Non-proxy Reductions of Eternalist Discourse

Fabrice Correia

Abstract Eternalists believe that there are past things and future things which are not present. In contrast, presentists hold that only present objects exist. In this chapter, I discuss presentist reductions of eternalist discourse which do not involve quantification over proxies—i.e. presentistically acceptable surrogates for merely past and merely future entities.

Introduction

Say that an object is *merely past* if it is past but not present, i.e. if it was present but is not so anymore, and that an object is *merely future* if it is future but not present, i.e. if it will be present but is not yet so.[1] *Presentists* hold that everything—absolutely everything—is present, i.e. that there are no merely past or future objects. *Eternalists* deny it; they hold that there are objects which are merely past and others which are merely future.[2] They are willing to claim, for instance, that there are things which

---

[1] There is an issue about how, in the present context, the notion of being present is to be understood. My own take on the issue involves a tensed notion of existence, and runs simply as follows: for an object to be present is for it to exist. I will have that account in mind, but accepting other accounts would not affect the points made in this chapter. Notice that I will assume that all the views discussed in this chapter agree that there are present things.

[2] Eternalism is not the only alternative to presentism. The so-called *growing block theorists*, for instance, agree with the eternalists that there are merely past objects but deny that there are merely future ones. The growing block theory is quite unpopular, and other alternatives to presentism and eternalism are implausible. Although my focus in this chapter will exclusively be on the divide between presentists and eternalists, it will be somewhat obvious how the discussion could be extended to other views on temporal ontology.

F. Correia (✉)
University of Geneva, Geneva, Switzerland
e-mail: fabrice.correia@unige.ch

were born in year 1800 which are no longer present and that there are things which will be born in year 2100 but which are not yet present. What are presentists to make of such claims?

Of course, *qua* presentists, they must say that these claims, *literally understood*, are false. Yet, pre-theoretically, we would naturally take them—at any rate, some of them—to be true. Leaving aside the fact that the use of 'there are things' to quantify over inhabitants of this world which are capable of having a birth may sound odd to the layman, this is arguably the case of the two examples just given. Some presentists may simply ignore that pre-theoretic attitude. But some will hold that the claims in question—at any rate, some of them—are in fact true, although they have to be understood in some nonliteral way. These presentists face the task of 'reducing' talk of merely past and future objects, i.e. of proposing appropriate translations or paraphrases of the relevant quantified claims which can be taken to be true on the background assumption that presentism holds.

Such paraphrases can be of two sorts. Using Kit Fine's (2005) terminology, they can be *proxy* or *non-proxy*. Proxy paraphrasing translates alleged quantification over non-present things into quantification over presentistically acceptable proxies or surrogates—*Ersatzen*, as David Lewis calls them. Non-proxy paraphrasing translates the target claims but without invoking proxies. Many objections have been raised against the viability of actualist proxy reduction of talk of mere possibilia,[3] and many of them carry over the possibility of proxy reduction of the non-present which concerns us here. I find these objections serious, and accordingly I think presentists who want to reduce eternalist discourse should seriously consider non-proxy reduction.

Some studies (Fine 1985, 2005; Forbes 1989; Correia 2007) have been (at least partly) dedicated to non-proxy reduction of possibilist discourse. Three methods of paraphrase can be found there, the *Peacockean*, the *Vlachian*, and the *Finean* methods, as I will call them. In contrast, the literature on non-proxy reduction of eternalist discourse is quasi nonexistent (the only publication on the topic I know of is Correia 2009). In some cases (see sections "Linear Time: The Peacockean and the Vlachian Methods" and "Linear Time: The Finean Method" below), the three methods of translation just mentioned can be applied to the case of eternalist discourse in a relatively straightforward way, whereas in some other cases (see section "Branching Time"), some nonobvious modifications are required. This chapter is a discussion of these methods in the temporal context and of a new method I call the *metric* method, which, unlike the others, is not applicable to the case of possibilist discourse.

I take the metric method to be of great interest, because, as I will argue, it escapes certain difficulties met by the other three methods. However, this chapter is not a wholehearted defence of that method against the others, for, as I will stress, unlike the latter methods, the metric method does not deliver what we want given certain (debatable, but nevertheless not implausible) assumptions about eternalist

---

[3] See Fine (2005) for a recent example.

quantification and the logic of tense. The aim of this chapter is rather to discuss the scope and relative merits of various techniques of non-proxy paraphrasing which can be used in the present context and indirectly to advertise such techniques—which I think is worth doing since, strangely enough, the three techniques already used and discussed in the literature are widely ignored by the philosophers who work on the philosophy of time or modality.[4]

The scope of such a study is bound to be limited. Let me here mention three limitations of this chapter.

1. The primary targets of non-proxy reduction are sentences of a language, e.g. English, which can reasonably be taken to be true by eternalists and which entail the existence of something which is not present. As we shall see, the adequacy of a method of paraphrase is highly sensitive to the class of sentences it is applied to. Any study of the present kind must focus on restricted classes of sentences (which should nevertheless be expressively rich enough to be of interest). I will assume that the target of presentist reduction is certain interpreted first-order tense-logical formal languages, leaving aside many other languages, e.g. languages with further tense-logical operators and higher-order languages.
2. I will assume that the tense logic for the languages to be considered, be it the logic accepted by eternalists or the one endorsed by presentists, can be characterized by some standard Kripke-style semantics.[5] This is an assumption I take to be fairly weak, and in any case, it is one that many, in both camps, are happy to accept.
3. The adequacy of a method of paraphrase may turn on which conception of the structure of time in the relevant Kripke models is countenanced. I will only take into consideration two such conceptions: the conception of time as linear and the conception of time as branching towards the future but linear towards the past. This is a limitation, but not a very drastic one since many philosophers endorse one or the other. And, in any case, it will be somewhat obvious how the discussion would go if certain other conceptions were taken into account.

The plan of the chapter is as follows. In section "Adequate Translations", I elaborate on what I take adequate presentist translations of possibilist talk to be. In section "The Target Language, Its Semantics and the Corresponding Logics", I present the language I take to be the target of the presentist translations, as well as semantic and logical material relative to that language. In the three sections that follow, I discuss the four methods of translation under the assumption that the logic for the target language is determined by linear models. And finally in section "Branching Time", I discuss these same methods under the alternative assumption that the logic of that language is determined by branching time models.

---

[4] Sider (2006) provides a striking recent example.

[5] On Kripke-style semantics for tensed languages one may consult, e.g. Burgess (2002) and Hodkinson and Reynolds (2006).

Adequate Translations

I will take the target of reduction of eternalist discourse to be the quantified tense-logical language *L* defined in the next section, and I will take it that the task of the presentists I am interested in here is to provide adequate non-proxy translations of the sentences of *L* into some language, *L* itself or another language, which we may call the *home language*.

There are several eternalist views as to what the correct logic for *L* is, and likewise there is potential disagreement amongst presentists about the logic of a candidate home language. The home languages of the translation functions to be discussed below are all extensions of *L*, and they exploit the tense-logical operators and the existential quantifier. Accordingly, there is no reason to expect that a translation function will be adequate whatever the eternalist view on the logic for *L* and the presentist view on the logic of the home language.

I will take the following as a constraint on adequacy:

Logicality constraint. Given an eternalist view about the logic for *L* and a presentist view about the logic for a language *L′*, for a translation function from *L* into *L′* to be adequate, the following condition should hold: for every sentence ϕ of *L*, ϕ is deemed to be logically true according to the eternalist view iff ϕ's translation is deemed logically true according to the presentist view.

This constraint is quite strong, and some might take them to be too demanding. I do not want to discuss this point here. As we will see, however, the methods of translation to be discussed fare rather well with respect to the constraint.

The fulfilment of the logicality constraint is certainly not sufficient for adequacy. The point of reducing eternalist discourse is to 'approximate' talk of merely past and future objects in terms acceptable to presentists. An adequate translation function should be such that what, according to the eternalist, the quantifier performs in a given sentence ϕ is 'mimicked' by syntactic elements in ϕ's translation: what our presentist aims at is defining pseudo-quantifiers over merely past and merely future objects. It is hard to come by with a rigorous characterization of that aspect of adequacy, and I have none to offer here. Yet hopefully what has been said will be enough for present purposes.

The Target Language, Its Semantics and the Corresponding Logics

I take the target of presentist non-proxy reduction to be an interpreted first-order tense-logical language *L*, with primitive truth-functional connectives ¬ (negation) and ∨ (disjunction), primitive quantifier ∃ (absolutely unrestricted existential quantification), and primitive tense-logical operators *P* (for 'sometimes in the past'), *F* (for 'sometimes in the future') and *N* (for the non-redundant 'now'[6]).

---

[6] See Prior (2003).

We assume that L's vocabulary comprises the identity predicate =, as well as the predicate Pres for presentness (and the predicates I will use in examples of sentences of L).[7]

Conjunction ($\wedge$), material implication ($\supset$), material equivalence ($\equiv$), and absolutely unrestricted universal quantification ($\forall$) are defined in a standard way in terms of the chosen primitive vocabulary. H (for 'always in the past'), G (for 'always in the future'), A (for 'always') and S (for 'sometimes') are defined as follows:

- $H\phi := \neg P\neg\phi$.
- $G\phi := \neg F\neg\phi$.
- $A\phi := H\phi \wedge \phi \wedge G\phi$.
- $S\phi := \neg A\neg\phi$.

Standard notational conventions will be followed throughout, and standard syntactical notions will be assumed to be known.

We define a *frame* as a tuple $\langle T, < \rangle$, where

- T (times) is a non-empty set
- $<$ (temporal precedence) an asymmetric, transitive relation on T

A *model* for L is then defined as a tuple $\langle \pi, T, <, D, I \rangle$, where

- $\langle T, < \rangle$ is a frame
- π (the present time) a member of T
- D (domain) a function taking each $t \in T$ into a set D_t (the domain of t), such that $\bigcup_{t \in T} D_t \neq \varnothing$
- I (interpretation) a function which takes each n-place predicate and member of T into a set of n-tuples of members of $\bigcup_{t \in T} D_t$, with the condition that (1) $I(=, t)$ is always the set of all $\langle d, d \rangle$ such that $d \in \bigcup_{t \in T} D_t$, and (2) $I(\text{Pres}, t)$ is D_t for all $t \in T$

Relative to a model, an *assignment* (to the variables) is a function which takes every variable of L into a member of the union of all the time domains of that model. If ρ is such a function, x a variable of L and d an element of the union of all the time domains of the model, $\rho^{x \to d}$ is ρ itself if $\rho(x) = d$, and the assignment just like ρ except that it assigns d to x otherwise.

Up to section "Branching Time", I will focus on eternalists and presentist positions which assume that the logic for L is a logic characterized by Kripke models whose temporal precedence relation is linear. I here present the corresponding semantics, leaving the alternative conception of the logic as characterized by models whose precedence relation is branching for section "Branching Time".

We define a *linear frame* as a frame $\langle T, < \rangle$ such that

- For all $t, t' \in T$, either $t < t'$, or $t = t'$, or $t' < t$.

A *linear model* for L is a model $\langle \pi, T, <, D, I \rangle$ where $\langle T, < \rangle$ is a linear frame.

---

[7] For the sake of simplicity, L's vocabulary is supposed not to comprise individual constants.

Given the divide on temporal ontology which is the focus of this chapter, we need to define two notions of truth in a linear model, an eternalist notion (*e-truth*) and a presentist notion (*p-truth*). *e*-truth at a time $t \in T$ in a linear model $M = \langle \pi, T, <, D, I \rangle$ relative to assignment ρ is defined recursively as follows:

1. $M, \rho, t \models^e \varphi(x_1 ... x_n)$ iff $\langle \rho(x_1), ..., \rho(x_n) \rangle \in I(\varphi, t)$.
2. $M, \rho, t \models^e \neg \phi$ iff $M, \rho, t \not\models^e \phi$.
3. $M, \rho, t \models^e \phi \vee \psi$ iff $M, \rho, t \models^e \phi$ or $M, \rho, t \models^e \psi$.
4. $M, \rho, t \models^e \exists x \phi$ iff for some $d \in \bigcup_{t \in T} D_t$, $M, \rho^{x \to d}, t \models^e \phi$.
5. $M, \rho, t \models^e P\phi$ iff for some $t' \in T$ such that $t' < t$, $M, \rho, t' \models^e \phi$.
6. $M, \rho, t \models^e F\phi$ iff for some $t' \in T$ such that $t < t'$, $M, \rho, t' \models^e \phi$.
7. $M, \rho, t \models^e N\phi$ iff $M, \rho, \pi \models^e \phi$.

p-truth at t in M relative to ρ is defined in just the same way, except that the truth clause for the quantifier is replaced by

4′. $M, \rho, t \models^p \exists x \phi$ iff for some $d \in D_t$, $M, \rho^{x \to d}, t \models^p \phi$.

A sentence—i.e. a closed formula—of L is said to be *e*-true (*p*-true) in linear model M iff it is *e*-true (*p*-true) at the present time of the model (relative to any assignment we like).

So far for the basic semantic apparatus. Where $\mathfrak{M}$ is a class of models, we shall call the set of all sentences of L which are *e*-true in all members of $\mathfrak{M}$ *the logic e-determined by* $\mathfrak{M}$, and the set of those which are *p*-true in all members of $\mathfrak{M}$ *the logic p-determined by* $\mathfrak{M}$. Throughout the next three sections, I will focus on eternalists who take the logic for L to be the logic *e*-determined by so me class of linear models and presentists who take it to be the logic *p*-determined by some such class of models.[8]

Linear Time: The Peacockean and the Vlachian Methods

Eternalists claim that there are objects which are past but not present, i.e. that

1. $\exists x(P\mathrm{Pres}(x) \wedge \neg\mathrm{Pres}(x))$.

---

[8] *e*-truth in a model M need not depart from *p*-truth in M: the two notions are coextensive if the time domains of M are all the same. If that condition on M is satisfied, I shall say that M is *flat*. An eternalist cannot hold the view that the logic for L is the logic *e*-determined by some class of models which are all flat. For the sentence $\forall x\mathrm{Pres}(x)$, which is deemed false by eternalists, is *e*-true in any flat model and so belongs to the logic *e*-determined by any class of such models. A presentist can in principle take the logic for L to be the logic *e*-determined by some class of flat models, but the resulting view is problematic. For the sentence $\forall x A\mathrm{Pres}(x)$ is *p*-true in any flat model and thus belongs to the logic *p*-determined by any class of such models. Yet the view that this sentence is true is very implausible: surely, I was not yet present a 100 years ago, and I will not be present anymore a 100 years hence. (The presentist view I deem problematic is a temporal version of the 'new actualist' position defended by Linsky and Zalta (1994), which I find equally problematic.) It will accordingly be (tacitly) assumed that the classes of linear models at stake here comprise at least some non-flat models, and the same will go for the classes of branching models presented in section "Branching Time".

There is a natural suggestion for a paraphrase which a presentist may put forward, which does not use expressive resources foreign to L itself. The idea is to translate (1) into

2. $P \exists x N \neg \text{Pres}(x)$,

i.e. 'there was something (present) which now is not present'. For the presentist, the complex expression $P\exists x$ acts like a quantifier over past objects, and N forces the predication to be evaluated at the time at which (2) as a whole is evaluated.

The foregoing considerations suggest a general translation procedure which runs as follows:

Take ϕ in L.

1. If ϕ contains no occurrence of $\exists$, then ϕ's translation is ϕ.
2. Otherwise, ϕ's translation is the result of replacing each sub-formula of type $\exists x\psi$ in ϕ by $S\exists x N\psi$.

For the presentist, in a formula of type $S\exists x N\phi$ the expression $S\exists x$ acts like a quantifier over past, present and future objects, and N forces ϕ to be evaluated at the time at which $S\exists x N\phi$ as a whole is evaluated.

The proposed translation scheme is simple and elegant, but it is very easy to see that it does not deliver adequate results in all cases (which is why I did not give it a name). For take the sentence

3. $P\exists x(x$ is walking on the moon $\land N(x$ is not walking on the Moon)).

Granted that Neil Armstrong was walking on the moon sometimes in 1969 and is not doing it now, everyone—eternalists and presentists alike—must take (3) to be true. Arguably, a presentist should then be able to take its translation to be true. But by the proposed translation scheme, (3) translates into

4. $PS\exists x N(x$ is walking on the Moon $\land N(x$ is not walking on the Moon)),

and on any reasonable tense logic, (4) cannot be true. For on any such logic, (4) is indeed equivalent to

5. $PS\exists x N(x$ is walking on the Moon $\land x$ is not walking on the Moon),

and (5) is a claim to the effect that sometimes in the past, either there was even more in the past, or there was then, or there would be, something such that now, a certain contradiction about that thing is true.

The problem, informally speaking, is that when (4) is evaluated at the present time, N points to that time, whereas in order for the translation to be correct it would have to 'follow' the temporal shift induced by the occurrence of P.

One way of getting things right invokes the temporal analogues of the indexing device introduced in Peacocke (1978) for the modal and the actuality operators and discussed in Forbes (1989, p. 87ff) and Correia (2007) in the context of non-proxy reduction of possibilist discourse. The idea is to index the tense-logical operators P, F and N, say by means of the numerals '1', '2', …, and to interpret an occurrence of the indexed presentness operator N_i as 'following' the temporal shift induced by the occurrence of P_i or F_i under certain syntactic conditions.

Let us be more precise. Languages containing these indexed operators can be given a Kripke-style model theory akin to the model theory for L presented above. In order to define a translation function for sentences of L, only one index is actually needed. Let then L^P be the language resulting from L by adding the operators P_1, F_1 and N_1. A linear model for L^P is simply a linear model of the sort at work in the semantics for L. The notion of truth for L^P is not just a notion of truth at a time of evaluation in a linear model relative to an assignment to the variables. An extra parameter—a second time, which we may call the *stored time*—is involved, which is acted upon by P_1 and F_1. As in the case of L, two notions of truth, one for eternalists and one for presentists, can be defined, but here we concentrate on the latter notion since L^P is to be used as the language into which the translation is to be carried out. Using '$M, \rho, u, t \models^p \phi$' for '$\phi$ is p-true at time of evaluation t given stored time u in linear model M relative to assignment ρ', the truth clauses for the indexed operators run as follows (T is M's set of times and $<$ its precedence relation):

- $M, \rho, u, t \models^p P_1 \phi$ iff for some $t' \in T$ such that $t' < t$, $M, \rho, t', t' \models^p \phi$.
- $M, \rho, u, t \models^p F_1 \phi$ iff for some $t' \in T$ such that $t < t'$, $M, \rho, t', t' \models^p \phi$.
- $M, \rho, u, t \models^p N_1 \phi$ iff $M, \rho, u, u \models^p \phi$.

The other truth clauses are like those of the definition of p-truth for L. A sentence of ϕ of L^P is then said to be *p-true* in a linear model M whose present time is π iff ϕ is true at time of evaluation π given stored time π in M (relative to any assignment we like).

Thus, the effect of P_1 and F_1 is twofold: (1) they shift the time of evaluation, in the same way as P and F, respectively, and, (2) unlike what happens with P and F, this shift is 'recorded' in the stored time position. What N_1 does is simply make the stored time the time of evaluation. Given this behaviour of the indexed operators, the following translation procedure suggests itself:

THE PEACOCKEAN TRANSLATION PROCEDURE:

Take ϕ in L.

1. If ϕ contains no occurrence of $\exists$, then ϕ's translation is ϕ.
2. Otherwise, ϕ's translation is the result of applying the following procedure to each occurrence o of a sub-formula of type $\exists x \psi$ in ϕ:

   (a) If o is not within the scope of an occurrence of P or F, replace o by $S\exists x N\psi$.
   (b) If o is within the scope of an occurrence of P or F, replace o by $S\exists x N_1 \psi$ and replace the occurrence of P or F which immediately governs o by its indexed counterpart (if this has not yet been done).[9]

---

[9] The reason for the bracketed qualification stems from the fact that more than one occurrence of a quantifier may be immediately governed by a tense-logical operator. Thus consider the formula $FP\exists x \exists y \phi$. Applying the procedure to $\exists x$ yields $FP_1 S\exists x N_1 \exists y \phi$. Since P has already been replaced by P_1, applying the procedure to $\exists y$ consists only in replacing $\exists y \phi$ by $S\exists y N_1 \phi$, which yields $FP_1 S\exists x N_1 S\exists y N_1 \phi$.

(An occurrence o' of P or F immediately governs an occurrence o of $\exists x\psi$ iff o is within the scope of o', and there is no occurrence o'' of P or F such that o is within the scope of o'' and o'' within the scope of o'.)

The proposed translation scheme eschews the kind of difficulty met by the previous one. By the new scheme, (3) translates into

(4′) $P_1 S \exists x N_1(x$ is walking on the Moon $\wedge N \neg(x$ is walking on the Moon)),

which is certainly not problematic in the way (4) was. N_1 is bound by P_1, while N is not: it points to the time at which (4′) as a whole is evaluated.

It can be shown that

Proposition 1. *Given any linear model M and sentence ϕ of L, ϕ is e-true in M iff ϕ's Peacockean translation is p-true in M.*

As a result, given any class of linear models $\mathfrak{M}$, and any sentence ϕ of L, ϕ belongs to the logic *e*-determined by $\mathfrak{M}$ iff ϕ's translation belongs to the logic *p*-determined by $\mathfrak{M}$. Therefore, the proposed translation function satisfies the logicality constraint on adequacy, granted an eternalist view of the logic for L as the logic *e*-determined by some class of linear models and a presentist view of the logic for L^P as the logic *p*-determined by the same class.

Before going further, let me present an alternative method—the *Vlachian* method, as I called it—which is in some important respects similar to the Peacockean method.

The new method makes use of two operators $\uparrow$ and $\downarrow$ introduced in Vlach (1973)[10] to formalize certain sentences containing the expressions 'once' and 'then' in their temporal senses and is the temporal counterpart of the method discussed in Forbes (1989, pp. 27ff) and Correia (2007) in the context of the reduction of possibilist discourse.[11] Both $\uparrow$ and $\downarrow$ are unary sentential operators and can be given a bi-dimensional semantics exactly like the one presented above for the Peacockean indexed operators but with the following truth clauses for Vlach's operators:

- $M, \rho, u, t \models^p \uparrow\phi$ iff $M, \rho, t, t \models^p \phi$.
- $M, \rho, u, t \models^p \downarrow\phi$ iff $M, \rho, u, u \models^p \phi$.

Thus, the effect of $\uparrow$ is to store the time of evaluation and that of $\downarrow$ is to make the stored time the time of evaluation.

It is clear from the proposed semantics for the Peacockean and the Vlachian operators that the former are definable in terms of the latter together with P and F: P_1 is definable as $P\uparrow$, F_1 as $F\uparrow$ and N_1 simply as $\downarrow$. Therefore, the Peacockean translation scheme immediately yields an equivalent translation scheme from L to the language L^V obtained from L by adding $\uparrow$ and $\downarrow$. But there is a simpler translation scheme from L to L^V, and it is that scheme I dubbed Vlachian. It is defined by the following procedure:

---

[10] Vlach uses instead K and R, respectively.

[11] Fine (1977) mainly deals with (proxy) reduction of possibilist languages which are extensional, i.e. which do not contain modal operators but rather quantifiers intended to range over merely possible worlds, but he nevertheless mentions the Vlachian method of reduction of modal languages on page 144.

The Vlachian Translation Procedure:

Take ϕ in L.

1. If ϕ contains no occurrence of $\exists$, then ϕ's translation is ϕ.
2. Otherwise, ϕ's translation is the result of replacing each sub-formula of type $\exists x \psi$ in ϕ by $\uparrow S \exists x \downarrow \psi$.

The scheme is indeed simpler, because here we do not have to treat occurrences of type $\exists x$ in a different manner according to whether or not they are within the scope of an occurrence of P or F. Notice that the Vlachian scheme is syntactically different from the Peacockean scheme, but these two schemes are equivalent from the point of view of the formal semantics, in the sense that for every sentence ϕ of L and every linear model M, ϕ's Vlachian translation is p-true in M iff ϕ's Peacockean translation is. Consequently,

Proposition 2. *Given any linear model M and sentence ϕ of L, ϕ is e-true in M iff ϕ's Vlachian translation is p-true in M,*

and as a consequence, the Vlachian translation function also satisfies the logicality constraint on adequacy given an eternalist view of the logic for L as the logic e-determined by some class of linear models and a presentist view of the logic for L^V as the logic p-determined by the same class.

Both the Peacockean and the Vlachian methods of translation appear to be promising. Yet, one might argue, presentists who want to use these methods face a serious difficulty. The argument for the case of the Peacockean method runs as follows[12]:

All we have so far by way of an explanation of the meaning of the sentences of the Peacockean language L^P is the model-theoretic characterization presented above. But that model-theory is a purely formal theory, where the 'times' it invokes are entities whose nature is not specified, the 'temporal precedence' relations are relations between such entities satisfying certain formal constraints and the domains of the various 'times' are sets of objects whose nature is again left open. This formal model-theory can at best characterize certain logical features of the sentences of L^P, but this is certainly not sufficient to tell us what the *truth-conditions* for these sentences are. Unless truth-conditions are given, the use of the indexed operators is illegitimate.

An eternalist can in an obvious way exploit the proposed model-theory in order to give truth-conditions for the sentences of L^P. He can tell us (i) that there is a special linear model M whose 'times' are just the real times, whose 'present time' is the real present time, whose 'temporal precedence' relation is the real earlier-later relation, whose 'time-domains' are the sets of objects existing at the corresponding real times, and finally whose interpretation function is 'faithful to reality', i.e. assigns to each predicate Π of L^P and time t the extension which Π has in fact at t, and (ii) that for a sentence of L^P to be true is for it to be true at the present time in M.

---

[12] This is a variant on a temporal analogue of an objection to Forbes' use of the Peacockean method for the purpose of reducing possibilist discourse in his (1989), an objection which is presented by Forbes himself (pp. 90ff) and made again in Melia (1992). The objection is against the view that a modalist can use the Peacockean method for the purpose in question, where a modalist holds that our modal concepts are not to be analysed in terms of quantification over possible worlds.

It is evident that a presentist cannot go the same way. For presentists believe that there is only one time, the present time, and consequently, from their own perspective no sentence of type $P\phi$ or $F\phi$ is true at the present time in the model defined in point (i) above, and so, were they to accept point (ii), they would be committed to the view that no sentence of that sort is true. That view is extreme, and it can be assumed that it is not one a presentist would be willing to endorse. Presentists thus face the challenge of providing us with an alternative semantic story, and it is hard to see which story this could be.

The argument for the case of the Vlachian method is exactly similar.

I take these arguments to have a great strength provided that it is assumed, as the arguments do assume, that truth conditions for the Peacockean or the Vlachian operators must be provided in order for them to acquire a meaning. Yet this assumption is questionable. One possible line of response is that the operators are not artificial devices whose meaning needs to be established; they merely help regiment an unproblematic linguistic phenomenon of which competent English speakers have a pre-theoretic understanding. Consider the following sentence, taken as an example by Vlach himself (1973, p. 2):

6. Jones was once going to cite everyone then driving too fast.

This sentence clearly differs in meaning from the following other sentence:

7. Jones was once going to cite everyone now driving too fast.

The response I have in mind runs as follows.[13] At any time of evaluation for (7), 'now' in (7) points to what is happening at that time. In contrast, in (6) 'then' is bound by 'was' (or 'was once') in a semantically relevant way. Where $D(x)$ is short for 'x drives too fast' and $C(j,x)$ for 'Jones cites x', (7) can be formalized using the rigid presentness operator as follows:

$$PF\forall x(ND(x) \supset C(j,x)).$$

As for (6), it can be formalized by

8. $P_1 F\forall x(N_1 D(x) \supset C(j,x))$

using the Peacockean operators, and by

9. $P\uparrow F\forall x(\downarrow D(x) \supset C(j,x))$

using Vlach's operators. Since (6) is clearly meaningful, both (8) and (9), which simply regiment (6), are also meaningful. More generally, since the Peacockean and the Vlachian operators are nothing but devices used to represent in formalized languages the kind of binding phenomenon at work in (6), the sentences of L^P and L^V are meaningful, and their meaningfulness stems from the meaningfulness of their counterparts in plain English.

This response leaves untouched the question of how one could formulate a proper semantics for 'once' and 'then' without invoking an ontology of past, present and

---

[13] This is the kind of response Forbes (1989, 1992) gives to the objection alluded to in footnote 12.

future times. Some will insist that this cannot be done, and accordingly that the use of the temporal 'once' and 'then' does commit one to quantification over times other than the present time. I disagree, but I will leave the issue at this point here. Yet at the end of the next section, I present a way of interpreting the Peacockean and the Vlachian operators I find particularly attractive.

Linear Time: The Finean Method

Fine (1985, 2005), drawing on the pioneering work of Arthur Prior, proposed a very nice alternative to the previous methods of paraphrase (in the modal case, but here I just focus on a variant of its temporal counterpart).[14] A crucial presupposition of the method is that presentists have no problem with the existence of times: they reject merely past and merely future objects, but they are happy to accept the existence of one time, viz. the present time. The Finean translation function takes the sentences of L into sentences of language L^T, which is L enriched with special variables for times τ, τ', τ'', ...[15] The presentist formal semantics for L^T is just like the presentist semantics for L but with the following extra conditions: (1) each time domain D_t of a model contains t, and no time distinct from t belongs to D_t, and (2) given any model M, an assignment assigns times of M to the temporal variables. I shall call linear models which satisfy condition (1) *temporalized*.

The Finean translation scheme is best presented via a détour. The Finean takes it that the locution

It is true at time τ that ϕ,

or 'at τ, ϕ', for short, can be defined as

$A(\text{Pres}(\tau) \supset \phi)$

(or alternatively, as $S(\text{Pres}(\tau) \wedge \phi)$). Let then '$\mathfrak{n}$' be a (temporally) rigid name for the present time. If the Finean is right, then given that 'Now, …' is equivalent to 'at $\mathfrak{n}$, …', $N\phi$ can be defined as $A(\text{Pres}(\mathfrak{n}) \supset \phi)$. A translation procedure akin to the first one we met in the previous section, but from L to L^T enriched with $\mathfrak{n}$, is accordingly available:

Take ϕ in L.

1. If ϕ contains no occurrence of $\exists$, then ϕ's translation is ϕ.
2. Otherwise, ϕ's translation is the result of replacing each sub-formula of type $\exists x\psi$ in ϕ by $S\exists xA(\text{Pres}(\mathfrak{n}) \supset \psi)$.[16]

---

[14] To be accurate, in these two papers the target languages are, like the languages (Fine 1977) mainly focuses on, extensional languages (see footnote 11 above). The method of translation presented in Fine (1985, 2005) already appears in Fine (1977), but in a different form: instead of quantification over worlds, Fine (1977) follows Prior and exploits quantification over world propositions, which go proxy for possible worlds (see in particular the end of p. 144).

[15] Instead of variables for times, one could add a predicate for times and express quantification over times by means of standard quantification restricted to the objects which satisfy that predicate.

[16] Of course, given the definability of 'now', a better translation procedure would be from L to L^T minus N enriched with $\mathfrak{n}$ and would translate N itself.

Of course, the resulting translation function is as problematic as its mate, for similar reasons.

The Finean fix is somewhat straightforward: instead of using a rigid designator for the present time, use a non-rigid definite description for it, namely, 'the present time', and give it wide scope. More precisely, the suggestion is that the second clause of the translation scheme should be (here I am using a bastard notation):

Otherwise, ϕ's translation is the result of replacing each sub-formula of type $\exists x \psi$ in ϕ by 'the present time τ is such that $S\exists xA(\text{Pres}(\tau) \supset \psi)$'.

Assuming that there is always only one present time,

$$\text{the present time } \tau \text{ is such that } S\exists xA(\text{Pres}(\tau) \supset \psi)$$

is equivalent to $\exists \tau(\text{Pres}(\tau) \wedge S\exists xA(\text{Pres}(\tau) \supset \psi))$, and since for presentists being present and being in the range of the existential quantifier are always equivalent, the latter is equivalent to $\exists \tau S\exists xA(\text{Pres}(\tau) \supset \psi)$. The Finean translation scheme can thus be defined by means of the following procedure:

THE FINEAN TRANSLATION PROCEDURE:

Take ϕ in L.

1. If ϕ contains no occurrence of $\exists$, then ϕ's translation is ϕ.
2. Otherwise, ϕ's translation is the result of replacing each sub-formula of type $\exists x \psi$ in ϕ by $\exists \tau S\exists xA(\text{Pres}(\tau) \supset \psi)$ (τ any variable for times).[17]

The way pseudo eternalist quantification is achieved is more complicated than in the case of the previous two translation schemes, but a great advantage of the Finean method of translation is that it is untouched by the objection levelled at the other schemes I discussed in the last section. For the vocabulary involved in translating is just part of the unproblematic vocabulary of L plus quantifiers over times, and, again, there is nothing to stop a presentist to use such quantifiers as long as he respects his view that everything is present.

The Finean translation function is *almost* logically as good as the Peacockean and the Vlachian scheme. Remember that the semantics for L^T involves exclusively *temporalized* models. An eternalist need not consider the logic for L to be *e*-determined by a class of models of that sort,[18] and accordingly there is no hope that we can establish that the logicality constraint can be fulfilled by the Finean function starting from any eternalist conception of that logic. We need to focus on conceptions of the logic for L as *e*-determined by some class of temporalized models—more accurately, given the background assumption at work at the moment, by some class of temporalized linear models. In fact, it can be established that

---

[17] This is a translation scheme from L to L^T, but of course it could be turned into a more elegant translation scheme from L to L^T minus N enriched with $\mathfrak{n}$ (see previous footnote). But for the sake of simplicity I leave things as they stand.

[18] An eternalist may be a nihilist or a monist about times. See below.

Proposition 3. *Given any temporalized linear model M and sentence φ of L, φ is e-true in M iff φ's Finean translation is p-true in M.*

Consequently, the Finean translation function satisfies the logicality constraint given an eternalist view of the logic for *L* as the logic *e*-determined by some class of temporalized linear models and a presentist view of the logic for L^T as the logic *p*-determined by the same class.

So far so good. The Finean method of translation is nice, but it presupposes a certain view about the ontology of time which may be rejected by some presentists, and hence is not available no matter what once presentism is taken for granted. The logics for L^T *p*-determined by classes of temporalized linear models all validate

(E) $A\exists\tau(\tau=\tau)$,

which we can read 'There is always a time', and all instances of

(P) $A\forall\tau(\phi\supset A(\mathrm{Pres}(\tau)\supset\phi))$.[19]

That (E) and all instances of (P) should be accepted is in fact a necessary condition for the Finean method to be adequate. This is obvious for (E). As to (P), remember that the starting point of the Finean story was to define the locution 'at τ, …' as $A(\mathrm{Pres}(\tau)\supset\ldots)$. The requirement that all instances of (P) should be accepted is a direct consequence, given that definition, of the requirement that the following principle should itself be accepted:

Always, for every object *x*, if *x* is a time and so and so is the case, then at *x*, so and so is the case.

And that *this* should be accepted by a presentist is obvious, since presentists believe that 'for every object *x*' and 'for every present object *x*' are always equivalent.

Now, the view that presentists should accept (E) and all instances of (P) as true is objectionable. Presentism is compatible with two (mutually incompatible) views about the ontology of time which have some plausibility, *nihilism* and *monism*, and endorsing either view is incompatible with accepting the conjunction of (E) and of all the instances of (P).

The nihilist claims that there is no time at all, not even a present time, and so rejects (E). It is not too difficult to appreciate that a presentist can be a nihilist. A presentist whose ontology makes room only for elementary particles and events involving such particles, for instance, is arguably a nihilist. A presentist may indeed countenance an ontology which is so poor that nothing in that ontology could even remotely deserve to be called 'the present time'.[20]

The monist, in contrast, holds that always there is a unique time. He indeed endorses the stronger view that the present time—call it again '𝔫'—is such that

---

[19] Compare Fine (2005, p. 226).

[20] Notice that it is natural for a nihilist to endorse the view that there never was and never will be any time (what would be so special about present ontology, as opposed to past or future ontology, in respect to the existence of times?), and in so doing he would take all instances of (P) to be trivially true.

there never was, and there never will be, any time distinct from it. For him, 24 h hence will be different from now insofar as what will be then true at n is different from what is presently true at n, not insofar as there will then be a time distinct from n at which different propositions (or other truth-bearers) will be true. A monist cannot accept all instances of (P)—in fact, there are infinitely many instances he must reject. Suppose ϕ is true but only temporarily so: either it was false sometimes in the past or it will be false sometimes in the future. It is easy to see that monism is incompatible with the truth of the corresponding instance of (P). For suppose the instance is true. Then $\phi \supset A(\mathrm{Pres}(n) \supset \phi)$ is true as well, and given the truth of ϕ, $A(\mathrm{Pres}(n) \supset \phi)$ is true. By monism, there is always a unique time, and that time is identical with n, so that $A\mathrm{Pres}(n)$ is true. It follows that $A\phi$ is true, in contradiction with the assumption that ϕ is only temporarily true.

Thus the Finean method of translation cannot be used by a nihilist or a monist. A presentist who wants to use that method must be a *pluralist*, i.e. he should endorse, like the monist, the view that always there is a unique time, plus the following view:

(Pl)   Always, for every time τ, the following holds: (always in the past, for every time τ', $\tau \neq \tau'$) and (always in the future, for every time τ', $\tau \neq \tau'$).

(Pl) is indeed validated on any view according to which the logic for L^T is the logic p-determined by some class of temporalized linear models. For a pluralist, the monistic way of accounting for the difference between now and 24 h hence is all wrong; 24 h hence will be different from now because there will then be a time distinct from n at which different propositions (or other truth-bearers) will be true.[21]

Digression: Interpreting the Peacockean and the Vlachian Operators

Fine (1977, p. 144) suggests that the modal Vlachian operators are definable in terms of quantification over world propositions and the necessity operator. Fine's definition can straightforwardly be turned into a definition in terms of (actualist) quantification over worlds and the necessity operator, and this definition has a straightforward temporal counterpart for Vlach's temporal operators. Given the definability of the Peacockean operators in terms of Vlach's, the Finean definition straightforwardly yields a definition of the former. In what follows, I focus on Vlach's operators, but it will be clear how the case of the Peacockean operators should be dealt with.

---

[21] The nihilism/monism/pluralism distinction is also relevant to eternalism, although in this case, the characterization of monism as well as that of pluralism should be slightly modified. Both the monist eternalist and the pluralist eternalist hold that always, there is a unique *present* time. And while the monist holds in addition that there never was, and there never will be, any time distinct from n, the pluralist accepts the result of modifying (Pl) by replacing all occurrences of 'for every time' by 'for every *present* time'.

In a nutshell, the Finean idea is to define ↑… as ∃τ… and ↓… as $A(\text{Pres}(\tau) \supset \ldots)$. In order to be more precise, some definitions are needed. Say that in a formula of L^V, an occurrence o of ↑ *binds* an occurrence o' of ↓ iff (a) o' is within the scope of o, and (b) there is no occurrence o'' of ↑ within the scope of o and having o' within its scope. Let us then say that a formula of L^V is *nice* iff in that formula, every occurrence of ↑ binds some occurrence of ↓ and every occurrence of ↓ is bound by some occurrence of ↑. Given the role played by Vlach's operators, formulas of L^V which are not nice are deviant and hence can be ignored.

The Finean suggestion is that Vlach's operators can be defined in L^T via the following translation procedure:

Take ϕ nice in L^V.

1. If ϕ contains no occurrence of Vlach's operators, then ϕ's translation is ϕ.
2. Otherwise, ϕ's translation is the result of replacing each sub-formula of type ↑ψ in ϕ by ∃τψ , where ψ results from ψ by replacing each sub-formula of type ↓χ where the occurrence of ↓ is bound by the indicated occurrence of ↑ by $A(\text{Pres} (\tau) \supset \chi)$ (τ any variable for times).

It can be shown that for every nice sentence ϕ of L^V and any temporalized linear model M, ϕ is *p*-true in M iff it's translation under that scheme is *p*-true in M. Notice that the translations of the formulas of L via the Vlachian scheme are nice, and that combining the Vlachian scheme with the translation procedure just described yields the Finean scheme.

The Finean definition can be seen as a special take on an interpretation of Vlach's operators in terms of the notion of truth at a time, which I find particularly attractive, on which ↑… is to be understood as 'there is a time such that …' (or 'the present time is such that …') and ↓… as 'at that time, …': the definition can be seen as resting on that interpretation plus a special view as to how 'at that time, …' is to be understood. I like this interpretation, but I am not so happy with the Finean take on expressions of type 'at that time, …', because its adequacy turns on which tense logic is taken for granted. I would prefer to regard 'at time τ, …' as a primitive. Be it as it may, the availability of both the Finean interpretation and the one which takes 'at time τ, …' to be primitive turns on which conception of the ontology of time is countenanced—these interpretations are incompatible with nihilism and monism, they require pluralism—and this may be taken to speak against them.

Linear Time: The Metric Method

All the methods of translation presented so far have their drawbacks. The Finean method is not neutral regarding the ontology of time since it requires pluralism. Both the Peacockean and the Vlachian methods make use of linguistic devices which raise doubts: the complaint is that these devices need to be given a proper semantics, and it is hard to see how this can be done without appealing to times

other than the present time—which is of course something presentists cannot do. In response to this worry I pointed to two ways of interpreting the Vlachian operators, and so indirectly the Peacockean operators, but I stressed that they too require pluralism to hold and hence are not ontologically neutral. Can we do better?

We can. Philosophers of time often use and discuss tense-logical operators, and most of the time the operators used or discussed are the so-called 'Priorean operators' 'sometimes in the past' and 'sometimes in the future' (and those definable in terms of these operators and truth-functional connectives). Yet, as Prior himself taught us, there is a wide variety of tense-logical operators, differing in various ways, in particular in their logical properties. Among the operators which have been largely neglected since Prior first discussed them in some details in his (1957, Chap. II) are the so-called 'metric operators', and it is these operators that what I called the *metric* method of translation invoke.

The Priorean operators allow us to talk about what happened in the past and the future, but not about what happened or will happen *a certain amount of time* in the past or in the future. The metric operators do. A metric operator is associated with a unit for measuring temporal intervals, e.g. the second or the day, and it takes a name for a number and a sentence to make a sentence. Choosing the day as unit, the metric operators are two, '— days in the past, …' and '— days in the future, …'. Which numbers (e.g. the positive natural numbers, the positive rationals or the positive reals) one allows to be designated by what fills in the '—' slot in a metric operator depends on one's take on the logic of these operators. Obviously, the Priorean operators are definable in terms of their metric mates: 'sometimes in the past' can be defined as 'some number of days in the past' and 'sometimes in the future' as 'some number of days in the future'.

Each of the methods of translation we met so far interprets the eternalist's 'there are objects *x*' by means of 'sometimes, there are objects *x*' and has its own way of cancelling the temporal shift induced by 'sometimes'. The shifts induced by the Priorean operators, and hence by 'sometimes', are of unspecified length. In contrast, the metric operators induce shifts of definite length, towards the past or towards the future, and accordingly they provide a nice way of inducing and subsequently cancelling shifts. The basic idea of the metric method of translation should be straightforward: understand the eternalist's 'there are past objects such that …' as 'for some *n*, *n* days in the past, there were (then present) objects such that, *n* days in the future, …', and in a similar way, understand the eternalist's 'there are future objects such that …' as 'for some *n*, *n* days in the future, there will be (then present) objects such that, *n* days in the past, …'.

We can take the home language L^M of the metric reduction to be L enriched with variables for numbers n, n', n'', …, and for each such variable n, a pair of operators P_n (for '*n* days in the past') and F_n (for '*n* days in the future').[22] The metric translation procedure can then be specified as follows:

---

[22] Given the definability of the Priorean operators in terms of the metric operators, the home language could be taken to be L^M minus P and F, but in order to keep things simple I leave things as they stand.

THE METRIC TRANSLATION PROCEDURE:

Take ϕ in L.

1. If ϕ contains no occurrence of $\exists$, then ϕ's translation is ϕ.
2. Otherwise, ϕ's translation is the result of replacing each sub-formula of type $\exists x \psi$ by $(\exists n P_n \exists x F_n \psi) \vee (\exists x \psi) \vee (\exists n F_n \exists x P_n \psi)$ (n any numerical variable).

The semantics for L^M is slightly more complicated than the semantics for the other languages we already met, due to the need to interpret the numerical variables. It can be described as follows[23]:

A subset N of $\mathbb{R}^+$ (the set of all non-negative reals) is said to be *closed* if $0 \in N$ and for all $\mu, \nu \in N$, $\mu + \nu \in N$. Where $F = \langle T, < \rangle$ is a linear frame, a *measure* on F is a tuple $\langle N, d \rangle$, where $N \subseteq \mathbb{R}^+$ is closed and d is a total function from T^2 to N, such that:

- $d(t, u) = 0$ iff $t = u$.
- $d(t, u) = d(u, t)$.
- $d(t, u) + d(u, v) \geq d(t, v)$.
- If $t < u$ and $u < v$, then $d(t, u) + d(u, v) = d(t, v)$.
- For all $t \in T$ and $\mu \in N$ distinct from 0, there is a $u \in T$ such that $u < t$ and $d(t, u) = \mu$, and there is a $v \in T$ such that $t < v$ and $d(t, v) = \mu$.

A *measurable linear frame* is a linear frame on which there exist measures, and a *measurable linear model* a linear model whose frame is measurable. An *m-linear frame* is a tuple $\langle T, <, N, d \rangle$, where $\langle T, < \rangle$ is a measurable linear frame and $\langle N, d \rangle$ a measure on it. And finally, an *m-linear model* is a tuple $\langle \pi, T, <, N, d, D, I \rangle$, where $\langle \pi, T, <, D, I \rangle$ is a linear model and $\langle T, <, N, d \rangle$ an *m-linear frame.*

Let $\langle \pi, T, <, N, d, D, I \rangle$ be an *m-linear model. Given the intended interpretation of the subscripted Ps and Fs, we require the assignments to assign members of N distinct from 0 to the numerical variables. The truth predicate $\models^p$ for L^M is then defined in the same way as for L, with the following truth clauses for the indexed operators:

- $M, \rho, t \models^p P_n \phi$ if for some $t' \in T$ such that $t' < t$ and $d(t, t') = \rho(n)$, $M, \rho, t' \models^p \phi$.
- $M, \rho, t \models^p F_n \phi$ if for some $t' \in T$ such that $t < t'$ and $d(t, t') = \rho(n)$, $M, \rho, t' \models^p \phi$.

p-truth in an *m-linear model is defined as before by reference to its present time.
One can show that

Proposition 4. *Given any measurable linear model M, any measure on its underlying frame and any sentence ϕ of L, ϕ is e-true in M iff ϕ's metric translation is p-true in M endowed with that measure.*

Where $\mathfrak{M}$ is any class of measurable linear models, say that a class $\mathfrak{M}*$ of *m-linear models is *based on* $\mathfrak{M}$ iff $\mathfrak{M}*$ is obtained from $\mathfrak{M}$ by endowing each

---

[23] For the sake of simplicity the way I present the semantics is not completely general. For a more general treatment, see Hajnicz (1991). The adequacy result mentioned below still holds on the more general approach.

model it contains with some measure on that model. Then for any class $\mathfrak{M}$ of measurable linear models and any class $\mathfrak{M}*$ of m-linear models based on it, for every sentence ϕ of L, ϕ belongs to the logic e-determined by $\mathfrak{M}$ iff ϕ's metric translation belongs to the logic p-determined by $\mathfrak{M}*$. The metric translation function thus satisfies the logicality constraint, granted an eternalist view of the logic for L as the logic e-determined by some class of measurable linear models and a presentist view of the logic for L^p as the logic p-determined by a class of m-linear models based on it.

The metric method of translation is of great interest, since it altogether eschews the problems met by the three other methods previously discussed. True, it relies on the assumption that the metric operators make sense and on certain associated eternalist and presentist views about their logic, which, from the point of view of the formal semantics presented above, amount to the view that models for the languages involved should be measurable. Yet this assumption does not appear to be very substantial. The fact that this formal semantics involves models which informally represent past, present and future times at various temporal distances from one another is no more problematic than the fact that the presentistic formal semantics for language L presented in section "The Target Language, Its Semantics and the Corresponding Logics" involves models which informally represent past, present and future times: these are *formal* semantics whose role is to characterize logical validity, and one is not supposed to take seriously what their models informally represent.

It might be argued, though, that the metric method does not fare well in one respect when compared to the other methods: unlike those methods, it relies on quantification over numbers, which is ontologically costly. But this argument is very weak. First, notice that quantification over numbers is not, unlike quantification over past and future times, per se problematic for a presentist. There is nothing to prevent a presentist, e.g. to endorse a Platonist view about numbers and take them to always exist or even to tenselessly exist. Secondly, and more importantly, number talk—especially of the rudimentary sort needed to make sense of the metric operators—appears to be unavoidable, e.g. in empirical science, but also in ordinary activities, and so *if* such talk could be shown to be intelligible only if an ontology of numbers were countenanced, then the argument would be straightforwardly undermined. Finally, whether the intelligibility of number talk—again, especially of the rudimentary sort in question—*does* require an ontology of numbers is of course a question open to philosophical debate.

Branching Time

So far we have focused on eternalists and presentists who take the logic for L to be determined by some class of linear models. Let us now turn to conceptions of that logic as determined by some class of backward linear and forward branching

models. As the reader may have already noticed, this alternative conception will make an important difference at various junctures. Given the background provided by the previous sections, I will omit certain points of detail.

Basic Notions

We define a *branching frame* as a tuple $\langle T, < \rangle$, where

- T (times) is a non-empty set.
- $<$ (temporal precedence) an asymmetric, transitive relation on T, which has the following extra properties:
  - It is backward linear: for any t, t' and t'' in T such that both $t < t''$ and $t' < t''$, either $t < t'$ or $t = t'$ or $t' < t$.
  - It is connected: for any t and t' in T such that $t \neq t'$, either $t < t'$ or $t' < t$ or for some t'' in T, both $t'' < t$ and $t'' < t'$.[24]

A branching frame is by definition backward linear, but it need not be forward branching: linear frames are all branching according to the definition. Given the views of interest to us in this section, the classes of models relevant for character-izing the logics put forward by these views will comprise at least some, maybe only, models based on branching frames which genuinely branch towards the future.

Where $\langle T, < \rangle$ is a branching frame, a *history* relative to that frame is a non-empty set $h \subseteq T$ such that

- For any t and t' in h, either $t < t'$ or $t = t'$ or $t' < t$.
- For any t in h and t' in T, if either $t < t'$ or $t' < t$, then $t' \in h$.

Notice that by the connectedness of $<$, any two histories overlap. Let $<_h$ be the restriction of $<$ to history h. Then by the first condition in the definition of a history, $\langle h, <_h \rangle$ is a linear frame.

A *measure function* on branching frame $\langle T, < \rangle$ is a function μ which takes each history h of the frame into a measure $\mu_h = \langle N_h, d_h \rangle$ on $\langle h, <_h \rangle$, such that

- For all histories h and h' and all times t and t', if both $t \in h \cap h'$ and $t' \in h \cap h'$, then $d_h(t, t') = d_{h'}(t, t')$.

This condition is imposed in order to ensure that the distance between two times does not depend on which history comprising these times is considered. A *measurable branching frame* is a branching frame on which there exists a measure function. Finally, an *m-branching frame* is a tuple $\langle T, <, \mu \rangle$, where $\langle T, < \rangle$ is a measurable branching frame and μ a measure function on it.

---

[24] This condition ensures e.g. that a branching frame cannot be composed of two nonoverlapping trees.

Simple Semantics

There is a simple branching semantics for *L* which looks a lot like its linear counterpart. A *branching model* for *L* is a model for that language whose underlying frame is required to be branching rather than linear. *p*-truth is defined exactly like it was defined in the linear case. The case of eternalist truth is not so straightforward.

One can distinguish between two eternalist views on (absolutely unrestricted) existential quantification, a *strong* and a *weak* one. Say that a time *t* in a branching model is *accessible* from a time *t′* of that model iff either *t* = *t′* or *t* is before or after *t′* relative to the temporal precedence relation of the model. On the strong view about existential quantification, the range of the quantifier at any time in a model is the same, and it is the union of all time domains. On the weak view, the range of the quantifier at a time *t* is limited to the union of the domains of the times accessible from *t*, and this typically changes from one time to another.[25] We thus distinguish two eternalist notions of truth, *se-truth* and *we-truth*. *se*-truth is defined exactly like *e*-truth, and *we*-truth as well except for the truth clause for existential quantification, which has to be modified in order to conform to the weak conception. The remaining semantic notions are defined as before.

The branching models for L^P and L^V are just the branching models for *L*, and *p*-truth for these languages is defined in the same way as for *L*, but by going two dimensional. The notion of a *temporalized branching model* is defined in the obvious way, and L^T is now interpreted by means of these models in the same way as before. Finally, the models for L^M are the *m-branching models*, i.e. models of type $\langle \pi, T, <, \mu, D, I \rangle$, where $\langle \pi, T, <, D, I \rangle$ is a model and $\langle T, <, \mu \rangle$ is an *m*-branching frame, and the rest of the semantics goes the same way as before.

The Peacockean, the Vlachian and the Finean translation schemes still fare well given a weak eternalist view on existential quantification. In fact,

Proposition 5. *Given any branching model M and any sentence ϕ of L, ϕ is we-true in M iff ϕ's Peacockean translation is p-true in M,*

Proposition 6. *Given any branching model M and any sentence ϕ of L, ϕ is we-true in M iff ϕ's Vlachian translation is p-true in M,*

and

Proposition 7. *Given any temporalized branching model M and any sentence ϕ of L, ϕ is we-true in M iff ϕ's Finean translation is p-true in M.*

In contrast, the metric scheme is inadequate. This can be seen from the fact that $\exists n P_n \exists x F_n$, which is supposed to mimic quantification over past objects, induces problematic temporal shifts. Take for instance a monadic predicate Π of *L* distinct from = and Pres. Then the sentence $\exists x \Pi(x)$ is *we*-true in a branching model iff

---

[25] The distinction between the strong and the weak views on quantification can also be drawn in the modal case: on the strong view, the range of a quantifier at a world *w* is the union of all world domains, and on the weak view, it is the union of the domains of the worlds accessible from *w*. Forbes (1989, p. 29) advocates a weak conception in that context.

$\exists xN\Pi(x)$ is. In contrast, $\exists nP_n\exists xF_n\Pi(x)$ can be p-true in an m-branching model without $\exists nP_n\exists xN\Pi(x)$ being true in that model. No modification of the scheme can lead to something acceptable. The best that can be done is to mimic quantification over future objects by means of $\exists nF_n\exists xP_n$.

None of the four translation schemes is adequate if a strong eternalist view on existential quantification is assumed. The problem, in the case of the Peacockean and the Vlachian schemes, is that at a time t in a model M, $S\exists x$ acts like an existential quantifier over the union of the domains of all times of M *accessible from t*, whereas what is needed is an expression which acts like an existential quantifier over the union of *all* time domains. A correct result is obtained if we replace S in the Peacockean and the Vlachian translation procedures by SS: it can be shown that

Proposition 8. *Given any branching model M and any sentence ϕ of L, ϕ is se-true in M iff ϕ's modified Peacockean translation is p-true in M*

and that

Proposition 9. *Given any branching model M and any sentence ϕ of L, ϕ is se-true in M iff ϕ's modified Vlachian translation is p-true in M.*

The Finean scheme faces the same problem on the strong eternalist view, and the translation scheme should also be modified by replacing S by SS. This is not yet enough, though. The rendering of 'at τ, …' as $A(\mathrm{Pres}(\tau)\supset…)$ is no longer adequate, because where M is a temporalized branching model and ρ an assignment which takes τ into a time t of the model, all sentences of type $A(\mathrm{Pres}(\tau)\supset\phi)$ are vacuously true at any time t' which is not accessible from t. An adequate rendering of 'at τ, …' is as $AA(\mathrm{Pres}(\tau)\supset…)$. These modifications are sufficient to ensure that the version of Proposition 7 for the strong eternalist view holds:

Proposition 10. *Given any temporalized branching model M and any sentence ϕ of L, ϕ is se-true in M iff ϕ's modified Finean translation is p-true in M.*

The metric scheme is still bad on the strong view, for the same reason as before, and there is no way to fix the problem.

Alternative Semantics

On the simple branching semantics, truth is relative to times only, rather than to pairs comprising a history and a time of that history. Alternative semantics, e.g. the semantics presented in Prior (2002, pp. 126–127 and p. 132) and the standard supervaluational semantics as put forward in Thomason (1970) do relativize truth to such pairs. In this section, I will not discuss in details adequacy results for the various methods of translation we met once such alternative semantics are adopted. I will rather focus on the notion of truth at a history-time pair, which is common to all these semantics, and put forward some considerations which should be enough to see how things go.

In the simple semantics, the original nonmetric translation schemes achieve pseudo *weak* eternalist quantification by means of the expression $S\exists x$. Now consider a model, a history h relative to the underlying frame and a time t in this history. Relative to $\langle h,t \rangle$, P acts like an existential quantifier over the times earlier than t, and F like an existential quantifier over the times later than t *in history h*.[26] As a consequence, when a formula of type $S\exists x\phi$ is evaluated at $\langle h,t \rangle$, $S\exists x$ reaches only all past, present and future objects of h: the future objects of alternative histories which are not in the domain of a time in h (if any) are left aside. In order to remedy this problem, we need a way of shifting histories.

This can be done by using a historical possibility operator $\Diamond$ which, semantically, acts like a quantifier over histories containing the time of evaluation: a formula of type $\Diamond\phi$ is p-true at $\langle h,t \rangle$ relative to an assignment ρ in a model M (given stored time u if the language is Peacockean or Vlachian) iff for some history h' such that $t\in h'$, ϕ is true at $\langle h',t \rangle$ relative to ρ in M (given stored time u). Define $\bar{S}\phi$ as $P\phi \vee \phi \vee \Diamond F\phi$. The idea is to use $\bar{S}\exists x$ instead of $S\exists x$. But then we need a way to cancel the shifts in histories induced by the possibility operator.

Consider first the Vlachian method. The natural thing to do here is to invoke modal Vlachian operators $\Uparrow$ and $\Downarrow$, and go for the following procedure:

Take ϕ in L.

1. If ϕ contains no occurrence of $\exists$, then ϕ's translation is ϕ.
2. Otherwise, ϕ's translation is the result of replacing each sub-formula of type $\exists x\psi$ in ϕ by $\Uparrow\uparrow\bar{S}\exists x\Downarrow\downarrow\psi$.

As for the Finean method, the natural idea is to invoke variables for histories in addition to variables for times, plus an actuality predicate Act—the modal counterpart of Pres—and adopt the following procedure ($\Box$ is defined as $\neg\Diamond\neg$):

Take ϕ in L.

1. If ϕ contains no occurrence of $\exists$, then ϕ's translation is ϕ.
2. Otherwise, ϕ's translation is the result of replacing each sub-formula of type $\exists x\psi$ in ϕ by $\exists\eta\exists\tau\bar{S}\exists xA\Box(\text{Act}(\eta)\wedge\text{Pres}(\tau)\supset\psi)$ (η any variable for histories and τ any variable for times).

Finally, the natural thing to do on the Peacockean side is to invoke an actuality operator @ —the modal counterpart of N—and go for the following procedure:

Take ϕ in L.

1. If ϕ contains no occurrence of $\exists$, then ϕ's translation is ϕ.
2. Otherwise, ϕ's translation is the result of applying the following procedure to each occurrence o of a sub-formula of type $\exists x\psi$ in ϕ:
   (a) If o is not within the scope of an occurrence of P or F, replace o by $\bar{S}\exists xN\psi$.
   (b) If o is within the scope of an occurrence of P or F, replace o by $\bar{S}\exists xN_1\psi$ and replace the occurrence of P or F which immediately governs o by its indexed counterpart (if this has not yet been done).

---

[26] The qualification is useless in the case of P, since the times earlier than t are bound to be in h.

Of course, in each case, the semantics should be modified in order to handle the new modal vocabulary. The details are somewhat obvious up to a certain point, but there are some subtleties.

The modified Peacockean language contains the actuality operator @, and accordingly its semantics will have to endow each model with a distinguished actual history comprising the present time of the model—a 'thin red line', as Belnap and Green (1994) would call it. In addition, the adequacy of the proposed translation scheme requires that *p*-truth in a branching model be defined by reference to the present time and the actual history of the model.

In contrast, the semantics for the modified Vlachian and the modified Finean languages can, but need not endow the models with distinguished histories. If they do, then can define *p*-truth in the same way as in the semantics for the modified Peacockean language. Alternatively, an option which does not require the introduction of actual histories is to go supervaluationist: define *p*-truth in a model as *p*-truth relative to the present time of the model whatever the history which comprises it. Another such option is to define *p*-truth in a model as *p*-truth relative to every history-time pair $\langle h,t \rangle$ such that $t \in h$. (The latter notion of truth is actually a notion of logical validity.)

In the simple branching semantics, the modified nonmetric translation schemes achieve pseudo *strong* eternalist quantification by means of $SS\exists x$. When a formula of type $SS\exists x\phi$ is evaluated at a history-time pair $\langle h,t \rangle$, $SS\exists x$ reaches only all past, present and future objects of h: $SS\exists x$ is indeed equivalent to $S\exists x$. A possibility operator $\Diamond$ can be invoked again. The idea is now to use $S\overline{S}\exists x$ instead of $SS\exists x$, and then to appropriately introduce mechanisms to cancel historical shifts. The Vlachian and the Peacockean translation procedures described above should be modified simply by replacing $\overline{S}$ by $S\overline{S}$. As to the Finean procedure, $\overline{S}$ should also be replaced by $S\overline{S}$, and in addition, $A\square$ should be replaced by $A\square A$. The previous considerations on *p*-truth in a model still apply here.

Interestingly, the metric method fares well on the new branching semantics in the case of weak eternalist quantification. One can adequately translate $\exists x\phi$ into

$$\exists nP_n\exists xF_n\phi \vee \exists x\phi \vee \Diamond\exists nF_n\exists xP_n\phi.$$

When that sentence is evaluated at a history-time pair $\langle h,t \rangle$, $\exists nP_n\exists x$ acts like a quantifier over all the objects of the domains of the times preceding *t*, and given that the history is fixed, F_n appropriately cancels the temporal shift induced by $\exists nP_n$; and in addition, $\Diamond\exists nF_n\exists x$ acts like a quantifier over all the objects of the domains of the times following *t*, and given that the relevant frames are backward linear, P_n appropriately cancels the historico-temporal shift induced by $\Diamond\exists nF_n$. In contrast, the method is not effective in the case of strong eternalist quantification.

A few remarks before concluding this section.

The complaints levelled against the use of Vlach's operators or the temporal Peacockean operators by presentists discussed in section "Linear Time: The Peacockean

and the Vlachian Methods" have counterparts which concern the use of the modal mates of these operators. It will be said that these modal operators cannot be understood if not in terms of quantification over merely possible histories. But notice here that if the claim is taken for granted, then the problem is for *actualists* and does not concern those presentists (if any) which are possibilist. In any case, actualist presentists can avail themselves of a line of response akin to the one put forward in section "Linear Time: The Peacockean and the Vlachian Methods": the Vlachian and the Peacockean operators are devices which formalize binding phenomena between locutions expressing historical possibility or necessity and 'actually' in one of its uses.[27] And of course, the interpretations of the temporal Vlachian and Peacockean operators presented in section "Linear Time: The Finean Method" have counterparts for the corresponding modal operators.

The distinction between nihilism, monism and pluralism put forward in section "Linear Time: The Finean Method" has a modal counterpart: the modal nihilist claims that there is no history; the modal monist holds that, (1) necessarily, there is just one history and (2) the actual history is such that, necessarily, every history is identical to it; and finally the modal pluralist accepts (1) and denies (2). Clearly, the interpretation of the Vlachian and Peacockean operators just alluded to requires modal pluralism, and the Finean translation procedure put forward in this section is available only to presentists which are pluralists in both the temporal and the modal sense. In contrast, of course, the last metric procedure introduced in this section does not require any particular take on these issues.

References

Belnap, N., and M. Green. 1994. Indeterminism and the Thin Red Line. *Philosophical Perspectives* 8: *Logic and Language*: 365–388.

Burgess, J.P. 2002. Basic tense logic. In *Handbook of philosophical logic*, ed. D. Gabbay, and F. Guenthner. Dordrecht: Kluwer.

Correia, F. 2007. Modality, quantification, and many Vlach-Operators. *Journal of Philosophical Logic* 36: 473–488.

---

[27] This is the point made by Forbes (see footnote 13)—although he does not have *historical* modality in mind.

Correia, F. 2009. Commentary on Arthur Prior. In *Past, Present and Future*, ed. R. Ciuni *Models of Time, Humana.Mente* 8: 177-184.

Fine, K. 1977. Postscript. In *Worlds, times and selves*, ed. A. Prior, and K. Fine. London: Duckworth.

Fine, K. 1985. Plantinga on the reduction of possibilist discourse. In *Alvin plantinga, reidel profile series 5*, ed. J.E. Tomberlin and P. van Inwagen. Dordrecht: Reidel.

Fine, K. 2005. The problem of possibilia. In *Modality and tense*, ed. K. Fine. Oxford/New York: Oxford University Press.

Forbes, G. 1989. *Languages of possibility*. Oxford: Basil Blackwell.

Forbes, G. 1992. Melia on modalism. *Philosophical Studies* 68: 57–63.

Hajnicz, E. 1991. *Time structures*. Berlin/Heidelberg/New York: Springer.

Hodkinson, I., and M. Reynolds. 2006. Temporal logic. In *Handbook of modal logic*, ed. P. Blackburn, J. van Benthem, and F. Wolter. Amsterdam: Elsevier.

Linsky, B., and E. Zalta. 1994. In defense of the simplest quantified modal logic. In *Philosophical Perspectives 8: Logic and Language*, ed. J. Tomberlin, 431–458. Atascadero: Ridgeview.

Melia, J. 1992. Against modalism. *Philosophical Studies*, 68: 35–56.

Peacocke, C. 1978. Necessity and truth theories. *Journal of Philosophical Logic* 7: 473–500.

Prior, A. 1957. *Time and modality*. Oxford/New York: Oxford University Press.

Prior, A. 2002. *Past, present and future*. Oxford/New York: Oxford University Press.

Prior, A. 2003. 'Now'. In *Papers on time and tense*, ed. P. Hasle, P. Øhrstrøm, T. Brauner, and J. Copeland. Oxford/New York: Oxford University Press.

Sider, T. 2006. Quantifiers and temporal ontology. *Mind* 115: 75–97.

Thomason, R.H. 1970. Indeterminist time and truth-value gaps. *Theoria* 36: 264–281.

Vlach, F. 1973. 'Now' and 'Then': a formal study in the logic of tense and anaphora. Ph.D. dissertation, UCLA.