Chapter 11
Relativistic Energy Approach to Cooperative Electron-γ-Nuclear Processes: NEET Effect

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Abstract A consistent relativistic energy approach to the calculation of probabilities of cooperative electron-gamma-nuclear processes is developed. The nuclear excitation by electron transition (NEET) effect is studied. The NEET process probability and cross section are determined within the S-matrix Gell-Mann and Low formalism (energy approach) combined with the relativistic many-body perturbation theory (PT). Summary of the experimental and theoretical works on the NEET effect is presented. The calculation results of the NEET probabilities for the $^{189}_{76}$Os, $^{193}_{77}$Ir, and $^{197}_{79}$Au atoms are presented and compared with available experimental and alternative theoretical data. The theoretical and experimental study of the cooperative electron-gamma-nuclear process such as the NEET effect is expected to allow the determination of nuclear transition energies and the study of atomic vacancy effects on nuclear lifetime and population mechanisms of excited nuclear levels.

11.1 Introduction

Methods for influencing the radioactive decay rate have been sought from early years of nuclear physics. Nuclear transmutation (i.e., change in the nuclear charge) induced by nuclear reactions is often accompanied by a redistribution of the electrons and positrons around the final transmuted nucleus. Electrons and positrons (other particles) originally in the ground state of the target atom can be excited reversibly either to the bound or continuum states. The rapid progress in laser technology even opens prospects for nuclear quantum optics via direct laser-nucleus coupling [1–5]. A principal possibility of storage of the significant quantities of the

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metastable nuclei in the nuclear technology processes and their concentration by chemical and laser methods leads to problem of governing their decay velocity.

The elementary cooperative e-,α-,β-,γ-nuclear processes in atoms and molecules were considered in the pioneering papers by Migdal (1941), Levinger (1953), Schwartz (1953), Carlson et al. (1968), Kaplan et al. (1973–1975), Goldanskii-Letokhov-Ivanov (1973–1981), Freedman (1974), Law-Campbell (1975), Martin-Cohen (1975), Isozumi et al. (1977), Mukouama et al. (1978), Batkin-Smirnov (1980), Law-Suzuki (1982), Intemann (1983), and Wauters-Vaeck et al. (1997) [5–17]. Naturally, in this context, the known Mössbauer, Szilard-Chalmers, and other cooperative effects should be mentioned [7].

The elementary cooperative electron-γ-nuclear processes were considered in the papers by Levinger (1953), Hansen (1974), Watson (1975), Law (1977), Anholt-Amundsen (1982), and Mukoyama-Ito et al. (1988) [6–13]. The cooperative "shake-up" electron-γ-nuclear processes in atoms and molecules are qualitatively in the nonrelativistic approximation considered by Goldanskii et al. and Kaplan et al. in Refs. [1, 5, 9]. In Ref. [16], a consistent relativistic energy approach combined with the quantum-electrodynamics (QED) perturbation theory (PT) has been developed and applied to calculation of the electron-γ-transition spectra of nucleus in the neutral atoms and multicharged ions. The intensities of satellites are defined in the relativistic version of the energy approach (S-matrix formalism). The results of the relativistic calculation for the electron-nuclear γ-transition spectra (set of electron satellites) of the nucleus in a number of the neutral atoms and multicharged ions have been presented. The possible experiments for observation of the cooperative "shake-up" effects in the thermalized plasma of the Ne- and O-like ions are discussed. In Ref. [16], it has been also presented a consistent quantum approach to calculation of the electron-nuclear γ-transition spectra (set of vibration-rotational satellites in molecule) of nucleus in the diatomic and multiatomic molecules, which generalizes the well-known Letokhov-Minogin model [2]. Estimates are made for vibration-rotation nuclear transition probabilities in a case of the emission and absorption spectrum of nucleus ^{127}I ($E_\gamma{}^{(0)} = 203$ keV) linked with molecule H^{127}I and the nucleus ^{191}Ir ($E_\gamma{}^{(0)} = 82$ keV) linked with molecular system IrO_4 and spectrum of nucleus ^{188}Os ($E_\gamma{}^{(0)} = 155$ keV) in molecule of OsO_4. In Ref. [17], the cooperative electron-β-nuclear processes in atomic systems (e-β-nuclear spectroscopy), including the processes of excitation, ionization, and electronic rearrangement induced by nuclear reactions and β-decay, are discussed. The relativistic many-body PT with the optimized Dirac-Kohn-Sham zeroth approximation and taking into account the nuclear, radiation, and exchange-correlation corrections is used to calculate the β-decay parameters for a number of allowed (super allowed) transitions (^{33}P-^{33}S, ^{241}Pu-^{241}Am, etc.) and study the chemical bond effect on β-decay parameters. The half-life periods for β-transition in some systems are estimated by taking into account the bound β-decay channel correction and some other accompanying cooperative effects.

In Ref. [18], we have presented a generalized energy approach in the relativistic theory of discharge of a metastable nucleus with emission of γ quantum and further muon conversion, which initiates this discharge. The numerical calculation

of the corresponding cooperative process probabilities is firstly carried out for the scandium nucleus (A = 49, N = 21) with using the Dirac-Woods-Saxon model. It has been noted that the theoretical and experimental study of the cooperative electron-muon-γ-nuclear interaction effects opens prospects for nuclear quantum optics, probing the structural features of a nucleus and muon spectroscopy in atomic and molecular physics.

This chapter goes on our work on studying the cooperative electron-gamma-nuclear processes [16–18]. The important example of the cooperative electron-gamma-nuclear process is so-called NEET (nuclear excitation by electron transition) effect [1, 19–23]. Naturally, the similar NEEC (nuclear excitation by electron capture) process should be reminded too. In both NEEC and NEET, which are at the borderline between atomic and nuclear physics, electronic orbital energy is converted directly into nuclear energy. These effects offer therefore the possibility to explore the spectral properties of heavy nuclei through the typical atomic physics experiments. In this chapter, a new, consistent relativistic energy approach to calculation of probabilities of the NEET is presented. In our approach, the NEET process probability and cross section are determined within the S-matrix Gell-Mann and Low formalism (energy approach) combined with the relativistic many-body perturbation theory (PT) [24–31]. Further, a summary of the experimental and theoretical works on the NEET effect is presented. The calculation results of the NEET probabilities for the $^{189}_{76}$Os, $^{193}_{77}$Ir, and $^{197}_{79}$Au atoms within different theoretical models are presented and compared with available experimental data.

11.2 Review of Theoretical and Experimental Work on the Process of Nuclear Excitation by Electron Transition

In fact, the NEET is a fundamental but rare mode of decay of an excited atomic state in which the energy of atomic excitation is transferred to the nucleus via a virtual photon. This process is naturally possible if within the electron shell there exists an electronic transition close in energy and coinciding in type with nuclear one. In fact, the resonance condition between the energy of nuclear transition ω_N and the energy of the atomic transition ω_A should be fulfilled. Obviously, the NEET process corresponds to time-reversed bound-state internal conversion. Correspondingly, the NEEC process is the time-reversed process of internal conversion. Here, a free electron is captured into a bound atomic shell with the simultaneous excitation of the nucleus.

Let us remind that firstly the NEET and NEEC effects have been postulated in 1973 by Goldanskii-Letokhov-Namiot and Morita [1, 19]. Unlike the NEEC effect, the NEET process has been observed experimentally in $^{197}_{79}$Au by Kishimoto et al. (Institute of Material Structure Science, KEK, and Japan Synchrotron Radiation Research Centre, Japan) and in $^{189}_{76}$Os by Ahmad et al. (Argonne National Laboratory, USA) [21, 22]. In Table 11.1, we present a summary of the experimental works on NEET in $^{189}_{76}$Os.

Table 11.1 Experimental data on the NEET probabilities P_{NEET} for the isotope of $^{189}_{76}$Os

	Year	Experimental techniques	P_{NEET}
Otozai et al.	1973	e^- Bombardment 75–85 keV	$1{\cdot}10^{-6}$
Otozai et al.	1978	e^- Bombardment 72–100 keV	$(1.7 \pm 0.2){\cdot}10^{-7}$
Saito et al.	1981	200-keV bremsstrahlung	$(4.3 \pm 0.2){\cdot}10^{-8}$
Shinohara et al.	1987	"White" synchrotron radiation	$(5.7 \pm 1.7){\cdot}10^{-9}$
Lakosi et al.	1995	300-keV bremsstrahlung	$(2.0 \pm 1.4){\cdot}10^{-8}$
Ahmad et al.	2000	Monochromatic 100-keV X-rays	$<9{\cdot}10^{-10}$

It should be noted that each of the experimental techniques has certain inherent difficulties. Analysis of this problem has been presented by Ahmad et al. [22]. It explains quite large difference between the results of different experiments. Saying briefly, the cited difficulties are reduced to the problem of revealing a NEET signal among the surrounding other effects. Really, use of an electron beam can cause direct Coulomb excitation of a nucleus. In this case, it is hardly possible to distinguish this component from that due to the NEET process. Use of a broad continuous spectral distribution of synchrotron or bremsstrahlung X-rays results in contribution from a direct nuclear photoabsorption into the nuclear state or into a range of nuclear levels that can feed that state or the lower-lying metastable state (look for more details in Refs. [22]). The theoretical models for the NEET effect were developed in Refs. [1, 19–23]. The first estimates of P_{NEET} for various atomic/nuclear systems have been received beginning Goldanskii-Letokhov-Namiot and Morita [1, 19]. Many of the early estimates involved the use of simplifying approximations that led to results at considerable variance. More recently, Tkalya [23] has proposed a model for description of the NEET process near the K-shell ionization threshold of an atom. The QED PT with empirical estimates of the nuclear and electron matrix elements and the Dirac-Fock code by Band and Fomichev (taking into account the finite nuclear size) were used. New theoretical approach by Ahmad et al. [22] is based on using the time-dependent amplitude coupled equations. These authors calculated electron wave functions using the GRASP code and tabulated values of the nuclear transition matrix elements. In Table 11.2, we present a summary of the theoretical work on NEET in $^{189}_{76}$Os (data till 2000 from Refs. [22]). Therefore, the theoretical models involved the use of different consistency level approximations led to results at quite considerable variance.

It is obvious that more sophisticated relativistic many-body methods should be used for correct treating the NEET effect. Really, the nuclear wave functions have the many-body character (usually, the nuclear matrix elements are parameterized according to the empirical data). The correct treating of the electron subsystem processes requires an account of the relativistic, exchange-correlation, and nuclear effects. Really, the nuclear excitation occurs by electron transition from the M shell to the K shell. So, there is the electron-hole interaction, and it is of a great importance a correct account for the many-body correlation effects, including the intershell correlations, the post-act interaction of removing electron and hole,

Table 11.2 Theoretical estimates of probabilities P_{NEET} for the isotope of $^{189}_{76}$Os

Model	Year	NEET (M1 transition)	NEET (E2 transition)
Morita	1973	–	$1.0{\cdot}10^{-6}$
Okamoto	1977	–	$1.5{\cdot}10^{-7}$
Pisk et al.	1989	$2.3{\cdot}10^{-7}$	$1.8{\cdot}10^{-8}$
Bondarkov et al.	1991	$1.1{\cdot}10^{-7}$	$2.5{\cdot}10^{-9}$
Ljubicic et al.	1991	$1.06{\cdot}10^{-7}$	$1.25{\cdot}10^{-7}$
Tkalya	1992	$1.1{\cdot}10^{-10}$	$7{\cdot}10^{-13}$
Ho et al.	1993	$2.1{\cdot}10^{-9}$	–
Ahmad et al.	2000	$1.3{\cdot}10^{-10}$	$3.8{\cdot}10^{-13}$
Tkalya	2007	$1.2{\cdot}10^{-10}$	–
Present work	2009	$1.9{\cdot}10^{-10}$	$8.5{\cdot}10^{-13}$

and possibly the continuum pressure [23, 29, 30]. In any case, the theoretical calculations for NEET occurring in scattering measurements are particularly useful, especially in finding candidate isotopes and transitions suitable for experimental observation.

11.3 Relativistic Energy Approach to the Process of Nuclear Excitation by Electron Transition

The relativistic energy approach is based on the S-matrix Gell-Mann and Low formalism combined with the relativistic many-body PT [24–31]. Let us remind that in atomic theory, a convenient field procedure is known for calculating the energy shifts ΔE of the degenerate states. Secular matrix M diagonalization is used. In constructing M, the Gell-Mann and Low adiabatic formula for ΔE is used. A similar approach, using this formula with the electrodynamical scattering matrix, is applicable in the relativistic theory. In contrast to the nonrelativistic case, the secular matrix elements are already complex in the PT second order (first order of the interelectron interaction). Their imaginary parts relate to radiation decay (transition) probability. The total energy shift of the state is usually presented as follows:

$$\Delta E = Re\Delta E + \mathrm{i}\ \mathrm{Im}\Delta E, \tag{11.1a}$$

$$\mathrm{Im}\ \Delta E = -\frac{\Gamma}{2}, \tag{11.1b}$$

where Γ is interpreted as the level width and the decay possibility $P = \Gamma$. The whole calculation of energies and decay probabilities of a nondegenerate excited state is reduced to calculation and diagonalization of the complex matrix M. To start with the Gell-Mann and Low formula, it is necessary to choose the PT zero-order approximation. Usually, the one-electron Hamiltonian is used, with a central

potential that can be treated as a bare potential in the formally exact PT [25, 29]. The total probability of radiative decay (excitation, de-excitation) is connected with imaginary part of ΔE of the system "atom plus field." It corresponds to the retarding effect in an interaction and self-interaction for radiative processes and can be calculated within the relativistic PT [24, 32]. The corresponding corrections of the PT for Im ΔE can be represented as a sum on the virtual states. In the lowest PT, the separated terms of these sums correspond to the additive contributions of different physical channels into the total decay probability. Naturally, the channel's interference effects will appear in the next PT orders.

The fundamental parameter of the cooperative NEET process is a probability P_{NEET} (cross section) of the nuclear excitation by electron transition. In fact, it can be defined as the probability that the decay of the initial excited atomic state will result to the excitation of and subsequent decay from the corresponding nuclear state. Within the energy approach, the decay probability is connected with an imaginary part of energy shift for the system (nuclear subsystem plus electron subsystem) excited state. An imaginary part of the excited state $_I$ energy shift in the lowest PT order can be in general form written as [18, 26]

$$\begin{aligned}\mathrm{Im}\Delta E &= e^2 \mathrm{Im} i \cdot \lim_{\gamma\to 0} \iint \mathrm{d}^4 x_1 \mathrm{d}^4 x_2 e^{\gamma(t_1+t_2)}\cdot \\ \cdot\{D(r_{\mathrm{N}1t1}, r_{\mathrm{N}2t2}) &< \Psi_I|(\hat{J}_{\mathrm{N}}(x_1)\hat{J}_{\mathrm{N}}(x_2))|\Psi_I> \\ +D(r_{\mathrm{e}1t1}, r_{\mathrm{e}2t2}) &< \Psi_I|(\hat{j}_{\mathrm{e}}(x_1)\hat{j}_{\mathrm{e}}(x_2))|\Psi_I>\}\end{aligned} \tag{11.2}$$

Here, $D(r_1t_1,r_2t_2)$ is the photon propagator; $\hat{J}_{\mathrm{N}}$ and $\hat{j}_{\mathrm{e}}$ are the four-dimensional components of a current operator for the nuclear and electron (hole) subsystems; $X=(r_{\mathrm{n}},\ r_{\mathrm{e}},\ t)$ is the four-dimensional space-time coordinate of the particles, respectively; and γ is an adiabatic parameter.

One should use the exact electrodynamical expression for the photon propagator (the Lorenz gauge):

$$D(r_1t_1, r_2t_2) = -\frac{1}{8\pi^2 r_{12}} \int\limits_{-\infty}^{\infty} \mathrm{d}\omega \exp(i\omega t_{12} + i\,|\omega|\, r_{12}). \tag{11.3}$$

The nuclear current can be written as follows:

$$J^{\mathrm{P}}(R,t) = \psi^{+}_{\mathrm{N}*}\hat{J}^{\mathrm{P}}\psi_{\mathrm{N}}, \tag{11.4}$$

where $\hat{J}^{\mathrm{P}}$ is the operator of an nuclear electromagnetic transition and ψ_{N} is a nuclear wave function. The current operator for electron is

$$\bar{j}^{\mu}_{\mathrm{e}} = \bar{\hat{\psi}}_{\mathrm{e}}\gamma^{\mu}\hat{\psi}_{\mathrm{e}}, \tag{11.5}$$

where γ^{μ} are the Dirac matrices. The Hamiltonian of the interaction of the electronic hole current j^{μ}_{fi} and the nuclear current $J^{v}_{fi}(R)$ is written as

$$H_{\text{int}} = \int \mathrm{d}^3 r \; \mathrm{d}^3 R \; j^{\mu}_{fi} \; D_{\mu v} \; (\omega_{\text{N}}, \; r - R) \; J^{v}_{fi}(R). \tag{11.6}$$

Below, we are limited by the lowest order of the QED PT. The energy shift can be further represented as the PT set. After integration transformations, the final expression for the imaginary part of energy shift can be represented as a sum of the corresponding nuclear-electron (hole) contributions:

$$\text{Im}\Delta E = \text{Im}E_{\text{e}} + \text{Im}E_{\text{N}}, \tag{11.7}$$

$$\text{Im}E_a = -\frac{z_a^2}{4\pi} \sum_{\text{F}} \int \int \mathrm{d}r_{\text{e}1} \mathrm{d}r_{\text{e}2} \int \int \mathrm{d}r_{\text{N}1} \mathrm{d}r_{\text{N}2} \cdot \tag{11.8}$$

$$\Psi^*_{\text{I}}(1)\Psi^*_{\text{F}}(2)\hat{T}_a(1,2)\Psi_{\text{F}}(1)\Psi_{\text{I}}(2), \tag{11.9}$$

$$\hat{T}_a(1,2) = \frac{\sin(\omega_{\text{IF}} r_{a12})}{r_{a12}} \tag{11.10}$$

Here, as usual, $r_{a12} = |r_{a1} - r_{a2}|$, ω_{IF} is the energy of transition between the initial I and final F states; the sum on F means the summation on the final states of a system, i.e., the total level width is represented as sum of the partial contributions, connected with radiative decay into the concrete final states of a system. These contributions are proportional to the probabilities of the corresponding transitions. Naturally, the form of operator in (10) is determined by a gauge of the photon propagator (look discussion in Ref. [26]). In the zeroth approximation, the dependence Ψ_{F}, Ψ_{I} on the nuclear and electron coordinates (R_{N}, $R_{\text{e(h)}}$) is factorized ($\sim \Phi_{\text{e}}\Phi_{\text{N}}$). Therefore, the combined electron (hole)-nuclear one-photon transitions occur as each of the operators T_{N} and T_{e} in (10) contains the combination of the nuclear and electron variables. After factorization and some transformations, the expression (10) can be presented in the following form:

$$\text{Im}E_a = -\frac{z_a^2}{4\pi} \sum_{\text{F}_\text{e}\text{F}_\text{N}} \int \int \mathrm{d}R_{\text{N}1} \mathrm{d}R_{\text{N}2} \int \int \mathrm{d}R_{\text{e}1} \mathrm{d}R_{\text{e}2} \cdot \Phi^*_{\text{Ie}}(R_{\text{e}1}) \Phi^*_{\text{IN}}(R_{\text{N}1}) \Phi^*_{\text{Fe}}(R_{\text{e}2}) \cdot$$

$$\cdot \, \Phi^*_{\text{FN}}(R_{\text{N}2}) \frac{\sin \omega_{\text{IF}} R_{a\,12}}{R_{a\,12}} \Phi_{\text{Fe}}(R_{\text{e}1}) \Phi_{\text{FN}}(R_{\text{N}1}) \Phi_{\text{Ie}}(R_{\text{e}2}) \Phi_{\text{IN}}(R_{\text{N}2}). \tag{11.11}$$

The expansion of the operator $\sin(\omega_{\text{IF}} R_{a12}) / R_{a12}$ on the spherical harmonics generates the decay probability multipole expansion. It can be written in the following known form:

$$\frac{\sin|\omega|\,R_{12}}{R_{a12}}=\frac{\pi}{2\sqrt{R_1R_2}}\sum_{\lambda=0}^{\infty}(\lambda)\,J_{\lambda+\frac{1}{2}}\,(|\omega|\,R_{a1})\,J_{\lambda+\frac{1}{2}}\,(|\omega|\,R_{a2})\,P_{\lambda}\,(\cos R_{a1}R_{a2}),\tag{11.12}$$

where J is the Bessel function of the first kind and $(\lambda)=2\lambda+1$. In fact, this expansion coincides with the known power expansion; naturally, the strict decreasing contribution on multipolarity corresponds to them. In our problem, the power expansion parameters are the combinations $\omega_{\mathrm{IF}}^{a}R_{\mathrm{e}}$ and $\omega_{\mathrm{IF}}^{\mathrm{N}}R_{\mathrm{N}}$. Further, the effects of purely nuclear transition, purely electron (hole) transition, and combined electron-nuclear transition in (11) can be distinguished. The corresponding technique of work with these expansions is well developed [24–26] and often used in our precious chapters (look [16–18, 27–29]). Finally, the NEET probability P_{NEET} is connected with the imaginary part of energy of the excited nuclear-electron state. It can be shown that P_{NEET} can be presented in the following form [23]:

$$P_{\mathrm{NEET}}=\left(1+\frac{\Gamma_{\mathrm{i}}}{\Gamma_{\mathrm{f}}}\right)\frac{M_{\mathrm{int}}^{2}}{(\omega_{\mathrm{N}}-\omega_{\mathrm{A}})^{2}+(\Gamma_{\mathrm{i}}+\Gamma_{\mathrm{f}})^{2}/4}.\tag{11.13}$$

Here, as usual, $_{\mathrm{i,f}}$ are the widths of the initial and final electron states and M^2 is averaged over initial states and summed over the final states the square modulus of the Hamiltonian of the electron hole current-nuclear current interaction. It can be written ($\mathrm{M_I}$-K transition) as follows (see details in Ref. [23]):

$$M_{\mathrm{int}}^{2}=\frac{1}{2j_{\mathrm{hK}}+1}\frac{1}{2J_{\mathrm{N}}+1}\sum_{mhK}\sum_{m_eh\,M1}\sum_{mN,mN*}|H_{\mathrm{ion}}|^{2}\tag{11.14}$$

or

$$M_{\mathrm{int}}^{2}=4\pi e^{2}\omega_{\mathrm{N}}^{2(\lambda+1)}\frac{\left(j_{\mathrm{i}}\frac{1}{2}\lambda O|j_{\mathrm{f}}\frac{1}{2}\right)^{2}}{[(2\lambda+1)!!]^{2}}\left|R_{\lambda}^{E/M}(\omega_{\mathrm{N}})\right|^{2}B(E/M\,\lambda;J_{\mathrm{i}}\to J_{\mathrm{f}}).\tag{11.15}$$

Here $B[E/(M)\lambda;J_{\mathrm{i}}{-}J_{\mathrm{f}}]$ is the reduced nuclear probability, $\left|R_{\lambda}^{E/M}(\omega_N)\right|$ are the atomic radial matrix elements of electric (magnetic) [E/M] multipolarity λ; $j_{i,f}$ and $J_{\mathrm{i,f}}$ are the angular momenta of the electronic and nuclear states correspondingly. The atomic radial matrix elements $\left|R_{\lambda}^{M}(\omega_N)\right|$ of electric (magnetic) [E/M] multipolarity λ are expressed by means of the integral:

$$\int_{0}^{\infty}\mathrm{d}r\,r^{2}Z_{\lambda}{}^{(1)}(\omega r)[g_i(r)f_f(r)+f_i(r)g_f(r)],\tag{11.16}$$

where, as usual, $f(r)$ and $g(r)$ are the large and small components of the Dirac electronic wave functions; Z function is usually defined as follows:

$$Z_{\lambda}^{(1)} = \left[\frac{2}{|\omega|\,\alpha Z}\right]^{\lambda+\frac{1}{2}} \frac{J_{\lambda+\frac{1}{2}}\left(\alpha\,|\omega|\,r\right)}{r^{\lambda}\,\Gamma\left(\lambda+\frac{3}{2}\right)}. \tag{11.17}$$

Other details can be found in Refs. [24–31].

11.4 Results on Probabilities of the Nuclear Excitation by Electron Transition

In concrete calculation of the NEET probabilities for different atomic/nuclear systems, one should calculate the corresponding matrix elements. As we will consider below M1 (E2) transition from the ground state to the first excited state in the nuclei $^{189}_{76}$Os, $^{193}_{77}$Ir, and $^{197}_{79}$Au, it should be noted that the values of $B[E/(M)\lambda; J_i{-}J_f]$ are usually taken from the nuclear data tables [33] or can be estimated according to the known formula (look [34, 35]). In order to calculate the electronic wave functions and matrix elements, we have used the relativistic many-body PT formalism [24–26, 28–30]. It allows to take into account accurately the relativistic, exchange-correlation, nuclear, and radiative corrections (the PC code "Superatom-ISAN"). The detailed description of the method is given in a number of Refs. (look, e.g., [24–31]). Here we are limited by a brief comment. The corresponding code contains the atomic and nuclear blocks. The zeroth approximation electronic wave functions are found from the Dirac (or Dirac-Kohn-Sham) equation with potential, which includes the SCF potential and the electric and polarization potentials of a nucleus. As an account of the finite nuclear size has a sensitive effect on the energy levels of the bound electron, we usually use the smooth Gaussian (or Fermi) function of the charge distribution in a nucleus. The correlation corrections of the second and high orders are taken into account within the Green function method (with the use of the Feynman diagrams technique). They have taken into account all correlation corrections of the second order and dominated classes of the higher order diagrams (electron screening, particle-hole interaction, mass operator iterations). The magnetic interelectron interaction is accounted in the lowest (on α^2 parameter, α being the fine-structure constant). The radiative corrections are taken into account effectively, namely, the Lamb shift self-energy part is accounted within the generalized Ivanov-Ivanova non-perturbative procedure and the polarization part—in the generalized Uehling Serber approximation. The important feature of the whole method is using the optimized one-quasiparticle representation in the zeroth approximation, which is constructed within the method [26]. The nuclear part of the general method includes a set of the nuclear shell models, including the relativistic mean-field approach [36] and the Dirac-Bloumkvist-Wahlborn and Dirac-Woods-Saxon models [35–37].

The calculation results on the NEET probability for the $^{189}_{76}$Os, $^{193}_{77}$Ir, and $^{197}_{79}$Au atoms together with the alternative theoretical (by Tkalya and Ahmed et al.) [22, 23] and experimental data [21, 22] are given in Table 11.3.

Table 11.3 Theoretical and experimental data on probabilities P_{NEET} (M1) for the isotopes of $^{189}_{76}$Os, $^{193}_{77}$Ir, and $^{197}_{79}$Au

Nucleus	Energy of nuclear excitation (keV)	Experimental values	Theory	Present work
$^{189}_{76}$Os	69.535	$<9.5\cdot10^{-10}$	$1.2\cdot10^{-10}$	$1.9\cdot10^{-10}$
			$1.3\cdot10^{-10}$	
$^{193}_{77}$Ir	73.04	$(2.8\pm0.4)\cdot10^{-9}$	$2.0\cdot10^{-9}$	$2.7\cdot10^{-9}$
$^{197}_{79}$Au	77.351	$(5.7\pm1.2)\cdot10^{-8}$	$3.4\cdot10^{-8}$	$4.6\cdot10^{-8}$
		$(4.5\pm0.6)\cdot10^{-8}$	$4.5\cdot10^{-8}$	

Let us note that in $^{189}_{76}$Os during the NEET process, the initial K-vacancy state decays via an electronic transition from the M shell. The KM_I (70.822 keV, M1), KM_{IV} (71.840 keV, E2), and KM_V (71.911 keV, E2) atomic transitions can give the contribution. The corresponding nuclear state at 69.535 keV can be excited via M1 or E2 transitions from the $3/2^-$ nuclear ground state. The following energy parameters $\omega_N = 69.535$ keV, $\omega = E_{MI} - E_K = 70.822$ keV, $\Gamma_K = 42.6$ eV, and $\Gamma_M = 12.8$ eV are used for the $^{189}_{76}$Os atom. Correspondingly, the energy parameters for $^{197}_{79}$Au are as follows: $\omega_N = 77.351$ keV, $\omega_A = 77.325$ keV, $\Gamma_K = 52$ eV, and $\Gamma_M = 14.3$ eV and for $^{193}_{77}$Ir, $\omega_N = 73.04$ keV, $\omega = 72.937$ keV, $\Gamma_K = 45$ eV, and $\Gamma_M = 12.8$ eV. Analysis of all presented theoretical data shows that these results are consistent with each other and are in physically reasonable agreement with the experimental results [21, 22].

11.5 Conclusion

In this chapter, a brief review of the experimental and theoretical works on the NEET effect is given. A new, consistent relativistic energy approach to calculation of the cooperative electron-gamma-nuclear NEET process combined with the relativistic PT is presented. It should be noted that the presented approach can be naturally generalized in order to describe the physics of the NEEC and other similar cooperative processes. The calculation results are listed for the heavy atomic systems $^{189}_{76}$Os, $^{193}_{77}$Ir, and $^{197}_{79}$Au and compared with available theoretical and experimental data. It is important to note that the theoretical and experimental study of the cooperative electron-gamma-nuclear process such as the NEET effects is expected to allow the determination of nuclear transition energies and the study of atomic vacancy effects on nuclear lifetime and population mechanisms of excited nuclear levels. The cooperative e-γ-nuclear spectroscopy of atomic/nuclear systems opens new prospects in the bridging of nuclear physics and atomic spectroscopy. These possibilities are significantly strengthened by quickly developed nuclear quantum optics [1, 3, 5, 14, 17]. Really, a superintense laser (raser) field may provide a definite measurement of the change in the dynamics of the cooperative electron-γ-nuclear processes.

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