Chapter 7
Periodic Oscillations on Angular Velocity with Maximum Brake Torque ABS Operation

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Abstract The appearance of oscillatory processes is inherent to the antilock braking system (ABS) operation, that can represent a problem on performance and comfort, that's why the oscillatory behavior represents an important study area, since in can lead to significant advances in ABS performance. In this paper we show that the ABS operation while the longitudinal contact force applied in a pneumatic system is near to the maximum value produces an oscillatory effect on the angular velocity of the vehicle′s wheel, and that for the time intervals that the system operates the oscillation can be considered periodic.

Keywords Antilock brake system · Contact force · Mathematical model · Periodic oscillation · Pneumatic brake system · Slip rate

7.1 Introduction

Security in modern automotive systems represents important criteria for design, for that reason, research in security systems has been increased in the last years, one of the concerns is brake systems, and more ABS, one of the problems to solve with ABS is the appearance of high frequency vibrations in the angular velocity of the

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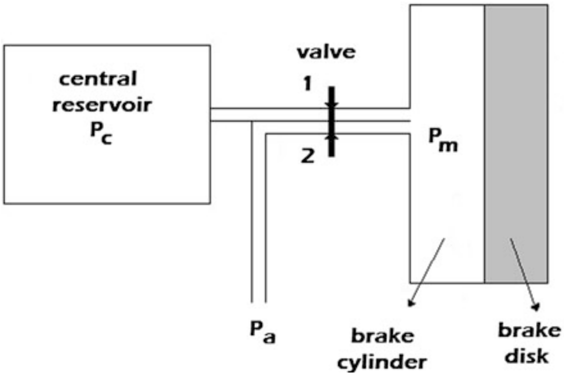

Fig. 7.1 Pneumatic brake model

wheel's rotation, which has been studied by Clover [1], Jansen [2], Kruchinin [3, 4], and Gozdek [5] among others. Modeling and research of forced oscillations in deformable wheel as a result of ABS activity has been discussed by Clover [1], and Jansen [2], while Kruchinin [3, 4] has analyzed the processes of appearance of vibrations during the pressure's relief phase in the brake cylinder of the ABS are analyzed, as well as the algorithms to suppress such vibrations. Gozdek [5] studied the possibility of longitudinal vibrations in the chassis of an airplane during the active phase of ABS is discussed.

The modern ABS systems very often use sliding modes control [6–11] with switching of ABS valves. Simultaneously the nonlinear character of ABS dynamics can lead to specific periodic regimes of angular velocity change for this manner of control algorithms that make programmed switch of the valve with a given period and duty cycle. The condition of existence of periodic changes in the angular velocity of the wheel's rotation due to the presence of specific ABS regimes is discussed in this paper.

The model of a pneumatic brake system is under consideration. The specific configuration of this system includes the next: brake disks, which hold the wheels, as a result of the increment of the air pressure in the brake cylinder (Fig. 7.1). The entrance of the air trough the pipes from the central reservoir and the expulsion from the brake cylinder to the atmosphere is regulated by a common valve. This valve allows only one pipe to be open, when 1 is open 2 is closed and vice versa. The time response of the valve is considered small, compared with the time constant of the pneumatic systems.

We study the case of wheel's rotation control, such that the longitudinal force, due to the contact of the wheel with the road, is near from the maximum value in the period of time valid for the model. This effect is reached as a result of the ABS valve's throttling.

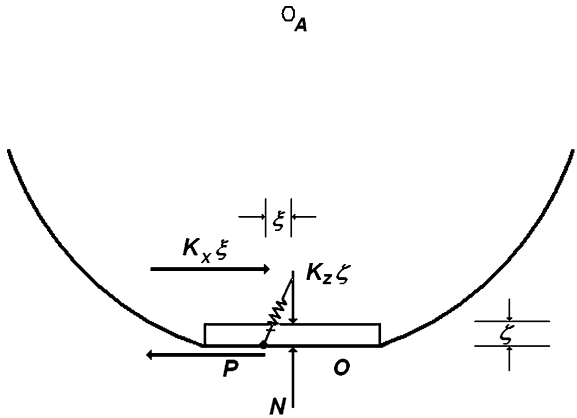

Fig. 7.2 Model for the contact element of the wheel

7.2 Mathematical Model

7.2.1 Wheel Motion Equations

To describe the wheel's motion we use a partial mathematical model of the dynamic system [3, 12]. Let's write the equation of the angular momentum change relative to the rotation axis (Fig. 7.2).

$$I_y \frac{d\Omega_y}{dT} = FR + L \tag{7.1}$$

where I_y—wheel's inertia moment, Ω_y—wheel's angular velocity, R—wheel's radius, F—contact force, L—brake torque.

The expression for longitudinal component of the contact force in the motion's plane according to experimental results [13] is equals

$$F = -\nu N\varphi(s) \tag{7.2}$$

ν is the friction coefficient between the wheel and the road, N—normal reaction.

$$s = \frac{V_x + \Omega_y R + \frac{d}{dT}\breve{\xi}}{V_x} \tag{7.3}$$

s—slip rate, V_x—longitudinal velocity of the wheel mass center, $\breve{\xi}$—longitudinal deformation of the tire's contact area element. The function $\varphi(s)$ is defined experimentally, and it looks like Fig. 7.3.

The motion equation of the contact element with mass M_c is described by the tire longitudinal deformation. The interaction between this element and the rigid part of the wheel can be described with a viscoelastic forces model. The movement equation for the contact element is the next

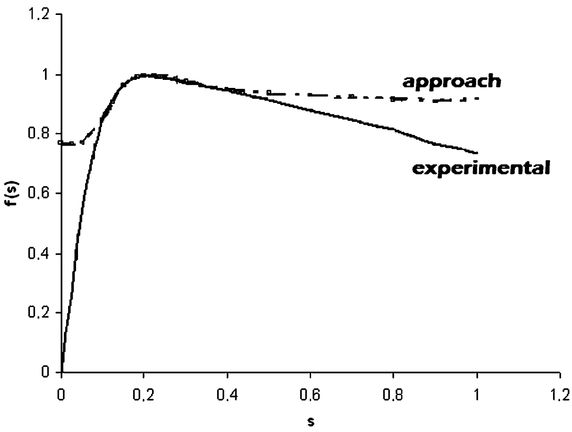

Fig. 7.3 Characteristic function for slip rate

$$M_c \frac{d}{dT}\left(V_x + \Omega_y R + \frac{d}{dT}\breve{\xi}\right) = F - C_x \frac{d}{dT}\breve{\xi} - K_x \breve{\xi} \tag{7.4}$$

Here C_x and K_x are longitudinal constants of viscous and elastic behavior of tire's model. The model to be used is the similar to description of first waveform in model [2].

The Eqs. (7.1)–(7.4) characterize wheel motion. This system is closed if we assume longitudinal velocity V_x and normal reaction N as constants. This approximation is correct for time lag about seconds if longitudinal velocity and normal reaction changes slowly and their variations are small [14].

Model proposed was previously used to describe the wheel's vibration for small values of slip ratio $s < 0.1$ when dependence $\varphi(s)$ is approximately linear $\varphi(s) = K_0 s$ [3]. Under these conditions, it is possible to consider that natural period of contact element vibrations in (7.4) is much smaller than the characteristic time of change of angular velocity and break torque. The fractional analysis method [14] can be used to reduce Eq. (7.4) to terminal form and write approximated relation $F = K_x \xi$. The wheel motion equations in this case is equivalent to pendulum equation [2, 3] with viscous friction. Natural frequency of this pendulum is

$$\omega_n = \sqrt{\frac{K_x R^2}{I_y} - \frac{K_x^2 V_x^2}{4 v^2 N^2 K_0^2}}$$

Such as been shown [3], this result is consistent with experimental effects detected in the process of ABS control algorithm tests.

Further we consider the behavior of the system around the maximum value of the brake torque, it means in the region of $\varphi(s)$ maximum. The Tikhonov's theorem [14] condition used for reduction in previous paragraph is correct too, but reduced equations has singularities for $\varphi'(s) = 0$. The analytic and numerically solution of this equations is difficult. Therefore it is necessary to study full system

(7.1)–(7.4) properties in order to analyze periodic oscillation of the angular velocity.

We use the next approximation for $\varphi(s)$

$$\varphi(s) = \frac{a_1 s^2 + a_2 s + a_3}{s^2 + a_4 s + a_5} \tag{7.5}$$

The parameters $a_1 \ldots a_5$ were calculated with the least squares method [15]. We use for calculation the values:

$a_1 = 0.8886$
$a_2 = -0.1776$
$a_3 = 0.0155$
$a_4 = -0.2226$
$a_5 = 0.0201$

These values approximates top neighborhood of tire characteristics, used by Mogamedov [10].

7.2.2 Pneumatic Brake System Equations

We suppose that the brake torque L is proportional to the pressure P_m in the brake cylinder.

$$L = K_L P_m \tag{7.6}$$

For the brake system we use an approximated model of pressure changes in the brake cylinder due to the opening of the valve with a first order relation [1, 16].

$$T_e \frac{dP_m}{dT} + P_m = P_* \tag{7.7}$$

Let's suppose opening and closing of valve is momentary and the parameters of the Eq. (7.7) are given by the next rules:

(a) $P_* = P_c = \text{const}$ $T_e = T_{\text{in}}$ when 1 is opened and 2 is closed
(b) $P_* = P_a = 0$ $T_e = T_{\text{out}}$ when 2 is opened and 1 is closed

Here P_c—pressure inside the central reservoir, P_a—atmospheric pressure, that we'll consider 0. T_{in} and T_{out}—time constants of internal and external pipelines.

7.3 Dimensionless Equations

We desire to rewrite Eqs. (7.1)–(7.3), (7.5) in a more useful form, by ignoring changes in V_x. Taking $\frac{d\Omega_y}{dT}$ from (7.1), and writing in (7.5) we have:

$$\begin{cases} I_y \frac{d\Omega_y}{dT} = vNR\varphi(s) - L \\ M_c \frac{d^2\breve{\xi}}{dT^2} + C_x \frac{d\breve{\xi}}{dT} + K_x\breve{\xi} = -\frac{M_c R}{I_y} L + \left(\frac{M_c R^2}{I_y} - 1\right) vN\varphi(s) \\ s = 1 + \Omega \frac{R}{V_x} + \frac{1}{V_x}\frac{d\breve{\xi}}{dT} \end{cases} \tag{7.8}$$

Equation (7.7) can be modified to following form:

$$T_e \frac{dL}{dT} - K_L P_* + L = 0 \tag{7.9}$$

To reduce the number of parameters we take the variables to a dimensionless form

$$l = \frac{L}{NR}, \ \omega = \frac{\Omega_y R}{V_x}, \ \xi = \frac{\breve{\xi}}{V_x T_1}, \ t = \frac{T}{T_1}$$

where

$$T_1 = \frac{I_y V_x}{NR^2}$$

is the characteristic time of the angular velocity changes, according to (7.1).

The system (7.1), (7.8), (7.9) has the next dimensionless form

$$\begin{cases} \frac{d\omega}{dt} = l - v\varphi(s) & s = 1 + \omega + \frac{d\xi}{dt} \\ \frac{d^2\xi}{dt^2} + q\frac{d\xi}{dt} + p\xi = -l - vk\varphi(s) & \\ \frac{T_e}{T_1}\frac{dl}{dt} = l_s - l & (a) l_s = l_c = \text{const} \quad T_e = T_{\text{in}} \\ & (b) l_s = 0 \quad T_e = T_{\text{out}} \end{cases} \tag{7.10}$$

where

$$q = \frac{C_x T_1}{M_c}, \quad p = \frac{K_x T_1^2}{M_c}, \quad k = \frac{I_y}{M_c R^2} - 1 \quad l_c = \frac{K_L P_c}{L_*}.$$

7.4 Periodic Solutions Finding

The main goal of this work is the study of periodic regimes produced by programmed switching of the valve with a given period and duty cycle [16].

To search for periodic regimes we analyze an auxiliary task: control with a relay feedback built such that the system switches the valve when the slip ratio s reaches the arbitrary limit values s_1 and s_2. We analyze the values s_1, s_2 for which the function $\varphi(s)$ changes around the maximum value (Fig. 7.3). In this region the contact force has a value less or equal than 10 % down the maximum value, for a constant normal reaction between the wheel and the road.

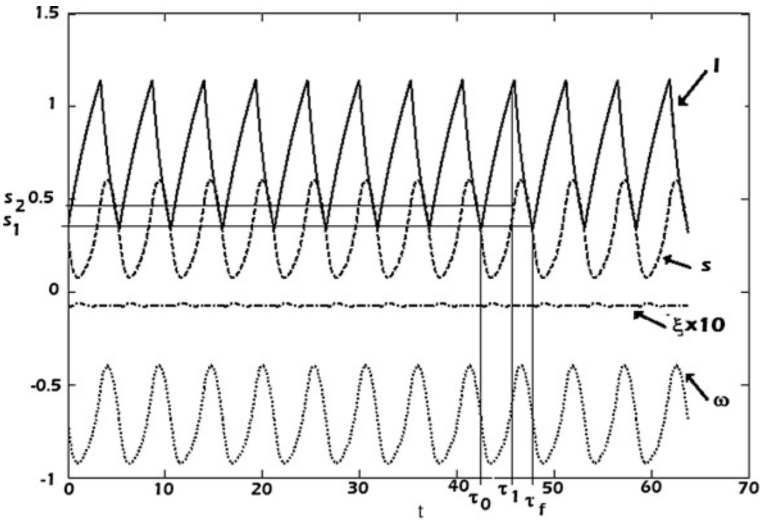

Fig. 7.4 Periodic solution

To find periodic solutions $[l_P, \xi_P, \omega_P]$ we integrate numerically the equation system (7.10) for initial conditions that can be present in real systems [3]. As a result of this integration we have solutions for which the values (a) work in the interval $\Delta_1 = \tau_1 - \tau_0$, and the values (b) in the interval $\Delta_{fr} = \tau_f - \tau_1$ (Fig. 7.4).

We consider that a periodic regime was found if the integration if the next criteria is true

$$\max\left(l_f - l_{P_0}, \xi_f - \xi_{P_0}, \frac{d\xi_f}{dt} - \frac{d\xi_{P_0}}{dt}, \omega_f - \omega_{P_0}\right) \leq 0.01$$

Here $\left(l_{P_0}, \xi_{P_0}, \omega_{P_0}\right)$ and $\left(l_f, \xi_f, \omega_f\right)$ are the variables in two successive periods at the moment of valve's opening. $\left(l_{P_0}, \xi_{P_0}, \omega_{P_0}\right)$— are the initial conditions of computed periodic solution.

All the possible values Δ_1, Δ_f and the corresponding initial conditions of the periodic solutions at the opening moment were obtained by solving the system for different pairs (s_1, s_2) inside the interval $\left(s_{1\,\min}, s_{2\,\max}\right)$. The region of founded values Δ_1, Δ_f for different friction coefficient value v can be seen in Fig. 7.5.

The parameters for calculations are

$T_{in} = 0.0043$ s
$T_{out} = 0.0085$ s
$p = 1000$
$q = 100$

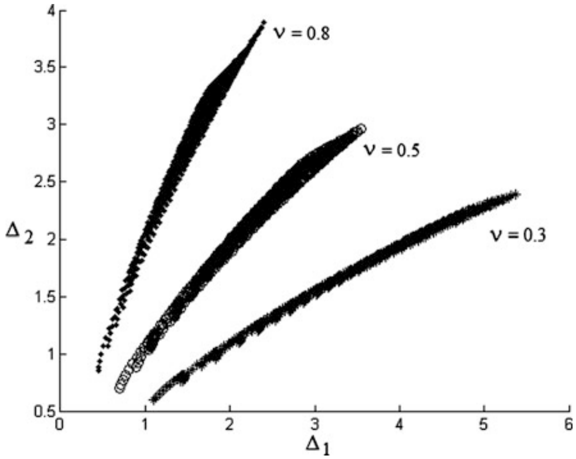

Fig. 7.5 Regions founded for different friction coefficients

$k = 10$
$l_s = 0.4755$
$T_1 = 0.0848$ s

7.5 Conclusion

ABS has become standard equipment in most of the modern vehicles since they can provide a good control response in direction during extreme braking situations. ABS operation is based on a switching process, oscillatory affects are produced, and the results can have consequences on performance, security and comfort of the vehicle, for that reason it is important to analyze the properties of such oscillations. The case of maximum longitudinal force before the wheel locks was considered because operation of ABS starts when this condition occurs. The simulation showed that the oscillations on the angular velocity of the wheel have a periodic behavior for some regions of the analysis, that information can be helpful to design control algorithms, either, to suppress vibrations or to take advantage of the periodic oscillation on the switching process, to increase performance.

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