Technology Integration in Secondary School Mathematics: The Development of Teachers' Professional Identities

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Abstract This chapter reports on research into the impact of digital technologies on Australian mathematics teachers' classroom practice. The aim of the study was to identify and analyse individual and contextual factors influencing secondary mathematics teachers' use of technology, and compare ways in which these factors come together to shape teachers' pedagogical identities. The first section of the chapter examines the teacher's role in terms of their pedagogical identities as users of technology, and introduces two theoretical frameworks for investigating trajectories of identity development. One framework classifies ways in which technology can change teaching and learning roles and mathematical practices. The other is concerned with teacher learning and development, and explains why teachers might embrace or resist technology-related change. The sections that follow provide case studies of two beginning teachers of secondary school mathematics who were integrating digital technologies into their classroom practice. Analysis of these case studies highlights issues related to identity development and demonstrates that identity trajectories are neither random nor fully determined, but instead are constrained by person-environment relationships.

Keywords Identity • Sociocultural perspectives • Technology metaphors • Valsiner • Zone theory • Zone of proximal development • Zone of free movement • Zone of promoted action

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Introduction

In the twenty-first century, young people live in a world where digital technologies have become essential for managing work and leisure activities. Communication, entertainment, manufacturing, transport, finance, medicine, weather forecasting and many other aspects of life now depend on sophisticated technological systems, much of which is invisible to the user (Confrey et al. 2010). Digital technologies are often personalised and seamlessly integrated into young people's daily lives through use of handheld devices such as mobile phones or tablets that offer a wide range of Web-enabled applications. Mathematics underpins many of these modern-day applications of technology, and yet, despite its ubiquitous presence in the world outside school, technology still plays only a marginal role in many mathematics classrooms (Artigue 2010).

A significant body of research has examined the effects of computer and calculator use on students' mathematical achievement and attitudes and their understanding of mathematical concepts (e.g., see Ellington 2003; Penglase and Arnold 1996 for reviews on calculator use). More recently, research has begun to examine the potential for learning mathematics within digital game environments such as Nintendo and Pokemon (Jorgensen and Lowrie 2011; Lowrie 2005). These studies suggest that engagement with the game, and especially in the repetition of moving back and forth through its different sites, may help players develop complex visualisation and problem solving skills. A second strand of inquiry is focusing on other kinds of technology-immersive environments created via digital mathematical performances. Gadanidis and Borba (2008) introduced this notion to highlight the social and multimodal affordances of new digital media. They noted that the Web offers a medium for sharing mathematical performances using texts, pictures and videos, and suggested that as a result the 'performers' – whether students or teachers – develop new mathematical understandings and new aesthetic appreciation of the power and beauty of mathematics.

In contrast to the longstanding research focus on how students learn mathematics with technology, less attention has been given to teachers' technology-mediated classroom practices and the role of the teacher in technology integration. Internationally there is research evidence that simply improving teachers' access to technology has not, in general, led to increased use of or movement towards more learner-centred teaching practices (Burrill et al. 2003; Cuban et al. 2001; Wallace 2004). Windschitl and Sahl (2002) identified two factors that appear to be crucial to the ways in which teachers adopt (or resist) digital technologies. First, teachers' use of technology is influenced by their beliefs about learners, about what counts as good teaching in their institutional culture, and about the role of technology in learning. Secondly, school structures, especially those related to the organisation of time and resources, often make it difficult for teachers to take up technology-related innovations. These are some of the issues that are considered in this chapter, which draws on the findings from a 3-year research study that sought to identify and analyse individual and contextual factors influencing secondary mathematics teachers' use of technology, and to compare the ways in which these factors come together to shape teachers' pedagogical identities.

The next section of the chapter theorises the teachers' changing roles in technology-enriched learning environments in terms of their pedagogical identities as users of technology, and introduces some theoretical frameworks for investigating the trajectories of their identity development. The sections that follow detail the case studies of two beginning teachers of secondary school mathematics who were integrating digital technologies into their classroom practice. The analysis of these case studies points to the factors that influence the development of their pedagogical identities.

Theorising About Technology-Enriched Mathematics Teaching

Two types of theoretical framework are needed to study implications for teachers of the impact of digital technologies on mathematics education. One type of framework represents ways in which technology can change classroom roles and mathematical practices. The other framework is concerned with teacher learning and identity development, and helps explain why teachers might embrace or resist technology-related change. The research study informing this chapter used both types of framework to investigate implications for technology-enriched mathematics teaching. This research drew on sociocultural theories of learning involving teachers and students in secondary school mathematics classrooms (see Goos 2009a, b). Sociocultural theories view learning as the product of interactions with other people and with material and representational tools offered by the learning environment. Because it acknowledges the complex, dynamic and contextualised nature of learning in social situations, a sociocultural perspective can offer rich insights into conditions affecting innovative use of technology in school mathematics.

Teaching and Learning Roles

In technology-enhanced learning environments, students experience mathematics in new ways that may challenge the traditional role of the teacher as transmitter of knowledge. Technology is not merely an add-on or supplement for pencil and paper work in such environments; instead, it becomes a "conceptual construction kit" that provides access to "new understandings of relations, processes, and purposes" (Olive et al. 2010, p. 138). If digital technologies have the potential to change mathematical knowledge and practices in the classroom, the role of the teacher also changes. The first framework for theorising technology-enhanced mathematics teaching, developed by Goos et al. (2000), classifies ways in which technology can change the teacher's role.

Goos et al. (2000) analysed the effects of digital technologies as cultural tools that both amplify and re-organise mathematical thinking. Mathematics learning is amplified when technology is used to speed up tedious calculations or to verify results obtained first by hand. However, a more profound cognitive re-organisation occurs when students' mathematical thinking is qualitatively transformed through interaction with technology as a new system for meaning-making. Goos et al. developed four metaphors to describe how digital technologies can act as tools that transform teaching and learning roles. Technology can be a *master* if students' and teachers' knowledge and competence are limited to a narrow range of operations. Students may become dependent on the technology if they are unable to evaluate the accuracy of the output it generates. Technology is a *servant* if used by students or teachers only as a fast, reliable replacement for pencil and paper calculations without changing the nature of classroom activities. Technology is a *partner* when it provides access to new kinds of tasks or new ways of approaching existing tasks to develop understanding, explore different perspectives, or mediate mathematical discussion. Technology becomes an *extension of self* when powerful and creative uses are seamlessly integrated into the teacher's mathematical and pedagogical repertoire to support and enhance a teaching program. Although this framework classifies more and less sophisticated uses of technology, it does not imply that only one type will be observed in a lesson or series of lessons (see Goos et al. 2000, for an example of a lesson in which all four technology roles were evident). However, the framework does provide a way of tracing changes in teachers' classroom roles as they appropriate digital technologies into their practice.

Teacher Learning and Development

The second theoretical framework is based on an adaptation of Valsiner's (1997) zone theory of child development to study interactions between teachers, students, technology and the teaching-learning environment. The zone framework extends Vygotsky's concept of the zone of proximal development (ZPD) to incorporate the social setting and the goals and actions of participants. Valsiner describes two additional zones: the zone of free movement (ZFM) and zone of promoted action (ZPA). The ZFM represents constraints that structure the ways in which an individual accesses and interacts with elements of the environment. The ZPA comprises activities, objects, or areas in the environment in respect of which the individual's actions are promoted. The ZFM and ZPA are dynamic and inter-related, forming a ZFM/ZPA complex that is constantly being re-organised by adults in interactions with children. However, children remain active participants in their own development because they can change the environment to achieve their emerging goals. Thus the ZFM/ZPA complex does not fully determine development; instead, development is 'canalised' along a set of possible pathways jointly negotiated by the child in interaction with the environment and other people.

Valsiner (1997) noted that the ZFM/ZPA complex is also observable in educational contexts, and he provided examples of how teachers can set up broad or narrow ZFM/ZPA systems that allow students different choices in completing tasks.

He additionally argued that zone theory is applicable to any human developmental phenomena where the environment is structurally organised, and so it seems reasonable to extend his theory to the study of teacher learning and development in structured educational environments. When considering teachers' professional learning involving technology, the ZPD represents a set of possibilities for developing new knowledge, beliefs, goals and practices. The ZFM is an inhibitory mechanism that structures the teacher's environment, and so could include perceptions of students (their behaviour, social backgrounds, motivation, perceived abilities), access to resources and teaching materials, curriculum and assessment requirements, and organisational structures and cultures. Whereas the ZFM might suggest which teaching actions are *permitted*, the ZPA represents activities, objects, or areas of the environment in respect of which certain teaching approaches are *promoted*. The ZPA could include pre-service teacher education programme formal professional development, and informal interaction with colleagues at school.

Previous research on technology use by mathematics teachers has identified a range of factors influencing uptake and implementation. These include: skill and previous experience in using technology; time and opportunities to learn; access to hardware and software; availability of appropriate teaching materials; technical support; organisational culture; knowledge of how to integrate technology into mathematics teaching; and beliefs about mathematics and how it is learned (Forgasz 2006; Simonsen and Dick 1997; Tharp et al. 1997; Thomas 2006). In terms of the zone framework outlined above, these different types of knowledge and experience represent elements of a teacher's ZPD, ZFM and ZPA, as shown in Table 1. However, in simply listing these factors, previous research has not necessarily considered possible relationships between the teacher's setting, actions and beliefs, and how these might influence the extent to which teachers adopt innovative practices involving technology. In the research discussed in this chapter, zone theory provides a framework for analysing these dynamic relationships.

Taken together, the two theoretical frameworks provide a way of investigating the development of teachers' pedagogical identities as users of digital technologies. From a sociocultural perspective, teachers' learning is understood as changing participation in practices that develop their identities as teachers (Lerman 2001). Wenger (1998) described identity as "a way of talking about how learning changes who we are" (p. 5). He argued that identity has a temporal dimension: because we continually re-negotiate our identities they form trajectories incorporating past, present and future. It is this sense of "learning as becoming" (Wenger 1998, p. 5) that the following analysis attempts to capture.

Research Design and Methods

Participants in the research study were four Australian secondary school mathematics teachers acknowledged by their peers as effective and innovative users of technology. The teachers were selected to represent contrasting combinations of factors known to influence technology integration (see Table 1). They included two

Table 1 Factors affecting teachers' use of technology

Valsiner's zones	Factors influencing teachers' use of digital technologies
Zone of proximal development (Possibilities for developing new teacher knowledge, beliefs, goals, practices)	Mathematical knowledge Pedagogical content knowledge Skill/experience in working with technology General pedagogical beliefs
Zone of free movement (Environmental constraints that limit freedom of action and thought)	Perceptions of students Access to hardware, software, teaching materials Technical support Curriculum and assessment requirements Organisational structures and cultures
Zone of promoted action (Activities, objects, or areas of the environment in respect of which teaching actions are promoted)	Pre-service teacher education Professional development Informal interaction with teaching colleagues

beginning teachers who experienced a technology-rich pre-service programme (described in Goos 2011) and two experienced teachers who developed their technology-related expertise solely through professional development experiences or self-directed learning. This chapter focuses on the professional formation of the two beginning teachers.

The aim of the research was to carry out highly contextualised investigations of how and under what conditions the participating teachers integrated digital technologies into their practice. There were four main sources of data. First, a semi-structured scoping interview invited the teachers to talk about their knowledge and beliefs (which influence their ZPDs), professional contexts (elements of their ZFMs), and professional learning experiences (providing ZPAs) in relation to technology. Thus the structure of the interview was based on the relationship of each zone to factors known to influence technology integration, as outlined in Table 1. For example, teachers were asked about their reasons for using technology in mathematics lessons, their views on how technology influenced student learning and attitudes towards mathematics, their perceptions of any constraints or opportunities in their schools that might affect their use of technology, and their formal and informal experiences in learning to teach mathematics with technology. Interviews were transcribed and teachers' responses were used to 'fill in' the abstract zones of proximal development, free movement, and promoted action with details that were relevant to their own professional histories and contexts.

A second source of data provided additional information about the teachers' general pedagogical beliefs via a Mathematical Beliefs Questionnaire (described in more detail in Goos and Bennison 2002). The questionnaire consisted of 40 statements to which teachers responded using a Likert-type scale based on scores from 1 (Strongly Disagree) to 5 (Strongly Agree).

Third, a snowballing methodology (described by Cobb et al. 2003), involving two rounds of audio-recorded interviews, was used to further probe sources of influence on teaching mathematics. The first round asked participating teachers to identify people who significantly influenced how they taught mathematics, and second round interviews were subsequently conducted with people identified via this process to determine how they attempted to influence how mathematics was taught.

The fourth source of data was lesson cycles comprising observation and video recording of at least three consecutive lessons in which digital technologies were used to teach specific subject matter, together with teacher interviews at the beginning, middle, and end of each cycle. A single video camera was placed on a tripod towards the rear of the classroom and focused on the teacher, the whiteboard, or the data projector screen on which the teacher's computer or calculator output was displayed. Interviews sought information about teachers' plans and rationales for the lessons and their reflections on the factors that influenced their teaching goals and methods.

The next section draws on the data outlined above to present case studies of two beginning teachers, Geoff and Susie (pseudonyms) in order to develop a picture of each teacher's pedagogical identity with respect to technology integration.

Teacher Case Studies

Introducing Geoff

Since graduating from his university pre-service programme in 2000, Geoff had been teaching at an independent girls' school with an enrolment of over 1,000 students in Grades 8–12. This is an academically-oriented school that charges expensive tuition fees, with students who come mainly from upper middle class professional families. Although he was qualified to teach English as well as mathematics, Geoff was assigned only to mathematics classes.

When interviewed, Geoff said that his teaching philosophy was influenced by his love of mathematics, instilled in him as a secondary school student by a mathematics teacher he admired for his 'command of the subject'. Geoff acknowledged that this teacher had been a conservative and a traditionalist at heart, but his 'quirky sense of humour' conveyed a sense of eccentricity that made learning mathematics exciting.

Geoff's passion for mathematics was reflected in responses to the Mathematical Beliefs Questionnaire, where he expressed strong agreement with the statements "Mathematics is an evolving, creative human endeavour in which there is much yet to be known" and "Doing mathematics involves creativity, thinking, and trial-and-error". Questionnaire responses also indicated that Geoff held student-centred views about mathematics teaching and learning; for example, he strongly disagreed that in mathematics something is either right or wrong and that mathematics problems can be solved in only one way, and agreed that teachers should allow time for students to find their own methods for solving problems.

Geoff had been interested in computers since his childhood and was an experienced Excel spreadsheet user when he started his pre-service teacher education programme at university. As a teacher, he enjoyed developing mathematical modeling tasks that were embedded in real life scenarios or stories. (One such task is described in the next section.) In these tasks, he used various types of digital technologies to collect and analyse real data, skilfully blending empirical and analytical approaches to help students develop mathematical models to fit the data.

Illustration of Geoff's Practice

Geoff had participated in an earlier research project that documented the modes of working with technology adopted by pre-service and beginning teachers and investigated personal and contextual factors that shaped their pedagogical identities (see Goos 2005). In his first year after graduation, he taught a Grade 8 mathematics class that was the focus of the research. This was his first experience of using a motion detector in conjunction with a graphics calculator and screen projection unit to teach students how to interpret distance-time graphs. He called on individual students to walk towards or away from the motion detector so as to match a pre-selected distance-time graph displayed on the calculator and view screen. In the following lesson, when he did not have access to the same technology, he devised a simulated graph matching activity in which students 'walked' the graph he had drawn on the whiteboard as he moved his pen along the horizontal (time) axis. In terms of the technology metaphors framework introduced earlier in the chapter, Geoff was using technology as a *partner* because he wanted to provide students with access to a new kind of task that developed their understanding of scale and gradient. This task engaged students in a physical experience that gave instant feedback on the match between the graphical representation and their movement.

When interviewed after the lesson, Geoff explained that he was looking for further challenges in learning to teach mathematics with digital technologies:

> I know what things the graphics calculator can do, and I have a pretty good knowledge of Excel, but really now that teachers know how to include this in their pedagogy, I suppose the emphasis would be now on getting the most out of it. Instead of just knowing what to do, how to really take this technology and explore it to its fullest extent and use all of the resources that [it] has to offer instead of taking bits and pieces that might be good. I suppose unlocking the potential … of what this technology has to offer.

Geoff's interest in integrating powerful uses of digital technologies into his pedagogical repertoire suggests that his trajectory for development was leading him towards using technology, and especially Excel spreadsheets, as an *extension of self*.

Although Geoff's approach in the Grade 8 lessons was taken directly from the teaching materials accompanying the motion detector, the activity resonated with his more creative interests in using drama, songs and story-telling in teaching mathematics as well as English. This mathematical performance approach was evident in a lesson that was observed during the subsequent research project, 5 years later.

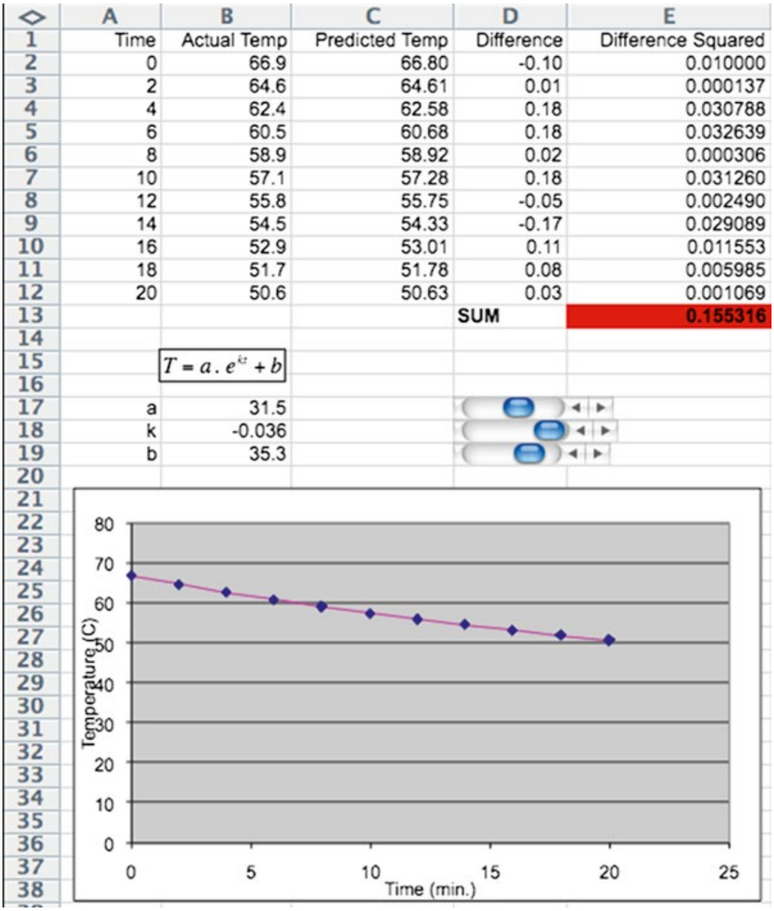
Geoff was teaching an advanced mathematics subject to a Grade 12 class. In a series of lessons on differential equations he planned to introduce Newton's Law of

Cooling via a 'murder mystery' that his students were to solve with the aid of data logging probes and Excel spreadsheets. His approach was aligned with the state-mandated syllabus that emphasised using mathematically-enabled technologies to allow students "to tackle more diverse, life-related problems" (Queensland Board of Senior Secondary School Studies 2000, p. 10). As part of the study of calculus, students were to "appreciate the importance of differential equations in representing problems involving rates of change" (p. 17), through learning experiences such as investigating "life-related situations that can be modeled by simple differential equations such as growth of bacteria, cooling of a substance" (p. 18).

In the first part of the lesson, Geoff introduced differential equations of the type $\frac{dy}{dx} = f(y)$ and reminded students that they had dealt with equations of this type in their earlier studies of mathematics. He noted that there were many instances of such equations in real life and asked students to suggest examples. They remembered that this equation could represent exponential growth or decay, such as in bacterial growth or measuring rates of cooling. One student recalled that the rate of change of temperature was a function of the difference between the object's temperature and room temperature.

Geoff worked through some examples, including one that illustrated exponential decay. He then set the students to work on textbook exercises. During this segment of the lesson another teacher, who had been recruited by Geoff to help set up the modeling scenario, burst into the room and announced that a 'murder' had been committed in a nearby classroom. Not knowing whether to believe the teacher or not, the class followed Geoff to the 'crime scene' where they found an outline of the 'victim' chalked on the floor and two cups of coffee that were still warm. Geoff told the class that police had arrested two suspects who admitted to being in the room making coffee some time earlier but denied committing the crime. According to the 'police report' that Geoff distributed to students, the time of death had been fixed at 11:45 am, 15 min before Geoff's colleague announced the 'murder'. The task for the class was to analyse the cooling rate of the coffee, given the time it was poured and initial temperature, in order to work out whether the suspects could have been in the room at the time the 'murder' was committed.

This task is an application of Newton's Law of Cooling, $T = (T_0 - T_R)e^{kt} + T_R$, where T = temperature of an object undergoing cooling, t = time, k = decay constant, T_0 = initial temperature of object, and T_R = room temperature. Geoff set up temperature probes in each coffee cup to collect temperature and time data while he developed the necessary theory with input from the class. This involved eliciting from students the differential equation for the relationship between the rate of cooling and the difference between object temperature and room temperature, $\frac{dT}{dt} = k(T - T_R)$, which was then re-written as $\frac{dt}{dT} = \frac{1}{k}\left(\frac{1}{T - T_R}\right)$. Students integrated this equation to give $t = \frac{1}{k}\log_e(T - T_R) + C$, and found the value of the constant of integration, C, by substituting the initial values $t = 0$ and $T = T_0$. This gave $t = \frac{1}{k}\log_e(T - T_R) - \frac{1}{k}\log_e(T_o - T_R)$, a function that the students then expressed in exponential form $T = (T_0 - T_R)e^{kt} + T_R$.



	A	B	C	D	E
1	Time	Actual Temp	Predicted Temp	Difference	Difference Squared
2	0	66.9	66.80	-0.10	0.010000
3	2	64.6	64.61	0.01	0.000137
4	4	62.4	62.58	0.18	0.030788
5	6	60.5	60.68	0.18	0.032639
6	8	58.9	58.92	0.02	0.000306
7	10	57.1	57.28	0.18	0.031260
8	12	55.8	55.75	-0.05	0.002490
9	14	54.5	54.33	-0.17	0.029089
10	16	52.9	53.01	0.11	0.011553
11	18	51.7	51.78	0.08	0.005985
12	20	50.6	50.63	0.03	0.001069
13				SUM	0.155316
14					
15		$T = a.e^{kt} + b$			
16					
17	a	31.5			
18	k	-0.036			
19	b	35.3			

Fig. 1 Newton's Law of Cooling spreadsheet

Geoff transferred the temperature (T) and time (t) data to a spreadsheet (http://extras.springer.com*) that plotted the exponential function $T = a.\ e^{kt} + b$. He had set up the spreadsheet to allow the user to change the values of a, k and b and observe changes in the 'goodness of fit' of the model (square of the difference between actual and predicted temperatures for each data point) and also in the corresponding graph superimposed over the scatterplot of temperature versus time data (Fig. 1). At the end of the lesson Geoff emailed this spreadsheet to the students. They finished the modeling task for homework and emailed Geoff their completed spreadsheets overnight.

Geoff's use of the modeling task allowed students to engage with a practical application of differential equations at the same time as they were developing an understanding of the associated mathematical concepts. His use of multiple technologies – graphics calculator, temperature probes, Excel spreadsheet – allowed

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him to combine empirical and analytical approaches to give meaning to these concepts. Five years into his teaching career, it seemed that Geoff was creatively integrating a range of technologies into his mathematical and pedagogical repertoire as an *extension of self*, as foreshadowed in the interview conducted during his first year of teaching (see above).

While it is not uncommon for teachers to use digital technologies such as spreadsheets and graphics calculators with data probes to illustrate Newton's Law of Cooling, Geoff's approach was distinguished by his creativity in embedding the modeling task in a dramatic 'murder' scenario that aroused his students' curiosity and guaranteed their attention as the underlying theory was developed. Although not a fully digital mathematical performance in the sense described by Gadanidis and Borba (2008), this was still a technology-enriched performance that had the potential to generate an emotional response to the murder and police investigation as well as a cognitive response to the mathematical problem.

Geoff's Developing Pedagogical Identity

According to Valsiner's (1997) zone theory, the zone of proximal development entails a set of possible 'next states' of the developing system's relationship with the environment, given the current state of the ZFM/ZPA complex and the individual's developmental state. Thus the ZPD captures development that lies between the possible and the actual. In Geoff's case, his ZPD as a recently graduated teacher included an appreciation of mathematics as a creative human endeavour, some student-centred understandings of how mathematics is learned, and considerable interest and skill in using digital technologies for learning and teaching mathematics. Thus his ZPD offered possibilities for development as a teacher who uses digital technologies as a 'conceptual construction kit' (Olive et al. 2010, p. 138) rather than only as a replacement for calculations that can be done by hand. Geoff found employment in an apparently well-resourced school that was beginning to implement a policy emphasising technology use in all subjects. In mathematics, this meant that all students from Grade 9 upwards had to buy their own graphics calculator, and the school had invested in data logging peripherals and screen projection units as well as fitting out several mathematics classrooms with data projectors and computers connected to the internet. External curriculum and assessment requirements in senior secondary mathematics included mandatory use of computer software or graphics calculators. On the surface, then, it seemed that Geoff's professional environment offered a zone of free movement with broad boundaries for action that permitted experimentation with digital technologies for teaching mathematics. Similarly, the teaching actions promoted by the school administration – the zone of promoted action – seemed to lie within the ZFM. For example, school-based professional development was provided whenever new technology resources were purchased, and the Head of the Mathematics Department encouraged Geoff to incorporate digital technologies into all of his mathematics teaching. The apparent

ZFM/ZPA complex therefore *promoted* teaching actions that were *permitted* within the school and external curricular environment.

However, the ZFM/ZPA complex that Geoff actually experienced within the school worked to constrain his development in subtle ways. Even though the school employed technical support staff to help teachers integrate technology into their lessons, Geoff said that they responded slowly, if at all, to his frequent requests for new mathematical software to be installed over the school's intranet, and he had been obliged to install programs himself on individual computers in order to teach some lessons. This was the case for the Newton's Law of Cooling lesson described earlier, where temperature and time data had to be manually entered into the modeling spreadsheet because the software that can do this automatically had not yet been installed on the classroom computer. The problem was exacerbated by having limited access to computer laboratories that were regularly booked out to other, non-mathematics, classes. Timetabling practices often allocated mathematics classes to rooms in which the teachers rarely used the available technology, while other mathematics teachers who wished to use these resources could not gain access. Geoff also referred to an organisational culture that was not conducive to risk taking, and especially to the conservative influence of parents who expected mathematics to be taught in traditional ways not involving technology. Despite the support of his Head of Department, Geoff's school-based ZPA was characterised by passive acceptance of technology on the part of the other mathematics teachers. He said he believed that he had brought more ideas to colleagues, in terms of technology, than they had been able to teach him.

Valsiner (1997) pointed out that children can negotiate changes to the ZFM/ZPA complex in order to achieve their emerging goals. Likewise, Geoff was able to find a zone of promoted action outside the school that mapped onto his ZPD in developmentally productive ways. There were three aspects to this external ZPA. The first involved participating in university research projects such as the one described here. Geoff noted that the press for innovation that he felt as a consequence of his participation was beneficial because he was motivated to turn 'a germ of an idea' into a real lesson. Discussing his ideas for the coffee-cup murder mystery some weeks before this lesson, he acknowledged:

> This project is good because it gives me the impetus to do something like that which … otherwise still might just be a happy thought.

The second aspect to the external ZPA saw Geoff looking for formal professional development opportunities, such as the intensive, week-long conference that had recently introduced him to advanced features of Excel. Nevertheless, Geoff was selective about what he took from these professional development experiences:

> The majority of things I see that I'd like to use I don't get to use, probably because I see so much of it. I've got to be a bit choosy about what I plan to do.

The third element of his external ZPA came from his increasing participation in the activities of his local mathematics teacher professional association, and in particular the professional growth he experienced by presenting workshops and seminars on teaching mathematics with digital technologies. Although Geoff had little control over his material circumstances at the school – his ZFM – his decision to access an external ZPA helped him to take charge of his own development and re-interpret the limitations imposed by timetabling rigidity, lack of technical support, and a conservative school culture as not necessarily preventing him from adopting innovative teaching practices. This zone theory analysis provides some sense of Geoff's identity trajectory in 'becoming' a teacher who confidently integrated digital technologies into his practice, and his role in negotiating that trajectory. The other way to observe the development of his pedagogical identity is to recognise that his modes of working with technology became more sophisticated over time, progressing towards *extension of self* as he integrated the range of resources available to him into the mathematical practices of the classroom.

Introducing Susie

At the start of the research study, Susie was in her third year of teaching in an independent secondary school with an enrolment of around 600 students in Grades 8–12. The student population was fairly homogeneous with respect to cultural and socio-economic background, with most students coming from white, Anglo-Australian middle class families. Susie was qualified to teach mathematics and music, but at this school she was assigned to teach only mathematics classes.

Susie's own experience of learning mathematics at school was structured and content based, and this influenced the ideas about mathematics teaching that she brought to the pre-service programme:

> I thought it would be great if I could just put stuff on the board and let them do their work and answer questions if they needed it and write exams, tick, cross and that's my job.

According to Susie, these ideas were first challenged by her mathematics curriculum lecturer at university who opened her eyes to different approaches to teaching mathematics. She was now trying to implement these approaches herself. For example, when interviewed, she explained that in her classroom "we spend more time on discussing things as opposed to just teaching and practising it", and that for students "experiencing it is a whole lot more effective than being told it is so".

Susie's responses to the Mathematical Beliefs Questionnaire were consistent with the student-centred approaches that she was now trying to implement in her teaching. For example, she expressed strong agreement with statements such as "In mathematics there are often several different ways to interpret something", and she disagreed that "Solving a mathematics problem usually involves finding a rule or formula that applies". Other questionnaire responses were strongly supportive of cooperative group work, class discussions, and use of calculators, manipulatives and real life examples.

Aged in her mid-20s, Susie said she felt she was born into the computer age and this contributed to her comfort with using digital technologies in her teaching.

She recognised that technology saved time with calculations and graphing but also saw it as providing opportunities for mathematical exploration:

> You make progress so much quicker than having to do things by hand and you can just do examples like … what does this rule look like? What does this linear function look like? And they can put it into their calculator and check and have a look […] So it's just quicker to explore things.

Illustration of Susie's Practice

Observations of Susie's Grade 10 mathematics class in the first year of the research study illustrate her preference for using digital technologies to explore mathematical concepts. In one lesson cycle, she introduced quadratic functions via a graphical approach involving real life situations and followed this with algebraic methods to assist in developing students' understanding. Lessons typically engaged students in one or two extended problems rather than a large number of practice exercises. For example, students used the regression function on their graphics calculators to investigated quadratic models for data on the growth of babies, the path of a tennis ball as it is hit over the net, the height of an object dropped from the top of a building, and the cross sectional dimensions of a railway tunnel arch. They then used their models to make predictions that went beyond the data. A characteristic of these tasks was that students were asked to comment on the strengths and limitations of their models in relation to the real life data rather than just accepting the calculator regression output as an indicator of goodness of fit.

The assessment task for this unit of work required students to investigate projectile motion as a practical application of quadratic functions. The task made use of a computer game in which the Sesame Street character Gonzo was shot from a cannon towards a bucket of water some distance away (http://www.funny-games.biz/flying-gonzo.html; see Figs. 2 and 3). The game allows players to vary the angle of projection and the cannon 'voltage' (a proxy for muzzle velocity) and observe the effects on the distance Gonzo travelled as they 'aim' him at the bucket of water.

Susie had discovered this game at a professional development workshop run by the local mathematics teacher association. The presenter was Geoff, the teacher profiled in the previous section of the chapter. Geoff found the game when searching on the internet for applications of projectile motion that he could use with his Grade 12 class. During this Grade 12 lesson, which was observed as part of the research study, Geoff introduced the parametric equations for projectile motion

$$x(t) = Vt\cos\vartheta \text{ and } y(t) = Vt\sin\vartheta - \frac{gt^2}{2}$$

where $x(t)$ is the horizontal displacement, $y(t)$ the vertical displacement, ϑ the angle of projection, V the initial velocity, t the time in flight and g acceleration due

Fig. 2 Opening screen of Flying Gonzo game

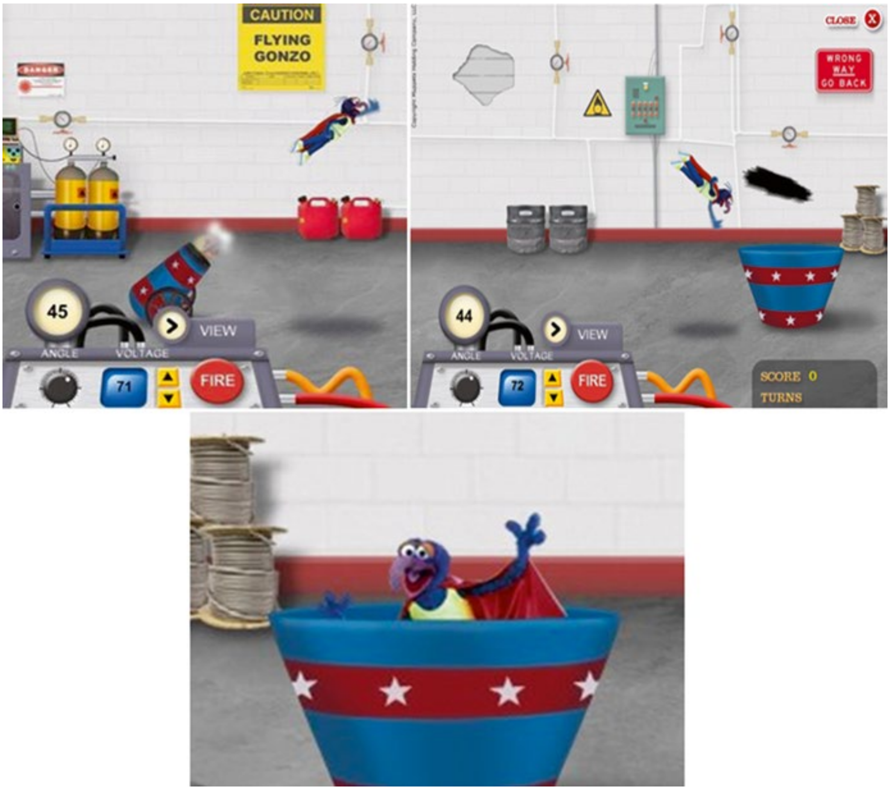


Fig. 3 Firing Gonzo to land in the bucket of water

to gravity. Noting that the y-component is zero when Gonzo lands, Geoff then solved $Vt\sin\vartheta - \frac{gt^2}{2} = 0$ to obtain $t = 0$ (at the start of flight) or $t = \frac{2V\sin\vartheta}{g}$ (when Gonzo lands). Substituting the latter value for t into the equation for $x(t)$ gives the range equation, $x(t) = \frac{2V^2\sin\vartheta\cos\vartheta}{g}$ or, using the double angle formula, $x(t) = \frac{V^2\sin 2\vartheta}{g}$. Geoff's Grade 12 students were to test a range of angles and velocities to predict the range, with the aim of landing Gonzo in the bucket of water. Because the real velocity and range were unknown, students instead recorded the cannon 'voltage' and estimated the range by counting the number of tiles on the wall in the screen background during Gonzo's flight. They entered all these values into a spreadsheet and compared the predicted range, calculated from the range equation, with the actual range expressed in 'tiles'. They then averaged the ratio of predicted to actual range to produce a constant factor (~140) that could be applied to subsequent tests to accurately predict Gonzo's range.

When she tried out the game at the professional development workshop that Geoff presented, Susie wondered whether she could adapt the mathematical content to suit her Grade 10 class. She devised an assessment task in which students used their graphics calculators or *TI-Interactive* software to tabulate and plot data that would allow them to find a mathematical model for the relationship between the range and the muzzle velocity. Algebraic methods were then to be used to determine the best cannon settings for Gonzo to hit a target at a given distance. Students were given a low voltage setting and high voltage setting. Keeping each constant in turn, they fired Gonzo at eight different angles and recorded the range for each trial. They then entered the data into their graphics calculators or *TI-Interactive* and found a quadratic model that gave the best fit. Note that when voltage (velocity) is kept constant the model is trigonometric rather than quadratic because the range varies with the angle of projection. Susie could perhaps have designed the task differently, to keep angle constant and vary the voltage, which would yield a true quadratic model. Nevertheless, a quadratic model fitted to the data as collected gives a good approximation and allowed students to practise finding critical points (intercepts and turning points) algebraically.

Interview and lesson observation data suggest that Susie was interested in having students use technology for mathematical exploration, and not just for checking calculations or making graphing quicker. In terms of the framework for teaching and learning roles introduced earlier, she was working with technology as a *partner* to develop students' understanding of mathematical concepts. Susie's and Geoff's use of a computer game to develop mathematical understanding of quadratic functions and projectile motion connects with Jorgensen and Lowrie's (2011) argument that immersion in digital game environments engages learners and reshapes their thinking. Although the game provided a dynamic image of Gonzo's motion, the effect was similar to 'panning' a camera so that the background seemed to move while Gonzo stayed in the centre of the computer screen. Students had to visualise his parabolic path and find an efficient method for measuring the horizontal distance travelled, which required many repetitions of the game. As Jorgensen and Lowrie noted, the purpose of this repetition was not to achieve fluency with taught skills as is often the case with practice on textbook exercises, but to gain a better understanding of the problem situation and solution strategies.

Susie's Developing Pedagogical Identity

During interviews, Susie referred to a range of people and environmental influences that shaped her development as a teacher of mathematics. Unlike Geoff, who found alignment between his mathematics learning experiences as a school student and the practices promoted by the pre-service teacher education programme, Susie's understanding that mathematics is learned and taught through memorisation and practice was challenged by her pre-service experience. It seemed that there was enough overlap between Susie's ZPD, representing her possibilities for development, and the zone of promoted action offered by the university teacher education programme to canalise her development towards more student-centred, exploratory approaches as she began her teaching career. But a teacher's identity trajectory is also influenced by the relationship between ZPA and ZFM and the meanings ascribed to different aspects of the school environment by the people who organise that environment. Development can be constrained when the environment seems not to permit teaching actions that are ostensibly promoted. However, this seemed not to be the case at Susie's school.

When Susie started working at the school she came under the influence of the Head of the Mathematics Department, who had developed a culture where mathematics was taught as much as possible in *context*, where students worked *collaboratively* and available *technologies* were used extensively. He had been the driving force behind the introduction of technology to the school during the 1990s, before the external curriculum had made the use of technology mandatory. When interviewed, he said he was able to achieve this cultural change because the school administration supported his teaching philosophy and provided funds for resources. Initially, however, even though he developed technology-based activities and provided teachers with professional development, there was not a great uptake of digital technologies by the mathematics teaching staff. To overcome this inertia he introduced technology into assessment tasks that had to be implemented by all teachers:

> You actually had to design activities that you ask all teachers to do or you build it into assessment and teachers will tend to engage a bit more because they always want their students to do the best they can. And it took a long time before it got to the point where it is now where people just pick it up and use it and there are still people that resist anything that's new, even in that culture.

Thus Susie started her teaching career in a school where there was a strong culture within the mathematics department that emphasised integrating digital technologies into everyday classroom practice, resulting in an expectation that she would teach in the same way. Susie described the approach at her school as "This is what we do here". She said it made sense to her, and she "learned so much in the first year about [her] personal understandings of maths, let alone to do with the teaching of it, but also the different approach to it". At this stage of her development, lesson observations indicated that Susie's main mode of working with digital technologies was as a *partner* in providing new ways for students to develop understanding of mathematical concepts.

The zone of free movement offered by the school supported technology innovation through an organisational culture that expected teachers of mathematics to make use of the substantial resources in which the school had invested. Students in Grades 9–12 had constant access to graphics calculators obtained through the school's hire scheme, there were additional class sets of CAS calculators for senior secondary classes, and data logging equipment compatible with the calculators was freely available. Computer software was also used for mathematics teaching; however, as is common in many Australian secondary schools, computer laboratories were difficult to access and had to be booked well in advance. Susie preferred to use graphics calculators so that students could access technology in class whenever they needed it. The data projector installed in her classroom also made it easy for her to display the calculator screen for viewing by the whole class.

The ZFM/ZPA complex that influenced Susie's development as a teacher featured an expansive zone of free movement with few constraints limiting her choice of actions and a zone of promoted action set up by the school administration and Head of Department that encouraged her to explore the resources that were available to her. As Susie explained, "Anything I think of that I would really like to do [in using technology] is really strongly supported". Susie's pedagogical identity was taking shape as she constructed meaning from her person-environment relationship. It seemed that the ZFM/ZPA set up by the school mapped exactly onto her ZPD, so much so that she evaluated the external ZPAs offered by formal professional development workshops in terms of how well they matched the teaching approaches permitted by her environment and promoted by the people who organised that environment. She had attended many conferences and workshops in the 3 years since beginning her teaching career, but found that most of them were not helpful "for where I am". She explained: "Because we use it [technology] so much already, to introduce something else we'd have to have a really strong basis for changing what's already here".

One of the risks in continually judging the fit of an external ZPA in terms of its match with existing people-environment relationships within a school is that it may limit possibilities for envisioning and adapting to change. A school's organisational culture and resources can change over time, as can the teaching approaches promoted if there is turnover of key staff. Susie was already aware that not all of the mathematics teaching staff were enthusiastic users of digital technologies. One experienced teacher who had been a longstanding staff member at the school expressed concerns that sometimes technology could be used "just because it's there" and cited as an example the use of dynamic geometry software in junior secondary classes at the expense of using concrete materials: "I think it's good to draw things and measure things". This teacher was willing to question the value of using technology in certain circumstances, and Susie acknowledged the teacher's influence in making her more discerning in her own use of technology with her classes. When the Head of the Mathematics Department left the school, Susie was promoted to the position of coordinator of the junior secondary mathematics programme. Now she noticed that some of the more recently appointed mathematics teachers were neutral and passive in their attitudes towards technology. Although they were willing to use technology in their teaching if pressed or shown how to, they rarely asked questions or engaged in discussions about improving existing tasks and technology-based teaching practices. Thus the ZPA implicitly set by the example of colleagues was contracting, and one might predict that Susie's identity trajectory of 'becoming' a creative user of digital technologies – perhaps as an *extension of self* – would be impeded unless she deliberately sought out external ZPAs consistent with her pedagogical beliefs and goals.

Conclusion

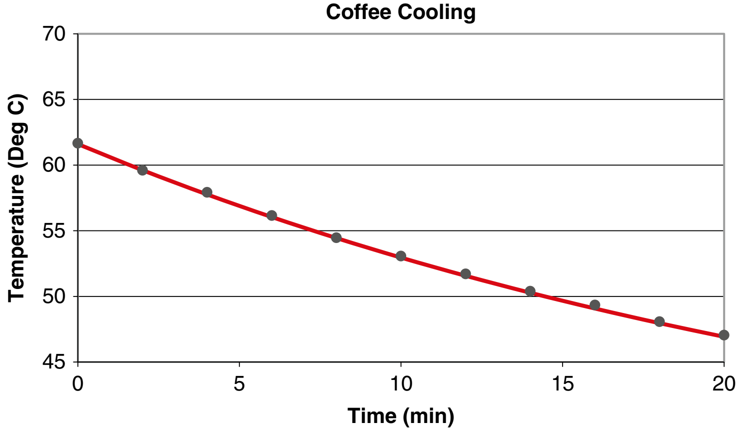
This chapter has focused on how mathematics teachers develop new practices in technological environments. Mathematics education researchers have been interested in the mathematical potential of technology and its effects on student learning for at least the last 30 years (Hoyles and Lagrange 2010), but only recently has there emerged a trend towards investigating how technology changes the professional work of mathematics teachers (Artigue 2010). The research reported in this chapter examined relations between factors known to influence ways in which teachers use digital technologies to enrich secondary school mathematics. Based on sociocultural theories that view learning as increasing participation in practices and constructing identities in relation to these practices, two frameworks were used to analyse the development of teachers' pedagogical identities as users of technology. The first framework classifies different ways of working with technology and provides evidence of 'what' changes in teachers' practice, while the second allows for investigation of teacher-environment relationships to explain the 'how' and 'why' of developing practice in terms of Valsiner's (1997) zone theory.

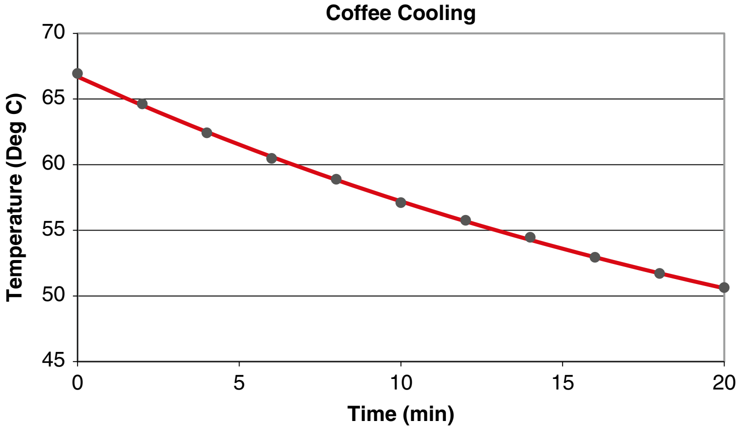
The analysis of two cases of beginning teachers illustrated several issues related to identity development. The first issue concerns the temporal dimension of identities, in that teachers are on a trajectory of 'becoming' a different practitioner. Zone theory is useful for conceptualising not only possibilities for development, but also the ongoing process of development as changing relationships between the zone of free movement, zone of promoted action, and zone of proximal development. Other issues are related to how trajectories of teacher development are constrained rather than fully determined. Teachers' knowledge and beliefs, on their own, do not determine how they will approach the classroom use of digital technologies. Neither does it make sense simply to 'add up' the positive or negative effects of institutional constraints or professional development opportunities to predict whether teachers will embrace or resist technology. Instead, an analysis is called for that gives attention to relationships amongst Valsiner's (1997) three zones, bearing in mind that the developing person is able to re-negotiate these relationships to some extent to achieve their emerging goals.

Susie and Geoff were regarded as innovative users of technology; however, they differed in the degree of fit between their respective ZPDs, ZFMs and ZPAs. The zone of free movement offered by their schools was important in allowing them some leeway to explore technology-enriched teaching approaches consistent with their pedagogical knowledge and beliefs. In both schools there was good access to most forms of technology, and an externally mandated mathematics curriculum that made it obligatory for teachers to use computers or graphics calculators. Yet, despite the availability of appropriate material resources, other institutional constraints worked against technology integration. In Geoff's case, constraints arose from the school's conservative academic culture and somewhat inflexible organisational structures that were not wholly conducive to experimentation with new technologies. For Susie, passive resistance from other mathematics teachers was beginning to undermine an organisational culture that had previously supported innovative technology integration. The zones of promoted action set up for and accessed by these two teachers also differed. Susie found that her school's ZPA enabled her to fully exploit the possibilities provided by the ZFM, although these circumstances were changing due to staff turnover at the time of the research study. In contrast, Geoff's school-based ZPA did not provide him with enough opportunities to develop and extend his teaching repertoire. Instead he sought an external ZPA through varying combinations of formal and informal professional development. This analysis shows that neither professional learning experiences, time, resources, curriculum mandates, nor supportive organisational structures and cultures are sufficient, on their own, to lead to a higher level of technology integration in mathematics classrooms. Instead, it is the dynamic relationships between these factors, and the teacher's active reshaping of their professional environment, that develop their professional identities as users of technology.

The extent of overlap between the ZFM/ZPA complex and the ZPD may be critical in supporting beginning teachers in further developing the innovative practices they typically encounter in pre-service programmes. Susie and Geoff experienced different combinations of factors known to influence technology integration, but both had a 'region' of overlap between their respective ZPD, ZFM and ZPA where they were able to find sources of assistance that supported their ongoing development as teachers of mathematics, and this in turn enabled them to integrate technology into their professional practice in a variety of ways. Some of these uses of technology went beyond the familiar applications of computer software and graphics calculators to incorporate elements of mathematical performance and digital gaming that may offer new ways of learning mathematics. Susie and Geoff developed these activities themselves without any intervention from the researcher, and the account in this chapter of how they implemented these activities provides an authentic picture of what is possible in a typical independent secondary school classroom. With new generations of students coming to school familiar with using digital technologies to organise their daily lives, provide entertainment, find information, and maintain social networks, mathematics education research needs to find better ways to understand the impact of such technologies on teachers' professional work and learning.

Police Crime Scene Coffee Analyser





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