A Multiphase Model for Assessing the Overall Yield Strength of Soils Reinforced by Linear Inclusions

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Abstract Conceived as an extended homogenization procedure, a multiphase approach for ascertaining the macroscopic behavior of reinforced soil structures has been developed in the last years. This contribution is dedicated to the evaluation of the yield strength properties of soils reinforced by linear inclusions by making use of a homogenization procedure, in which the reinforced soil is regarded as a periodic composite, as a first calculation, and using the multiphase model. It appears from such a calculation that only the multiphase model is able to capture scale and boundary effects, which may play an important role in the yield design of reinforced structures. The decisive element is the introduction of a parameter characterizing the strength of the interaction between two continuous media ("phases") representing the soil and the reinforcing inclusions, respectively. A preliminary analysis suggests that such a parameter varies in direct proportion to the inverse of a scale factor.

1 Introduction

A large range of soil reinforcement techniques used to improve soil structures stiffness and strength consist in incorporating into the soil mass a distribution of unidirectional inclusions made of steel or concrete. Beyond the differences as regards the construction mode of such reinforced structures, they undeniably exhibit some common futures which can be summarized as follows:

- The reinforcing inclusions usually take the form of linear structural elements (metal or polymeric strips or bars, concrete piles, ...) incorporated into the soil mass following a regular (periodic) arrangement and one or several preferential orientations, in much the same way as for industrial fibre composite materials, although at a quite different scale.

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- The mechanical properties of the reinforcing material are considerably higher than those of the native soil: concrete or steel yield strength is 1000–10000 times greater than that of a soft clay or a sand.
- The volume fraction of the reinforcing material is quite small, remaining in most cases lower than few percents.

The strong heterogeneity of the composite reinforced soil associated with the relatively high number of the reinforcing inclusions involved in such reinforcement techniques, makes it very difficult to set up appropriate design-oriented calculation methods in which the inclusions would be treated as individual elements embedded in the soil. Indeed, a fully three dimensional analysis to take into account the cylindrical shape of the reinforcements and a locally refined mesh to capture with sufficient accuracy the complex interaction between the soil and the inclusions would be required. This would lead to oversized numerical problems and thus a time consuming calculation methods incompatible with an engineering design approach.

As an alternative approach to direct numerical simulations, the periodic homogenization technique [1, 2] appears to be a good alternative since the heterogeneous composite material is replaced by a homogeneous anisotropic medium. Another way to set up design methods for soil structures reinforced with linear inclusions consists in the application of the multiphase model, which has been developed in the last decade.

The objective of this paper is to point out the shortcomings of such a homogenization procedure and to show how a multiphase approach, perceived as an extension of the homogenization concept, is able to capture "scale" as well as "boundary effects", which may have important consequences in the yield design of reinforced soils structures.

2 Macroscopic Strength Condition of a Unidirectionally Reinforced Soil

The determination of the macroscopic strength condition of a material reinforced by one single family of parallel cylindrical inclusions could be performed by making use of the homogenization theory for periodic media implemented in the context of yield design (limit analysis). It relies upon the solution to a yield design boundary value problem defined over the reinforced soil's representative unit cell sketched in Fig. 1 [1, 2].

Denoting by s the spacing between two neighboring inclusions, and by R and t the radius and the thickness of the reinforcing inclusions, respectively, the reinforcement volume fraction is equal to the ratio between the inclusion and the unit cell cross sectional areas:

$$\eta = \frac{2\pi Rt}{s^2} \tag{1}$$

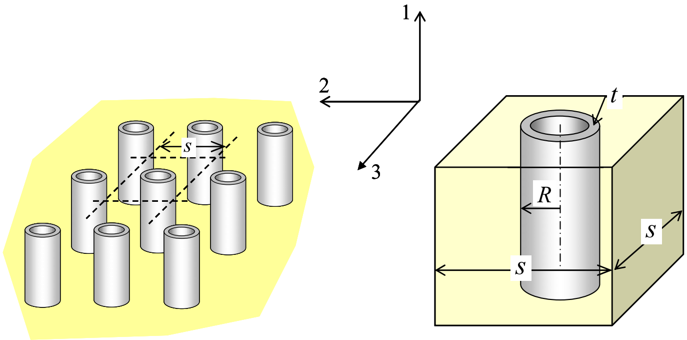


Fig. 1 Representative unit cell of a soil reinforced by tubular inclusions

As regards most types of reinforced soil structures, the volume fraction η is very small, rarely exceeding 5 %, whereas the strength of the reinforcing material is considerably higher than that of the soil. This situation can be mathematically obtained by making the volume fraction tend to zero while the product of this volume fraction by the reinforcing material's uniaxial yield strength σ_r^{Y} is kept constant:

$$\eta \to 0 \quad \text{as} \quad \eta \sigma_r^{\mathrm{Y}} = \sigma_0 = ct \tag{2}$$

where σ_0 may be interpreted as the tensile (compressive) resistance of the reinforcing inclusions per unit transverse area. Under such circumstances, it can be shown [3] that, assuming perfect bonding at the interface between the inclusion and the surrounding soil, the macroscopic strength condition of the reinforced soil simply reduces to:

$$F(\underline{\underline{\Sigma}}) \le 0 \quad \Leftrightarrow \quad \begin{cases} \underline{\underline{\Sigma}} = \underline{\underline{\sigma}}^s + \sigma \underline{e}_1 \otimes \underline{e}_1 \\ f(\underline{\underline{\sigma}}^s) \le 0, \quad |\sigma| \le \sigma_0 \end{cases} \tag{3}$$

where $f(.)$ denotes the soil's strength condition. The above simplified criterion proves also valid for plane strain-loaded multilayered materials under the same condition as (2) [4, 5]. For a purely cohesive soil (soft clay) characterized by a cohesion or undrained shear strength equal to C, the macroscopic strength condition, expressed under plane strain conditions parallel to the reinforcement direction, writes [2]:

$$F(\underline{\underline{\Sigma}}) \le 0 \quad \Leftrightarrow \quad \Sigma_{\mathrm{M}} - \Sigma_{\mathrm{m}} \le 2C^{\mathrm{hom}}(\alpha) \tag{4}$$

where Σ_{M} (resp. Σ_{m}) is the major (resp. minor) principal stress and α its orientation with respect to the reinforcement direction. The reinforced soil thus appears to be a purely cohesive anisotropic medium, with its cohesion, represented in Fig. 2(b) in the form of a polar diagram, varying from that of the native soil (C) for $\alpha = \pm 45°$ to a maximum value equal to $C + \sigma_0/2$ for $\alpha = 0°, 90°$.

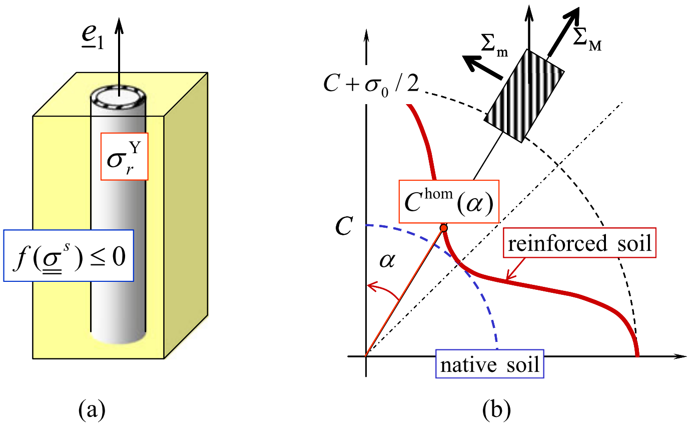


Fig. 2 (**a**) Representative unit cell of reinforced soil. (**b**) Polar diagram for a unidirectionally reinforced purely cohesive soil

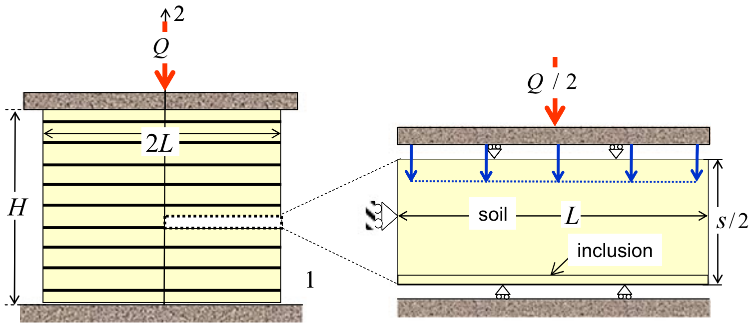


Fig. 3 Compressive strength of a purely cohesive reinforced block: initial and auxiliary problems

3 A Partial Validation of the Homogenization Approach

The problem under consideration is that of a block of height H and half-width L, subjected to a compression test in plane strain conditions in the Oxy-plane. This block has been reinforced with regularly placed horizontal inclusions (Fig. 3) and placed between two rigid planes in smooth contact with its upper and bottom sections. The upper plane is moving down and then applying a compressive loading Q to the block whereas the lower plane is fixed. The two lateral sides are stress free.

According to the homogenization procedure, the composite material is modeled as a homogeneous anisotropic purely cohesive medium for which the corresponding yield function is expressed by (4). Referring to the lower bound static approach of yield design for the above problem, a homogeneous stress field of the form:

$$\underline{\underline{\Sigma}} = \Sigma_{22}\underline{e}_2 \otimes \underline{e}_2 + \Sigma_{33}\underline{e}_3 \otimes \underline{e}_3, \quad \Sigma_{22} \leq \Sigma_{33} \leq 0 \tag{5}$$

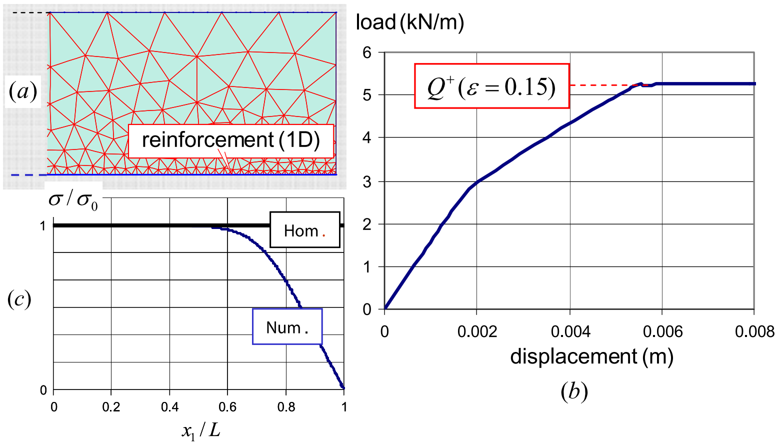


Fig. 4 Results of elastoplastic simulation for $\varepsilon = 0.15$: (**a**) finite element mesh; (**b**) computed load–displacement curve; (**c**) "at failure" stress distribution in the reinforcement

is considered, where the major principal stress is then equal to zero ($\Sigma_{11} = \Sigma_M = 0$) while the minimum (maximum compressive stress) is $\Sigma_m = \Sigma_{22}$, so that $\alpha = 0$. The strength condition (4) may thus be written:

$$\Sigma_M - \Sigma_m = 0 - \Sigma_{22} \leq 2C^{hom}(\alpha = 0°) = \sigma_0 + 2C \tag{6}$$

It follows immediately that a lower bound value to the compressive strength of the reinforced block is:

$$Q^+_{hom} \geq 4CL[1 + \sigma_0/2C] \tag{7}$$

which turns out to be the exact value upon applying the upper bound kinematic approach.

The validity of such a procedure is now assessed by comparing the obtained compressive strength (7) with a direct numerical simulation of the same problem, where for the sake of simplicity, but without any loss of generality, the reinforced soil is modeled as a multilayered material in which the reinforcements are treated as 1D beam elements, equally spaced by a distance s throughout the block, so that a "scale factor" defined by the spacing to half-width ratio, may be introduced:

$$\varepsilon = s/L \tag{8}$$

Making use of the symmetry and the periodicity conditions, it could be easily proved that the numerical simulation of the plane strain compression test could be performed by solving the boundary value problem attached to a representative "slice" of the reinforced block (Fig. 2). The corresponding limit loads Q^+ have been evaluated numerically by means of the finite element computer code PLAXIS [6]. As sketched in Fig. 4, the soil mass is discretized into 6-noded triangular elements whereas the reinforcing inclusion is modeled as a beam. An elastoplastic calculation

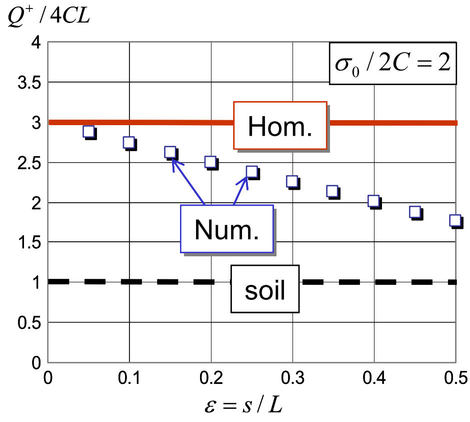


Fig. 5 Homogenization vs. f.e.m. numerical simulations

is performed, until failure, for several values of the scale factor ε ranging between 0.05 and 0.5. It is worth noting that the computational time for each elastoplastic calculation up to failure, represented by a load-displacement curve (Fig. 4(b)), does not exceed one minute on any standard PC.

Figure 4(c) displays the uniaxial stress distribution in the inclusion at failure and the corresponding distribution predicted by the homogenization theory. This comparison shows a perfect agreement of the results obtained by both methods in the central part of the reinforced structure. However, the f.e.m. and homogenization results strongly diverge when approaching the lateral sides of the block.

The variation of the non-dimensional parameter $Q^+/4CL$ as a function of the scale factor is represented in Fig. 5. The comparison between the numerical and homogenization method results clearly shows that the latter fails to capture the "scale effect" due to the variation of the scale factor. Indeed, the f.e.m numerical results converge to that predicted by the homogenization method as the scale factor tends to zero:

$$\lim_{\varepsilon \to 0} Q^+(\varepsilon) = Q^+_{\text{hom}} \tag{9}$$

thus confirming the well known convergence result of the homogenization approach, but the latter may significantly overestimate the actual value of the compressive resistance if the scale factor is not sufficiently small. Such a "scale effect" is obviously of no consequence as far as industrial composite materials are concerned (leaving aside purely local effects associated with brittle failure, such as delamination phenomena), but remains a relevant question for reinforced soils, since the scale factor is generally of the order of 0.1–0.3 for this kind of composite material.

4 Multiphase Model as an Extended Homogenization Method

An extension of the classical periodic homogenization method, namely the multiphase model, has been proposed in the last decade, allowing to assess the macro-

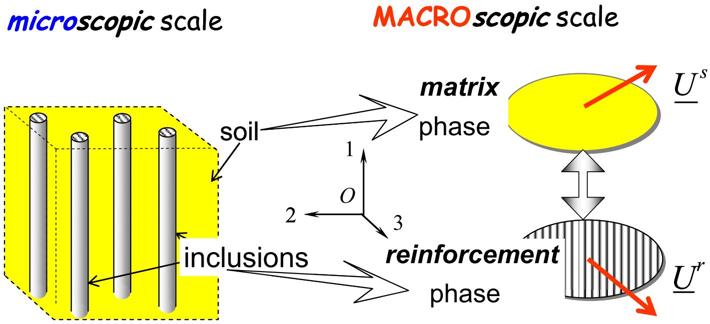


Fig. 6 Principle of the multiphase model for reinforced soils

scopic behavior of reinforced soil structures taking "scale" as well as "boundary" effects into account.

The intuitive idea of the multiphase model is to homogenize separately the soil on the one hand and the array of reinforcing inclusions, on the other hand. The thus obtained interacting continuous media, called the "matrix" and the "reinforcement" phases, are given two different kinematics, namely a velocity field $\underline{U}^s$ for the matrix, representing the soil mass, and $\underline{U}^r$ for the reinforcement phase (Fig. 6). The multiphase model could be derived from the virtual work method (see [7] for more details) and leads to the decomposition of the macroscopic total stress $\underline{\underline{\Sigma}}$ as a sum of the "partial" stresses relating to the soil and the reinforcement.

A more detailed presentation of the multiphase model may be found in [7] or [8], in the context of an elastic behavior of the different constituents. The general governing equations of the model, will now be presented in the context of the yield design theory.

The equilibrium equations are written for each phase separately, that is in the absence of any external body force, as:

$$\operatorname{div} \underline{\underline{\sigma}}^s + I \underline{e}_1 = 0 \tag{10}$$

for the matrix phase, representing the soil, and:

$$\operatorname{div}(\sigma \underline{e}_1 \otimes \underline{e}_1) - I \underline{e}_1 = 0 \tag{11}$$

for the reinforcement phase, where I denotes the interaction body force density. These equations are completed by stress conditions defined on the boundary surface of each phase independently. Referring to a yield design boundary problem for any such two-phase system, it is necessary to specify the strength condition at any point of each phase, namely:

$$f\left(\underline{\underline{\sigma}}^s\right) \leq 0 \quad \text{and} \quad |\sigma| \leq \sigma_0 \tag{12}$$

for the individual phases, along with an interaction strength condition of the form:

$$|I| \leq I_0 \tag{13}$$

In the situation of “perfect bonding”, characterized by the fact that the interaction strength parameter I_0 takes an infinite value, it is quite apparent from summing up Eqs. (10) and (11), and thus eliminating the interaction force density, that the yield design homogenization method is recovered.

The stability analysis of a block of reinforced soil, previously considered in the light of the homogenization method, is now revisited within the context of the multiphase model. The compressive strength of the reinforced block is thus defined as the maximum value of Q for which it is possible to exhibit a couple of stress fields, $\underline{\underline{\sigma}}^s$ in the matrix phase and σ in the reinforcement phase, along with an interaction force density I, satisfying both the equilibrium equations (10) and (11) along with the boundary conditions specified for each phase independently, and the respective strength conditions (12) and (13).

It is to be noted that the strength properties of the multiphase system depend on the strength properties of the different constituents: the soil’s cohesion C for the matrix and the reinforcement uniaxial strength density σ_0 for the reinforcement phase, whereas the interaction strength parameter I_0 depends on several parameters and could be determined through a numerical procedure which is presented in Sect. 5.

The kinematic approach of yield design is based on the “dualization” of the equilibrium equations of the multiphase system by making use of the virtual work principle. Denoting by $\{\underline{\hat{U}}^s, \underline{\hat{U}}^r\}$ any virtual velocity field, kinematically admissible for the boundary value problem, this principle writes:

$$W_e\left(\underline{U}^s, \underline{U}^r\right) = W_i\left(\underline{U}^s, \underline{U}^r\right) \tag{14}$$

where W_e (resp. W_i) represents the virtual work of external (resp. internal) efforts for the two-phase system. It is worth noting that the interaction body force density I exerted on the matrix phase must be considered as an external effort for the latter but, as regards the multiphase system as a whole, this volume density is an internal effort since it corresponds to an interaction between two subsystems (matrix and reinforcement phases) of the reinforced volume Ω.

In the case of a loading depending on n parameters, the virtual work of external forces writes:

$$W_e\left(\underline{U}^s, \underline{U}^r\right) = \int_{\partial\Omega} \left(\underline{T}^s.\underline{U}^s + \underline{T}^r.\underline{U}^r\right) \mathrm{d}S = \underline{Q}.\underline{\dot{q}} \tag{15}$$

where $\underline{Q}$ is the vector of the loading parameters (compressive resultant force in the above problem) and $\underline{\dot{q}}$ the associated kinematic parameters (vertical velocity of the reinforced block upper section).

On the other hand, the virtual work of internal forces is equal to the sum of the contribution of each phase and the interaction prevailing between them:

$$W_i\left(\underline{U}^s, \underline{U}^r\right) = \int_{\Omega} \left(\underline{\underline{\sigma}}^s : \underline{\underline{d}}^s + \sigma d + I\dot{\Delta}\right) \mathrm{d}\Omega \tag{16}$$

where $\underline{\underline{d}}^s$, d and $\dot{\Delta}$ are the strain rate variables, defined as:

$$\underline{\underline{d}}^s = \frac{1}{2}\left(\underline{\underline{\mathrm{grad}}}\,\underline{U}^s + {}^T\underline{\underline{\mathrm{grad}}}\,\underline{U}^s\right), \quad d = \frac{\partial \hat{U}_1^r}{\partial x_1}, \quad \dot{\Delta} = \hat{U}_1^r - \hat{U}_1^s \tag{17}$$

The *maximum resisting work* defined as the maximum of the work of internal efforts, satisfying the strength conditions (12) and (13), in the virtual velocity field $\{\hat{\underline{U}}^s, \hat{\underline{U}}^r\}$:

$$W_{mr}\left(\underline{U}^s, \underline{U}^r\right) = \int_{\Omega} \left(\pi^s\left(\underline{\underline{d}}^s\right) + \pi^r(d) + \pi^I(\dot{\Delta})\right) \mathrm{d}\Omega \tag{18}$$

where π^m, π^r and π^I denote the support functions of the matrix, the reinforcement phase and the interaction, respectively:

$$\begin{aligned} &\pi^s(\underline{\underline{d}}^s) = \sup\left\{\underline{\underline{\sigma}}^s : \underline{\underline{d}}^s;\ f\left(\underline{\underline{\sigma}}^s\right) \leq 0\right\} \\ &\pi^r(d) = \sup\left\{\sigma d;\ |\sigma| \leq \sigma_0\right\} \\ &\pi^I(\dot{\Delta}) = \sup\left\{I\dot{\Delta};\ |I| \leq I_0\right\} \end{aligned} \tag{19}$$

Combining the virtual work principle (14) and (15) with the definition of the maximum resisting work given by Eqs. (18) and (19), the necessary condition of stability may be written:

$$\forall\left\{\underline{U}^s, \underline{U}^r\right\}\text{K.A.}, \qquad \underline{Q}.\underline{\dot{q}} \leq W_{mr}\left(\underline{U}^s, \underline{U}^r\right) \tag{20}$$

4.1 Lower Bound Static Approach

The lower bound static approach is implemented by making use of the following stress field in the reinforced block modeled as a two-phase system:

$$\sigma = \sigma(x_1) \quad \text{with} \quad \sigma(x_1 = \pm L) = 0 \tag{21}$$

for the reinforcement phase,

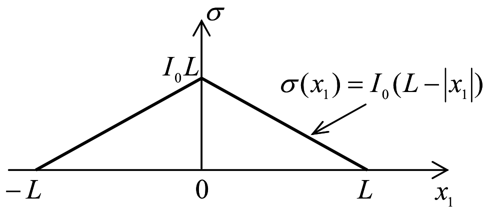
$$\begin{cases} \sigma_{11}^s(x_1) = -\sigma(x_1) \\ \sigma_{22}^s(x_1) = \sigma_{33}^s(x_1) = -2C - \sigma(x_1) \\ \sigma_{ij}^s = 0 \quad \text{if } i \neq j \end{cases} \tag{22}$$

for the matrix phase, and

$$I = \frac{\mathrm{d}\sigma}{\mathrm{d}x_1} = \sigma'(x_1) \tag{23}$$

for the interaction.

Fig. 7 Stress distribution in the reinforcement phase ($\chi \leq 1$)



It can be easily shown that this stress field complies with the equilibrium equations (10) and (11), along with the strength conditions (12) and (13). The corresponding compressive force in equilibrium with such a stress field is given by:

$$Q = -\int_{-L}^{L} \sigma_{22}^{s}(x_1)\,\mathrm{d}x_1 = 4CL + \int_{-L}^{L} \sigma(x_1)\,dx_1 \tag{24}$$

The optimal (i.e. maximum) value of this compressive force depends on the relative importance of the reinforcement phase uniaxial strength density with respect to the interaction strength parameter. Introducing the non dimensional parameter:

$$\chi = \frac{I_0 L}{\sigma_0} \tag{25}$$

the following two different cases, depending on the value of χ, are considered:

- $\chi \leq 1$ ($I_0 L \leq \sigma_0$). The interaction force density I is chosen so as to be equal to the corresponding strength I_0. Combining Eqs. (21), (22) and (23), it comes out that the internal efforts in the multiphase system are of the following form (Fig. 7):

$$\begin{cases} \sigma(x_1) = -\sigma_{11}^{s}(x_1) = 2C - \sigma_{22}^{s}(x_1) = I_0(L - |x_1|) \\ |I| = I_0 \end{cases} \tag{26}$$

  which comply with the equilibrium and the strength conditions. It follows that:

$$Q_{\text{mult.}}^{+} \geq 4CL + I_0 L^2 \tag{27}$$

- $\chi \geq 1$ ($I_0 L \geq \sigma_0$). The generalized stress field defined on the reinforced block given by (Fig. 8):

$$\sigma(x_1) = -\sigma_{11}^{s}(x_1) = 2C - \sigma_{22}^{s}(x_1) = \mathrm{Min}\left\{I_0(L - x_1); \sigma_0\right\} \tag{28}$$

  which complies with the above equilibrium and strength requirements, leading to the following lower bound for the reinforced block compressive resistance:

$$Q_{\text{mult.}}^{+} \geq 4CL + 2\sigma_0 L\left[1 - \frac{\sigma_0}{2 I_0 L}\right] \tag{29}$$

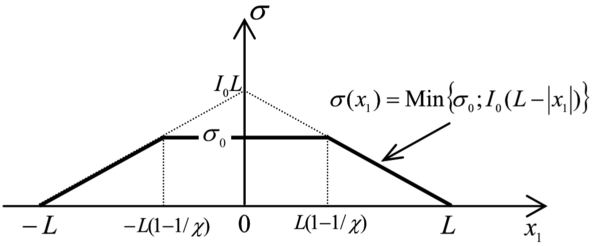


Fig. 8 Stress distribution in the reinforcement phase ($\chi \geq 1$)

4.2 Upper Bound Kinematic Approach

The upper bound kinematic approach is implemented by considering the following virtual velocity field defined for each phase separately:

$$\underline{\hat{U}}^s = \frac{U}{H}(x_1\underline{e}_1 - x_2\underline{e}_2) \tag{30}$$

for the matrix phase, and

$$\underline{\hat{U}}^r = \begin{cases} -\frac{U}{H}x_2\underline{e}_2 & \text{if } \chi \leq 1 \\ \begin{cases} \frac{U}{H}(x_1\underline{e}_1 - x_2\underline{e}_2) & |x_1| \leq L(1-\chi^{-1}) \\ \frac{U}{H}(L(1-\chi^{-1})\underline{e}_1 - x_2\underline{e}_2) & |x_1| \geq L(1-\chi^{-1}) \end{cases} & \text{if } \chi \geq 1 \end{cases} \tag{31}$$

for the reinforcement phase.

The calculation of the maximum resisting work leads to the following expression:

$$W_{mr}\left(\underline{\hat{U}}^s, \underline{\hat{U}}^r\right) = 4CLU + \begin{cases} I_0L^2U & \text{if } \sigma_0 \geq I_0L \\ 2\sigma_0LU[1 - \frac{\sigma_0}{2I_0L}] & \text{if } \sigma_0 \leq I_0L \end{cases} \tag{32}$$

On the other hand, the work of the external forces in the considered mechanism is equal to the product of the applied effort Q by the corresponding velocity U of the upper section of the block. The upper bound kinematic approach of yield design finally leads to the following upper bound value for

$$Q^+_{\text{mult.}} \leq 4CL + \begin{cases} I_0L^2 & \text{if } \sigma_0 \geq I_0L \\ 2\sigma_0L[1 - \frac{\sigma_0}{2I_0L}] & \text{if } \sigma_0 \leq I_0L \end{cases} \tag{33}$$

hence the exact value of the compressive resistance predicted by the multiphase model.

It can be observed that in the situation of perfect bonding the above expressions reduce to that derived from the homogenization approach:

$$Q^+_{\text{mult.}}(\chi \to \infty) = Q^+_{\text{hom}} = 4CL[1 + \sigma_0/2C] \tag{34}$$

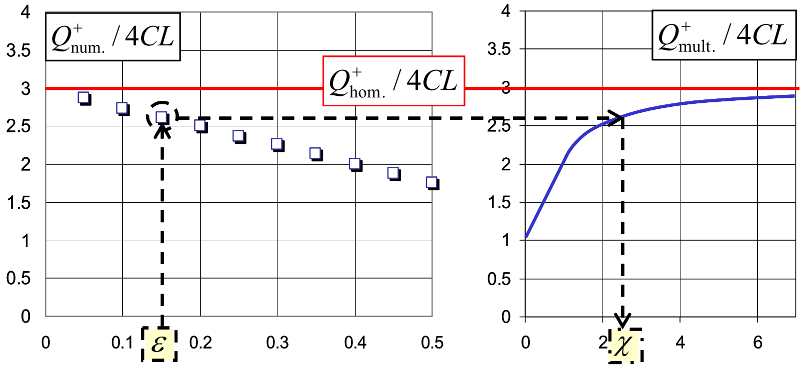


Fig. 9 Identification procedure for the interaction strength parameters ($\sigma_0 = 4C$)

5 Identification of the Interaction Strength Parameter

The curve sketched on the right-hand side of Fig. 9 represents the results of the multiphase approach expressed in terms of variation of the non dimensional compressive resistance $Q^+/4CL$ as a function of the parameter χ, for $\sigma_0 = 4C$. On the left-hand side of the same figure, are reported the results of the f.e.m.-based numerical simulations performed by using Plaxis, expressed in terms of the variation of $Q^+/4CL$ as a function of the scale factor ε.

Starting from these representations, the relationship between the scale factor ε and the parameter χ can be established. The first step of this procedure consists in representing the evolution of $\chi(\varepsilon)$ which could be then approximated by an analytical expression (Fig. 10). It appears that the obtained series of points is best fitted by an analytical curve which obeys the following approximate equation:

$$\chi \cong 0.4\varepsilon^{-1} \tag{35}$$

which means that χ, and hence the interaction strength parameter I_0, is inversely proportional to the scale factor. It is important to notice that the coefficient of proportionality (equal to 0.4 in the present case), and then the interaction strength parameter I_0, can therefore be determined from one single numerical simulation. A more thorough and detailed analysis (which is beyond the scope of the present paper) would certainly show that this coefficient of proportionality depends on the soil's cohesion, since no failure is considered at the soil-inclusion interface at the microscopic scale.

The combination of the relationships (33) and (35) finally leads to the following expression of the compressive strength as a function of the scale factor:

$$\frac{Q^+_{\text{mult.}}}{4CL} = \frac{Q^+_{\text{num.}}}{4CL} = \begin{cases} 3 - 2.5\varepsilon & \text{if } \varepsilon \le 0.4 \\ 1 + 0.4/\varepsilon & \text{if } \varepsilon \ge 0.4 \end{cases} \tag{36}$$

This prediction, obtained from the application of the multiphase approach in the field of yield design, tends to the results of the homogenization approach for very

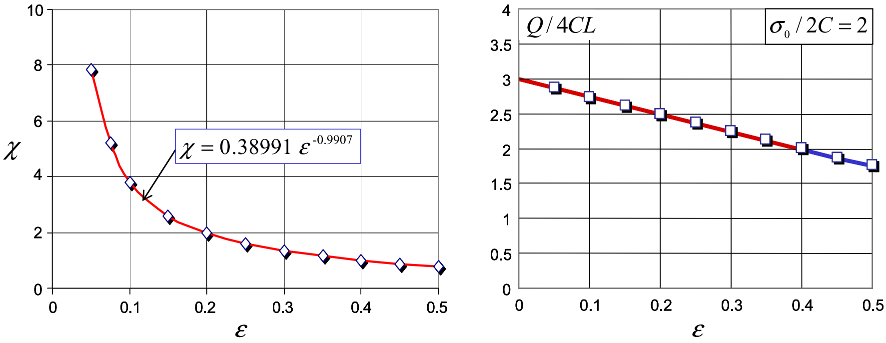


Fig. 10 Identification of the interaction strength parameter and comparison between the numerical results and the multiphase model-based predictions

small values of ε and appears to linearly decrease down to a value of the scale factor equal to 0.4 (Fig. 10).

6 Concluding Remarks

It has been shown in this contribution that the multiphase model, developed in the context of yield design, is not subject to the limitations of the classical periodic homogenization method, since it allows to capture scale and boundary effects, which may play a decisive role in the reinforced-soil structures design. This is achieved through the introduction of a matrix-reinforcement interaction strength parameter, accounting, at the macroscopic scale, for a possible slippage between the reinforcing inclusion and the surrounding ground. Such a parameter could be identified, as shown in Sect. 5, through one f.e.m.-based elastoplastic calculation performed on a unit cell. The homogenization results could be recovered as a particular case of the multiphase approach, when the interaction strength parameter I_0 tends to infinity (perfect bonding assumption), which corresponds to a vanishing scale factor ε.

It is worth noting that a limited interaction strength between phases, at the *macroscopic* scale, should be taken into account even in the case of *perfect bonding at the microscopic scale*, that is unlimited strength, at the soil-inclusion interface, thus assuming that the soil is perfectly adherent to the reinforcement.

From an engineering design and optimization viewpoint, the multiphase model is a robust tool, combining the decisive advantages of the classical homogenization method with its ability to capture scale and boundary effects, in order to analyse the stability of reinforced soil structures, as illustrated in [9] for reinforced earth retaining walls.

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