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QUANTUM OBSERVER, INFORMATION THEORY AND KOLMOGOROV COMPLEXITY

ABSTRACT

The theory itself does not tell us which properties are sufficient for a system to count as a quantum mechanical observer. Thus, it remains an open problem to find a suitable language for characterizing observation. We propose an information-theoretic definition of observer, leading to a mathematical criterion of objectivity using the formalism of Kolmogorov complexity. We also suggest an experimental test of the hypothesis that any system, even much smaller than a human being, can be a quantum mechanical observer.

1. INTRODUCTION

A few years after Carlo Rovelli proposed a relational interpretation of quantum mechanics (Rovelli 1996), it received a sharp rebuke from Asher Peres. The issue that Peres addressed was Rovelli's claim to the universality of the quantum mechanical observer. According to Rovelli, all systems should be seen as observers insofar as their degrees of freedom are correlated with the degrees of freedom of some other system. Information contained in such a correlation is the information possessed by the observer about the observed system. Nothing else is needed, not even a limit on the size of systems or the number of their degrees of freedom. This is where Peres objected: "The two electrons in the ground state of the helium atom are correlated, but no one in his right mind would say that each electron 'measures' its partner" (Peres 1986). The controversy is still unresolved: Is the capacity to serve as quantum mechanical observer universal and extends to all systems? Or is it true that only some systems, but not others, can be observers, and if there is a limitation, then what is it precisely?

I will argue that in order to give an answer to this question, we need to revolutionize our idea of physical observation. For this, I'll first briefly review the history of thinking on quantum mechanical observers and then I'll propose a new conceptual toolkit with which to approach this question. This toolkit will involve the notion of information and the Kolmogorov complexity as its quantitative measure.

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2. Observer in the interpretations of quantum mechanics

2.1 Observer in the Copenhagen orthodoxy

Bohr's lecture at Como in 1927 was a foundation of what later came to be known as the Copenhagen interpretation of quantum mechanics. Despite being a common reference among physicists, this interpretation has a variety of slightly different formulations. Its main point, however, is clearly stated by Bohr:

> Only with the help of classical ideas is it possible to ascribe an unambiguous meaning to the results of observation.... It lies in the nature of physical observation, that all experience must ultimately be expressed in terms of classical concepts. (Bohr 1934, p. 94)

Two different readings of this statement are possible, divided by what exactly is meant by "classical". The first reading is a straightforward *sine qua non* claim about quantum and classical mechanics:

> It is in principle impossible to formulate the basic concepts of quantum mechanics without using classical mechanics. (Landau and Lifschitz 1977, p. 2)

The second reading is that quantum mechanical experiments can only be described by classical language. Even if classical language later leads us to classical mechanics, it is the *language* – not any form of mechanics – that becomes a crucial ingredient:

> Bohr went on to say that the terms of discussion of the experimental conditions and of the experimental results are *necessarily* those of 'everyday language', suitably 'refined' where necessary, so as to take the form of classical dynamics. It was apparently Bohr's belief that this was the only possible language for the *unambiguous communication* of the results of an experiment. (Bohm 1971, p. 38)

The first reading implies that the world consists of mechanical systems only, whether quantum or classical, and no observer external to physical theory is necessary. Contrary to this, the second reading assumes that the formulation of the problem includes an agent possessing classical language: the experimenter. The latter prepares and measures the quantum system, thereby acting as a quantum mechanical observer.

2.2 London and Bauer

First published in 1932, John von Neumann's magisterial book on quantum mechanics offered what were to become a standard theory of quantum measurement (von Neumann 1932). But von Neumann's musings about the place of the observer during measurement were not entirely satisfactory. The mathematics worked perfectly, however its meaning required further clarification. Writing as early as 1939, London and Bauer set the tone of the conceptual debate. They noted that quantum mechanics didn't ascribe properties to the quantum system in itself, only in connection to an observer. For London and Bauer such an observer had to be human: "it seems that the result of measurement is intimately linked to the consciousness of the person making it" (London and Bauer 1939, p. 48). The cut between the observer and the observed system introduced by von Neumann and Dirac was pushed to the extreme, leaving all physical systems – even the human eye and the visual nerve – on one side, and only leaving the observer's 'organ' of awareness, namely consciousness, on the other.

If this were true, why would objectivity be possible at all and why have physicists not yet become solipsists? Why do two physicists agree on what constitutes the object of their observation and on its properties? According to London and Bauer, the reason is the existence of something like a "community of scientific consciousness, an agreement on what constitutes the object of the investigation"(London and Bauer 1939, p. 49). The exact meaning of this assertion remained a mystery.

2.3 Wigner

Bohr emphasized that a linguistic faculty is necessary for observers because they must communicate unambiguously. This was further developed by Eugene Wigner. The consciousness of the observer "enters the theory unavoidably and unalterably" and corresponds to an impression produced by the measured system on the observer. The wave function "exists" only in the sense that "the information given by the wave function is communicable":

> The communicability of information means that if someone else looks at time t and tells us whether he saw a flash, we can look at time $t + 1$ and observe a flash with the same probabilities as if we had seen or not seen the flash at time t ourselves. (Wigner 1961)

The observer "tells us" the result of his measurement: like for Bohr, communication for Wigner is therefore linguistic. But do observers actually *have* to communicate or is it enough to require that they simply *could* communicate? On the one hand, Wigner says, "If someone else somehow determines the wave function of a system, he can tell me about it…", which requires a mere possibility of communication but no sending of actual information. On the other hand, he famously analyzes the following 'Wigner's friend' situation:

> It is natural to inquire about the situation if one does not make the observation oneself but lets someone else carry it out. What is the wave function if my friend looked at the place where the flash might show at time t? The answer is that the information available about the *object* cannot be described by a wave function. One could attribute a wave function to the joint system: friend plus object, and this joint system would have a wave function also after the interaction, that is, after my friend has looked. I can then enter into interaction with this joint system by asking my friend whether he saw a flash. …The typical change in the

wave function occurred only when some information (the *yes* or *no* of my friend) entered *my* consciousness. (Wigner 1961)

Although he calls this situation natural, Wigner is the only one among the founding fathers of quantum theory to have addressed it explicitly. Here Wigner's agreement with his friend is clearly possible thanks to the linguistic communication between them, but this communication itself is not a quantum measurement: whatever the situation, Wigner always knows the question he should put to his friend and fully trusts the answer, always *yes* or *no*. Communication from the friend must actually occur before the wave function could be known by Wigner; it is not enough that this communication be merely possible. The question remains open as for the exact mechanism, whether a human convention or a physical given, of the agreement between observers.

Wigner also touches on the question of belief and trust in his discussion of repeatability of experiments in physics. To explore the statistical nature of the predictions of quantum mechanics, it is necessary to be able to produce many quantum systems in the same state; subsequently these systems will be measured. One can never be absolutely sure, as Wigner stipulates, that one has produced the same state of the system. We usually "believe that this is the case" and we are "fully convinced of all this"(Wigner 1976, p. 267), even if we have not tried to establish experimentally the validity of the repeated preparation of the same state. What is at work here is again a convention shared by all physicists. How do they know that repeated preparations produce the same state if they do not measure each and every specimen in order to verify it? The answer is that they have common experience and a convention on what a 'controlled experiment' amounts to, and their respect of this commonly shared and empirically validated rules enables them to postulate the existence of repeated states even in the situations which had never been tested before. This is how physical theory with its laws and a precise methodology arises by way of abstraction ('elevation', as Einstein or Poincaré would say (Friedman 2001, p. 88)) from the physicist's empirical findings and the heuristics of his work.

2.4 Everett

The need to refer to consciousness exists insofar as only consciousness can distinguish a mere physical correlation, e.g. of an external system with the observer's eye, from the information actually available to the observer, i.e. the observer's knowledge on which he can act at future times. Other characteristics are irrelevant: jokingly, London and Bauer tell us that "there is little chance of making a big mistake if one does not know [the observer's] age" (London and Bauer 1939, p. 43). Treating the observer as an informational agent requires that we say precisely what property authorizes different systems possessing information to be treated as observers. In other words, what is the nature of a convention shared by all observers? Brillouin was among the first to believe that information in physics must be defined with the exclusion of all human element (George 1953, p. 360). This was continued by Hugh Everett (1957), for whom observers are physical systems that possess memory. Memory is defined as "parts... whose states are in correspondence with past experience of the observers". Thus observers do not have to be human: they could be "automatically functioning machines, possessing sensory apparatus and coupled to recording devices".

Everett was the first to explicitly consider the problem of several observers. The "interrelationship between several observers" is an act of communication between them, which Everett treats as establishing a correlation between their memory configurations. He listed several principles to be respected in such settings:

1. When several observers have separately observed the same quantity in the object system and then communicated the results to one another they find that they are in agreement. This agreement persists even when an observer performs his observation after the result has been communicated to him by another observer who has performed the observation.
2. Let one observer perform an observation of a quantity A in the object system, then let a second perform an observation of a quantity B in this object system which does not commute with A, and finally let the first observer repeat his observation of A. Then the memory system of the first observer will *not* in general show the same result for both observations. ...
3. Consider the case when the states of two object systems are correlated, but where the two systems do not interact. Let one observer perform a specified observation on the first system, then let another observer perform an observation on the second system, and finally let the first observer repeat his observation. Then it is found that the first observer always gets the same result both times, and the observation by the second observer has no effect whatsoever on the outcome of the first's observations. (Everett 1957)

As we shall see, the problem of agreement between different observers and the need for memory as a defining characteristics of observation are intimately connected.

3. Information-theoretic definition of observer

3.1 Observer as a system identification algorithm

What characterizes an observer is that it has information about some physical system. This information fully or partially describes the state of the system. The observer then measures the system, obtains further information and updates his description accordingly. Physical processes listed here: the measurement, updating of the information, ascribing a state, happen in many ways depending on the physical constituency of the observer. The memory of a computer acting as an observer, for instance, is not the same as human memory, and measurement devices vary in their design and functioning. Still one feature unites all observers: that whatever they do, they do it to a *system*. In quantum mechanics, defining an observer goes hand in hand with defining a system under observation. An observer without a system is a meaningless nametag, a system without an observer who measures it is a mathematical abstraction.

Quantum systems aren't like sweets: they don't melt. Take a general thermodynamic system interacting with other systems. Such a system can dissipate, diffuse, or dissolve, and thus stop being a system. If at first a cube of ice gurgling into tepid water is definitely a thermodynamic system, it makes no sense to speak about it being a system after it has dissolved: the degrees of freedom that previously formed the ice cube have been irreparably lost or converted into physically non-equivalent degrees of freedom of liquid water. Quantum systems aren't like this. The state of a quantum system may evolve, but the observer knows how to tell the system he observes from the environment. An electron in a certain spin state remains an electron after measurement even if its state has changed, i.e., it remains a system with a particular set of the degrees of freedom which we call an "electron". Generally speaking, the observer maintains system identity through a sequence of changes in its state. Hence, whatever the physical description of such 'maintaining' may be, and independently of the memory structure of a particular physical observer, first of all every observer is abstractly characterized as a system identification machine. Different observers having different features (clock hands, eyes, optical memory devices, internal cavities, etc.) all share this central feature.

Definition 1. An observer is a system identification algorithm (SIA).

Particular observers can be made of flesh or, perhaps, of silicon. 'Hardware' and 'low-level programming' are different for such observers, yet they all perform the task of system identification. This task can be defined as an algorithm on a universal computer, e.g., the Turing machine: take a tape containing the list of all degrees of freedom, send a Turing machine along this tape so that it puts a mark against the degrees of freedom that belong to the quantum system under consideration. Any concrete SIA may proceed in a very different manner, yet all can be modelled with the help of this abstract construction.

The SIAs with possibly different physical realization share one property that does not depend on the hardware: their algorithmic, or Kolmogorov, complexity. Any SIA can be reconstructed from a binary string of some minimal length (which is a function of this SIA) by a universal machine. As shown by Kolmogorov, this minimal compression length defines the amount of information in the SIA and does not depend (up to a constant) on the realization of the SIA on particular hardware (Kolmogorov 1965).

3.2 Quantum and classical systems

Each quantum system has a certain number of degrees of freedom: independent parameters needed in order to characterize the state of the system. For example, a system with only two states (spin-up and spin-down) has one degree of freedom and can be described by one parameter $\sigma = \pm 1$. If we write these parameters as a binary string, the Kolmogorov complexity of this string is at least the number of the degrees of freedom of the system. Consequently, for any system S and the Kolmogorov complexity of the binary string s representing its parameters

$$K(s) \geq \mathrm{d}_S, \tag{1}$$

where d_S is the number of the degrees of freedom in S. In what follows the notation $K(s)$ and $K(S)$ will be used interchangeably.

When we say that observer X observes quantum system S, it is usually the case that $K(S) \ll K(X)$. In this case the observer will have no trouble keeping track of all the degrees of freedom of the system; in other words, the system will not ‘dissolve’ or ‘melt’ in the course of dynamics. However, it is also possible that X identifies a system with $K(S) > K(X)$. For such an observer, the identity of system S cannot be maintained and some degrees of freedom will fall out from the description that X makes of S.

Definition 2. System S is called quantum with respect to observer X if $K(S) < K(X)$, meaning that X will be able to maintain a complete list of all its degrees of freedom. Otherwise S is called classical with respect to X.

Suppose that X observes a quantum system, S, and another observer Y observes both S and X. If $K(Y)$ is greater than both $K(X)$ and $K(S)$, observer Y will identify both systems as quantum systems. In this case Y will typically treat the interaction between X and S as an interaction between two quantum systems. If, however, $K(X)$ and $K(Y)$ are close, $K(X) \gg K(S)$ and $K(Y) \gg K(S)$ but $K(X) \simeq K(Y)$, then Y will see S as a quantum system but the other observer, X, as a classical system. An interaction with a classical system, which we usually call ‘observation’, is a process of decoherence that occurs when the Kolmogorov complexity of at least one of the involved systems approaches the Kolmogorov complexity of the external observer. In this case Y cannot maintain a complete description of X interacting with S and must discard some of the degrees of freedom. If we assume that all human observers acting in their SIA capacity have approximately the same Kolmogorov complexity, this situation will provide an explanation of the fact that we never see a human observer (or, say, a cat) as a quantum system.

4. Elements of reality

4.1 Entropic criterion of objectivity

Ever since the Einstein-Podolsky-Rosen article (1935), the question of what is real in the quantum world has been at the forefront of all conceptual discussions about quantum theory. The original formulation of this question involved physical *properties*: e.g., are position or momentum real? This is however not the only problem of reality that appears when many observers enter the game. Imagine a sequence of observers X_i, $i = 1, 2, \ldots$, each identifying systems S_n, $n = 1, 2, \ldots$. System identifications of each S_n do not have to coincide as some observers may have their Kolmogorov complexity $K(X_i)$ below, or close to, $K(S_n)$, and others much bigger than $K(S_n)$. If there is disagreement, is it possible to say that the systems are real, or objects of quantum mechanical investigation, in some sense? We can encode the binary identification string produced by each observer in his SIA capacity as some random variable $\xi_i \in \Omega$, where Ω is the space of such binary identification strings, possibly of infinite length. Index i is the number of the observer, and the values taken by random variable ξ_i bear index n corresponding to "i-th observer having identified system S_n". Adding more observers, and in the limit $i \to \infty$ infinitely many observers, provides us with additional identification strings. Putting them together gives a stochastic process $\{\xi_i\}$, which is an observation process by many observers. If systems S_n are to have a meaning as "elements of reality", it is reasonable to require that no uncertainty be added with the appearance of further observers, i.e., that this stochastic process have entropy rate equal to zero:

$$H(\{\xi_i\}) = 0. \tag{2}$$

We also take this process to be stationary and ergodic so as to justify the use of Shannon entropy.

Let us illustrate the significance of condition (2) on a simplified example. Suppose that $\theta_1, \theta_2, \ldots$ is a sequence of independent identically distributed random variables taking their values among binary strings of length r with probabilities q_k, $k \leq 2^r$. These θ_k can be seen as identifications, by different SIAs, of different physical systems, i.e., a special case of the ξ_i-type sequences having fixed length and identical distributions. For instance, we may imagine that a finite-length string, θ_1, is a binary encoding of the first observer seeing an electron and θ_2 is a binary string corresponding to the second observer having identified a physical system such as an elephant; and so forth. Then entropy is written simply as:

$$H = -\sum_k q_k \log q_k. \tag{3}$$

Condition (2) applied to entropy (3) means that all observers output one and the same identification string of length r, i.e., all SIAs are identical. This deterministic system identification, of course, obtains only under the assumption that the string length is fixed for all observers and their random variables are identically distributed, both of which are not plausible in the case of actual quantum mechanical observers. So, rather than requiring identical strings, we impose condition (2) as a criterion of the system being identified in the same way by all observers, i.e., it becomes a candidate quantum mechanical "object of investigation".

4.2 Relativity of observation

Let us explore the consequences of condition (2). Define a binary sequence α_n^i as a concatenation of the system identifications strings of systems S_n by different observers:

$$\alpha_n^i = \overline{(\xi_1)_n}\,\overline{(\xi_2)_n}\dots\overline{(\xi_i)_n}, \tag{4}$$

where index i numbers observers and the upper bar corresponds to "string concatenation" (for a detailed definition see Zvonkin and Levin 1970). Of course, this concatenation is only a logical operation and not a physical process. A theorem by Brudno (1978, 1983) conjectured by Zvonkin and Levin (1970) affirms that the Kolmogorov complexities of strings α_n^i converge towards entropy:

$$\lim_{n\to\infty}\lim_{i\to\infty}\frac{K(\alpha_n^i)}{i} = H(\{\xi_i\}). \tag{5}$$

For a fixed i and the observer X_i who observes systems S_n that are quantum in the sense of Definition 2, variation of $K(\alpha_n^i)$ in n is bounded by the observer's own complexity in his SIA capacity:

$$K(\alpha_n^i) < K(X_i) \qquad \forall n, \qquad i \text{ fixed}. \tag{6}$$

Hence eqs. (2) and (5) require that

$$\lim_{i\to\infty}\frac{K(\alpha_n^i)}{i} = 0. \tag{7}$$

This entails that the growth of $K(\alpha_n^i)$ in i must be slower than linear. Therefore the following:

Proposition 3. *An element of reality that may become an object of quantum mechanical investigation can be defined only with respect to a class of not very different observers.*

To give an intuitive illustration, imagine adding a new observer X_{i+1} to a group of observers $X_1,\dots,X_i$ who identify systems S_n. This adds a new identification string that we glue at the end of concatenated string α_n^i consisting of all X_i's identifications of S_n, thus obtaining a new string α_n^{i+1}. The Kolmogorov complexity of α_n^{i+1} does not have to be the same as the Kolmogorov complexity of α_n^i; it can grow, but not too fast. Adding a new observation may effectively add some new non-compressible bits, but not too many such bits. If this is so,

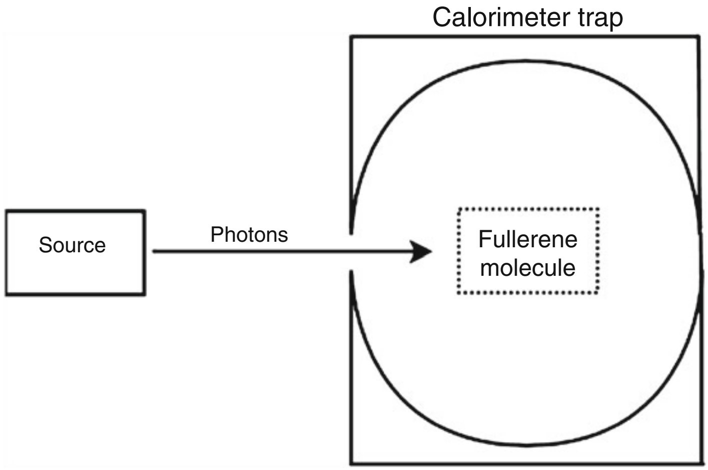

Figure 1: Experiment leading to heat production when observer's memory becomes saturated.

then $H = 0$ still obtains. Although observers $X_1, \ldots, X_i, X_{i+1}$ produce slightly different identification strings, they will agree, simply speaking, that an atom is an atom and not something that looks more like an elephant.

The above reasoning applies only to quantum systems S_n in the sense of Definition 2. This is because, in the case of non-quantum systems, different observers may operate their own coarse-graining, each keeping only some degrees of freedom. System identification strings may then differ dramatically, and one cannot expect $K(\alpha_n^i)$ to grow moderately.

5. Experimental test

A previously suggested experimental connection between thermodynamics and theories based on Kolmogorov complexity is based on observing the consequences of a change in the system's state (Zurek 1989, 1998; Erez et al. 2008). Zurek (1989) introduced the notion of physical entropy $\mathcal{S} = H + K$, where H is the thermodynamic entropy and K the Kolmogorov entropy. If the observer with a finite memory has to record the changing states of the quantum system, then there will be a change in $\mathcal{S}$ and it will lead to heat production that can be observed experimentally. We propose here a test independent of the change of state.

An individual fullerene molecule is placed in a highly sensitive calorimeter and bombarded with photons, which play the role of quantum systems with low $K(S)$ (Figure 1). The fullerene is a SIA, or a quantum mechanical observer, with $K(X) > K(S)$. Thus the absorption of the photon by the fullerene can be described as measurement: the fullerene identifies a quantum system, i.e. the photon, and observes it, obtaining new information. Physically, this process amounts to establishing a correlation between the photon variables (its energy) and the vibrational degrees of freedom of the fullerene. From the point of view of an observer external to the whole setting, the disappearance of the photon implies that the act of observation by the fullerene has occurred, although the external observer of course remains unaware of its exact content.

Informationally speaking, the same process can be described at storing information in the fullerene's memory. If measurement is repeated on several photons, more such information is stored, so that at some point total Kolmogorov complexity of concatenated identification strings will approach $K(X)$. When it reaches $K(X)$, the fullerene will stop identifying incoming photons as quantum systems. Any further physical process will lead to heat production due to memory erasure, as prescribed by Landauer's principle (Landauer 1961). Physically, this process will correspond to a change of state of the carbon atoms that make up the fullerene molecule: the calorimeter will register a sudden increase in heat when C_{60} cannot store more information, thereby ending its observer function.

Actual experiments with fullerenes show that this scenario is realistic. A fullerene molecule "contains so many degrees of freedom that conversion of electronic excitation to vibrational excitation is extremely rapid". Thus, the fullerene is a good candidate for a quantum mechanical observer, for "the molecule can store large amounts of excitation for extended periods of time before degradation of the molecule (ionization or fragmentation) is observed" (Lykke and Wurz 1992). The experiments in which fullerenes are bombarded with photons demonstrate that "the energy of the electronic excitation as a result of absorption of a laser photon by a molecule is rapidly converted into the energy of molecular vibrations, which becomes distributed in a statistical manner between a large number of the degrees of freedom of the molecule... The fullerene may absorb up to 10 photons at $\lambda = 308$ nm wavelength before the dissociation of the molecule into smaller carbon compounds" (Eletskii and Smirnov 1995). We read these results as a suggestion that there should be one order of magnitude difference between $K(S)$ and $K(X)$ and that this allows the fullerene to act as a quantum mechanical observer for up to 10 photons at 308 nm wavelength. What needs to be tested experimentally in this setting is heat production: we conjecture that if the same process occurs inside a calorimeter, the latter will register a sudden increase in heat after the fullerene will have observed 10 photons (Figure 2). What we predict here isn't new physics, but an explanation of a physical process on a new level: that of information. We suggest that heat production deserves special attention as a signature of the fullerene's role as quantum mechanical observer.

As a side remark, imagine that the photon's polarization state in some basis were fully mixed:

$$\frac{1}{2}(|0\rangle + |1\rangle).$$

While only the energy of the photon matters during absorption, the external observer records von Neumann entropy $H = \log 2$ corresponding to this mixture (the initial state of the fullerene is assumed fully known). After absorption, it is manda-

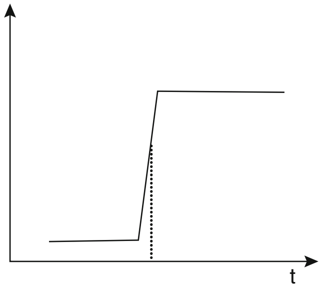

Figure 2: Conjectured time dependence of heat production in the calorimeter (vertical axis). A sharp increase occurs when the fullerene's memory is erased as it stops 'observing' photons quantum mechanically.

tory that this entropy be converted into Shannon entropy of the new fullerene state, corresponding nicely to the uncertainty of the external observer in describing the "statistical manner" of the distribution over a large number of the degrees of freedom. From the internal point of view, we may assume perfect 'self-knowledge' of the observer, which puts his Shannon entropy equal to zero. However, his Kolmogorov entropy will increase as a result of recording the measurement information (Zurek 1998). Heat produced during the erasure of measurement information is at least equal to the Kolmogorov complexity of the string that was stored in observer's memory; but, according to quantum mechanics, this heat will not reveal to the external observer any information about the precise photon state observed by the fullerene.

6. Concluding remarks

Information-theoretic treatment of quantum mechanical observer provides a formal result that encapsulates the Einstein-Podolsky-Rosen notion of "element of reality". We have shown how to make sense of a system existing independently of observation, with respect to a class of observers whose Kolmogorov complexities may differ, even if slightly. Equation (7) provides a mathematical criterion. It remains an open problem to find out whether the information-theoretic definition of observer will yield useful insights in other areas of quantum mechanics. We are currently pursuing this research program for studying quantum mechanical non-locality.

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