Chapter 2
Review of Frontier Models and Efficiency Analysis: A Parametric Approach

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Abstract The parametric frontier approach to efficiency measurement has been extensively used in applied research. Within this conceptual framework, techniques for econometric frontier analysis will be described. The purpose of this paper is to present an overview of parametric frontier methods related to the measurement of economic efficiency, focusing on both deterministic and stochastic perspectives. In addition, development and extension of the cross-sectional and panel data context associated with specification of functional forms are also revisited.

Keywords Efficiency analysis • Parametric frontier models • Functional forms • Stochastic and deterministic specification

2.1 Introduction

In the actual context of global economy, with some economies experiencing slow and decelerating growth, accompanied by high levels of unemployment, sustainable economic recovery of states emerges as a priority issue of world development strategy. It is in this paradigm that organizations' competitiveness, allied to efficiency analysis, must be allocated as tools to improve societies well-being. Also at the micro-level of analysis, efficiency is associated to sustainable development as the concept evolves the parsimony use of economic resources in order to reach cost minimization, output and profit maximization. Measurement of firm efficiency represents one of the most important subjects of investigation at the microeconomic level, either in the context of developing and developed countries or within different contexts of analysis. This is supported by the amount of empirical studies dealing

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with analysis of organizational efficiency emerging over the past 30 years in benchmarking scientific literature, covering a wide range of fields. Agriculture, banking, environmental economics, finance, transport, education, forestry, tourism and sport are examples of sectors where such evaluation has been applied. The research of reasons behind firm inefficiency is very important as it may be used to correct fragilities and to improve efficiency in the production context. According to the literature, organization' efficiency measures may be obtained through the estimation of an efficient frontier, with distance to this frontier being an indicator of the organization inefficiency (Kopp and Diewert 1982). Although traditional regression analysis has been widely applied in economic analysis, for the study of firm efficiency, it is consensual among the scientific community that frontier methodology is much more appropriate than least-squares methods, because the adjustment of a function through the middle of a cloud of points can only obtain average parameter estimates of the production structure rather than those associated with 'best practice' technology (Greene 1997). The concept of (in)efficiency is associated with the theory of optimization and with the extent to which an economic unit fails to achieve a theoretically ideal level of production possibilities (Forsund et al. 1980).

The research of reasons behind firm inefficiency is very important as it may be used to correct fragilities and to improve efficiency in the production context. Efficiency is associated with (1) technical efficiency if the goal of the analysis is to obtain maximum output given a set of inputs, (2) cost-efficiency if the aim is the minimum cost of producing that output given the input prices or (3) profit efficiency in the case where interest is in the maximum profit attainable given the inputs, outputs and price of the inputs (Greene 2005).

In order to measure organizations' economic (in)efficiency, two main alternative approaches have been developed and applied in empirical studies: a parametric and a non-parametric method, or stochastic frontier approach (SFA) and data envelopment analysis (DEA) (Charnes et al. 1978), as the most efficient frontier is best described by a parametric function involving econometric methods for estimation purposes or constructed through the use of a mathematical programming model applied to observed data, respectively.

Frontier models and the associated measurement of efficiency have a long history in the field of applied economics. Indeed and although papers by Debreu (1951) and Koopmans (1951) had marked the origin of discussion on measurement of efficiency, it was the work of Farrell (1957), extending Debreu and Koopmans researches that suggested to measure technical (in)efficiency as the realized deviation from a frontier isoquant. After Farrell's seminal article on efficiency measurements, several other approaches have been developed. It is consensual that only after the 1950s was the threshold between micro- and macro-level production/cost analysis developed, expanding economic analysis at firm level.

For contextualization of the parametric approach to economic frontier modelling at the microeconomic level, the contributions, in estimating production functions, of Cobb and Douglas (1928), Samuelson (1938), Dean (1951), Shephard (1953), Johnston (1959), Arrow et al. (1961) and Nerlove (1963) represent a significant point in econometrical literature. Still in the paradigm of empirical analysis at the micro context, extension and identification of the term inefficiency, with the usual disturbance component of a regression model, have been frequently applied.

This chapter presents an overview of the parametric frontier approach to efficiency measurement. Section 2.2 refers to the econometric approach to efficiency, Sect. 2.3 reviews cross-sectional frontier models, Sect. 2.4 deals with frontier analysis with panel data and, finally, Sect. 2.5 extends the theory to new statistical developments and applications.

2.2 Econometric Approach to Efficiency

The econometric approach to efficiency results from the estimation of frontier models which deal with parametric representations of technology along with a one-sided error term or a two-part error term (composed error), depending on how the functional form has been specified as being a deterministic or a stochastic frontier (Kalirajan and Shand 1999; Murillo-Zamoran and Vega-Cervera 2001). As the parametric approach to efficiency states an econometric specification, models may also be classified according to the function form, production, cost or profit, or functional form, which describes the relationship between inputs and outputs. Additionally, other criteria may be assumed to classify frontier models, such as the sample context (cross-sectional or panel data), the temporal variation of inefficiency and the functional specification of the inefficiency term of the composed error.

Introduced by Koopmans (1951), extended by Debreu (1951) and developed in the empirical econometric field through the seminal paper by Farrell (1957), the concept of economic efficiency has been of interest to modern researchers. Both (1) the conceptual contribution of Koopmans (1951), defining technical efficiency as a feasible input–output combination where it is not possible to increase output (decrease input) without simultaneously increasing input (decreasing output), and (2) the Debreu coefficient of resource cost allocation for the measurement of technical and allocative inefficiency, supported by resources, technology and preferences and resulting from the ratio between minimized resource costs of obtaining a given consumption bundle and actual costs, for given prices and a proportional contraction of resources (Forsund and Sarafoglou 2002), represent significant contributions to interpretation of efficiency. Augmenting the Debreu coefficient of a proportionate input contraction and associating technical inefficiency with deviations from an idealized frontier isoquant, Farrell' work has been greatly analysed and discussed in the context of empirical literature on production and cost functions. Extending the standard definition of production function, a production frontier model defines the boundary of the former as it gives the maximum possible output for a given set of inputs. So, an efficient organization will be located on the frontier of production, reflecting technical or allocative efficiency. Technical efficiency occurs when a firm produces at the maximum level of output given inputs or as it uses the minimum level of inputs given output. Allocative efficiency occurs when the marginal rate of substitution between inputs equals the input price ratio, or as it operates at an optimal proportion use of inputs, given the prices.

2.3 Cross-Sectional Frontier Models

In a cross-sectional context and depending on the specification of the parametric frontier function error term (one or two component), models used for the estimation of technical efficiency can be classified as deterministic frontier models or as stochastic frontier models. The parametric approach to efficiency measurement estimation appears in the production theory with Aigner and Chu (1968) being the first to follow Farrell's suggestion, but it is in the 1970s that the study of economic efficiency in the sectional context develops most. In the econometric literature, various criteria exist for classifying frontier models (Forsund et al. 1980).

The models are statistical when the error term is specified by a given distribution of probability and the estimators have statistical properties. On the contrary, models are not statistical when this term does not have statistical properties. In nonstatistical deterministic models, linear programming and quadratic programming techniques are used to construct the frontier.

In the category of statistical models, they are deterministic or stochastic according to the error term of the respective objective function being formed by one component, which only translates inefficiency of the process or by two components, an inefficiency term and a white noise, respectively. Parametric deterministic models evolve both goal programming and econometric techniques in order to either calculate the parameter vector or estimates of inefficiency.

With goal programming techniques, the technology parameter vector is calculated through the solution of a deterministic optimization method (Aigner and Chu 1968; Timmer 1971; Forsund and Hjalmarsson 1979; Nishimizu and Page 1982; Forsund 1992). This type of frontier was subsequently abandoned and replaced by another type more suited to statistical analysis of the results, that is, by deterministic parametric frontiers with statistical properties.

With the deterministic statistical approach, the parameters are estimated rather than calculated, allowing the use of additional statistical inference procedures. This new type of model, initially developed by Afriat (1972), was later enhanced by the contributions of Richmond (1974), Gabrielsen (1975), Schmidt (1976) and Greene (1980a). At the end of the 1970s and as an alternative to deterministic frontiers, stochastic frontiers appeared, allowing deviation in relation to the frontier to be also explained by a factor outside the firm's control (Lee and Tyler 1978). Stochastic frontier specification includes a two-sided error term, capturing not only the effects of the classical statistical noise but also technical inefficiency or the magnitude of the shortfall of output from its maximal possible value.

2.3.1 Deterministic Frontier Models

Under statistical deterministic models, all the deviations from the frontier are assumed to be the result of technical inefficiency of the production process, and no account is taken of measurement errors or statistical noise. The error term is completely due to inefficiency which may be specified according to a given asymmetric distribution of probability, such as a semi-normal, a truncated normal, an exponential (Schmidt 1976) or a gamma distribution (Greene 1990).

Aigner and Chu (1968) were the first authors to estimate a parametric and deterministic frontier model through a Cobb–Douglas function, which would express behaviour's heterogeneity of firms in the production context. According to them, differences captured from a cross-sectional group of units would be explained by technological reasons associated to the industry where they operate, by the scale of operations and by different options of management structures. When a firm operates at the frontier of production it is expected a zero disturbance in its model specification, meaning that it operates at the potential level of production.

By contrary, when a firm operates under the potential level of production, it is expected that the disturbance captures this fact, through a distance from the frontier, or the inefficiency magnitude. A deterministic parametric frontier may be specified as $Y_i = f(X_i; \beta)\mathrm{TE}_i$, where i indicates the producer, Y the scalar output, X a vector of inputs, $f(\cdot)$ the production frontier, β the parameter vector and TE technical efficiency. This last formulation is obtained through the ratio of the observed output to the maximum feasible output, or

$$\mathrm{TE}_i = \frac{Y_{i\,(\text{observed})}}{f\left(X_{i\,(\text{frontier})}, \beta\right)} \tag{2.1}$$

This formulation suggests that technical efficiency is assessed, for each productive unit, through $\mathrm{TE}_i = \exp(-ui)$, with $0 < \mathrm{TE_i} \leq 1$, reflecting the distance of each unit from the production frontier. So, the deterministic frontier formulation may be expressed through:

$$y_i = f\ (X_i;\beta) \exp(-u_i) \quad \mu_i \geq 0 \tag{2.2}$$

where y_i represents the dependent variable and translates the production observed for a productive unit i, with $i = 1, 2, \ldots\ldots N, \beta$ represents a vector of unknown technological parameters, x_i represents a vector of non-stochastic productive factors for observation i and u_i is the model's error component translating technical inefficiency and restricted to be $\geq$0, in order to guarantee that TE ≤ 1.

$$\ln y_i = \beta_0 + \sum_{n=1}^{N} \beta_n \ln x_{ni} - u_i$$

$$u_i \geq 0 \tag{2.3}$$

where the observations in u for each productive unit are non-negative, independent and identically distributed random variables, with an expected value to be positive and constant and the variance finite. It is also assumed that these random variables are not correlated with the regressors. Although the slope parameters in the deterministic frontier models can be consistently estimated by ordinary least squares (OLS) method, the constant term cannot be consistently estimated as the error term is not normality distributed. Considering some additional hypotheses or including adjustments on the specification of the error term of the frontier model, efficiency measures can also be obtained just from OLS as the adjustments restrict the frontier specification to be similar to the classical regression model.

Two methods for evaluating efficiency from OLS estimator involve the corrected ordinary least squares (COLS) and the modified least square (MOLS) as the estimated average-practicing frontier had been shifted up by the maximum amount of residuals (Gabrielsen 1975) or by the mean of residuals (Richmond 1974). Proposed by Winsten (1957) and developed by Gabrielsen (1975) and Greene (1980a, b) to estimate the frontier, COLS method adjusts the OLS line upwards or downward, by the maximum or by the minimum of the residuals, for a production function or for a cost function, respectively, in a way that COLS line rests parallel to the OLS line. Bounding all the units from above, inefficiency may be measured as a distance function from the COLS line so that all the unknown parameters may be consistently estimated.

In a first stage, OLS is used to obtain estimates of the slope parameters and a consistent but biased estimate of the intercept. In the second step, the estimated intercept is shifted up by the maximum value of the OLS residuals $\hat{\beta}_{\mathrm{COLS}} = \hat{\beta}_{\mathrm{OLS}} + m\acute{a}x\, \hat{e}_i$ so that the resulting COLS intercept is consistently obtained. Individual efficiency measures result from subtracting to an individual OLS residual the maximum sample residual such as $\hat{u}_i = \hat{e}_{\mathrm{ols};i} - \max\, \hat{e}_{\mathrm{ols};i}$.

Under a parametric context and proposed by Afriat (1972) and by Richmond (1974),[1] the modified least-squares method (MOLS) represents an alternative technique to OLS procedure in the frontier context, consisting in the correction of the intercept with the expected value of the error term such that the estimated frontier function could be displaced upwards by the estimate of $E[u_i]$, or, $\hat{u}_i$. As in the context of the COLS method, the residuals also provide consistent estimates of individual measures of efficiency, provided the estimated mean of the error term is subtracted: $-\hat{u}_i = \hat{e}_i - \hat{e}_i - \hat{u}_i$ instead of the maximum residual.

Indeed, this difference between two methods does not ensure that all units under MOLS estimation procedure are bounded from above by the estimated production frontier, although both frontiers (COLS and MOLS) led to parallel line to the OLS regression. The inconvenient of this parallelism is associated to the fact that it restricts the structure of the production technologies, best practice (frontier models) and mean practice (classical mean regression) to be equal. Also in the deterministic

[1]These two authors proposed an exponential and a half-normal to model the error term of the model, respectively.

frontier context, Greene (1980b) propose maximum-likelihood (ML) estimation, which represents the most popular and widely used technique in the estimation of frontier models. Kumbhakar and Lovell (2000) present an excellent review of the various distributions for modelling technical efficiency, necessary for maximum-likelihood estimation.

The main advantage of deterministic statistical models is the ease of obtaining individual estimates of efficiency for productive units although the estimation of a deterministic frontier, common to all productive units, assumes that all the deviations from the frontier are entirely interpreted as inefficiency. Indeed, in a deterministic frontier model context, maximum production is given by a function whose error term only reflects the firm's technical efficiency.

However, there are other factors outside its control which affect its behaviour and which are also 'captured' by the unilateral error term. So, the residuals of estimation provided by deterministic methods are therefore overvalued. Assessment of the performance of productive units and their comparison from efficiency measurement based on these residuals is consequently harmed. Summarizing, a deterministic and statistical frontier of production means that all observations (except one) are situated below the production frontier (or in the case of a cost function, above the cost frontier).

This restriction is the main limitation of using deterministic frontiers. The non-existence of a symmetric component in the error term able to capture random or uncontrollable shocks is the principal criticism of statistical deterministic frontier models.

2.3.2 Stochastic Frontier Models

The origins of parametric stochastic frontier analysis are in the efforts made to overcome the limitations imposed by deterministic models in assessing efficiency. This new approach assumes that the frontier varies randomly between productive units, that is, incorporating in its specification an additional error term that captures the effects caused by factors outside the productive unit's control. With a stochastic frontier to model an economic process, the error term is structured according to two components: a first component that would capture statistical noise and a second component that would translate the effects of technical inefficiency.

Both components of error term are specified from probability distributions (an asymmetric one for modelling inefficiency component and a symmetric/normal distribution for modelling the stochastic error component). From an econometric perspective, the stochastic statistical method refers to estimation of models based on functional forms that allow observations on both sides of the frontier.

This method was proposed almost simultaneously in three continents: Meeusen and Van den Broeck (June 1977), Aigner et al. (July 1977) and Battese and Corra (1977), in an attempt to overcome the disadvantages caused by deterministic frontiers in assessing the individual efficiency of productive units.

These models possess not only a component reflecting the distance from the frontier due to producers' technical inefficiency but also a new component that absorbs the impact of random shocks on production. These models are characterized by having an error term with two elements $\varepsilon = v - u$. The first component of the error, usually assumed to follow a Gaussian distribution, reflects disturbances or factors which affect the production level but cannot be controlled (climatic variations, equipment breakdown, illnesses ...).

The second component of the error captures the inefficiency in managing production and is assumed to follow a one-sided distribution, such as an half-normal (Aigner et al. 1977), a truncated normal (Stevenson 1980), an exponential (Meeusen and van den Broeck 1977) and a gamma (Greene 1990). Considering a stochastic formulation for a frontier model, the following Cobb–Douglas log-linear function will include a compound error term and a deterministic part as is showed in (2.4):

$$\ln Y_i = \beta_0 + \sum_{n=1}^{N} \beta_n X_{ni} + v_i - u_i \tag{2.4}$$

where Y_i is the logarithm of production concerning producer $i, i = 1, \ldots N$, X_i is a vector of productive factors used by producer i, β is a vector of technological parameters to be estimated, (v_i) are random i.i.d variables with zero mean and independent of u_i and of the regressors and $u_i \geq 0$ are non-negative random i.i.d variables, independent of v_i and of the regressors. Both error components, v and u, have constant means (0 and μ) and variances, σ_v^2 and σ_u^2, respectively, over all observations.

The joint density of these two error components will underlie likelihood functions. So, according to the distribution assumed for the asymmetric component of the error term, model designations will be given by normal-half-normal, normal-truncated-normal, normal-exponential and, finally, normal-gamma. Initially, stochastic frontiers only allowed estimation of one measurement for the sample's average efficiency.

> The main limitation of the stochastic frontier model is the impossibility of separating the two components from the individual residual, i.e., it does not allow estimation of individual technical inefficiency. (Forsund et al. 1980)

Two years later, Jondrow et al. (1982) presented a method which was able to overcome this major limitation of stochastic frontiers. They showed that for the half-normal case, the expected value of u_i conditional on the composed error term is

$$E\left[\frac{u_i}{\varepsilon_i}\right] = \frac{\sigma\lambda}{(1+\lambda^2)}\left[\frac{\phi(e_i\lambda/\sigma)}{\Phi(-e_i\lambda/\sigma)} - \frac{e_i\lambda}{\sigma}\right] \tag{2.5}$$

where $\phi(\cdot)$ represents the density of the standard normal distribution and $\Phi(\cdot)$ the cumulative density distribution,

$$\lambda = \frac{\sigma_u}{\sigma_v}, \quad e_i = v_i - u_i \quad \text{and} \quad \sigma = \left(\sigma_u^2 + \sigma_v^2\right)^{1/2} \tag{2.6}$$

The basic idea of this method consisted in using the mean and mode of conditional distribution to obtain estimates for each producer as $\text{TE}_i = 1 - E[u_i/e_i]$. Jondrow et al. (1982) applied the methodology to models with the u_i component specified from a semi-normal and an exponential distribution. For the models described through a Cobb–Douglas function and in the form of logarithm, Battese and Coelli (1988) suggested the use of another estimator adapted to sectional data and for normal-truncated-normal and for normal-half-normal distributions and expressed as

$$E\left[\exp\frac{(-u_i)}{e_i}\right] = \left[\frac{1-\Phi\left(\delta+(\gamma e_i/\delta)\right)}{1-\Phi(\gamma e_i/\delta)}\exp\left(\gamma e_i+\left(\frac{\delta^2}{2}\right)\right)\right] \tag{2.7}$$

where

$$\delta = \frac{\sigma_u\sigma_v}{\sigma}; \quad \gamma = \frac{\sigma_u^2}{\sigma^2}. \tag{2.8}$$

With sectional data, two methods of estimating stochastic frontiers are generally analysed: maximum likelihood (ML) – Afriat (1972), Greene (1980b) and Stevenson (1980); and modified least squares (MOLS). The option between methods based on OLS and the ML method also depends on the distribution asymmetry intensity of u_i: when this is not very pronounced, distribution of the error term is approximately symmetric and normal. In these circumstances, results of estimation are similar to those obtained with OLS. The efficiency gains attained by using the ML method only occur if the joint distribution of the error term is clearly asymmetric.

Calculation of the sample's mean technical efficiency can be made from the mean of the residuals from stochastic model estimation: with v_i being a random variable normally distributed with zero mean, the value of the sample's mean efficiency is identical to the mean of the asymmetric component of the error or $E[\varepsilon_i] = E[v_i - u_i] = E[-u_i]$. So, $\overline{\text{ET}} = 1/N\left[\sum_i(-\hat{u}_i)\right]$ or $\overline{\text{ET}} = 1/N\left[\sum_i \exp(-\hat{u}_i)\right]$ if the model is presented in logarithmic form.

The expressions for the sample's mean efficiency and for the respective expected value depend on the probability distribution assumed for u_i (Jondrow et al. 1982). Therefore, if a semi-normal or exponential distribution is assumed, the sample's mean efficiency will be given by $\overline{\text{ET}} = -\hat{\sigma}_u\sqrt{2/\pi}$ and by $\overline{\text{ET}} = 1/\hat{\gamma}$ respectively.

The main disadvantages arising from estimating stochastic frontiers from sectional data for assessment of technical efficiency of productive units are associated with the requirement of major restrictions, such as the absence of correlation between regressors and the term of technical efficiency, suitability of the chosen distributions for modelling the asymmetric component of the compound error and the impossibility of ensuring consistency of the estimators when a productive unit is observed only once. The variance of distribution assumed for the component of technical efficiency conditional in the entire error term does not disappear when the size of the sample increases. Advantages of the panel data stochastic frontier models versus cross-sectional data are presented and explored in Schmidt and Sickles (1984).

2.4 Frontier Models in Panel Data Framework

The literature is very rich with regard to theoretical and empirical use of data set in a panel (see Baltagi 1995). The first parametric approach to frontier models with panel data for estimating measurements of technical inefficiency is due to Pitt and Lee (1981), who combined the potential of analysing time series with the advantages of sectional analysis to estimate frontier models through maximum likelihood. This was followed immediately by innovative studies in this domain, responsible for the later development of this type of model.

The contributions of Schmidt and Sickles (1984), Cornwell et al. (1990), Battese and Coelli (1988, 1992, 1995) and Kumbhakar et al. (1991) stand out particularly. For a review of the literature on using panel data in the context of stochastic frontiers, see the studies by Kumbhakar (1990) and Kumbhakar and Lovell (2000). In the literature referring to frontier models estimated from panel data, we find different econometric specifications, resulting from the various hypotheses assumed for the term of technical inefficiency (Ahmad and Bravo-Ureta 1996). Regarding the hypotheses assumed for the term of technical inefficiency, these hypotheses can be summarized as follows:

(a) Absence of correlation between the term of efficiency and regressors
(b) Correlation between the term of efficiency and regressors
(c) Temporal invariance of the term of technical efficiency
(d) Temporal variance of the term of technical efficiency

The models are classified according to the hypotheses assumed for the term of technical inefficiency and according to the method of estimation adopted. The following categories may be therefore considered:

A – Models with time-invariant inefficiency term

- Fixed effect models
- Random effect models
- Maximum-likelihood models

B – Models with time-variant inefficiency term

- Models based on least-squares correction (fixed effect model and random effect model)
- Maximum-likelihood models
- Exogenous influence function models

2.4.1 Fixed Effect Model with Time-Invariant Inefficiency Term

In the non-frontier context of estimation with panel data, the fixed effect model was introduced by Mundlack (1961) and developed by Hock (1962) among others.

As the starting point, it is consider the general model (Cobb–Douglas), of just one product, for panel data:

$$\ln y_{it} = \beta_0 + \sum_{n=1}^{N} \beta_{nit} \ln x_{nit} + \varepsilon_{it} \tag{2.9}$$

with $i = 1, \ldots, N$ producers; $t = 1, \ldots, T$ periods and $n = 1, \ldots N'$, explanatory variables. The level of production for individual i in period t is represented by $\ln y_{it}$, the independent term is given by β_0 and the β_{nit} regression coefficients may or may not vary in i or in t. It is noted that in a traditional panel data model, the error term has an expected value of $E[\varepsilon_{it}] = 0$ and a constant variance $V[\varepsilon_{it}] = \sigma_\varepsilon^2$. Schmidt and Sickles (1984) proved that, in a context of using panel data to estimate efficiency measurements, it was possible not to specify a particular distribution for the effects of inefficiency, since the model's parameters could be estimated using traditional estimation methods with panel data, where the fixed effect method is included.

Schmidt and Sickles (1984) considered models where individual effects are constant parameters that can be correlated with the explanatory variables. Coefficients are estimated from the idea of variation in productive units, within-firm variation (Farsi et al. 2005, 2006), not being affected by the existence of correlation between regressors and individual effects. The specification underlying this type of model suggests that the differences found in terms of productive factors are simply explained by a set of individual factors, constant over time, which in frontier models translate technical inefficiency. Productive structure is identical for all firms. The estimation techniques adopted depend on the absence, or not, of correlation between regressors and the technical inefficiency term and on the imposition, or not, of a specific distribution for the technical inefficiency term.

The frontier model with time-invariant technical efficiency is therefore expressed as follows:

$$y_{it} = \beta_0 + \sum_n \beta_n \ln x_{nit} + v_{it} - u_i \quad \mu_i \geq 0 \tag{2.10}$$

where $i = 1, \ldots, N$, $t = 1, \ldots, T$, $n = 1, \ldots, N'$ are, respectively, the index distinguishing the different productive units, the time index and the index describing the N' regressors used by producer i. $v_{it} - u_i$ represents the compound term of model disturbance, where the first component of the error is an i.i.d random variable, $E[v_i] = 0$ and $V[v_i] = \sigma_v^2$, which varies over units and time and the second component (u_i) is the asymmetric inefficiency error term, assumed to vary only over units.

This last error component is treated as firm-specific constants or as fixed effects or individual effects. It is assumed that $u_i \geq 0$, that is, the non-negativity of this component for any i, and also that $E[u_i] = \mu$ and $V[u_i] = \sigma_u^2$. It is also considered that u_i is distributed independently of v_{it} and that v_{it} is not correlated with the regressors. The model presented in (2.10) may be adapted for OLS estimation proposes, by eliminating the intercept term and adding a dummy variable for each sample element:

$$y_{it} = \beta_{0i} + \sum_{n} \beta_n \ln x_{nit} + v_{it} \tag{2.11}$$

where $\beta_{0i} = (\beta_0 - u_i)$ represents the specific N intercepts associated with each producer. In this new model, named the fixed effect frontier model, no distribution is specified for the asymmetric error term (u_i) which can be correlated with the regressors or with v_i. As the component u_i is estimated together with the specific intercept of each producer, it is treated as a fixed effect (not random). OLS estimation procedure is applied, and individual estimates are obtained for $\hat{\beta}_{0i}$ from the mean of the within estimation residuals and by productive unit: $\hat{\beta}_{0i} = \overline{\hat{\varepsilon}_{\iota W}}$ where $\overline{\hat{\varepsilon}_{\iota W}} = \bar{y}_\iota - \beta' (x_{it} - \bar{x}_\iota)$ with $\bar{y}_\iota = \sum_{t=1}^{T} y_{it}/T$ and $\bar{x}_\iota = \sum_{t=1}^{T} x_{it}/T$ and $\bar{v}_\iota = \sum_{t=1}^{T} v_{it}/T$. The estimate for u_i is obtained from the following correction (Gabrielsen 1975 and Greene 1980a method): $\hat{u}_i = \hat{\beta}_0 - \hat{\beta}_{0i}$. This correction ensures the positivity of the individual effects and is done on the assumption that the most efficient firm is 100 % efficient, that is, it presents when $\hat{\beta}_0 = \max \hat{\beta}_{0i}$. The estimates are consistent when the respective variances tend towards 0 as the number of observations $(N \times T)$ tends towards infinity. Individualized estimates for the measurement of technical efficiency are given by $\mathrm{TE}_i = \exp\{-\hat{u}_i\}$ with $\hat{u}_i = \hat{\beta}_0 - \hat{\beta}_{0i}$, that is, it is given by the difference between the global estimate for the intercept and the estimates obtained for producers' specific intercepts.

The global estimate for the intercept is the result of $\hat{\beta}_0 = \max\left\{\hat{\beta}_{0i}\right\}$. The producer situated on the frontier is considered 100 % efficient presenting $\max\left\{\hat{\beta}_{0i}\right\}$, and the efficiency of the others is assessed in relation to this producer. Estimation with a fixed effect frontier model (within estimator) generates consistent estimates for the technological parameters $\hat{\beta}_n$ when T or N tends towards infinity, without the need to assume absence of correlation of the asymmetric error term with the regressors or normality of the symmetric error term distribution. As for the consistency property for the estimates of the specific intercepts for each producer, $\left\{\hat{\beta}_{0i}\right\}$, this is only found when $T \to \infty$. Estimates for the asymmetric error term are only consistent if N and $T \to \infty$. Another possible transformation to the previously specified model (2.10) consists in estimating a frontier model (2.12) by OLS after the within-groups transformation (or after all observations have been transformed in order to be expressed in terms of deviations from the individual means):

$$y_{it} - \overline{y_{it}} = \beta\prime (x_{it} - \overline{x_i}) + v_{it} - \overline{v_i} \tag{2.12}$$

The possibility of obtaining consistent estimators of individual technical efficiency, even faced with the hypothesis of correlation between regressors and individual effects, is the main attraction of estimation based on fixed effect models. Another relevant advantage lies in the fact of u_i is fixed, and so, the specification of the respective distribution is not necessary.

The main limitations arising from using these models can be expressed in the following points: (a) estimation of the model means the variables must present sufficient time variations, since the within technique assumes the respective parameters estimated from the deviations of variables from the respective means. If the variation is small, the accuracy of the estimates is in doubt; (b) the measurements of technical inefficiency reflect not only the inefficiency of the productive process but also the effect of other factors which are invariable over time and variable between productive units; (c) the productive unit with the smallest intercept is understood to be the efficient unit with which the other units are compared.

Among many other studies with fixed effect production frontier models, that of Ahmad and Bravo-Ureta (1996) stands out, comparing the effects on measuring the efficiency of fixed effect models and stochastic frontier models with different specifications for the asymmetric error component (semi-normal and truncated normal). It should be mentioned that the first fixed effect frontier models only considered balanced data panels. Some years later, these models were developed and adapted by Battese and Coelli (1988) to integrate also unbalanced panels.

2.4.2 Random Effect Model with Time-Invariant Inefficiency Term

The first developments in the sphere of random effect models were the work of Pitt and Lee (1981). The authors considered a model with distributional assumptions about the error term where $v_{it} \cap \text{i.d.}N\left(0, \sigma_v^2\right)$ represents noise and $u_{it} \cap \text{i.d.}N^+\left(0, \sigma_u^2\right)$ represents distribution of the non-negative component which translates the inefficiency of the model.

For the respective estimation, Pitt and Lee (1981) proposed the ML technique. Several years later, Battese and Coelli (1988) adopted this formulation, proposing truncated-normal distribution for modelling the component of technical inefficiency and using ML for estimation purposes. Schmidt and Sickles (1984) used another random effect model aiming to avoid the drawbacks arising from the Pitt and Lee specification which assumed a particular distribution for the inefficiency component and regressors variable over time. Assuming now independence of the inefficiency term and the regressors and that u_i are random than fixed results new modification of the initial model (2.10) or the random effect model given by the expression:

$$\ln Y_i = \beta_0^* + \sum_{n=1}^{N} \beta_n \ln X_{nit} + v_{it} - u_i^* \tag{2.13}$$

where $\beta_0^* = \beta_0 - E(u_i)$ and $u_i^* = u_i - E(u_i)$ and zero mean for u_i^* and v_i. With the introduction of this transformation, zero mean for the error term, GLS (generalized least squares) technique can be applied to estimate the model (2.13). The random effect model operates in exactly the same way as the error component (one-way) model described in the literature on panel data. To estimate this model, the GLS technique in two steps is used.

The method involves, at the first stage, OLS estimation of all the model's parameters. When the matrix of covariances of the error $v_{it} - u_i^*$ is known, that is, σ_v^2 and σ_u^2 are known, the GLS estimator for β_0^* and for β_n is BLUE (best linear unbiased estimator), and consistency is ensured either when $N \to \infty$ or when $T \to \infty$. However, usually σ_v^2 and σ_u^2 are not known. In this situation, it is appropriate to use the FGLS (feasible generalized least squares) method to estimate the variance of the compound error term $\hat{V}[\varepsilon] = \hat{V}[u_i] + \hat{V}[v_i] = \hat{\sigma}_u^2 + \hat{\sigma}_v^2$. The estimate for the variance of the symmetric error term is given by the variance of the residuals of the fixed effect model (within[2] residuals), $\hat{\sigma}_v^2 = \hat{\varepsilon}'\hat{\varepsilon}/[N(T-1)-K]$ and the estimate for the variance of the asymmetric error term is given by the combination of the residuals of the between[3] estimation with the residuals of the within estimation $\hat{\sigma}_v^2 = \{\hat{\varepsilon}'\hat{\varepsilon}/[N-K] - \hat{\sigma}_v^2\}/T$. At a second stage and after estimation of β_0 and β_n (with GLS or FGLS), the measurements of technical efficiency are given by $\mathrm{TE}_i = \exp\{-\hat{u}_i\}$ with $\hat{u}_i = \max\{\hat{u}_i^*\} - \hat{u}_i^*$ and u_i^* resulting from the mean residuals of FGLS estimation:

$$\hat{u}_i^* = \frac{1}{T}\sum_t \left(\ln y_{it} - \hat{\beta}_i^* \sum_n \hat{\beta}_n \ln x_{nit}\right)^4 \tag{2.14}$$

In these conditions, the estimates obtained for individual inefficiency translate, just as in the case of the fixed effect model, the distances between the intercept of each productive unit and the greatest intercept relating to the productive unit considered efficient. The frontier is then moved to the greatest intercept estimated in the sample. The BLUP (best linear unbiased predictor) by Lee and Griffiths (1979) is an alternative estimator to u_i^* and is given by

$$\tilde{u}_i^* = \frac{-\hat{\sigma}_u^2}{T\hat{\sigma}_u^2 + \hat{\sigma}_u^2} \sum_{t=1}^{T} \left(\ln y_{it} - \hat{\beta}_{0\,\mathrm{FGLS}}^* - \hat{\beta}_{n\,\mathrm{FGLS}} \ln x_{it}\right) \tag{2.15}$$

[2] The residuals of the within estimation are given by $\hat{\varepsilon}'\hat{\varepsilon} = \sum_{i=1}^{N}\sum_{t=1}^{T}\left[y_{it} - \overline{y_i} - \hat{\beta}'_{\mathrm{Within}}(x_{it} - \overline{x_i})\right]^2$.

[3] $\varepsilon^{*\prime}\varepsilon^* = \sum_{i=1}^{N}\left(\overline{y_i} - \hat{\beta}^*_{\mathrm{Between}} - \hat{\beta}'_{\mathrm{Between}}\overline{x_i}\right)^2$. The latter residuals are the result of applying the OLS technique to the model: $\overline{y}_{it} = \beta^* + \beta'\overline{x_i} + \overline{v_i} - \overline{u_i^*}$.

[4] These estimates are consistent as long as N and $T \to \infty$.

The GLS[5] estimator for β_0^* and for β_n^* is consistent when simultaneously N and $T \rightarrow \infty$, and the variances of the two components of the error term are known. When these are unknown, it is necessary that $T \rightarrow \infty$ for the variance of u to be estimated consistently and that N or $T \rightarrow \infty$ for the FGLS estimator of the variance of v to be consistent. The estimators of $\tilde{u}_i$ and $\hat{u}_i$ are consistent when N and $T \rightarrow \infty$. The FGLS estimator is suitable when N is large and when the hypothesis of the existence of correlation of u with the regressors is rejected. Opting for the FGLS estimator or the within estimator depends on the hypothesis of absence of correlation between technical inefficiency and the regressors being confirmed or not.

Hausman and Taylor (1981) developed an alternative estimator (HT) which shares characteristics with within and FGLS estimators, being indicated for testing this hypothesis of absence of correlation between technical inefficiency and the regressors. Equally, adoption of a random effect model also involves limitations. Firstly, these models do not allow us to distinguish the inefficiency of non-observable heterogeneity (only one parameter is specified to capture that heterogeneity).

As in the fixed effect model context, to check consistency of the asymmetric error term, it is necessary that $T_i \rightarrow \infty \forall_i$. Secondly, the frontier associated with a random effect model is constructed from moving the frontier to the intercept relating to the most efficient unit in the sample. However, if the sample is small, it may not include any efficient unit or one close to optimal production.

2.4.3 Temporal Variation of Efficiency Term

The fixed effect, random effect and maximum-likelihood models share the assumption of temporal invariance in the component of technical inefficiency. However, we find that, when analysing the efficiency of a productive process, it is often more appropriate to consider the time effect on this component of the error, principally when there are sufficient data on the same productive unit observed in various periods. In these circumstances, it is improbable that productive units continue to present a constant measurement of inefficiency in all the periods of observing their production.

Indeed, knowledge of the level of technical inefficiency over time necessarily causes interventions in the production process that affect results in the following periods, invalidating the initial hypothesis of individual effects being constant in t. Cornwell et al. (1990), Lee and Schmidt (1993), Heshmati and Kumbhakar (1994),

[5]When the variances of the components of the error term are unknown, the FGLS estimator of σ_v^2 is consistent with N or $T \rightarrow \infty$, whereas for σ_u^2 consistency is ensured only with $T \rightarrow \infty$.

Kumbhakar and Heshmati (1995) and Battese and Coelli (1992, 1995) were pioneers in studying frontier models with time variation of technical efficiency, and Cuesta (2000) enhanced the literature on this subject with new developments.

The different models, which in the respective specifications integrate the time variation of technical efficiency, can be classed in three categories: (1) fixed effect models and random effect models (based on correction of least squares), (2) models estimated using ML and (3) exogenous influence function models. The main difference between the categories of models with time-variable efficiency lies in how this is modelled. In fixed and random effect models, efficiency is modelled from the independent term, whereas in maximum-likelihood models and those incorporating exogenous influences, this is modelled from the error term, therefore meaning imposition of joint probability distribution.

For each of the first two categories mentioned, they are also differentiated by the specifications where the standard of time-variable efficiency is common to all producers from those specifications where the efficiency standard varies between producers. Prominent among the first type of model are those by Cornwell et al. (1990), with a distinct standard of time variation for each productive unit,[6] and by Lee and Schmidt (1993), with a standard of time variation common to all productive units, both estimated using COLS techniques. In these models, it is not possible to separate the effect of technological progress from the effect of technical inefficiency in the independent term.

In the context of the second group of models, assuming simultaneously the conditions of independence of u regarding the regressors and the distributional suppositions mentioned concerning the two components of the respective errors, this includes those satisfying the conditions required for maximum-likelihood estimation. Of special note are the models suggested and developed by Kumbhakar (1990), Battese and Coelli (1992, 1995) and Cuesta (2000). While the models by Kumbhakar (1990) and Battese and Coelli (1992) have a standard of time-variable efficiency common to all firms, the models proposed by Battese and Coelli (1995) and Cuesta (2000) present a rate of time-variable inefficiency which varies between firms. In this second group of models, it is possible to separate the effect of technological progress from the effect of inefficiency on the productive process.

The third category of models concerns a type of frontier model where the term of inefficiency is a vector of observable factors. Kumbhakar et al. (1991) developed a model in a sectional context, and Battese and Coelli (1995) generalized it for panel data context.

[6]The effects where the inefficiency is contained are given by the product of time effects (common to all firms) and individual effects: $\beta_{it} = \theta_t \delta_i$.

2.4.3.1 Fixed and Random Effect Models

According to the literature on the subject, in this type of model, time-variable efficiency is incorporated from the independent term also named individual effects. In these models, the independent term incorporates two components: one reflecting the effect of technological progress and the other the effect of technical inefficiency on the productive process. Concerning the estimation method, as in the case of time-invariant inefficiency, this is based on corrections carried out on the results of estimation obtained with the least-squares method.

Two subgroups of models are distinguished, however: those for which a standard of efficiency variation is assumed for each productive unit and models where this standard is common to all productive units. The Cornwell et al. (1990) model is included in the first type of models, and the model proposed by Lee and Schmidt (1993) falls into the second type.

The Cornwell et al. (1990) Model

Cornwell et al. (1990) developed and estimated a frontier model in whose specification they introduced the component of technical inefficiency varying over time. The functional form is based on a Cobb–Douglas technology of production where the independent term and some regression coefficients vary with the individuals and with time.

This model generalizes the one by Schmidt and Sickles (1984), considering in the production function, a function of variable coefficients which is quadratic in t and contemplates the individual and time variation of technical efficiency which is identical for all productive units. The model can therefore be presented with the following formulation:

$$\begin{aligned} y_{it} &= \beta_{0t} + \sum_k \beta_n \ln x_{nit} + v_{it} - u_{it} \\ &= \beta_{it} + \sum_k \beta_n \ln x_{nit} + v_{it} \end{aligned} \tag{2.16}$$

with $\beta_{it} = \beta_{0t} - u_{it}$ expressing the intercept of unit i in period t and with β_{0t} translating the common frontier intercept in period t. The authors assume a quadratic function to explain, or $\mu_{it} = \Phi_{1i} + \Phi_{1i}t + \Phi_{3i}t^2$, where Φ's are specific producer parameters. With this new specification, production levels vary between firms and over time, as well as technical efficiency. For estimation proposes, authors applied GLS random effect estimator, as they assumed time-invariant regressors, time-varying technical efficiency in the specified model and independence between the asymmetric error term and regressors.

Lee and Schmidt (1993) Model

The main difference in relation to the last model lies in the fact that these authors have imposed a standard time variation for u_i, which is identical for all the sample's productive units. They propose an alternative specification for the asymmetric error component $\mu_{it} = \delta_t \mu_i$ or the product between the time effects (dummies) and individual producers' inefficiencies.

2.4.3.2 Maximum-Likelihood Models

For a second group of models, there is simultaneous assumption of the conditions of independence of u_i with regard to the regressors and distributional suppositions about the two components of the respective errors. These models can then be estimated by ML, highlighting the models suggested and developed by Kumbhakar (1990) and Battese and Coelli (1992), with a standard of time variation common to all producers, and by Battese and Coelli (1995) and Cuesta (2000), where the standard time variation is specific for each producer.

Kumbhakar (1990) Model

Kumbhakar (1990) was the first to suggest and use a stochastic frontier model with a time variation standard of levels of technical inefficiency common to all productive units with estimation by maximum likelihood. The author suggests that the time variation of u assumption may be defined through an exponential function of time given by $u_{it} = f(t) \cdot u_i$ or the product of a function in t or

$$f(t) = \left[1 + \exp\left(\alpha t + \beta t^2\right)\right]^{-1} \tag{2.17}$$

by u_i which is modelled as a truncated-normal distribution $u_i \cap$ i.i.d.$N^+(0, \sigma_u^2)$, independent of regressors. The estimation of the time-varying efficiency effect model is realized in a random effect framework and using ML method.

Battese and Coelli (1992) Model

Battese and Coelli (1992) generalized Kumbhakar's idea to unbalanced (or incomplete panel data) models, proposing an alternative frontier model to that of Kumbhakar, also assuming time varying for efficiency and restricted to be common to all individuals (productive units).

The author suggests that the time variation of u should be defined from a function given by $u_{it} = f(t) \cdot u_i$, where

$$f(t) = \exp\left[\eta\left(t - T\right)\right], \quad f(t) \geq 0; \quad f(T) = 1 \tag{2.18}$$

involving only one unknown parameter and so, less flexible (Coelli et al. 1998, pp. 278). Since the rate of time variation is identical for all units, the value estimated for the additional parameter allows analysis of the tendency of efficiency over time: efficiency increases (for all units) if the value estimated for this parameter is positive, diminishing if it is constant over time if the parameter assume a zero value.

With this new specification only in the last year of analysis, productive units present a specific standard of efficiency. In the other years, the standard is also common to all productive units. Battese and Coelli (1992) used the method of maximum likelihood together with a truncated-normal distribution for the asymmetric error term modelling.

2.4.3.3 Models with Exogenous Influences

The third category of models is used as the result of research for an answer to the existence of inefficiency, and it is associated with a type of frontier where the inefficiency term is a vector of observable factors. Kumbhakar et al. (1991) developed this methodology in the context of sectional data, and Battese and Coelli (1995) generalized it to panel data.

For Kumbhakar et al. (1991), technical inefficiency would be composed by a deterministic part with exogenous variables and by a stochastic component $u_i = \gamma' m_i + \theta_i$ with m_i being a vector of observable qualitative factors, γ a vector of parameters and non-observable random component (inefficiency model error term).

Additionally, Kumbhakar et al. (1991) assuming (1) $v_i \cap$ i.i.d. $N(0, \sigma_v^2)$, (2) $u_i \cap N^+\left(\gamma' m_i, \sigma_u^2\right)$ and (3) v and u independently distributed, suggested ML estimation procedure. According to this specification, technical inefficiency only varies between productive units and depends on specific exogenous variables.

This type of model has been criticized due to the fact that it tries to explain differences in inefficiency through variables that already appeared in the models, as productive factors.

Battese and Coelli (1995) Model

Battese and Coelli proposed a model where the inefficiency term varies over time, follows truncated-normal distribution and is a function of certain explanatory variables, such as $u_{it} = z_{it}\delta + W_{it}$. δ is a unknown vector of coefficients, z represents a vector of observable explanatory variables and W the error term of the inefficiency model.

These explanatory variables (also called exogenous variables or inefficiency effects) of the inefficiency effects model include determinant factors for understanding the magnitude of the distance of the observed production in relation to the corresponding production situated on the respective stochastic frontier. The term

(ε) or error term of the Battese and Coelli (1995) model is given by two random variables, $\varepsilon_{it} = v_{it} - u_{it}$, which translate two types of effects or disturbances affecting the productive process.

The first component is the error term that captures effects caused by errors of measurement and by all factors outside the productive unit's control and is modelled through a Gaussian (0, 1). The second component is a non-observable measurement of technical inefficiency which may be time varying and controllable by the unit. This component measures the magnitude of the effort made to reduce the distance from the technological frontier.

These variables are therefore non-negative variables representing the technical inefficiency of production. Values for the inefficiency measurements are derived from generalization of the Jondrow et al. (1982) methodology, that is, deduced by means of the expected conditional value of the estimated error value, and the other parameters of the model are estimated in just one step using maximum likelihood.

The size of the panel has quality estimation implications whatever the method used. The most favourable situation, or when N and $T \rightarrow \infty$, allows consistent estimators. When T is high, but N is low, the within estimator subject to hypotheses of correlation between regressors and technical inefficiency and absence of distribution for u_i is the most appropriate. When N is high, but T is low, consistency is not guaranteed whatever the method used.

2.5 New Developments

When firm characteristics are not taken in account and they are erroneously estimated as being inefficiency, it may cause serious biases in efficiency results. Modelling heterogeneity among organizations represents an important field of research. Indeed, in the conventional panel data context, firm-specific heterogeneity was incorrectly considered as inefficiency.

In the fixed effects model of Schmidt and Sickles (1984), time-invariant unobserved heterogeneity was captured by the inefficiency component until Greene's (2005) suggestion of the true fixed effects model which restricts fixed effects to only represent the unobserved firm heterogeneity and not inefficiency. Research in this field has been conducted by Farsi et al. (2005, 2006) and Greene (2005). Modelling heterogeneity has been extended to the Bayesian context by Caudill et al. (1995), Tsionas (2001, 2002) and Huang (2004).

Nowadays, stochastic frontier approach to organizations efficiency measurement may play an important role in the field of economical sustainable development of societies, as it allows managements to reach high levels of performance restricting resources, minimizing undesired outputs or optimizing desired outputs. More research is needed in order to enlarge the knowledge in this issue.

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