tau} - 1\big)\right].$$

Proof Let $x(t,\phi) = (x_1(t,\phi), x_2(t,\phi), \dots, x_n(t,\phi))^T$ and $x(t,\varphi) = (x_1(t,\varphi), x_2(t,\varphi), \dots, x_n(t,\varphi))^T$ be an arbitrary pair of solutions of Eq. (1). Let

$$\Delta x_i(t,\phi,\varphi) = x_i(t,\phi) - x_i(t,\varphi),$$
$$\Delta g_j(x_j(t,\phi,\varphi)) = g_j(x_j(t,\phi)) - g_j(x_j(t,\varphi)),$$

$$V(t) = \sum_{i=1}^{n} \alpha_i \Bigg\{ |\Delta x_i(t,\phi,\varphi)| e^{-\varepsilon t} + \sum_{j=1}^{n} \int_{t-\tau_j(t)}^{t} \frac{L_j |c_{ij}| |\Delta x_j(s,\phi,\varphi)|}{1 - \dot{\tau}_j(\psi_j^{-1}(s))} e^{\varepsilon(s+\tau_j(\psi_j^{-1}(s))} ds \Bigg\}. \tag{2}$$

We proceed by considering two possibilities.

Case 1. $t \neq t_k$ for all $k \in \{1,\dots,p\}$. From the second condition in Theorem 4, there is a small positive number ε such that

$$\alpha_i(a_i - \varepsilon) > L_i \sum_{j=1}^{n} \alpha_j \big(|b_{ji}| + \delta_j |c_{ji}| e^{\varepsilon\tau}\big), \tag{3}$$

where $i = 1,\dots,n$. Calculating the derivatives of $V(t)$ along the solutions of Eq. (1), we get

$$
\begin{aligned}
D^{+}V(t) = \sum_{i=1}^{n} \alpha_i \Bigg\{ & e^{-\varepsilon t} D^{+} |\Delta x_i(t,\phi,\varphi)| \\
& + \sum_{j=1}^{n} \Bigg[\frac{L_j |c_{ij}|}{1-\dot{\tau}_j(\psi_j^{-1}(t))} |\Delta x_j(t,\phi,\varphi)| \cdot e^{\varepsilon(t+\tau_j(\psi_j^{-1}(t))} \\
& - L_j |c_{ij}| |\Delta x_j(t-\tau_j(t),\phi,\varphi)| e^{\varepsilon t} \Bigg] \\
& + \varepsilon e^{-\varepsilon t} |\Delta x_i(t,\phi,\varphi)| \Bigg\}. \qquad (4)
\end{aligned}
$$

Note that for $i = 1, \ldots, n$,

$$
\begin{aligned}
\dot{x}_i(t,\phi) - \dot{x}_i(t,\varphi) = & - a_i \Delta x_i(t,\phi,\varphi) \\
& + \sum_{j=1}^{n} b_{ij} \Delta g_j(x_j(t,\phi,\varphi) \\
& + \sum_{j=1}^{n} c_{ij} \Delta g_j(x_j(t-\tau_j(t),\phi,\varphi),
\end{aligned}
$$

which plus (H_1) yields

$$
\begin{aligned}
D^{+}|x(t,\phi) - x(t,\varphi)| \leqslant & - a_i |\Delta x_i(t,\phi,\varphi)| + \sum_{j=1}^{n} L_j |b_{ij}| |\Delta x_j(t,\phi,\varphi)| \\
& + \sum_{j=1}^{n} L_j |c_{ij}| |\Delta x_j(t-\tau_j(t),\phi,\varphi)|. \qquad (5)
\end{aligned}
$$

Substituting Eq. (5) into Eq. (4), we obtain

$$
\begin{aligned}
D^{+}V(t) = \sum_{i=1}^{n} \alpha_i \Bigg[& - a_i e^{-\varepsilon t} |\Delta x_i(t,\phi,\varphi)| \\
& + e^{-\varepsilon t} \sum_{j=1}^{n} L_j |b_{ij}| |\Delta x_j(t,\phi,\varphi)| \\
& + e^{-\varepsilon t} \sum_{j=1}^{n} L_j |c_{ij}| |\Delta x_j(t-\tau_j(t),\phi,\varphi)| \\
& + \varepsilon e^{-\varepsilon t} |\Delta x_i(t,\phi,\varphi)| \\
& + \sum_{j=1}^{n} \frac{L_j |c_{ij}|}{1-\dot{\tau}_j(\psi_j^{-1}(t))} |\Delta x_j(t,\phi,\varphi)| e^{\varepsilon(t+\tau_j(\psi_j^{-1}(t))}
\end{aligned}
$$

$$
\begin{aligned}
&\quad - \sum_{j=1}^{n} L_j |c_{ij}| |\Delta x_j(t-\tau_j(t), \phi, \varphi)| e^{\varepsilon t}\Bigg] \\
&\leqslant e^{-\varepsilon t} \sum_{i=1}^{n} \alpha_i \Bigg[(\varepsilon - a_i) |\Delta x_i(t, \phi, \varphi)| \\
&\quad + \sum_{j=1}^{n} L_j |b_{ij}| |\Delta x_j(t, \phi, \varphi)| \\
&\quad + \sum_{j=1}^{n} L_j \delta_j |c_{ij}| e^{\varepsilon \tau} |\Delta x_j(t, \phi, \varphi)| \Bigg] \\
&= e^{-\varepsilon t} \sum_{i=1}^{n} \Bigg[\alpha_i (\varepsilon - a_i) \\
&\quad + L_i \sum_{j=1}^{n} \big(|b_{ji}| + \delta_i |c_{ji}| e^{\varepsilon \tau} \big) \Bigg] \cdot |\Delta x_i(t, \phi, \varphi)| \leqslant 0. \qquad (6)
\end{aligned}
$$

Case 2. $t = t_k$, for some $k \in \{1, 2, \ldots, p\}$. Then

$$
\begin{aligned}
V(t+0) = \sum_{i=1}^{n} \alpha_i \Bigg[&|\Delta x_i(t+0, \phi, \varphi)| e^{-\varepsilon t} \\
&+ \sum_{j=1}^{n} \int_{t-\tau_j(t)}^{t} \frac{L_j |c_{ij}| \Delta x_j(s, \phi, \varphi)}{1 - \dot{\tau}_j(\psi_j^{-1}(s))} e^{\varepsilon (s + \tau_j(\psi_j^{-1}(s)))} ds \Bigg].
\end{aligned}
$$

According to Eq. (2) and (H_4), we obtain

$$
\begin{aligned}
V(t+0) - V(t) &= e^{-\varepsilon t} \sum_{i=1}^{n} \alpha_i \Big(|\Delta x_i(t+0, \phi, \varphi)| - |\Delta x_i(t, \phi, \varphi)| \Big) \\
&= -e^{-\varepsilon t} \sum_{i=1}^{n} \alpha_i \big(1 - |1 + \gamma_{ik}| \big) |\Delta x_i(t, \phi, \varphi)| \leqslant 0.
\end{aligned}
$$

Namely, $V(t+0) \leqslant V(t)$.

Combining the above discussions, we obtain $V(t) \leqslant V(0)$ for all $t \geqslant 0$. This plus the inspections that

$$
\begin{aligned}
V(t) &\geqslant e^{\varepsilon t} \sum_{i=1}^{n} \alpha_i |\Delta x_i(t, \phi, \varphi)| \\
&\geqslant \min_{1 \leqslant j \leqslant n} \alpha_j e^{\varepsilon t} \sum_{i=1}^{n} |\Delta x_i(t, \phi, \varphi)|
\end{aligned}
$$

$$\geqslant \min_{1\leqslant j\leqslant n} \alpha_j e^{\varepsilon t} \big\| x_i(t,\phi) - x_i(t,\varphi) \big\|_\infty, \tag{7}$$

and

$$\begin{aligned}
V(0) &= \sum_{i=1}^{n} \alpha_i \Bigg[|x_i(0,\phi) - x_i(0,\varphi)| \\
&\quad + \sum_{j=1}^{n} \int_{-\tau_j(0)}^{0} \frac{L_j|c_{ij}|}{1-\dot{\tau}_j(\psi_j^{-1}(s))} \cdot \Delta x_j(s,\phi,\varphi) e^{\varepsilon(s+\tau_j(\psi_j^{-1}(s)))} ds \Bigg] \\
&\leqslant \sum_{i=1}^{n} \alpha_i \Bigg[\big|\phi_i(0) - \varphi_i(0)\big| \\
&\quad + \sum_{j=1}^{n} \int_{-\tau}^{0} L_j \delta_j |c_{ij}| e^{\varepsilon\tau} \big|\phi_j(s) - \varphi_j(s)\big| e^{\varepsilon s} ds \\
&\leqslant \sum_{i=1}^{n} \alpha_i \Bigg[1 + \frac{L_j\delta_j}{\varepsilon} |c_{ij}| \big(e^{\varepsilon\tau} - 1\big) \Bigg] \|\phi - \varphi\|_\infty.
\end{aligned} \tag{8}$$

This implies that the conclusion of the Lemma hold by using Eqs. (6)–(8). □

Proof of Theorem 4. First, we prove that Eq. (1) possesses an ω-periodic solution. For each solution $x(t,\phi)$ of Eq. (1) and each $t \geqslant 0$, we can define a function $x_t(\phi)$ in this fashion:

$$x_t(\phi)(\theta) = x(t+\theta,\phi), \ \text{for } \theta \in [-\tau, 0].$$

On this basis, we can define a mapping $P : C^* \to C^*$ by

$$P\phi = x_\omega(\phi).$$

Let $x(t,\phi)$, $x(t,\varphi)$ be an arbitrary pair of solutions of Eq. (1). Let ε be a positive number satisfying Eq. (3). Let $m \geqslant \frac{1}{\varepsilon\omega} \ln(2M(\varepsilon)) + 1$ be a positive integer. It follows from Lemma 5 that

$$\begin{aligned}
&\|P^m\phi - P^m\varphi\| \\
&\quad = \sup_{-\tau\leqslant\theta\leqslant 0} \|x(m\omega+\theta,\phi) - x(m\omega+\theta,\varphi)\|_\infty \\
&\quad \leqslant M(\varepsilon) \sup_{-\tau\leqslant\theta\leqslant 0} e^{-\varepsilon(m\omega+\theta)} \|\phi - \varphi\|_\infty \\
&\quad \leqslant M(\varepsilon) e^{-\varepsilon(m-1)\omega} \|\phi - \varphi\|_\infty \leqslant \frac{1}{2} \|\phi - \varphi\|_\infty,
\end{aligned}$$

which shows that P^m is a contraction mapping on the Banach space C^*. According to the contraction mapping principle, P^m possesses a unique fixed point $\phi^* \in C^*$. Note that

$$P^m(P\phi^*) = P(P^m\phi^*) = P\phi^*.$$

which indicates that $P\phi^* \in C^*$ is also a fixed point of P^m. It follows from the uniqueness of fixed point of P^m that $P\phi^* = \phi^*$, viz. $x_\omega(\phi^*) = \phi^*$. Let $x(t, \phi^*)$ be the solution of Eq. (1) with initial condition ϕ^*, then

$$x_{t+\omega}(\phi^*)(\theta) = x_t(x_\omega(\phi^*)) = x_t(\phi^*) \quad \text{for} \quad t \geqslant 0.$$

which implies

$$x(t+\omega, \phi^*) = x_{t+\omega}(\phi^*)(0) = x_t(x_\omega(0)) = x(t, \phi^*).$$

Thus, $x(t, \phi^*)$ is ω-periodic of Eq. (1).

On the other hand, it follows from Lemma 5 that every solution $x(t, \phi)$ of Eq. (1) satisfies

$$\|x(t,\phi) - x(t,\phi^*)\|_\infty \leqslant M(\varepsilon)\|\phi - \phi^*\|_\infty e^{-\varepsilon t},$$

for all $t \geqslant 0$. This shows that $x(t, \phi)$ is globally exponentially periodic. □

3 An Illustrative Example

Consider the impulsive Hopfield neural network with time-varying delays:

$$\begin{aligned}
\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} &= \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} \\
&+ \begin{pmatrix} 0.6 & 0.3 \\ 0.3 & -0.5 \end{pmatrix} \begin{pmatrix} \sin \frac{1}{\sqrt{2}} x_1(t) \\ \sin \frac{1}{2\sqrt{2}} x_2(t) \end{pmatrix} \\
&+ \begin{pmatrix} 0.8 & -0.5 \\ -0.6 & 0.6 \end{pmatrix} \begin{pmatrix} \sin \frac{1}{\sqrt{2}} x_1(t - \tau_1(t)) \\ \sin \frac{1}{2\sqrt{2}} x_2(t - \tau_2(t)) \end{pmatrix} \\
&+ \begin{pmatrix} 1 - \cos 2\pi t \\ 1 + \sin 2\pi t \end{pmatrix}, \\
\Delta x_1(t_k) &= \gamma_{1k} x_1(t_k), \\
\Delta x_2(t_k) &= \gamma_{2k} x_2(t_k).
\end{aligned} \tag{9}$$

Obviously, the right hand side of Eq. (9) is 1-periodic(i.e. $\omega = 1$). Now we investigate the influence of the delay and the period T of impulsive effect on the Eq. (9). If $\tau(t) = \frac{1}{5}\pi$, $T = 1$, $\gamma_{1k} = \gamma_{2k} = 0.1$, then $p = 1$ in (H_2). According to Theorem 4, impulsive Hopfield neural networks Eq. (9) has a unique 1-periodic solution which is globally asymptotically stable (see Figs. 1, 2, 3, 4). In order to clearly observe the

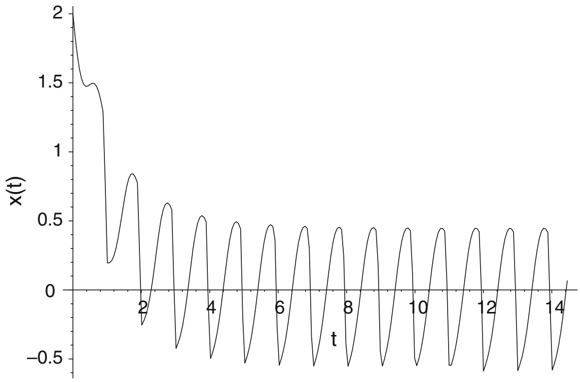

Fig. 1 Time-series of the $x_1(t)$ of Eq. (9) for $t \in [0, 16]$

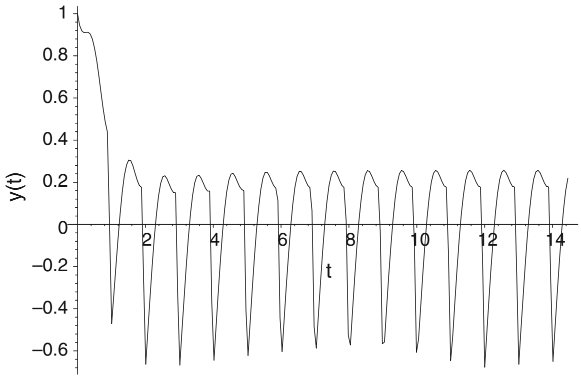

Fig. 2 Time-series of the $x_2(t)$ of Eq. (9) for $t \in [0, 16]$

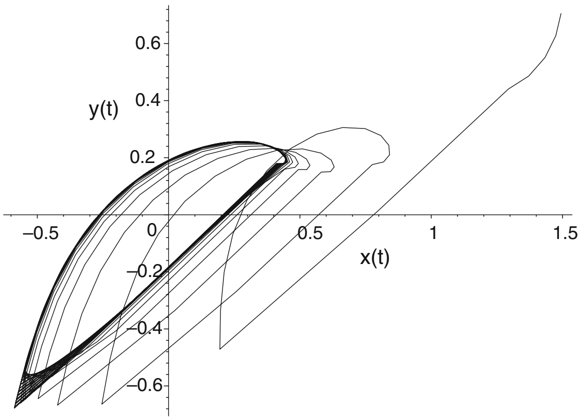

Fig. 3 Phase portrait of 1-periodic solutions of Eq. (9) for $t \in [0, 42]$

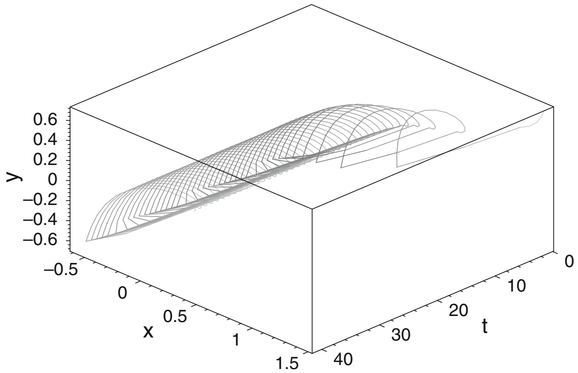

Fig. 4 Space figure of 1-periodic solutions of Eq. (9) by adding a time coordinate axes t

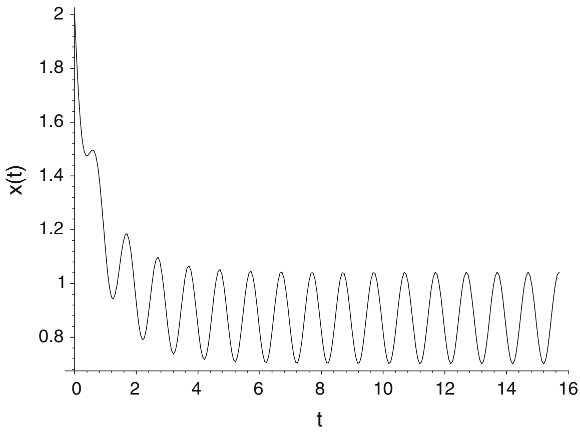

Fig. 5 Time-series of the $x_1(t)$ of Eq. (9) for $t \in [0, 16]$ with $\tau(t) = \frac{1}{5}\pi$

change trend of the solutions, we add a time coordinate axes to the Fig. 4 and change 2-D plan (Fig. 3) into 3-D space (Fig. 4).
If the effect of impulse is ignored, i.e. $\gamma_{1k} = 0, \gamma_{2k} = 0$, then Eq. (9) becomes periodic system. Obviously, the right hand side of Eq. (9) is 1-periodic. Numeric results show that Eq. (9) has a 1-periodic solution Fig. 5. Figures 6, 7, 8 show the dynamic behavior of the Eq. (9) with $\tau(t) = \frac{1}{5}\pi$.

Furthermore, If $\tau(t) = \frac{1}{5}\pi$ rises to $\tau(t) = \pi$ gradually, then periodic oscillation of Eq. (9) will be destroyed. Numeric results show that Eq. (9) still has a global attractor which may be gui chaotic strange attractor (see Figs. 9, 10, 11, 12). Every solutions of Eq. (9) will finally tend to the chaotic strange attractor.

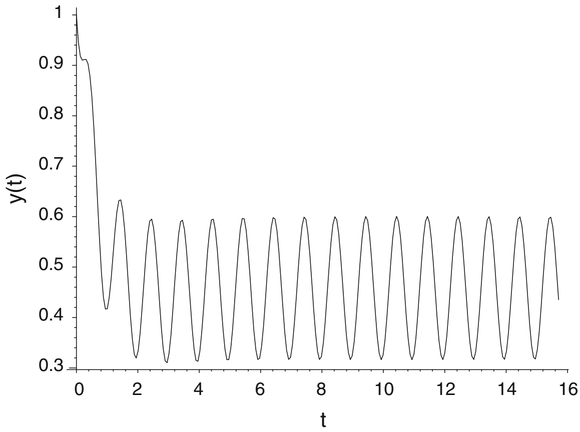

Fig. 6 Time-series of the $x_2(t)$ of Eq. (9) for $t \in [0, 16]$ with $\tau(t) = \frac{1}{5}\pi$

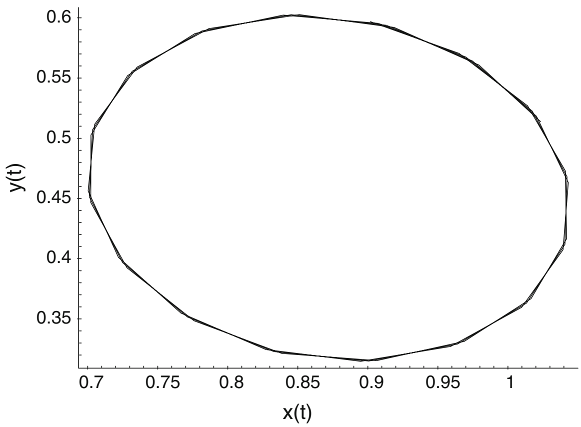

Fig. 7 Phase portrait of 1-periodic solutions of Eq. (9) with $\tau(t) = \frac{1}{5}\pi$

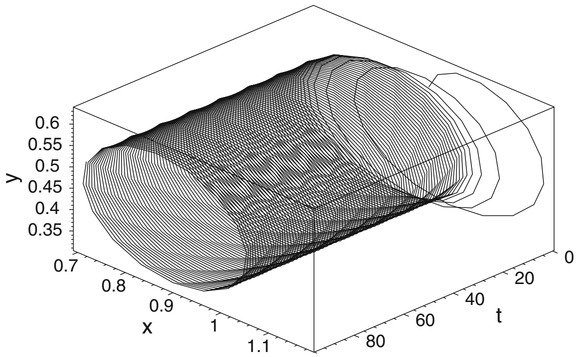

Fig. 8 Space figure of 1-periodic solutions of Eq. (9) by adding a coordinate axes t

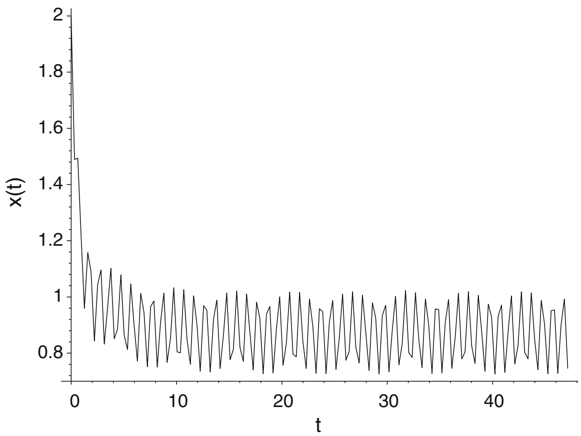

Fig. 9 Time-series of the $x_1(t)$ of Eq. (9) for $t \in [0, 48]$ with $\tau(t) = \pi$

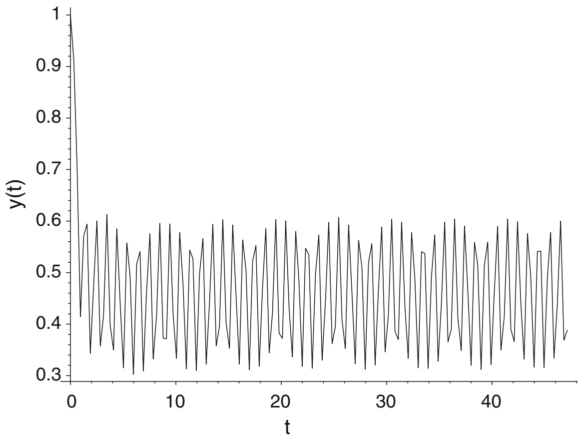

Fig. 10 Time-series of the $x_2(t)$ of Eq. (9) for $t \in [0, 48]$ with $\tau(t) = \pi$

4 Conclusion

We have established a sufficient condition for the existence and global exponential stability of a unique periodic solution in a class of HNNs with time-varying delays and periodic impulses, which assumes neither the differentiability nor the monotonicity of the activation functions.

Our condition extends and generalizes a known condition for the global exponential periodicity of pure continuous Hopfield neural networks with time-varying delays. Further the numerical simulation shows that our system can occur many forms of complexities including chaotic strange attractor and periodic solution.

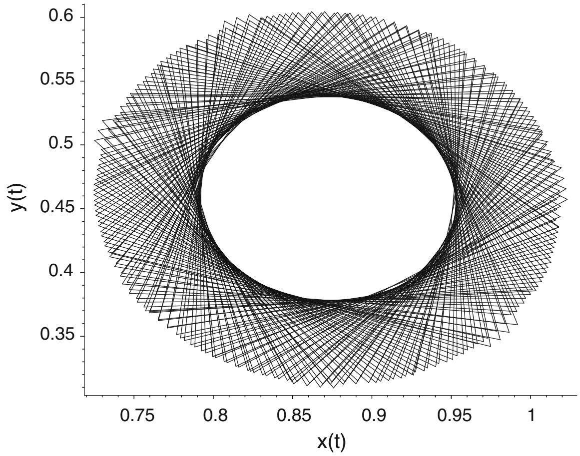

Fig. 11 Phase portrait of chaotic strange attractor of Eq. (9) with $\tau(t) = \pi$

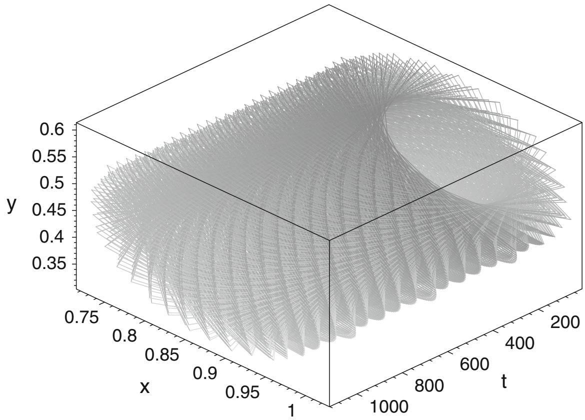

Fig. 12 Space figure of attractor of Eq. (9) by adding a coordinate axes t

In recent years, numerous results have been reported on the stability of discrete as well as continuous neural networks. It is worthwhile to introduce various impulsive neural networks and then establish the corresponding stability results that include some known results for pure discrete or continuous neural networks as special cases.

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