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Chapter 1
Introduction

What lies in front of you, the reader, is a monograph appropriately called 'Dynamic Semantics'. After reading it, it will, I hope, be clear to you why it is called so, but before reading it the title may need some clarification. A most obvious interpretation of the term "Dynamic Semantics" may be that it is concerned with a dynamic style of semantic linguistic theorizing. If one would think of this monograph as exhibiting such a type of linguistic theorizing I would be very much flattered, but I don't think I could agree with the qualification, really. The monograph is dull, and intended to be dull. A less likely, but equally inappropriate, interpretation of the title would be that this monograph is concerned with a semantic theory of dynamic objects, like actions, processes, arrows and pop stars. This monograph, and the subjects it covers, are definitely not about that, even though arrows and pop stars, like stamps, numbers, and thoughts, are not excluded from the domains this monograph wants to include in its semantics. But in no way do these dynamic objects figure as primary targets. A more likely interpretation of the title would be that the monograph exemplifies a sort of semantic theory according to which meanings are dynamic. This, as well, is not the correct interpretation, but it deserves some special attention.

There are good, philosophical, linguistic, and computational, reasons for thinking of meanings, whatever they are, as some sort of dynamic entities. Meanings can be conceived of as proofs, or processes, or computations, or patterns, or potentials. There is a whole variety of dynamic objects which, as has been argued for in the literature, constitutes the kinds of things we are concerned with if we talk about meanings. Honestly, I am very sympathetic to these ideas, and I subscribe to them, but this conception of meaning is still not the one intended when I talk about dynamic semantics in this monograph. Such a dynamic conception of meaning will be left untouched throughout this monograph, even though it may, throughout, replace the static conception employed or assumed in the monograph. The monograph is NOT about dynamic meanings, even though you can, if you want, construe it that way.

So, then, what is the dynamic thing about the semantics which this monograph talks about, granted that we have some understanding of the term semantics? I will adopt a very conservative understanding of the term "semantics", which deals with the

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interpretation of “expressions” in some independently motivated domain of “meanings”. This assumes an abstract domain of expressions, to be uncovered by some syntactic theory, and a domain of meaning and use, to be covered by a pragmatic theory. The two assumptions are highly controversial, and will also not be left undisputed in the remainder of the monograph, but they are not the target of discussion. They provide a good starting point, if only from the current theoretical linguistic state of the art, and for the moment I would like to leave it at that. The “dynamics” of the semantics in this monograph does not lie in the meanings assigned to well-formed expressions, but, rather, in the composition of these meanings.

In accordance with a very intuitive and well-established tradition, and appropriately attributed to the mathematician and philosopher Gottlob Frege, linguistic constructions are composed of their parts, and so are their meanings. There are various reasons to get bewildered by this quite obvious observation. One of these is the equally obvious observation that the same constituent expression may figure in different compound constructions; another is that one and the same (compound) construction may contain multiple occurrences of one and the same constituent expression. (This does not happen with houses and the bricks they are built from.) This means that, even though we can agree on the idea or the notion of a constituent expression, and of its meaning (provided that we can make sense of these notions anyway) we still can question and discuss the various ways in which an expression with its meaning can combine or conjoin with another constituent expression. What some have labeled the dynamics of natural language, comes down to precisely this dynamic composition, or conjunction, of expressions and their meanings.

Almost all semantic theories are, willingly or unwillingly, dynamic. All theories consciously or inadvertently agree that interpretation is dependent on context, if only on the agent performing the interpretation, or on the language employed. Some may have doubts about the significance of this, and prefer to abstract away from this type of context dependence, but it can hardly be denied that interpretation processes, and co-occurring belief states are essentially indexical. We don’t want to go as far as proving that one exists from the premise that one believes, but it surely seems to be a presupposition that a spoken word cannot do without a speaker, that a written word doesn’t come without a writer, and that an interpretation requires an interpreter. Some universal features of natural language essentially reflect these facts. It appears that all natural languages have either grammaticalized their personal, spatial and temporal dependence, if they haven’t made it part of their default meaning. And also all interpreted formal languages, if fully and appropriately specified in the right handbooks, have to relate their key concepts relative to a language and a model for that language—normally the language with its interpretation *currently* discussed in the handbook. Some things are so essential that they easily go unnoticed.

First and second person pronouns, indexicals or demonstratives, third person pronouns, tenses and temporal adverbs, all display essentially contextual aspects of meaning, which, nevertheless, work in a cross-contextual way. We can quite successfully state the meaning of the Dutch first person pronoun “ik” by saying that it, always, refers to the speaker. This may be many speakers, an in principle unbounded number of them, but it is still one meaning. Yet it appears that “ik” never means, or should be intended to mean, what "the speaker" means. When I say that it is not surprising that I am short-sighted, for instance because it is not surprising that I am, I do not mean that it is not surprising that the speaker of my utterance is short-sighted, because why should a short-sighted person suddenly say so? Basically the same observations pertain to the temporal reference in Arthur N. Prior's "Thank Goodness that's over." Or to take an example from Peter T. Geach, if everybody thinks that he is clever, so if I think I am clever, and you think you are clever too, and everybody thinks so, then what exactly is the very same thing that everybody is thinking?

The pronouns from natural languages, and the variables from formal languages, share the feature of being so context-driven that they seem to be basically useless. In practice, they are so essential that it is difficult to do without them. It may require some first graduate training to indeed *read* the predicate logical formula "$\exists x(STUx \wedge \neg\exists y(PROy \wedge ADMxy))$" as a way of rendering the meaning of "Some student admires no professor." It takes, it seems, a genius to read Willard van Orman Quine's variable free equivalent $\mathcal{E}(\mathcal{R}(STU \times \mathcal{N}(\mathcal{E}(\mathcal{R}(\mathcal{I}(PRO \times ADM))))))$ the same way. (Quine's rendering only involves a couple of logical operations on the predicates STU, PRO and ADM.) Pronouns, I believe, are not only essential, but also essentially practical.

Historically, the discussion about the dynamic composition of meanings has focused on linguistic constructions with pronominal elements, or with open places, or expressions which are otherwise incomplete. Surely it is easy to make fun of a dynamic semantic enterprise by saying it deals only with pronouns which are words of length 3 ('she') or less ('he', or 'I', or 'Ø'). But once one realizes the 'essential indexical' nature of natural language, as e.g., Saul Kripke, John Perry, David Lewis and recently François Recanati have observed, then the indexical, or referential, or anaphoric potential of expressions is not at all so trivial. The quite obvious fact that one and the same expression, even under one and the same analysis, may have different interpretations in different contexts has far-reaching logical consequences. Aristotle's most beloved syllogism Barbara fails in the presence of pronouns as we will also see in Chap. 2 of this monograph. If we act like those who followed Gottlob Frege, but not like Frege himself, we might blame natural language and its anaphoric devices for being imperfect, and get them out of the way; if we, however, want to live with our situated nature, we may have to face the logical complications of the practical merits of having pronouns. This is what this monograph is about. I hope to show to the reader that even a very superficial analysis of pronouns does complicate our logic, yet does not make it illogical, and that the phenomena do not to force us to change our concept of meaning, even if one may of course find other reasons to do so.

Formally speaking I do little more than the following. In Chap. 2 I extend the architecture of interpretation of predicate logic with a category of pronouns. Why do I do this? In the first place, first order predicate logic is the most minimal, well-behaved and well-studied logical formalism that can be taken to model natural language structures besides those of its logical connectives, or their counterparts. Taking a liberal view on the kinds of things one may quantify over, its expressive power is quite impressive indeed. Adding pronouns essentially means adding context dependence, context change, and indexical reasoning. The major endeavour in this chapter is to see the logical consequences and practical merits of extending a standard architecture in a systematic and precise way.

The resulting system is dynamic, not because the meanings are dynamic, but because the composition of meanings is dynamic. In the basic system only the propositional conjunction is dynamic, and, as a consequence, the derived notion of implication is dynamic, as well as the ensuing notion of entailment. I will extensively discuss the logical consequences of extending first order predicate logic this way.

The next Chaps. 3 and 4 show that such a conservative and minimal extension paves the ground for generalizations in the same spirit: minimal and conservative. In Chap. 3 the reader will find an account of pronouns in updates of information and a speaker's support for the same kind of information. The extensional architecture from the first chapter is lifted to an intensional one in a fairly standard way, and it automatically generates a first order analysis of content, information and information exchange. One benefit is a fully formalized account of what has become known as "Peirce' Puzzle."

Chapter 4 discusses extensions with generalized quantifiers and first focuses on the dynamic composition of set denoting expressions. The main aim here is to show that, in spite of what is suggested by most rival approaches, no further complications need to arise from such extensions with generalized quantifiers. The so-called dynamics of generalized quantifiers entirely resides in the dynamics of composing meanings, if it resides anywhere, and not in their meanings.

In Chap. 4 also modal expressions are discussed, especially attitudinal operators and epistemic modals. This chapter heavily draws from Maria Aloni's sophisticated use of individual concepts, which are set to use in accounts of puzzles surrounding *Knowing Who*, *de re* knowledge and beliefs, including a treatment of Ortcutt-sentences and also Hob/Nob-examples and similar creatures. The chapter concludes with a classical treatment of modalities and a substantial treatment of (epistemic) modalities in discourse.

In the final Chap. 5 I will try and collect the findings of the previous ones. They are, first and foremost, that not only a Montagovian approach to the meanings of natural language expressions stands up to several challenges leveled against it, implicitly, or explicitly, but moreover that the challenging data brought to the debate are best tackled indeed from the given old-fashioned paradigms. Against all odds, the data can be handled without needing to resort to fancy conceptions of meanings that are dynamic, sentences which denote situations, or pronouns which are variables. A claim that is difficult to make hard and precise, but I hope which this monograph succeeds in communicating, is that the old paradigms even help in formulating the relevant issues more transparently than fancy alternatives do. Again I must qualify the last moral in the sense that I do sincerely believe that dynamic meanings and situations belong to a future we cannot escape from; my only point is that they should be conceptualized properly first.

This book is meant for both graduate students and colleagues working in logic, language and AI. The prerequisites are familiarity with first order predicate logic and with intensional semantics. How to read (or not read;-) this book? Chap. 2 is crucial for the reader to decide to read or not read further. Most of the topics discussed in the next two chapters can be read independently of each other, but it may hamper the understanding of formal details—though hopefully not of the accompanying prose. Chapter 5 summarizes the findings as they have been alluded to in this introduction.

Chapter 2
Predicate Logic with Anaphora

Dynamic semantics has often been preoccupied with pronouns. Pronouns are essential in interpretation and they succeed in oiling the wheels of efficient linguistic information exchange. They are naturally involved in all basic features of ordinary language, like reference, coreference, indexicality, modality, belief, and presupposition. Pronouns are ubiquitous. (In English, and closely related languages, that is.) But they almost go unnoticed when they are, as they usually are, conveniently accommodated in a context. Taken out of their natural habitat, however, they speak up. Dynamic semantics provides the means to get a grip on their role in the flow of information.

In this first substantial chapter of this monograph I lay out the basic framework, the system of Predicate Logic with Anaphora (*PLA*). It is a system of interpretation which extends ordinary first order predicate logic just to capture the results on anaphoric relationships achieved within *Discourse Representation Theory* (*DRT* , Kamp 1981; Kamp and Reyle 1993), *File Change Semantics* (*FCS*, Heim 1982), *Situation Semantics* (*SitS*, Barwise and Perry 1983), *Game Theoretical Semantics* (*GTS*, Hintikka 1983), proof-theoretic semantics (*PTS*, Sundholm 1984), and *Dynamic Predicate Logic* (*DPL*, Groenendijk and Stokhof 1991). I will take time and space to explain and motivate this system even though it covers worn-out results. (The first presentation of *PLA* dates from 1994 (Dekker 1994), and related formal literature dates back to the early eighties of that century.)

The mentioned interpretational architectures, *DRT* , *FCS*, *SitS*, *GTS*, and *DPL*, all have their own sound, formal and intuitive, motivation for deviating from what may have been the paradigm of truth-conditional semantics, the prime formal semantic paradigm of the latter half of the twentieth century, conceived by Alfred Tarski, conceptualized by Donald Davidson, rigourously formulated by Richard M. Montague and implemented by others, e.g., Theo M.V. Janssen. The reasons for departing from this paradigm have been manifold, ranging across the intuitive insights that meanings are computed (*DRT*), that interpretation is essentially tied to the update of information (*FCS*, *DPL*), and that its use is essentially tied to situated agents (*SitS*, *GTS*).

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There is no reason to complain about any of these enterprises. Still, it seems that all five of them come with the implication, suggestion, or even slogan, that meaning is something dynamic. Again, I do not at all object to such a supposition. On the contrary I do hold the same supposition, but I do believe that it is not motivated by any of the linguistic data these theories aim to account for.

Meaning is not dynamic in any intuitive sense of the word. In ordinary language, if we talk about the meaning of an expression, simple or compound, it is simply just that: the meaning, if any, of the expression. Meanings do not do things. If meanings would be able to do things, change my commitments or beliefs, I would not want to use them, and advise you not to do so either. Interpretation, on the other hand, is, in a certain specific sense, dynamic. If interpretation is conceived of as the assignment of meanings, it is, by this very description, a dynamic thing. It involves an agent, the interpreter, and a process, the assignment of meanings.

Logically speaking, and also from a theoretical linguistic perspective, there does not seem to be independent reason to be interested in this dynamic process. It is the concern of the psychologist, or sociolinguist, or ethnographer, to see how agents actually interpret utterances. This is not, or does not seem to be, the objective of the theoretical linguist. Surely, any serious linguist wants to say something about the meanings of expressions, and how they relate to how these expressions are interpreted, or could be interpreted, or should be interpreted. In this sense, indeed, the so-called ‘interpretation’ is a matter of concern for the theoretical linguist, but it comes to be something of a fossilized notion. The idea of ‘the interpretation of an expression’ is something put forward, be it in a normative or descriptive way, but once conceived thus it is no longer anything dynamic. Besides, that is, from one fundamental insight. The fact that meanings really have to do with actual interpretations, intended interpretations, or conventionalized interpretations means that there are or can be systematic aspects of use that a semantic theory should account for, and such a theory, then, may appropriately be called ‘dynamic’.

Peter F. Strawson rightly claimed that names do not refer. How could they, linguistic types or tokens, do that? It is normally humans or other agents that refer, and significantly they do this by using names—or variables. If, on an account of proper names like that of John Stuart Mill, or Saul Kripke, we say that a certain type of name refers, rigidly, if you want, and statically, to a certain type of individual, you can, if you want, equate this with saying that it is used to refer, generally, rigidly, but dynamically, to that particular individual. Not much seems to be gained by stating what seems to be the same fact in a static or dynamic way. It is, or seems to be, quite a different fact that certain expressions, in different contexts, can be used to refer to different individuals in each of these contexts. As indicated above, indexical expressions can be typically used so, and pronouns, in general do so all over the place. More intriguingly, the kinds of context which seem to define, or at least determine, the specific interpretations of these terms, can be entirely linguistic as well.

This point, doubtlessly, must have contributed to the fascination of linguists for the typical use of certain terms to ‘introduce’ discourse referents, and for the use of other terms, pronouns, to typically pick them up. Any elementary presentation of *DRT* , *FCS*, or *DPL*, focuses on especially this phenomenon, originally observed by Lauri Karttunen. In certain contexts certain terms, like indefinite descriptions, introduce discourse referents, and subsequent expressions refer back to them. It is interesting to see that this may happen without there being a decisive answer to the question which individuals are actually referred to. Often it is just "the one previously mentioned", also referred to as "the such and such who did so and so" without there being any implication that there was one and only such and such who did so and so. No matter how these discourse referents, their introduction, revitalization, and death are actually conceived of, they have shown to be a pertinently useful device in the treatment of a vast number of phenomena involving quantifiers, tense, aspect, and models, as shown in the works of Hans Kamp, Nicholas Asher, Reinhard Muskens, Maria Bittner, and Adrian Brasoveanu.

While the idea of introducing and picking up discourse referents has been very appealing and successful in the presentation of the dynamic systems of interpretation mentioned above, Henk Zeevat (1989) has shown at a quite early stage that it does not presuppose a dynamic notion of meaning in the ordinary sense. The results of *DRT* , like those of *FCS*, and *DPL*, can be fully captured by means of standard algebraic operations on suitably structured meanings, statically conceived. More recently, Max Cresswell has defended the point, from both a philosophical and linguistic point of view. In response to certain phenomena the analysis of which has been argued to require a dynamic treatment he sets out: "The purpose of this article is (…) to question the need for any change in the basic aims of semantics." (Cresswell 2002). And even though Zeevat's algebraic operations are not dynamic, intuitively speaking, a proper assessment of the system presented in this chapter will show that there really are dynamic ways of composing meanings. Arguably, Cresswell's observations point in the same direction. This intuition then will be key to the system presented in this chapter, as well as a more technical insight from Kees Vermeulen. In one or another way, the three explicitly dynamic systems of *DRT* , *FCS*, and *DPL*, deal with updates of meanings which are sets of variable assignments. For certain technical reasons Vermeulen has shown we can do with sequences of values as our semantic objects, and define dynamic notions of compositions on those. Actually, this is what we will do in the remainder of this chapter.

Before we really start let me briefly note on the status of the 'examples' in this chapter. The system of *PLA* is meant to eventually evolve into a formal language adequate for the formulation of the meanings, or structurally interesting aspects of meanings, of expressions of natural language. With this objective in mind, I suggest the basic language (i.e., that of *PLA*) to already model some such aspects, with a formal conjunction as a model for coordinating operations in natural language, and, significantly, an existential quantifier modeling the contribution of indefinite descriptions. Needless to say how poorly this is done with first order predicate logical means only. Even so, I will also use natural language paraphrases to illustrate such deliberately defective uses of our logical devices. In this chapter, these paraphrases only serve an expository purpose, to explain the basic devices. The examples, therefore, will be rather stilted.

Paying debt to the prehistory of *PLA*, and in order to set the stage, I will start with a very concise overview of the architecture of *DRT* , and the system of *DPL*. These systems have evidently served as the main inspiration for the system presented in this book and more in general they have been extremely influential in the relevant literature of the past 20 years or so.

2.1 Static and Dynamic Semantics

The system of *DRT* was first presented in the innovative (Kamp 1981), appropriately called ‘a theory of truth and semantic representation’. It is a two-layered architecture in which, first, sentences from a natural language discourse are mapped into so-called *discourse representation structures* (*DRS* s), which represent the semantic contents of the current discourse, and extend it with the contents of the sentence next processed. On a second level these *DRS* s are assigned Tarski-style truth conditions, specifying under which circumstances the discourse representations are true, and also in what way.

I will not go into the way in which a discourse formulated in a stylized fragment of natural language gets mapped into these *DRS* s, and refer the reader to the handbooks (Kamp and Reyle 1993; Kamp et al. 2011) for an extensive treatment. I do want to say something about the language in which these *DRS* s are formulated, because that will enhance the comparison, technically, and philosophically, with the system presented later in this section.

In most run-of-the-mill applications of *DRT* the *DRS* -language is that of a special kind of a first order predicate logical language, in which atomic conditions, negations, disjunctions and implications, are always preceded by a (possibly empty) sequence of existential quantifiers. (Many practical applications employ a multi-sorted language, which not only quantify over a first order domain of individuals, but also over states, events, times, worlds, and what have you. More theoretically inspired implementations of *DRT* also have second and higher order reference and quantification, as well as lambda-abstraction. For the purposes of this monograph we can safely assume such generalizations.) These ‘formulas’, or structures, are very perspicuously and conveniently displayed, pictorially, as boxes, consisting of two parts, representing a domain of individuals under discussion, and a series of conditions imposed on them. They are typically (and schematically) displayed as follows:

- $$\begin{array}{|c|}\hline x_1, \ldots, x_n \\ \hline \phi_1 \\ \vdots \\ \phi_m \\ \hline \end{array}$$

In this pictorial representation, the $x_1, \ldots, x_n$ are ‘discourse markers’, which represent or keep track of the subjects of a discourse under consideration. These subject are also called ‘discourse referents’. They are no ordinary referents, because their specific identity may be left undecided, and in more involved discourse representation structures they may become quantified ‘from the outside’. (For a more logically oriented person, they are, hence, like variables.) The $\phi_1, \ldots, \phi_m$ are conditions on the values of these discourse markers, conditions on the discourse referents exposed in the discourse at issue. These conditions may atomically ascribe properties of and relations between the discourse referents, but they can also be compound, for instance the negation of an embedded *DRS* , or the statement of an implication relation between two *DRS* s. The *DRS* s are supposed to be partial models of reality, as construed on the basis of a discourse under construction, but the whole language is easily seen to be as expressive as first order predicate logic itself.

The statement of the *DRS* -language just now given is quite unspecific, but I hope sufficient for the remainder of this monograph. One thing, however, needs to be said about how it is used, especially in the original (Kamp 1981). The idea, in that paper, is that the interpretation of a discourse starts from an empty representation—because nothing has been said yet. (Obviously, it could be a representation of the shared assumptions of the participants in the discourse.) The idea about what happens is that any syntactically analyzed sentence uttered in a discourse gets added to the representation of the preceding discourse, and then decomposed into new discourse markers, atomic conditions, and compound ones. Thus the interpretation algorithm, also called 'discourse representation construction algorithm', yields a discourse representation structure of the contribution of the utterance relative to the preceding context, with an update (*DRS*) of the contents of the discourse. This of course sounds very natural, but there is a very interesting twist to it. By manipulating discourse representational structures, the system is capable of suitably establishing connections between subjects or discourse referents mentioned in one contribution, and subjects brought up earlier. Because the *DRS* s, as models, are partial, these connections can be efficiently established, even though the identity of the specific referents is still unresolved. Interestingly, this is the major, and sophisticated, innovation of *DRT* , and by the same token one of the main sources of criticism.

Jeroen Groenendijk and Martin Stokhof have argued that *DRT* commits itself, formally, to an indispensable level of representation in the interpretational architecture. (Hans Kamp has himself motivated this commitment, but does not seem to think it unconditionally indispensable for all purposes any longer.) The complaint, in their paper (Groenendijk and Stokhof 1991), is not that a level of representation is unrealistic, but that a semantic architecture should not be committed to it—or, more to the point, that the data that motivated the inception of *DRT* , do not force us to a representational architecture.

The paper by Groenendijk and Stokhof takes a constructivist stance. (The term 'constructivist' here is used in its ordinary, not in its logical or mathematical sense.) They present an interpretation of the language first order predicate logic, which is dynamic, but arguably not representational, and which accounts for the kind of discourse phenomena that *DRT* was originally meant to account for. The philosophical, or methodological, discussion will be postponed to Sect. 2.4. Here I want to briefly survey the main features of the system of *DPL* as presented in Groenendijk and Stokhof (1991).

Groenendijk and Stokhof combine, in a very interesting and sophisticated way, philosophical insights from Stalnaker (1978), with tools and results from the field of artificial intelligence, in particular, the semantics and verification of programming languages. The leading idea is that sentences, or better, descriptive utterances, are not just independent means to characterize the world as being a certain way. Rather, they are context dependent acts, which are meant to change the contexts. As in *DRT* , assertions are taken to be contributions to what is assumed to be the content of the discourse in which they occur, but unlike the way in which this is done in *DRT* , these contributions are not taken to be updates of representations of the content, but updates of the content itself. In order to settle the main theoretical point, and also guided by the preceding linguistic discussions, Groenendijk and Stokhof deliberately restrict themselves to the dynamics of establishing anaphoric relationships in discourse. However, as I have argued above, and will argue below, the dynamics of establishing these relationships can be taken to be paradigmatic for the dynamic establishment of structure in discourse in general.

One way of introducing the *DPL* concept of meaning consists in taking a procedural view upon the interpretation of ordinary first order predicate logic. Consider the following example, with its first order rendering.

(1) Mary borrowed a copy of *Naming and Necessity* from a professor in linguistics.
$\exists x(CNx \wedge \exists y(PLOyx \wedge BORmxy))$.

The translation key, simplified of course, runs as follows.

- $CNx := x$ is a copy of *Naming and Necessity*;
  $PLOyx := y$ is a professor in Linguistics who owns x;
  $BORmxy := m$ borrowed x from y.

How can we evaluate this sentences in a model $M = \langle D, I\rangle$, where D is the domain of individuals under discussion, and I gives us an extensional interpretation of the individual and predicate/relational constants? We try and find a valuation of the variable x by means of a variable assignment g, so that it satisfies CNx; precisely, so that $g(x) \in I(CN)$. As long as we interpret x as something which is not a copy of *Naming and Necessity*, we try and find another value of x to see whether that is one that is such a copy. If we never find any such copy in our model, we report, "this does not work, the formula is false." However, if we can find a value for x which is such a copy, we continue. We then try and find a value for y in our model, such that is a professor in linguistics; moreover, one that owns the value of x, the copy of *Naming and Necessity*. If we do not find such a professor, we give up on the chosen value for x, and try and find another copy of *Naming and Necessity*, as a value for x, and a professor in linguistics, as a value for y, who owns that copy of the book. If all of this fails, we render the formula false again, but if we can get through, we continue. We finally test whether our value for x ($g'(x)$) is something that Mary ($I(m)$) borrowed from the professor ($g'(y)$). If it does not, we have to redo the whole procedure again, and if the procedure never succeeds, we, again, have to say, "Sorry, the formula is false." (In this model.) However, if we do succeed, we can happily report that the formula is true in the model, and that is all we need to hear—in our first order predicate logic. *DPL*, however, does not stop here with only returning the truth-value true. Rather, it remembers that our $g'(x)$ is that copy of *Naming and*

Necessity, which is owned by $g'(y)$, who is a professor in linguistics, and such that particular professor $g'(y)$ owns that copy $g'(x)$. Having these witnesses $g'(x)$ and $g'(y)$ is useful, because we can refer back to them. Consider a continuation of the discourse, with the following sentence, for instance.

(2) It$_x$ was covered with comments and exclamation marks. The professor$_y$ must have studied it$_x$ extensively.
$(Cx \wedge SIyx.)$

(Cx rendering that x was covered with all these marks, and $SIyx$ that y must have studied x intensively.) Because *DPL*, in a sense, "remembers" the witnesses for x and y that satisfy the previous sentence, it can take up on that. It merely adds the condition that what we have found as satisfying values for x and y, also satisfies the next sentence. If they do not, we have refuted the continuation, but not without further ado. We might go back to find other values for x and y, which in the end do satisfy the continuation, and then we can go on, simply and happily, with subsequent discourse.

What distinguishes *DPL* from ordinary first order predicate logic, is that, first, it keeps track of the witnesses $g'(x), g'(y), \ldots$ of variables that have been quantified over in a previous discourse, and, second, and for good reasons, it keeps track of all of these possible satisfying values. Maybe this is all one needs to know about *DPL*, but for a proper understanding of the main contents of this monograph, it is useful to specify *DPL*'s formal details.

The language of *DPL* is that of first order predicate logic, and its semantics is specified as a relation between variable assignments g and h. Intuitively, a pairs of assignments $\langle g, h\rangle$ stands in that relation, for any given formula ϕ, iff g figure as a possible valuation of the free variables in ϕ, and h as a possible evaluation of the bound, existentially quantified, variables in ϕ. Given any such pair $\langle g, h\rangle$ in the interpretation of ϕ, the assignment g may count as a possible input for truthfully interpreting the formula ϕ, and h a possible output, satisfying, also, constraints on variables introduced in ϕ. Formally, the definition can be given as follows. Interpretation is stated relative to a standard model $M = \langle D, I\rangle$, with a domain D of individuals, or objects, and an interpretation function I for the individual and predicate logical constants of the language.

Definition 1 (*DPL Interpretation*)

- $g[\![Rx_1 \ldots x_n]\!]_M h$ iff $g = h$ & $\langle g(x_1), \ldots, g(x_n)\rangle \in I(R)$;
  $g[\![\neg\phi]\!]_M h$ iff $g = h$ & $\neg\exists h$: $g[\![\phi]\!]_M h$;
  $g[\![\exists x\phi]\!]_M h$ iff $\exists k$: $g[x]k[\![\phi]\!]_M h$;
  $g[\![\phi \wedge \psi]\!]_M h$ iff $\exists k$: $g[\![\phi]\!]_M k[\![\psi]\!]_M h$.
- A formula ϕ is true in a model M and relative to assignment g, $M, g \models \phi$, iff $\exists h$: $g[\![\phi]\!]_M h$

Apart from the revolutionary shift from (sets of) satisfying variable assignments, to (sets of) *pairs* of satisfying input-output variable assignments, nothing really freaky goes on in this definition. An input assignment, and the very same output, satisfies an atomic formula if the formula is satisfied in the classical way. Satisfying a negation $\neg\phi$ simply comes down to not being able to satisfy the negated sentence ϕ. A good input for $\neg\phi$ is a variable assignment relative to which (the interpretation of) ϕ is unable to render a satisfying output. Existentially quantified formulas $\exists x\phi$ are interpreted as usual, trying to find a satisfying instance of the variable x quantified over—but for the fact that the satisfying output assignment may remember which lucky choice of the interpretation of the variable x made ϕ get satisfied. (Let me emphasize the *may* in the previous sentence; it so happens in *DPL* that a variable x gets 'reintroduced', with the effect that previous knowledge about its possible values have to be discarded. This is an annoying technical issue, which will be readdressed below.) Since *DPL*'s interpretations are possible input/output pairs of variable assignments, conjunction cannot be considered other than as relation composition. If, given input g, ϕ may bring me to output k, and ψ, with input k, may produce output h, then, of course, with g as input, should produce h as output to $(\phi \wedge \psi)$.

It is not expedient to illustrate *DPL*'s interpretation function in full detail here—for this, consult (Groenendijk and Stokhof 1991; Dekker 2011)—but it is interesting to see the major difference with ordinary first order predicate logic. It can be argued that the following observation, Egli's Theorem, completely characterizes the main difference, and its corollary the useful result of that Egli (1979).

Observation 1 (Egli's Theorem)

- $(\exists x\phi \wedge \psi) \Leftrightarrow \exists x(\phi \wedge \psi)$.

Observation 2 (Egli's Corollary)

- $(\exists x\phi \rightarrow \psi) \Leftrightarrow \forall x(\phi \rightarrow \psi)$.

What is not prominently visible in observation (1), which is a standard equivalence, is that it *lacks* the proviso, that x should not be free in ψ. This is, however, *DPL*'s distinctive feature. The existential quantifier $\exists x$ in the first conjunct indeed, effectively, binds free occurrences of the variable x in the second conjunct ψ. The second observation is a direct consequence of this, if we employ the standard definition of $\rightarrow$ and $\forall x$ using negation and conjunction. Observation (1) can account for the following examples.

(3) A man is riding through the park. He is whistling.
(4) A man who is riding through the park is whistling.

Apart from the different ways in which information is presented in (3) and (4), the two examples seem to be (truth-conditionally) equivalent. But indeed, their fairly natural translation in a predicate logical language is, fully, equivalent as well in *DPL*. For Egli's Theorem gives us the following result.

- $(\exists x(Mx \wedge Rx) \wedge Wx) \Leftrightarrow \exists x((Mx \wedge Rx) \wedge Wx)$

Moreover, Egli's Corollary gives us the following result. Consider the following examples

- If a farmer owns a donkey, he beats it.

- Every farmer beats every donkey he owns.

These sentences had previously been argued to be equivalent as well, and this equivalence is rendered by the *DPL*-equivalence of the following formulas.

- $(\exists x(Fx \wedge \exists y(Dy \wedge Oxy)) \rightarrow Bxy) \Leftrightarrow$
  $\forall x((Fx \wedge \exists y(Dy \wedge Oxy)) \rightarrow Bxy) \Leftrightarrow$
  $\forall x(Fx \rightarrow (\exists y(Dy \wedge Oxy) \rightarrow Bxy)) \Leftrightarrow$
  $\forall x(Fx \rightarrow \forall y((Dy \wedge Oxy) \rightarrow Bxy))$

DPL thus deals with a number of anaphoric relationships, which had puzzled logicians and philosophers for years. In addition, it can be argued that *DPL*'s distinctive feature is that it *only* does this: rendering Egli's Theorem a real theorem. For there is a reduction algorithm that takes any predicate logical formula as input, and possibly changes that formula into one with the illuminating property that, while it has exactly the same meaning as the original formula in *DPL*, the reduced formula has also exactly the same truth-conditions in first order predicate logic and *DPL*. The steps in the algorithm thus reveal what is 'standard', and what is 'new' in *DPL*, and, interestingly, the step where conjunctions are reduced along the lines of Egli's algorithm is the only non-standard step. [For a full specification of the algorithm, see (Groenendijk and Stokhof 1991; Dekker 2011).]

The above characterization of *DPL* is of course not all there is to say about the system. Of course, changing things in a logic system have repercussions. Some of them maybe might have been expected, but it is worth mentioning them. For one thing, conjunction is not commutative and not idempotent. That is, it is not in general the case that $(\phi \wedge \psi) \Leftrightarrow (\psi \wedge \phi)$ or that $\phi \Leftrightarrow (\phi \wedge \phi)$. Thinking from a classical perspective this may sound strange, but from a dynamic perspective it makes sense. If interpretation is dynamic, that is, if it is both context dependent *and* context altering, then of course one can expect that the interpretation of one occurrence of a formula ϕ may affect a second, subsequent, occurrence of it. Thus, if one formula ϕ is repeated, as in $(\phi \wedge \phi)$, the second occurrence of that formula may have an interpretation different from that of the first, so that the whole may mean something different from what a single occurrence of the formula means. As for commutativity, if a formula ψ occurs after a formula ϕ has occurred, as in $(\phi \wedge \psi)$, its interpretation may be different from the one that it is obtained without first establishing ϕ, as in $(\psi \wedge \phi)$; moreover, the interpretation of ϕ itself may be different if it 'occurs out of the blue', as in $(\phi \wedge \psi)$, or after ψ, as in $(\psi \wedge \psi)$.

DPL also employs a dynamic notion of entailment, which we will not formally specify here, but which is easily seen to lack the structural properties of reflexivity, monotonicity, and transitivity. It is dynamic, because it builds on the idea that an entailment is valid if the conclusion is true, not always *in case*, but always *after* the premises have been accepted. Entailments come as an ordered sequence of formulas, in which (anaphoric) dependencies may get established in a dynamic way. I will not go into the (non-)structural properties of this entailment here, because very much the same holds of the entailment relation of *PLA*, extensively discussed in Sect. 2.3. For now, it suffices to observe that if a formula ϕ is accepted, it may as well have changed the context of interpretation, also if that is a context in one wants to conclude that ϕ. Since, due to the premise occurrence of ϕ, it may have changed the context of interpretation of the conclusion, the same formula but in a different context of interpretation, it may make the conclusion, on its second interpretation, no longer guaranteed, or even acceptable. Similar arguments can be given to show that because of the possibility of context change, and the (im-)possibility of establishing anaphoric relationships, the *DPL*-entailment relation is not monotone or transitive either.

2.2 First Order Satisfaction in PLA

It is time now to turn the main subject of this chapter, the system of *PLA*. I will argue that most of the findings and results achieved in *DRT* and *DPL* can be achieved in a more classical fashion by means of an arguably static semantics, a Tarskian satisfaction semantics, which accommodates dynamic interpretation as the dynamic composition of meanings. The language with which we will be concerned from now on is literally that of a predicate logic with anaphora, or anaphoric pronouns. It is a minimal first order predicate logical language that, apart from employing a primitive category of pronouns as terms, is built up like a language of ordinary predicate logic. As I said above, I will assume throughout that the reader is familiar with predicate logic.

Definition 2 (*PLA Language*) The *PLA*-language $\mathcal{L}_{PLA}$ is built up from relational constants $R^n \in \mathcal{R}^n$, individual constants $c \in C$, variables $x \in V$ and pronouns $\mathtt{p}_i$, for $i \in \mathbb{N}$.

- $t ::= c \mid x \mid \mathtt{p}_i$;
  $\phi ::= R^n t_1 \ldots t_n \mid \neg\phi \mid \exists x\phi \mid (\phi \wedge \phi)$.

(Later on we will also assume identity formulas ($t_1 = t_2$) to be among the atomic formulas.) As can be seen from the definition, any constant c, variable x, and pronoun $\mathtt{p}_i$ counts as a term t. Terms are those things which fill argument positions of predicates in atomic formulas. An individual constant simply names an individual as the argument; a variable indeed is a variable ranging over possible interpretations of it, to be controlled by corresponding variable binding quantifiers; and a pronoun refers back to something which has been mentioned before it. Together with n such terms, an n-place predicate may be saturated in an atomic formula, and it says, when it occurs, that the n terms have referents which stand in the corresponding n-place relation.

Constants and variables will be interpreted like they are in ordinary first order logics; the new category of pronouns will be interpreted in the style of "de Bruijn indices" (de Bruijn 1972). The interpretation of a pronoun $\mathtt{p}_i$ totally depends on the context of its occurrence. In the style of a variable free semantics (Jacobson 1999; Szabolcsi 1989) it is interpreted like the identity function, which take a contextually supplied individual as an argument, and delivers the very same individual as a value.

Pronouns in general are assumed to be a bit more flexible, though. In *PLA* they are functions taking a sequence of individuals as an argument selecting a designated one of them as a value. The sequences of individuals here are the possible values of terms in previous discourse, and any selective pronoun p_i will serve to refer to the ith term occurring before the pronoun's occurrence, in a manner detailed below.

The categories of constants, variables, and pronouns are, thus, all dependent on context, but in different manners. A constant's interpretation depends on the model, or the world of interpretation, if you want, in which a name has been defined, or an object has been baptized; a variable's interpretation depends on the quantifier (or other operator) which controls it, or binds it; variables are very useful technical devices, which are eliminable, though; a pronoun is a device to correlate actual pieces of discourse and its interpretation serves to establish coreference between actually occurring linguistic material. Eventually, pronouns are eliminable, too, but in the practice of structured exchange of information they are indispensable. We will come back in more detail on the relation between, especially, variables and pronouns, in Sect. 2.4.

To keep generalizations within easy reach, the language consists of formulas ϕ, built up from atomic formulas using negation ($\neg$), existential quantification ($\exists x$) and conjunction ($\wedge$). Other useful operators like the universal quantifier, and other connectives like disjunction ($\vee$) and (material) implication ($\rightarrow$) can be defined in the usual way. I.e., $\forall x \phi$ can be taken to be short for $\neg \exists x \neg \phi$, $(\phi \vee \psi)$ for $\neg(\neg \phi \wedge \neg \psi)$ and $(\phi \rightarrow \psi)$ for $\neg(\phi \wedge \neg \psi)$. With some of the required hand-waving, the operators of the language can be taken to correspond to natural language expressions like 'not', 'some', 'and', 'all', 'or', and 'if ... then'; predicates can be taken to stand in for nouns and verbs, individual constants for proper names, and pronouns, indeed, for pronouns. This, in short, is the layman's translation manual for expressing natural language meanings in first order logic. The only difference with textbook first order logic is the use of pronouns. As we will see, they are eliminable, but useful as well.

Pronouns are essentially indexical. Like I said, pronouns are a device to refer to contextually given entities. We cannot state out of the blue what the referent of a pronoun is, because one and the same pronoun may have different referents in different contexts. In natural language such a context is always a context of use, and this means a context of an occurrence of a pronoun. In *PLA*, for the time being, I identify the uses of a pronoun with its occurrences, and thereby employ the formulas in which they occur as their contexts of utterance. It is through these formulas that pronouns may find their referents. And when I say that a pronoun refers to a contextually given entity, I mean it relates to something that is 'given' at its point of occurrence, something given by an expression that literally occurs to the left of the pronoun's occurrence in a formula. (We do make generalizations here, which is clear from the fact that we can talk about the *occurrences* of a formula in which a pronoun *occurs* at a certain place; and these formulas may occur in larger constructions which have their own occurrences in even larger ones. The notion of an 'occurrence', therefore, needs to be understood relativistically, or contextually, or, indeed, indexically.) Moreover, the specific choice of the previous 'antecedent' is determined in an entirely indexical way: it is determined by the index i on a pronoun p_i by counting back from the location of the occurrence of the pronoun in a formula ϕ, to the ith antecedent term before that occurrence of p_i. This is exactly how de Bruijn's indices in the lambda calculus work: they are indices i, who indicate the $lambda$-occurrence that binds them, by counting up, from the location of the occurrence of i, to the ith λ that binds them.

Pronouns take their input, more specifically, from previously used existentially quantified formulas, typically the expressions that formally render the interpretation of indefinite noun phrases in natural language. Existentially quantified formulas are selected for this purpose, partly for theoretical and partly for practical reasons. Like I said, the formal language in which I want to start to state our investigations is kept as simple as possible. The very first level to stage discussions of reference and coreference then is at the level of sentences in what must be at least a first order logic and the very first operators which we find there are the first order quantifiers; among these the existential quantifier lends itself best for the present purposes. (From a philosophical and an historical perspective one could in principle choose among four quantifiers: one saying that a formula is satisfied by everything, one saying that it has a satisfying instance, one saying that it has an exception, and one saying that it is not instantiated; for practical reasons I take the second option as the basic one, even though philosophically I'd prefer the third, in the spirit of Herakleitos.) There is also an historical reason. Most of the formal semantic work on the interpretation of pronouns in *DRT* , *FCS* and *DPL*, has concentrated on anaphoric relationships of pronouns and preceding indefinite noun phrases. The reason is that these posed the greatest challenge for both syntactic and semantic treatments of anaphora. As we will see, this is not a bad motivation, since anaphoric relationships with terms other than indefinite noun phrases can be easily modeled after those with indefinite noun phrases.

Before I start with the semantics of the above language, I need to introduce two syntactic notions with semantic impact. The following definition first spells out how many potential antecedents a formula may provide for subsequent pronouns yet to come, and next how many of such antecedents it presupposes itself. The first is called the *domain* $n(\phi)$ of a formula, which equals the number of 'active' existential quantifier occurrences in ϕ; the second is called its *range* $r(\phi)$, which is determined by occurrences of pronouns in ϕ.

Definition 3 (*Domain* $n(\phi)$ *and Range* $r(\phi)$ *of a Formula* ϕ)

- $n(Rt_1 \dots t_m) = 0$; $\quad n(\neg\phi) = 0$;
  $n(\phi \wedge \psi) = n(\phi) + n(\psi)$; $\quad n(\exists x\phi) = n(\phi) + 1$.
- $r(Rt_1 \dots t_m) = MAX\{j \mid \mathrm{p}_j \in \{t_1, \dots, t_n\}\}$; $\quad r(\neg\phi) = r(\phi)$;
  $r(\phi \wedge \psi) = MAX\{r(\phi), (r(\psi) - n(\phi))\}$; $\quad r(\exists x\phi) = r(\phi)$.

If $n(\phi) = 0$, ϕ is called *closed*; if $r(\phi) = 0$, ϕ is called *resolved*.

(Later in this chapter we will also talk about the domain and range of sequences of formulas $n(\phi_1, \dots, \phi_n)$ and $r(\phi_1, \dots, \phi_n)$. The sequences then are conceived of as their conjunction.)

The domain of a formula originates from existentially quantified formulas. Each quantifier occurrence contributes one item, and in a conjunction they are summed up. There is one proviso: an existential quantifier should not occur in the scope of a negation. Negations annul the discourse contribution of existential quantifiers, as is generally assumed. So $n(\neg\phi) = 0$ no matter what $n(\phi)$ is. The reason is that, in general, a negation has the effect of denying the existence of witnesses for existentially quantified variables in its scope. In this sense, ironically, existentially quantified variables behave like free variables, as Heim in the spirit of Quine has rightly claimed. The same effect is observed with indefinite noun phrases in the scope of a negation in natural language.

The range of a formula originates from pronouns. A pronoun p_i literally claims coreference with the ith existential quantifier before its occurrence, so that it requires a pre-existing discourse where at least i items have been introduced. From an atomic formula, the loudest, or most far reaching claim will be heard. If the most demanding pronoun in a formula is honoured, which requires at least i previous existentials, then the claims of less demanding pronouns are automatically satisfied.

As one can see from the definition, the range of $\neg\phi$ and $\exists x\phi$ equals that of ϕ, which means that pronominal claims are preserved under negation and quantification. (Notice that if a pronoun is in the scope of an existential quantifier $\exists x$, then that occurrence of $\exists x$ does not count as a term which *precedes* the pronoun; for one thing, this is correct because the pronoun itself contributes to establishing the possible witnesses for the variable quantified over. We will learn more about this in due course.) A conjunction, however, allows pronominal demands from the second conjunct to be weakened. For instance, if the second conjunct ψ in a conjunction $(\phi \wedge \psi)$ requires a number of existential terms before it, and if the first conjunct ϕ actually contributes such terms, then the demand of the whole conjunction may be weakened or even annulled. Of course, pronouns in the first conjunct ϕ may impose further, and even stronger, constraints on the discourse preceding the conjunction. The range of a conjunction, therefore, equals the strongest demand among that of the first conjunct $r(\phi)$ and that of the second conjunct $r(\psi)$, as it is weakened by the contribution of the first conjunct, so minus $n(\phi)$. Here already we can see that the dynamics of our system of interpretation originates from its notion of conjunction. Pronominal demands may get satisfied, in a conjunction, where else? And if they do, that is, if any pronouns at all occur in a formula, and if they are, where they occur, satisfied by existentials preceding them in the very same formula, then these pronouns counts as resolved, and the formula counts as resolved as well then.

With the definition of the range of a formula, in terms of the occurrence of existential quantifiers, and with their use in resolving pronouns, in whatever way this is going to be spelled out, we face again the issue already mentioned. I could have chosen to allow individual constants (proper names) and pronouns to contribute to the domain of a formula, because, for one thing, they are most natural antecedents for pronouns themselves. For the moment I have chosen not to do so. For expository reasons it is better to start with one, typical, category of introductory terms only. Besides, the anaphoric potential of names and pronouns is easily dealt with by similar means. [A simple, quite crude, but effective, way of implementing this potential is to assume that names and pronouns always come with an existential quantifier themselves, so that, after all, they *do* introduce items. Instead of rendering *Anne/She walks* as Wa or $W\mathrm{p}_1$, one could render it as $\exists x(x = a \wedge Wx)$ or $\exists x(x = \mathrm{p}_1 \wedge Wx)$. Actually this is also how it works, roughly, in, for instance, (Kamp and Reyle 1993).]

We may now turn to the semantics of *PLA*. Existentially quantified expressions (like indefinite descriptions) are evaluated relative to possible satisfying witnesses for their eventual truth. In standard logic $\exists x\phi$ is deemed true iff ϕ is true under at least one evaluation of the variable x. That is, if for some individual d from the domain, $\phi[x/d]$ is true under the usual rendering of $\phi[x/d]$. The same will be required to be the case in *PLA*, but for the fact that such a witness d of the truth of $\exists x\phi$ is remembered, so to speak. If $\exists x\phi$ is true because a couple of instantiations of x make ϕ true, then, in *PLA*, $\exists x\phi$ is satisfied relative to each of these witnesses. The witnesses associated with (occurrences of) existentially quantified formulas will figure as the intended referents of the (occurrences of) coreferential pronouns *later* in that formula. Notice that, since any formula may introduce any number of items (that is, $n(\phi)$ may be any number m), we cannot just speak of 'a possible witness' of a formula, but we have to speak of 'sequences of possible witnesses', as Tarski has observed. If $n(\phi) = 0$, a possible witness is the empty sequence. These things being said, we may now turn to the definition itself.

Formally, the semantics of *PLA* is spelled out as a Tarskian satisfaction relation, in which the formulas of our language are said to be 'satisfied' (or not satisfied) relative to the relevant parameters. It is defined, first, relative to the usual first order models $M = \langle D, I\rangle$, consisting of a domain of individuals D and an interpretation function I for the relational and individual constants and such that $I(R^n) \in \mathcal{P}(D^n)$ and $I(c) \in D$. Satisfaction is defined also relative to variable assignments g such that $g(x) \in D$ for all variables x, and relative to sequences of witnesses $\widehat{e} = e_1 \ldots e_n \in D^n$. The values of variables, given by assignment functions g, are controlled by variable binding operators, the existential quantifier in the first place, and this is done in the usual way known from predicate logic. The witness sequences $\widehat{e}$ establish the interpretation of pronouns; they are controlled by the linguistic environment of the pronouns, and essentially keep track of the witnesses of terms (read here: existentially quantified phrases) which have, or may have, occurred before these sequences are actually employed. (It may surprise, or even worry, the reader that I propose a separate treatment of variables and pronouns, and do this in even in formally distinct ways: the first in terms of variable assignments g; the other in terms of sequences of objects $\widehat{e}$. I will come back to this issue in Sect. 2.4 where I give formal, theoretical, and methodological arguments for proceeding this way.)

In the definition below, and throughout the monograph, I employ the following conventions. Formulas ϕ are in general evaluated relative to sequences $\widehat{ce}$, or $\widehat{ace}$, or $b\widehat{ce}$, where:

- $\widehat{ce}$ is the concatenation of the sequences $\widehat{c}$ and $\widehat{e}$,
  $\widehat{ace}$ the concatenation of the sequences $\widehat{a}$, $\widehat{c}$ and $\widehat{e}$,
  and $b\widehat{ce}$ the concatenation of an individual $b \in D$ and the sequence $\widehat{ce}$.

All sequences $\widehat{a}$, $\widehat{c}$ and $\widehat{e}$ may be empty; their length depends on the formal properties of the formula ϕ at issue.

- If ϕ is evaluated relative to a sequence $\widehat{ce}$ (or $\widehat{ace}$), $\widehat{e}$ has *at least* length $r(\phi)$; the sequence is long enough to satisfy pronominal demands;
- $\widehat{c}$ (or $\widehat{ac}$) is supposed to have *exactly* length $n(\phi)$; the sequence supplies exactly the right number of witnesses for existentially quantified formulas in ϕ.
- For the evaluation of a conjunction $(\phi \wedge \psi)$ the sequences $\widehat{c}$ and $\widehat{a}$ are used to distinguish the contributions of ϕ, and ψ, respectively, so that $\widehat{c}$ is a sequence of length $n(\phi)$ and $\widehat{a}$ a sequence of length $n(\psi)$; thus $\widehat{ac}$, the concatenation of $\widehat{a}$ and $\widehat{c}$ has length $n(\phi \wedge \psi)$;
- $\widehat{ce}$ has length at least $r(\psi)$, and $\widehat{e}$ has length at least $r(\phi \wedge \psi)$, so it is at least $r(\phi)$ elements long and $r(\psi) - n(\phi)$ elements long; thus, $\widehat{e}$ directly satisfies the pronominal demands of ϕ, and, together with witnesses for existentials in ϕ, it indirectly satisfies the pronominal demands of ψ.

Here is a specific simple example to illustrate these conventions:

(5) A Canadian farmer, whose horse was ill, went to see his veterinarian. She lent him her donkey.

Neglecting many details, the whole sequence can be translated using the following abbreviations. 'CFx' stands short for 'x is a Canadian farmer', '$OHxy$' for 'x owns horse y', 'Iy' for 'y is ill', '$SVxu$' for 'x see x's veterinarian u', '$ODuv$' for 'u owns donkey v', and '$Luvx$' for 'u lends v to x'; the choice of the indices on the pronouns are explained below.

- $(\exists x((CFx \wedge \exists y(OHxy \wedge Iy)) \wedge \exists uSVxu) \wedge \exists v(OD\mathrm{p}_2v \wedge L\mathrm{p}_2v\mathrm{p}_1))$

This formula is a conjunction $(\phi \wedge \psi)$, the first conjunct of which:

- $\phi := \exists x((CFx \wedge \exists y(OHxy \wedge Iy)) \wedge \exists uSVxu)$

contains no pronouns, and the second one

- $\psi := \exists v(OD\mathrm{p}_2v \wedge L\mathrm{p}_2v\mathrm{p}_1))$

contains three occurrences of two pronouns. So $r(\phi) = 0$, and $r(\psi) = 2$, the highest index of a pronoun here. We also see that ϕ contains three existential quantifiers, not in the scope of a negation, so that $n(\phi) = 3$; and ψ contains one, so that $n(\psi) = 1$, and $n(\phi \wedge \psi)$, the domain of the whole formula is 4. The range of the whole formula, $r(\phi \wedge \psi)$ is the largest among $r(\phi) = 0$ and $(r(\psi) - n(\phi)) = (2 - 3) = -1$ which is 0.

Putting things together, I will, according to the stated conventions, evaluate such a conjunction relative to a sequence $\widehat{ace}$, where:

- the length of $\widehat{e}$ is a least 0; the length of $\widehat{c}$ is 3; the length of $\widehat{a}$ is 1.

It will turn out that the above formula, for such a sequence $\widehat{ace}$ of length 4, of course, imposes no requirements on $\widehat{e}$; it will require $\widehat{c}$ to consist of 3 elements, a Canadian Farmer c, a veterinarian v that the farmer c sees, and an ill horse h that the farmer owns; the sequence $\widehat{a}$, a one element sequence only, is furthermore required to consists of a donkey d which the veterinarian owns, and which she lends to the farmer. If one tries to evaluate a formula ϕ relative to a sequence $\widehat{e}$ which cannot be split along the above lines, that is, a sequence that is too short, then interpretation crashes. Let us now turn to the definition which actually gives us this notion of satisfaction.

Definition 4 (*PLA Satisfaction and Truth*)

- $[c]_{M,g,\widehat{e}} = I(c)$; $[x]_{M,g,\widehat{e}} = g(x)$; $[\mathrm{p}_i]_{M,g,\widehat{e}} = \widehat{e}_i$;
- $M, g, \widehat{e} \models Rt_1 \dots t_n$ iff $\langle [t_1]_{M,g,\widehat{e}}, \dots, [t_n]_{M,g,\widehat{e}} \rangle \in I(R)$;
  $M, g, \widehat{e} \models \neg\phi$ iff there is no $\widehat{c} \in D^{n(\phi)}$: $M, g, \widehat{ce} \models \phi$;
  $M, g, b\widehat{ce} \models \exists x\phi$ iff $M, g[x/b], \widehat{ce} \models \phi$;
  $M, g, \widehat{ace} \models (\phi \wedge \psi)$ iff $M, g, \widehat{ce} \models \phi$ and $M, g, \widehat{ace} \models \psi$;
- ϕ is true relative to M, g and $\widehat{e}$ iff there is a $\widehat{c} \in D^{n(\phi)}$: $M, g, \widehat{ce} \models \phi$.

(In case ϕ is true relative to M, g and $\widehat{e}$, I also write $M, g, \widehat{e} \models \phi$. This notation is the same as the one used for the satisfaction of atomic formulas and negations; this should not be problematic since for these formulas truth and satisfaction coincide.)

Atomic formulas are evaluated in the Tarskian way, relative to sequences of individuals. A pronoun p_i selects the ith individual in the sequence $\widehat{e}$, claiming, and as a matter of fact effectuating, coreference with a variable existentially quantified over i-terms ago. With an atomic formula $Rt_1 \dots t_n$ the references of the sequence of terms $t_1 \dots t_n$ are required to stand in the said relation R. (With an identity formula $t_1 = t_2$ the references of t_1 and t_2 are required to be identical.)

An existentially quantified formula $\exists x\phi$ behaves like an ordinary quantifier in the sense that it binds free occurrences of the variable x in its scope. In addition, the first element of the sequence parameter is required to be a 'witness' of x for the satisfaction of the embedded formula ϕ. We may understand this best in a constructive, or 'dynamic' way. If ϕ can be satisfied relative to a sequence $\widehat{ce}$ with b as a witness for x, then $\exists x\phi$ can be said to contribute b to the sequence of possible witnesses, thus delivering $b\widehat{ce}$ as a witness sequence. With the notation conventions employed this means that $b\widehat{c}$ is a sequence of witnesses contributed by $\exists x\phi$, relative to M and g, and relative to a sequence of witnesses $\widehat{e}$ brought up by preceding discourse. Notice that the witness b for the existential quantifier is put in the front of the sequence $\widehat{ce}$. This reflects the fact that the item introduced most recently is the first to be selected for pick-up afterwards. If the next formula ψ uses a pronoun p_1 to refer to the item introduced last, i.e., to the witness of the last term which has occurred before the pronoun, it will indeed pick up the witness b from the sequence $b\widehat{ce}$ relative to which ψ is evaluated. The combined interpretation of existential quantifiers and pronouns thus displays the truly indexical nature of the pronouns. A pronoun's reference can only be determined from the location in the formula where it occurs.

Here is an elementary example in which two typical formulas are combined in a conjunction. (Conjunction will be explained in more detail below.)

(6) There is a boy in the garden. He sneezes.
$(\exists x BGx \wedge S\mathrm{p}_1)$.

In a model M, and relative to a sequence $\widehat{e}$ (which turns out to be irrelevant), the first conjunct is satisfied by an individual b iff b is a boy_in_the_garden in the model, i.e., iff $b \in I(BG)$. For, $M, g, b\widehat{e} \models \exists x BGx$ iff $M, g[x/b], \widehat{e} \models BGx$ iff $[x]_{M,g[x/b],\widehat{e}} = g[x/b](x) = b \in I(BG)$. Relative to this sequence $b\widehat{e}$ the second conjunct is true iff the witness b for the first conjunct sneezes, i.e., iff $b \in I(S)$. For, $M, g, b\widehat{e} \models S\mathrm{p}_1$ iff $[\mathrm{p}_1]_{M,g,b\widehat{e}} = (b\widehat{e})_1 = b \in I(S)$. In a model M the whole conjunction is, thus, satisfied by a sequence $b\widehat{e}$ iff b is a boy_in_the_garden who sneezes in M, and the conjunction is true iff there is such a sequence, that is, iff there is a boy_in_the_garden who sneezes in M, i.e., iff $I(BG) \cap I(S) \neq \emptyset$.

Real quantificational effects in *PLA* are obtained by negation (and by the corresponding notion of truth). The negation of a formula ϕ requires that there are no witnesses for the existentially quantified variables in ϕ, thus obtaining the usual effect of denying the quantified variables a satisfying instantiation. (In the definition $D^{n(\phi)}$ is the set of $n\phi$)-tuples of individuals, the set of sequences that might potentially be witnesses for existentially quantified terms in ϕ.) A formula like $\neg\exists x Fx$ thus means, as usual, that no individual has the property F. Saying that a formula ϕ is true is claiming that there *are* witnesses for the existentially quantified variables in ϕ, thus stating that ϕ *can* be satisfied. The only difference with plain satisfaction is that in stating the truth of a formula one does not keep track of the witnesses by means of which it is satisfied.

Let us now inspect the clause defining satisfaction of conjunctions. If such a conjunction $(\phi \wedge \psi)$ is evaluated relative to a sequence $\widehat{e}$, the first conjunct ϕ is evaluated relative to $\widehat{e}$ and may contribute a sequence $\widehat{c}$, so that $\widehat{ce}$ satisfies ϕ. The second conjunct ψ is then evaluated relative to $\widehat{ce}$, that is the original context plus the contribution $\widehat{c}$ from ϕ, and it may contribute its own sequence $\widehat{a}$ next. Such a conjunction is thus taken to be satisfied by a sequence $\widehat{ace}$ where $\widehat{a}$ satisfies ψ relative to $\widehat{ce}$, and ϕ is satisfied by $\widehat{c}$ relative to $\widehat{e}$. It may be noticed, as example (6) already illustrates, that the interplay between existential quantifiers and pronouns only comes off the ground through the conjunction of the respective elements. (This is interesting, because we may as well choose to correlate the conjuncts in a different way. Nothing stands in the way of a notion of conjunction in which pronouns in the first conjunct get related to quantifiers in the second, as it also may happen in natural language by the way. Notice that such an alternative way of conjoining information contents is hardly conceivable in rigid systems of dynamics semantics, in which the left to right interpretation is taken to be constitutive of the meanings of formulas.)

Let us consider a slightly more involved example in some detail.

(7) There once was a king. He lived in a castle.
$(\exists x Kx \wedge \exists y(Cy \wedge LI\mathrm{p}_1 y))$.

The whole formula $(\phi \wedge \psi)$ is a conjunction of two formulas ϕ and ψ each of which contains one existential quantifier. Neither of them occur in the scope of a negation, so $n(\phi \wedge \psi) = (n(\phi) + n(\psi)) = (1 + 1) = 2$. The second conjunct contains a pronoun, and its range $r(\psi) = 1$, but this is met by the item introduced by the first conjunct, because $n(\phi) = 1$, so the range $r(\phi \wedge \psi)$ of the whole is 0. Unlike the second conjunct, the whole conjunction counts as resolved. The conjunction is satisfied by two individuals c and k iff k satisfies the first conjunct $(\exists x Kx)$ and ck satisfies the second conjunct $(\exists y(Cy \wedge LI\mathrm{p}_1 y))$. This is the case if k is a king and c a castle which the king lives in. Formally this is spelled out as follows.

- $M, g, ck \models (\exists x Kx \wedge \exists y(Cy \wedge LI\mathrm{p}_1 y))$ iff
  $M, g, k \models \exists x Kx$ and $M, g, ck \models \exists y(Cy \wedge LI\mathrm{p}_1 y)$ iff
  $M, g[x/k] \models Kx$ and $M, g[y/c], k \models (Cy \wedge LI\mathrm{p}_1 y)$ iff
  $M, g[x/k] \models Kx$ and $M, g[y/c], k \models Cy$ and $M, g[y/c], k \models LI\mathrm{p}_1 y$ iff
  $k \in I(K)$ and $c \in I(C)$ and $\langle k, c\rangle \in I(LI)$.

The conjunction of the two formulas, with an existential quantifier in the first and a pronoun in the second, is true if we can find a witness sequence for this example. That is, the conjunction is true in a model M iff that model hosts a king k (an element of $I(K)$) and a castle c (an element of $I(C)$) such that the first lived in the second ($\langle k, c\rangle \in I(LI)$). The connection between the existential quantifier in the first conjunct and the pronoun in the second is properly taken care of.

The observations on *PLA*'s notion of conjunction bring to bear on the *PLA* notion of material implication. Recall the classical definition of $(\phi \rightarrow \psi)$ as $\neg(\phi \wedge \neg\psi)$. Observe that, according to this definition, the implication inherits the combinatory effects of *PLA*'s dynamic conjunction, but with a different force. Working through the clauses for negation and conjunction, an implication is seen to be satisfied under the following conditions.

Observation 3 (Implication Satisfaction)

- $M, g, \widehat{e} \models (\phi \rightarrow \psi)$ iff for all $\widehat{c} \in D^{n(\phi)}$ there is $\widehat{a} \in D^{n(\psi)}$:
  if $M, g, \widehat{ce} \models \phi$ then $M, g, \widehat{ace} \models \psi$.

A formula $(\phi \rightarrow \psi)$ is satisfied in a context $\widehat{e}$ iff the consequent clause ψ is satisfied relative to *all* witness sequences satisfying the antecedent ϕ in that context. In effect, this implies that variables existentially quantified over in the antecedent are read with universal force. Consider the following simple example.

(8) If someone is a king, he lives in a castle. $(\exists x Kx \rightarrow \exists y(Cy \wedge LI\mathrm{p}_1 y))$.

The implication requires that for every witness of the antecedent, i.e., for every king, there is a witness for the consequent, a castle which he lives in. So for the implication to be satisfied, every king must live in some castle.

From the explanatory remarks so far it may be clear that (1) *PLA*'s semantics is a proper extension of a classical semantics in both form and content, and (2) that the dynamics resides in the actual conjunction of information. The format of the clauses in the above definition is classical, *but* for the use of an additional parameter, that of witness sequences; and the contents are classical as well, *but* for the additional requirements (dynamically) imposed on these witness sequences. These two features are persistent. In the extensions proposed below the basic definitions remain classical and the only additional machinery relates to the additional parameter of witnessing sequences.

Let us state the first feature mentioned in a precise manner. Let *PL* stand for ordinary first order predicate logic.

Observation 4 (PLA and PL) For all formulas ϕ without pronouns

- for all M, g: $M, g \models_{PL} \phi$ iff $M, g \models_{PLA} \phi$.

Since pronouns, the additional category of terms in *PLA*, relate to existential quantifiers, but do not interfere with them, the usual laws of quantification remain valid. In particular, α-conversion is a meaning preserving substitution operation under the usual conditions, and throughout the *PLA*-language.

Observation 5 (α-conversion) If y is free for x in ϕ, then

- for all $M, g, \widehat{ce}$: $M, g, \widehat{ce} \models \exists x\phi$ iff $M, g, \widehat{ce} \models \exists y[y/x]\phi$,

where $[y/x]\phi$ is obtained from ϕ by replacing all free occurrences of x in ϕ with y.

This fact may rightly surprise only those who are familiar with *DPL*, in which this fact fails to obtain. Since pronouns are not conflated with variables in *PLA*, α-conversion holds unconstrained, and this fact will turn out to be important when we turn to *PLA*'s resolving binding forms below.

Conjunction and Resolution in PLA

In this section I describe in more detail how anaphoric connections are established in *PLA*, so as to pave the ground for a fully general comparison with other, static and dynamic, systems of interpretation. I consider a number of constructions which display the dynamic properties of *PLA*'s conjunction and implication in a relatively natural way. Conjunctions (or implications) with pronouns in them are shown to be equivalent to certain quantified constructions which are pronoun free. Consider the following example.

(9) A man is walking in the park. ($\exists x(Mx \wedge Wx)$.) There is a dog. ($\exists y Dy$.) It frightens him and he chases it. ($(F\mathtt{p}_1\mathtt{p}_2 \wedge C\mathtt{p}_2\mathtt{p}_1)$.)

As noted above, I use rather stilted examples, and for the purpose of exposition I neglect all temporal and situational aspects of the interpretation of these sentences. The above sequence of sentences, with the given translation, fits the equation below. I use $\Leftrightarrow$ to indicate that two formulas are the same in meaning, that is, by definition, $\phi \Leftrightarrow \psi$ iff for all $M, g, \widehat{ce}$: $M, g, \widehat{ce} \models \phi$ iff $M, g, \widehat{ce} \models \psi$.

- $((\exists x(Mx \wedge Wx) \wedge \exists y Dy) \wedge (F\mathtt{p}_1\mathtt{p}_2 \wedge C\mathtt{p}_2\mathtt{p}_1)) \Leftrightarrow$
  $\exists y \exists x(((Mx \wedge Wx) \wedge Dy) \wedge (Fyx \wedge Cxy))$.

The equivalence can be easily calculated. The second formula requires valuations of the variables y and x as d and m, respectively, by means of which all of the formulas Mx, Wx, Dy, Fyx, and Cxy are satisfied; if these requirements are met, then dm constitutes a witness sequence for the formula. The first formula, the one from example (9), achieves precisely the same effect, but in a step by step manner. First it seeks a valuation of x by means of which Mx and Wx are satisfied; any such valuation m counts as a witness; then it seeks a valuation of y by means of which Dy is satisfied, and such a witnessing dog d is paired with the man m in the sequence dm; finally it is required that the first of dm (i.e., the dog d) frightens the second (i.e., the man m) and that the man m chases the dog d. If the tests succeed, then the sequence dm satisfies the whole conjunction. The satisfaction conditions are, thus, identical to those of the existentially quantified formula. The equivalence above shows that the same can be expressed with a pronoun free formula, or, rather, that what can be expressed by a rather involved existentially quantified formula can be expressed in a constructive, step by step, manner as well.

The next example is a variant of the previous one with an implication.

(10) If a man is walking in the park ($\exists x(Mx \wedge Wx)$), and there is a dog ($\exists y Dy$), then ($\rightarrow$) it frightens him and he chases it ($F\mathtt{p}_1\mathtt{p}_2 \wedge C\mathtt{p}_2\mathtt{p}_1$).

- $((\exists x(Mx \wedge Wx) \wedge \exists y Dy) \rightarrow (F\mathtt{p}_1\mathtt{p}_2 \wedge C\mathtt{p}_2\mathtt{p}_1)) \Leftrightarrow$
  $\forall y \forall x(((Mx \wedge Wx) \wedge Dy) \rightarrow (Fyx \wedge Cxy))$.

As we have seen in Sect. 2.2, an implication requires its consequent clause to be satisfied relative to all witness sequences for its antecedent clause. The above implication, thus, is satisfied if relative to *all* pairs dm of a dog d and a man m who is walking in the park, the consequent clause ($F\mathtt{p}_1\mathtt{p}_2 \wedge C\mathtt{p}_2\mathtt{p}_1$) is satisfied. That is, it requires of *any* such pair dm that the first (the dog d) frightens the second (the man m), and that the man m chases the dog d. These truth conditions are typically rendered by the (equivalent) formula $\forall y \forall x(((Mx \wedge Wx) \wedge Dy) \rightarrow (Fyx \wedge Cxy))$, which is pronoun free.

Notice that the pronouns $\mathtt{p}_1$ and $\mathtt{p}_2$ in the examples (9) and (10) refer to the last and the penultimate introduced term with reference to the location where these pronouns occur. Thus, $\mathtt{p}_1$ picks up the dog there, introduced *last* in these examples, and $\mathtt{p}_2$ the man, introduced *earlier*. The order of appearance is the reverse of the order of salience so to speak, due to the essentially indexical interpretation of the pronouns. In the next two examples this pattern is different.

(11) A diver found a pearl. ($\exists x(Dx \wedge \exists y(Py \wedge Fxy))$.) She lost it again. ($L\mathtt{p}_1\mathtt{p}_2$.)

We find two existentially quantified formulas again, both of which are picked up by a subsequent pronoun. This time, however, the quantified expressions do not properly succeed one another, but one of them is embedded in the other. The two existential quantifiers require a pair of witnesses, of a diver d and a pearl p the diver found. If one carefully inspects the satisfaction clause for existentially quantified formulas, this pair of witnesses will appear in the order of presentation, as dp. For, first $(Dy \wedge Oxy)$ will have to be evaluated relative to $g[x/d][y/p]$ for some diver d and pearl p, and if this is successful p is added to the contextual sequence as a witness for $\exists y(Py \wedge Fxy)$. Only then $(Dx \wedge \exists y(Py \wedge Fxy))$ can be evaluated relative to $g[x/d]$ and if this is successful indeed the diver d is added to the sequence creating dp as a witness for $\exists x(Dx \wedge \exists y(Py \wedge Fxy))$. For this reason, the next conjunct

$L\mathrm{p}_1\mathrm{p}_2$ states that the last witness constructed, which is the diver d, not the pearl p, lost the one constructed earlier, i.e., the pearl, not the diver. For this reason the sequence of sentences turns out to be equivalent to “A diver lost a pearl she found.”

- $(\exists x(Dx \wedge \exists y(Py \wedge Fxy)) \wedge L\mathrm{p}_1\mathrm{p}_2) \Leftrightarrow$
  $\exists x \exists y((Dx \wedge (Py \wedge Fxy)) \wedge Lxy)$.

Although the terms ‘A diver’ and ‘a pearl’ literally occur in this order, and that one first hears of a diver, and then of a pearl, the witnesses they introduce are raised in reverse order, because we have to have a pearl first, to make up the property of finding that pearl, before we can set up a witness for a diver, who has that property of finding that pearl. Since, in this sense, the diver comes *after* the pearl here, it is the first in prominence when the pronouns are interpreted.

With the witness order in mind, we may now inspect the museum-piece donkey sentence, which is an implicative variant of example (11).

(12) If a farmer owns a donkey, he beats it.
$(\exists x(Fx \wedge \exists y(Dy \wedge Oxy)) \rightarrow B\mathrm{p}_1\mathrm{p}_2)$.

The antecedent of the implication $(\exists x(Fx \wedge \exists y(Dy \wedge Oxy)))$ is satisfied by any pair of witnesses fd, where f is a farmer who owns a donkey d. By the satisfaction clause for implications (Observation 3), the formula requires $B\mathrm{p}_1\mathrm{p}_2$ to be satisfied by any such sequence. That is, for any such pair fd the first (the farmer) is required to beat the second (the donkey). The truth-conditions of this example then are adequately rendered by the equivalent sentence “Every farmer beats every donkey he owns.”

- $(\exists x(Fx \wedge \exists y(Dy \wedge Oxy)) \rightarrow B\mathrm{p}_1\mathrm{p}_2) \Leftrightarrow$
  $\forall x \forall y((Fx \wedge (Dy \wedge Oxy)) \rightarrow Bxy)$.

The preceding observations are summed up in the following example, in which three items are introduced.

(13) Once there was a queen. $(\exists x Qx.)$ Her son fell in love with a frog. $(\exists y(Sy \wedge \exists z(Fz \wedge Lyz)).)$ He kissed it, and she got mad. $((K\mathrm{p}_1\mathrm{p}_2 \wedge M\mathrm{p}_3).)$

The first conjunct may be satisfied by a (any) (former) queen q. The second is satisfied by any pair sf of a son s and a frog f the son fell in love with. The first two conjuncts thus will be satisfied, if by anything, by triples sfq of a son, a frog and a queen, the mother of the son. In this order these contextually given entities are addressed by the last conjunct $K\mathrm{p}_1\mathrm{p}_2 \wedge M\mathrm{p}_3$, requiring the first (the son) to have kissed the second (the frog), to the effect that the third (the mother / queen) got mad. Again the order of the witnesses is the reverse of the order of appearance. The truth conditions are captured in an equivalent existentially quantified formula.

- $((\exists x Qx \wedge \exists y(Sy \wedge \exists z(Fz \wedge Lyz))) \wedge (K\mathrm{p}_1\mathrm{p}_2 \wedge M\mathrm{p}_3)) \Leftrightarrow$
  $\exists y \exists z \exists x((Qx \wedge (Sy \wedge (Fz \wedge Lyz))) \wedge (Kyz \wedge Mx))$.

It is interesting to see, then, that the contents of rather involved existentially quantified formulas can always be displayed in a step by step manner using the pronominal devices of *PLA*.

So far we have been looking at some examples of sentences or sequences of sentences in which pronouns occur, and which are equivalent to ordinary pronoun free first order formulas. These 'normalized', or 'binding' alternatives can be defined fully generally. I will present a 'normalization' algorithm below, which essentially draws from the following equation presented first. (Here, and in what follows, if $\widehat{x}$ is a sequence of variables $x_1 \ldots x_n$, then $\exists\widehat{x}\phi$ abbreviates $\exists x_1 \ldots \exists x_n\phi$.) (The notion of a (normal) binding form is given in the normalization algorithm.)

Observation 6 (Binding Conjunctions) If $\widehat{x}$ is a sequence of variables $x_1 \ldots x_n$, and ϕ and ψ are closed and in binding form, and if the variables in $\widehat{y}$ respectively $\widehat{x}$ do not occur free in ϕ respectively ψ, then:

- $(\exists\widehat{x}\phi \wedge \exists\widehat{y}\psi) = \exists\widehat{y}\exists\widehat{x}(\phi \wedge [\widehat{x}/\mathrm{p}_i]\psi)$, where
  - $[x_1 \ldots x_n/\mathrm{p}_i]\psi = [\mathrm{p}_{r(\psi)-n}/\mathrm{p}_{r(\psi)}] \ldots [\mathrm{p}_1/\mathrm{p}_{n+1}][x_n/\mathrm{p}_n] \ldots [x_1/\mathrm{p}_1]\psi$, and
  - the variables x_i are free for the occurrences of p_i in ψ.

The conditions on this equation are that ϕ and ψ themselves are closed and in binding form, i.e., they do not contain 'active' existential quantifiers, and no locally resolved pronouns. (Cf., the binding algorithm below.)

Even though the conditions may appears rather complicated at first glance, the idea behind them is conceptually and computationally pretty simple. Observation (6) shows that quantifiers $\exists x_i$ from the left conjunct may take scope over the right conjunct, *if* pronouns p_i coreferential with them are replaced by variables x_i now bound by the quantifier. (The details of this are explained in a second.) When the scope of these existential quantifiers $\exists\widehat{x}$ is, thus, extended, we have to make sure that existential quantifiers $\exists\widehat{y}$ from the second conjunct, which were introduced later, still make their contribution after the $\exists\widehat{x}$ have done so. This is why the $\exists\widehat{y}$ from the second conjunct now also are assigned scope over the existential quantifiers $\exists\widehat{x}$. In order to make sure that no further existential quantifiers and resolved pronouns are left in the embedded formulas ϕ and ψ, they are required to be closed and in binding form before this resolution takes place.

In order for this binding reformulation to be equivalent, two kinds of things have to happen in the second conjunct ψ. Since ψ is assumed to be closed and resolved, there are no internally resolved pronouns there. So all pronouns either relate back to terms in the first conjunct ϕ, or they relate back to terms before the conjunction. Pronouns of the first kind, viz., those pronouns p_i with $i \leq n(\phi)$, are eliminated and replaced by the variable x_i bound by the ith existential quantifier from $\exists\widehat{x}$, which comes to take scope over ψ. Pronouns of the second kind, viz., those pronouns $\mathrm{p}_{n(\phi)+j}$ lose $n(\phi)$ preceding terms as an antecedent (the $n(\phi)$ existential quantifiers from ϕ) so their index reduces to j in p_j. They thus remain coreferential with the jth term before the whole conjunction. I will supply examples of the above observation after I have presented the *PLA*-binding algorithm.

Observation (6) can be used to produce the (normal) binding forms of a *PLA*-formula, which is an equivalent formula from with resolved pronouns removed. We obtain the binding form of a formula ϕ by questioning it as $[\phi]^?$, which will return an answer $[\psi]^!$ to the effect that ψ indeed is the binding form of ϕ, also written as $\phi^\bullet$.

Definition 5 (*Binding Algorithm*) The (normal) binding form $\phi^\bullet$ of a formula ϕ is the formula ψ such that $[\phi]^? \mapsto [\psi]^!$, where:

- $[(\phi \wedge \psi)]^? \mapsto ([\phi]^? \wedge [\psi]^?)$; $\quad ([\exists\widehat{x}\phi]^! \wedge [\exists\widehat{y}\psi]^!) \mapsto [\exists\widehat{y}\exists\widehat{x}(\phi \wedge [\widehat{x}/\mathrm{p}_i]\psi)]^!$ [1];
  $[\exists x\phi]^? \mapsto \exists x[\phi]^?$; $\quad \exists x[\phi]^! \mapsto [\exists x\phi]^!$;
  $[\neg\phi]^? \mapsto \neg[\phi]^?$; $\quad \neg[\phi]^! \mapsto [\neg\phi]^!$;
  $[Rt_1 \ldots t_m]^? \mapsto [Rt_1 \ldots t_m]^!$.

The questioning procedure moves us directly, top down, to the atomic subformulas of a formula ϕ, in order to observe that these are in binding form: $[Rt_1 \ldots t_m]^? \mapsto [Rt_1 \ldots t_m]^!$. This reflects the fact that in a solitary atomic formula pronouns cannot get resolved. Once a formula ϕ is in binding form, its negation $\neg\phi$ is as well, and also the existentially quantified $\exists x\phi$. In a negation and in an existentially quantified formula there is nothing to be resolved besides what can be resolved in the embedded (negated or quantified) formula. This means that the feature of being in binding form percolates bottom up through quantifiers and negations, until it reaches a conjunction. And such a conjunction, then, will always be of the form $(\exists\widehat{x}\phi \wedge \exists\widehat{y}\psi)$, where ϕ and ψ are closed and in binding form. The resulting translation is like the one given in Observation (6). From this definition it is directly clear where the resolution/binding does happen, that is, in a conjunction, in which pronominal relations are after all established. This happens in the way we have seen in Observation (6).

Observe that, of course, it may happen that the side conditions on observation (6) fail to hold, if variables in $\widehat{y}$ or $\widehat{x}$ *do* occur free in ϕ or ψ, respectively, or if an x_i is not free for a p_i in ψ. However, in all of these cases we can always use an α-converted variant of the formula to be resolved, which, by observation (5) above, is fully equivalent.

I now present an application of the Binding Algorithm to example (13), repeated here for convenience:

(13) Once there was a queen. Her son fell in love with a frog. He kissed it, and she got mad. $((\exists xQx \wedge \exists y(Sy \wedge \exists z(Fz \wedge Lyz))) \wedge (K\mathrm{p}_1\mathrm{p}_2 \wedge M\mathrm{p}_3))$.

The algorithm applies as follows.

- $[((\exists xQx \wedge \exists y(Sy \wedge \exists z(Fz \wedge Lyz))) \wedge (K\mathrm{p}_1\mathrm{p}_2 \wedge M\mathrm{p}_3))]^? \mapsto$
  $((\exists x[Qx]^? \wedge \exists y([Sy]^? \wedge \exists z([Fz]^? \wedge [Lyz]^?))) \wedge ([K\mathrm{p}_1\mathrm{p}_2]^? \wedge [M\mathrm{p}_3]^?)) \mapsto$
  $((\exists x[Qx]^! \wedge \exists y([Sy]^! \wedge \exists z([Fz]^! \wedge [Lyz]^!))) \wedge ([K\mathrm{p}_1\mathrm{p}_2]^! \wedge [M\mathrm{p}_3]^!)) \mapsto$
  $(([\exists xQx]^! \wedge \exists y([Sy]^! \wedge \exists z[(Fz \wedge Lyz)]^!)) \wedge [(K\mathrm{p}_1\mathrm{p}_2 \wedge M\mathrm{p}_3)]^!) \mapsto$

[1] provided that the conditions in observation (6) obtain.

$$
\begin{aligned}
&(([\exists x Qx]^! \wedge \exists y([Sy]^! \wedge [\exists z(Fz \wedge Lyz)]^!)) \wedge [(K\mathrm{p}_1\mathrm{p}_2 \wedge M\mathrm{p}_3)]^!) \mapsto \\
&(([\exists x Qx]^! \wedge \exists y[\exists z(Sy \wedge (Fz \wedge Lyz))]^!) \wedge [(K\mathrm{p}_1\mathrm{p}_2 \wedge M\mathrm{p}_3)]^!) \mapsto \\
&(([\exists x Qx]^! \wedge [\exists y \exists z(Sy \wedge (Fz \wedge Lyz))]^!) \wedge [(K\mathrm{p}_1\mathrm{p}_2 \wedge M\mathrm{p}_3)]^!) \mapsto \\
&([\exists y \exists z \exists x(Qx \wedge (Sy \wedge (Fz \wedge Lyz)))]^! \wedge [(K\mathrm{p}_1\mathrm{p}_2 \wedge M\mathrm{p}_3)]^!) \mapsto \\
&[\exists y \exists z \exists x((Qx \wedge (Sy \wedge (Fz \wedge Lyz))) \wedge (Kyz \wedge Mx))]^!.
\end{aligned}
$$

All in all, we find that:

- $((\exists x Qx \wedge \exists y(Sy \wedge \exists z(Fz \wedge Lyz))) \wedge (K\mathrm{p}_1\mathrm{p}_2 \wedge M\mathrm{p}_3))^\bullet =$
  $\exists y \exists z \exists x((Qx \wedge (Sy \wedge (Fz \wedge Lyz))) \wedge (Kyz \wedge Mx))$.

Before we carry on, it is interesting to observe that indeed all the action in the binding algorithm takes place in the rule dealing with conjunctions—because conjunctions are the place where connections get established and pronouns may get resolved. Also, this is quite the same as what happens in the translation of *DPL* into a static predicate logic as given in Cresswell (2002). There, as well, the crucial work is done in the clause for conjunctions, in which, too, major substitutions take place. Notice, however, that Cresswell's specification of the *DPL* notion of conjunction, in terms of a number of substitutions, is a way of rendering a dynamic interpretation of discourse in terms of a static first order translation. The translation, however, is stipulated, and it is not based on an independently specified semantics for the discourse itself. Distinctively, the substitutions we find in Observation (6) are sound with respect to the semantics of *PLA*, which has been independently specified above. The *PLA*-semantics itself, of course, is in no need of any substitutions.

The following observation shows the resolution to be correct.

Observation 7 (Resolution Correctness)

- For all formulas ϕ: $\phi^\bullet \Leftrightarrow \phi$.

This observation enables a straightforward comparison of *PLA* with kindred systems.

Observation 8 (PLA, PL, DRT and DPL) For all resolved formulas ϕ

1. for all M, g: $M, g \models_{PL} \phi^\bullet$ iff $M, g \models_{PLA} \phi$;
2. a *DRS* representing the contents of ϕ is isomorphic to $\phi^\bullet$, up to alphabetic equivalence;
3. for all M, g: $g[\![\phi]\!]_M g[\widehat{x}/\widehat{c}]$ iff $M, g, \widehat{c} \models \phi$,

where $\phi^\bullet = \exists\widehat{x}\psi$ with ψ closed, and there are no repetitions of quantifiers $\exists x$ in ϕ.

The first observation is a direct consequence of the Observations (4) and (7). The resolution of any resolved formula is pronoun free, and, hence, classical.

The second observation builds on the fact that ordinary discourse representation structures (*DRS* s) are of the form $\langle D, C\rangle$, where D is a set or sequence of discourse referents, and C a set or sequence of conditions, and the conditions are atomic formulas, or negations *DRS* s (or, for that matter, disjunctions of, or implications between *DRS* s). The resolution of a *PLA*-formula consists of a sequence of existential quantifiers $\exists\widehat{x}$ followed by a sequence of conjunctions of atomic formulas and negations, where in each negation $\neg\phi$ the formula ϕ is in resolved form, i.e., a *DRS* . Notice that while the architecture reaching normal binding forms of *PLA*-formulas is not stated in a compositional fashion, neither is the construction algorithm yielding *DRS* for natural language, or natural discourse. However, the interpretation function for the *PLA*-formulas themselves gives us the very same results, but in a direct and compositional way. If we put these facts in a illustrative row, for instance for the previous example (13), we get the following picture. First we repeat the sentence; then we translate in a fairly standardly fashion into the language of predicate logic (with anaphora); next we show the result of applying the normal binding algorithm, which we have seen before, and finally we get the corresponding *DRS* that would have come out of the *DRS* construction algorithm for example (13).

(13) Once there was a queen. Her son fell in love with a frog. He kissed it, and she got mad.
(13′) $((\exists x Qx \wedge \exists y(Sy \wedge \exists z(Fz \wedge Lyz))) \wedge (K\mathrm{p}_1\mathrm{p}_2 \wedge M\mathrm{p}_3))$.
(13′•) $\exists y \exists z \exists x((Qx \wedge (Sy \wedge (Fz \wedge Lyz))) \wedge (Kyz \wedge Mx))$.
(DRC(13))

xzy
Qx Sy Fz Lyz Kyz Mx.

Notice, first, that the order of discourse markers in (*DRC*(13)) is opposite to that of the existential quantifiers in (13′•), but this is immaterial; second, that the round brackets, superfluous here, have disappeared—but they are pieces of structure that are not omitted in more sophisticated versions of *DRT* . For the rest we can easily construct isomorphisms, and the reader is invited to do so with examples of his own, using the definitions stated above.

The comparison with *DPL* in Observation (8) is a bit more involved. *DPL*'s input variable assignment g plays the ordinary role of an assignment in *PL*, and *PLA*, of the free variables in a formula. A so-called *DPL*-'output' assignment h, which is $g[\widehat{x}/\widehat{c}]$ in the case above, encodes possible values of variables quantified over in ϕ, viz., the variables in the sequence $\widehat{x}$. Their possible values are the witnesses $\widehat{c}$ of the corresponding quantified constructions in *PLA*, whence the equivalence in observation (8).

Notice that the stated equivalence cannot be maintained if a formula does not have a binding form itself and we have to resort to α-conversion in *PLA*. Since α-conversion is typically not allowed in *DPL*, the correspondence between *DPL* and *PLA* breaks down here. Notice that it is precisely in these cases that *DPL*'s elegance breaks down as well. In these cases where we would have to resort to α-conversion in *PLA*, we find so-called 'variable-clashes' in *DPL*, where information about items introduced in a discourse gets destroyed because a variable in use is re-used for another purpose. This nasty and well-known problem in *DPL* basically originates from the choice to equate pronouns with variables.

It appears that Vermeulen (1993) provides an interesting alternative motivation for moving from *DPL* to *PLA*. In order to solve the mentioned problem with variable clashes, Vermeulen argues that one can model the repeated use of one and the same variable if different uses are taken to invoke *stacks of values*, instead of singular ones. Variables indicate whether they relate to the last, or any earlier element of the stack associated with the variable. As it happens, *DPL* can thus be modeled by stacking all discourse referents on one variable, so that, as a matter of fact, the variable is useless, and we are left with stacks alone, as in *PLA*.

A final note concerns the restriction in Observation (8), that we only deal with resolved formulas ϕ, which have no unresolved pronouns. For a logical system, which cannot itself look outside its context of use, this is a natural restriction. For, similarly, a classical system of predicate logic cannot say much about the resolution or interpretation of free variables, besides assuming they have some value.

2.3 Logical Properties of PLA

The last part of this section is devoted to the general logical properties of *PLA*. Not much would be gained if *PLA* did not fail *some* classical logical properties, and it fortunately does do so. *PLA*-conjunction, the typically 'dynamic' *PLA*-operation, fails two characteristic properties of its standard kin: idempotence and commutativity.

Observation 9 (Non-idempotence and Non-commutativity)

- *PLA* conjunction is not idempotent and not commutative.

Since the idea embodied in a system of dynamic semantics is that the occurrence of a formula is not only context dependent, but also context changing, clearly these two properties are under attack. If a formula ϕ contains pronouns (and its interpretation is, hence, dependent on context) and if it also contributes items (and, hence, changes the context), ϕ may have a different impact before and after it has occurred, i.e. in $(\phi \wedge \phi)$. Let us label ϕ_1 the first occurrence of ϕ in $(\phi \wedge \phi)$, and ϕ_2 the second. Then we can say that the context for ϕ_2 is the context for ϕ_1 *plus* what ϕ_1 has contributed to the context in the meantime; and the contribution of ϕ_1 is present, but put behind what ϕ_2 contributes. The conjunction $(\phi \wedge \phi)$ thus may be stronger than ϕ itself. Here is a stilted example.

(14) She is seeing a woman. She is seeing a woman.
$(\exists y(Wy \wedge S\mathtt{p}_1 y) \wedge \exists y(Wy \wedge S\mathtt{p}_1 y)) \Leftrightarrow$
$\exists y \exists x((Wx \wedge S\mathtt{p}_1 x) \wedge (Wy \wedge Sxy))$.

The effect of the first occurrence of $\phi = \exists y(Wy \wedge S\mathtt{p}_1 y)$ consists in the contribution of a woman seen by *she*, and this creates a context in which that woman, not the original antecedent of *she*, figures as a target referent for the use of *she* in the second occurrence of ϕ. The repetition of the sentence, *She is seeing a woman*, requires that she, whoever she is, is seeing a woman who is (also) seeing a woman, something which was not asserted by one occurrence of the sentence ϕ alone. Therefore, conjunction is not idempotent.

By the same token, if two formulas ϕ and ψ are both context dependent and context changing, then $(\phi \wedge \psi)$ and $(\psi \wedge \phi)$ are also deemed to be different as well. The context relevant for ψ in $(\phi \wedge \psi)$ is whatever is the context for ψ in $(\psi \wedge \phi)$ *with* the contribution of ϕ added, and the same goes *mutatis mutandis* for the context for ϕ. Moreover, the contributions of ϕ and ψ are ordered differently in both conjunctions. So, conjunction is not commutative.

Of course, this dynamic extension of *PL* affects the accompanying notion of entailment. As we will see, however, besides solicited effects, there are no unsolicited ones. Let me first state the required notion of entailment. (For completeness I add the standard static notion of entailment, which will become useful later.) In the following a sequence of formulas is naturally understood as its conjunction.

Definition 6 (*PLA Entailment*) A sequence of formulas $\phi_1, \ldots, \phi_m$ dynamically entails a formula ψ, denoted by $\phi_1, \ldots, \phi_m \models \psi$, iff

- for all $M, g, \widehat{e} \in D^{r(\phi_1,\ldots,\phi_n,\psi)}$ and $\widehat{c} \in D^{n(\phi_1,\ldots,\phi_m)}$ there is $\widehat{a} \in D^{n(\psi)}$: if $M, g, \widehat{ce} \models (\phi_1, \ldots, \phi_m)$, then $M, g, \widehat{ace} \models \psi$.

Naturally, *PLA* accommodates a notion of entailment involving sequences, not sets, of premises, which, together with the conclusion, are evaluated as to logical consequence. It is required, relative to any model M and variable assignment g, and relative to any potentially required sequence $\widehat{e}$ for unresolved pronouns in $\phi, \ldots, \phi_n, \psi$, in that order, that if any sequence of witnesses satisfies the premises, in the order given, then it provides a context in which ψ is true—always. The idea is classical, and the only difference is that possible witnesses may be passed through in the order of premises to serve as target reference points for subsequent pronouns. Obviously, when restricted to the pronoun free fragment the *PLA*-entailment relation is classical.
Observation 10 (Conservative Entailment) The *PLA* entailment relation $\models$ is classical relative to the pronoun free formulas of $\mathcal{L}_{PLA}$.

Behaving classically in all classical cases does not mean that *PLA* entailment is classical in all respects. The notion is dynamic not only in the rather trivial sense that anaphoric connections may get established between terms in different premises; anaphoric connections are also possible between terms in premises and pronouns in the conclusion. The following inference is modeled after Heim (1982); the translation is mine.

(15) If a man is from Athens, he is not from Rhodes. There is a man from Athens here. So, he is not from Rhodes.
$(\exists x(Mx \wedge Ax) \rightarrow \neg R\mathrm{p}_1), \exists x(Mx \wedge Ax) \models \neg R\mathrm{p}_1.$

The translation, as entailment, is valid in *PLA*. A similar, more natural, example Geach (1962) has taken from Peter Strawson.

(16) A: A man has just drunk a pint of sulphuric acid.
B: Nobody who drinks sulphuric acid lives through the day.
A: Very well then, *he* will not live through the day.
$\exists x(Mx \wedge DPSAx), \neg\exists z(DPSAz \wedge LTDz) \models \neg LTD\mathrm{p}_1.$

The transcription is a valid entailment in *PLA*. We observe natural entailments here, with pronouns in the conclusion, which relate back to terms figuring in the premises, and which are valid entailments.

In the examples from Heim and Strawson we observe, not surprisingly perhaps, a strong connection between premises and conclusion. Heim's man from Athens can be concluded to be not from Rhodes because *no* man from Athens is from Rhodes, at least, this is what one of the premises claims. Likewise, Geach's man cannot live through the day, because he has just a drunk a pint of sulphuric acid, and an additional premise has it that no man who does so will live through the day. The inferential step from an arbitrary man to a conclusion about *him* can only be made because the step is universal, by the logic, and/or by additional premises. This strong (universal) connection between terms in the premises of an entailment, and pronouns in its conclusion, is reminiscent of the strong (universal) connection between terms in donkey sentences. This, again, need not come as a surprise, since it is easily seen that a valid entailment directly corresponds with an unconditionally valid implication.

Observation 11 (Deduction Theorem)

- $\phi_1, \ldots, \phi_m, \chi \models \psi$ iff $\phi_1, \ldots, \phi_m, \models (\chi \rightarrow \psi)$.

Heim's inference directly follows from the following, valid, identity inference.

- $(\exists x(Mx \wedge Ax) \rightarrow \neg R\mathrm{p}_1) \models (\exists x(Mx \wedge Ax) \rightarrow \neg R\mathrm{p}_1)$.

Geach's inference requires some more work, but it can be remodeled to the same form.

The fact that $\models$ subsumes all classical validities (Observation 10), does not imply that it obeys classical structural laws. Obviously it does not, since, as we have seen, entailment is dynamic, and it will not generally be preserved under permutation of the premises. Order matters. Moreover, like notions of entailment from other systems of dynamic interpretation, the *PLA*-entailment relation fails some other structural properties, characteristic of, and sometimes deemed essential for, standard logical systems. Entailment is not a reflexive, monotone, and transitive relation. Good reason exist for these failures though, as I will argue, and I will next show that the consequences are not that bad after all, in *PLA* that is.

Let us inspect the rationale behind a non-reflexive, non-monotone, and non-transitive entailment relation. Like I said before, interpretation is context dependent and context changing. From this it directly follows that, in principle, a formula which is satisfied in a certain context, may also change the context into one which no longer satisfies it. This observation relates to the fact that conjunction is not idempotent. A clear, but artificial, example is the following.

(17) He is an Irish boy, and he wrote a non-Irish friend. So$^?$, *he* (i.e., the friend) is an Irish boy and he wrote a non-Irish friend.
$(IB\mathrm{p}_1 \wedge \exists y(\neg IBy \wedge W\mathrm{p}_1 y)) \not\models (IB\mathrm{p}_1 \wedge \exists y(\neg IBy \wedge W\mathrm{p}_1 y))$.

The formula at issue (or the rendering of it under the intended interpretation) does not entail itself, because that would require a non-Irish friend to be Irish. (Worse, the conjunction of the contingent formula with itself is inconsistent.) Non-monotonicity comes about for basically the same reason. A conclusion may follow from a sequence of premises, because the premises always set up a context in which the conclusion is satisfied, but then an additional premise may undo precisely the relevant contextual effects. Consider the previous example again. From *He is an Irish boy* it follows that

He is an Irish boy. The conclusion, however, does not follow from *He is an Irish boy, and he wrote a non-Irish friend.*

(18) $IB\mathrm{p}_1 \models IB\mathrm{p}_1$, but $IB\mathrm{p}_1, \exists y(\neg IBy \wedge W\mathrm{p}_1 y)) \not\models IB\mathrm{p}_1$.

The non-transitive aspects of entailment already can be witnessed from Johan van Benthem's "dynamic" counterexample to Aristotle's prime example of a valid syllogism, Barbara. Barbara relies on the transitivity of the universal quantifier, and in van Benthem's counterexample the cutting of Barbara's middle term causes the break down of an anaphoric connection.

(19) All men who have a garden sprinkle it on Saturdays.
All men who have a house are men who have a garden.
So$^?$, all men who have a house sprinkle it on Saturdays.

The example has the impact of a practical joke, but it does show a serious problem, which we have to take to heart. For, inference schemes like Barbara, like basically all logical schemes, are supposed to be valid because of their form, and the form of van Benthem's example is impeccably Barbarian. With the following example we see essentially the same problem arise for our notion of entailment.

(20) If Jane has a garden, she sprinkles it right now and if Jane owns a house, she has a garden. Now Jane actually owns a house. So, she has a garden, and, so, she sprinkles it right now.
$((\exists y(Gy \wedge Hjy) \rightarrow Sj\mathrm{p}_1), (\exists x(Hx \wedge Ojx) \rightarrow \exists y(Gy \wedge Hjy)), \exists x(Hx \wedge Ojx) \models (\exists y(Gy \wedge Hjy) \wedge Sj\mathrm{p}_1).)$

In this example the second and the third premise yield the first conclusion, that Jane has a garden, and this conclusion together with the first premise yields the goal conclusion that she sprinkles it. By 'cutting' the inference, taking out the intermediate conclusion that Jane has a garden, we would like to conclude directly from the three premises that she sprinkles it.

(21) If Jane has a garden, she sprinkles it right now and if Jane owns a house, she has a garden. Now Jane actually owns a house. So, she sprinkles it right now.
$((\exists y(Gy \wedge Hjy) \rightarrow Sj\mathrm{p}_1), (\exists x(Hx \wedge Ojx) \rightarrow \exists y(Gy \wedge Hjy)), \exists x(Hx \wedge Ojx) \models Sj\mathrm{p}_1.)$

This obviously sounds absurd and indeed the inference is invalid in *PLA*. By cutting the middle term in the inference we lose the appropriate anaphoric connection gets lost, and the pronoun appears to resolve with the house. The conclusion then, which is *not* entailed, would be that Jane sprinkles the house. Naturally, this inference comes out wrong in *PLA*, but then it shows that it is not in general allowed to 'cut' inferences this way.

Even though the *PLA* entailment relation fails the three mentioned properties, for good reasons I say, it nevertheless retains the valuable aspects of these properties. That entailment is reflexive and monotone is attractive, or even pertinent, because an assumption should, if anything, at least entail itself, and a conclusion ought to remain valid once it is obtained. In *PLA*, *these* facts do remain beyond doubt, however. If a conclusion is established as an assumption or conclusion at a certain point in an argument, then it remains as a valid assumption or conclusion throughout the whole argument; the point is that the same conclusion may have to be reformulated, in order to adjust it to the fact that the context may have changed in the meantime. [The point is nicely put by Frege: “Wenn jemand heute dasselbe sagen will, was er gestern das Wort ‘heute’ gebrauchend ausgedrückt hat, so wird er dieses Wort durch ‘gestern’ ersetzen.” (Frege 1918, p. 62)] As we will see shortly, such a reformulation can always be given, a possibility which is not obvious in a system like, for instance, *DPL*. Transitivity is also a useful property, because it allows us to re-use inferences made earlier, and not to have to redo every inference time and again. In *PLA* we can transport the results of one inference from one context to another, as long as we keep track of the various changes in context and make suitable reformulations.

In order to state the right structural rules in *PLA*, we need a general device to adapt formulas to the fact that the context around it may have been expanded or reduced. The relevant facts pertain to the number of existentials that have occurred, and that may have increased or decreased. Obviously, such changes in the context should be reflected by corresponding increases and decreases of the index on the relevant pronouns. I therefore define the increase $[^{n+}_{j}]$ with n and a decrease $[^{-n}_{j}]$ with n of the index of pronouns which are selected by the auxiliary device j. The definition is a bit tedious, yet easily computable.

Definition 7 (*Pronoun Update and Downdate*) $[^{n+}_{j}]\phi$ and $[^{-n}_{j}]\phi$ are recursively defined for variables $\widehat{x}$ free for the pronouns in ϕ.

- $[^{n+}_{j}]c = c = [^{-n}_{j}]c$; $[^{n+}_{j}]\mathrm{p}_i = \mathrm{p}_{n+i}$, if $(j < i)$;
  $[^{n+}_{j}]x = x = [^{-n}_{j}]x$; $[^{-n}_{j}]\mathrm{p}_i = x_{i-j}$, if $(j < i \leq (j+n))$;
  $[^{n+}_{j}]\mathrm{p}_i = \mathrm{p}_i = [^{-n}_{j}]\mathrm{p}_i$ if $(i \leq j)$; $[^{-n}_{j}]\mathrm{p}_i = \mathrm{p}_{i-n}$, if $((j+n) < i)$;
- $[^{n+}_{j}]Rt_1 \ldots t_n = R[^{n+}_{j}]t_1 \ldots [^{n+}_{j}]t_n$; $[^{n+}_{j}]\neg\phi = \neg[^{n+}_{j}]\phi$;
  $[^{-n}_{j}]Rt_1 \ldots t_n = R[^{-n}_{j}]t_1 \ldots [^{-n}_{j}]t_n$; $[^{-n}_{j}]\neg\phi = \neg[^{-n}_{j}]\phi$;
  $[^{n+}_{j}](\phi \wedge \psi) = ([^{n+}_{j}]\phi \wedge [^{n+}_{j+n(\phi)}]\psi)$; $[^{n+}_{j}]\exists x\phi = \exists x[^{n+}_{j}]\phi$;
  $[^{-n}_{j}](\phi \wedge \psi) = ([^{-n}_{j}]\phi \wedge [^{-n}_{j+n(\phi)}]\psi)$; $[^{-n}_{j}]\exists x\phi = \exists x[^{-n}_{j}]\phi$.

The instruction $[^{n+}_{j}]$ on ϕ updates ϕ to the fact that n more terms have been used, and $[^{-n}_{j}]$ indicates the update of ϕ to the fact that n less terms have been used. The j indicates *where* in the past n more or n less terms have been used, the break-even point so to speak. This means that pronouns are left untouched if they relate to witnesses up to j terms back. Pronouns, however, that refer to witnesses beyond that point will either have to gain a wider reach (increase their index by n) or lower it (decrease their index by n). If less terms have been used than before, and if a pronoun initially targeted its referent from among the terms that have gone, then it will be removed and replaced by a variable. (A variable which eventually is existentially closed.)

In Definition (7), the first half states the required substitutions on terms. In the left column nothing happens, because it concerns individual constants, variables, and pronouns with an index up to j, which are not affected. In the right column the upgrade and downgrade get accounted for, in the manner just sketched. The second half of the definition states the required effects for any formula in a recursive manner. Except for the second conjunct of a conjunction, this effect involves a simple matter of distribution. In a conjunction $(\phi \wedge \psi)$, however, the instruction on the second conjunct ψ has to adapt to the fact that $n(\phi)$ more terms have occurred between ψ and the break-point j, which means that the break-even point has to be updated to $j + n(\phi)$.

Armed with this notational device we can state acute versions of identity, monotonicity and a cut rule, the rules which motivate the reflexive, monotone and transitive nature of entailment.

Observation 12 (Acute Identity and Monotonicity)

- $\phi_1, \ldots, \phi_i, \ldots, \phi_m \models [^{k+}_{0}]\phi_i$
  - for $k = n(\phi_i, \ldots, \phi_m)$, and
- if $\phi_1, \ldots, \phi_m \models \psi$, then $\phi_1, \ldots, \phi_{i-1}, \chi, \phi_i, \ldots, \phi_m \models [^{k+}_{j}]\psi$,
  - for $k = n(\chi)$ and $j = n(\phi_i, \ldots, \phi_m)$.

Any formula ϕ entails itself, or can be repeated, provided that, when it is used as a conclusion, it is updated with the information that more terms have occurred than when it was used before. The conclusion drawn from ϕ is the one obtained from ϕ by replacing unresolved pronouns p_i by p_{k+i}, with k the number of terms that have occurred in the meantime. In the example (17) discussed above, indeed $(IB\mathrm{p}_1 \wedge \exists y(\neg IBy \wedge W\mathrm{p}_1 y))$ does not entail itself:

- $(IB\mathrm{p}_1 \wedge \exists y(\neg IBy \wedge W\mathrm{p}_1 y)) \not\models (IB\mathrm{p}_1 \wedge \exists y(\neg IBy \wedge W\mathrm{p}_1 y))$.

However, it does entail $[^{1+}_{0}](IB\mathrm{p}_1 \wedge \exists y(\neg IBy \wedge W\mathrm{p}_1 y))$ which is $(IB\mathrm{p}_2 \wedge \exists y(\neg IBy \wedge W\mathrm{p}_2 y))$:

- $(IB\mathrm{p}_1 \wedge \exists y(\neg IBy \wedge W\mathrm{p}_1 y)) \models (IB\mathrm{p}_2 \wedge \exists y(\neg IBy \wedge W\mathrm{p}_2 y))$.

Also, if ψ is entailed by a sequence of premises, then the same sequence with an additional premise χ still entails ψ, but after the conclusion is updated with the fact that $n(\chi)$ more terms have occurred. So every pronoun p_i in ψ which has to 'bridge' the interfering terms in χ, is replaced by $\mathrm{p}_{n(\chi)+i}$.

The statement of the cut-inference pattern is a bit more involved, because a mediating conclusion which serves as a premise gets cut out, even though it may have supplied witnesses for the goal conclusion. For this reason, we have to use the $[^{-n}_{j}]$ instruction which deletes pronouns which get rid of their antecedent witness, and replace these with existentially bound variables.

Observation 13 (Acute Cut Rule)

- If $\phi_1, \ldots, \phi_{m-1}, \phi_m \models \psi$ and $\chi_1, \ldots, \chi_l \models \phi'_m$,
  then $\phi_1, \ldots, \phi_{m-1}, \chi_1, \ldots, \chi_l \models \psi'$, where

– $\phi'_m = [^{n(\chi)+}_{0}]\phi_m$, with $\chi = \chi_1, \ldots, \chi_l$ and
$\psi' = [^{n(\chi)+}_{0}]\exists\widehat{x}[^{-n(\phi_m)}_{0}]\psi$, with $\widehat{x} = x_1 \ldots x_{n(\phi_m)}$.

In the second condition for the cut rule the conclusion ϕ' says what the premise ϕ says in the first condition if $\chi_1, \ldots, \chi_l$ had not occurred. The goal conclusion ψ', is the original conclusion ψ updated to the fact that the premise ϕ_m has been cut out, and $\chi_1, \ldots, \chi_l$ have been added. Pronouns which related back to antecedents in ϕ_m are replaced by variables which get bound by $\exists\widehat{x}$. Here is how the van Benthem-style inference gets cured. We find that (a) and (d) entail (e), and that (b) and (c) entail (d). But we do not find that (a) and (b) and (c) entail (e), but (e′).

(a) If Jane has a garden, she sprinkles it right now. $(\exists y(Gy \wedge Hjy) \rightarrow Sj\mathrm{p}_1)$;
(b) If Jane owns a house, she has a garden. $(\exists x(Hx \wedge Ojx) \rightarrow \exists y(Gy \wedge Hjy))$;
(c) Jane actually owns a house. $(\exists x(Hx \wedge Ojx))$;
(d) Jane has a garden. $(\exists y(Gy \wedge Hjy))$;
(e) She sprinkles it right now. $(Sj\mathrm{p}_1)$;
(e′) She sprinkles something right now. $([^{1+}_{0}]\exists x[^{-1}_{0}]Sj\mathrm{p}_1 \equiv \exists xSjx)$.

Clearly the (derived) cut rule will never fail for any practical purpose, since the conditions on identity or cut never fail. The standard structural properties of logical consequence are thus preserved in a suitably adapted form, and the logic remains well-behaved.

2.4 On the Representation of Information

The achievements of *DRT* , *FCS*, and *DPL*, and all of their offspring, can be characterized as follows. Agents have representations of the world, or of stories, dreams, or impressions they have made up, or have been told, or have experienced. These representations are very often rather large structured wholes and communicating them involves cutting them into pieces. When these wholes have been cut into pieces, and other agents have to glue the pieces together, the structural relations between (the parts of) the pieces have to be re-established of course. The three frameworks mentioned present typical ways in which this may be done, for the typical kind of relationships which, in natural language, we encounter as identity anaphora.

The way in which this task is achieved in classical *DRT* , that of (Kamp 1981; Kamp and Reyle 1993), may be pictured as follows. I will, again, use a rather stilted example, but it displays the essential ingredients. Suppose someone has a picture of the following situation, in which a man, who was walking in the park, ran away from a dog he saw there. To simplify things a bit more we neglect the park and all temporal aspects and then the description, as given, can be represented by means of the following predicate logic formula:

(22) $\exists x((Mx \wedge Wx) \wedge \exists y((Dy \wedge Sxy) \wedge Rxy))$. (A man is walking and there is a dog he sees and which he runs away from.)

In *DRT* , the corresponding representation is the following discourse representation structure (*DRS*):

(23)

x y
Mx Dy
Wx Sxy Rxy

Surely, there is a way of communicating this little ‘story’ in one sentence, like I did above, but it can be cut in parts, as in the following little discourse.

(24) A man is walking in the park.

(25) He sees a dog.

(26) He runs away from it.

The most interesting part of this little discourse is the sentence in the middle. The sentence contains a pronoun (‘he’), which needs to be resolved with something mentioned in the previous sentence, and it contains an indefinite (‘dog’) which may license subsequent anaphoric pronouns. Classical *DRT* deals with this sequence of sentences in the following way. The first sentence (24) gives rise to a preliminary *DRS* :

(27)

x
Mx, Wx

This *DRS* represents the information that some man was walking in the park, and the *representation* serves as the context of interpretation for the second and subsequent sentences. The second sentence (25) is then literally *plugged* into this representation, yielding the *DRS* :

(28)

x y
Mx Dy
Wx Sxy

This *DRS* represents the information that some man who was walking through the park saw a dog, and the representation again serves as the context of interpretation for the final sentence (26). This third sentence is also plugged into the current discourse representation and yields the final *DRS* , which is the same as the original one (23), and thus captures the information we started out with. In this way, a structured representation has been cut into pieces, and reconstructed, in a sound and information preserving way. All more involved structured representations which agents may have, with structural relationships other than identity, and which cannot that easily be formulated into one sentence, can similarly be decomposed and reconstructed along essentially similar lines.

While any occurring discourse representation in the example above comes with its own content, or meaning, specified in terms of its Tarskian truth-conditions if you want, one of the major criticisms of *DRT* has been that almost none of the sentences of natural language themselves are assigned a meaning under this approach. In the example above, the middle sentence (25), does not have a meaning of its own, but it is associated with an instruction to changes one *representation*, *DRS* (27), into another one, *DRS* (28). And although each of these *DRS* 's have their own truth conditions, the change from the one into the other cannot be given a truth-conditional interpretation. For this reason, many authors hold that, although meaning may in the end be truth-conditional, practically a level of representation is necessary for the actual interpretation of structured discourse, and, hence, of meaning in general.

However, one of the merits of the *DRT* interpretation procedure, and also a reason for its success, is that this representational level is, or can be, extremely rich, much richer and more fine-grained than a corresponding domain of meanings, so that a lot of interpretational effects can be modeled there. *DRT* indeed lends itself most naturally as an architecture in which to formulate an account of phenomena which seem to require quite a lot of computation, such as those involving plurals, tense, ellipsis, and many other phenomena. This wealth, however, also carries a risk.

While it may be obvious to many that this level of representation is called for, as Groenendijk and Stokhof remark, such a conclusion does not come without philosophical pitfalls (Groenendijk and Stokhof 1991, p. 97). For if such representationalist conclusions are implemented in the formal semantic architecture, it may become vulnerable to cognitive psychologist findings about how agents *actually* represent things. If natural language meanings are spelled out, partly, in terms of how they interact with given discourse representation structures, then these ought to be to some extent realistic structures, in order for the analyses to be tenable, or at least so it seems. Furthermore, if it is not the expressions of an arguably *public* language, like a natural language, whose meanings we learn in life, but if these are eventually, equally arguably, *private* representations, then how at all could we learn them, or how could there be anything to learn about them at all?

Groenendijk and Stokhof therefore conceive of the decomposition and reconstruction task differently, in *DPL*, and as a matter of fact, recent formulations of *DRT* , as in van Eijck and Kamp (1997) and Kamp et al. (2011), appear to agree with their conception. This alternative conception is nicely displayed as follows. In order to deconstruct a representation like (23), we can suggestively cut it apart as follows:

(29)

x	y	
Mx	Dy	
Wx	Sxy	Rxy

The first part in (29) indicates that we start the story with a man who is walking in the park, and suggests there may be more to be told about such a man, which is indicated by omitting the right- and bottom-lines from (23). [Actually, this is the type of representation employed by Pieter Seuren (1985).] The second part states the condition that *he* (x) sees a dog (y), where the omitted top- and left-lines indicate that x seeks a resolution in previous discourse, and the omitted right- and bottom-lines indicate that both x (here: the man) and y (the dog) may be elaborated further upon in subsequent discourse. The third, and final part, adds the condition that x runs away from y, that both need to be resolved by previous discourse, and the concluding right- and bottom-line indicate that this is the end of the story.

Notice that the elegant simplicity of *DPL* may obscure the remarkable achievement that it succeeds in associating all three parts of (29), that is each of the sentences (24), (25), and (26), with an independent and uniform type of meaning. As we indicated above, this is done in terms of input- output-conditions. Most significant is again the middle part. It requires as an input an assignment which associates x with someone who sees a dog, and renders as output an assignment with the same value for x, and which associates y with a dog x sees. Basically, this is the technique *DPL* employs to model the decomposition and reconstruction of structured representations, in terms of a truly compositional semantics.

It can be argued, though, that *DPL* does not really answer the anti-representationalist challenge, mentioned above. For while *DPL* can model the interpretations of *DRS* parts as in (29), and, thus, of the type of representations employed in recent versions of *DRT* like that in Kamp et al. (2011), it inherits some of *DRT* 's representational nature. For one thing, taking the *DPL* metaphor of interpretation literally, a (pseudo-)formula like $\exists y((Dy \wedge Sxy)$, which corresponds to the second part of (29), carries the information, or presupposition, that something has been previously mentioned under the label, or by means of the variable x. It establishes "a fact about the conversation, and not about the subject matter," as Stalnaker (1998, p. 13) puts it. In the words of Groenendijk et al. (1996, p. 183): "When one is engaged in a linguistic information exchange, one (…) has to store *discourse information*. …. Discourse information of this type looks more like a book-keeping device, than like real information." Of course, there is no point in denying that many devices of natural language have a typical discourse role to play, and whose meaning partly or even entirely consists in its function in whole-scale discourses. But it is quite debatable that such should constitute the core-idea of linguistic meaning, as displayed in the typically dynamic slogan that meanings are context change potentials.

With Frege, the early Wittgenstein, and Tarski, it seems we can make some sense of a truth-conditional concept of meaning, or at least of truth-conditional roots of meaning, and with the radical translations and interpretations of Quine and Davidson, a truth-conditional methodology in the theory of interpretation may stand up against relativistic threats. Against such a background, however, the idea that meanings are context change potentials is hard to hold. The contexts employed in dynamic semantics, and the changes brought about in them, are very abstract objects, and not just because they belong to the linguist's theoretical ontology, but also if they are conceived of as real objects which these abstracts objects are supposed to model. As abstract objects, they will not provide the radical translator, or the language learner, any input. And if they are concrete, say representations or information states of the individuals involved in a conversation, they are again subject to the anti-representationalist challenge, and, indeed, provide little or no input to the radical translator.

How does *PLA* stand in the face of these conceptual qualms? In the abstract model of *PLA* the witnesses, together with a dynamic notion of conjunction, are used to establish anaphoric connections. If we generalize over structural relationships other than identity anaphora only, represented connections have to be established at the level of meaning, in the real relations between witnesses. Indeed all of this is done in terms of the models, worlds, and its ingredients relative to which interpretation takes place, and it makes no representational commitments. In addition to this, as we have seen, everything that does not belong to establishing meaningful connections, everything that does not contribute to decomposing and reconstructing meanings, is totally classical. So while I do not want to deny *DRT* or *DPL* any of their respective merits and benefits, I claim that *PLA* may serve the same purposes without deviating from conceptually well-established paradigms.

Variables and Pronouns Another issue different from but also related to representation and information, is the use of (free) variables to hang discourse information upon, or pronouns, or neither. The literature on discourse, quantification, and anaphora displays a variety of positions one may take in this issue. We can ultimately deal only with bound variables (classical logic, *DPL*); we can deal only with free variables (Kamp, Heim, *FCS*); we can do without variables (Quine, Jacobson); and we can deal with bound variables and pronouns (*PLA*). Mixed logical and theoretical motivations are given for either of these stances.

First observe that anything that can be said in *PLA*, with resolved formulas, can be said by (quantified) formulas which are pronoun-free. This is essentially what Observation (4) tells us. The converse, however, holds as well. Anything that can be said in *PL*, with formulas without free variables, can be said with *PLA*-formulas without any (distinct) variables. For, the resolved form $\exists x_1 \ldots \exists x_n \phi$ of any such formula can be written, equivalently, as $((\ldots(\exists x_n(x_n = x_n) \wedge \exists x_{n-1}(x_{n-1} = x_{n-1}))\ldots) \wedge \phi')$ where ϕ' is obtained from ϕ by replacing all the x_i's with suitable pronouns. The proof is left to the reader. Notice that the variables $x_1 \ldots x_n$ in the resulting formula are completely immaterial, so that we might have written $((\ldots(\exists \wedge \exists)\ldots) \wedge \phi)$ instead. The trade-off between variables and pronouns sheds an interesting light on several discussions about variables and pronouns in the logico-linguistic literature. The differences among the various approaches must lie in the details of implementing the various types of semantics coming with them.

The claim that the variables can be dispelled with has been convincingly argued for in Quine (1960), Jacobson (1999) and Szabolcsi (1989). Without going into details, the main moral here consists in the fact that the meanings of sentences can be conceived of in full generality as formulas with n open places, so that their meanings are functions from sequences of n individuals to the truth values the sentences obtain when they ate evaluated when the n open places are filled with the n individuals in these sequences. As a matter of fact, this is how Alfred Tarski conceived of spelling out the truth conditions of a formal language (Tarski 1923, 1956).

Similarly, while considering variables and pronouns in a dynamic semantic setting, we can identify, on methodological grounds for instance, the use of variable assignments and that of sequences of individuals. In the framework of a dynamic semantics, Irene Heim first cashed out the trade-off between variable assignments on the one hand (assuming an enumeration of the variables), and sequences of individuals, on the other (Heim 1982). Kees Vermeulen and Jan van Eijck proceeded along similar lines (Vermeulen 1993; van Eijck 2001). Quite recently, Cresswell (2002) has argued for a reformulation of *DPL* in terms of static first order predicate logic. It is worthwhile to inspect his reformulation in some detail.

Max Cresswell builds his *DPL*-interpretation of a sentence "A man entered the house." on the insight that it is supposed to mean something like, "A man, namely y, entered the house." Even though y may be left unspecified, it serves the purpose of anaphorical pick-up in a continuation "He was broke," meaning, that he, viz. y, was broke. Notice that this "namely y" interpretation of uses of indefinite noun phrases is very much like its *PLA* interpretation, where we employ the witness d, which can be supposed to be the interpretation of y. As we mentioned above already, Cresswell's main moral is like the one argued for in this monograph. Dynamic semantic observations do not force us to adjust our notion of meaning, but, as Cresswell also shows, that they require a more involved, or dynamic notion of conjunction, and of other coordinating expressions. Notice, too, that the output value of variables in the static reformulation suggested by Cresswell, is stored under a name (the variable y) which is required not to belong to the language of *DPL* itself. Like our witnesses, one might say, it is 'alien'.

The difference between Cresswell's approach and the one offered in this monograph, which one may classify as a formal semantic implementation of Cresswell's insights, bears on the present discussion. While the title suggests otherwise, Cresswell does not really supply a static semantics for dynamic discourse. What he eventually offers is a translation of *DPL* into a static first order language, where, like in *FCS* and *DRT* , and *PLA*, the terms that seems to be existentially bound variables are treated as free variables, or free variable copies. The translation is effective, but it is not stated in a compositional way, though. For instance, the (crucial) translation of a conjunction $(\phi \wedge \psi)$ from *DPL*, is stated, among other things, in terms of the translation of ϕ along with a substitution of that translation, of certain variables that ϕ—not the translation of ϕ!—has in common with ψ. To define the interpretation of the modified translation of ϕ from the interpretation of the translation of ϕ itself i surely not easy.

Nevertheless, the task seems to succeed, but arguably in a rather unrevealing manner. The main technical result of Cresswell's paper can be phrased, in more mundane words, as follows. Where a pair $\langle g, h\rangle$ of assignments to the variables in a *DPL*-language satisfies a certain formula ϕ, the standard predicate logical interpretation of Cresswell's translation of the formula is an assignment μ, which encodes both $g(x)$'s possible input value of a variable x, and $h(x)$'s possible output value of that variable. In formal semantic terms, it translates a set of pairs of assignments $\langle g, h\rangle \in [\![\phi]\!]_{dpl} \subseteq (D^V \times D^V)$ into a set of assignments $\mu \in [\![\phi']\!] \subseteq D^{V'}$, where V' is the disjoint union of V with itself. The two domains are isomorphic, as is easily seen. Furthermore, I have given the semantic implementation of Cresswell's ideas in Dekker (1998, 2000), where the 'static' meanings are assignments of both input- and output values of variables.

While very similar in spirit the approach with *PLA* is more attractive than the one suggested by Cresswell. In the first place, *PLA* does employ a direct and ordinary interpretation of conjunction, as intersection, whereas Cresswell's reformulation of

DPL conjunction requires a translation algorithm involving elaborate substitution and quantification over the input- and output-values of variables.

As may be obvious from the preceding deliberations, we can easily deal with variables in terms of sequences of individuals, like Tarski did, or in terms of variable assignments, which may be the standard way now, and which is extensively studied and motivated by Theo Janssen (1986). The same holds for the interpretation of pronouns, which can be dealt with by sequences of witnesses of terms that have occurred previously, as in *PLA*, or by assignments to variables that name these previous occurrences, like, e.g., in Groenendijk et al. (1996), among many others. The fact that variables and pronouns are dealt with by means of different techniques in *PLA* is totally immaterial. That they are handled separately, I believe, has some substance.

For as far as I can make sense of them, variables are theoretical devices. They are a logician's or a theoretical linguist's invention, used to indicate argument places, binding open slots of predicates and relation expressions, but do not seem to be realized in natural language. In contrast, pronouns, as I think of them, really occur in natural language, even if invisible. Sure enough, obvious connections exist between the two, because what can be formulated, in a formal language, using variables, can be formulated, in a natural language, using pronouns, and vice versa, as most of the literature on the subjects presupposes. But there is no a priori, or self-evident, justification that the theorist's variables really are the natural language user's pronouns. The very, valuable, existence of a variable free semantics bear witness to that fact: we can have a logic without variables, but we cannot deny that natural languages do accommodate pronouns.

A distinction between pronouns and variables helps to explain some fundamental features of syntax, having to do with locality constraints on reflexives, and non-locality constraints on pronouns. [The following observation are taken from Butler (2003).] Consider the following examples, typically attributed to conditions A and B of Binding Theory:

(30) David/Every boy shaved himself.

(31) David/Every boy shaved him.

In example (30), the reflexive "himself" must be interpreted as, or bound by, David, or every boy, in its most local domain. In example (31), such a reading is hardly/not possible for the pronoun "him." The pronoun can be interpreted as David, if it can be construed as being anaphorically dependent on David when David has been mentioned before, but it cannot get a bound reading as in example (30). The reason is that "him" really is a pronoun, which cannot be bound by a controlling term; it can only be co-valuated with terms mentioned before it. "Himself" is not a pronoun, or variable, at all, but it is best conceived of as an operator which turns a relation into a reflexive predicate. Indeed, this is something we can represent very well by using variables in a language with lambdas. For any relation R, we can take "R him/her-self" to translate as $\lambda x\ Rxx$. In this way, the argument position occupied by "himself" gets bound by the term this predicate is ascribed to. Likewise, consider:

(32) Only David voted for himself.

(33) Only David voted for him.

In example (32) we get the reading that only David is a self-shaver, nobody, except David, is said to shave himself. Conversely, in example (33) we only get the reading that nobody else but David only shaved "him", a figure that needs an antecedent from the context—and maybe this is David himself. Even in the latter case, the reading is that nobody else but David shaved David.

The above observations are accounted for if we maintain that pronouns cannot be bound by governing quantifiers, but that they are there in order to pick up a subject already established in the wider context of their occurrence. Variables, if there are any [cf., (Szabolcsi 1989)], must be bound and they are, hence, eliminable in their local context.

Pronouns behave like bound variables in two types of cases:

(34) Every boy thinks he is smart.

(35) No boy brought his umbrella.

However, Butler (2003) argues that sentential complements as in (34) and possessive constructions as in (35) involve a shift to a structural subdomain where information about variables is stored on the contextual sequence parameter. Thus, after all, in the relevant *quantified* context, it is again contextual information, and not (directly) the quantifier, which determines the interpretation of the pronoun—*not* a variable. The very shift is brought about by a BAR-operator, which, Butler convincingly shows, can be used to account for condition C effects, locality constraints on different types of movement (A-bar and A), and strong crossover violations.

In this monograph I do not want to maintain or defend certain syntactic principles, but at least Butler's work supports a firm distinction between pronouns and variables. Maybe it does not so much provide an argument for making the distinction, but it seems a harmless thing indeed. There may be a methodological argument, so certainly not a knock-down argument, for preceding the way we do, to distinguish the two categories of terms. Adding pronouns to the classical, static machinery does not interfere with it, and not with its quantificational apparatus. Whatever holds with respect to quantifiers, variables, and binding, continues to hold after we have introduced our new category of pronouns.

In this context it is interesting to see de Bruijn's motivation for his variable free notation, which employs indexical de Bruijn indices.

> Manipulations in the lambda calculus are often troublesome because of the need for re-naming bound variables. (…) It seems to be worth-while to try to get rid of the re-naming, or, rather, to get rid of names altogether. Consider the following criteria for a good notation: (i) easy to write and easy to read for the human reader; (ii) easy to handle in metalingual discussions; (iii) easy for the computer and for the computer programmer. The system we shall develop here is claimed to be good for (ii) and good for (iii). It is not claimed to be very good for (i) …. (de Bruijn 1972, pp. 381–382)

The kind of complications mentioned under (i) may certainly obtain for λ-terms his calculus generates, and the same goes for Quine (1960)'s rendering of the sentence "Some student admires no professor which we saw in the introduction.". $(\mathcal{E}(\mathcal{R}(STU \times \mathcal{N}(\mathcal{E}(\mathcal{R}(\mathcal{I}(PRO \times ADM)))))))$ It may be a prejudice due to my specific logical training, but I believe the average natural language user would very much prefer to read a first order predicate logical rendering of the sentences, *with* (bound) variables, indeed.

When it comes to pronouns, one may feel slightly uneasy if we interpret them indeed in the de Bruijn fashion. Reformulating formulas into their binding forms requires, motivated, but tedious, substitution operations, and pronouns which carry the very same index only occasionally select the same antecedent. One of the major attractive features of *DPL* is indeed that it is prima facie obvious from the logical form of a formula which quantifier semantically binds which variable, and whether the quantifier syntactically binds the variable or not. The worrying down-side of this feature is that it severely damages structural logical properties of the system, e.g., it does not license α-conversion. In response to such effects, Jan van Eijck has resorted to a variable free notation for pronouns, in terms of inverse de Bruijn indices, which select an antecedent by counting from the start of a discourse.

Even so, van Eijck's inverse de Bruijn indices are not well motivated intuitively. For one thing, such a treatment neglects the inherently indexical nature of pronouns. Related to this, it obscures the obvious fact that what is most salient is introduced last in a discourse. In *PLA* this is always the first and foremost element of a sequence that a pronoun may take a value from. A default pronoun, roughly, is $\mathtt{p}_1$. In van Eijck's system of incremental dynamics, the default value of a pronoun has to be found by first counting how many items have been introduced in the discourse, and then determining for such a number n, the individual e_n which is the last element of a contextually supplied sequence.

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Chapter 3
Information Update and Support

The previous chapter on *PLA* gives an impression of the impact of putting a simple idea to work in a well-defined setting. It presents an extension of ordinary predicate logic with pronouns, which are taken to be anaphorically related to existential quantifiers and shows the consequences to be quite manageable. However, the reader may be concerned that such extensions generate complications at any further level of generalization. This chapter together with the following one serve to eliminate those concerns.

In this chapter I lift the extensional system from the previous chapter to an intensional level, showing that some welcome opportunities and applications result. Such a lift enables a study of information update and information support. I focus on an update algorithm and a support calculus and show that these are well-behaved in a very intuitive sense. The intensional lift turns out to be unproblematic, precisely because the *PLA* system does not confuse the semantics with the pragmatics of information exchange.

Before I begin, however, it is expedient to start with a concise overview of some previous systems which have extended *DPL*, in its indeed dynamic spirit, and which have inspired our treatment of the relevant phenomena in this chapter and the following, adopting a static notion of meaning, with a dynamic notion of conjunction.

3.1 Coreference and Modality

Interpretation in *DPL* (Sect. 2.2), and in many of its off-spring, has been characterized as an update function on states of information about the values of variables. The selling metaphor is that certain noun phrases, in particular indefinite descriptions, set up discourse referents and this is covered by saying that they 'declare variables', in computer science jargon. If an indefinite noun phrase is used, it declares a variable as its marker, and in subsequent interpretation one may learn more about the possible values of that variable, or, rather, put more constraints on the values.

P. J. E. Dekker, *Dynamic Semantics*, Studies in Linguistics and Philosophy 91,
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The relational format of *DPL*, in which interpretation is modeled as a relation between variables assignments, is not directly fit for the intuitive format of update of information. Interpretation in *DPL* is a *relation*, not a *function*, and variable assignments characterize total information about the values of variables. This kind of information is not in any need of update at all, because it is total.

Simultaneously with the development of *DPL*, and of very similar interpretative architectures, insights from Robert Stalnaker and Irene Heim led to the development of formal systems of interpretation which also characterize 'update of information'. At first, however, these were not concerned with information about the values of variables, but about what concerns language users in the first place: about reality, or the world.

The basic idea taken from Stalnaker (1978) is that interlocutors in a discourse distinguish between relevant states of the world, which the interlocutors conceive possible, or desirable, and who seek to diminish this set of possibilities to (one of those of) the one the *actual* word could or should be. The idea is quite simple, indeed. If we look at situations of exchange of information, our partial state of information about the world can be modeled as a set of possibilities, which represent the world as being (like one) in that set; and information update consists in reducing that set, which effectively means reducing the set of alternative ways the world may be, and thus getting more information about what or how it actually is. As a matter of fact, this kind of approach to stating the interpretation of the sentences of natural language as 'update functions' on 'information states' has led to intriguing new perspectives on presupposition and epistemic modality. (Most prominently Heim 1991; Beaver 1995; Chierchia 1995; Groenendijk et al. 1996.)

Now it seems it would be an attractive thing, to combine the mentioned two notions of dynamic interpretation—at least it could look like a missed opportunity if one dynamic set-up would not, in a sense, generate descriptions, generalizations, and explanations by means of the same mechanisms. This turned out not to be an easy matter, though. Various attempts have been made (van Eijck and Cepparello 1994; Chierchia 1995; Groenendijk et al. 1996; Aloni 1998) all of whom develop idiosyncratic architectures, the differences between which hardly ever become substantial. One of the sources of this proliferation, and also of its solution, was the acknowledgment that the type of information related to the establishment of anaphoric relationships, is after all orthogonal to the type of information we are really interested in. The real world is, of course, real; but variables, and the values of particular variables, are virtual. Variables do not themselves show up in natural languages, and where they do in syntactic/semantic structures, they are interchangeable. (One should not talk about the real world in that, technical, way. One could do so, in a philosophical way.)

Coherent systems of interpretation that combine the two types of interpretation, or, rather, the two types of information, indeed distinguish the two carefully. As we have already seen, (Groenendijk et al. 1996) has it that: "Discourse information of this type looks more like a book-keeping device, than like real information." The other authors just mentioned also use distinct methods and tools to model update of information about variables, versus update of information about the world. In order for the reader to get a glimpse of what is going on here, technically and critically, I give a tiny outline of the prominent system of Groenendijk et al. (1996) in the remainder of this section.

The task set out in Groenendijk et al. (1996) (from now on: *GSV*) is to develop a coherent system of *dynamic* interpretation that deals with anaphoric dependencies, epistemic modality, and, possibly, presupposition. Two types of information are involved, which are kept separate, conceptually, but which are technically intertwined nonetheless.

The first kind of information which the authors focus on is what they call discourse information, which is information about the (representation of) 'discourse items', a technical concept, relating to the series of subjects or objects that have been introduced in a discourse. Building on Vermeulen (1996), this type of information is encoded in so-called referent-systems. In the presentation of *GSV*, referent systems are partial functions (injections) from variables to natural numbers. The domains of these functions r typically consist of the variables x employed in (the transcription of) a discourse, and the numerical value $r(x)$ assigned to them. If the value $r(x)$ is i, then the variable is associated with the i-th discourse item in a currently evolving discourse. We have already touched upon the redundancy of this kind of information in Sect. 2.4, and even on it figuring as a source of logical and computational complications.

The second kind of information extends these referent systems with information about the world, and about the discourse items figuring in these worlds. This type of information can be summed up as follows. Once a discourse has introduced n discourse items, and attributed several properties to them, this kind of information is encoded as an 'n-ary relation in intension'. The relation consists of precisely those sequences $d_1, \ldots, d_n$ of n objects, paired with a possible world w, iff these sequences in w have the properties attributed to them in the discourse, and also stand in the asserted relations there.

It is not very useful to inspect the relevant definitions here, but the following illustration may gives a sense of the right idea. Consider the following middle-of-the-road utterance.

(36) She$_x$ insulted a$_y$ nurse.

Relative to the current state of the discourse, the referent system r assigns x something that has been introduced and which is thus assigned a discourse item, say the ith discourse item of the current discourse. The indefinite "a$_y$ nurse" introduces a new discourse item—an insulted nurse—which comes under the label y. If, and only if, the current stage of discourse is inhabited by n discourse items, this nurse is the $(n + 1)$th item, so $r(y) = (n + 1)$. At this point in the discourse $r(y)$ is a nurse who is said to be insulted by $r(x)$. This kind of information is modeled by means of the sequences of individuals $d_1, \ldots, d_{n+1}$, that, paired with a world w, have the properties and stand in the relations asserted of the n discourse items in the discourse, such that, in w, the $r(x)$th individual in the row (i.e. $d_{r(x)} = d_i$) stands in the insult-relation relation with the $r(y)$th individual in the row (i.e. $d_{r(y)} = d_{n+1}$). In this sketchily presented example we find two types of information, and two types of updates. There is information about the ‘discourse items’, and there is information about their properties and the world they live in. The first can be updated by *adding*, or *introducing*, discourse items; the second by ascribing them, or the world they live in, certain properties.

Even though in the presentation of *GSV* thus far things look cumbersome, nevertheless, these details are well-motivated, intuitively. That is, as long as one buys the idea that updates in discourse involve updates of information about the values of specific variables. To be sure: nothing can be said against the invaluable use of variables in any theoretical or rational enterprise, be it physics, economics, humanities, linguistics, or even the question whether to go shopping or not. Here, however, the variables themselves, or their names, are getting subject of discussion, and this should not be.

In the first place, what index or name we assign to a parameter, or linguistic- or discourse- constituent, should not matter as long as we do it systematically. Variables are, and ought to be, replaceable by other variables. As argued in the previous chapter, however, they are not interchangeable in *DPL*; neither are they in *GSV*. In the second place, conflating variables or pronouns in actual discourse with the variables quantified or abstracted over in theoretical linguistics leads to troubling, and uninvited, questions about whether quantifiers do or do not bind them. Part of the reaons that made it take so long to develop comprehensive dynamic systems of interpretation had to do precisely with this issue.

Interpretation in *GSV* is presented in the following format. For any formula in the language—a language of first order predicate logic with an epistemic modality operator, and possibly an operator indicating presuppositional material—an interpretation is given that ‘updates’ ‘information states’. For any such formula ϕ, and for any given ‘input state’ s, the update of s with ϕ is a new information state, written $(s)[\![\phi]\!]$; the result of applying the update function $[\![\phi]\!]$ to the information state s. The update function normally produces an *extension* of information, in the two ways. Updates may consist in a state s getting more informed about the discourse items already kept track of in s, or about the world in general; and it may consist in the introduction of new discourse items in state s. Of course, in general the two kinds, or aspects, of updating information proceed in tandem.

One of the most important, and most debated, clauses in the definition of the interpretation function is the one dealing with existentially quantified formulas, i.e., the definition of $(s)[\![\exists x\phi]\!]$. An interpretation that assumes existential quantifiers correspond to random assignments would interpret $\exists x\phi$ in s, first, as an extension $s[x]$ of s with a random assignment to x, and then an update of the state $s[x]$—state s with a new discourse item labeled x, about which there is no information—with ϕ. As a matter of fact, this is not the interpretation that *GSV* advocates, but it is worth spelling it out in some detail. The definition could be given as follows.

- $(s)[\![\exists x\phi]\!] = (\bigcup_{d\in D} s[x/d])[\![\phi]\!]$.

In this definition, $s[x/d]$ is the set of extensions of possibilities i in s, with a new discourse item x, evaluated as d. Given the referent system of s, this is one with a new, most recent discourse item, and the state $\bigcup_{d\in D} s[x/d]$ conceives of all theoretically possibly interpretations $d \in D$ of that discourse item, relative to any conceived possibility in s. Thus, $\bigcup_{d \in D} s[x/d]$ contains no information whatsoever about x, and relative to this state of information of ignorance about x, ϕ provides a first update.

Notice that, upon the just now suggested definition of $\exists x\phi$, the existential quantifier can be conceived of as an atomic action, which could stand alone in a discourse. Upon this interpretation $\exists x\phi$ really says: "Take any x. Now ϕ." The definition that *GSV* actually supports does not allow for such a paraphrase. Formally the difference is subtle because only the round brackets are organized in a different way.

- $(s)[\![\exists x\phi]\!] = \bigcup_{d \in D}(s[x/d])[\![\phi]\!]$.

This time, the update provide by the embedded formula ϕ is carried out distributively over extensions $s[x/d]$ of the information state s with the information that x is a new discourse item, with the value d, for any d in the domain D. After all these updates have been carried out, the results are summed up, by means of the big union $\bigcup_{d \in D}$. The subtle difference with the previous definition is that, in the former, ϕ is interpreted, locally, with no information whatsoever about the value of x; while in the latter, ϕ is interpreted every time with *total* information about the value of x, that it is d—and this for any $d \in D$. Now if ϕ is an ordinary extensional formula, the two definitions will not make any difference. However, if ϕ contains epistemic operators or presuppositional devices, the updates may be different if these devices have a free occurrence of x in their scope.

Let us, like *GSV*, and for the time being, interpret $\Diamond$ as an epistemic operator, to the effect that $\Diamond\phi$ says that, according to the current information, it is possible that ϕ—that ϕ has not been excluded—formally, that an update $(s)[\![\phi]\!]$ with ϕ in the current state s does not lead to contradictions. In that case $\exists x\Diamond Hx$ ("Someone might be hiding.") may yield different results according to the two definitions. The first, the one not used by *GSV*, says that you take may a random interpretation of x, and then test whether you think it is possible that x be hiding. All in all, this says that you don't exclude that anybody is hiding—you conceive it is possible that anybody is hiding. According to the second definition, you test, as a value for the new discourse item labeled by x, whether any specific value d of x might be an individual which, for as far as you know, might be hiding. All in all, this gives you information about the identity of x: it can only be an individual you have not excluded to be hiding.

One of the examples *GSV* discusses, and where this difference matters, concerns the following two statements, with associated translations.

(37) Someone is hiding in the closet. He might have broken the vase.
$((\exists x HCx \wedge \Diamond Dx).)$

(38) Someone who is hiding in the closet might have broken the vase.
$(\exists x(HCx \wedge \Diamond Dx).)$

In case you have no idea who is hiding in the closet, but do know that someone broke the vase, and that *he* might be hiding in the closet, then (37) is fine. However, if there are also some persons whom you know have not broken the vase, and if one of them might be hiding in the closet, then (38) is not supported. In that case none of the ones who might have done it are in the closet. Or, so the argument goes.

If the above observation is correct, Egli's theorem has to be qualified again, after all. According to *GSV*, at best the following holds.

Observation 14 (Modified Egli's Theorem)

- In *GSV*, $(\exists x\phi \wedge \psi) \Leftrightarrow \exists x(\phi \wedge \psi)$, provided that x does not occur free in the scope of a modal operator $\Diamond$ in ψ.

Related observations can be made about the interaction between quantifiers and presuppositions. One way of formulating presuppositions is by using a sentential presupposition operator ∂, to the effect that $\partial\phi$ says that, in the current state of information s, ϕ is presupposed to hold (Beaver 1995). Quite a long time ago this conception was argued to lead to what has become known as Heim's problem. Consider the following example, with schematic translation.

(39) A fat man was pushing his bicycle.
$(\exists x(FMx \wedge \exists y(\partial BOxy \wedge Pxy))).)$

The first definition of $\exists x$ [the one not chosen by Groenendijk et al. (1996)] induces a random assignment. The effect of interpreting this formula will be as follows. First it says, take any random value of x, so go to a state of information, containing the previous information, and add a new discourse item, labeled by x, about which you have no information. Then add the information that x is a fat man. Next, perform another random assignment to a new discourse item labeled by y, and interpret $\partial BOxy$, i.e., that your current state already contains the information that a/any fat man x has this bicycle y. This presupposes that all fat men have bicycles, and even worse, that you know them, the men and the bicycles, which doesn't seem to be a reasonable presupposition of (39).

The interpretation of (39) gets much better when using the ordinary distributive notion of existential quantification that *GSV* adopts. Doing so, the interpretation involves local processing of the embedded clauses relative to specific possible values of x and y; once a specific value of x is known to be a man, and y is known to be a bike of his, we can test or update with the information that that fat man was pushing that bike. The only presupposition that remains after processing example (39) is that at least one man is known to have a bike, which is fairly acceptable.

The *GSV* system does get around a couple of annoying problems raised in the literature, but uses ad hoc devices to circumvent the main problem, which consists in conflating variables with pronouns. The heart of the problem, which is not answered anywhere in this type of dynamic semantics, is that this type is bound to fail α-conversion as a valid logical principle. If, in a logic with quantifiers, or variable binding operators, it *does* semantically matter which are the variables (names) actually bound, this will hamper all kind of logical reductions. The laws of $lambda$-conversion, the Church-Rosser property of the λ-calculus, but also completeness proofs of first order predicate logic, depend on α-conversion. It is pretty undigestible to throw this away only for the purpose of treating natural language pronouns.

Another unsolicited effect of conflating pronouns with variables in the *GSV* system is the update with formulas containing real, quantified, variables, different from the update with those containing pronouns, which are free variables in their system. Consider a formula $\Diamond BVx$. If, on the one hand, the variable x in this formula is eventually bound, the evaluation of the formula is rigid. It tests whether the current interpretation of x, a concrete individual in the domain, might possibly have the property BV. It tests particular individuals whether they, according to the current information, might be blamed for having broken the vase. If, on the other hand, the variable is 'free'—a pronoun, so to speak—the update is different. Because it is no longer about individuals, but about discourse items the formula tests whether whatever information is gathered about the discourse item labeled by x is consistent with that item, whoever he is, having broken the vase. An significant consequence of this is that the law of universal instantiation no longer holds: in the *GSV* system, $\forall x \Diamond BVx$ does not even entail $\Diamond BVx$.

In addition to the problems with the interpretation of, or update with, modal formulas of the *GSV*-language, the informational support people bring to bear upon their utterance can be questionable. Here is (a variant of) an example brought forward by Maria Aloni in 1994, with associated translation.

(40) Someone might have broken the vase. She didn't do it.
$((\exists x \Diamond BVx \wedge \neg BVx).)$

If 'might' in the first sentence expresses an alethic modality nothing is wrong with the sequence of sentences. But the idea is to interpret it as an epistemic modality, so that the first sentence says: "There is someone whom, as far as my information is concerned, is not excluded to not having broken the vase." The next sentence asserts that person didn't break the vase. This sequence of two sentences, under this interpretation, is pretty weird. A hearer might indeed have no problem in, first, acknowledging, that some person *might* be the culprit, and, next, accepting, that *that* person *did not* do it after all. The problem with this example in the *GSV* system is that one and the same speaker is attributed information that supports his utterance that someone might, for as far as he knows, done it, and also that he didn't do it. While this indeed sounds, intuitively, very inconsistent, it is not only consistent in the *GSV*-system, it can even be coherently supported in the very same system.

The *GSV* system is intriguing because it combines a dynamic treatment of anaphora and modality in a single system, with some good results. It can be disputed, however, that it is in any sense explanatory, or even satisfactory. The dynamics of establishing anaphoric relationships, the treatment of epistemic modalities and the conflation of variables with pronouns have not improved our understanding of the empirical issues involved, but rather raise problems of their own. The moral of the current findings is that anaphora, quantification, modality and presupposition are, or can be, or have to be, all related, but they are, after all, distinct subjects. We want to be able to study them in their own right, without any non-classical bias towards what a solid formalism is needed for stating generalized interpretations. As argued in Chap. 2, a classical and solid basis for the treatment of anaphoric relationships may consist in a satisfaction semantics in the style of Tarski. We will pursue this idea in the remainder of this chapter.

In the remainder of this chapter I show how a well-motivated system of interpretation can be fine-tuned to accommodate talk about 'contents' of utterances, 'updates' of information and 'support' for assertions, without having to give up classical intuitions about meaning. As a matter of fact, I argue that this modest point of view allows for a perspicuous treatment of anaphoric dependencies which have been—mistakenly—taken to motivate a dynamic notion of meaning. In Chap. 4 I also show how this perspective lends itself to systematic extensions of its scope to quantification and modality, without the kind of ad hoc moves that rival systems appear to have propagated.

3.2 Update and Support

The primary aim of this section is to present an update semantics which makes *PLA* more closely resemble the familiar update systems from Heim (1982), Veltman (1996), Groenendijk et al. (1996), and others. Before this is done an intensional notion of the contents of *PLA*-formulas will be specified. This is achieved by lifting the extensional Tarskian satisfaction system to a Kripkean possible worlds model, and thus get our fingers at what one may label a formula's propositional or cognitive content.

The propositional contents of formulas are formalized as sets of possible worlds, those that are consistent with the information expressed by these formulas. In the style of *PLA* they are defined relative to what one may call the pragmatic parameter of witnesses, which are now not just sequences of individual witnesses, but sequences of individual *concepts*, that is, sequences of intensional witness functions. The specification of the contents of *PLA*-formulas then simply consists in a relativization of all ingredients of the *PLA* system to possible worlds, and corresponding abstractions over these worlds. This is by and large a notational exercise.

In the intensional setting a model $M = \langle W, D, I\rangle$ consists of a set of possibilities, or worlds W, a domain of individuals D, and an interpretation function I for the constants of the language. For predicate and n-ary relational constants R^n we have that $I(R^n) \in (\mathcal{P}(D^n))^W$, i.e., a function from worlds to sets of n-tuples of individuals; for individual constants c it is required that $I(c) \in D_w$, by means of which I indicate the domain of *partial* functions from W to D. Individual constants, and variables and pronouns are interpreted as partial functions because they may fail to have a denotation in some worlds. Interpretation is defined relative to variable assignments g such that for any variable x: $g(x) \in D_w$, and sequences $\widehat{\epsilon}$ of witnesses $\epsilon_1 \ldots \epsilon_n \in D^n_w$. Everything is, thus, made parametric upon the set of worlds. (A formal side-remark. The domain of functions $(D^W)^n$ is isomorphic, of course, to the domain $(D^n)^W$, so that any witness sequence of total individual concepts can always be identified with a total function from worlds to witnessing sequences of individuals. It may sometimes be helpful to think of these sequences this way.)

The interpretation of terms (constants c, variables x and pronouns $\mathtt{p}_i$) is always world dependent. They are given as follows.

- $[c]_{M,w,g,\widehat{\epsilon}} = I(c)(w)$, $[x]_{M,w,g,\widehat{\epsilon}} = g(x)(w)$, $[\mathtt{p}_i]_{M,w,g,\widehat{\epsilon}} = \widehat{\epsilon}_i(w)$.

In what follows the elementary satisfaction clause is abbreviated as follows.

- $M, w, g, \widehat{\epsilon} \models Rt_1 \ldots t_n$ iff $\langle [t_1]_{M,w,g,\widehat{\epsilon}}, \ldots, [t_n]_{M,w,g,\widehat{\epsilon}} \rangle \in I(R)(w)$,

and similarly for identity statements $t_1 = t_2$, of course. The notation conventions for $\widehat{e}$, $\widehat{c}$, $\widehat{ce}$, $b\widehat{ce}$, $\widehat{a}$, and $\widehat{ace}$ from Sect.2.2 now apply to intensional objects $\widehat{\epsilon}$, $\widehat{\gamma}$, $\widehat{\gamma\epsilon}$, $\beta\widehat{\gamma\epsilon}$, $\widehat{\alpha}$, and $\widehat{\alpha\gamma\epsilon}$. With these notational conventions reformulating the *PLA*-satisfaction semantics in the intensional setting is fairly easy. All parameters are functionally dependent upon a world of evaluation. The contents of *PLA*-formulas, specified as sets of possible worlds, are recursively defined as follows. ($[\![\phi]\!]_{M,g,\widehat{\epsilon}}$ here specifies the contents of ϕ in a model M relative to assignment g and sequence of witnessfunctions $\widehat{\epsilon}$.)

Definition 1 *(PLA Contents)*

- $[\![Rt_1 \ldots t_n]\!]_{M,g,\widehat{\epsilon}} = \{w \in W \mid M, w, g, \widehat{\epsilon} \models Rt_1 \ldots t_n\}$;
  $[\![\neg\phi]\!]_{M,g,\widehat{\epsilon}} = W \setminus (\bigcup_{\widehat{\gamma} \in D_w^{n(\phi)}} [\![\phi]\!]_{M,g,\widehat{\gamma\epsilon}})$;
  $[\![\exists x\phi]\!]_{M,g,\beta\widehat{\gamma\epsilon}} = [\![\phi]\!]_{M,g[x/\beta],\widehat{\gamma\epsilon}}$;
  $[\![(\phi \wedge \psi)]\!]_{M,g,\widehat{\alpha\gamma\epsilon}} = [\![\phi]\!]_{M,g,\widehat{\gamma\epsilon}} \cap [\![\psi]\!]_{M,g,\widehat{\alpha\gamma\epsilon}}$.

The content of an atomic formula is as usual given by the set of worlds in which it is satisfied. The ons significant addition is that the associated sequences of witness functions must provide the satisfying sequences of witness in each world. Relative to each particular world, satisfaction is identical to satisfaction in the extensional system. The content of a negated formula also captures that of the extensional system, but in a different format. According to the definition $\neg\phi$ gives us those worlds w for which no satisfying sequence of witnessing functions $\widehat{\gamma}$ for ϕ can be found, that is, relative to the given sequence $\widehat{\epsilon}$. The definition for the existential quantifier is virtually identical to that in the extensional system. The content of ϕ relative to an interpretation of x as individual concept β, is that of $\exists x\phi$ with the concept β as its witness. Where witnesses in the extensional set up figured as satisfiers, or truth makers, they are now conceived of as concept builders. The content of $\exists x\phi$ is the content of ϕ relative to a particular, contextually given, concept of x.

The definition for conjunction is the most remarkable one, since it stands in contrast with standard *dynamic* notions of conjunction: it is classic. Conjunction boils down to the intersection of information content. The operation is still dynamic, of course, like that of extensional *PLA*, but the dynamics does not reside in the contents, but in the contextual parameter and the fregean treatment of witnessing function sequences. A quick and easy comparison reveals that this treatment of the witness parameters is exactly like that in extensional *PLA*. The contents of the second conjunct ψ are given by its own sequence of witness functions $\widehat{\alpha}$, relative to the sequence of witness functions $\widehat{\gamma}$ for the first conjunct ϕ, when it itself is evaluated relative to $\widehat{\epsilon}$ in its turn.

One argument against systems of dynamic semantics is that they fail to provide an independent notion of information content (cf., e.g., Moltmann (2006, p. 228ff), who calls this the "propositional content problem." Brasoveanu (2010) makes a similar observation.). We now have a system here that deals with the dynamics of interpretation and whose independently motivated contents have been demonstrated, by being defined. It does not serve a great purpose to illustrate the assignment of contents in this system with worked out examples, because they are easily derived from the extensional formulation of *PLA* in the previous chapter, as shown in the following observation. Let M_w be the extensional model corresponding to w in M, g_w the assignment such that for all x: $g_w(x) = g(x)(w)$, and $\widehat{\epsilon_w} = \epsilon_1(w) \dots \epsilon_n(w)$ if $\widehat{\epsilon} = \epsilon_1 \dots \epsilon_n$. Thus, $M_w, g_w, \widehat{\gamma\epsilon_w}$ constitutes a sequence of parameters for extensional *PLA* satisfaction.

Observation 15 (Proper Contents)

- $[\![\phi]\!]_{M,g,\widehat{\gamma\epsilon}} = \{w \in W \mid M_w, g_w, \widehat{\gamma\epsilon_w} \models \phi\}$.

Although $[\![\bullet]\!]$ (contents) and $\models$ (satisfaction) are each given an independent compositional definition, we now see they amount to exactly the same thing. The intensional content of ϕ consists of the extensional models for ϕ, relative to the right parameters, of course. One may observe that, like I announced before, nothing unusual is required in the intensional lift of the extensional system, neither does anything unexpected happen when doing so. The dynamics of interpretation, which has been seen at work in the Sect. 2.2 on the extensional satisfaction system, has been transposed to an intensional level, and we see again, and maybe more clearly, that the dynamics essentially resides in the dynamic conjunction of information contents, and not in the notion of contents themselves. Of course, this rather conservative moral does not prevent us from making a rather standard shift to a 'dynamic', or 'update' notion of meaning, a task which I now turn to.

Apart from the question whether lifting *PLA* is necessary there are some benefits of turning *PLA* into an update semantics in the style of Stalnaker (1978), Heim (1982) and Veltman (1996), where update semantics seems to have received its name. The basic idea is to give a recursive specification of what happens to an information state on a particular occasion if it is made to accept what has been said on that occasion. Interestingly, although the notion of an update is every now and then assumed to be the primitive, basic, notion, it always gets explained in terms of the update of a state *with the news or information conveyed by an utterance*. Conceptually, then, something more primitive underlies the update, and, conservatively speaking, that notion can very well be equated with our notion of information content, as we will see shortly. The information state may be or represent any hearer's state or a model of the common ground, a matter of debate which I will come back to later.

An update semantics for *PLA*-formulas is given by the specification of $(\tau)[\![\phi]\!]$, which is the information state that results from accepting ϕ in information state τ. It is specified here relative to the by now familiar parameters of a model, assignment, and a sequence of witness functions.

Definition 2 *(PLA Update)*

- $(\tau)[\![Rt_1 \dots t_n]\!]_{M,g,\widehat{\epsilon}} = \{w \in \tau \mid M, w, g, \widehat{\epsilon} \models Rt_1 \dots t_n\}$;
  $(\tau)[\![\neg\phi]\!]_{M,g,\widehat{\epsilon}} = \tau \setminus (\bigcup_{\widehat{\gamma} \in D_w^{n(\phi)}} (\tau)[\![\phi]\!]_{M,g,\widehat{\gamma\epsilon}})$;
  $(\tau)[\![\exists x\phi]\!]_{M,g,\beta\widehat{\gamma\epsilon}} = (\tau)[\![\phi]\!]_{M,g[x/\beta],\widehat{\gamma\epsilon}}$;
  $(\tau)[\![(\phi \wedge \psi)]\!]_{M,g,\widehat{\alpha\gamma\epsilon}} = ((\tau)[\![\phi]\!]_{M,g,\widehat{\gamma\epsilon}})[\![\psi]\!]_{M,g,\widehat{\alpha\gamma\epsilon}}$.

Information states are conceived of as set of possible worlds, like the contents of formulas, where sets of worlds characterize information, beliefs, and also desires in modal logical frameworks. A set of possible worlds models some agent's information if they are consistent with what that agent believes, or, put differently, if each of them might be the actual world according to what that agent knows and believes. All the things the agent believes must be true in the worlds in such a set, and if all these beliefs *are* true in a world, then that world belongs to the set. With this intuitive understanding, the first two clauses of the update definition are fairly obvious. If an atomic formula is asserted, and accepted, then one rules out those worlds, initially considered possible, in which that formula is not satisfied. Only those worlds that do satisfy the atomic formula are preserved. This is standard in an update semantics, but here, as well, we have to update relative to the right contextual parameters: sequences of witness functions. An update with a negated formula $\neg\phi$ involves ruling out those worlds which would have been preserved if a satisfying sequence of witness functions had been found for ϕ. If the update under no such sequence turns out to preserve a world w from the initial state τ, then w is preserved in the update of that state with $\neg\phi$.

The clause for the existential quantifier is basically copied from the content format to the update format. Relative to a witness function β, $\exists x\phi$ amounts to the update with ϕ with x rendered as β. Update with such a formula does not need a satisfying or instantiating witness, but a satisfying or supporting witness function, and the update then proceeds smoothly. Of course, in actual situations a hearer will often fail to know what *is* the witness function at issue, or what kind of individual concept an indefinite description is associated with. As we will see in the next section, the speaker can or should be taken to know this, but she is not required to provide the hearer with a clue to the witness. The hearer thus may be ignorant of the witness function and have to take into consideration a whole array of possible witnessing concepts, and consequently allow for a whole array of corresponding updates, one for each of these concepts. A more realistic notion of an update is therefore not deterministic, or it ought not to be conceived of as a function, but rather as a relation. [As a matter of fact, this moral has, for similar reasons, already been drawn in Groenendijk et al. (1996), Aloni (2000), van den Berg (1996), and Brasoveanu (2008).]

For a system dealing with the dynamics of interpretation, the notion of conjunction is the most obvious one. Updating with a conjunction $(\phi \wedge \psi)$ amounts to updating with ϕ first, and ψ next. The conjunction of two update functions, thus, consists in their composition.

We could again inspect a couple of examples to see this update system at work, but it would not really serve an independent purpose. For, as we already have seen in the slogan motivating an update semantics, an update of an information state τ with a formula ϕ consists in the update of τ with the information conveyed by ϕ. This is made precise in the following observation.

Observation 16 (Proper Update)

- $(\tau)[\![\phi]\!]_{M,g,\widehat{\gamma_\epsilon}} = (\tau \cap [\![\phi]\!]_{M,g,\widehat{\gamma_\epsilon}})$.

This observation shows that we can define updates in terms of contents, but it follows that we can also define contents in terms of updates, for the observation implies $(W)[\![\phi]\!]_{M,g,\widehat{\gamma_\epsilon}} = (W \cap [\![\phi]\!]_{M,g,\widehat{\gamma_\epsilon}})$, where the latter of course equals $[\![\phi]\!]_{M,g,\widehat{\gamma_\epsilon}}$.

Two interesting points arise from the last observation, which formally maybe trivial, but conceptually are significant. In the first place information update is persistent. Whatever information is there in an information state, an update of that state always contains at least the same information, or more. For any state τ and formula ϕ: $(\tau)[\![\phi]\!]_{M,g,\widehat{\gamma_\epsilon}} \subseteq \tau$. In the second place, while updates and contents can be (that is to say: are) independently specified in a compositional manner, the present observation shows that they bring us the same result. Whatever reason one may have for formatting one's semantics in a standard static way (by defining contents), or in a standard dynamic way (by defining updates), the above observation shows that for the purposes at hand the choice is immaterial. Robert Stalnaker's analysis of assertion has already been based on this idea (Stalnaker 1978; Stalnaker 1998), and Max Cresswell has made it an explicit issue (Cresswell 2002). For the purpose of this chapter, the question whether conjunction should be thought of as intersection or as composition is no more than a frame question.

Support for Statements

A hearer's update of his information state with the contents of utterances made does not make much sense if there was any reason to suspect the information were false. Since nature shows itself, but doesn't talk, we have to rely on the agents that make utterances, and if we have sufficient reason to trust them, we have good reason to believe what they say. This observation motivates a shift of attention to the speaker's role in the exchange of information. Of course we cannot blame a speaker for saying something false if she is deluded, and sincerely believes what she says. A minimal demand is that she only provides information which she has. We may require her information state to support what she says. (In actual exchanges, and also in the exchange situations set out in Stalnaker (1978), the situation is a bit less demanding. People can be justified to only pretend, or act as if, their information supports what they say. The only reasonable requirement then would be that a speaker at least *could have had* a non-absurd information state to support all the things that she says, and then the baseline is one should not be inconsistent, as Groenendijk (2007) formulates the requirement. Even if this is the requirement, however, it does build on the underlying idea of an information state supporting what is said, which will be developed in the remainder of this section.)

In order to make this precise I use the simplifying assumption that information, the information of a speaker this time, can be represented by a set of non-excluded possibilities. In terms of such information states it is easy to define a support relation for the *PLA* language, $\sigma \models \phi$, which holds if σ contains the information which ϕ provides. (This, again, relative to a model M, variable assignment g and sequence of witness functions $\widehat{\epsilon}$, of course.) This relation is recursively defined again. The relevant parameters are those of a model, an assignment, and a sequence of witness functions.

Definition 3 *(PLA Support)*

- $\sigma \models_{M,g,\widehat{\epsilon}} Rt_1 \ldots t_n$ *iff for all* $w \in \sigma$: $M, w, g, \widehat{\epsilon} \models Rt_1 \ldots t_n$;
  $\sigma \models_{M,g,\widehat{\epsilon}} \neg\phi$ *iff there is no* $\emptyset \subset \rho \subseteq \sigma, \widehat{\gamma} \in D_w^{n(\phi)}$: $\rho \models_{M,g,\widehat{\gamma\epsilon}} \phi$;
  $\sigma \models_{M,g,\beta\widehat{\gamma\epsilon}} \exists x\phi$ *iff* $\sigma \models_{M,g[x/\beta],\widehat{\gamma\epsilon}} \phi$;
  $\sigma \models_{M,g,\widehat{\alpha\gamma\epsilon}} (\phi \wedge \psi)$ *iff* $\sigma \models_{M,g,\widehat{\gamma\epsilon}} \phi$ *and* $\sigma \models_{M,g,\widehat{\alpha\gamma\epsilon}} \psi$.

An atomic formula is supported by an information state iff it is supported throughout the state, relative to the sequence of witness functions. This requires that the formula is satisfied in each possibility in the state by the corresponding witness sequence. A negation $\neg\phi$ is supported by a state if no update or extension of the state supports the negated formula ϕ by means of any sequence of witness functions, except, that is, the absurd, or empty state. Support for $\exists x\phi$ relative to a witness function β consists in support for ϕ under a valuation of x as β. Support for a conjunction mimicks a conjunction's satisfaction. If the second conjunct ϕ can be supported with a sequence of witness functions $\widehat{\alpha}$ relative to a sequence $\widehat{\gamma\epsilon}$, of which the first part $\widehat{\gamma}$ helps to support the first conjunct relative to sequence $\widehat{\epsilon}$, then the whole $\widehat{\alpha\gamma\epsilon}$ serves to support $(\phi \wedge \psi)$ relative to $\widehat{\epsilon}$. The format of this clause is virtually the same as the one defining the satisfaction of a conjunction in the extensional set up of *PLA*.

Once again, it does not really serve a purpose to illustrate the above definition with some worked out examples, because the outcomes can be derived from the propositional contents or the update system, that is, ultimately, from the extensional satisfaction relation discussed above. In the following observation the various systems show to be appropriately related to each other.

Observation 17 (Proper Support)

- $\sigma \models_{M,g,\widehat{\gamma\epsilon}} \phi$ iff $\sigma = (\sigma)[\![\phi]\!]_{M,g,\widehat{\gamma\epsilon}}$ iff $\sigma \subseteq [\![\phi]\!]_{M,g,\widehat{\gamma\epsilon}}$.

This observation shows that an information state supports a formula *iff* it already has incorporated the update with it, that is, *iff* it subsumes its contents. As with the observation about proper updates, the observation about proper support raises a couple of interesting points, which, again, are formally trivial but conceptually significant. In the first place, the above observation implies that what we can do with an independently and compositionally defined notion of speaker support, we can do with propositional contents or updates as well. This, in the second place, means that complicated existentially quantified statements, the satisfaction conditions and contents of which can be laid out in a step by step fashion, can also be supported in a step by step manner. This is a truly non-trivial point, to which I return in the next section.

The close connection between update and support also gives us the following equivalences.

- $\sigma \models_{M,g,\widehat{\epsilon}} \neg\phi$ iff for all $\widehat{\gamma} \in D_w^{n(\phi)}$:
  - $(\sigma)[\![\phi]\!]_{M,g,\widehat{\gamma\epsilon}} = \emptyset$;
- $\sigma \models_{M,g,\widehat{\epsilon}} (\phi \rightarrow \psi)$ iff for all $\widehat{\gamma} \in D_w^{n(\phi)}$ there is $\widehat{\alpha} \in D_w^{n(\psi)}$:
  - $(\sigma)[\![\phi]\!]_{M,g,\widehat{\gamma\epsilon}} \models_{M,g,\widehat{\alpha\gamma\epsilon}} \psi$.

These equivalences agree with intuition of course. An information state supports the negation of a formula iff any update with the formula is ruled out; and it supports a conditional sentence $(\phi \rightarrow \psi)$ if an update of it with ϕ yields a state that supports ψ. Indeed, the matter boils down to the so-called 'Ramsey-test' for conditional sentences.

A final observation here puts the above-developed notions of update and support 'en rapport' in situations of information exchange.

Observation 17 (Supported Updates)

- If $\sigma \models_{M,g,\widehat{\gamma\epsilon}} \phi$ then $(\sigma \cap \tau) \subseteq (\tau)[\![\phi]\!]_{M,g,\widehat{\gamma\epsilon}}$.

Given the previous observations, this result is formally trivial. For if we take $\upsilon = [\![\phi]\!]_{M,g,\widehat{\gamma\epsilon}}$, then by Observations (16) and (17) Observation (18) claims that if $\sigma \subseteq \upsilon$ then $(\sigma \cap \tau) \subseteq (\tau \cap \upsilon)$, an instance of a distribution law for $\subseteq$. Conceptually it is extremely appealing though, because it may counts as a real felicity condition on ordinary discourse. The observation says that the update with supported utterances does not corrupt the information exchanged. If the information of speaker and hearer is correct, that is, if the actual world w_0 is excluded by neither of them, then it is also not excluded in any update they accept provided the update is supported. If I believe what you tell me, and you believe what you tell me, then we can end up wrong only if at least one of us was already wrong in the first place.

It may be noticed that this feature is typically threatened when it comes to discussions about epistemic modality. An epistemic possibility or necessity can be made correctly, and be supported by the speaker's information state, but taken in a different way by the hearer, so that almost automatically misunderstanding and conflict arise. Not so, as we will see, in the treatment of epistemic modalities in the final part of this monograph. What should be emphasized, here, is that the confusion from statements of epistemic modality does not arise from the differences in content a sentence may have on different occasions. Rather the confusion arises by the use of a sentence in a very specific circumstance which may lead to divergent assessments by speaker and hearer, respectively. Such a confusion is due to a misguided analysis, I claim.

The requirement that information may get corrupted through information change of course cannot be expelled throughout. For one thing, Stalnaker (1978) states very explicitly that assertions are acts which change the world. Therefore, something which was true before an assertion may be false afterwards. (For instance, the fact that no assertion has been made.) As soon as we allow assertions to be about assertions themselves, or about their consequences, then the very assertion of one true statement may make itself false. Here is a natural example.

(41) I haven't told you before, but your wife is cheating on you.

An assertion made with this sentence may be true and supported, but once it is made, it is rendered false. After you have told me that my wife is cheating on me, it is no longer true that you haven't told it. However, as long as we talk about independent facts of the world, and not about the peculiarities of our own discourse and information, the supported update fact stated in Observation (18) is preserved by all means.

The exposition about update and support could stop here, but this would leave neglected an issue which will turn out to be more and more relevant in the following sections. The three notions of content, update and support are defined relative to arbitrary sequences of witness functions, that is, relative to sequences of arbitrary individual concepts. The full domain of individual concepts is normally unreasonably large. This doesn't matter so much for the *PLA* notion of content. For the *PLA* notion of an update, such a large domain may already seem to be a bit too liberal to be reasonable, but that's maybe to be decided by the person who has to act out the update. For the notion of support, however, unrestricted access to all individual concepts is awkward.

In actual situations of information exchange, quite a few things are conceptually organized and coordinated, and among these is a common assumption about the domain under discussion. Not only do people involved in an exchange situation assume a common domain, but also a common conception of it. Interlocutors do not act as if the things they talk about happen to be the same things, as if by coincidence. This implies, among other things, that when we are concerned with the actual support one agent has for an utterance, or the update another one extracts from it, both the support and the update are assumed to be based on witness functions which naturally belong to a shared, suitable and accessible conception of the domain.

These deliberations imply most concretely that if we want to say that a speaker's information state supports the utterance of a certain formula, the sequence of witness functions is not arbitrary, but these witness functions should be taken to belong to a suitable set of individual concepts. An appealing characterization of such a set has been given by Maria Aloni in terms of what she aptly dubbed 'conceptual covers' (Aloni 2000). Conceptual covers are special sets of individual concepts. The concepts in the sets constitute a way of 'seeing' a domain in the sense that each individual in any relevant situation, is covered by exactly one concept. Formally they are defined as follows. In the following definition W is the set of worlds assumed in a model, and relative to a subset U of them we have a set of concepts C, and we define whether this set C constitutes a conceptual cover for, at least, a subset V of U, which can be taken to constitute a characterization of what the world is taken to be like. (The intermediary subset U is needed to allow concepts to be defined for possibilities not believed to be actual.)0

Definition 4 *(Conceptual Covers)*

- *A set of concepts $C \subseteq D^U$ is a conceptual cover for $V \subseteq U \subseteq W$ ($V \neq \emptyset$) iff for all $c, c' \in C$, if $c \neq c'$ then for all $v \in V$: $c(v) \neq c'(v)$;*
- *a conceptual cover for V covers $C_u = \{c(u) \mid c \in C\}$ in $u \in V$.*

The above definition of a conceptual cover is more general than Aloni's because it allows covers of proper subsets of the domain and for proper subsets of worlds. (As a matter of fact, for practical purposes, the total covers from Aloni, which are defined for all possible worlds, and which cover all individuals, are quite unrealistic. We will come back to this issue below.)

A conceptual cover is a set of individual concepts, at least defined for a particular set of possible worlds V—an information state, or a situation if you want. They serve to uniquely identify a number of individuals, in each possible world in the set V. Different concepts are assumed to identify different individuals in every world in V, even though we could have different concepts that, counterfactually, do identify the same individual in a world $u \notin V$. (In that case, however, the two extended concepts could not both be concepts in one and the same cover for $V \cup \{u\}$.)

If we take any world v from V, then a cover C for V will determine a set of individuals, which are the 'real' individuals identified by C in v. These may be different sets of individuals in different worlds v and v', even though, even if $c(v)$ and $c(v')$ are different, they 'count as the same', from the perspective of the cover, that is. It is like the temperature in my house, which may change.

Before we discuss an example some formal properties of conceptual covers will be given which follow from the definition above.

Observation 19 (Restricting and Extending Conceptual Covers)

1. If C covers C_u in $u \in V$ then $|C| = |C_u|$;
2. a cover C for V, is a cover for $X \subseteq V$ ($X \neq \emptyset$);
3. if C covers C_u in u then $\{c \in C \mid c(u) \in C'_u\}$ covers $C'_u \subseteq C_u$ in u;
4. a cover C' for $V' \supseteq V$ is an extension of a cover C for V iff for all $c \in C$ there is $c' \in C'$ such that for all $v \in V$: $c(v) = c'(v)$;
5. every cover of C_u for $u \in V$ has an extension which covers D for $u \in W$.

According to the first point in Observation (19), the number of concepts in a cover C for V equals the number of objects that it covers in any world $v \in V$. Relative to an information state or situation V, a cover, thus, is numerically well-behaved. The other points have to do with extensions and restrictions of the information states and the domains covered. If we restrict the set of worlds V, so if we gain or add more information, then we can keep the cover, for a more limited domain of worlds. Due to the distinctness clause in the definition of a cover, the number of concepts will remain the same. (That is, as long as we don't take U to be the empty set, which counts as an absurd state of information.) What does change, under such an 'update' is not the number of concepts, but the number of alternative individual values that each concept may take. That is to say, if we reach a state $\{v\} \subseteq V$, then the restriction to $\{v\}$ of a cover C for V will be a cover that totally defines C_v. Actually, in such an ideal state of total information no further question exists about the identity of the individuals covered.

The third item shows that one can take any restriction of a domain of individuals covered by a cover C, and get a cover $C' \subseteq C$ that covers exactly that domain. The fourth item is actually a definition of a cover-extension. An extension of a cover C for V is a cover C' for $V' \supseteq V$ consisting of concepts which are defined for a possibly larger domain of worlds. It may also cover a larger domain of individuals in every world $v \in V$, but, of course, only as long as C' still is a cover satisfying distinctness. The final line states that every partial cover can be extended to a total cover, for all possible worlds, and such that it covers the whole domain of individuals D in each world. Thus, Aloni's own total covers are, directly, or indirectly still in play.

While it is true that one conceptual cover for V may cover different sets of individuals in different worlds in V—so that for some $u \neq v$: $C_u \neq C_v$–we can as well have covers for V that cover exactly the same domain $C_u = C_v \subseteq D$, for any u and v in V. In that case one might say that such a cover is one cover of, or perspectives on, a specific subset of the domain of individuals.

Let us now turn to some covers in action. A typical covering situation may arise in a soccer stadium, e.g., Nou Camp in Barcelona. Suppose you are there, you know Barça plays in red/blue shirts and this team consists of 11 players. You see indeed 11 players in the right colours, but they are too far away for you to even distinguish them. Also, you are given a list of the names of the players: Valdés, Alves, Piqué, Mascherato, Abidal, Sergio B., Xavi, Iniesta, Pedro, Villa, Messi. You are also given a list of the squad numbers, a subset of the numbers ranging from 1 to 22. In addition to this you are carrying a photograph of the team playing (winning) today, August 14, 2011. Focusing on the single domain of players seen at the field, you can conceive of them from various perspectives, each corresponding with a way of 'seeing' that domain. The list of eleven names corresponds one-to-one with the eleven figures seen on the field, even though you may fail the right or complete mapping—or even be mistaken about it. What you do suppose, though, is that any two names name a different person on the field, and every two persons in the field have a different name. Believing what you are told, the same holds between the 11 squad numbers, and the names of the players. (Just FYI, the association runs as follows: 1 Valdés, 2 Alves, 3 Piqué, 14 Mascherato, 22 Abidalm, 16 Sergio B., 6 Xavi, 8 Iniesta, 17 Pedro, 7 Villa, 10 Messi. Of course, one may fail to know part of this information.) Consequently, the squad numbers are another way of uniquely determining the domain—even though you still need not be able to tell who is who. You also believe the persons depicted in the photograph are mapped one-to-one to the players on the field, the list of names and the list of numbers.

Now all this information is captured by an information state σ that only consists of worlds that satisfy the stated assumptions. Then we can formally identify (at least) four relevant conceptual covers for σ, all of which cover the eleven players in the field.

1. A direct perception cover C_1, let's call it a 'Kaplan cover'. This cover consists of eleven constant functions. For any individual d among the eleven players in the field, there is a concept assigning d to any world in σ.
2. A naming cover C_2, let's call it a 'Kripke cover'. This cover consist of eleven functions which individuate the domain by their names. So there is a concept assigning to each world $w \in \sigma$ the person named 'Valdés in w, and similarly for the other ten names. Notice, that given the stated assumptions, the individuals assigned to the names are always one of the persons on the field.
3. The squad number cover C_3, let's call it an 'Evans cover'. This is the cover with functions which, for any squad number, e.g., 17, assigns every world in σ the person who is carrying number 17 on his back. (Which you might be unable to see.)
4. The picture cover C_4, let's call it a 'Lewis cover'. This cover identifies a person by his resemblance with a person on the picture. Since the faces on the picture are dissimilar enough, and this information is present in σ, in every world there is exactly one among the players in the field who resembles, for instance, the person in the middle of the front row.

The main point about, and motivation for, these conceptual covers, is that one may know, for instance, who scored, if one sees a person on the field making a goal, and one employs the Kaplan cover; at the same time, you may as well wonder who actually scored, not knowing this from the perspective of the Kripke cover. At the same time one may know again who did, from an Evans cover, because one could see his squad number. Yet, one may again not know, from the Lewis cover, which of persons on the picture scored—whether it was this most handsome guy, or the guy with the funny hair. All kinds of possibilities of knowing who and not knowing who are possible. In Sect. 4.2 we will come back to these issues in more detail.

A note-worthy point that anticipates some issues to be discussed in Sect. 4.2 is, in a slogan, that conceptual covers cover each other. [(This point is also, formally, addressed in Aloni (2005).] First, observe, that conceptual covers are initially assigned the status of a perspective on a given domain of individuals, *without* any relevant properties of themselves. They are all 'Dinge an Sich', just elements of any set. These objects, however can be equated with the concepts given in a direct perception cover, since for any set of individuals $E \subseteq D$, we can establish an isomorphism with the set of *constant* functions from any non-empty set of worlds V to E. It is then, conceptually, a simple step to conceive of any cover (C) of the same domain, as a cover of the set of corresponding set of constant concepts, i.e., as a cover of the direct perception cover (DPC). For, if two concepts in C are different, then, for any world v in the set of worlds covered will have a different value, and are also associated with different constant concepts from DPC in v. And this holds in general. For any two covers C_1 and C_2 for sets of worlds V, which both cover $E \subseteq D$, we find that for any $v \in V$, there is a one-to-one mapping from C_1 to C_2, relating $c_1 \in C_1$ with $c_2 \in C_2$ iff $c_1(v) = c_2(v)$. Assuming, for the sake of the argument, cover C_1 to be 'basic', we may assume C_2 to be a cover of a domain consisting of the concepts in C_1, and so that in any world v, the value of c_2 is, not the individual C_2, but the concept c_1 from C_1

that has the same value as c_2 in v. Of course we can also conceive of C_1 as providing a cover of C_2. In this sense, any pair of covers C_1 and C_2 for the same set of worlds, and with the same cover, can also be seen to cover each other. Philosophically this may be a delicate point, although, eventually, it boils down to the observation that no logic or semantics can distinguish between two models which are isomorphic to it.

Returning now to the issue of *PLA*-support, we could implement the idea that the individual concepts relevant for the support of linguistic utterances all come from one and the same contextually given conceptual cover of the domain. This, however, would prevent a flexible use of them which appears to be required to model the type of ignorance expressed by *But I don't know who is who.* (I will come back to this type of locution in the Chap. 4.) The solution is to allow each quantifier to draw its concepts from its own cover. Formally this requires us to label quantifiers with an index specifying which covers they draw from, and for the intensional system of *PLA* this solution requires us to add the qualification that the witness for an existential quantifier be in the cover associated with the quantifier. The relevant clauses in the definition now read as follows.

Definition 5 *(PLA Satisfaction and Support)*

- $M, w, g, \beta\widehat{\gamma\epsilon} \models \exists x_C \phi$ *iff* $\beta \in C$ *and* $M, w, g[x/\beta], \widehat{\gamma\epsilon} \models \phi$;
- $\sigma \models_{M,g,\beta\widehat{\gamma\epsilon}} \exists x_C \phi$ *iff* C *is a cover for* σ, $\beta \in C$, *and* $\sigma \models_{M,g[x/\beta],\widehat{\gamma\epsilon}} \phi$.

The only relevant change here is that the witnesses for an existentially quantified formula, are witness functions from a given conceptual cover C. This may invoke contextually restricted quantification, depending on which set of individuals is covered by C. As indicated above, if needed we can always assume this to be the whole domain D. Notice that I also have not specified which set of worlds the cover C is supposed to be a cover for. In the definition of the support relation of an existentially quantified formula by a state σ it is supposed to be a cover for at least σ, but this is not always sufficient, for instance, if x eventually has to be evaluated in a counterfactual world. However, as we have seen above, we can always assume such a cover to be 'large enough', to make sure that the individual concepts involved will never be undefined for any world they have to apply to, including counterfactual ones.

The definition of quantification relative to conceptual covers is still very liberal, yet it enables us to state further requirements on a conceptual cover whenever it shows up. In general, for instance, we can assume the employed conceptual covers to be 'natural covers'. Because, most probably, there *are* no really natural concepts, it is inappropriate to require specific individual concepts to be natural. However, to require particular concepts to be taken from a natural cover does make sense. (This is like asking whether a cover fits in a natural conception of the domain, which might be given independent motivation. Compare this, for instance, with compass points. Southwest is a fine concept if is assumed to belong to a natural 'cover' which includes N, E, S and W, as well as NE, SE, and NW; an unnatural cover, however, can be argued to be one which hosts north, north northeast, northeast, east northeast, east and southwest.) In the sequel I will silently assume such a 'naturalness' assumption, without making it very specific, though. The benefits of doing so, and, hence, of the present adaptation, will become clear in the subsequent sections.

3.3 Information Exchange

Even though the preceding sections were not entirely trivial, one may wonder why we went through this exercise of defining alternative but basically equivalent semantic notions like that of content, update and support for *PLA*-formulas. In the first place, this exercise shows, by demonstration I claim, that it is immaterial whether the treatment of singular anaphoric pronouns is or should be stated in the fashion of a satisfaction semantics, an assignment of propositional contents, an update algorithm, or a support calculus. Second, it shows a real payoff when we start talking about contents in multi-agent situations, typically the kind of situations in which information is exchanged by linguistic means. In this section I want to apply the concepts developed above to a couple of phenomena that seem to resist a treatment in purely semantic terms.

While, intuitively, the role of sequences of individuals in the extensional setting consists in providing satisfying witnesses for truth, the role of sequences of individual concepts in the intensional support setting is to embody the referential intentions instrumental in a formula's support. When terms, among which I include indefinite noun phrases, are used in discourse, it is assumed they are used with referential intentions. This claim is actually quite old, and it has relatively recently been picked up in van Rooy (1997), Kamp (1990), Stalnaker (1998), and Zimmermann (1999). Let me substantiate it a bit further first, before we see how it fits in the picture developed in this section.

A first and compelling example for the assumption of referential intentions dates from Peter Strawson 1952. Strawson mentions two possible responses, (43) and (44), to statement (42) (Strawson 1952, p. 187, in the 1960 edition).

(42) A man fell over the edge.
(43) He didn't fall; he jumped.
(44) It wasn't a man; it was a woman wearing trousers.

These examples pose a challenge to standard syntactic and semantic approaches, because a speaker here starts talking about an individual with an indefinite noun phrase, which the respondent refers back to by means of a pronoun. What information is exchanged here, and how remains a mystery if the relation between the indefinite noun phrase and the anaphoric pronoun is one of syntactic or semantic binding. However, on the simple assumption that the first speaker intends his indefinite noun phrase to speak of a certain man (or woman) in the vicinity of both interlocutors, what the little exchange is about, and what the exchanged contents are, is clear. The actually present man (or woman) can be used to resolve the pronoun. Notice that this analysis perfectly fits in Cresswell (2002)'s proposal, at least in concept. The opening statement of this little discourse would be read as "A man, namely x, fell over the edge." and the response could be "He, namely x, did not fall, etc." On this interpretation x gets a value from the non-linguistic context of the discussion, and two different, together inconsistent, attributions are made about this value. Notice, though, that this analysis does not easily fit Cresswell's formal rendering of *DPL*, as it does not fit in the system of *DPL*. For, the analysis of the reply "He, namely y, didn't fall, etc." then cannot be achieved without binding y to a man (or woman) who is said to have fell.

One might think that the actual presence of the man (or woman) in Strawson's example may be used to explain the case as one of direct reference. However, indefinites can also be used to report on individuals which are not actually present. Imagine the following situation. One person, Liz, was visited by two students yesterday, who both asked for the secretary's office. One of them (Wilburt) did, and the other one (Norbert) did not wear pink pumps. Liz is fully aware of all this, except that she never knows the students' names. Zil knows that Wilburt always bothers people with nonsense questions, and that he wears pink pumps. Now Liz starts the following exchange with Zil:

(45) *Liz:* Yesterday, a student ran into my office who inquired after the secretary's office.
Zil: Was he wearing pink pumps?
Liz: He was indeed.

So far the conversation seems perfectly fine, if Liz had indeed started out talking about Wilburt. But notice that Zil can ask questions about the student Liz talked about, without (yet) having an idea of who it was. Yet, there is a proper answer to the question. Notice, too, that if Liz had simply meant to assert that the set of students that visited her yesterday was not empty, then Zil's question would have been odd indeed, or impossible for Liz to answer. For suppose that her reply to Zil's question was the following. (This version of the scenario is due to Ede Zimmermann, p.c.)

(46) *Liz*: I don't know. What do you mean? If it was the first one who came in, he was wearing pink pumps; if it was the other, he was not.

Liz could motivate her 'ignorance', by claiming that, when she earlier said that a student ran in, she had not yet decided which of the two she was talking about. It seems obvious that this would be a very odd reply. Liz is expected to have had someone in mind when she started to talk about a student. So, if one starts talking about an individual, who is introduced by means of an indefinite description, there still is a fact of the matter of who it is, about whom we can sensibly ask and answer questions, like whom it was about.

Such questions can also be genuine identity questions. Imagine the following situation. Weird rumours have it that young girls get battered in Gotham city, and that city representatives like, e.g., Bernard J. Ortcutt are involved. Bor and Cor exchange the latest findings in the tabloids.

(47) *Bor:* A magistrate from Gotham city has confessed to battering young girls.
Cor: They say he suspected them of sorcery. Do you know if more magistrates

confessed?
Bor: I don't know.
Cor: Do you know who confessed?
Bor: No idea, the police didn't disclose his identity.

There is nothing incoherent about a dialogue like this. It is an exchange of information between two inhabitants of Gotham city, who—perhaps naively, but not incoherently—take their tabloids seriously. The example shows that the two interlocutors can exchange information about an individual without knowing exactly whom they are talking about. Yet, the two interlocutors apparently assume the discourse to be about a certain individual, otherwise the question *who he is* wouldn't make sense in the first place. The explanation of this issue, like any issue of identity, is that a person or object may be given in a certain way, for instance as 'the person whom the tabloids are reporting on'. While one fails other means to identify that person, there still is a referential intention involved, viz., the intention to relate to precisely that person.

The upshot of these examples is that regular indefinite noun phrases are used with referential intentions, even when legitimate questions about the identity of the actual referents remain, but that their use is justified conceptually. The concept may be intensional, in the sense that it invokes the concept of an individual someone else intended to refer to, and its use may engage in a causal intentional chain in the sense of Kripke (1972), Chastain (1975), Donnellan (1978), and Evans (1982). In any case, and that is relevant here, it is a concept which can always be rendered, formally, as a function from worlds to individuals.

The preceding observations then neatly fit the compositional notion of support developed above. Like I said, the sequences of witness functions embody the referential intentions with which the indefinite noun phrases or corresponding existentially quantified formulas may be used. So, follow up on example (42) from Strawson above, the speaker says that *a man fell over the edge*, we may assume that the speaker's state σ supports an indexical individual concept β of a person who goes down the edge in the speaker's presence. Notice that such a concept makes perfect sense in a natural conception of the situation, a conceptual cover indicated as S here.

- $\sigma \models_{M,g,\beta} \exists x_S(Mx \wedge Fx)$.

The hearer, in this situation, may be able to pick up that individual concept, and notice that the propositional content expressed, $[\![\exists x_S(Mx \wedge Fx)]\!]_{M,g,\beta}$ is false according to his information state τ.

- $(\tau)[\![\exists x_S(Mx \wedge Fx)]\!]_{M,g,\beta} = \emptyset$.

The hearer corrects the speaker by saying that *he didn't fall; he jumped*, as in (43), because that's the information he has.

- $\tau \models_{M,g,\beta} \neg F\mathrm{p}_1$ and $\tau \models_{M,g,\beta} J\mathrm{p}_1$.

The information of speaker and hearer, and the communicated contents, are clearly centered around the same individual $\beta(w_0)$ in the actual world w_0, if any such person indeed went down the edge. Here we see that the notions of information content, information support and information update fit well together in the analysis of a situation in which one speaker employs a pronoun to refer to an individual introduced in the discourse by another. The other scenarios we have discussed above fit our notion of support in an analogous fashion. The point made here about supporting witness functions for indefinite noun phrases and pronouns, also applies to names (individual constants), of course. While it may be hard to sort out which specific witness function a speaker has in mind when she uses an indefinite noun phrase, for a proper name this process is often easy, for each individual constant comes with its own individual concept which is a very natural concept to use it for. This is not to say, though, that we always use a name this way; not when we say, for instance, that Fred might not have been called 'Fred'.

To conclude this section, I apply the concept of support to illuminate a puzzle raised by Charles Saunders Peirce. (Also the following analysis neatly fits in the proposal made in Cresswell (2002); this time formally as well.) The puzzle, very concisely, is this. The following two formulas are equivalent in first order predicate logic.
(α) $(\exists x Wx \leftarrow \forall x TPx)$;
(β) $\exists x(Wx \leftarrow TPx)$.
In Read (1992), Stephen Read presented a situation to evaluate two natural sentences, the meanings of which appear to be captured by the above two formulas. The situation is that a sweepstakes takes place, a thousand people are in the position to participate, and each participant brings in one dollar. So if n people participate, one of them wins n dollars. Now consider the following two statements.

(a) Someone wins $1,000 if everyone takes part.
(b) Someone wins $1,000 if he takes part.

Statement (a) is obviously true in the situation as described. Statement (b), however, may raise people's eyebrows. This statement provides or suggests inside information that does not directly follow from the description of the situation. The puzzle is, how can that be, if (α) and (β) render the meanings of (a) and (b)? If the first two are equivalent, then the second two should be equivalent, too. But they don't seem to be. What is going wrong here? Our judgment of the sentences (a) and (b)? Their translation into (α) and (β)? Or perhaps the equivalence of the latter?

A *PLA* style analysis of the situation reveals what is going on. First observe that an utterance of (b), but not of (a), naturally raises the question: *Who?* For it suggests that there is someone with the special property that if he takes part, he wins. (The property is even more special, since his taking part would make all other take part as well, otherwise he doesn't win the full $1,000.) If we know who has this special property, we have good enough reason to convince him to take part. Where does this suggestion come from? Consider what concept δ of the speaker could make her information state σ support her utterance of (b), rendered as (β) above.

- $\sigma \models_{M,g,\delta} \exists x(Wx \leftarrow TPx)$ iff
  $\sigma \models_{M,g[x/\delta]} (Wx \leftarrow TPx)$ iff
  $\forall w \in \sigma$: $\delta(w) \in I(W)(w)$ if $\delta(w) \in I(TP)(w)$.

We find that the speaker should indeed have a concept δ at her disposal of someone who, according to her information, wins \$1,000 if he takes part. What concept can that be? It can be the concept of a person of which the speaker thinks he is not going to take part anyway. It can also be the concept of a person of which the speaker thinks he is going to win \$1,000 anyway. Gricean reasoning disallows these explanations, because if they provide the speaker's evidence for his assertion, the assertion would violate Grice's quantity maxim. Another concept is one of a person of which the speaker doesn't know whether he will participate or not, but of whom she does believe that if he will participate, he will win this \$1,000. Here is our lucky bird. The speaker seems to know of a predestined winner, or she has inside information that the whole sweepstakes is set up.

The story does not happily end here, though. Read's sweepstakes situation supplies another concept that enables an information state to support formula (β) or example (b). This is a hybrid concept that corresponds to the concept of the winner in all possibilities in which everybody participates, and to the concept of an arbitrary drop out if at least one person doesn't take part. This witness function renders (β) supported by any information state that incorporates the description of the sweepstakes situation. However, even though the concept itself cannot be ruled out as unnatural, it doesn't seem to fit in a reasonable conception of the sweepstakes situation, that is, it does not belong to any natural conceptual cover of the situation. The amended version of the support relation thus rules this out, so the story does happily end after all.

The overal conclusion of this discussion is that (α) and (β) do render the meanings of examples (a) and (b), and that the two formulas do have the same truth conditions, in *PL* and, hence, in *PLA*. In *PLA* however, (β) and (b) have additional satisfaction conditions. They require a supporting witness together with the cover it is drawn from, and on the level of information support these combine with pragmatic information to yield the stronger reading which Pierce and Read have observed.

3.4 On the Contextualist Debate

With our use of witnesses, the notion of a dynamic conjunction, and our focus on issues of update, support, and information exchange, we have obviously, and deliberately, incorporated arguably pragmatic aspects of meaning in the interpretational architecture. 'Dubious business', some may claim, as for instance those who, like Gareth Evans maybe, live in Plato's, Frege's, Russell's or Carnap's heaven. Going this way, you spoil the autonomy of semantics, of logic, and put much of philosophy up for grabs. Others may be less discomforted, like those who have felt uneasy about the old-fashioned practice of throwing all unsolvable syntactic/semantic problems in the proverbial pragmatic wastebasket (Bar-Hillel 1971). One may rejoice that we continue cleaning up that bin. Yet others again may welcome this pragmatic move, and welcome me at their side, arguing this is just a first step into seeing that everything is pragmatic after all, and once this is properly acknowledged, we all see there is no room left for semantics proper.

With the last reaction we enter the so-called 'Contextualist debate'. I say 'so-called', because the discussion does not consist much of arguments supporting conclusions. The debate is often presented as one between contextualists on the one hand, who claim that the interpretation of linguistic utterances is so inherently context-dependent that there is no sensible room left for any context-independent semantics proper; and on the other hand so-called minimalists, who fight for the right of something like a formal semantics, delivering minimal propositions as core meanings associated with linguistic constructions. In what follows I will attempt a constructive reply to the contextualists' arguments. I accept almost all of their arguments, without any inclination to agree with their destructivist conclusions.

The contextualist challenge consists in series of arguments showing to what a massive extent pragmatic and contextual phenomena determine the interpretation of utterances on all syntactic levels. Let us review some of these types of observations with regards to the crucial realms of reference, predication, quantification, and construction.

Reference There exists quite established consensus, since Donnellan (1966, 1978), that the following example can be used to convey information about a man who hasn't murdered Smith.

(48) Smith's murderer is insane.

If uttered in the right context, with people sharing the same, mistaken, assumptions, a person who is not Smith's murderer can be taken to be so, and thus serve as the referent of the predication that he is insane. Saul Kripke, not convinced that the phenomenon addressed by Donnellan is semantically relevant, agrees that an utterance can be used in such a way, and he adds two more relevant examples (Kripke 1979b):

(49) Jones is raking the leaves.
(50) *A:* Her husband is nice to her.
B: He is nice to her, but he is not her husband.

The man actually said to be raking the leaves can be mistakenly identified as Jones, but even so an utterance of (49) may successfully communicate that *he*, not Jones, is raking the leaves. Example (50) can be used in relation to a wandering couple, whom the speaker assumes to be married, while, apparently, the hearer knows better. Yet, what is communicated, and what the hearer agrees upon, clearly, is that the man who is not her husband is nice to her. What a semantic theory would say about the meaning of the uttered sentences in these examples is orthogonal to some extent to what is communicated. The intended reference is accomplished, but not, or not obviously, by semantic means. An even more striking case is one from Nunberg (1979):

(51) The ham sandwich wants to pay.

The definite noun phrase "the ham sandwich" is used to pick out the person who ordered a ham sandwich in a restaurant, while, of course, strictly speaking the sentence itself is not about a person, but expresses a somewhat unexpected inclination of a ham sandwich. If language is a means for communication, then, in the above examples, context plays a larger role in determining the referents spoken about than say the literal meanings of the denoting words employed.

Predication Consider the following utterance, at the edge of a lake, on a very hot August afternoon.

(52) The water is polluted.

It may be clear what the referent of "the water" is (the water in the lake). The intention of the utterance may also be clear: you better not go swimming in there, let alone drink from it. All of this can be very clear, even though many questions can be asked as to what the speaker means by saying that the water to be polluted. Is every bit of the water polluted? Or most, or just an unknown part of it? Or is there just a couple of barrels filled with chemicals at the bottom of the lake? The speaker may fail to know the answer, and may not even care a bit about these questions. Does this mean that the speaker doesn't know what she says? Or is what she says the disjunction of all affirmative responses to these questions, a proposition like "The water contains some substance which might be dangerous to your health"? Prima facie, this proposition is too weak to make any sense, and, on second thought, it may seem to stand in need of further analysis itself. Still, even without deciding upon a specific proposition, the purpose of the message is clear enough: don't go in the water and don't drink it. Something essentially similar holds for a statement like:

(53) The mountains are dangerous.

One may ask, "dangerous, in what sense?" "Are the slopes slippery? Are there mountain lions? Will the desert rocks have a fatal effect on my desperate mood?" In a given context, any such thing can be meant, or, as in the case of the polluted water, not even such a single thing. Still, again, the message can be clear: stay in the valley.

A typical example often discussed in the literature concerns the apparent 'simple' qualification of something as red.

(54) The car is red.

François Recanati remarks:

> "(…) in most cases the following question will arise: what is it for the thing talked about to count as having that colour? *Unless that question is answered, the utterance ascribing redness to the thing talked about (John's car, say) will not be truth-evaluable*. It is not enough to know the colour that is in question (red) and the thing to which that colour is ascribed (John's car). To fix the utterance's truth-conditions, we need to know something more—something which the meanings of the words do not and cannot give us: we need to know *what it is for that thing (or for that sort of thing) to count as being that colour*. What is it for a car, a bird, a house, a pen, or a pair of shoes to count as red? To answer such questions, we need to appeal to background assumptions and world knowledge." (Recanati 2005, p. 183)

The issue is pushed further in an example from Anne Bezuidenhout.

(55) Chris has been sorting red apples.

> "We're at a county fair picking through a barrel of assorted apples. My son says 'Here's a red one,' and what he says is true if the apple is indeed red. But what counts as being red in this context? For apples, being red generally means having a red skin, which is different from what we normally mean by calling a watermelon, or a leaf, or a star, or hair, red. But even when it is an apple that is in question, other understandings of what it is to call it 'red' are possible, given suitable circumstances. For instance, suppose now that we're sorting through a barrel of apples to find those that have been afflicted with a horrible fungal disease. This fungus grows out from the core and stains the flesh of the apple red. My son slices each apple open and puts the good ones in a cooking pot. The bad ones he hands to me. Cutting open an apple he remarks: 'Here's a red one.' What he says is true if the apple has red flesh, even if it also happens to be a Granny Smith apple." (Bezuidenhout 2002, p. 107)

Now one may wonder, can we say that Chris was sorting apples yesterday, and today he did it again? Can we say that what he did today was the same thing that he did yesterday? Surely, one can say he has been sorting apples both days, so, yes, it was the same thing, or the same kind of thing that he was doing. But if it comes to his sorting *red* apples, are we still inclined to say he has been doing that both days? It doesn't seem so. It also doesn't seem to be appropriate to say he was doing the same thing in a different way. It seems he was really doing a different thing these two days, but both activities are appropriately described as sorting red apples. Shouldn't we conclude from this, then, that 'sorting red apples' is ambiguous, at least? And when it comes to the core term in this example, 'red', shouldn't we conclude that it is ambiguous between at least the readings 'red on the outside' and 'red on the inside'? And, so the arguments goes, shouldn't we conclude that 'red' is ambiguous in many unexpected ways, so that, eventually, it cannot be assigned a meaning?

Quantification Quantifiers are typically context-sensitive as well.

(56) Some philosophers are not from France.

Besides the probably not uniquely answerable question who or what to count as a philosopher, a statement like this is often used with a specific domain of quantification in mind, so that it means something like "some philosophers attending the present conference," or "some philosophers we have been studying lately." The point here is, or so it has been argued, that one doesn't seem to construe or understand a most general and context independent proposition first, like "Some philosophers in the entire universe are not from France," and then derive from this a more specific, and more informative one, to the effect, for instance, that some of the philosophers at this conference are not from France. Rather, it seems, that in a given context one directly interprets the phrase "some philosophers" as "some philosophers at this conference," or any of the other specifications. But then, contextualists ask, what role would the most general, and trivial proposition have to play here in the first place?

While many facts about natural language determiners are basically well-understood, with the work originating from Barwise and Cooper (1981) and many subsequent studies, these very same expressions also interact with their linguistic context in ways which have not been explored and clarified fully yet. Here are two examples, a careful inspection of which requires an adaptation of the respective head determiners "All," "Five," "No," and "Most":

(57) { All / Five / No } boy scouts teased girl scouts and covered each other.

(58) Most orphans who got a present from their tutor thanked him for it.

Without going in the details a suitable interpretation of (57) has to adapt the basic interpretation of the noun phrase(s) "{ All / Five / No } boy scouts" to something which they may have done group-wise, and such that "cover each other" can be predicated of them. Likewise, the head determiner "Most" in example (58) must allow the restrictive property of being an orphan who gets a present from his or her tutor, to interact, or co-vary, with the scopal property of thanking the tutor for the present. This is not the place to actually see how these adaptations work out, but just to note that various proposals for sophisticated modified interpretations of these determiners have been made, by Kamp and Reyle (1993), Chierchia (1995), Nouwen (2003), and Brasoveanu (2008). These proposals do not converge on a most basic meaning of these determiners, but, rather, if combined, they create highly complicated interpretations apt for worst case scenarios, in which plurals and anaphora conspire, with who knows what else types of phenomena. It seems unlikely that under such an adaptive approach the 'Real Meanings' of the determiners are going to be uncovered. But then, or so it is argued, the sophisticated adapted interpretations should be properly conceived of as contextual adaptations themselves. Not shying the word, as 'pragmatically infected' interpretations.

Construction Simple numeral determiner phrases can be combined with a simple transitive verb and produce an equally simple cumulative reading:

(59) Twenty-five Dutch firms own three-hundred fourty-six Irish cows.

The, rather likely, cumulative, interpretation is that, if we look at the relation between x and y such that x are Dutch firms owning Irish cows y, then we count 25 owners, and 346 ownees. The example, and its interpretation, has been presented in Scha (1984) and Keenan (1992) has argued that it guides us beyond the Frege boundary—very roughly speaking, beyond first order logic. What this means, essentially, is that we have a construction, or context, in which the basic, or literal, noun phrase meanings are provably insufficient to produce the intended interpretation of the whole construction.

A related point arises when we reflect upon an example from Hendriks (1993), which is, deliberately it seems, rather unusual:

(60) Thomas seeks a fish or a bike.

This relatively simple seven word sentence can be assigned a large number of interpretations, using various tools developed and motivated in formal semantic systems. Here I list five of them, with corresponding glosses, but more can be generated.

- $SEEK(t, \lambda P(A\_FISH(P) \vee A\_BIKE(P)))$ ("Thomas is looking for whatever is a fish or bike; any such thing suits his purpose; hell knows why.");
- $(A\_FISH(\lambda x SEEK(t, \lambda P\ P(x))) \vee A\_BIKE(\lambda y SEEK(t, \lambda P\ P(y))))$ ("There is particular fish Thomas is looking for, or a particular bike; I don't know which of the two he has set set his mind on.");
- $(SEEK(t, A\_FISH) \vee SEEK(t, A\_BIKE))$ ("Thomas is looking for whatever fish, or he is looking for whatever bike, I don't really know which of the two.");

- $A\_FISH(\lambda x\, A\_BIKE(\lambda y\, (SEEK(t, \lambda P\, (P(x) \vee P(y))))))$ ("There is a (particular) fish and a (particular) bike such that Thomas seeks whatever is either one of the two. God moves in a mysterious way.");
- $(SEEK(t, A\_FISH) \vee A\_BIKE(\lambda y\, SEEK(t, \lambda P\, P(y))))$ ("Thomas seeks whatever fish, or a particular bike. I am so stoned.")

No matter how unlikely the sentence or one of its glosses is, with a bit of creative imagination one can think up a situation exactly described by the gloss, and the idea then is that one might be tempted to report that situation by uttering (60). If so, and I believe I could do so, one might be tempted to, after all, conclude, that the sentence (60) is so multiply ambiguous, and that it has all these readings. This would be correct, if by "multiply ambiguous" we mean "subject to a multitude of interpretations." However, it seems very hard indeed to believe that such a, relatively simple, sentence has all these readings—that knowing its meaning boils down to realizing all these readings, and that in any context of utterance we have to select an appropriate one of them. Rather, or so the contextualist can argue, the sentence has only one analysis, e.g.:

(61) $[_S[_{NP}$ THOMAS$][_{VP}$ SEEK$[_{NP}[_{NP}$ A_FISH$]$ OR $[_{NP}$ A_BIKE$]]]]$

It is only in a specific context that the building blocks get put together, in a specific way, which hearer and speaker may quarrel about and negotiate.

Considerations like those given above, and many others can be supplied at will, lead, unavoidably it seems, to the conclusion that there is an indefinite number of fairly unanswerable questions about the core mechanisms of meaning in actual practice—viz., reference, predication, quantification, and construction; moreover, assuming that language derives its meaning, not because it is a God given heavenly instrument, but from its actual use, to claim that these meanings are well established and firmly given seems preposterous and it seems that in certain, or maybe all, contexts their specific details escape our notice, or are derived *despite* these firmly given meanings.

As a matter of fact, rather destructivist conclusions have been drawn from such observations. "The evidence in favour of contextualism is provided by indefinitely many examples in which the same sentence, which does not seem to be ambiguous, is used in different contexts to say different things (Recanati 1994, p. 9)". "According to these philosophers, sentences can never express complete propositions independent of context, however explicit speakers try to be. In other words, content is always under-determined by the linguistic material (Recanati 2006, pp. 22–23)." Emma Borg summarizes the current feeling as follows: "These days, the natural descendent of the formal approach, known as minimalism, has been consigned to the margins: not everyone rejects minimalism, but lots of people do. Minimalism is rejected in favour of contextualism: roughly, the idea that pragmatic effects are endemic throughout truth-evaluable semantic content (Borg 2007, p. 339)."

Even minimalists who defend the formal semantic semantic enterprise tend to agree with these negative conclusions. Emma Borg, for instance, submits that "according to the minimalist (as I construe her) there is an entirely formal route to meaning. This means not only that every contextual contribution to semantic content must be grammatically marked but also that those features contributed by the context must themselves be formally tractable (Borg 2007, p. 358).” This amounts to saying that there is hardly any formal meaningful structure which is not pragmatic. Cappelen and Lepore, who also defend semantic minimalism, as well as ‘Speech Act Pluralism’ (‘SPAP’), conclude that “SPAP is not really a theory; it’s a collection of observations, one of which is that there can be no *systematic theory* of speech act content ((Cappelen and Lepore, 2005)[p. 190]).” In the same spirit, Pagin and Pelletier (2007) aims to preserve a classical compositional architecture of interpretation, but one that allows Recanati’s pragmatic processes of enrichment, loosening, and semantic transfer to interfere on every node in the construction tree, or in the corresponding meaning-tree. This, obviously, means taking the contextualist’s arguments to heart, and surrender to the pragmatic hegemony which practically may overrule anything that has been called semantic ever before in the ensuing architecture.

Giving things a thought, however, I don’t believe the situation is so dramatic. The preceding discussion may paint a bleak future for semanticists, if one believes, along with the early Wittgenstein and other analytic philosophers, in something like the ultimate analysis of a sentence. Some such thing may speak from the following propositions [(from Wittgenstein (1922)]:

> 2.0201 Jede Aussage über Komplexe lässt sich in eine Aussage über deren Bestandteile und in diejenigen Sätze zerlegen, welche die Komplexe vollständig beschreiben.
> 2.0211 Hätte die Welt keine Substanz, so würde, ob ein Satz Sinn hat, davon abhängen, ob ein anderer Satz wahr ist.
> 2.0212 Es wäre dann unmöglich, ein Bild der Welt (wahr oder falsch) zu entwerfen.

The reduction ad absurdum in proposition 2.0211 relies on the assumption, not disputed there, that sentences in the end have meanings, independent of further analysis—that is, independent of other sentences being true or not. The later Wittgenstein, has quite nicely reformulated this intuition, in order to seriously qualify it next. In §99, Wittgenstein restates one of the main tenets from the Tractatus as follows:

> §99 Der Sinn des Satzes – möchte man sagen – kann freilich dies oder das offen lassen, aber der Satz muß doch einen bestimmten Sinn haben. Ein unbestimmter Sinn, – das wäre eigentlich gar kein Sinn. – Das ist wie: Eine unscharfe Begrenzung, das ist eigentlich gar keine Begrenzung. Man denkt da etwa so: Wenn ich sage “ich habe den Mann fest im Zimmer eingeschlossen – nur eine Tür ist offen geblieben” – so habe ich ihn eben gar nicht eingeschlossen. Er ist nur zum Schein eingeschlossen. Man wäre geneigt, hier zu sagen: “also hast du damit garnichts getan”. Eine Umgrenzung, die ein Loch hat, ist so gut, wie gar keine—Aber ist das denn wahr?

But is this true? Eventually, he conceives of it as a misconception in the *Philosophische Untersuchungen*. His picture of the road indicator in paragraph §85 is revealing in this context.

§85 Also kann ich sagen, der Wegweiser läßt doch keinen Zweifel offen. Oder vielmehr: er läßt manchmal einen Zweifel offen, manchmal nicht. Und dies ist nun kein philosophischer Satz mehr, sondern ein Erfahrungssatz.

Many questions can be raised as to what a Wegweiser (road indicator) exactly indicates, what direction it shows exactly, and with what amount of freedom. Reflecting on all kinds of deviant contexts, it seems that we don't have a full answer to the question what the road indicator in all of these contexts *really* means. (For instance, what is its meaning in the middle of the desert, or in five-dimensional space?) But as a matter of fact, in all normal circumstances, any non-deviant road indicator is perfectly clear, and we all know how to read it, and how to act upon what it signals in all or most of our run of the mill activities in everyday life. The observation that the indicator works well as long as it is embedded in our regular practices, is maybe all that needs to be stated. It doesn't require a Jack-in-the-Box meaning that makes clear to us what its function in a particular context is. Already in the first paragraph of the *Philosophical Investigations*, Wittgenstein has pointed out that every analysis comes to an end, not because it is the final analysis, but because we simply stop there. "Nun, ich nehme an, er handelt, wie ich es beschrieben habe. Die Erklärungen haben irgendwo ein Ende (Wittgenstein 1953, §1)."

Jason Stanley defends the formal semantic paradigm as follows:

> Competent English speakers know the meanings of the words in the sentence 'Some philosophers are from New York.' They also know how to combine the meanings of each of the words in this sentence to arrive at what is said by the utterance of the sentence, 'Some philosophers are from New York.' It is that linguistic competence that seems to be the source of their ability to report correctly about the truth of what is said by that sentence relative to different possible circumstances (…). (Stanley 2005, p. 221)

In response to the observation that more enters into the meanings of utterances, Stanley notes: "The first response (…) is to attempt to preserve the clear and elegant explanation in the face of the apparently recalcitrant data. The second is to abandon the clear and elegant explanation of the source of our truth-conditional intuitions in favor of a different one (Stanley 2005, p. 222)." Clearly, he prefers the first, not wanting to abandon "the project of giving a systematic explanation of the source of our intuitions" and instead "appeal to unconstrained and non-explanatory notions or processes." Even with the contextualist observations in the back of our minds to keep on carrying out such a program makes sense.

We all have an idea of objects we refer to, even though we are unable to define what an object is; we all know how to predicate stuff of things, even though we fail to explain what any specific predication exactly amounts to; we all know what it means to say that everybody, or most of my fellow students, have a certain property, even though we cannot say for each occasion what it exactly says that the property applies to all or most members of a certain group. In formal semantics one tries and lays out the things which are known, without committing oneself to the things which are unknown. Nobody denies that there are all kinds of structural properties of languages, also on the level of meaning, 'meaning' intuitively understood here. A semanticist can be justified to claim to work on revealing such structural properties, by simulating meaningful languages in formal models. This is not to say that these models, or some entities or constructions in there, depict the ‘Real Meanings’ of natural language utterances. Rather, they are coarse-grained simulations of parts of meaningful language, displaying structural properties of meaning found in real languages, or so we hope. Nothing in this picture entails that they should be like basic meanings, which language users have to grasp, in order to put them into practice. And why should one be tempted to do so? Surely one should not want to draw analogous conclusions from the models economists, or sociologists, or biologists, build to simulate, more or less realistically, economical, sociological, or biological processes.

Language is an organism, which is alive, in action and in evolution, and which is inherently social. Meaning, originally and eventually, lies in use. From a semantic point of view, where semantics is conceived of as the (study of the) theory of meaning, we cannot seriously close our eyes to pragmatic intrusions of meaning. No autonomous semantics of natural language seems to be forthcoming, which can serve as the input to a pragmatic theory of interpretation. But then, even if we adopt this (radical) pragmatic perspective, we are still confronted with all kinds of systematic, structural, meaningful aspects of natural language. Devices which systematically (but not unavoidably) serve certain purposes, and certain structural purposes. Even from this pragmatic perspective, bringing such aspects of use to light, and bring them together in an orderly system is certainly worthwhile. Indeed, no system accomplishes this better than a formal system, and, then preferably, a system that reflects, or models those systematic aspects of use. This reflection, set up in a fully general form, by itself is not a theory of meaning, as it is not a system of interpretation. It does not mimick or aim to mimick the ways in which agents produce and interpret language, or their capacity to do so. It does, however, bring to light these systematic properties, no matter how non-robust, contingent, and non-persistent they may be in reality. Yet, bringing these structures to light, brings important aspects of language to light. Conceived in this manner, semantics does not constitute a theory of meaning; but, turning the table, I wouldn’t see either how pragmatics would ever offer anything like a theory of the envisaged kind, if it has not extrapolated a job for the semanticists.

The upshot of this whole discussion is that from the old-fashioned semanticists’ point of view, the pragmaticists may have won, or are in the process of winning the battle, and the semanticist may have to surrender. But surely the pragmaticists, who claim victory, like they did in the fifties and sixties of the previous century, have nothing on offer instead. Besides case studies of single expressions in singular contexts, they cannot supply us with any more than entirely contingent descriptions, which are always subjective, or at least not objectifiable. If any real work is to be done, they will have to agree that the old fashioned semanticists are the ones that do the job, and build models, which do generalize to some extent. With them only, theories will come off the ground. Upon further reflection the situation is even worse. Herman Cappelen and Ernest Lepore argue that radical contextualism appears to be internally inconsistent. “To interpret the sentences that express RC [Radical

Contextualism, PD] you have to assume that RC is *not* true.” (Cappelen and Lepore, 2005)[extensively discussed on pp. 128–139]

In this whole picture dynamic semantics plays a peculiar role. It has been put forward as a radical departure from old-fashioned theories of meaning, it has been advertised as the new semantics which steals ground from the pragmaticists, and cultivates it. This may be correct, historically speaking, but the situation is conceived from a different perspective now. All semantics finds its roots in pragmatics, and flourishes from there. Yet, there is good reason to say that there are semantic structures and regularities to be revealed. Dynamic semantics can then be conceived of as the kind of *real* semantics which wears its pragmatic roots on its sleeves.

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Chapter 4
Quantification and Modality

In this chapter I show that *PLA* allows for a rather straightforward extension of its scope, so as to cover the contents of natural language quantifiers and modalities. I could have chosen to offer a sketch of an account of plurals, tense, and events as well. I believe, however, that, on the one hand, an account of these phenomena in a dynamic setting does not require any substantially different means besides those pertaining to their treatment in a standard setting, so nothing would be really gained; on the other hand, the treatment of plurals, tense and eventualities comes with too many complications of its own to be covered well in one monograph. So I decided to skip this endeavour.

The case for quantifiers and modalities is rather different. Both phenomena have been shown to need a dynamic treatment of them (Chierchia 1995; Groenendijk et al. 1996) and an appropriate dynamic way of handling these phenomena has raised a branch of semantic theorizing of its own. The purpose of this chapter, then, is both rather modest but also provocative. I aim not to argue, but merely demonstrate, that a standard and independently motivated analysis of quantification and modality can be directly imported into a *PLA* -style semantics. Doing so does not just serve a theoretical or methodological purpose. This is shown by the fact that the dynamic extensions with quantifiers and modalities come along with uninvited, but very welcome further results.

4.1 Terms and Quantifiers

Before we turn to a treatment of quantifiers in the *PLA* system, we must, first, reflect a bit on the status of the type of terms we have been dealing with so far. Definite and indefinite descriptions can be subsumed in a special category of terms, which names and pronouns belong to, but genuine quantifiers do not. This idea traces back to the pre-analytical opinions that Bertrand Russell (Russell 1905) has objected to, like Stephen Neale and Peter Ludlow did lately, see also (Montague 1974; Neale

P. J. E. Dekker, *Dynamic Semantics*, Studies in Linguistics and Philosophy 91,
DOI: 10.1007/978-94-007-4869-9_4,

1990; Ludlow and Neale 1991). The idea has been rehabilitated, though, with the invention of *DRT* and of choice function approaches to indefinites. The idea in these rather recent approaches is that indefinite noun phrases behave like, or really are, free variables, or epsilon terms, which, like definite descriptions and names, constitute a category of terms that may directly provide arguments for verbs and predicates. (See, among many others, (Heim 1982; Slater 1986; Szabolcsi 1997; Reinhart 1997; Winter 1997; Kratzer 1998; von Heusinger 2004; Kempson and Meyer-Viol 2004) for related views.)

As a matter of fact, these ideas align with the system of *PLA* . For we can, without loss of generality, define satisfaction of terms, be they definite or indefinite descriptions, proper names or pronouns, relative to witnesses, and define satisfaction of atomic formulas in terms of (1) the satisfaction of the involved sequences of terms by witnesses, and (2) the satisfaction of the predicates by these sequences of witnesses. These definitions constitute a referential, not quantificational, treatment of terms. The payoff of such an approach explains the behavior of indefinites, like that of definites and names, on so-called scope islands. Indefinite noun phrases, like definite noun phrases and names, seem to escape scope island constraints on quantifiers, in the sense that their informative contribution can be reformulated as if they had wide scope. This phenomenon can be explained without allowing violations of island constraint and simply by assuming that indefinites, like definites and names, are scopeless so to speak, and that their use is governed by intuitive pragmatic principles. (We will touch upon this issue in the next section.)

. Let me now turn to a proper (second order) extension of the expressive power of our language with generalized quantifiers. Generalized quantifiers have to be introduced in the language first, of course. In keeping with standard treatments, I assume an extension of the *PLA* language so that if ϕ and ψ are formulas, D a quantifier, and x is a variable, then $Dx(\phi)(\psi)$ is a formula. A quantifier D may stand for a determiner like *Some*, *Every*, *Not all*, *Most*, *Many*, *Few*, *More than/Less than/Exactly five*, etc. A formula $Dx(\phi)(\psi)$ corresponds to a natural language structure $[[D\ A_{CN}]_{NP}\ B_{IV}]_S$, which says that D (every, all, most, ...) A's have the property of being B. I assume the standard interpretation of these quantifiers D as a relation $\mathcal{D}$ between sets of individuals. In this section I will first deal with doubly upward monotonic quantifiers only, that is, quantifiers $\mathcal{D}$ for which it holds that if $\mathcal{D}(A)(B)$, $A \subseteq A'$ and $B \subseteq B'$, then $\mathcal{D}(A')(B')$. To facilitate the exposition I start with a definition of extensional satisfaction, and turn to an intensional version at the end of this section.

Before we can actually state the satisfaction conditions for quantified constructions, we have to decide on their domain and range, in the technical sense introduced in Chap. 2. A natural assumption is that the embedded formulas ϕ and ψ in a quantified construction impose the same constraints they would impose otherwise, in the same order, so that $n(Dx(\phi)(\psi)) = n(\phi \wedge \psi)$ and $r(Dx(\phi)(\psi)) = r(\phi \wedge \psi)$. It would in addition not be inappropriate to also assume that the whole construction adds an anaphoric requirement itself, a topical domain of quantification, which also contributes a witness for the whole construction. For, a quantified construction $D(A)(B)$ in natural language generally presupposes a domain of A's under discussion, and the whole construction generally raises a witness set of A's that are B. Such additional structure is relatively easily incorporated in the definition below (see, e.g., (Dekker 2008)), and we therefore leave it out, just to focus on the main aspects of the definition. Domain and range are thus tentatively defined in the way stated above.

Having established the presupposed and contributed numbers of witnesses for quantified constructions, this doesn't mean we have established their type yet. The constituent formulas in a quantified construction $Dx(\phi)(\psi)$ are of course quantified over, which means they will be evaluated relative to all possible valuations of the quantified variable x. This means that if we talk of witnesses for these formulas ϕ and ψ, they may by coincidence be the same for different valuations of x, but generally they will vary with different valuations of x. In turn, this means that potential witnesses for these formulas are functionally dependent on these valuations, so that the real witnesses are not real individuals, but functions from individuals to individuals. Since we may have to deal with quantifiers embedded in other quantified constructions as well, we have to allow for witnesses which are functions from individuals quantified over to (functions from individuals quantified over ...) to ordinary witnesses, i.e., individuals.

In the definition below I continue using the same types of variables $\widehat{a}, b, \widehat{c}, d$ and $\widehat{e}$ as before, but in what follows they range over (sequences of) witnesses of all levels (that is, over individuals and over functions over the domain of individuals). I will also speak of functions from individuals to sequences of witnesses when we are concerned with sequences of functions from individuals to witnesses. This is harmless for the relevant domains $(E^n)^D$ and $(E^D)^n$ are isomorphic, of course. (For this reason, I employ the obvious notation $\widehat{e}_d$ for $e_1(d) \dots e_n(d)$, if $\widehat{e} = e_1 \dots e_n$.) A final convention is that if e is *not* a function, but an individual, then the application $e(d)$ of e to d is simply e itself. (This may look like a hack, but it is conceptually sound.)

Let us turn to a preliminary, unrestricted and extensional, definition of the satisfaction of quantified constructions.

Definition 1 [*Generalized Quantification* (*Extensional*)]

- $M, g, \widehat{ace} \models Dx(\phi)(\psi)$ iff $\mathcal{D}(A)(B)$, where

$$-A = \{d \mid d \epsilon D \;\&\; M, g[x/d], \widehat{ce}_d \models \phi\}, \textit{and}$$
$$B = \{d \mid d \epsilon D \;\&\; M, g[x/d], \widehat{ace}_d \models \psi\}.$$

A quantified structure $Dx(\phi)(\psi)$ is true or satisfied iff the associated quantifier $\mathcal{D}$ applies to the pair of sets of valuations of x, for which the restrictive clause ϕ and the nuclear scope ψ are satisfied. The first set A here is the set of valuations of x as individuals d under which ϕ is satisfied by a witness sequence assigned to d by $\widehat{c}$, relative to the sequence assigned to d by $\widehat{e}$; the set B is computed likewise, except that ψ is taken to be satisfied by a witness sequence ($\widehat{a}_d$) assigned to d relative to the sequence previously associated to d by $\widehat{c}$, and $\widehat{e}$, of course.

The interpretation given in definition (12) ensures a proper treatment of the anaphoric connections between the restrictive clause (ϕ) and the nuclear scope (ψ) of the quantifier. In order to see this, consider the following donkey-style example.

(62) At least five men who bought a farm, built a barn next to it.
($AT\_LEAST\_FIVEx(Mx \wedge \exists y(Fy \wedge BOxy))(\exists z(BAz \wedge BUxz\mathrm{p}_1))$.)

This example comes out true, if we have at our disposal a function f assigning to at least five men d a farm $f(d)$ that d bought, and a function g assigning to each d a barn $g(d)$ that d built next to $f(d)$. The witness functions f and g are thus required to relate (correlate) the men with farms bought and barns built, and they are required to be there for at least five men in the domain. Besides, if there are such men d, farms $f(d)$ and barns $g(d)$, the witness functions may be picked up by subsequent anaphoric pronouns. Consider the following continuation, which elaborates on a witness set of at least five men who bought a farm and built a barn.

(63) They all painted it red. ($ALLx(\Pi_1 x)(PAINT\_REDx\mathrm{p}_2)$.)

As I stated at the beginning of this chapter, I don't deal with plurals here, or with the (plural) antecedents that quantifiers set up, but if we assume that the pronoun Π_1 here picks up the set of men who bought a farm and who built a barn next to it, which is the intersection of sets which serve as the interpretation of A and B above, then we get the following truth conditions of this continuation. For each of these men d it ought to hold that d paints the barn $g(d)$ assigned to d by g. The connections between terms in the restrictive clause and pronouns in the nuclear scope of a quantified construction, and between terms in such a construction and terms outside of it, are thus handled in a uniform manner. All of these interconnections come about by a simple generalization of the treatment of pronominal relationships in *PLA* .

The very same treatment works for a classical example from Gabriel Sandu (Sandu 1997).

(64) Most men had a gun, but only a few used it.

This first part of the example is satisfied if indeed most men had a gun, by a witness function f which assigns to gun-owning men the gun they own; the second is satisfied if only a few of these men d used the gun $f(d)$ they own. We see that donkey-type dependencies in quantified constructions are handled in a fairly straightforward way, and also that further anaphoric dependencies are, or can be, smoothly accounted for. Notice, though, that the basic definition is entirely classical. The quantifier is applied to the sets of valuations of x under which ϕ, respectively ψ, are satisfied.

Because the extension of the *PLA* system with quantifiers is a conservative one, with only witness stuff added, all the standard concepts and results from the theory of generalized quantifiers apply to the 'dynamic' ones specified here. This means that classical properties of quantifiers like, e.g., monotonicity and conservativity, reappear in our framework in the usual way, by only attuning the constituent expressions to new contexts. The meanings of quantifiers in quantified constructions don't have to be changed in any substantial sense, in order to adapt them to anaphoric relationships that actually occur. And rightly so. They only have to adapt to more context sensitive constituent phrases and resolve their anaphoric needs. Except for the witnesses involved everything is classical.

The above benefit does not maintain when we adopt the 'dynamic' definition of generalized quantifiers of other proposals. In order to obtain binding relations between the restriction and scope of a quantifier, those working in an update semantic framework have had to build conservativity into the dynamic meaning of quantifiers by force. (See, e.g., (Chierchia 1992; Kanazawa 1994), among many others.) Since our notion of satisfaction does not assume a bound variable treatment of pronouns, nor an update semantic implementation, such methodologically driven adaptations of meaning don't have to be made. For instance, I do not unconditionally claim that 'only' is a determiner, but the above definition does not *prevent* it from being one, something which the aforementioned alternatives do indeed exclude.

The previous discussion focused on the interpretation of upward monotonic quantifiers. Once quantifiers enter the picture which license other, or no, entailment patterns, the definition has to be strengthened, in two ways. The quantifier 'every', for instance, is downward entailing in its restrictive clause. It is easily seen that, according to the above definition, and without further constraints, it would be too easy to satisfy a donkey sentence, under its properly quantified construal.

(65) Every farmer who owns a donkey beats it.
$(ALLx(Fx \wedge \exists y(Dy \wedge Oxy))(Bx\mathrm{p}_1).)$

If we employ a witness function $\widehat{c}$ that assigns to every farmer a goat he doesn't own, then the example would be deemed true, independent from what farmers do with the donkeys they do own. For the set of d such that d is a farmer and such that $\widehat{c}(d)$ is a donkey he owns, would be empty, because $\widehat{c}(d)$ is a goat d doesn't own. The whole statement would, thus, be vacuously true. A very effective fix to this problem is to require that the witness function $\widehat{c}$ be truthful, in the sense that if there is a way of satisfying the restrictive clause, then $\widehat{c}$ captures it. So here is the first fix, which is assumed in the definition of the satisfaction of quantified constructions in what follows.

- For all $d \epsilon D$, $\widehat{z} \epsilon D^{n(\phi)}$: if $M, g[x/d], \widehat{ze}_d \models \phi$ then $\widehat{c}_d = \widehat{z}_d$, that is, $\widehat{c}_d$ is *the* witness, if any, for ϕ relative to $M, g[x/d], \widehat{e}_d$, for any $d \epsilon D$.

This requirement ensures that the witness function assigns to every individual d in the domain of quantification the unique witness sequence which satisfies ϕ for d, if any such sequence does. By means of this requirement subsequent anaphora are rendered defined. For if for some of these individuals there were more than one satisfying sequence, the choice of the sequence may get relevant for the evaluation of the truth or satisfaction of the whole construction, and give conflicting results. It seems that these structures are only interpretable if no such choices interfere with the interpretation. I should emphasize that this additional requirement is not part of the truth conditional content of a quantified construction, but serves as a presupposition.

A little digression is in order here. The present subject relates to a huge discussion in the literature which started at least with (Evans 1977), which revived with (Kadmon 1990 and Heim 1990), and which continues till today, see, e.g., (Elbourne 2005).

Once several choices for witnesses are available one can say that the satisfaction of the whole formula requires satisfaction by all witnesses, or by some. The fact that such a choice may be relevant have led some to posit an ambiguity between so-called 'strong' and 'weak' readings of quantifiers, and raised empirical studies which readings are favoured for which constructions. However, if the above uniqueness condition is not met both the literature and experimental settings (Geurts 2002) show that people's intuitions are severely blocked, or even that interpretation crashes. The above restriction removes all questions about weak and strong readings because quantification over *all* or over *some* witnesses doesn't make a difference when there is exactly one. End of digression.

With the uniqueness restriction we fixed the problem of wrong witnesses for the restrictive clause of generalized quantifiers, and something similar ought to be done for their nuclear scope. The quantifier 'no', for instance, is downward monotonic in its second argument as well, and a suitable choice of wrong witnesses would render constructions with 'no' too easily true as well. Consider the following sentence.

(66) No boy likes a girl. ($NOx(Bx)(\exists y(Gy \wedge Lxy))$.)

The sentence comes out trivially true, if we employ a witness function *his father* for the indefinite a girl, because no boy likes a girl who is his father. (Trivially, because, arguably, no girl is any boy's father.) Yet, the sentence should be false, of course, if there *is* a boy who likes a girl. The cure is fairly easy, again. When we compute the nuclear scope of a downward monotonic quantifier, we gather all the individuals in the satisfaction set for which there is *some* witness sequence satisfying the nuclear scope, so we do not use the witness parameter for the whole quantified construction here. Here is the required second fix, also assumed in the definition of the satisfaction of quantified constructions.

- if $\exists \widehat{z} \epsilon D^{n(\psi)}$: $M, g[x/d], \widehat{zce}_d \models \psi$ then $M, g[x/d], \widehat{ace}_d \models \psi$, that is, $\widehat{a}_d$ is *a* witness, if any, for ψ relative to $M, g[x/d], \widehat{ce}_d$, for any $d \epsilon D$.

The constraint on witnesses for the nuclear scope ψ is weaker than the one on those for the restriction of a quantifier. The witness sequence relevant to the nuclear scope of a quantified construction is not, for any individual, *the* witness sequence for ψ, if any at all, but merely *some* such sequence, if any at all. The stronger requirement is not necessary, and is not desirable either. Still this requirement serves its purpose by providing a witness that allows for subsequent anaphoric pronouns, even in downward monotonic contexts. Consider the downward monotonic quantifier *At most five*.

(67) At most five students handed in a cake.
$AT\_MOST\_FIVEx(Sx)(\exists y(Cy \wedge Hxy))$.)

If in fact at most five students handed in a cake, so if indeed there are at most five students for whom we can find a witness that is a cake they handed in, then the construction relates a set of students who handed in a cake, by means of a witness function from that set of students to the cakes they handed in. Notice that the sentence may be satisfied if no students handed in a cake. However, normally, the sentence can be followed by one with subsequent anaphoric pronouns.

(68) They baked it themselves. ($ALLx(\Pi_1 x)(Bx\mathrm{p}_1)$.)

This effectively means that each student baked the cake (s)he handed in, and thus, provides an existential claim that even it were at most five students who handed in a cake, there was also at least one, or maybe even two, who did so. In any case, if this additional presupposition is satisfied, the continuation claims that all of the students who handed in a cake (at most five), baked that cake themselves. This construal also serves to explain why corresponding anaphoric take up is excluded in constructions with the quantifier 'no'.

(69) No boy brought a cake along.

If no witness function exists from boys to cakes they brought along, then the sentence is satisfied, and it is satisfied by *any* witness function. This may seem to be a problem, but really it is not. If one were to try and employ such a function, one should have to pick up the referent set for the boys who brought a cake in the first place, and the required take up would be vacuous because that set is claimed to be empty by the first sentence.

By now we have almost set the stage for the final definition of quantification. Two more modifications are required, but they come almost automatically. The above extensional definition has to be lifted to the intensional level which is a notational exercise. In the second place, this lift requires one to quantify over individuals, not as they are given, but as they are given by sets of individual concepts. Therefore, the definition is relativized to certain conceptual covers, which are simultaneously restricted to bodies of knowledge or information states in the way I did above and to domains of quantification in the way of Dag Westerståhl (Westerståhl 1984).

Definition 2 • $M, w, g, \widehat{\alpha\gamma\epsilon} \models D_C x(\phi)(\psi)$ iff $\mathcal{D}(A)(B)$, where

– $A = \{\delta(w) \mid \delta \epsilon C \ \& \ M, w, g[x/\delta], \widehat{\gamma\epsilon}_\delta \models \phi\}$, and
  $B = \{\delta(w) \mid \delta \epsilon C \ \& \ M, w, g[x/\delta], \widehat{\alpha\gamma\epsilon}_\delta \models \psi\}$, assuming

$\widehat{\gamma}_\delta$ is the witness, if any, for ϕ relative to $M, w, g[x/\delta], \widehat{\epsilon}_\delta$, and
$\widehat{\alpha}_\delta$ is a witness, if any, for ϕ relative to $M, w, g[x/\delta], \widehat{\gamma\epsilon}_\delta$, for any $\delta \epsilon D_w$.

Comparing this definition with the preliminary (extensional) version, we can see that the w's are added at all relevant places, and that Greek letters have replaced roman ones; furthermore, there is a contextual restriction of quantification to a set of individual concepts from a conceptual cover C. Here, again, it is not specified which set of worlds C is supposed to be a cover for, but we can always assume it to be 'large enough', to make sure that the individual concepts involved will never be undefined for any world they apply to. These covers will be put to use in Sect. 4.2.

In the early youth of dynamic semantics Jeroen Groenendijk and Martin Stokhof conceived of a dynamic notion of meaning as involving abstracting over 'possible continuations' of a discourse (Groenendijk and Stokhof 1990, p. 19); the idea is not only conceptually very inspiring, but it is shown to be derivable by general principles of type-lifting. (Hendriks 1993) Not surprisingly the idea has been caught up again, in recent work from numerous authors (Asher and Pogodalla 2010; Barker 2002;

Barker and Shan 2008; Barker et al. 2010; de Groote 2001, 2006; Szabolcsi 2003). In particular, Barker and Shan have adapted a continuations-based view of meaning to the analysis of generalized quantifiers in natural language. Their approach is very appealing because it builds on classical analyses of generalized quantifiers, like we do in this monograph, while extending it with novel linguistically (syntactically and semantically) interesting features. As Shan puts it: "Integrating variable-free semantics and dynamic semantics gives rise to interactions that make new empirical predictions (...)." (Shan 2001, p. 204) This monograph is not the place to discuss the delicate merits and potential deficits of these approaches, but it is worthwhile to point out that they do conservatively build on received wisdom about quantifiers, that they do achieve new results by adopting a dynamic perspective, and that we are now capable of framing the involved type of dynamic interpretation in a conservative, classical, semantics. Putting it concisely: what is new in dynamic semantics is good, or even better.

4.2 Knowing Who and Believing What

The previous sections have laid out intensional *PLA* , and the *PLA* treatment of quantifiers and have paved the way for an analysis of attitude ascriptions. This subject is full of logical, philosophical and linguistic pitfalls, and I do not aim to address many of them. For instance, I say nothing about logical omniscience or the deductive closure of belief states. I do think, however, that Willard van Orman Quine's problems with quantification into beliefs contexts, and Peter Geach's problems of anaphora across attitudes, are straightforwardly handled if one imports a classical treatment of attitude reports into intensional *PLA* . The only thing that has to be carefully dealt with is the settlement of the domains of quantification and their conceptualization. For this purpose Aloni's conceptual co498 vers prove entirely adequate, as has already been shown in (Aloni 2005).

I need to make one proviso before we start. In what follows I do not give an account of indexical belief or belief *de se*, something which would be most in the spirit of the present investigations, which after all involve indexical pronouns, and, in the next section, an indexical modality operator 'might'. The reason for excluding them is that I want to show that the *PLA* system is compatible with all of the standard extensions, so I adopt what I believe is the standard analysis of belief, which is non-indexical. Let me just mention that if we really go essentially indexical, as we should, I would advocate the superior treatment given by Lewis (1979), involving beliefs about one's personal location in logical space. See (Dekker 2001) for an attempt at an analysis along these lines.

What I believe to be a standard analysis of belief ascriptions roughly goes along the following lines. An agent's beliefs are characterized by a set of possibilities, the possible ways the world might be, according to how the agent thinks the world is. Such sets of possibilities are like the information states we have employed above, but now they are not assumed to be just given, but are the subject of description themselves.

Our models, thus should distinguish between possibilities in which an agent does have certain beliefs from possibilities in which she does not have them. For this purpose, each epistemic agent a is assigned an epistemic accessibility relation $\mathcal{R}_a$ between worlds, such that the set of worlds $\mathcal{R}_{a,v} = \{w \mid \langle v, w\rangle \epsilon \mathcal{R}_a\}$ characterizes the beliefs of agent a in world v. A model $M = \langle W, D, (\mathcal{R}_a)_{a \epsilon A}, I\rangle$ is like an ordinary intensional model, then, but with a set of accessibility relations $\mathcal{R}_a$ added, for each agent $a \epsilon A \subseteq D$. Saying that agent a in v believes a formula ϕ boils down to claiming that ϕ is true throughout the state $\mathcal{R}_{a,v}$, which means that if a's beliefs are correct in v, then ϕ is true in v. This much is standard.

Let me try to rephrase the given satisfaction conditions within the *PLA* system. First, then, we have to extend the language with a belief operator $\mathcal{B}$, such that if ϕ is a formula, and t a term, $\mathcal{B}(t, \phi)$ is a formula. Next we have to settle the domain and range of the formula. The range of $\mathcal{B}(t, \phi)$ is determined by ϕ, so that $r(\mathcal{B}(t, \phi)) = r(\phi)$, if t is not a pronoun p_i where $i > r(\phi)$; in the latter case, $r(\mathcal{B}(\mathrm{p}_i, \phi)) = i$. We may speculate about the domain $n(\mathcal{B}(t, \phi))$ of $\mathcal{B}(t, \phi)$. Belief ascriptions, and statements of modality in general, can be 'about' propositions, propositions believed to be true, or propositions deemed possible. Undeniably, such attitude and modality statement license propositional anaphora to the contents of the attitudes or modalities. Therefore we can require these assertions to relate to a (propositional) witness which and agent believes or deems possible. Since I am not aiming at an account of propositional anaphora, though, I will set this matter aside. (The issue has been addressed by (Roberts 1989; Frank 1997; Geurts 1999; Stone and Hardt 1999; Brasoveanu 2006), among many others.) Indefinites under belief operators also seem to bring their supporting witness functions. These will be exploited in what follows, and for this reason I render $n(\mathcal{B}(t, \phi))$ as $n(\phi)$. A witness sequence for a belief report provides the witnesses for the belief reported. Given these preliminaries, the following definition is quite straightforward.

Definition 3 [*Belief Reports*]

- $M, w, g, \widehat{\gamma\epsilon} \models \mathcal{B}(t, \phi)$ iff $\mathcal{R}_{d,w} \models_{M,g,\widehat{\gamma\epsilon}} \phi$, with $d = [\![t]\!]_{M,g,w,\widehat{\epsilon}}$.

The satisfaction condition for a belief report is essentially classical, so it doesn't need further comments. Its logic is the one one wants to dress it up with in terms of associated properties of the associated accessibility relation, which is interesting, but not our concern here. The *only* difference with a completely standard definition resides in the addition of the use of witnesses, which are totally transparent. The witnesses for a belief report are those for the belief reported. In the next paragraphs we show that this allows us to account for the correlations between terms in the scope of belief operators, and terms outside of them. This concerns both issues of quantifying in and existential export, and anaphoric relationships across attitude contexts.

First consider a very natural entailment pattern.

(70) Ralph believes that Ortcutt is a spy. So Ralph believes about Ortcutt that he is a spy. ($\mathcal{B}(r, So) \models \exists x_C(x = o \wedge \mathcal{B}(r, Sx))$.)

This entailment pattern is valid if the *Ortcutt*-concept $I(o)$ is an element of the relevant cover C. This is a fairly natural assumption, but the inference can be qualified in various ways. Ortcutt may fail to exist, and the speaker may be aware of that. Or, Ralph may have mistaken somebody else for Ortcutt, as, for instance, according to the following report.

(71)Ralph (mistakenly) believes that the man with the brown hat is Ortcutt. And Ralph believes that he is spying.
$((\mathcal{B}(r, \exists x_C(BHx \wedge x = o)) \wedge \mathcal{B}(r, S\mathtt{p}_1)).)$

Even though the reported situation licenses the conclusion that Ralph has a belief concerning Ortcutt, that he is a spy, to say that Ralph believes about somebody who is *not* Ortcutt, that he is a spy is more accurate. The previous example renders the following conclusion valid.

- $\exists x_{C'}((BHx \wedge (x \neq o)) \wedge (\mathcal{B}(r, (x = o)) \wedge \mathcal{B}(r, Sx)))$.

The point is that the concept of x which supports this formula is that of a man with the brown hat, which coincides with the *Ortcutt*-concept in Ralph's belief worlds, but not in the worlds incompatible with Ralph's beliefs, or at least not in the actual world.

These previous examples give an idea of how one can describe Ralph's beliefs, as they are reported in (Quine 1956).

(72) Ralph believes that the man in the brown hat is a spy. $(\mathcal{B}(r, \exists x(BHx \wedge Sx)).)$
(73) Ralph believes that the man seen at the beach is not a spy. $(\mathcal{B}(r, \exists x(SBx \wedge \neg Sx)).)$

Furthermore the man in the brown hat is the man seen on the beach, who is actually Bernhard J. Ortcutt. So from (72) one may conclude (74) and from (73) one may conclude (75).

(74) Ralph believes of Ortcutt that he is a spy.
$(\exists x((x = o) \wedge \mathcal{B}(r, Sx)).)$
(75) Ralph believes of Ortcutt that he is not a spy.
$(\exists x((x = o) \wedge \mathcal{B}(r, \neg Sx)).)$

When one takes these two conclusions together, they are inconsistent, or that Ralph has inconsistent beliefs. However, reporting the situation this way is slightly incoherent. Let us see what exactly is at stake. The following formula gives the essentials of the situation put forth by Quine.

- $((\exists x_{BH}(BHx \wedge \mathcal{B}(r, Sx)) \wedge \exists x_{SB}(SBx \wedge \mathcal{B}(r, \neg Sx))) \wedge (\mathtt{p}_1 = o = \mathtt{p}_2)))$.

Ortcutt is known, also to the speaker, as the man with the brown hat, and under that description, that is, under the cover *BH* of the domain including the man with the brown hat, Ralph believes Ortcutt to be a spy; but he (Ortcutt) is also known under the cover *SB* of the domain, under which Ralph does not believe him to be a spy. As long as Ralph doesn't realize that the person seen in these two ways is one and the same person, there is no inconsistency on his behalf, nor on that of the speaker.

Believe it or not, the above formula is equivalent to the following one.

- $((\exists x_{BH}(SBx \wedge \mathcal{B}(r, Sx)) \wedge \exists x_{SB}(BHx \wedge \mathcal{B}(r, \neg Sx))) \wedge (\mathrm{p}_1 = o = \mathrm{p}_2)))$.

Since the man in the brown hat, which is Ortcutt, is the man seen on the beach, it follows that what Ralph believes about the man in the brown hat, he also believes about the man seen on the beach, that is, under the cover of that man as a man with a brown hat; the same holds, mutatis mutandis about the man seen on the beach. The correct interpretation of the two existential quantifiers involves a shift in conceptual cover which generates incorrect results if carried out inappropriately.

Aloni's successful use of conceptual covers in the analysis of quantified sentences of course depends on the mediating use of individuals concepts. Quine already pointed out that this may seem to lead to overgeneration. If we render example (76) in the way indicated, it licenses the conclusion (77).

(76) Ralph believes there are spies. ($\mathcal{B}(r, \exists x Sx)$.)
(77) There is someone whom Ralph believes to be a spy. ($\exists x \mathcal{B}(r, Sx)$.)

Quine clearly claims that we shouldn't reason like this. "The difference is vast; indeed, if Ralph is like most of us, [(76)] is true and [(77)] is false." (Quine 1956, p. 178) This is correct, and what is the problem here is that the following inference is valid in a system in which quantifiers are defined to range over individual concepts.

Observation 20 (**Quantifier Import and Export**)

- $\mathcal{B}(r, \exists x_C Sx) = \exists x_C \mathcal{B}(r, Sx)$.

This equation is valid, but it shows that $\exists x_C \mathcal{B}(r, Sx)$ does not really say so much. As a matter of fact it says nothing, indeed, more than that Ralph beliefs that there are spies, or that the set of spies is non-empty. It does not even say that there is *any* relation between Ralph's beliefs and anything in the real world.

The above observation implies that if one wants to make a more substantial *de re* belief report, like the one given in (77), stronger means are required than the simple formulas given as the translation. This can be easily done by imposing constraints on the conceptual cover under which these attitude reports are made. If Ralph is said to believe somebody x to be a spy, with $\exists x_C \mathcal{B}(r, Sx)$, under a cover C of the domain relevant to the speaker and hearer, then the formula is not trivial. The report then would have to be satisfied by a concept which identifies an individual or type of individual in both the actual world and Ralph's belief worlds, in a non-trivial way.

One might be inclined to render a true *de re* belief report one which establishes some intrinsic relation between the contents of an epistemic agent's beliefs and objects in the real world. I believe such is unsupportable, although there is no problem in doing so formally. We might require a *de re* report to be one of the form $\exists x_D \mathcal{B}(r, \phi)$, where D is the cover which rigidly identifies all the objects in the domain, a cover under which the identity of all the objects in the domain is fully known. (Roughly, like in a Kaplan cover.) Such a report would, however, require that Ralph is totally knowledgable of the identity of the *res* in question in the following sense. It would be said to be impossible to think of a possibility or situation consistent with Ralph's beliefs, in which the witness of x in ϕ is presented to Ralph, and in which he fails to see that the witness has the property expressed by ϕ. Not only would this charge

Ralph with incredible mental powers, also the speaker's assumptions would be rather overdone. She would be supposed to be speaking under the assumption that she has such a conception of the domain, that is, that she herself is totally knowledgable of the identity of all objects. This is something which I believe is factually impossible, methodologically untenable, and practically undesirable.

De re belief reports, and beliefs about entities, objects, events and things, are of a more modest nature. They do involve notions of knowledge of identity, but do not require such notions to enable much more than that of defining or identifying an object in certain almost always very limited situations. This is precisely the idea of a conceptual cover as defined above: a cover is a set of concepts which in a given situation, or in a corresponding information state, serve to uniquely identify a (limited) set of objects. There is no requirement that the whole domain, in all possible circumstances, is totally identified, let alone that it puts us 'en rapport' with the 'Dingen an sich'. The Ortcutt examples we have discussed already serve to show how this works.

The way in which referential belief reports work becomes clearer if we inspect an intriguing example due to Bas van Fraassen, as adapted from (Aloni 2005). The situation is the following.

- "Susan's mother is a successful artist. Susan goes to college, where she discusses with the registrar the impact of the raise in tuition on her personal finances. She reports to her mother." (van Fraassen 1979)

Now imagine that the following discussion develops between Susan and her mother.

(78) *S*: He said that I should ask for a larger allowance from home.
M: He must think I am rich.
S: I don't think he has any idea who you are.
M: I am your mother.

The discussion is truly intriguing. Apparently there is some misunderstanding between Susan and her mother. The confusion is resolved by a statement of the mother ("I am your mother.") which is clearly known to be true by both of them already at the start of the whole discussion. How can this be? Let us suppose the mother is Irene (i), and let us neglect the modals "must" and "I don't think." We can render the last three statements above as follows then. (Here, A indicates Susan's mother's initial conceptual cover, and B that of Susan.)

(79) *M*: The registrar believes I am rich.
$(\exists x_A(x = i \wedge \mathcal{B}(r, Rx)).)$
S: He doesn't; he does not know who you are.
$((\neg\mathcal{B}(r, R\mathfrak{p}_1) \ \wedge \neg\exists x_B \mathcal{B}(r, (x = i))).)$
M: He knows who I am, I am your mother.
$((\exists x_A(x = i) \wedge \mathcal{B}(r, (x = \mathfrak{p}_1))).)$

As can be seen from the final revelation by Susan's mother, when she says the registrar believes her to be rich, she adopts a cover A which includes the concept of being Susan's mother 'sm'. That the registrar believes the proposition Rsm, about Susan, that her mother must be rich, is enough for the mother to conclude that the registrar has a belief about her, Irene, that she is rich. Susan denies this claim, but under a different construal, from the perspective of cover B. From the fact that the registrar cannot at all identify Irene under cover B, Susan concludes he cannot believe anything about Irene, like, for instance, being rich. The cover B here may include a concept of being directly acquainted with Irene, or the concept of 'that famous artist'. Presenting herself as given by the concept sm of being Susan's mother, Irene clarifies the case. The registrar's situation is quite like that of Ralph. There is a sense in which he surely does, and one in which he certainly does not believe about Irene that she is rich. What plays up in the discussion between Susan and Irene is under which conception Irene is known to be rich.

In the situation presented by van Fraassen I adopted a notion of (not) knowing who. The notion of knowing who is that of being able to identify an individual among alternative individuals. The idea is that one knows who Irene is, if one knows that $(x = i)$ for all and only the possible values of x that is i. Surely, such a notion of knowing who is relative to the alternatives, and the way they are given. This type of relativity shows itself in epistemic situations involving Hesperus (h) and Phosphorus (p) raised to our attention already in (Frege 1892).

Observation 21 (**Knowing and not Knowing Who**) There are conceptual covers C_e and C_m such that the following is consistent.

- $\forall x_{C_e}((x = h) \leftrightarrow \mathcal{B}(r, (x = h))) \wedge \exists x_{C_m}((x = h) \not\leftrightarrow \mathcal{B}(r, (x = h)))$.

This relates to the situation of the early Babylonians. A Babylonian could have been said to know who Hesperus is, if he were able to point out the right object when looking at the evening sky, so from the perspective of the evening sky cover C_e. By the same token he could have been taken not to know who Hesperus was, since he was unable to pick out the right object (Phosphorus, which is Hesperus) in the morning sky, so from the morning sky cover C_m. This epistemic situation is laid down in the observation above. By means of Aloni's conceptual covers, the context dependent nature of knowledge and ignorance is neatly accounted for.

The previous examples show the use of transparent beliefs by means of which the contents of beliefs are related to the objects in the actual world. But this does not even need actually existing objects. In Peter Geach's examples of 'intentional identity' the beliefs of two agents may be correlated by means of a concept, even though the concept does not meaningfully relate to an object in the actual world. Geach introduced the concept of intentional identity as follows. "We have intentional identity when a number of people, or one person on different occasions, have attitudes with a common focus, whether or not there actually is something at that focus" (Geach 1967, p. 627). Here is his example, and a transcription in which I employ $\mathcal{Q}$ for the attitude of *wondering*.

(80) Hob thinks a witch has blighted Bob's mare, and Nob wonders whether she (the same witch) killed Cob's sow.
$((\mathcal{B}(h, \exists x_C(Wx \wedge Bxm)) \wedge \mathcal{Q}(n, K\mathrm{p}_1 s)).)$

The anaphoric connection here is formally easily accounted for, since the first conjunct is equivalent with $\exists x_C \mathcal{B}(h, (Wx \wedge Bxm))$, and the pronoun under Nob's wonder simply picks up the witness concept associated with the existential quantifier. Surely, this raises the big question how a speaker can have evidence or support for a concept figuring in two different persons' attitude contexts. The select branch of literature devoted to this question, see for instance (Edelberg 1992) for a relatively recent approach, can be considered to be trying to answer exactly this question. In the absence of any real witch on which Hob's beliefs, and Nob's questions, are coordinated, one may think of a postulated or hypothetical witch which Hob and Nob have exchanged their opinions about; it may be the fictional subject of a rural story; it may be something imposed on Hob and Nob by some external force. The possible number of explanations is as a matter of fact innumerable.

The truly positive point about the approach presented here is that it doesn't depend on any such explanation and that it only requires the users, speakers and interpreters of intentional identity statements to make some pragmatical sense of them. According to the present proposal, and, I claim, according to our natural intuitions, interpreting example (80) poses no problem. If someone asserts (80), we may of course wonder, whom he means, which witch, and ask *What are you talking about?* The speaker then may provide many possible explanations, like someone she heard about, some witch Hob and Nob talked about or invented, a decent vet who was subject to their disgusting gossip, a deamonly inspiration if you want, whatever; many answers are possible. The presented proposal only requires there to be some such explanation.

A very common opinion among those not terribly interested in belief reports is that examples like (80) are very marginal. But example (81) is a very similar one about an alarm at the Dutch airport Schiphol which has been actually reported on the Dutch radio on the morning of January 6, 2010.

(81) A man is said to have entered the terminal without being checked. He has not been found.
(The Dutch version in the newspaper De Telegraaf, Tuesday January 5, 2010, reads: "Zondagavond (lokale tijd) was een terminal van de internationale luchthaven van Newark, bij New York, ontruimd. Een man zou zonder te zijn gecontroleerd langs een verkeerde route de hal zijn binnengegaan, waarop iedereen opnieuw moest worden gecontroleerd. De man is nog altijd spoorloos.")

Indeed, there may have been a specific man who has caused all these troubles, for instance a man seen on a security tape, at least that is what the TSA officials make us believe. But, not having identified the man, or such a man, how can we tell it is a man and not a woman? How can we tell there was a man, or a woman? What can assure us that there really was someone being monitored on a tape the TSA officials have only told us about? Nothing, I believe, and we don't need any such assurance, not even for saying that "He is unfindable."

Notice that the present account of *de re* beliefs and of intentional identity comes without any claim on the structure of beliefs, and without any specific moral about how to treat belief reports. It is a direct consequence of the use of satisfying witnesses, and their natural transposition to a notion of information support.

4.3 Alethic and Epistemic Modality

Much attention in dynamic semantics has focused on epistemic modalities, as expressed by *might*, for instance, and their interaction with other operators, mainly quantifiers. This section presents the results of using *PLA* to improve on the framework involving quantification and epistemic modality of (Groenendijk et al. 1996). In order to set the stage, however, I must first discuss alethic modalities and show how they fit in the general system of intensional *PLA* .

In this section I refrain from taking a particular stance on a notion of ontological modality, possibility, or necessity, except for its consequences for a notion of identity. I agree with, e.g., (Kripke 1972) that if we think of individuals (times, events, ...) in other possible situations or configurations we do take their identity for granted. We think that the world could have been different in some, or zillions, different ways. However, if one thinks of an individual in other possible circumstances, then one cannot imagine or think of a possibility in which that thing is not itself; neither, I believe, can we think of a possibility in which two things we conceive of, literally two things, are one and the same thing. (Of course, I am not now talking about 'appearances' of things here, or parts of things.) Identity appears to be unquestionable. (Already in (Wittgenstein 1922) identity shows up as one of those entirely logical things that cannot be stated, but only shown.)

If I attribute a property to an object, and if I am confronted with the very same object in other circumstances, without realizing it is the same object, it still remains beyond doubt that I have attributed the property to that very same object. No discussion. Logically speaking, when one's purposes don't go beyond a first order model, or even a typed hierarchical model theoretic universe, identity is non-negotiable. This does not deny, as Kripke does *not* deny (cf., (Kripke 1979)), that we, human beings, or any kind of epistemic agents, may be mistaken about the identity of things.

How do we account for these intuitions about possibility, and about the related idea of necessary identity? With Kripke we may distinguish intended ontological notions of possibility from the ways we conceptually get a grip on them. Individuals are given to us through our conceptualization of them, but once they are identified through such conceptualizations, their identity is necessary, even though their identity may escape us. We can account for this in our system by declaring some of our conceptual covers of the domain 'ontological covers', the idea being that the things we attribute to the individuals conceived that way, are the real individuals. The individual concepts from an ontological cover then must respect equivalence classes of worlds under an ontological accessibility relation $\mathcal{R}_o$, the epistemic accessibility relation of 'ontology', or God. Formally this is rendered as follows.

Definition 4 [*Alethic Necessity*] A model M comes with an accessibility relation $\mathcal{R}_o$ such that for every ontological cover C_o:

- for all $c \epsilon C_o$ and for all $\langle w, v\rangle \epsilon \mathcal{R}_o$: $c(w) = c(v)$;
- $M, w, g, \widehat{\gamma\epsilon} \models \Box\phi$ iff $\mathcal{R}_{o,w} \models_{M,g,\widehat{\gamma\epsilon}} \phi$.

Under this rendering of possibilities and necessities, the following observations are immediate consequences.

Observation 22 (**Necessary Identity**)

- $\models \forall x_{C_o} \forall y_{C'_o}((x = y) \rightarrow \Box(x = y))$;
  $\models \forall x_{C_o} \forall y_{C'_o}(\Diamond(x \neq y) \rightarrow (x \neq y))$.

We may fail to see that an individual presented one way is, or is not, identical to an object presented another way. But if they are identical this is necessarily so, and if they can possibly not be identical, then they simply are not. Here we find Kripke's ideas about identity fully warranted.

Here I need to take another small digression. An alternative definition in the style of Lewis' philosophy, rather than that of Kripke, is possible. We could require that for all ontological covers C and C' and concepts $c \epsilon C$ and $c' \epsilon C'$, we have for all $\langle w, v\rangle \epsilon R_o$ that $c(w) = c'(w)$ iff $c(v) = c'(v)$. This removes the annoying assumption that epistemic agents are in unmistakenable acquaintance with the real individuals involved. (Groenendijk et al. 1996) claims that we cannot do without such identification of real objects, or that in dynamic modal semantics some questions of identification cannot be solved unless the language contains demonstratives (see also (Stokhof 2002)). Ironically, this also holds for this system, in which quantification is defined to be over individuals which otherwise could not be accessed. In general, however, such an identification requirement is as unreasonable as rigid designation is unattestable. Kripke says "Let's call something a *rigid designator* if in every possible world it designates the same object (...)", and argues that there is no problem in finding out what counts as the same object (in different circumstances) "(...) *because* we can refer (rigidly) to Nixon, and stipulate that we are speaking of what might have happened to *him* (under certain circumstances) (...)" (Kripke 1972, pp. 48, 49). This is all very intuitive, but question-begging, too. It does not, and cannot, for instance, establish the issue whether or not, unbeknownst to us, the objects we believe to be talking about are imperceptibly replaced by others objects every five minutes. Put differently, from a logico-philosophical perspective, the inhabitants of a modal universe cannot find out whether they live in Kripke models rather than Lewis models, because these models are indistinguishable from within. For this reason, then, it also doesn't matter whether we adopt the Lewisian definition suggested here, or simply stick to the definition given above. End of digression.

It may be clear that *epistemic* modalities are different from their alethic counterparts. Epistemic modal operators like 'might' and 'must' in English, and semantically related verbs, adverbs, and adnominals, express a kind of possibility or necessity relative to some body of knowledge or evidence. A sentence formalized as $Might(\phi)$ (or: $\mathcal{M}\phi$) is used to express that ϕ is not excluded relative to some source of evidence, and $Must(\phi)$ (or: $\mathcal{W}\phi$) that it is or seems to be entailed by it. In a Kratzer-style semantics such a body of knowledge or evidence K is conceived of as a set of possibilities (situations, worlds, ...), relative to which $\mathcal{M}\phi$ ($\mathcal{W}\phi$) is true iff ϕ is true with respect to some (all) possibilities in K (Kratzer 1977, 1981).

Many authors observe that epistemic modality statements indexically reflect on a current information state. This idea is appealing and worth pursing, but not uncritically, because to lay our hands on the idea of a current information state is notoriously difficult, and the opinions about that issue diverge both explicitly and implicitly. Let us first take our clue from (Veltman 1996), according to which 'might' is used to claim that the update with ϕ does not produce inconsistency. It is not excluded that ϕ. The very idea of an update semantics suggests that we are talking of an inconsistency with the state of a hearer here. If the hearer updates with the elicited test, the utterance amounts to the claim that the hearer's information state can be consistently updated with the sentence ϕ. Even though this *is* a sensible claim to make in certain circumstances, it does not seem to be what an utterance of $\mathcal{M}\phi$ conveys. For one thing, it is perfectly consistent to say:

(82) *You* still think it is possible that Mary will not come to the party, but it is ruled out already. She has decided to come, and nothing can change her mind and stop her.

In this situation to say that Mary might come seems very inappropriate. It may seem, then, that an utterance of $\mathcal{M}\phi$ is, rather, concerned with the information state of a speaker, who reflectively says: my information does not exclude the possibility that ϕ. Again, this is a sensible thing to say, but it does not appear to be what one uses 'might' for. Normally, a speaker is taken to be an expert on the issue of what it is that she believes and what she doesn't rule out. Still it is very well possible to counter a claim that Mary might come as follows:

(83) No, maybe you think that it is still an option that Mary will, but I happen to know that this possibility is ruled out.

This time the addressee of the original utterance points out a fault in a *might* claim of a speaker has said that Mary might come. The, rather airy, suggestion then is that what *Might*-utterances reflect on is 'the situation in discourse' or 'the common ground'. Now this may well be true, but, on the one hand, it would require a conceptual reinterpretation of an update semantics as a semantics for updating, not a hearer's information state, but a common ground—something which is not without its logical and philosophical pitfalls. (See, for instance, (Gerbrandy 1999) for relevant discussion.) In the second place, it would render *Might*-claims vacuous, intuitively speaking. If anything, a common ground should rule in and rule out what the interlocutors rule in and rule out, and what they mutually know the others to rule in and rule out. Thus conceived, if ϕ is consistent with the common ground, this is something which everybody is aware of, so that the statement that *Might A* can only be vacuous. A final retreat then, in defense of Veltman's *Might*, consists in conceiving of $\mathcal{M}\phi$ to eventually serve to correct one's possibly mistaken picture of what the common ground is.

An interpretation of Veltman's *Might* can be implemented in the *PLA* system in the following way. A Veltman-style epistemic modality statement $\mathcal{M}_V\phi$ is attested against a contextually given information state τ, and claims that that state can be consistently updated with ϕ. If that information state is to serve as a picture of the common ground, it should be updated in the course of interpretation, something which effectively happens under a conjunction. The following definition spells out the relevant clauses for the definition of $M, w, g, \tau\widehat{\epsilon} \models \phi$, the satisfaction of ϕ in w in M relative to g and relative to a sequence of witness functions $\widehat{\epsilon}$ and common ground τ.

Definition 5 [*Veltman's Might*]

- $M, w, g, \tau\widehat{\epsilon} \models \mathcal{M}_V\phi$ iff there is $\widehat{\gamma}\epsilon D_w^{n(\phi)}$: $(\tau)[\![\phi]\!]_{M,w,g,\tau\widehat{\gamma\epsilon}} \neq \emptyset$;
- $M, w, g, \tau\widehat{\alpha\gamma\epsilon} \models (\phi \wedge \psi)$ iff $M, w, g, \tau\widehat{\gamma\epsilon} \models \phi$ and $M, w, g, \upsilon\widehat{\alpha\gamma\epsilon} \models \psi$,

– where $\upsilon = (\tau)[\![\phi]\!]_{M,g,\tau\widehat{\gamma\epsilon}}$.

Actually, this definition is the formal implementation of (Cresswell 2002)'s version of Veltman's dynamic semantics in a static semantics. A formula $\mathcal{M}_V\phi$ tests whether the contextually given state τ can be consistently updated with ϕ; the amended clause for conjunction has it that, if τ is to serve the purpose of a common ground, then after ϕ has been accepted this common ground is supposed to be updated with ϕ, when it is supposed to serve as the common ground for a subsequent utterance of ψ.

Notice, first, that apart from the additional Stalnaker-style update of the common ground, $\mathcal{M}_V\phi$ is an ordinary Kratzer-style modality statement, which does justice to the dynamic intuitions from (Veltman 1996).

Observation 23 (**Dynamic Epistemic Might**$_V$)

- $(\mathcal{M}_V p \wedge \neg p)$ is consistent (satisfiable);
  $(\neg p \wedge \mathcal{M}_V p)$ is not consistent.

That is, employing Veltman's *Might*, the following example turns out consistent:

(84) Mary may be home now, but she isn't. $(\mathcal{M}_V Hm \wedge \neg Hm)$.

It is inconsistent, though, to say:

(85) Mary is not home now, but she may be. $(\neg Hm \wedge \mathcal{M}_V Hm)$.

This comes out as required. Differently from (Veltman 1996; Groenendijk et al. 1996), however, example (84) is rendered coherent. A sentence is called coherent iff there is an information state that supports the sentence. It is perfectly possible, of course, to have a common ground which does not exclude the possibility that Mary is home, while the speaker has information to the contrary. Thus a speaker may properly convey his observation that the common ground allows for the possibility that Mary is not home, and directly add his information that she is not. Of course, there is redundancy in stating things this way, but intuitively it is not incoherent to do so. (The reason why (Groenendijk et al. 1996) deem this example incoherent is that, after all, they do relate the epistemic modal *Might* to the speaker's information state. Under that interpretation we would have got an instance of Moore's paradox, *I don't believe it, but, I tell you, Mary is not home.* I agree that Moore's sentence is indeed an incoherent thing to utter, but this is *not* what example (84) intuitively says.)

Employing both alethic necessity and Veltman's *Might*, we can square Kripke's views on naming and necessity with those of Frege on Sinn und Bedeutung (Frege 1892; Kripke 1972).

Observation 24 (Possibly Necessary Identity)

- $(\mathcal{M}_V\Box(t_1 = t_2) \wedge \mathcal{M}_V\Box(t_1 \neq t_2))$ is coherent (supportable).

The following example brings out this fact neatly. Let C_e again be an (ontological) cover of the sky in the evening, and C_m one of the sky in the morning. Then it is possible to have an information state where both necessary identity and necessary non-identity are acceptable.

(86) Maybe Hesperus is necessarily identical with Phosphorus, maybe Hesperus is necessarily not identical with Phosphorus. I don't know.
$(\exists x_{C_e}((x = h) \wedge \exists y_{C_m}((y = p) \wedge (\mathcal{M}_V\Box(x = y) \wedge \mathcal{M}_V\Box(x \neq y)))).)$

The consistency of this statement is independent of the properties we assume the accessibility relation $\mathcal{R}$ to have. The attentive reader may notice that observation (24) summarizes (Kripke 1972) in a nutshell.

The following observations derive from (Aloni 2000), and as a matter of fact constitute one of her arguments for her style of quantification. Let $\mathcal{W}_V\phi = \neg\mathcal{M}_V\neg\phi$, so that $\exists x_C\mathcal{W}_V(x = t)$ says that t is known$_V$ under cover C.

Observation 25 (Quantified Epistemic Modality (1))
For any term t

- $\exists x_C\mathcal{W}_V(x = t), \forall x_C\phi \models \phi[x/t]$;
  $\exists x_C\mathcal{W}_V(x = t), \phi[x/t] \models \exists x_C\phi$.

(In this observation I have, uncautiously, neglected the possibility that dynamic effects of the premises may require an update of the conclusion. Nevertheless, we can be confident that a proper update can be defined.) Universal instantiation and existential generalization are always allowed as long as one remains looking at the domain from the same conceptual perspective. As soon as we make a switch between covers, this feature fails.

Observation 26 (Quantified Epistemic Modality (2))

- $(\forall x_C\mathcal{M}_V\phi \wedge \exists x_{C'}\mathcal{W}_V\neg\phi)$ is coherent.

The following example typically illustrates this pattern.

(87) Anybody around here might be Dr. Livingstone, and anybody might not be Dr. Livingstone. But of course, Dr. Livingstone cannot *not* be himself.
$((\forall x_C\mathcal{M}_V(x = l) \wedge \forall x_C\mathcal{M}_V(x \neq l)) \wedge (\exists x_{C'}\mathcal{W}_V\neg(x \neq l) \wedge (\mathtt{p}_1 = l)))$.

Also with respect to two different *ontological* covers one may fail any identificational information whatsoever. Take again the ontological cover of the sky in the evening C_e and the one in the morning C_m. For someone who doesn't even recognize Ursa Major, the following may hold.

Observation 27 (Identity Ignorance)

- $\forall x_{C_e}\forall y_{C_m}\mathcal{M}_V(x = y)$ is coherent.

Looking at the world (sky) from different perspectives, one in the evening, and one in the morning, a person may have no clue as to how these scenes relate, and moan the following.

(88) Anyone could be anyone.

For the linguistic examples discussed, Veltman's *Might* appears to work in an intuitively convincing way, and we would like to preserve these results. One question remains, however. If we conceive of *Might* as expressing consistency with a common ground, which we are all supposed to know perfectly, then no *Might* statement can make sense, really. Like the sentences modeled in propositional logic, which are satisfied or not satisfied, it would be a test which succeeds or does not succeed. The intuitive difference with propositional logic is that, while we may fail to know whether or not the actual world satisfies the sentences from propositional logic, we cannot fail to know whether a certain sentence or formula is consistent with our information. Any *use* of such a *Might* statement would therefore be useless.

Indeed we have already opted for an alternative interpretation of *Might* statements, as testing a picture of a common ground. All by itself, this is still quite unsatisfactory. Hardly anybody denies the relevance of facts about the actual world for our own acting and thinking, and normally one does not deny the relevance of other people's information about these facts. But what could sensibly be the use of knowing that someone thinks that a certain proposition is consistent with her current picture of a common ground? Without further explanation, if somebody says that Mary might come to the party, and only conveys that it is consistent with her pictures of the common ground that Mary comes to the party, the first and most likely reaction would be, I guess, "Good for you, or for your picture, so what?"

Several authors have suggested that epistemic modal statements additionally serve to "raise" possibilities and that they are used to bring us to "attend to" or "focus on" possibilities. (Hulstijn 1997; Groenendijk 2007; Yalcin 2008; Roussarie 2009; Brumwell 2009; Ciardelli et al. 2009). However, what exactly it means to raise a possibility, or for there to be one has remained unclear. As before, in response to a claim that $\mathcal{M}\phi$, one might agree that, "Yes, there is the possibility that ϕ." or that "No, there is not." but this will not all by itself serve to make $\mathcal{M}\phi$ any less pointless. Surely, $\mathcal{M}\phi$ can be taken to effectuate something like the presence or actuality of the possibility that ϕ in the common ground. The question then, however, becomes what these actually present possibilities are? One may also ask what is the difference between a state of information with the possibility that ϕ and the same state without that possibility. So far the only answer I have seen is that the first does not, and the second does support that $\mathcal{M}\phi$. What does it mean that ϕ is a possibility, other than that it makes *Might* ϕ true?

Nevertheless, hardly anybody would deny that such possibility statements serve a non-trivial purpose. For instance, because they have substance. Interestingly, Frank Veltman has himself presented kind of an answer to this question in his earlier work (Veltman 1984), cf., also, (Landman 1986). In that system a statement *Might* ϕ says, relative to a particular information state, that there is an extension of that state, in a given information space, in which ϕ is true. While it is true that this type of data-semantic treatment of *Might* failed to say much about the information spaces in which the epistemic modalities are to be evaluated, the basic idea can be fleshed out better nowadays. The relevant extensions of information states can be taken to be possible updates of them, as in Veltman's own update semantics in (Veltman 1996). And we could say that *Might* ϕ really says that there is a possible, reasonable, update of the present information state, in which ϕ holds. More specifically, *Might* is used to talk about a possible future state of the discourse, but it is not taken to be any theoretical possibility, but one that is likely to emerge from the current situation given the participant's information and prevailing questions. For a full treatment of *Might* along these lines, this requires us to explicitly relate to the current discourse situation, and indexically relate epistemic *Might*-statements to the current state of the discourse. In addition, we need access to the participants of the situation, and the (also indexical) beliefs and desires they have about it, and about each others beliefs. A full treatment of these matters goes well beyond the scope of this monograph, but we refer to (Dekker 2010) for further details.

The basic ideas are sketched below. Formally, $\mathcal{M}\phi$ states that ϕ holds in a possible resolution of the current discourse.

Definition 6 [*Epistemic Possibility*]

- $M, w, g, \tau\widehat{\epsilon} \models \mathcal{M}\phi$ iff $M, w, g, \tau'\widehat{\gamma\epsilon} \models \phi$ for a possible resolution τ' of τ.

Much, if not everything, in this 'definition' hinges upon the question what is a possible resolution. In principle, I favour allowing any reasonable update, which even includes possible revision of information and of questions pertaining in the given information state τ. In practice, however, we assume that not everything gets allowed as a possible update or resolution. As a short discussion of examples below demonstrates a lot hinges upon the beliefs and desires of participants in very specific discourse situations.

Before we turn to these examples, let us first adduce some general observations about epistemic *Might* as 'defined' above. The present definition directly accounts for a number of typical features of the use of *Might*.

Observation 28 (**Reasonable Updates**)

- Normally, $\mathcal{M}\phi$ doesn't make sense when ϕ has already been decided.
- Normally, $\mathcal{M}\phi$ doesn't make sense when the question whether ϕ is already an issue.
- Normally, $\mathcal{M}\phi$ is not persistent.
- Abnormally, $\neg\mathcal{M}\phi$ is not persistent.

In the first place $\mathcal{M}\phi$ doesn't make sense in situations where ϕ is an issue already, or where the issue whether ϕ has been resolved. For the use of $\mathcal{M}$ would be superfluous, hence excludable, on Gricean lines of reasoning. In the second place it is fully indexical. The truth of $\mathcal{M}\phi$ totally depends on the situation in the discourse where it is used, and on the information available there. For this reason, it is also non-persistent. Once new relevant information enters the common ground, the possibility that ϕ, once acknowledged, may eventually have to be given up. Thus, $\mathcal{M}\phi$ can be true at some point in the discourse situation and not true later, for instance, if one learns that $\neg\phi$ in the meantime. Also, what is less common, $\mathcal{M}\phi$ can be *false* at some discourse situation, and true later, for instance, if one revises the assumption that $\neg\phi$ in the meantime. Surely, this is a rather deviant use of $\mathcal{M}\phi$.

Apart from the last (fourth) observation, the present notion of $\mathcal{M}$ behaves like Veltman's $\mathcal{M}_V$, except for the fact that the behaviour of $\mathcal{M}_V$ is logically established, whereas the corresponding properties of the present notion of $\mathcal{M}$ are much more context-dependent, and defeasible. And rightly so. The fact that the discourse can develop into a state in which ϕ holds can be a warning, or a revelation, or a call for inquisitive action is certainly a contingent, and non-trivial thing to observe. Although our understanding of epistemic modality is rather different from Veltman's consistency *Might*, logically speaking, pragmatically it makes similar predictions as the following two observations show.

Observation 29 (**Discourse Might and Veltman's Might**)

- Normally, $\mathcal{M}\phi$ entails $\mathcal{M}_V\phi$.

The reason is that, normally, a possible outcome or resolution of the discourse is one which we haven't yet excluded. As a converse, we also find that the above entailment to go "practically" in the reverse direction.

Observation 30 (**Veltman's Might and Discourse Might**)

- Practically, $\mathcal{M}_V\phi$ entails $\mathcal{M}\phi$.

We have to say 'practically', because it is not really a fact about the utterance situation which renders $\mathcal{M}\phi$ satisfied when $\mathcal{M}_V\phi$ is, but the very fact that $\mathcal{M}_V\phi$ gets uttered. For any possibility that is raised and not immediately excluded is, for the moment at least, a "live" possibility. Once I say that it is not excluded that there are cockroaches in your coffee, you will, if only for a split second, have to think of that possibility. In this sense, $\mathcal{M}$ behaves almost like a self-verifying operator, which is easily abused by skeptical philosophers.

The above two observations show how close $\mathcal{M}$ and $\mathcal{M}_V$ actually are in their practical behavior, and that, generally, and practically, all of the previous observations about $\mathcal{M}_V$ can be applied to $\mathcal{M}$. With one general difference, viz., that $\mathcal{M}$ allows for exceptions in all cases. You can justifiably say $\mathcal{M}\phi$ and I can prove you wrong simply by disagreeing. So you may say "This might be it" and I may agree that neither you nor I have excluded that this be it, and yet formally disagree and say "No, but this is surely not going to be it." Surely this is not an instance of cooperative argumentative behavior, but in acting so I prove myself right and make you wrong.

So far we have listed some general properties of the present notion of *Might*, for as far as something general can be said about it. It still has two general, and interesting properties. The *Might*-operator can be used to bring possibilities to attention, like I said, in a current discourse situation, and thus steer or help further investigative actions. Consider the following little dialogue.

(89) A: Will Bernd be at the reception?
(90) B: He might have finished grading.
(91) A: So, what?
(92) B: If he has, he will definitely be there.

Upon this way of proceeding, the interlocutors have an incentive to go and find out whether Bernd has indeed finished grading, that is, a new question has emerged from the possibility statement. Similarly, if I wonder whether or not to go to the reception, and ask who will be there, the assertion that Bernd might be there would elicit a possibility that would directly decide my original question: if Bernd goes I wouldn't hesitate to go as well. Again the modal statement incites us to investigate or query whether Bernd indeed will go. Finally, if we are looking for the bicycle keys, with the major issue being where the keys are, we are possibly facing a whole lot of questions, viz., for any possible location l the question whether the keys are at l. The statement that they might be in the basement would turn the main question into a more tractable one, viz., whether they are in the basement, and we may find reason to try and find evidence for that possibility, among the interlocutors, by consulting an oracle, or, what may amount to the same thing, go down to the basement and look for the keys.

In each of the above cases, of course, no guarantee is given that the stated possibility will turn out true, or supported, and, hence, may help answer our question. Even so, each of the modal statements incite a specific investigative action, which may lead us to do at least something to achieve the required goal. By pointing at a possible resolution of the current discourse situation, one in which ϕ holds, this automatically raises the question whether we can reach that state. This, naturally, provides the incentive to go and find out.

Precisely for the reason that an utterance of:

(93) The keys of this room might be in the basement.

may provide the incentive to inquire about that possibility, or indeed go down and look for the keys, whereas an utterance of:

(94) The keys of this room might not be in the basement.

seems pointless. If we have no clue where the keys are, it is possible that they are not in the basement, but what point would a use of (94) make? Search all of the basement so as to conclude they are indeed not there? Rather not. But instead, (94) can be used, after all, to guide our inquiries. For if, indeed we have been looking for the keys, without success, and on the silent assumption that they are in the basement, indeed (94) may open our eyes to other possibilities, and provide a fresh start to the whole issue. This example may, again, show how sensitive our judgments of epistemic modality statements are.

The same goes for the case of defusing-the-bomb. I don't know why, but this always involves cutting one of two wires, one red and one blue, while cutting one of them will make the bomb explode, and cutting the other will defuse it. Now you may say to me "It may be the red wire which you have to cut." This is a helpful thing to say, to help to focus my actions, and $\mathcal{M}$, unlike $\mathcal{M}_V$ neatly accounts for this. However, if you subsequently add "It may also be the blue wire which you have to cut." the use of $\mathcal{M}$ eventually turn out as unhelpful as that of $\mathcal{M}_V$.

As defined, a statement of epistemic possibility has truth-conditions, even though its truth is very much context-dependent, unstable, and, hence, quite negotiable. Nevertheless, with this little bit of truth-conditions $\mathcal{M}\phi$ may non-trivially figure in attitude reports and questions. As a matter of fact, as (Gillies and von Fintel 2008; Brumwell 2009; Roussarie 2009) have observed, the following sentences do not just report or question (in)consistencies, but true worries, beliefs and questions:

(95) Timothy wonders whether he might go to the reception.
(96) Sybille believes that he might stay home.

(97) Might Timothy go somewhere else?

The present notion of discourse *Might* can neatly account for these cases. Before showing this I show why the other analyses of *Might* don't work. First observe that the interpretation of *Might* as just a consistency test appears to be quite inappropriate. When Tim is wondering whether he might go to reception, he is not just reflecting on his information. He is not inspecting his knowledge, with the question, "Well, is my information state consistent with this possibility?" Also, saying that Sybille believes that Tim might stay home does not just require that her information state be consistent with that possibility. The fact that her information does not exclude such a possibility is not sufficient for such an attribution to be true. (For, otherwise she could be attributed all kinds of epistemic possibilities about the whereabouts of my cousins whom she has never heard of.) Also, a question with *might* in it, as in (97) would really be no question. Assuming the common ground is public, we are all supposed to know whether it does or does not exclude the possibility that Tim goes somewhere else. Neither does it seem to ask for our beliefs about the common ground. (Like, "We are having a common ground together, but we don't know what it is.")

On the account presented in the previous section these statements gain full weight. Example (95) can be taken to state that Timothy indeed wonders whether there is a reasonably possible update of his current state into one in which he comes—or if there is no such update. This does not require deciding yet, it is more like deciding if it is still conceivable to possibly decide positive. (Of course, if the outcome is negative, he would consistently decide he will not go, we hope.) Likewise, example (96) can be taken to state that Sybille believes that there is a reasonably possible update of her state to one in which Timothy stays home. And finally, example (97) may be taken as a genuine question whether there is a reasonably possible update of the common ground in which ϕ holds. These observations fall right in place, as it should, in a straightforward combination of appropriate and independently motivated semantic treatments of questions, beliefs, and wonders on the one hand, and epistemic modalities on the other.

Like I said, much more needs to be done to formally elaborate the above proposals. As above, we need to take into account indexical beliefs about the actual discourse situation, the way the interlocutors think their dialogue may or may not develop, and so on.

The present proposal seeks to understand the discourse contributions as more or less reasonable attempts of agents to engage in the larger project of achieving an optimal inquisitive discourse. Only by assuming that the wider goal of communicating agents is that of effective and reliable communication of situated agents can we understand what the individual contributions mean or try to mean. In such a setting it is reasonable to raise questions and provide data which have been unsolicited, and, typically, to raise possibilities to attention like we do with epistemic modality statements. A global perspective on discourse, and I think this is the one Grice originally must have had in mind, seems to automatically make sense of these contributions.

4.4 On Situations and States

The proposals and results from this chapter and the previous ones are based on classical semantical intuitions, and have not forced us to adopt a representational notion of meaning, and not a dynamic one either. An, arguably, pragmatic notion of a dynamic composition of meanings has been advocated to deal with many kinds of anaphoric dependencies, a type of dependencies which we think is typical for the decomposition and reconstruction of meaning needed in any communicative endeavour. In this section I want to look at some rival approaches to pronouns advocated in the literature, which crucially depend on some notion of a 'situation' or an 'information state'. From an abstract point of view, the paradigms motivating situations and states can be easily aligned. For if one thinks of situations as parts of real worlds, they can be called 'partial worlds' and can be equated with the sets of their total extensions, i.e. information states, just like they have been above. Conceptually, and also more practically, whether one takes situations or information states as a starting point for analysis makes a difference.

From the outset I must say that there is no reason to exclude situations, or events, nor information states, from the ontology relevant to the semantics of natural language, on the contrary. Even so, I argue, that they don't show any special benefit when it comes to the data under discussion. In particular, I will argue that situation-based approaches only yield satisfactory results on the basis of unmotivated, ad hoc, and sometimes dubious, stipulations; and that information states, if not explicitly called for by modal operators designed for that task, are dispensable. I will first critically review the E-type (or D-type) tradition, from which the situation-based approach has emerged. I focus on the defense given in (Elbourne 2005). I next discuss the use of dynamic information states as proper objects as in (Geurts 1999; Frank 1997) and more recent approaches involving regular information states by Adrian Brasoveanu and Maria Bittner. I argue that the latter treatment fits neatly into the picture developed in this monograph, without this requiring us to frame the analysis in terms of (updates of) information states.

4.4.1 E- and D-type Pronouns

The basic idea from the E-type pronoun tradition is roughly this. Pronouns really are definite descriptions referring to individuals. In the way in which (Evans 1977,

1980) presents this analysis, it is virtually the same as the one advocated in this monograph, because the E-type referents can be taken as the witnesses in *PLA* , and the only difference then is that the E-type referents are to be determined by a description to be retrieved from the context, and *PLA* witnesses are simply assumed to be given, correctly or incorrectly, by previous discourse. The difference is not really substantial, and I will not expand on it. Most elaborations of Evans' proposal, however, make further assumptions. For instance, Irene Heim and Paul Elbourne think of the pronouns as disguised definite descriptions themselves, so that in a given context a pronoun 'it' has to be interpreted, really, as "the *A* who *B*", where *A* and *B* constitute syntactic material reconstructed from the linguistic context. (Heim 1990, p. 170) (Elbourne 2005, p. 2) An interpretation of pronouns along these lines is called a "D-type interpretation" in (Elbourne 2005). This analysis is appealing to the extent that it seems that pronouns can indeed be replaced by definite descriptions, without obvious changes in the interpretation of the sentences in which they occur, and that, hence, the task of interpreting pronouns is replaced by the more tractable task of interpreting definite descriptions. An impressive body of literature on that issue exists.

For the moment putting aside the question what are the kinds of definite descriptions, in syntactic or logical disguise, that pronouns have to stand in for, a D-type analysis of pronouns must somehow guarantee that the definite descriptions which ought to replace, or be used to interpret, pronouns, have a definite denotation whenever the corresponding pronouns can be used unproblematically. In an attempt to secure this phenomenon, (Heim 1990; Elbourne 2005), building on (Berman 1987; Kratzer 1989), call upon situations as the primary parameter of interpretation. The idea can be most easily (and maybe only) explained with the help of an illustrative example. Consider the paradigm donkey-sentence:

(98) If a farmer owns a donkey, he beats it.

Disregarding all kinds of subtle details, the sentence is held true in a situation s, if any minimal sub-situation s' of s, in which there is a farmer who owns a donkey, can be extended to an other sub-situation s'' of s, in which *the farmer in s'* beats *the donkey in s'*. Assuming that there is such a minimal sub-situation with just a farmer and a donkey owned by that farmer, for any farmer owning any donkey in s, this implies that in an actual situation s, every farmer beats every donkey he owns. I believe this suffices to illustrate the essentials of the D-type theory.

Even before reflection, it may be clear that the odds are quite against such a D-type analysis. (A critical point I would not like to emphasize is that a so-called analysis of anaphoric pronouns as definite descriptions, carries the danger of being very appropriate, but circular, if definite descriptions are conceived of as being anaphoric themselves. (Elbourne 2005, pp. 60–62) seems to be content with this i.e., though.) In the first place, substantial assumptions about the underlying structure of language are claimed to be based on all kinds of linguistic material which is invisible at the surface of the linguistic expressions used in natural language. While those working in the Chomsky paradigms may have no problems with this kind of reasoning, it must be noted that if it is not a dubious assumption, it is at least debatable, and certainly not 'proven'. Also, this move involves a syntactification of what appears to be a real pragmatic process, that of anaphora resolution. Gareth Evans has provided good motivation for this, because it allows us to speak of the (context independent) meaning of sentences containing pronominal elements. However, if there is indeed something to the contextualist arguments as presented in Sect. 3.4, then such a move goes in the wrong direction. In the third place, for the analysis to be at least remotely adequate, it must rely on a situation-based ontology. Notice that this is not an innocuous move. Those, who, like me, find a notion of 'possible worlds' a useful tool in semantic analysis, motivate this notion as only a reformulation of the idea that there are possibilities, "different ways things might have been." This has already been argued for in (Lewis 1979; Stalnaker 1976). In contrast, situations are not likewise grounded in common sense opinion. Generalizing over events, states and situations, the computational philosopher L.R.E. FraCaS has put it thus: "(…) events and states are among the most problematic ontological categories, the identity criteria of which are difficult to apply not just in a few marginal cases, thought up by ill-meaning philosophers bent upon showing their ultimate fragility, but in perfectly ordinary, run-of-the-mill cases as well." (FraCaS 1994, p. 65) Choose any question you may wish to focus on, and consider the two questions: "How many possibilities are there?" and "How many situations are we talking about?" One may fail to know the answer to the first question, but agree that there is an answer to it. The second seems to be very dubious and quite unanswerable indeed.

In (Dekker 2004), originally from 1996, I have pointed out the problem that proposals for situation-based treatments of anaphora rely on assumptions about situations which are systematically not made explicit. This is not very appealing since it may hide inconsistencies, spotted in some approaches, or it may unintentionally make the intended intensional framework collapse into an extensional one, like, e.g., in that of Kai von Fintel. It may also show the approach to be vacuous, as it appears to be in (Elbourne 2005). The main idea of the situation-based approach to anaphora, one never really independently argued for, is that for every farmer and every donkey he owns, there is a minimal situation including only that farmer (MacDonald, say) and that donkey (say, Jesabel) and the two standing in the owning-relationship. And this should hold, maybe, for all sequences of n-types of individuals, their being of a certain type, and the corresponding tuple standing in any sort of relation. Or not. The closest Elbourne comes to such a requirement is that "A minimal situation such that p is the situation that contains the smallest number of particulars, properties, and relations that will make p true (intuitively speaking)." This is not very helpful. 'Intuitively speaking,' The situation in which I don't have a donkey is different from a situation in which I don't have any money, or one in which I don't have a mind. In Elbourne these are all the same situation, however, all situations just containing me, a thin particular, and nothing else.

Let me be clear that I have no desire whatsoever to argue against the incorporation of situations (or events, or states) in the regular domains of discourse, because obviously we do quantify over them. The main point of my paper (Dekker 2004) is that, in order to cure mistaken assumptions about anaphoric pronouns, situations provide no help. A lot of ink has been spilled on the notorious subject of "undistinguishable participants," and Elbourne does an excellent job to manoevre his account out of that problem by means of further ad hoc stipulations. But this is all waste of energy, because the indistinguishable participants in a situation do not cause the problem, but the whole assumption that situations, whatever they are, help in providing the ultimate analysis and fine-grainedness brought into force to rescue the on the face of it absurd idea that an innocent invisible pronoun stands in, syntactically, for a description which uniquely identifies an object. If a framework of situations is employed to guarantee that definite descriptions have unproblematic denotations, just in case the pronouns which they are supposed to represent are unproblematic, then the framework must satisfy constraints of 'minimality' and 'distinctness,' as they are called in (Dekker 2004). These constraints are easily motivated, if they are to serve the interpretation of definite descriptions standing in for pronouns, but they are not intuitively motivated in the situation-based philosophy.

The constraint of minimality is implicit in all situation-based approaches to anaphora. The idea is that, in order to be able to interpret a donkey sentence like "If a farmer owns a donkey, he beats it," we must have access, for any farmer and any donkey owned, to a minimal situation in which there is only that farmer and only that donkey present. But the same must hold, then, for corresponding sentences like "If a charmer lends a room-key, he reads it," "If a beamer shows a monkey, he highlights it." It seems, in view of our ability to construct donkey sentences, all 'literals' of natural language have to have their own minimal, and unique, satisfying situations.

The constraint of distinctness, intuitively says, that all distinct literals have to have distinct satisfying situations. The minimal situation in which farmer MacDonald owns donkey Jesabel is thought to be distinct from the minimal situation in which farmer MacGregor feeds goat Mirabel, and also distinct from the minimal situation in which Freddie Mercury hoovers the sofa, etc. All of this may sound intuitive, but as a matter of fact it is not the kind of constraint one should wish to hold in general.

Regarding distinctness, (Kratzer 1989) must definitely exclude it as a general constraint, because it precludes the main subject of her paper, that of 'lumping' propositions. If somebody painted an apple as part of her painting a still life, then the proposition that she painted a still life is said to lump the proposition that she painted an apple. The apple painting situation is part of, not distinct from, the situation of painting a still life. Thinking of it, all logical entailments, and also all practical implications, require some sort of non-distinctness which the situation-based approach to the interpretation of anaphoric pronouns demands. Consider, for instance, the following example from (Berman 1987):

(99) Usually, if a letter arrives for me, I am at home.

Imagine that the reporter of this sentence was always at home when a letter arrived for him, say fifty times, except this one time when sixty letters were delivered in one package. If all the other times the letters arrived one at a time, and if the adverb *usually* requires us to quantify over particular letter arrivals, then example (99) would turn to be false. However, most people are inclined to judge the sentence true, because it is taken to quantify over moments at which, or situations in which, letters are delivered. The idea is that the situation in which letter *a* arrives is not different from the situation in which letter *b* arrives if the arrivals are part of one delivery. Observe that this possibility is excluded by distinctness. Similar examples can be made at will.

(100) Usually, if a mafioso enters my restaurant, I offer everybody a drink on the house (before I call the police).

Also in this example it can be argued that what counts are enterings of one or more mafiosi (they usually come in fours), and visits by the police, *not* individual mafiosi enterings.

The minimality constraint, as a general constraint on situations, is also obviously at odds with situations which are states or activities or processes. For, up to some level of fine-grainedness, states and processes have the 'subinterval' (or 'substate', or 'subevent') property. If s is situation (state) in which Micky sleeps and s' is part of s then s' is situation (state) in which Micky sleeps. And if e is an event of Rob running, then, up to some somehow to be specified level of relevance, if e' is part of e, then e' is an event of running of Rob too. So much seems to be established wisdom in the field. Since states and processes hardly can be taken to divide up into natural minimal units, they conflict with the minimality constraint. Yet, we do find adverbial quantification over states and processes (or processes):

(101) When John is asleep, Mary is usually awake.
(102) When Robert is working he usually sings.

Even if there are minimal bits of work in Robert's way of working, and such that every proper part of them is not any longer to be considered a real 'Robert-work', still nobody will probably be inclined to hold that these are the kinds of things one quantifies over with example (102). Similarly, it doesn't make any sense to understand example (101) as quantifying over minimal smallest states of John's sleep.

The upshot of this discussion on minimality and distinctness is not so much that we cannot conceive of all required minimal and distinct situations, which, if pressed, I think we can, but that all the kinds of situation structures have to be assumed to be given on the situation-based approach, before they can render our quantified statements true or false. Here is an example in the spirit of (Kratzer 1989).

(103) If Angelika paints something, it is always a still life. She usually starts with painting an apple.

According to the situation-based doctrine of interpreting anaphora, if there are minimal situations of Angelika painting something, it is usually her painting just apples, which are still lifes. This suggest Angelika's painting are all still lifes consisting of just one apple. There is something crucially wrong about this. I will not offer a full and better interpretation, but it seems that the example intuitively forces us to interpret the first sentence of (103) as quantifying over more or less complete painting situations, and the second as expanding on these, stating that there is something like initial subsituations of Angelika painting a still life that are situations of her painting an apple. There is definitely some pragmatic reasoning involved in getting the domains of quantification right, and they definitely involve situations, or events or what have you. But surely this domain does not have to be given before we can even start to interpret the first sentence of example (103). Notice, though, that the situation-based approach to the interpretation of pronouns essentially requires exactly this: our quantified statements themselves come with and impose a structure on a rather unstructured domain. If this is correct then it cannot be a given situational structure which enables us to make sense of anaphoric pronouns in these constructions in the first place. It is all, as the contextualists would jubilate, emphatically pragmatic.

In order to conclude this critical examination of the situation-based approach to anaphora, let me quickly mention the three additional merits of the situation-based approach according to (Elbourne 2005). They are that such an approach can account for Bach-Peters sentences, modal subordination, and pay-check sentences, pp. 79–83. For one thing, these phenomena are handled well in the classical approach developed in this monograph. For another, the situation-based treatment of modal subordination is tellingly incomplete, if not entirely vacuous again. “We can leave aside, given our purposes, the problem of how *the fish* in the second sentence of (110) [“John wants to catch a fish. He hopes I will grill the fish for him.”] comes to talk about the putative fish that John may or may not catch.” (p. 81) This is revealing, for Elbourne here suggests that the pronoun in the target sentence [(109) “John wants to catch a fish. He hopes I will grill it for him.”] really refers to some intensional/intentional object, precisely the object our approach predicts: the concept of a fish which John catches in the worlds in which his wants are satisfied. It is not at all clear how Elbourne would get at this interpretation using the detour across situations, which after all, are only worked out for extensional (i.e. real world) purposes, cf., p. 50.

4.4.2 Information States

I would like to re-emphasize, and I hope this is clear from the previous discussion, that I have no qualms against accepting situations, states, events, or stuff in our natural language ontology. The main, critical, conclusion of the previous discussion is that situations do not help to solve the dubious intuition that pronouns are disguised definite descriptions. Like I stated earlier, information states can be theoretically compared to situations, but they have also been used with great success in many accounts of anaphoric dependencies. The reason is also rather trivial: information states, unlike situations, can be used, and have systematically been used, to code discourse information, typically the kind of discourse information required to deal with anaphoric dependencies.

In the remainder of this paragraph I briefly indicate some of the main features of these empirically successful analyses, and argue that indeed these analyses can be reduced to the type of architecture defended in this monograph. As we will see, however, this does not essentially invoke information states, nor a dynamic notion of meaning which many have thought to be key to the use of information states in the first place.

Major empirical extensions of the discourse oriented and dynamically inspired theories of anaphoric relationships have been provided by Bart Geurts and Anette Frank in a discourse representational framework. (Geurts 1999; Frank 1997). Their two PhD theses deal with all kinds of anaphoric dependencies in modal and quantified contexts, employing highly sophisticated methods which crucially involve reference to discourse markers for information states, which are essentially discourse representation structures. In the spirit of their proposals dynamic semantic formulations have been given in the work of Matthew Stone, Daniel Hardt, and more in particular Maria Bittner and Adrian Brasoveanu. (Stone and Hardt 1999; Bittner 2001; Brasoveanu 2006) I will focus on these most recent contributions.

One of the major empirical challenges in the treatments of anaphora referred to in this monograph, has always been to generalize it, not only beyond the scope of simple singular identity anaphora, but to plurals, generalized quantifiers, other modal and quantificational constructions, and, cross-linguistically. The body of empirical data, and of potential analyses, is huge, and stupifying, but with hindsight, and gratefully building on the pioneering work of Geurts and Franks and Stone and, not yet mentioned, (van den Berg 1996; Nouwen 2003), some results can be established and organized under three labels. Compositionality, linguistic data, and the dynamics of interpretation.

I won't say much on the issue of compositionality here, since the proposals I freely draw from are those from Brasoveanu and Bittner, which have already been given a compositional formulation. Looking at things from an analytical and formal perspective, maybe compositionality requires no more than a capable logician, in a philosophical mood, like Richard Montague, Theo Janssen, and for our present purposes Reinhard Muskens (Muskens 1996).

The linguistic data issue is difficult to address. Any domain of research like that of plurals, quantifiers, modals, tenses, already comes with its own never definitely solved question of what are the primary objects, what is the genuinely linguistic treatment of them, and how should a theory of that domain be designed. Because this monograph is not directly concerned with any of these topics in particular, I will have nothing to say about them, even though, in the end, the linguistic data are going to be empirically relevant. Any adequate understanding of the few examples on situations brought up in the previous paragraph will have to take a stand on these types of issues.

Once we have come to have settled on the empirical issues, however, the issue of the dynamics of interpretation may be less problematic than all of its implementations may suggest it to be. There are two basic questions related to this issue: the proper set up of discourse referents, for plurals, generalized quantifiers, modal bases, and, e.g., antecedents of conditional sentences, and the way in which they have to be used. Disregarding the impressive complications in the formulation of the solutions to this issue in particular examples, the required techniques appear to be surprisingly simple. Already the seminal work of (van den Berg 1996), perspicuously elaborated in (Nouwen 2003), and certainly in the extensions of Brasoveanu and Bittner, demonstrates that involved syntactic structures get analyzed, in the end, in terms of long sequences of dynamic conjunctions, in each of which the right structural dependencies get appropriately established. In the terminology of this monograph, they involve no more than discourse reference and dynamic conjunction.

Let me go through two examples, which appropriately exemplify the approaches of Brasoveanu and Bittner. Besides the highly sophisticated formally required technical details, the crucial ingredients are those advocated in this monograph. Bittner has ingeniously invented analyses of plurals, tense and modalities, for a variety of languages, in a theoretical framework which involves the compositional update of information states like those represented in *DRT*. Here is a typical example involving a counterfactual conditional.

(104) Kennedy was assassinated because he had many enemies. If Oswald hadn't killed him someone else would have.

To discuss the specific details of Bittner's analysis of this example would take us too far, so let me employ her own summary. In the context of the first sentence of (104), the second is analyzed as involving 25 elementary updates (some of which are grouped into a topic-comment sequence), to the effect that it

> projects an expected consequence of the aforementioned event of JFK's enemies reaching critical mass (…). The modal base for this expectation is the aforementioned class of worlds where this causal event is realized (…). The topical hypothesis introduced by the hypothetical mood is the sub-domain (…) where JFK is not assassinated by Oswald. The main attitudinal comment is that within this remote topical sub-domain (…), the worlds that best fit the speaker's (…)-current expectations are those where in the wake of JFK's enemies reaching critical mass (…) some other bad guy assassinates him. (Bittner 2010, p. 39)

The interesting thing to observe about this analysis, and many others proposed in the paper by Bittner, is that it is framed, entirely, in terms of elementary updates, and comments upon previously established topics and backgrounds.

The analyses of structural and anaphoric dependencies from Adrian Brasoveanu are fleshed out in the very same decompositional spirit. The next two examples discussed in (Brasoveanu 2010) display typical structural (anaphoric) relationships, across quantified and modal constructions.

(105) Harvey courts a woman at every convention. She always comes to the banquet with him.
(106) A wolf might come in. It would eat Harvey first.

Again I will not spell out the formal analysis, which is formulated in an extension of the framework of Compositional *DRT*, from (Muskens 1996), adapted to deal with plurals and modalities (IPCDRT). The (dynamic) components, however are this. Example (105) is interpreted as involving six updates:

1. There is someone (u_1) who is Harvey;
2. there is a (maximal) set (u_2) of conventions he visits;
3. relative to the conventions u_2, there is a (maximal) set of conventions (u_3) where u_1 (Harvey) courts a woman (u_4);
4. the set of conventions Harvey visited (u_2) is a subset of those (u_3) where he courts a woman;

5. there is a (maximal) set (u_5) of conventions where the woman u_4 comes to the banquet with u_1 (Harvey);
6. the set of conventions where Harvey courts a woman (u_3) is a subset of those (u_5) where the woman comes to the banquet with him.

Example (106) is analyzed in an analogous fashion.

1. There is a (maximal) set of worlds p_1 where a wolf (u_2) comes in;
2. in the actual world, and relative to a modal base, p_1 is possible;
3. there is someone (u_1) who is Harvey;
4. there is a (maximal) set of worlds (p_2) where u_2 (the wolf) eats u_1 (Harvey);
5. in the actual world, and relative to the same modal base, p_1 necessitates p_2.

The analysis of both examples (105) and (106) involves a sequence of updates of information states with discourse referents and conditions on them. (And a maximalization operator, deriving from old-fashioned static semantics.) By a mere shift in terminology we may label these discourse referents 'possible witnesses', and dynamically conjoin the purported updates in the *PLA* fashion, so that no talk of updating information states is required any longer. The metalinguistic statement of the interpretation of (105) and (106) is essentially a donkey-sequence, which we have learned how to deal with in a classical fashion in Chap. 2 Without in any way aiming to discredit the work of Bittner and Brasoveanu, to whom we owe the intricate, appropriate and indeed elegant analyses of the empirical and cross-linguistic phenomena, the proper conclusion is that, after all, the dynamics resides where we have claimed it to be in this whole monograph. In a discourse there is stuff talked about, decomposed and reconstructed, and a form of dynamic conjunction may be all we need to account for that.

I suggested that in particular Brasoveanu's account of anaphoric dependencies, can be easily cashed out in the classical framework advocated in this monograph. A minor theoretical point of debate remains though, because Brasoveanu explicitly presents his framework as superior to that of Matthew Stone, a framework which, if conservatively translated in a static fashion, resembles mine very much. The point of debate is whether to adopt structured information states, like Brasoveanu does, which directly encode and model relational dependencies between various co-occurring pronominal expressions, or to adopt possibly functional witnesses, like we have done here, essentially along the lines of Stone. Even though there may be different (marginal and hardly testable) empirical implications of the two approaches, their formal equivalence cannot, I believe, be underestimated, however.

Brasoveanu argues in favour of (plural) information states of the kind advocated in (van den Berg 1996), in view of examples like the following:

(107) Every boy made a paper flower and gave it to a girl.
They thanked them for the very nice gifts.

Besides the fact that the truth-conditions of example (107) have to be satisfied, truth conditions that are derived from the update it is taken to provide, the first sentence is argued to establish a dependency, modeled in terms of at least a three place relation between boys, paper flowers, and girls, that at certain indices, or other relevant parameters, stand in the relation asserted of them. This is required, because the second sentence can be used to say that any of the girls thanked the boy who gave her a flower for the flower he gave—so that the relevant relationships are preserved. But surely, this three-place relation can be recovered from the set of boys and the functions associating boys with paper flowers and girls, to that they deliver, for each boys, the girl and the flower which stand in the giving relation—and vice versa.

There may be some empirical debate about whether or not we want the second sentence to talk of *the*, or *a*, or *every* present which a certain boy has given to *the*, or *a*, or *every* girl, but this is something I claim we shouldn't have any intuitions about. (See also the discussion in Sect. 4.1.) What worries Brasoveanu, is that terms of a definite type b, may have to be lifted to a functional type $\langle a, b\rangle$, and this kind of conditionalization may have to be iterated. However, the various interpretations of the different types of terms need not be encoded in the lexicon, or in the contextual information states—which we don't need for these purposes anyway. As in Jacobson's variable free semantics (Jacobson 1999), and like in all categorial frameworks, this kind of type change is easily available as a form of Geach division, and it's use is perfectly controlled by the linguistic environment. A term in the scope of n quantifiers and operators, can be systematically, and compositionally, be turned into a term functionally dependent upon the n-types of things quantified over or operated upon. This is all very straightforward and innocent, and, as a matter of fact, this is exactly what we have done in Sect. 4.1 in the first place. To conclude, there does not seem to be a real issue here, so that, as long as we have witnesses, and conditionalization, (plural) information states are not really required either.

The last paragraph of this chapter has been built on the assumption that information state approaches to the interpretation of anaphora like that of Geurts, Frank, Stone, Brasoveanu, and Bittner, are empirically very adequate, or at least extremely promising. The point, however has been, in line with the main moral of this monograph, that something that may intuitively come under the label of 'information states' is not really called for. If you want, we need discourse referents, whom we prefer to call possible witnesses, and they come in many kinds, and they are of many sorts, but that is not what is key to a dynamic semantics. What we need, really, is some dynamic notion of conjunction, or of a composition of meanings, and then the relevant data of the formal semantic literature can be accounted for.

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Chapter 5
Conclusion

This small monograph has covered some extensive ground, or so I hope. Grasping all the definitions, and implications, in one go may be difficult, but I hope the moral got through to the reader. The intuitions about natural language meaning advocated here are quite pragmatic. The drive is theoretical, formal, and is intended to display, or model, structural properties of interpretation. The techniques are very classical. I believe nothing in this whole monograph hinges upon more than (1) a Tarskian satisfaction relation, extended with a pragmatic use of witnesses, (2) a very modestly implemented notion of the dynamic composition of meanings, and (3) a belief in possibilities. No further philosophical or technical assumptions have been made to cover the linguistic data addressed, or so I think. The reader is seriously invited to nail me down on more assumptions.

The main results of this monograph can, thus, be seen as negative. In order to account for a whole lot of phenomena over which much ink has been spilled the last 25 years or so, we don't need discourse representation structures, nor do we need to think of meanings as context change potentials, nor do we need a situation-based semantics—all tests are, or seem to be, negative. If I were your physician, you would be happy with such a result: no diseases, no cure, no pay. The situation in semantics is still a little bit different, though. I would be the last to claim that the so-called 'diseases' are evil. I believe it is, indeed, illuminating to work with discourse representation structures, and I believe that context change potentials do a wholesome job, and I am confident that situations of various kinds will flourish in any reasonably rich semantics, and I do think that a dynamic conception of meanings may be called for. However, dynamic semantics, can survive, and be fruitfully practiced, without any of these commitments.

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