Chapter 104
A Closed Form Solution for Pollutant Dispersion Simulation in Atmosphere Under Low Wind Conditions

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Abstract The present study proposes a mathematical model for dispersion of contaminants in low winds that takes into account the along-wind diffusion. The solution of the advection-diffusion equation for these conditions is obtained applying the 3D-GILTT method. Numerical results and comparison with experimental data are presented.

Keywords Low wind conditions • Pollutant dispersion

104.1 Introduction

In the last several years, special attention has been paid to the task of searching analytical solutions for the advection-diffusion equation in order to simulate the pollutant dispersion in the Atmospheric Boundary Layer (ABL). Focusing our attention in this direction, in this work we step forward, reporting an analytical solution for the three-dimensional advection-diffusion equation, applying the new 3D-GILTT method (Three-Dimensional Generalized Integral Laplace Transform Technique)

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[2], considering the longitudinal diffusion. Furthermore, in the turbulence parameterizations, the eddy diffusivities are functions of distance from the source and represent correctly the near-source diffusion in weak winds [1].

The importance of dispersion modelling in low wind conditions lies in the fact that such conditions occur frequently and are crucial for air pollution episodes. In such conditions, the pollutants are not able to travel far and thus the near-source areas are affected the most. The classical approach based on conventional models, such as Gaussian puff/plume or the K-theory with suitable assumptions, are known to work reasonably well during most meteorological regimes, except for weak and variable wind conditions. This is because (i) down-wind diffusion is neglected with respect to advection (ii) the concentration is inversely proportional to wind speed (iii) the average conditions are stationary and (iv) there is a lack of appropriate estimates of dispersion parameters in low wind conditions. In view of such restrictions, various attempts have been made in literature to explain dispersion in the presence of low wind conditions by relaxing some of the limitations.

104.2 The Closed Form Solution

The advection-diffusion equation of air pollution in atmosphere is essentially a statement of conservation of a suspended material, and it can be written as:

$$\frac{\partial \bar{c}}{\partial t} + \bar{u}\frac{\partial \bar{c}}{\partial x} + \bar{v}\frac{\partial \bar{c}}{\partial y} + \bar{w}\frac{\partial \bar{c}}{\partial z} = \frac{\partial}{\partial x}\left(K_x\frac{\partial \bar{c}}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y\frac{\partial \bar{c}}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z\frac{\partial \bar{c}}{\partial z}\right) \tag{104.1}$$

where $\bar{c}$ denotes the mean concentration of a passive contaminant (g/m^3), $\bar{u}$, $\bar{v}$, and $\bar{w}$ are the cartesian components of the mean wind (m/s) in the directions x $(0 < x < L_x)$, y $(0 < y < L_y)$ and z $(0 < z < h)$, K_x, K_y and K_z are the eddy diffusivities. Equation 104.1 is subjected to the usual boundary conditions $\boldsymbol{K}\nabla\bar{c}|_{(0,0,0)} = \boldsymbol{K}\nabla\bar{c}|_{(L_x,L_y,h)} = 0$ and initial and source conditions: $c\,(x, y, z, 0) = 0$; $\bar{u}c\,(0, y, z, t) = Q\delta(x)\delta\,(y - y_0)\,\delta(z - H_s)$, where Q is the emission rate (g/s), h the height of the ABL (m), H_s the height of the source (m), L_x and L_y are the limits in the x and y-axis and far away from the source (m) and δ represents the generalized Dirac delta function. The source position is $(0, y_0, H_s)$.

In order to solve the problem (104.1), we initially apply the integral transform technique in the y variable. For such, we expand the pollutant concentration as:

$$\bar{c}\,(x, y, z, t) = \sum_{m=0}^{M} \bar{c}_m\,(x, z, t)\,Y_m(y) \tag{104.2}$$

where $Y_m(y) = \cos(\lambda_m y)$ is a set of orthogonal eigen functions and $\lambda_m = \frac{m\pi}{L_y}$ for m = 0,1,2, ... are the respective eigen values. To determine the unknown coefficient $\bar{c}_m(x,z,t)$ for m = 0: M we began substituting Eq. 104.2 in Eq. 104.1 and then taking moments. This procedure leads to:

$$\frac{\partial \bar{c}_m(x,z,t)}{\partial t} + \bar{u}\frac{\partial \bar{c}_m(x,z,t)}{\partial x} - \frac{\partial}{\partial x}\left(K_x\frac{\partial \bar{c}_m(x,z,t)}{\partial x}\right) - \frac{\partial}{\partial z}\left(K_z\frac{\partial \bar{c}_m(x,z,t)}{\partial z}\right) + \lambda_m^2 K_y \bar{c}_m(x,z,t) = 0$$

The analytical solution of the above two-dimensional problem is obtained by the GILTT (Generalized Integral Laplace Transform Technique) approach [6]. The mean feature of the GILTT method comprehends the steps: solution of an associate Sturm-Liouville problem, expansion of the pollutant concentration in a series in terms of the attained eigen function, replacement of this expansion in the advection-diffusion equation and, finally, taking moments. This procedure leads to a set of second order differential ordinary equations, named the transformed equation. After an order reduction, the transformed problem is solved analytically by the application of the Laplace transform technique without any approximation along its derivation, except the round-off error. In the case $K_x \to 0$, on these assumptions, we obtain the solutions of Buske et al. [2] and Moreira et al. [6].

104.3 Turbulent Parameterization

To represent the near-source diffusion in weak winds the eddy diffusivities should be considered as functions of not only turbulence (e.g., large eddy length and velocity scales), but also of distance from the source [1]. Following this idea, Degrazia et al. [4] proposed for an algebraic formulation for the eddy diffusivities, which takes the form:

$$\frac{K_\alpha}{w_* h} = \frac{0.58 c_i \psi^{2/3} (z/h)^{4/3} X \left[0.55 (z/h)^{2/3} + 1.03 c_i^{1/2} \psi^{1/3} \left(f_m^*\right)_i^{2/3} X\right]}{\left[0.55 (z/h)^{2/3} \left(f_m^*\right)_i^{1/3} + 2.06 c_i^{1/2} \psi^{1/3} \left(f_m^*\right)_i X\right]^2}$$

where $\alpha = x, y, z, i = u, v, w, c_i = \alpha_i (0.5 \pm 0.05)(2\pi\kappa)^{-2/3}$, $\alpha_i = 1, \frac{4}{3}$ and $\frac{4}{3}$ for u, v and w components respectively, $\kappa = 0.4$ is the von Karman constant, $\left(f_m^*\right)_i$ is the normalized frequency of the spectral peak, h is the top of the convective boundary layer height, w_* is the convective velocity scale, ψ is the non-dimensional molecular dissipation rate function and $X = \frac{x w_*}{\bar{u} h}$ is the non-dimensional time, where $\bar{u}$ is the horizontal mean wind speed. More details on the paper [4].

The wind speed profile is described by a power law [8].

104.4 Application to Experimental Data

The performance of the 3D-GILTT model was evaluated against experimental ground-level concentration using SF_6 data from dispersion experiments in low wind conditions carried out by the Indian Institute of Technology (IIT Delhi), described in Sharan et al. [9, 10]. The pollutant was released without buoyancy of a height of 1 m and the concentrations of SF_6 were observed near the ground-level (0.5 m). The release rate of SF_6 tracer varied from 30 to 50 ml min^{-1}. The sampling period for each run was 30 min. Wind and temperature measurements were obtained at four levels (2, 4, 15 and 30 m) from a 30 m micrometeorological tower. In all the cases, the wind speed was less than 2 ms^{-1} at the 15 m level. The samplers were located on arcs of 50 m and 100 m radii.

Table 104.1 shows the performance of the new model compared with other models for the unstable experiments of IIT Delhi, using the statistical indices described by Hanna and Paine [5]. While the present approach (3D-GILTT) is based on a genuine three dimensional description an earlier analytical approach called GILTTG uses a Gaussian assumption for the horizontal transverse direction [6]. The ADMM approach [7], solves the two-dimensional advection-diffusion equation by adiscretisation of the ABL in a multilayer domain and also uses a Gaussian assumption for the horizontal transverse direction. The GIADMT [3] is a dimensional extension to the previous work, but again assuming the stepwise approximation for the eddy diffusivity coefficient and wind profile. The model of Arya [1] is obtained by the numerical integration of the Gaussian puff solution using dispersion parameters based on convective similarity scaling. The results obtained with this approach reveal a further under-prediction of concentration. The same happens with the model of Sharan et al. [9]. Better results are obtained by the model of Sharan et al. [10] that is an improvement of the previous one where the friction velocity is used instead of the convective velocity. The statistical indices of Table 104.1 indicate that, compared with other models, a good agreement is obtained between the K-model and observed near ground-level centerline concentrations.

Table 104.1 Statistical comparison between model results

	NMSE	COR	FA2	FB	FS
3D-GILTT	0.14	0.83	0.88	−0.05	−0.04
GILTTG	0.29	0.77	0.81	0.05	−0.25
ADMM	0.35	0.76	0.81	−0.01	−0.33
GIADMT	0.22	0.93	0.88	0.33	0.31
Arya 1995	13.86	0.77	0.00	1.68	1.59
Sharan 1996	7.11	0.76	0.00	1.49	1.32
Sharan 1996	0.37	0.91	0.75	0.45	0.40

104.5 Conclusions

A mathematical model for the dispersion of a pollutant from a continuously emitting near-ground point source in a ABL, with low wind conditions, has been described. Besides advection along the mean wind, the model takes into account the longitudinal diffusion. The closed form analytical solution of the proposed problem is obtained using the 3D-GILTT method. The present model has been evaluated in unstable conditions for concentration distributions. Particularly, the results obtained by the analytical dispersion model agree very well with the experimental concentration data, indicating that the model represents the diffusion process correctly under low wind conditions.

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