Developing and Enhancing Elementary School Students' Higher Order Mathematical Thinking with SimCalc

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1 Introduction

Nowadays, there is a greater demand for our schools to produce graduates who are highly creative and critical, and able to demonstrate more complex thinking processes (European Commission, 2011). The mathematics curricula of a number of countries stress that students must develop the mathematical knowledge and skills necessary to further their education, careers and everyday lives, as productive and independent members of society (Ministry of Education and Culture in Cyprus, Cyprus Pedagogical Institute, and Development Programs Service, 2010; National Council of Teachers of Mathematics [NCTM], 2000). Despite this realization, the results of several studies show that students have limited abilities in problem solving, lack conceptual understanding, and have inadequate critical thinking skills (Henningsen and Stein, 1997; Hiebert and Carpenter, 1992). It can be argued that we have not yet sufficiently developed the kinds of environments that facilitate students' content knowledge, critical, creative and complex thinking.

According to a number of researchers, such environments can be developed through the use of technology (Clements et al., 2008; Jonassen, 2000; Jonassen et al., 2008; Slangen et al., 2008). Clements et al. (2008) suggest that the various environments that support learning with the use of technology can enhance students' learning in new dynamic ways. Technology, as a mindtool, might offer students the opportunity to develop higher order thinking (Heid and Blume, 2008; Pea, 1987). However, it is not yet clear which pedagogical principles are necessary to maximize

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the potential that technology has to offer to mathematical thinking and learning (Lagrange et al., 2003).

In this chapter, we describe the design of six lessons, using SimCalc MathWorlds®(hereon called SimCalc), which were aimed to enhance elementary school students' higher order mathematical thinking. We illustrate the way in which this software and the organization of the lessons supported the development of elementary school students' higher order thinking in mathematics. To this end, Sect. 2 offers an overview of the advances that the use of technology might offer, and in particular the potential SimCalc creates for higher order mathematical thinking. Then, we concentrate on the pedagogical role of technology in the teaching of mathematics. Section 3 considers the purpose of this chapter and related research questions, while Sect. 4 provides information about the design of the lessons. The outcomes of the implementation of the proposed lessons are presented in Sect. 5, and in Sect. 6 we draw some conclusions and consider implications for teaching.

2 Theoretical Background

2.1 Technology as a MindTool: Changes in Students' Thinking and Learning in Mathematics

According to Jonassen (2000), the use of new technologies supports students' learning in a way that is meaningful to them. However, technology by itself cannot bring change or positive results in the learning and understanding of mathematics (Heid and Blume, 2008). It appears that it may be beneficial for students to be engaged in active, constructive, authentic activities and cooperative learning, and to be offered opportunities to explore situations and interpret the results of their interventions. This can be promoted by the use of technology as mindtools. Mindtools are technological tools and learning environments which have been developed or adopted so as to be used as intellectual partners, requiring students to think critically or use higher order thinking (Jonassen, 2000). Clements et al. (2008) and Tall et al. (2008) suggested that the use of appropriate technological tools gives students the opportunity for reflection, and allows them to understand in greater depth mathematical ideas and procedures. It is therefore important to investigate the types of thinking that it is possible to develop through the use of cognitive technology in mathematics.

Using the Integrated Thinking Model by Iowa Department of Education (1989), Jonassen (2000) suggested that cognitive technology can promote higher order thinking. More specifically, he suggested that students might develop more complex processes of thinking in a technological environment, which includes:

> The goal-directed, multi-step, strategic processes, such as designing, decision making and problem solving. This is the essential core of higher order thinking, the point at which thinking intersects with or impinges on action (Iowa Department of Education, 1989, p. 7).
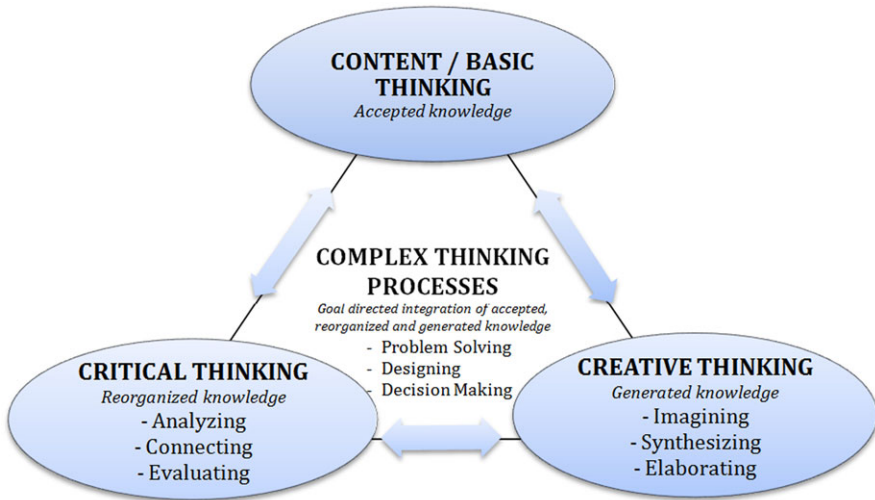
Fig. 1 Integrated Thinking Model (Iowa Department of Education, 1989)

For someone to reach higher order thinking, a combination of content/basic knowledge, critical thinking and creative thinking is necessary (see Fig. 1). These three components should be interrelated and dependent on each other. According to Jonassen (2000) and NAACE and BECTA (2001), these three components can be developed and supported by cognitive technology.

Students' content/basic knowledge is the knowledge that they can retrieve directly from what they have learned (Jonassen, 2000). This knowledge refers to both procedural and conceptual knowledge, i.e., what and why an operation is executed. Critical thinking is the ability to reorganize the knowledge using the processes of analyzing, connecting, and evaluating in accepted knowledge (Iowa Department of Education, 1989). The process of analyzing involves breaking a whole into meaningful parts, recognizing patterns and understanding interrelationships. Connecting refers to the ability to find similarities in and differences between things, and construct relationships within and between systems. The evaluation process involves making judgments based on criteria and information, and investigating the implications or results of a hypothesis in order to confirm or reject it (Iowa Department of Education, 1989). Creative thinking involves "using and going beyond the accepted and reorganized knowledge to generate new knowledge" (Iowa Department of Education, 1989, p. 7). More specifically, creative knowledge is the new knowledge brought about by imagining, synthesizing and elaborating processes (Iowa Department of Education, 1989). Creative thinking involves imagining processes, which require original ideas through intuition, visualization, prediction, and fluency. It also involves synthesizing skills, which depend on the ability to combine parts to form a new whole using analogies, to summarize key ideas succinctly, to hypothesize and to plan a process. Elaborating refers to the ability to develop an idea fully by expansion, extension and modification (Iowa Department of Education, 1989). Finally,

complex thinking, as mentioned above, combines the skills and knowledge types of the other three kinds of thinking, to produce an integration of accepted, reorganized and generated knowledge. In other words, complex thinking is not a separate kind of thinking, but incorporates critical and creative thinking skills in various ways. This kind of thinking includes processes such as problem solving, designing and decision making. Problem solving is the process of using systematic methods to reach a goal. Designing refers to the invention of any type of creation to fit some goal or purpose, while decision making is defined as the ability to choose from alternatives systematically (Iowa Department of Education, 1989).

This Integrated Thinking Model has been used in different ways in the research field of educational technology. For example, Jonassen (2000) used this model to evaluate different technological tools as mindtools. Michko et al. (2003) used the dimensions of critical thinking as criteria to analyze the potential of specific mathematical software. Slangen et al. (2008) used this model to develop a checklist in order to investigate the type of thinking skills that eight grade students applied when using the microworld Techno-Logica. They found that students tended to apply the evaluating skills rather than the connecting, synthesizing and analyzing skills. However, they suggested that further investigation is needed to examine the type of thinking skills stimulated by various microworlds.

Jonassen (2000) suggests that SimCalc, as well as a number of other microworlds, allow students to develop their critical, creative, and complex thinking processes. He also suggests that compared with other technological tools, microworlds engage students in more critical and creative procedures, such as the recognition of patterns, inductive reasoning, hypothetical reasoning, and flexible manipulation of situations. In particular, it is suggested that SimCalc can develop students' critical thinking by offering them the possibility to assess available information, decide upon the criteria for selecting information, recognize patterns, identify causal relationships and use logical reasoning to support their answers inductively or deductively (Dalton and Hegedus, this volume; Jonassen, 2000; Jonassen and Carr, 2000). For example, students who work in SimCalc can assess and connect information presented to them in different representations of the microworld: the world, the table and the distance-time graph or velocity-graph (Dalton and Hegedus, this volume). Furthermore, SimCalc can develop students' creative thinking by offering an environment where students can expand their thinking to something new: it has the fluency and flexibility to provide multiple solutions to a problem. For example, students can expand the given graphs in SimCalc to fit specific instructions. Such activities were used by Dalton and Hegedus (this volume) in their implementation in high school Algebra 2 classrooms. In particular, students had to edit the velocity function symbolically and graphically to control the motion of a rocket to fit given instructions. Finally, SimCalc supports the development of complex thinking by engaging students in situations which require competence in problem solving, designing solutions, and decision making (Jonassen, 2000; Mousoulides, this volume). For example, students can design a graph in SimCalc to fit specific instructions or to find the solution to a problem. In a similar way, complex modeling activities were used by Mousoulides (this volume). In particular, he invited elementary school students to construct models in the SimCalc environment to fit specific instructions. In

addition to this, Bishop (this volume) and Orrill (this volume) underlined that despite the opportunities offered by SimCalc environment to solve complex problem solving activities in a meaningful way, SimCalc offers students the opportunity to be engaged in a more productive discourse in the classroom.

2.2 Pedagogical Role of Technology in Mathematics Teaching and Learning

A technological tool is not sufficient in itself to become a mindtool. Heid and Blume (2008) argue that the effects of technology on teaching and learning are the result of a range of configurations of technologies, teachers' and students' actions, as well as the nature and organization of the curriculum and mathematical content. Based on this idea, Pierce and Stacey (2010) described the ways in which technology may be used to bring pedagogical advantages in terms of three levels—subject, classroom organization, and tasks.

Subject level refers to the opportunities that technology offers to "provoke or support new or changed goals or teaching methods for a mathematics course as a whole and to provide its users with new insights into the subject matter that they are teaching" (Pierce and Stacey, 2010, p. 10). Technology might be used to alter the balance in teaching skills, concepts and applications, and to build metacognition abilities and higher order thinking skills.

The second level of Pierce and Stacey's (2010) pedagogical map, the classroom level, focuses on the changes that occur in the interpersonal dimension of the classroom when compared with the traditional classroom. Pierce and Stacey (2010) argue, for example, that the use of technology changes classroom social dynamics, with teachers facilitating rather than dictating, and it also encourages group work, with students working collaboratively and engaging in mathematical discussion. In addition to this, technology engages students in expressive and exploratory activities (Doerr and Pratt, 2008), where students with the same technological tool can create their own construction or investigate using a prepared environment.

At the task level of their pedagogical map, Pierce and Stacey (2010) present different uses of technology, which may enhance mathematical activities in the classroom. It appears that the functionalities of some software provide an opportunity to use real world data or to explore regularity and variation. Moreover, some software offers the opportunity to elaborate simulations and/or link different representations, which promotes students' understanding.

3 Purpose and Research Questions

Taking into consideration both the possibilities that SimCalc may offer as a mindtool, as well as Pierce and Stacey' s (2010) pedagogical map, the aim of this study

was to examine whether SimCalc could be used as a mindtool to enhance students' higher order thinking in mathematics. Our aim was to design a learning environment and analyze the ways in which SimCalc enhanced students' content knowledge and critical, creative, and complex thinking in mathematics. In particular, we investigated the following research questions:

- Is SimCalc MathWorlds® software a useful mindtool to facilitate elementary school students' high order thinking in mathematics?
- Can we observe active high order mathematical thinking in the SimCalc environment?

4 Designing a Learning Environment with SimCalc

Many researchers stress that it is high time to search for the ways in which computers bring added value to and can transform education, supporting the development of different types of thinking and abilities (Heid and Blume, 2008; Papert, 2006). In the following section, we analyze the learning environment that we designed with SimCalc (Kaput and Roschelle, 1998). We will present the aims of the lessons, their structure and examples of activities employed.

4.1 Aims of the Lessons with SimCalc

The aims of these lessons addressed the four components: content knowledge, critical thinking, creative thinking, and complex thinking. The underlying assumption is that content knowledge is achieved through students' involvement with the interpretation and invention of linear graphs. Critical thinking is built with the requirements for students to find similarities and differences between different graphs and link the various representations in the SimCalc environment. Creative thinking is targeted through multiple solution tasks, and finally complex thinking is required in designing solutions to fit a given purpose. In particular, the aims of the lessons were to help students to: (a) interpret distance-time graphs and velocity-time graphs and identify similarities and differences between them; (b) link verbal descriptions with distance-time and velocity-time graphs; (c) provide multiple solutions to mathematical problems in the SimCalc environment; and (d) design graphs using SimCalc to fit given instructions.

4.2 Description of the Learning Environment with SimCalc

To design the learning environment using SimCalc, we adopted Pierce and Stacey's (2010) pedagogical map and concentrated on the subject, classroom organization,
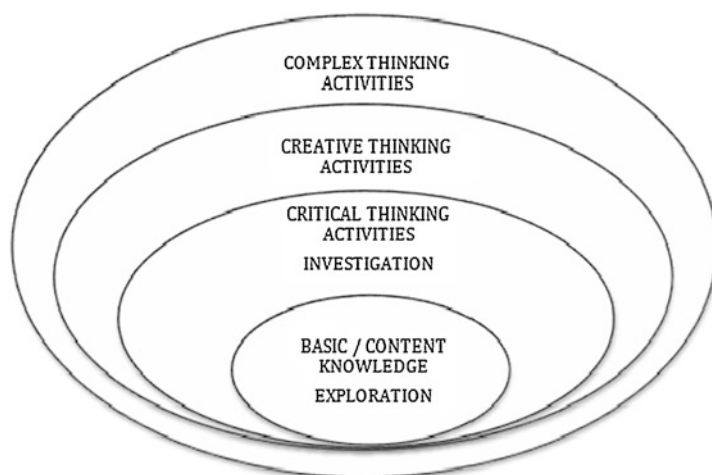
**Fig. 2** Structure of the learning environment with SimCalc

and tasks levels. By subject level, we mean the mathematics that students should learn, including mathematical thinking skills. We define classroom level as the way in which we organized the classroom setting, i.e., the role of students, teachers, and technology in the classroom. Finally, by task level, we refer to the advantages of SimCalc and the way in which these are incorporated in the activities of the learning environment.

4.2.1 Subject Level

Figure 2, offers a description of the way in which we designed our lessons in an attempt to address the subject. Our aim was that by the end of the six lessons, students would have developed their content knowledge of distance-time and velocity-time graphs, and also critical, creative, and complex thinking about these mathematical concepts. Thus, at the beginning of the lesson, students were asked to explore a real life situation. They were shown a video of a race from the 2008 Olympic Games without any commentary. Students were asked to find a way to present this race in a diagram. In the next stage, students were asked to think critically—analyzing the meaning of different representations presented in SimCalc—and investigate the similarities and differences between them (see, for example, "Distance-Time Graph Versus Velocity-Time Graph," Fig. 3). Once these activities were completed, students were asked to work with tasks that required creative thinking. In particular, students were asked to elaborate given distance-time graphs, to visualize the verbal instructions they were given, and to produce as many graphical solutions as possible (see, for example, "Time for Creation!!!," Fig. 3). Lastly, students were engaged in complex thinking activities. They were asked to sketch a graph with SimCalc in order to fit specific instructions (see, for example, "Article in the Sport Section of

DISTANCE-TIME GRAPH VERSUS VELOCITY-TIME GRAPH

Open the file «Runners4». This file contains two graphs. Describe what you see on the screen.
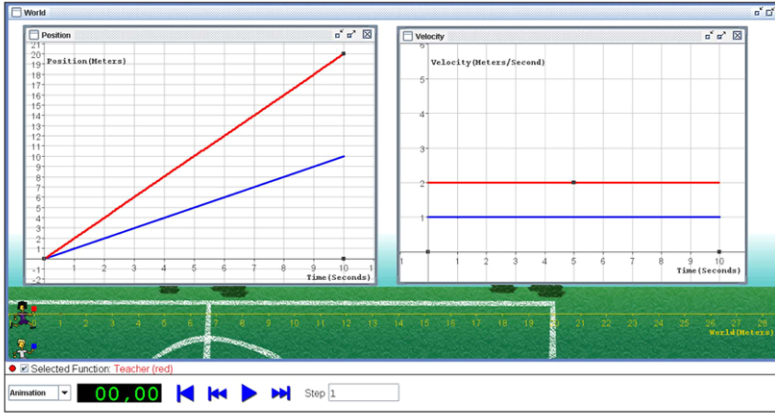
(a) What are the similarities and differences between these two graphs?
(b) Drag the two graphs and write your observations. For instance, what happens on the distance-time graph, when the velocity of a runner increases?

TIME FOR CREATION!!!

What changes can you make in the SimCalc environment so that the blue runner finishes first in the 20 m race?
Describe the way you worked.
Draw on the grid the graph that you constructed in the SimCalc environment.
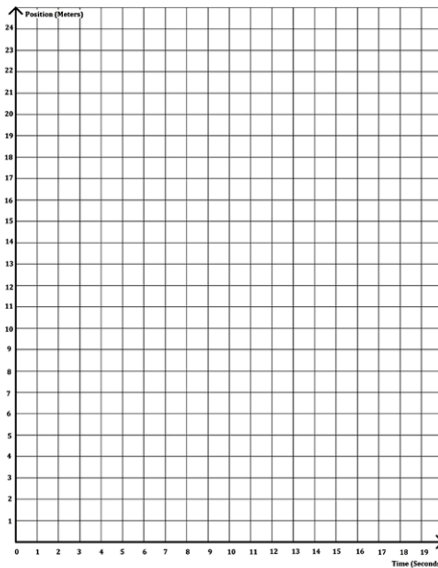
Draw graphs in which the blue runner finishes the 20 m race first. Draw as many solutions as possible.

Fig. 3 Critical thinking, creative thinking and complex thinking activity in SimCalc

ARTICLE IN THE SPORT SECTION OF A NEWSPAPER

Help me understand what the following sections of the newspaper describe. Use SimCalc to show me.

Three cars participated in a 20 km car race.

The first car finished the race in 8 minutes.

Although the second car led the race for the first 4 minutes, it eventually finished second in 10 minutes.

The third car started after 2 km, and stopped for 2 minutes due to a technical problem. It finished the race in 15 minutes.

OUR RACE

Describe your race and draw it in the SimCalc environment. Give your description to one of your fellow students to check.

Fig. 3 (Continued)

a Newspaper," Fig. 3) and to design their own race using SimCalc, and ask their partner to describe it (see, for example, "Our Race," Fig. 3).

4.2.2 Classroom Organization Level

With regard to the classroom organization level, students were asked to work in pairs. For the majority of the lesson, students were working without any guidance from their teacher. They were reading, exploring and completing the activities that they had been given in collaboration with their partner. The teacher only interfered if students needed clarification about a task. The teacher also addressed the whole class for a few minutes at the beginning and at the end of the lesson. This was done in order to give students the opportunity to present their results, make connections and comparisons between the various solutions, and exchange ideas and experiences gained while they were working with SimCalc.

4.2.3 Task Level

Finally, at the task level (Pierce and Stacey, 2010), students were given the opportunity to use real data about races—velocity, distance, and time. They had the opportunity to simulate these situations and explore the similarities and differences between simulations, graphs and tables. Students could link and make comparisons between the various representations and learn from the feedback provided by the computer. In addition, students were also asked to "teach" the computer to simulate a narrative story in a visual form.

5 Implementing the Designed Learning Environment with SimCalc in an Elementary School Classroom

5.1 Participants and Setting

Fifteen 5th and 6th grade students (10 and 11-year-olds, 6 boys and 9 girls) of a rural elementary school in Cyprus participated in the study. The students participated in six 40-minute lessons taught by one of the authors. These lessons were conducted during a 3-week period. The participants had never used SimCalc or any similar computer program beforehand.

To answer the research questions, we collected students' written responses and the electronic SimCalc files which they produced for each activity. For the purposes of the current chapter, all responses and collected material were translated into English. In addition to this, detailed observation notes were taken of students' actions and discussions, both with their peers and their teacher. Particularly, an observation protocol was used which was based on the Integrated Thinking Model (Iowa Department of Education, 1989). We recorded students' statements that showed: (a) critical thinking processes in terms of analysis, connection and evaluation (e.g., students' statements which suggested construction of hypothesis, prediction of outcomes, comparison of similarities and differences, analysis of situations, generalization, connection between concepts and procedures, logical thinking, verification of arguments etc.); (b) creative thinking processes in terms of imagining, synthesizing and elaborating (e.g., statements that showed students synthesis of data in order to provide answers and original ideas etc.); and (c) complex thinking processes (e.g., statements that showed the way that students solved complex thinking tasks). For each of the above processes, researcher indicated the task which students were engaged with and the time that these processes occurred.

In the following section, we will present the results of the implementation of the lessons with SimCalc on students' higher order thinking in mathematics. For the data analysis, the constant comparative method was used (Maykut and Morehouse, 1994). This method combines "inductive category coding with a simultaneous comparison of all units of meaning obtained" (Glaser and Strauss, 1967, as cited as Maykut and Morehouse, 1994, p. 134). In particular, we were informed by the literature about the components of conceptual understanding of linear graphs and the various types of processes involved in critical, creative and complex thinking. Based on these, we created categories, organized the data collected from multiple sources, and presented the results of our analysis.

5.2 Students' Performance

With regard to students' performance, at the end of the six lessons, we compared students' initial inventions of a distance-time graph and their final inventions, as
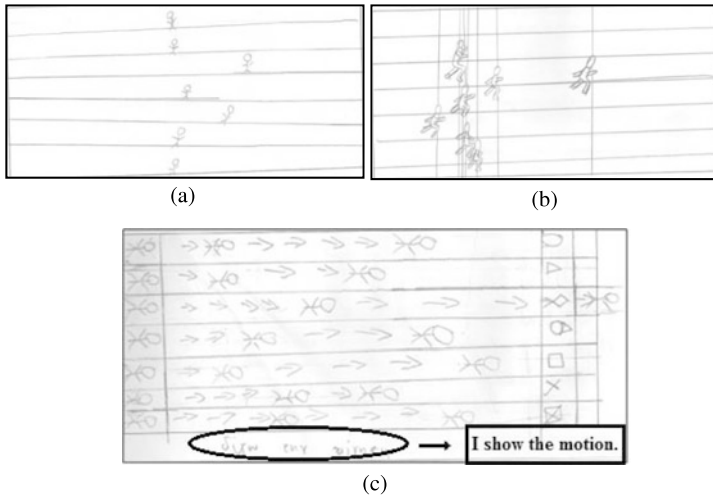
(a)   (b)

I show the motion.

(c)

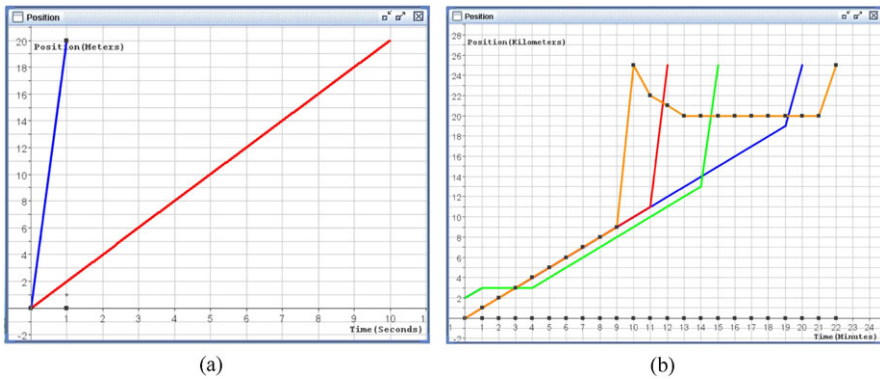**Fig. 4** Students' diagrams of Olympic Games race prior to the use of SimCalc
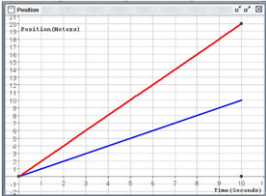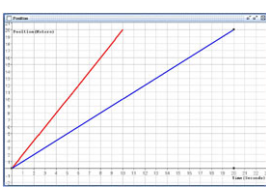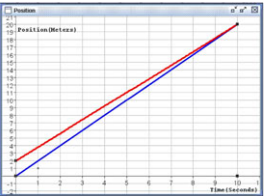

(a)   (b)

Fig. 5 One student's invention of distance-time graphs

well as students' initial interpretations of the distance-time graphs with their final interpretations. Students' responses and actions were grouped into the above categories using the constant comparative method analysis.

At the beginning of the first lesson, students were asked to represent the Olympic Games race, which they had watched on video, in a diagram. Six of the students presented the race in a static form (see Fig. 4a), five tried to show the motion of the athletes, drawing bent arms and legs (see Fig. 4b), and four tried to show the motion by presenting multiple figures for every runner (see Fig. 4c).

After students' engagement with SimCalc, all of them were able to invent distance-time graphs to represent a race of two or more athletes, both on paper and in the SimCalc environment. One student's drawings of distance-time graphs are displayed in Fig. 5. Figure 5a shows, as the student said, "the blue runner finishing

Table 1 Students' initial interpretations of distance-time graphs

(a)	(b)	(c)

Students' initial interpretation of distance-time graph (a)	Number of students	Students' initial interpretation of distance-time graph (b)	Number of students	Students' initial interpretation of distance-time graph (c)	Number of students
"I think that the graph shows the two runners' times and the number of meters they ran."	8	"I think that the graph shows that the blue runner is much further ahead than the red runner."	7	"I think that the graph shows that the red and the blue runners finished their race."	8
"I think that the graph shows the way in which the red and the blue athletes ran"	3	"I think that the graph shows the time that the two runners needed to finish the race."	3	"I think that the graph shows that the red runner started his race from the 2 meter mark."	3
"I think that the graph shows the movement of two runners."	2	"I think that the graph show the number of meters run by two runners."	3	"I think that the graph shows that the two athletes ran equally fast."	2
"I think that the graph shows that the blue line is on the 10th position, but the red line is much further away."	2	"I think that the graph shows that the red athlete ran 10 meters, while the blue athlete ran 20 meters."	2	"I think that the graph shows that the two runners started from two different points and finished at the same point."	2

a 20 meter race in 1 second, while the red runner needs 10 seconds to complete the race". Figure 5b shows another graph designed by the same student, which according to him is "a 25 kilometer race with four cars."

At the beginning of the study, students were also asked to interpret three distance-time graphs. Their initial interpretations of these graphs are presented in Table 1. It appears that, at the beginning of the lessons, students were not able to give accurate interpretations of the distance-time graphs. Most of them interpreted the graphs as if they were static pictures. For the distance-time graph (a), most students (8 out of 15) concentrated on the names of the axes, and claimed that they showed the two

runners' times and the number of meters they ran. The other students claimed that this graph showed the movement of two runners. For the distance-time graph (b), seven students hypothesized that the blue runner came first, since his line graph was "bigger." The others interpreted the graph as a picture—they concentrated on its characteristics (meters, position, seconds). For the distance-time graph (c), eight students said that the two athletes finished their race. We hypothesize that they may have reached this conclusion because the two line graphs intercepted. It is a classic graph interpretation given by students. Three students concentrated only on the starting point of the red runner, and two described the two runners beginning from two different points but finishing at the same point (they probably saw the black points on the graph), while two students maintained that the two athletes ran equally fast since they finished at the same point.

After students' engagement with SimCalc, they were able to interpret distance-time graphs. This is illustrated by the responses they provided when asked to write a description of the races that these graphs demonstrated. It is noteworthy that all students carried out this task correctly after the lessons with SimCalc (see, for example, Fig. 6).

## 5.3 Students' Higher Order Thinking in Mathematics

The second research question was about whether we could observe active higher order mathematical thinking in the SimCalc environment. The answer to this question is positive. In the following section, we demonstrate evidence of students' critical, creative, and complex thinking using the constant comparative method analysis of data. In particular, we looked carefully at the data collected from the classroom (students' responses, actions, and discussions) and identified thinking processes that fit into the three categories of thinking: (a) critical thinking processes in terms of analysis, connection and evaluation, (b) creative thinking processes in terms of imagining, synthesizing and elaborating, and (c) complex thinking processes.

5.3.1 Critical Thinking

During the lessons with SimCalc, students employed critical processes to respond to the tasks, namely analysis, connection, and evaluation of information.

5.3.1.1 Analysis

At the beginning of the first lesson, students were asked to watch the video of the Olympic race and explain "why" the runner came first. Six students claimed that, "This athlete ran faster than the rest," five students argued that "He trained very well and he was confident," two students commented that "This runner had more support from the spectators," and two students said that "His strides were bigger than those of the other runners." From these responses we can see that some students referred to velocity, whereas others had other ideas.
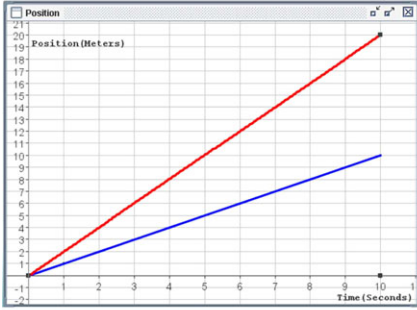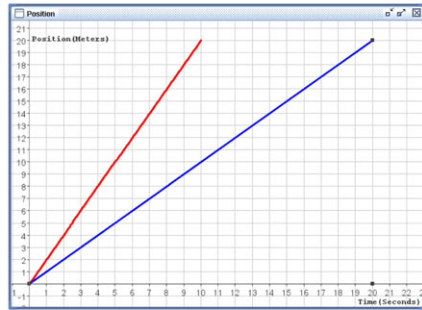
The red athlete came first in the 20 m race and his time was 10 seconds. The blue athlete ran 10 m during the same period.
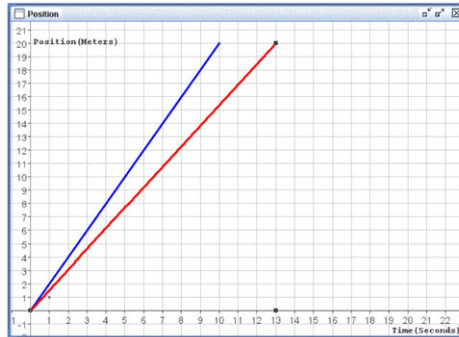
The blue athlete ran the 20 m in 10 seconds. The red athlete started 2 m ahead and finished the race at the same time as the blue athlete.
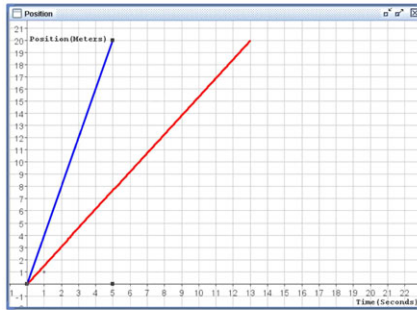
The blue athlete came first in the 20 m race with a time of 5 seconds. It took 13 seconds for the red athlete to run 20 m.

The red athlete finished the 20 m race first in 10 seconds. The blue athlete took 20 seconds to run the distance.

The blue athlete ran the 20 m in 10 seconds. The red athlete ran the same distance in 13 seconds.

Fig. 6 A student's answer in the assessment activity

As the lesson unfolded, students' explanations became more mathematical and also more specific. For example, in one of the tasks students were asked to talk about a 20 m race with two athletes that they saw in the SimCalc simulation. All students simply said that the red runner finished the race first. Eleven claimed that,

"This athlete came first, because he was running faster than the other runner," while four students added, "The red athlete came first and had a large distance difference from the blue athlete." As time elapsed students analyzed what they saw in the simulation using the various functionalities of SimCalc, and their responses became more specific. All students argued that, "The red athlete came first in the 20 m race. His time was 10 seconds. The blue athlete only managed to run 10 m during these 10 seconds." To facilitate students' understanding and highlight the difference between the two athletes, the teacher invited the students to use the *marking* option and describe what they saw in the simulation. Nine students commented that "The red athlete runs 2 m every second, while the blue runner runs 1 m every second. For example, when the red athlete has run 6 meters, the blue runner has run 3." In other words, the *marking* option gave them the opportunity to break the race into parts and see the pattern of the motion of the two runners. The other six students were not able to understand this pattern completely. They stated that: "The red athlete did long strides, while the blue athlete did short strides. This is why the red athlete covered twice the distance that the blue athlete covered."

When students were asked to discuss the numbers presented in the SimCalc table, they again started with more general comments. As time elapsed, they were able to identify patterns and understand interrelationships. All students' initial comments about the table were that "the table shows time and meters." Once the teacher suggested that they should pay more attention to the information provided on the computer screen, students started making connections between the data in the table and the simulation. All students were able to identify some patterns. Five students stated that: "The two athletes run at different speeds. The red athlete covers twice the distance that the blue athlete covers in the same time." Four argued that, "The time is the same for the two athletes. When the blue athlete has run 4 meters, the red athlete has run 8 meters," and six students commented that, "We noticed that the red athlete did strides of two meters, while the blue athlete did strides of one meter." In other words, all the students attended to the relationships between the simulation and the information in the table, but only five of them reached a conclusion about the different rates of the two athletes.

5.3.1.2 Connection

In another task, students were asked to identify the similarities and differences between three different distance-time graphs (see figures (a), (b), (c) in Table 1). All students claimed that the three distance-time graphs showed the distances that two athletes covered in a specific period of time, and all of them noticed that in figure (c) "the two athletes finished at the same time, while in the other two graphs the red athlete finished first."

When students were asked to compare these distance-time graphs with their respective velocity-time graphs (Fig. 3), they were able to say that the distance-time and the velocity-time graphs showed the same race but provided different information. They claimed that one of the graphs showed the distance that the athletes covered, whereas the other showed the athletes' velocity. Three pairs of students pointed out that "In the velocity-time graph the *lines* are horizontal and straight. This does

not appear in the distance-time graph." They also noticed that "In the distance-time graph the athletes' *lines* started from 0, while in the velocity-time graph the two runners' *lines* started from 2 and 1." It appeared that these students' comments were more descriptive than showing any evidence of conceptual understanding. For this reason, the teacher suggested that the students use the *marking* option and look for connections between the marks and the velocity shown on the graph. Six students commented that "The red athlete had velocity 2, since he covered 2 meters every second, while the blue athlete had velocity equal to 1, since he covered 1 meter every second." Moreover, two pairs of students noticed that the athletes' velocity was constant. They argued that "the distance that the runners covered each second did not change."

All students were able to understand the relationship between the two graphs. This was achieved by dragging the line of one of the graphs and observing the changes appearing on the other graph. Indicative of this was the following dialogue between two boys:

GS: Move further up the red straight line in the velocity-time graph. Move it to number eight. What happens to the simulation?

AP: I think that the red athlete will run faster now. I will press "play" to see it.

GS: You are right. When we drag the line upwords in the velocity-time graph the athlete runs faster than before… See, the red line in the distance-time graph also moves (He pointed with his hand).

AP: Yes. But the red athlete continues and finishes the race at 20 m.

GS: I think that we can conclude that every action on one graph influences the appearance of the other graph.

5.3.1.3 Evaluation

During these lessons all students frequently checked their hypotheses, explanations, and connections by "exploring the microworld." One student said that he was "watching very carefully what was happening on the screen to check my answers." In another incident, a student asked his partner: "How many seconds does the red athlete need to run 20 m in this graph (pointing to the distance-time graph) and how many seconds does he need in this graph (pointing to the velocity-time graph)?" "What are the distances that every athlete runs in this graph and in this graph? (He pointed to both graphs)." His partner responded that both graphs referred to the same distance. We could argue that discussions like these suggest that all students became more engaged in the activities, and examined the information presented to them critically.

5.3.2 Creative Thinking

During the lessons with SimCalc, students employed creative processes to respond to the tasks, namely: imagining, synthesizing, and elaborating.

According to Leikin and Lev (2007), multiple solution tasks are a useful instrument to develop and also measure students' creative thinking. This is the reason we

asked students to provide multiple solutions in two of the tasks presented to them. In one of these tasks, students were asked to draw distance-time graphs showing that the blue runner finishes the 20 m race first and the red runner second. The second creativity task required the students to add an actor (a car) in a given 2-car race (see Figs. 8a, 8b, 8c, 8d), and have this new actor overtake the previous two actors in the race. The two creativity tasks were presented to the students in two 40-minute lessons, which took place one week apart.

5.3.2.1 Imagining

From the students' behavior in the classroom, it appears that they all tried to imagine what they needed to do before designing the graphs. As the teacher moved around the students and asked them to describe what they had to do, all of them stated that they had to draw lines in such a way as to show that the blue runner would finish the 20 m race first. Students were moving their hands along the screen to indicate the directions of the two lines, frequently maintaining that there were many different solutions. Indicative of this is the following comment by one of the students: "I am sure that there are more than 20 graphs that I could sketch to show that the blue runner finished the race first." In the next section, we analyze the procedure students used to provide more than one solution in the two creativity tasks.

5.3.2.2 Synthesizing

With regard to students' creative processes in the first creativity task, we identified two strategies used. The first is illustrated in Fig. 7a. In particular, we observed that eight students initially moved the *graph* of the blue runner slightly away from the horizontal axis but did not change the *graph* of the red runner. Then, they dragged the *graph* of the blue runner up a little towards the *y*-axis and then even closer to it. When the teacher asked a pair of students what they observed, they claimed that, "When the blue runner runs very fast, the angle created by his "line" graph and the horizontal axis becomes bigger."

The second strategy is illustrated in Fig. 7b. In particular we observed that the rest of the students (seven) initially moved the blue line and covered the red line and then they dragged the red line "slightly below the *graph* of the blue runner." As one of the students said: "First, we set the blue runner to cover 20 m in 10 seconds. Then we moved the red runner slightly below the blue runner's line."

All students drew at least four different distance-time graphs to show the 20 m race with the blue runner winning and the red runner coming second, using one of the two strategies described above.

5.3.2.3 Elaborating

Elaboration processes were obvious in students' solution processes in the second creativity task. In this task, all students originally created an actor similar to those
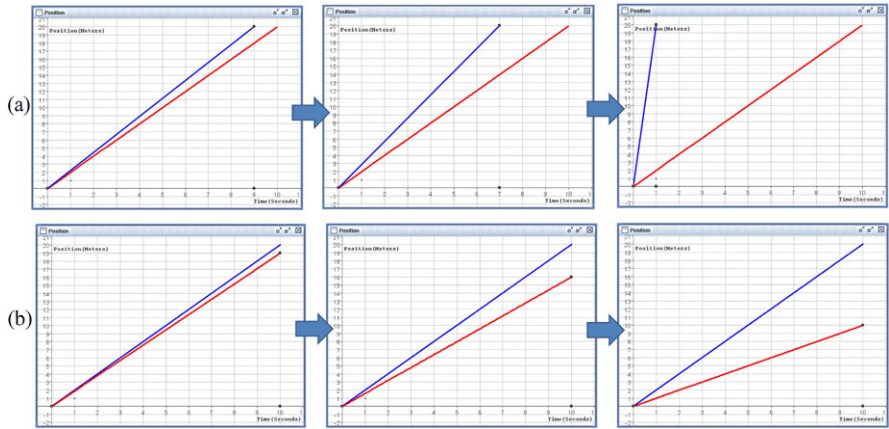
Fig. 7 The two strategies students used to draw distance-time graphs where the blue runner finishes the 20 m race before the red runner
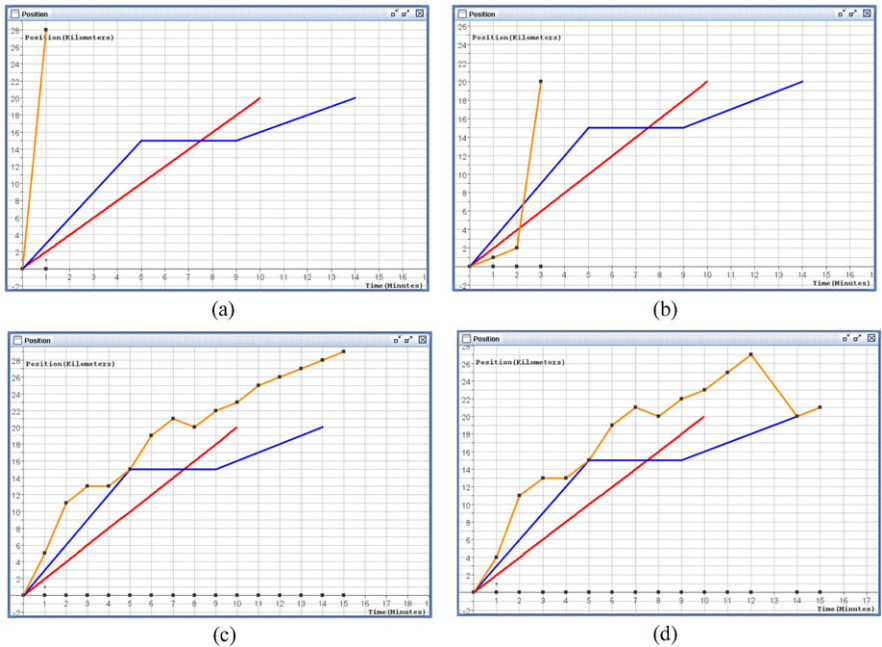

Fig. 8 Creations of a new actor who finishes the race first

that they had already observed. Indicative of this was one student's comment to her partner: "To finish first, the graph needs to be "steeper," nearer to the vertical axis. We need to do it in the same way as we did before" (in the first creativity task, see Fig. 8a). As students became more familiar with SimCalc and appeared to

gain a better understanding of the graphs, they started presenting more elaborated solutions. For example, three pairs of students set their car to initially move with small velocity and then increased its velocity and overtake the other two cars (see Fig. 8b). Two other pairs of students decided to set their cars to move forward for a few seconds, stop for a while, then move forward again and then backwards for some seconds (see Figs. 8c and 8d).

5.3.3 Complex Thinking Processes

In the final two activities, students were asked (a) to design a distance-time graph to fit given data, and (b) to design their own race in SimCalc. These activities require complex thinking processes, since students need to apply a combination of critical and creative approaches to provide solutions for them.

In the first design task, students identified the goal of the task and then analyzed the data that they had been given. Then one student in each pair drew a graph on the empty grid of SimCalc, which their partner then checked. The following dialogue between two students exemplifies the thinking involved:

> MC: It is easier to create a car that travels 20 km in 8 minutes.
> KX: OK. I will do it.... What does it mean when it says that the second car "in the first 4 minutes"? Do we draw it here? (He points at point (0,4) on the distance axes)
> MC: No! According to the instructions the second car is leading the race for the first 4 minutes. It is not written that the car starts from the 4th km. Then, this car moves for 10 minutes.
> KX: OK, like this (he draws the graph)?
> MC: Yes. I will press play, to check if we are correct.
> KX: It is cool!

These students were able to analyze the instructions step by step, design a graph to fit the given data, and check their solution by activating the simulation. In other words, they applied complex thinking processes to provide their solutions, since they used critical thinking processes (analyzing, evaluating) and creative thinking processes (synthesizing).

In the activity where students were asked to create their own race and describe it, four of them created three races, four more created two races and six created one race. Students' designs were quite elaborate and they used existing SimCalc Worlds, such as Cars, Fishy and Rockets. Their objects did not simply move forward; They also moved backwards or stopped for some time. During this task students produced races with many "changes." A pair of students put it nicely: "we designed a "crazy" race."

The processes that students followed to design their own races were common, but their products were very different. First of all, they set their goal, then they planned their design and simultaneously sketched it on the SimCalc grid. Once having finished, they evaluated their design by activating the simulation and checking whether it was presenting the story that they had in mind. This SimCalc functionality allowed students to co-act and verify their responses. Below is a dialogue between two boys:

VA: I want to design a car race.
RO: Me too. I want my car to stop for a while.
VA: Yes!!! We should sketch it in a similar way to the one we saw before.
RO: The car which will finish the race first we have to draw it near this axis (The boy points on the *y*-axis on the screen).

For every race that students designed a graph, they also had to write a story. All students were able to connect the visual forms (graphs and simulated motions) with narrative stories. One such story, written by a pair of students, is presented below.

The green fish starts at 6 m, then goes for 2.5 m and continues for 15 m. Then he stops for 1 second. Then he continues to 19 m and in 9 seconds he reaches his destination which is at 24 m. The blue fish goes straight to 24 m, then he returns to 15 m. He stops for 2 seconds and then continues for 3 m. He stops for 1 second. Then he goes to 24 m in 7 seconds. The red fish starts at 10 m. Then he goes for 5 m, stops for one second and then goes to 14 m. Then he goes for 3 m and stops for 1 second. Then he starts again to reach his destination at 24 m. He returns to 20 m, he makes a stop for one second and then returns to 24 m. This is the crazy ride we thought of!

6 Conclusions

The purpose of this chapter was to examine whether SimCalc could be used as a mindtool, and to describe and analyze the ways in which it might develop and enhance elementary school students' higher order thinking in mathematics. The results of the study showed that SimCalc can be used as a mindtool in elementary schools. SimCalc provides students with the opportunity to be engaged in activities, which may facilitate not only their content knowledge, but also their higher order thinking.

After the implementation of lessons with SimCalc, students showed evidence of development in their content knowledge, critical, creative, and complex thinking. With regard to content knowledge, students were able to interpret distance-time and velocity-time graphs flexibly and discuss their similarities and differences. The SimCalc environment also gave students the opportunity to develop their critical abilities. These seem to have been developed though the possibilities that SimCalc offered them to analyze visual (graphs and simulated motion) and linguistic (numbers and algebraic symbols) forms, to connect various types of representations, and to evaluate their responses by the feedback provided. Furthermore, SimCalc appears to enhance students' creative abilities, allowing them to expand their thinking to new answers, and offering them the fluency and flexibility to provide more than one solution. Finally, it developed students' complex thinking by allowing them to design solutions.

The findings from our study provide evidence that SimCalc offers students the potential to be actively engaged in authentic activities, and to construct knowledge in a meaningful way. Until now, these types of activities were absent from the Cypriot mathematics classrooms. More intensive implementation of SimCalc is needed in the Cypriot mathematics classrooms. It will be interesting to investigate if the positive effects of SimCalc appear both in quantitative and qualitative form. In particular,

our future work involves the investigation of the effects of SimCalc on elementary school students' thinking compared to students' thinking in regular classrooms.

As illustrated in this chapter, in order to achieve higher order thinking, it appears to be necessary to consider not only the subject to be taught, but also classroom organization and tasks. For this, it will be interesting if future research concentrates in more detail on the way that the mathematical subject is taught, classroom organization, and the impact of the various types of tasks. In particular, do different types of learning environments (subject, class organization and tasks) with SimCalc activate different aspects of thinking? It will be necessary to carry out further research on SimCalc to discover the specific effects of these learning environments. For example, do learning environments with SimCalc, which address the topic of proportional reasoning activate higher order thinking skills? In addition to this, it will be interesting to investigate the degree of impact on different student variables (e.g., students' efficacy, students' argumentation).

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