Chapter 8
Dubislav and Classical Monadic Quantificational Logic

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Walter Dubislav was born in Berlin-Friedenau in 1895, and ended his life under dramatic circumstances in Prague in 1937. Since 1927 he had belonged to the Berlin Group, i.e. the Society for Scientific (or: Empirical) Philosophy. His interests and contributions were broad, as shown by his work on various topics such as the writings of Bolzano, the Friesian School, natural philosophy and the philosophy of mathematics. The best known of Dubislav's work is *Die Definition*, first published in 1926 (under the title *Über die Definition*), then in a second edition in 1927, and finally in a completely revised and enlarged third edition by the publishing house of Felix Meiner, as the first supplement ("Beiheft") of the journal *Erkenntnis* edited by Reichenbach and Carnap in 1931. On the occasion of the 50th anniversary of its publication, Meiner published a reprint (Dubislav 1981) with a new preface by Wilhelm K. Essler.

The present paper deals with Dubislav's interesting, but contentually and technically problematic contribution to the philosophy of mathematics, or more precisely, to mathematical logic and metalogic. When Dubislav turned his eye to this field, Gödel had not yet written his dissertation on the completeness of the calculus of quantificational logic, nor his paper on the incompleteness of "*Principia Mathematica* and related systems", Church had not yet proved the undecidability of classical quantificational logic, and no "Hilbert-Bernays" was at hand. This said, beyond the *Principia Mathematica*, there were promising investigations on proof theory by Hilbert, Ackermann, Behmann, von Neumann (von Neumann 1927) and some others. And, the theoretical survey provided by the first two of those scholars (entitled *Grundzüge der theoretischen Logik*) would stimulate research by Gödel, Carnap and other great logicians of the 1930s. Yet, according to Hilbert and Ackermann, the "main problem of mathematical logic" at the time was the decision

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problem, i.e. the quest for a procedure "which permits us to decide for any given formula, by finitely many operations, whether it is universally valid (or satisfiable, respectively)" (Hilbert and Ackermann 1928, 77 and 73, respectively). With regard to the calculi of classical propositional logic and classical monadic quantificational logic (treating only of one-place propositional functions) the problem had already been solved by this time. However, for classical quantificational logic, such a resolution remained elusive, even for classes of formulas with a particular logical structure. Thus, it was no wonder that Dubislav also turned his attention to this question.

On his approach, the method of the so-called "quasi truth-tables" is of great importance. In short, they are an extension of the familiar truth-tables of classical propositional logic. Operations with truth-values had been known, at least in principle, to Peirce, MacColl, Frege and others, but they were explicitly propagated at the beginning of the twentieth century, in large part, by Wittgenstein's *Tractatus* (Wittgenstein 1921). In 1921, Emil L. Post supplemented the two truth-values "true" and "false" (T and F, + and –, 1 and 0 or vice versa) with further values, not all of which could now be interpreted to strictly pertain to the truth as such—hence the prefix "quasi".

Dubislav makes use of his method (inspired by Post) in two papers, published in 1928 and 1929. In the second of these, he explicitly mentions Wittgenstein and Post as his sources. As an aside, it is worth noting that in neither of the two papers are the terms "decidable", "decidability" and "decision problem" ever deployed. For, these works serve a different set of concerns that is made clear by their titles. Specifically, the paper from 1928 is called "Zur kalkülmäßigen Charakterisierung der Definitionen". Herein, Dubislav investigates the explicit definitions, eliminable in the sense of Pascal, and goes on to describe them as rules for the replacement of a definiendum by its definiens in all, or only in some places, of an expression containing them. Examples that had already been given by Peano in 1901 make it evident that special precautions have to be taken. For, while Dubislav shows that Peano's criterion is necessary, he also demonstrates that it is not sufficient. In order to rectify this situation, he introduces a property abbreviated by the letter "E". This property is meant to hold of a well-formed expression of the calculus if and only if its valuation, by means of the usual truth-tables or of certain quasi truth-tables, finally yields a column of exclusively designated values (i.e., "true", + or 1 or 0, respectively). This property is passed on, in propositional logic and in monadic quantificational logic, by every application of the rules of substitution and of the rule of detachment ("modus ponens"). Dubislav finds the criterion for the correctness of explicit definitions in the condition that the latter, if formulated as "additional substitution rules", also transmit the property E.

By 1928 he had already mentioned another and even more elementary application of this property. For, due to the fact that the truth table of negation replaces a designated value with a non-designated one, and given that Dubislav's quasi truth-tables, being conservative extensions of the truth-tables, have the same effect, the negation of an expression whose valuation leads to a column with exclusively designated values can never yield a column of the same kind. Therefore, since all expressions derivable in the calculus get designated values, an expression and its negation cannot both be derivable. This is to say, that the calculus—in our case the calculus of propositional logic and that of classical monadic quantificational logic, respectively—is consistent.

It is this point that is the subject of the second early paper of Dubislav's mentioned above, which was published under the title, "Elementarer Nachweis der Widerspruchslosigkeit des Logik-Kalküls" in the *Journal für die reine und angewandte Mathematik*, a journal highly esteemed as *Crelles Journal*, albeit not specializing in logic. Again, in this paper there is no mention of decidability. Rather, Dubislav first explains the method of evaluation for the validity of formulas composed by propositional connectives: if "plus" (+) designates the value "true", such a formula is valid if its evaluation by the truth-tables yields a plus-column. On account of the heredity property, mentioned previously, and of the fact that all axioms of the classical propositional calculus (taken by Dubislav from Hilbert and Ackermann) yield a plus-column, we may infer the consistency of the calculus. Dubislav extends this result (quoting Post 1921 as, at least formally, a predecessor, although with different tables) to formulas of classical monadic quantificational logic. In the same manner as before, Dubislav also establishes the consistency "for the calculus operating with 'all' and 'some'" (Dubislav 1929, 110). Here, it is worth noting that he actually only treated the monadic case, but he did so without drawing attention to this restriction. That is, the axioms of propositional logic are simply supplemented by an axiom for the universal quantifier and one for the existential quantifier, while the general calculus of quantificational logic is not even mentioned by name.

In the third edition of *Die Definition* (Dubislav 1931) we are met with a closely related presentation. The subject makes it necessary to include the purely calculatory criterion for correct explicit definitions established in "Zur kalkülmäßigen Charakterisierung der Definitionen" (Dubislav 1928), and the consistency proof emerges, as it were, *en passant*. This time it is somewhat clearer why we need three-valued tables, and why this is sufficient. But once more, the argument is given only for the monadic calculus, and its validity for the general calculus is merely asserted. Of course, for a one-place predicate and a one-place complex propositional function it seems simply evident that they have to be "always true", or "always false", or "sometimes true and sometimes false". Thus, the three cases yield the three values, and reflection on their content justifies the structure of the three-valued tables employed by Dubislav.

Tables of this kind are completely absent in a survey of the philosophy of mathematics in Germany published as "Les recherches sur la philosophie des mathématiques en Allemagne" (Dubislav 1931–32), (for the most part a French translation of extracts from Dubislav's book *Die Philosophie der Mathematik in der Gegenwart* announced for 1932). In the book as well as in its partial translation, Dubislav refers the reader to Gödel's papers on the completeness of quantificational logic and on the undecidability of *Principia Mathematica* (Gödel 1930, 1931, respectively), and mentions Löwenheim's discovery of the decidability of classical monadic quantificational logic as well as Behmann's decision procedure of 1922 (Behmann 1922), to which he adds another one developed by himself (curiously quoting his paper in *Crelles Journal* of 1929 where the tables served quite another purpose). The chapter entitled “Der wissenschafts-theoretische Problemkreis” appears *nolens volens* under the title “Les problèmes épistémologiques”. Incidentally, the French translation, which is at times somewhat freely done, but obviously competently, comes courtesy of Emmanuel Levinas.

The more comprehensive German monograph (Dubislav 1932) appeared under the title *Die Philosophie der Mathematik in der Gegenwart*, as Heft 13 of the series *Philosophische Forschungsberichte* of the publishing house Junker und Dünnhaupt in Berlin. Dubislav presents his quasi truth-valuation in Chap. 4 “Das Entscheidungsproblem und das Vollständigkeitsproblem” (op. cit., 22–27). It is here that, for the first time, he also offers his own appraisal of the procedure. On page 24, he explicitly calls it a decision procedure, and after recalling the classical truth-table method, announces its extension to the functional calculus (i.e. to quantificational logic): “to begin with [‘*zunächst*’], to the functional calculus in which the fundamental logical connectives join only formulas with one and the same variable” (loc. cit., 25). As a point of application and for the purpose of illustration, he examines the syllogistic inference from “all men are mortal beings” and “Caius is a man” to “Caius is a mortal being”. The schema of evaluation is presented on page 27 (with “M” for “Mensch”, “S” for “sterblich” and “c” for “Caius”):

$M\hat{z}$,	$St\hat{z}$	$Mx \supset Stx$	$(x)\{Mx \supset Stx\}$	Mc	Stc	$[(x)\{Mx \supset Stx\} \cdot Mc] \supset Stc$	
+	+	+	+	+	+	+	
+	−	−	−	+	−	+	
−	+	+	+	−	+	+	
−	−	+	+	−	−	+	
*	+	+	+	+; −	+	+	(bei allen durch die Mehrdeutigkeit entstehend. Kombinationen)
+	*	*	−	+	+; −	+	”
*	−	*	−	+; −	−	+	”
−	*	+	+	−	+; −	+	”
*	*	+; *	+; −	+; −	+; −	+	(bei allen durch die Mehrdeutigkeit entstehend. Kombinationen, die auf Grund der vorgängigen Wertungen nicht von selbst ausfallen).

The method by which the lines and columns are calculated will be explained later. For the moment, it is sufficient to realize that the last line shows a horizontal combination of plus signs at the ambiguous entries, and that the rightmost plus-column (the result of the evaluation) is flanked by two not very perspicuous marginal notes stating that in the last five lines the combinations caused by ambiguities either disappear automatically, or yield a plus.

Dubislav was reproached for this irritating opacity, in reviews of *Die Definition* (1931) and of the volume on the philosophy of mathematics. The consistency paper had been announced briefly but without complaints by Fraenkel in the *Jahrbuch über die Fortschritte der Mathematik* of 1929. But in the 1931/1932 edition of *Zentralblatt für Mathematik und ihre Grenzgebiete* a member of the Hilbert circle in Göttingen, Arnold Schmidt, reviewed the third edition of Dubislav's *Die Definition* and criticized some of its central points severely. In his words:

> The consistency proof given for the restricted functional calculus, using three-figure value tables [...] is not correct; to render it correct, one would not only have to modify and formally extend the tables given on page 84/85, but also to supply the (necessarily) ambiguous places of the tables with a special instruction for the distinction of values. (Schmidt 1932, 1)

What is more, he asserted that the validity of the procedure claimed by Dubislav for all formalizable disciplines, is lacking. Since even for the hitherto known consistency proofs for the elementary theory of numbers the method of valuation is insufficient. In Schmidt's opinion:

> The author overlooks the circumstance that his consistency proof for the functional calculus does not cover every axiom of a discipline that can be expressed by the symbols of this calculus. The *sufficiency* of the criterion becomes evident—with the proviso that the necessary consistency proofs are available at all—only if one may be sure (as in the case of the propositional calculus) the property E does not pertain to a formula which is expressible by the symbols of the corresponding calculus but is not (and ought not be) provable in it. But this is a condition far from being a matter of course, a condition the fulfilment of which (or its proof) will in many a case turn out to be a rather difficult task. E.g., the Hilbert-Ackermann consistency proof for the functional calculus mentioned by the reviewer does not fulfil the condition. (loc. cit., 2)

In the 1931/1932 edition of *Zentralblatt* we also find a review of Dubislav's *Die Philosophie der Mathematik in der Gegenwart.* Again, the reviewer is Arnold Schmidt, and, as such, it is not surprising that Dubislav's use of the procedure of 1931, for the restricted functional calculus, is the subject of criticism. So too is the continuing lack of support for the claim "that the criterion offered (the table of plus values) *suffices* for provability; the proof of the *necessity* is open to the objection [made by Schmidt in the review just quoted] that we miss particular instructions for distinguishing values at ambiguous places" (Schmidt 1933, 145). Schmidt also criticizes the opacity of this work, that has already been discussed herein. As he notes:

> [...] in an example (27) we are told of 'combinations which disappear by themselves thanks to valuations performed before'. But combinations of this kind do not exist in the procedure described by the author; rather, he refers to *contentual* considerations, whereas the aim of the procedure is just to make the decision independent from contentual considerations. (Ibid.)

On a more positive note, Schmidt tentatively presents a conceivable amendment to Dubislav's procedure, but not without immediately pointing out that even this augmentation would not be sufficient to remedy the situation in all possible cases.

With regard to Dubislav's book on the philosophy of mathematics, I will only mention the review published by Heinrich Scholz in the *Jahresbericht der Deutschen Mathematiker-Vereinigung* of 1934 (Scholz 1934). Scholz speaks of how Dubislav proposes "interesting tables of evaluation for the solution of the decision problem in the elementary predicate calculus". However, he closes his review "with a few critical remarks" (loc. cit., 89). The second of these reads:

> A stringent proof of the efficiency of the valuation tables on page 25 has not been given; neither for the rules of inference nor for the rules of substitution of the Hilbert-Bernays predicate logic (which are insufficient and therefore in need of a precise reformulation) has the heredity of the designated value with reference to these rules been demonstrated.

Scholz supplements this criticism with a reference to the critical passages in the two reviews by Arnold Schmidt just mentioned. The remark on the insufficient formulation of the rule of substitution relates to the insufficient version in the first edition of Hilbert-Ackermann (corrected in the second edition, not least on account of the criticism in Scholz's mimeo *Logistik* of 1932/1933). This correction was explicitly noted by Quine in his review of the second edition in the *Journal of Symbolic Logic* (Quine 1938) as well as in Bernays's preface to the first volume of *Grundlagen der Mathematik* (Hilbert and Bernays 1934, VI).

The only place in the subsequent logical literature where Dubislav's procedure, the problem of ambiguity and its partial clarification are discussed is Hans Reichenbach's *Elements of Symbolic Logic* (Reichenbach 1947). § 23 introduces "truth characters of one-place functions", i.e. quasi truth-values. In a footnote their interpretation as "necessity", "possibility" and "impossibility" in Russell's *Introduction to Mathematical Philosophy* (Russell 1919) is mentioned. Right after this we learn of the introduction of Dubislav's tables in the paper of 1929 with the application to "case analysis", completed by Reichenbach's use of the tables for the interpretation of modalities in 1932 (Reichenbach 1932).

After the "Definition of tautologies containing functions" in § 24, the subject of § 25 is "The use of case analysis for the construction of tautologies in propositional functions". This method, the introduction of which Reichenbach attributes to Dubislav, uses quasi truth-tables and because of this is restricted to monadic quantificational logic. To examine an expression correctly built up according to the rules of this logic means "going through all possible cases resulting for different truth characters of its constituents" (Reichenbach 1947, 131). Among the difficulties arising from ambiguities there are those that are harmless, and those that aren't. The harmless ones are those in which "the indeterminacy of the middle line drops out" (ibid.). This statement obviously refers to the combinations "disappearing by themselves" in (Dubislav 1932), although Reichenbach does not include a reference to that work. On the other hand, Reichenbach also gives several examples of cases which are not so harmless insofar as the ambiguity arising from the concurrence of two ambivalent pairs of values may be overcome only by forming a combination in which the first member of one of the pairs corresponds to the first member of the other, and likewise for the second members. This, however, can only be decided by "material thinking", i.e. contentual considerations, as indicated by Arnold Schmidt.

For his part, Reichenbach points out that in these sorts of cases we need to refer to an infinite amount of objects, though he does not regard such a maneuver as being illegitimate. Consequently, the situation is this: if the case analysis shows, without material considerations, that a formula is a tautology, then it certainly is a tautology. However, if the examination leaves the result underdetermined, the formula still may be a tautology. Thus, as Quine puts it within his review of Reichenbach's book in 1948, what is provided is "a partial test of validity in quantification theory, [. . .] adequate to a *portion* of *monadic* quantification theory" (Quine 1948, 162). While this limitation is clear to Reichenbach, he showed "no awareness that test methods have existed since 1915 for the *whole* of *monadic* quantification theory" (with reference to Löwenheim 1915; Quine 1945). Regardless of this weakness, the judgment is clear: Dubislav's procedure of case analysis by quasi-valuation is a sufficient, but not a necessary criterion for validity.

Let me finally illustrate this point by reference to three examples that, while leading back to the origin of my own occupation with Dubislav's procedure, also shed some light on the rather abstract explanation of the method discussed at the beginning of this paper. Let us first take Dubislav's case analysis in the Caius example:

(Λ_x	(Mx	$\rightarrow$	Sx)	$\wedge$	Mc)	$\rightarrow$	Sc
0	0	0	0	0	0	**0**	0
1	0	1	1	1	0	**0**	1
0	1	0	0	1	1	**0**	0
0	1	0	1	1	1	**0**	1
0	2	0	0	0/1	0/1	**0**	0
1	0	2	2	1	0	**0**	0/1
1	2	2	1	0/1	0/1	**0/1**	1
0	1	0	2	1	1	**0**	0/1
0/1	2	0/2	2	0/1	0/1	**0/1**	0/1

The compound formula to be tested is the result of a step-by-step construction beginning with the two elementary propositional functions Mx and Sx and continued by either instantiation, or quantification, or propositional connection. In the first step Mx and Sx are combined to form a conditional Mx → Sx which in the second step is universally quantified to yield Λ_x (Mx → Sx). In the third step we instantiate Mx and Sx for c ("Caius") to get Mc and Sc, respectively. In step 4, Λ_x (Mx → Sx) and Mc are combined within a conjunction Λ_x (Mx → Sx) ∧ Mc, which in step 5 is subjunctively joined with Sc, yielding (Λ_x (Mx → Sx) ∧ Mc) → Sc.

To calculate the final column, with the values of the compound formula, we start by assigning all possible values to the initial elementary functions Mx and Sx – i.e. first (0, 0), then (0, 1), etc. until we reach (2, 2). Writing the values under the components Mx and Sx of the compound formula on top, we get columns 2 and 4 of the schema. The further calculation precisely follows the steps of the construction of the compound formula, taking the values from Dubislav's quasi truth-tables (in which we have merely replaced + by 0, – by 1 and * by 2):

ax	at
0	0
1	1
2	**0/1**

ax	$\bigwedge_x$ ax
0	0
1	1
2	1

ax	$\bigvee_x$ ax
0	0
1	1
2	0

ax	¬ ax
0	1
1	0
2	2

ax ∨ bx

bx / ax	0	1	2
0	0	0	0
1	0	1	2
2	0	2	**0/2**

ax ∧ bx

ax / bx	0	1	2
0	0	1	2
1	1	1	1
2	2	1	**1/2**

ax → bx

ax / bx	0	1	2
0	0	1	2
1	0	0	0
2	0	2	**0/2**

The column reached in the final step (here printed in bold letters) contains, in the third line from bottom, the ambiguity 0/1. In view of the table for “sub” (→), the desired 0 could only be obtained if, in the ∧ – column of the same line, we had a 1 (otherwise the last step would result in 0 → 1, which yields 1). But to achieve this, also the right-hand component Mc of the conjunction would also have to be 0. Whereas this does not result unambiguously from the 2 of Mx since according to the table of cases we get 0/1. This shows that even Dubislav's own example does not work without recourse to material considerations.

As our second example we take the case analysis of the expression

			(Λ_x	ax	→	Λ_y	by)	→	Λ_z	(az	→	bz)
ax	bx											
0	0	:	0	0	0	0	0	**0**	0	0	0	0
0	1	:	0	0	1	1	1	**0**	1	0	1	1
1	0	:	1	1	0	0	0	**0**	0	1	0	0
1	1	:	1	1	0	1	1	**0**	0	1	0	1
2	0	:	0	2	0	0	0	**0**	0	2	0	0
0	2	:	0	0	1	1	2	**0**	1	0	2	2
2	1	:	0	2	1	1	1	**0**	1	2	2	2
1	2	:	1	1	0	1	1	**0**	0	1	0	2
2	2	:	0	2	1	1	2	**0/0**	0/1	2	0/2	2

The schema shows that we have only a single ambiguity, appearing in the last line in the column of the z-quantifier reached in the second last step. But here the ambiguity does indeed drop out as Dubislav had expected, since the antecedent of the conditional, reached in the last step, obtains the value 1, thereby assigning the value 0 to the conditional for each of the three possible values of the succedent.

With some embarrassment I confess that in the first draft of my logic script, my example was just this formula (which is indeed a tautology). As such, after demonstrating the smooth working of Dubislav's procedure, I claimed that his work did provide a *decision procedure* for classical monadic quantificational logic. However, the fact that it yields only a sufficient criterion of validity, and not a necessary one, soon became clear through counter-examples of a type that was also employed by Reichenbach (although I had not consulted his work at that time). Let me close with a very simple example, procured by one of the tutors of my logic course at Erlangen: the generalized tertium non datur.

Λ_x	(ax	V	¬	ax)
0	0	0	1	0
0	1	0	0	1
0/1	2	0/2	2	2.

In this case, the ambiguity remaining in the last line cannot be removed by way of mutual compensation of two ambiguities. For, in the pertinent last line, before reaching the final column, only a single ambiguity is extant. Thus, it seems that Dubislav's idea of a cancelling out of ambiguities cannot be saved, not even by Reichenbach's benevolent attempt at clarification, proffered in 1947.

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