Chapter 12
Multiple Forest Stocks and Harvesting Decisions: The Enhanced Green Golden Rule

Shashi Kant and Chander Shahi

Abstract The concept of the Green Golden Rule (GGR), introduced by Chichilnisky et al. (1995), which refers to the configurations of the economy that give the highest indefinitely maintainable level of instantaneous utility, is extended to forest resources. Generally, a forest has multiple types of stocks/cohorts—stocks of different ecological attributes and age classes—that provide different goods and services, and these goods and services are valued differently by different user groups. Hence, the aggregation of all stocks into a single stock is unable to capture the complexities of forest growth, user groups' preferences, and their implications for sustainable management of these resources. Sustainable management of forest resources requires optimal consumption as well as an optimal level of conservation of each type of stock separately. We develop optimal conditions for conservation and consumption for a forest comprised of three differentiable stocks, and generalizes these conditions for any number of stocks greater than three. We term these conditions as the Enhanced Green Golden Rule (EGGR). The EGGR provides more distinct optimality conditions than the GGR for all stocks except the terminal stock. We demonstrate the applications and implications of the EGGR for logistic growth functions of three types of forest stock having a Cobb-Douglas utility function of forest consumption and conservation.

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12.1 Introduction

The main focus of Faustmann forest economics has been on the determination of an optimal forest rotation, and the Faustmann formula has been termed as forestry's golden rule by Newmann (2002). Probably more than 500 papers have been published on this subject in the last 3 decades, and the most comprehensive approach has been presented in the generalized Faustmann model by Chang (1998). Most of these chapters have addressed an optimal forest rotation for a single stand and the value of timber only. Hartmann (1976) included non-timber values for a single stand, and many chapters have followed and extended Hartmann's model but mainly to a single stand case. In addition, Hartman's model and its extensions included only the consumption value, measured in monetary units, of non-timber products, and ignored any amenity values due to the stock of forest.

The importance of multiple stands (mainly in terms of multiple age classes) in a forest and interdependencies between multiple stands has been recognised by some resource economists. Swallow and Wear (1993) and Koskela and Ollikainen (2001) examined landowner decisions in the presence of interactions between two or more adjacent stands. Another stream of chapters has included age-class dynamics in studies of landowner behaviour; some focused on timber benefits only (Berck 1976, 1979; Mitra and Wan 1985, 1986; Sedjo and Lyon 1990; Salo and Tahvonen 2002, 2003; Khan and Piazza 2011) while others included timber and non-timber benefits (Bowes and Krutilla 1985, 1989; Uusivuori and Kuuluvainen 2005). In the age-class dynamics category, there is another stream commonly known as the economics of uneven-aged forest management or selection harvesting, and recent studies in this stream include Chang and Gadow (2010) and Xabadia and Goetz (2010). However, this last stream has focused only on consumption values from timber and in some cases non-timber products.

In short, most of forest economics literature addressing harvesting decisions is focused on the utility derived from the monetary value of either timber only or in some cases timber and non-timber products. There are very few exceptions that have included the utility from the amenity value of forest stock. For example, Bowes and Krutilla (1985, 1989) included the amenity value of forest stock in the objective function but simply added it to the revenue from forest harvesting, and implicitly assumed that there is no economic flaw in adding the amenity value's economic measure (willingness to pay) to the price of timber. In fact, market price is determined by the interactions between demand and supply while willingness to pay, irrespective of its other limitations, simply reflects the demand curve, and therefore the sum of the two is similar to the sum of apples and oranges. Uusivuori and Kuuluvainen (2005) separated utility from timber harvesting and the amenity utility from the forest stock, and assumed the amenity utility as a function of the volume of biomass, but did not distinguish between the possible differences in the amenity utility from different types of stands. In the forestry literature, it is well recognised that different forest stands, distinguished on the basis of either age or other physical and biological characteristics such as biodiversity and habitat, provide different amenity values, not due to the difference in biomass volume only but due to the different physical and biological features of each stand.

Second, all these studies have focused on the conceptualized state of the forest called the "normal forest", which means that forestland is evenly distributed over age-classes. This concept of a normal forest may be a useful concept, but it is an idealistic and unrealistic concept similar to the concept of a perfect market in economics. Even the usefulness of the normal forest concept is being restricted by the emerging concepts of the new forest management regime known as "ecosystem-based" forest management, "near-natural" forest management, "continuous cover" forest management, or sustainable forest management. This new forest management regime looks for a "near-natural" state and not for the ideal state of the normal forest. One of the common approaches of this new forest management regime is multiple-cohort forest management that attempts to emulate a natural age structure and composition of forest across a given land base. The rationale behind multiple-cohort management is that emulation of structures resulting from natural processes favours the maintenance of biodiversity and ecological functions (Bergeron et al. 1999).

Third, other most common feature of these studies is the maximization of the discounted net revenue or utility using a constant discount rate, a criterion known as the discounted utilitarian criterion. This criterion has been challenged by many economists in the context of sustainability, specifically with reference to intergenerational equity. Finally, in most of these chapters, the prices of timber are assumed to be the same for timber coming from different age classes, implying that the marginal utility from timber consumption from different age-classes is the same. This assumption is also far from reality. Hence, there is a need for a new economic approach for forest harvesting decisions that addresses the above discussed four limitations of existing approaches.

The concept of sustainability, which is the key in sustainable forest management, has attracted the attention of many mainstream economists. The contributions of Dasgupta and Heal (1974, 1979); Stiglitz (1974) and Solow (1974) are some of the early contributions, but Hartwick's rule or the Weak Sustainability approach (Hartwick 1977, 1978a, b) is one of the more common contributions despite its limitations demonstrated by Asheim (1986) and others. This early literature on sustainability considered utility to be a function of natural resources consumption only. The next wave of chapters included "resource stock" as a source of utility in addition to consumption of this stock. Krautkraemer (1985) developed a model for non-renewable resources and Beltratti et al. (1993)

extended Krautkraemer's work for both non-renewable and renewable resources using Chichilnisky's criterion of maximizing the weighted sum of the present and long run values of utility. In the case of non-renewable resources, they found that for maximum sustainable utility, conservation of the entire stock is the optimal solution, which leads to equal treatment for present and future generations. However, their analysis of a renewable resource suggests that the problem does not have a solution because of the conflict between benefits for present and future generations. Hence, they used a declining discount rate which asymptotically converges to zero to solve this problem. Chichilnisky et al. (1995) characterized this state of economy as the Green Golden Rule (GGR) and found that the solution to the discounted utilitarian criterion discriminates against future generations, whereas the GGR accounts for future generations using the solution to the problem of maximizing long-run utility.

The GGR is an extension of the Golden Rule (GR) of economic growth, established by Meade-Phelps-Robinson, which refers to a growth path of the highest maintainable level of consumption per head (Phelps 1961) while the GGR refers to the highest indefinitely maintainable level of utility—which includes utility from consumption as well as the stock of environmental and/or natural resources (Chichilnisky et al. 1995). Hence, the GGR is a valuable contribution to the economic literature on issues related to sustainability due to its incorporation of intergenerational equity and utility from the natural resource stock. However, the GGR is insensitive to the diversity of natural resource stocks and different utilities derived from them—either by consumption or by the stock itself; this results in the limited applications of the GGR to the real problems of natural resource management.

In this chapter, we enhance the GGR by incorporating the diversity of forest resource stocks and utilities from them, and apply that concept to determine harvesting rules for different types of forest stocks. We call this approach the Enhanced Green Golden Rule (EGGR). The EGGR addresses all the four limitations, identified in the previous paragraphs, related to the literature on optimal forest rotations. It recognizes multiple-cohorts or multiple stands of forests, and incorporates the utilities from timber consumption and amenities from the each stand/cohort separately. The EGGR model, presented in this chapter, is based on utilities and not on market prices and/or willingness to pay, and therefore it overcomes the problem of adding market prices and willingness to pay together. The EGGR model also avoids the use of the discounted utilitarian concept and the use of the same market prices for timber from different stands/cohorts.

The EGGR is, first, developed for a forest resource of three cohorts/stocks. For simplicity we categorize the forest in three cohorts/stocks based on age—young, mature and old cohorts/stocks. Using the results of the EGGR for three cohorts/stocks, a generalized EGGR is presented for any number of cohorts/stocks of the forest resource. The outcomes of the EGGR—optimum levels of consumption and conservation of multiple stocks of a forest resource—are illustrated by an example. We have used age as a criterion for classifying the cohorts for simplicity, and our results are applicable to the classification of cohorts/stocks based on any other criterion such as biodiversity, habitat, or any other useful characteristic.

Next, in Sect. 12.2, we introduce the growth structure of a three cohort forest resource. In Sect. 12.3, we present the EGGR for a forest resource of three cohorts/stocks, and the generalized EGGR for any number of cohorts. An illustration of the outcomes of the EGGR is provided in Sect. 12.4, and in the last section, some concluding remarks are put forward.

12.2 The Growth Structure of Multiple Cohorts of a Forest

Forest resources provide multiple products and services, such as timber, fuelwood, fodder, recreation, biodiversity, wildlife habitat, carbon sequestration, watershed services, esthetic values, cultural and spiritual values, and Aboriginal values. All these products and services are not equally valuable to different sections of a society. For example, environmentally-oriented groups and Aboriginal people may place higher values on biodiversity, wildlife habitat, and cultural values, recreation groups on recreation services, economic-growth oriented groups on timber values, and forest-dependent groups in developing countries on fuelwood and some non-timber products. The production of these goods and services depends on various attributes of forests such as composition in terms of species and size and distribution of trees, canopy cover, climatic conditions, and topographical conditions. Hence, classification of a forest into different stands or cohorts is an essential element of forest management, and age is the most common characteristic used to classify forests for management purposes. However, as stated earlier, in the multiple cohort approach of sustainable forest management, classification is done on the basis of compositional, structural, and age variables. Similarly, managers of national parks and wildlife sanctuaries use a forest stands/cohorts classification system based on a wildlife habitat suitability index. Hence, it is critical to consider the specific features of different cohorts/stands in forest harvesting decisions.

First, for simplicity reasons, we consider a forest of three types of stands or cohorts. In some cases of forest management, three cohorts may be enough, but in other cases a greater number of cohorts may be required. For example, the proponents of multi-cohort forest management (MCFM) in Ontario, Canada have classified forests in three cohorts: the first cohort of a young even-aged forest, the second cohort of a mid-successional forest, and the third cohort when virtually all the first cohort pioneer trees have died (Kuttner 2006). Generally, the number of cohorts/groups based on a wildlife habitat suitability index is more than three and varies across national parks.

Let us consider three types of forest stock/stands simply distinguished on the basis of age only—young, mature, and old stock. In terms of consumption, young stock is generally consumed for fuelwood, pole crop, and pulpwood, mature stock for agricultural equipments, small construction, and low-end furniture, and old stock for valuable construction and furniture. In terms of amenity values from forest stock, young stock provides wildlife habitat for small mammals, mature and old stocks provide recreation, wildlife-life habitat for big mammals, and existence values. The contributions of these three types of stocks to other values, such as carbon sequestration, watershed services, cultural values, are also not directly proportional to the volume of timber (cubic meters) in each age class. Hence, the utility, either from consumption or conservation, provided by a forest having a fixed growing stock, say 100 m^3 of wood, will depend not only upon the total timber stock, but also on the distribution of that timber stock in three classes of stock, and different types of forest stocks will have different utility functions for timber consumption as well as for amenity value from stock. Hence, the stocks and consumption from these three types of stands are not additive, whereas the utilities are additive.

In addition to the differentiation between the consumption and conservation utilities from different types of stocks, the dynamic relationship between the different types of stocks will also affect the economically optimal harvesting decisions. For example, with time the forest stock from the young class will move to the mature class and from the mature class to the old class, and this movement will influence the economically optimal conditions for harvesting and conservation. Hence, we propose a growth structure before we develop the EGGR for harvesting decisions.

We assume that the young forest stock is expressed as S_1, the mature forest stock as S_2 and the old forest stock as S_3. The proportions of these three forest stocks will vary from forest to forest, depending upon the biological features of each forest, natural disturbances, and forest management. The growth function of each stock is assumed to be logistic with S_1^*, S_2^*, and S_3^* as maximum possible stocks that can be preserved in the forest resource. As the forest resource grows, in a given period of time, some trees in the young stock remain in young stock and some cross over to mature forest stock. Similarly, within a given period, some trees in mature stock remain in mature stock and some cross over to old forest stock. The total growth of young forest stock is the difference between its own growth and its growth that crosses over to the mature forest stock; the total growth of mature forest stock is the sum of its own growth and the partial growth of young stock that crosses over to mature forest stock minus its growth that crosses over to the old forest stock; and the total growth of old forest stock is the sum of its own growth and the partial growth of mature stock that crosses over to old forest stock. If θ_1 is the proportion of the growth of S_1 that remains in S_1 and $(1 - \theta_1)$ is the proportion of the growth of S_1 that adds to the growth of S_2; and if θ_2 is the proportion of the growth of S_2 that remains in S_2 and $(1 - \theta_2)$ is the proportion of the growth of S_2 that adds to the growth of S_3, then the growth functions of the three stocks could be represented as:

$$R^1 = \theta_1 \rho_1 S_1 \left(1 - \frac{S_1}{S_1^*}\right) \qquad \text{where, } 0 < S_1 < S_1^* \tag{12.1}$$

$$R^2 = (1-\theta_1)\rho_1 S_1\left(1-\frac{S_1}{S_1^*}\right) + \theta_2\rho_2 S_2\left(1-\frac{S_2}{S_2^*}\right) \qquad \text{where, } 0<S_2<S_2^* \quad (12.2)$$

$$R^3 = (1-\theta_2)\rho_2 S_2\left(1-\frac{S_2}{S_2^*}\right) + \rho_3 S_3\left(1-\frac{S_3}{S_3^*}\right) \qquad \text{where, } 0<S_3<S_3^* \quad (12.3)$$

where, ρ_1, ρ_2 and ρ_3 are characteristic growth coefficients of young, mature and old stocks respectively. The values of ρ_1, ρ_2 and ρ_3 depend on the type of forest resource, and climatic, soil, and topographical features of the forest site.

12.3 The Enhanced Green Golden Rule

We modify the economic model of Chichilnisky et al. (1995) to incorporate three cohorts/stocks, and assume that the consumption and levels of three stocks contribute to utility. Suppose the utility function $U(C_{1t}, C_{2t}, C_{3t}, S_{1t}, S_{2t}, S_{3t})$ is strictly concave. For succinctness, we use the notation $U(C_t, S_t)$. We also assume that the utility function is additively separable in consumption and stocks. Suppose the production of man-made capital K_t occurs according to the linear homogeneous production function $F(K_t, S_{1t}, S_{2t}, S_{3t})$, and capital accumulation is expressed as

$$K_t = F(K_t, S_t) - C_t \quad (12.4)$$

The rates of change of the three stocks of forest are expressed as:

$$\dot{S}_{1t} = R^1 - C_{1t} \quad (12.5)$$

$$\dot{S}_{2t} = R^2 - C_{2t} \quad (12.6)$$

$$\dot{S}_{3t} = R^3 - C_{3t} \quad (12.7)$$

Similar to the GGR, in which society is concerned only with the long-run values of consumption and the levels of forest stocks, we seek a path to maximize the long-run utility, $\lim\limits_{t\to\infty} U(C_t, S_t)$ The solution is specified by the following proposition:

12.3.1 Proposition

There exist values of $(K^*, S_1^*, S_2^*, S_3^*, C_1^*, C_2^*, C_3^*)$ characterized by

$$\frac{U_{s_1}}{U_{c_1}} = -R^1_{s_1} - \frac{U_{c_2}}{U_{c_1}} R^2_{s_1},$$

$$\frac{U_{s_2}}{U_{c_2}} = -R^2_{s_2} - \frac{U_{c_3}}{U_{c_2}} R^3_{s_2}, \text{ and}$$

$$\frac{U_{s_3}}{U_{c_3}} = -R^3_{s_3},$$

such that $\lim_{t\to\infty} U(K_t,\ C_t,\ S_t) = \mathrm{U}(K^*, S_1^*, S_2^*, S_3^*, C_1^*, C_2^*, C_3^*)$ is a necessary and sufficient condition for a feasible path (K_t, C_t, S_t) for all t to be a solution of the problem that maximizes $\lim_{t\to\infty} U(K_t,\ C_t,\ S_t)$ over all feasible paths.

Proof: The indefinitely maintainable values of C_1, C_2, C_3 and S_1, S_2, S_3 satisfy $R^1 = C_1$, $R^2 = C_2$, and $R^3 = C_3$. Therefore, the problem. Maximize $\lim_{t\to\infty} U(K_t,\ C_t,\ S_t)$, over feasible paths, reduces to Maximize $U(C, S)$

Subject to the constraints given by Eqs. (12.8), (12.9), and (12.10).

$$R^1 = C_1 \tag{12.8}$$

$$R^2 = C_2 \tag{12.9}$$

$$R^3 = C_3 \tag{12.10}$$

Similar to the GGR, the stock of capital is not a concern because any stock of capital can be accumulated over a sufficiently long period. The set of (S, C) satisfying the constraint in (12.8), (12.9), and (12.10) is compact, so this problem is well-defined. Hence, the maximum is characterized by the first order conditions:

$$\frac{U_{s_1}}{U_{c_1}} = -R^1_{s_1} - \frac{U_{c_2}}{U_{c_1}} R^2_{s_1} \tag{12.11}$$

$$\frac{U_{s_2}}{U_{c_2}} = -R^2_{s_2} - \frac{U_{c_3}}{U_{c_2}} R^3_{s_2} \tag{12.12}$$

$$\frac{U_{s_3}}{U_{c_3}} = -R^3_{s_3} \tag{12.13}$$

This completes the proof of Proposition 1.

We term the solution provided by Eqs. (12.11), (12.12), and (12.13) as the Enhanced Green Golden Rule (EGGR), and the rule provides the optimal conditions for three stocks. The rule does not provide the actual levels of harvesting but it tells that harvesting should be done in a way that these conditions are satisfied. The rule presented above is for a forest with three cohorts/stocks, but the similarity between the Eqs. (12.11) and (12.12) and the difference between the Eqs. (12.11) and (12.13) or (12.12) and (12.13) can be used to generalize the EGGR for any number of cohorts/stocks greater than three. On the basis of these three equations, we can conclude that the optimality conditions for all stocks, except the terminal stock, will be the same, while the optimality condition for the terminal stock will always be given by the equation that is the same as Eq. (12.13). Hence, a generalized EGGR for n number of stocks/cohorts is given by n equations given below:

$$
\begin{aligned}
&\frac{U_{s_1}}{U_{c_1}} = -R^1_{s_1} - \frac{U_{c_2}}{U_{c_1}} R^2_{s_1} \\
&\frac{U_{s_2}}{U_{c_2}} = -R^2_{s_2} - \frac{U_{c_3}}{U_{c_2}} R^3_{s_2} \\
&\downarrow \\
&\downarrow \\
&\frac{U_{s_{n-1}}}{U_{c_{n-1}}} = -R^{n-1}_{s_{n-1}} - \frac{U_{c_n}}{U_{c_{n-1}}} R^n_{s_{n-1}} \\
&\frac{U_{s_n}}{U_{c_n}} = -R^n_{s_n}
\end{aligned}
$$

12.3.2 Economic Interpretation of the EGGR

The optimality condition for the terminal stock (old stock) is the same as the optimality condition given by the GGR, that is, the optimality condition when stock differentiation is not considered. In addition, the optimality conditions for other stocks (young and mature stocks) will also turn into the same optimality condition as given by the GGR if the growth of stocks from one type of stock to another type of stock, such as growth from young to mature and mature to old stock, is assumed to be zero. In other words, if different forest stocks are considered independent of each other, the optimality conditions for different stocks will be the same as given by the GGR. Hence, the key distinguishing factor between the EGGR and the GGR is not the multiplicity of stocks, but the growth dynamics between different stocks. In the case of a forest comprised of n distinct stocks where each stock is defined in a way that the growth of all stocks always remains part of the same stock, we will get n conditions for optimality, but all of them will be the same as Eq. (12.13) or the same condition as for the GGR. In other words, in the case of a forest with n totally independent stocks, the GGR will be applicable to each stock separately.

Next, let us examine the differences between the optimality conditions for the terminal stock and all other stocks. The left-hand side (LHS) of each equation (optimal condition) signifies the marginal rate of substitution (MRS) between consumption and stock-level, and the right-hand side (RHS) corresponds to the marginal rate of transformation (MRT) of the respective stock. The EGGR gives the same optimality condition for the old (or terminal) stock (Eq. 12.13) as for the GGR; the MRS between consumption and stock-level is equal to the MRT of the stock with respect to itself. However, the optimality conditions for young and mature stocks (Eqs. 12.11 and 12.12) are different than the GGR conditions. For young stock, the EGGR requires that the MRS between the consumption and stock-level is equal to the MRT of the young stock with respect to itself plus the MRT of mature stock with respect to young stock expressed in terms of young stock (normalized by the ratio of the marginal utilities of mature stock and young stock). We call the RHS of Eq. (12.11) as the Normalized Composite Marginal Rate of Transformation (NCMRT) (composite of the rate of transformation of the young and mature stocks and normalized to express in the units of young stock). The same interpretation applies to all other stocks except the terminal stock. We would like to remind readers that the three Eqs. (12.11), (12.12), and (12.13) are not independent, and the optimal levels of three stocks will be given by the solution of these three simultaneous equations. Hence, readers should avoid inferences based on each equation independently.

12.3.3 Welfare Implications

The welfare implications of the optimal conditions for multiple stocks can be understood by expressing Eqs. (12.11), (12.12) and (12.13) as follows:

$$\Delta C_1 U_{c_1} = \Delta C_1 [U_{s_1} + U_{c_1} R^1_{s_1} + U_{c_2} R^2_{s_1}] \tag{12.14}$$

$$\Delta C_2 U_{c_2} = \Delta C_2 [U_{s_2} + U_{c_2} R^2_{s_2} + U_{c_3} R^3_{s_2}] \tag{12.15}$$

$$\Delta C_3 U_{c_3} = \Delta C_3 [U_{s_3} + U_{c_3} R^3_{s_3}] \tag{12.16}$$

These welfare equations describe the equalities, at the optimal conditions, between the welfare gain and welfare loss due to marginal changes in consumption or the level of stock, and not the overall welfare gain or loss due to a change in consumption or level of stock. As expected, due to the incorporation of multiple stocks and growth dynamics between different stocks, the welfare implications are quite different than the case of a single-stock-based GGR. The growth dynamics make these implications quite interesting because a decrease/increase in the consumption of a particular stock, say young stock, not only affects the level of that stock, but it also affects the growth of the same stock as well as of the next class of stock, the mature stock. Three terms on the RHS of Eqs. (12.14) and (12.15) capture the effects of change in the stock level and the growth dynamics of two groups of stock. In the case of terminal stock (old stock), since there is no movement of the stock from this class to the next class of stock, there are only two terms on the RHS of the equation (one for the level of stock and the second for the growth in this class of stock).

Using Eq. (12.14), let us analyze the welfare implications of reducing the consumption of young stock by an amount ΔC_1. The reduction in consumption will increase the stock S_1 by the same amount, and due to this change in the level of stock, the growth of S_1 and S_2 will be affected. The LHS of this equation, $\Delta C_1 U_{c_1}$, signifies the welfare loss associated with reduced consumption. The right-hand side of the equation gives the welfare increase due to the change in the level of stock equal to an amount ΔC_1 of the young stock. However, the welfare increase due to the change in the stock level is composed of three components: (1) welfare change of $\Delta C_1 U_{s_1}$ from increased level of young stock, (2) welfare change of $\Delta C_1 U_{c_1} R^1_{s_1}$ due to the change in growth of stock S_1 resulting from the increase of stock S_1, and (3) welfare change of $\Delta C_1 U_{c_2} R^2_{s_1}$ due to the change in growth of stock S_2 resulting from the increase of stock S_1. Similarly, Eq. (12.15) can be interpreted for a reduction in the consumption of mature stock by ΔC_2, and its welfare implications are the same as for the young stock. In the case of the terminal stock, Eq. (12.16), the loss in welfare due to the reduction in consumption by ΔC_3 is equal to the welfare gains due to the increased stock of S_3 and change in growth of stock S_3.

12.3.4 User Groups' Specific Optimal Conditions

One of the key features of the EGGR, similar to the GGR, is that the conditions for the highest indefinitely maintainable level of utility–which includes utilities from consumption as well as the stock of the forest resource–depends on the marginal utilities of consumption and level of stocks of different types of stocks (economic characteristics of the user groups of the respective forest) and the rate of growth of different stocks (biological features of forest under consideration). We call these conditions "conditions of sustainability" or "sustainable forest management". In these conditions of sustainability, there is no direct role for the price of timber and the discount rate, but the price of timber may depend on the marginal utilities of consumption of timber. Hence, the EGGR provides the sustainability conditions which may provide different levels of stocks and consumption levels of different types of stocks across different user groups due to the possible differences in marginal utilities of different stocks across user groups. This means that for the same type of forest (a forest which has the same biological features), the sustainability configuration (composition of different types of stocks) may be different in different locations depending on the marginal utilities of the associated user groups and, accordingly, harvesting decisions for sustainability will also vary across user groups as per their marginal utilities. Hence, the outcome of the EGGR may be a compositional diversity of the same types of forests which is in contravention to the idealistic concept of a normal forest—the same configuration of forests (all age classes) groups have the same area) across all user groups.

In addition, any user group's utilities, either from stock or consumption of different stocks, may not remain the same forever; the shape of the utility function may change or the values of utility indices may change over time, and that will lead to a change in the sustainability configuration of a forest over time. Hence, the sustainability conditions, given by the EGGR, are not static or a permanent equilibrium concept, but an evolutionary concept which captures the dynamics of economic as well as biological features. This is also contrary to the concept of a normal forest. The dynamics of sustainability conditions are similar to the concept of dynamics of optimal forest regimes proposed by Kant (2000).

12.4 An Illustration of the Enhanced Green Golden Rule

Let us assume that the utility function for a three-stock forest resource, introduced in Sect. 12.2, is a standard logarithmic Cobb Douglas function. Further, let us assume the index of the utility of stock S_1 is α, stock S_2 is β, stock S_3 is γ, consumption C_1 is $(1-\alpha)$, consumption C_2 is $(1-\beta)$, and consumption C_3 is $(1-\gamma)$. The utility function can be represented as:

$$\begin{aligned} U(C,S) = {} & \alpha \ln S_1 + \beta \ln S_2 + \gamma \ln S_3 + (1-\alpha)\ln C_1 \\ & + (1-\beta)\ln C_2 + (1-\gamma)\ln C_3 \end{aligned} \tag{12.17}$$

Using Eqs. (12.11), (12.12) and (12.13) and solving these equations, the EGGR gives optimum values for the consumption and levels of three stocks $(C_1,\ C_2,\ C_3,\ S_1,\ S_2,\ S_3)$ as follows:

$$C_1 = \theta_1 \rho_1 S_1 \left(1 - \frac{S_1}{S_1^*}\right) \tag{12.18}$$

$$C_2 = (1-\theta_1)\rho_1 S_1 \left(1 - \frac{S_1}{S_1^*}\right) + \theta_2 \rho_2 S_2 \left(1 - \frac{S_2}{S_2^*}\right) \tag{12.19}$$

$$C_3 = (1-\theta_2)\rho_2 S_2 \left(1 - \frac{S_2}{S_2^*}\right) + \rho_3 S_3 \left(1 - \frac{S_3}{S_3^*}\right) \tag{12.20}$$

$$\frac{\alpha C_1}{(1-\alpha)S_1} = -\theta_1 \rho_1 \left(1 - \frac{2S_1}{S_1^*}\right) - \frac{(1-\beta)C_1}{(1-\alpha)C_2}\left[(1-\theta_1)\rho_1\left(1 - \frac{2S_1}{S_1^*}\right)\right] \tag{12.21}$$

$$\frac{\beta C_2}{(1-\beta)S_2} = -\theta_2 \rho_2 \left(1 - \frac{2S_2}{S_2^*}\right) - \frac{(1-\gamma)C_2}{(1-\beta)C_3}\left[(1-\theta_2)\rho_2\left(1 - \frac{2S_2}{S_2^*}\right)\right] \tag{12.22}$$

$$\frac{\gamma C_3}{(1-\gamma)S_3} = -\rho_3 \left(1 - \frac{2S_3}{S_3^*}\right) \tag{12.23}$$

In these six Eqs. (12.18–12.23), there are six unknowns (C_1, C_2, C_3, S_1, S_2, and S_3), that can be solved in terms of θ_1, θ_2, ρ_1, ρ_2, ρ_3, S_1^*, S_2^*, S_3^*, α, β, and γ. Here, for illustration purpose, we solve these equations for some assumed values of θ_1, θ_2, α, β, and γ.

Case 1: *A Forest Resource of Three Stocks in Which the Growth in Every Stock is Independent of the Other Stocks:* This means that all growth in the young stock remains in the same stock ($\theta_1 = 1$) and all growth in the mature stock remains within the mature stock ($\theta_2 = 1$).

Substituting $\theta_1 = 1, \theta_2 = 1$ in Eqs. (12.18–12.23) and solving, we obtain

$$S_1 = \frac{S_1^*}{[2 - \alpha]} \tag{12.24}$$

$$S_2 = \frac{S_2^*}{[2 - \beta]} \tag{12.25}$$

$$S_3 = \frac{S_3^*}{[2 - \gamma]} \tag{12.26}$$

These three equations clearly indicate that the optimal level of each type of stock (young, mature, and old) depends on the possible maximum level of that stock (S_1^*, S_2^*, and S_3^*) and the utility index (α, β, γ) for the level of that stock. As the utility index for the level of stock increases, the optimal level of the stock also increases. The maximum possible value of the utility index is 1 which means that forest user groups derive all utility from the level of stock and no utility from the consumption of that stock; in such cases, the optimum level of stock will be equal to the possible maximum level of the stock. On other hand, if user groups derive utility only from consumption and no utility from the level of stock, the utility index for the level of stock will be equal to zero, and in this case the optimal level of stock will be half of the possible maximum stock. Given our growth functions for the three types of stocks, the Maximum Sustained Yield (MSY) levels for young, mature, and old stocks are 0.5 S_1^*, 0.5 S_2^*, and 0.5 S_3^*, respectively. Hence, in the case of no utility from the level of stocks, the optimal solution will be equal to the MSY for all three stocks. However, if user groups derive any utility from the levels of stocks ($\alpha > 0$, $\beta > 0$, $\gamma > 0$), the optimal stock levels will be higher than the MSY levels of the stocks.

In essence, if we consider a forest composed of different stocks, but that all stocks are independent of each other (there is no movement from one type of stock to other types of stocks), the optimal level of each stock will depend on the utility index of the level of that stock (user groups' characteristic) and a biological characteristic of the forest—the possible maximum level of that stock. In the case of some societies, such as Aboriginal groups and other tribal groups, utility indices from the levels of stocks may be close to 1 and utility indices from the consumption of all stocks close to zero. In this case, the optimum level of all stocks will be the possible maximum levels of stocks, which means conserving all stocks and letting them reach the possible maximum levels. In the case of industrial-growth focused societies, utility indices from the consumption of all stocks may be close to one while the utility indices from the level of stocks may be close to zero. The optimum level of stocks in these cases will be equal to the MSY of each stock.

These are two extreme cases. In other cases, the optimum levels of stocks will be somewhere between the possible maximum level of each stock and the MSY of respective stock.

Case 2: *A Forest Resource of Three Stocks in Which All Growth in Young Stock Crosses Over to the Mature Stock* $(\theta_1 = 0)$ *and All Growth in Mature Stock Crosses Over to the Old Stock* $(\theta_2 = 0)$.

Substituting $\theta_1 = 0, \theta_2 = 0$ in Eqs. (12.18–12.23) and solving, we obtain

$$C_1 = 0$$

$$C_2 = \rho_1 S_1 \left(1 - \frac{S_1}{S_1^*}\right)$$

$$C_3 = \rho_2 S_2 \left(1 - \frac{S_2}{S_2^*}\right) + \rho_3 S_3 \left(1 - \frac{S_3}{S_3^*}\right)$$

$$\frac{\alpha}{S_1} = -\frac{(1-\beta)}{C_2}\left[\rho_1\left(1 - \frac{2S_1}{S_1^*}\right)\right]$$

$$\frac{\beta}{S_2} = -\frac{(1-\gamma)}{C_3}\left[\rho_2\left(1 - \frac{2S_2}{S_2^*}\right)\right]$$

$$\frac{\gamma C_3}{(1-\gamma)S_3} = -\rho_3\left(1 - \frac{2S_3}{S_3^*}\right)$$

These six equations can be solved for C_1, C_2, C_3, S_1, S_2, and S_3 only if we know the values of ρ_1, ρ_2, ρ_3, S_1^*, S_2^*, S_3^*, α, β, and γ. Hence, for illustration, we assume that the growth coefficients for all three stocks are equal to unity $(\rho_1 = \rho_2 = \rho_3 = 1)$, and utility indices for the level of stocks for all three stocks is equal to 0.5 $(\alpha = \beta = \gamma = 0.5)$. This means that the utility indices for the consumption of all three stocks are also equal to 0.5. In other words, this is the case in which elasticity of utility with respect to consumption and the level of stock are the same (0.5) for all three types of stocks. Using these values, we get the following solutions:

$$S_1 = \frac{2S_1^*}{3} \tag{12.27}$$

$$S_2 = \frac{3S_2^*}{4} \tag{12.28}$$

$$S_3 = \frac{3S_3^*}{4} \tag{12.29}$$

Equations (12.27), (12.28), and (12.29) provide the optimal levels of stocks for young, mature, and old stocks, respectively, and the optimal levels for all three stocks are greater than the respective levels of stocks for the MSY. The optimum level of young stock is only two-thirds of the possible maximum level of this stock, while the optimum levels of mature and old stocks are three-quarters of their possible maximum levels. This may seem strange but it is due to the fact that all growth in the young stock moves to the mature stock which results in no consumption of the young stock even though there is a positive utility from the consumption of young stock. Hence, this is an outcome of our assumption which we have to make to find a solution in the simplest way.

In brief, we can conclude that the optimal levels of different stocks will vary between the MSY levels of the stocks and the maximum possible stocks, while the actual level of an optimal stock will depend upon the utility indices of the levels of stocks and growth functions of different stocks. For any specific stock, the optimal level of that stock will be closer to the MSY level for a low utility index of the level of stock and a higher utility index for the consumption of that stock; while the optimal level of that stock will be closer to the possible maximum level of that stock for a high utility index of the level of stock and a low utility index for the consumption of that stock. We can obtain optimum levels of conservation of these stocks for different combinations of θ_1 and θ_2 depending on the type of forest.

12.5 Conclusions

The concept of the Green Golden Rule, introduced by Chichilnisky et al. (1995), gives a path of maximum sustainable long-run utility, accounts for future generations in determining the optimal solutions, and includes utilities from resource consumption as well as the level of resource stock or amenity values of the resource. The concept of GGR is much closer to the requirements of sustainable forest management as compared to the concept of the normal forest used by the Faustmann forest economist to determine harvesting decisions for a forest. Hence, in this chapter, we extended the concept of GGR to incorporate the diversity of forest stocks, and identified the optimal conditions for a forest of three types of stocks as well as a forest of n types of stocks, and illustrated the determination of optimal conditions using the Cobb-Douglas Utility function and logistic growth function of forest stocks. The results of this chapter provide many useful insights with respect to harvesting decisions for sustainable forest management.

The most significant result of this chapter is that the harvesting decisions for forests for sustainable forest management will not be the same across different user groups even for forests that are biologically same. Harvesting decisions will be affected by the user groups' utilities from consumption as well as the amenity values from the level of stocks (conservation of stock). User groups' utilities will vary across groups, and therefore harvesting decisions will vary across user groups. The variation in user groups' utilities also imply that the sustainability composition of the same type of forests will be different across user groups, and that means the concept of the normal forest is redundant. Hence, Post-Faustmann forest economics has to have its roots in the concept of multiple and evolving equilibria rather than in the concept of a single and permanent equilibrium such as a normal forest.

Second, the EGGR does not provide harvesting rules but it does provide the levels of different stocks that need to be conserved for maximum long-run sustainable utility. Hence, the harvesting rules have to be designed by forest managers according to the levels of stocks to be conserved. This is similar to current forest management practices in which forest managers design harvesting rules considering only the biological aspects of forests. However, sustainability includes ecological, social, and economic considerations, and therefore the inclusion of social and economic dimensions in forest harvesting decisions is critical. The outcomes of the EGGR provide a tool to forest managers to develop harvesting rules that incorporate different user groups' consumption preferences as well as their preferences for amenity values.

Third, the inclusion of multiple stocks or cohorts extends the application of the Green Golden Rule to all types of forest management, such as management for biodiversity, wildlife habitat management, near-natural forest management, continuous cover forest management, Aboriginal forest management and even forest management for industrial purposes only. Hence, the EGGR can be used to design harvesting rules for any type of forest management.

Finally, good knowledge of user groups' utilities and the growth functions of each stock of different types of forests is essential for the applications of the EGGR. Hence, for sustainable forest management, all agencies involved in forest management should focus their attention on studies of forest growth and user groups utilities.

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