Integrating Technology in the Primary School Mathematics Classroom: The Role of the Teacher

María Trigueros, María-Dolores Lozano, and Ivonne Sandoval

Abstract In this chapter, we analyse the role of the teacher when using digital resources in the primary school mathematics classroom in Mexico and its relation to students' mathematical learning. We carry out this analysis through the use of an instrument that we developed in which we relate five different aspects of the role of the teacher we consider important with the three different uses of technology classified by Hughes (*Journal of Technology and Teacher Education, 13*(2), 277–302, 2005) namely ***replacement, amplification*** and ***transformation***. We use an enactivist perspective that considers learning as effective action in a given context (Maturana, H., & Varela, F. *The tree of knowledge: The biological roots of human understanding*, Revised Edition, Boston: Shambhala, 1992) in order to describe the way in which differences both in the uses of technology and in the role the teacher assumes in the classroom contribute to creating classroom contexts in which mathematical learning is promoted to different degrees.

Keywords Digital technology • Enciclomedia • Role of the teacher • Enactivism • Mathematics learning • Uses of technology

M. Trigueros (✉)
Instituto Tecnológico Autónomo de México, Mexico D.F., Mexico
e-mail: trigue@itam.mx

M.-D. Lozano
Universidad de las Américas Puebla, Puebla, Mexico

I. Sandoval
Universidad Pedagógica Nacional, Mexico D.F., Mexico

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Introduction

The use of digital technology in the classroom has occupied the attention of researchers in mathematics education for several decades. Most of the studies focusing on the teaching of mathematics using specific software have been concerned with middle or high school students and teachers (e.g. Ruthven 2007; Drijvers et al. 2009). Since the teaching of mathematics in primary school has particularities and restrictions that make it very different from the work done at higher levels of education, it is important to carry out more research in order to investigate how the integration of technology occurs at this level. Through the work in this chapter, we intend to contribute in this direction by analysing the role of the teacher when using digital resources in the primary school mathematics classroom in Mexico and its relation to students' mathematical learning.

Background

The use of digital technologies in the primary school in Mexico has been influenced greatly by the introduction of the national teaching programme Enciclomedia, which was created in 2004 with the intention of complementing already existing materials in primary school classrooms – such as the mandatory textbooks – with computer programs and teaching resources designed to be used with an interactive whiteboard. It is a large-scale project, involving more than 7,000 schools and 170,000 classrooms, and is meant to support the teaching and learning of all subjects in grades 5 and 6 of primary school by working with one computer. Enciclomedia's programs were designed with the intention of motivating students to engage in mathematical problems by inviting them to take part in games and other activities and by providing them with interesting contexts. The use of interactive whiteboards was intended to promote classroom interaction and to enhance interactivity with computer programs. Evaluations of the Enciclomedia project have shown positive results in terms of resources' usability and interactivity, a high potential for promoting meaningful and high order operations learning, as well as high motivation of students (Holland et al. 2006; Díaz de C. et al. 2006; Trigueros et al. 2007). Infrastructure and teacher training were, however, found to be problematic (Loredo et al. 2010). Furthermore, difficulties were identified in terms of the integration of Enciclomedia's resources by the teachers in their everyday lessons. Some studies (e.g. Díaz de C. et al. 2006; Sagástegui 2007) report that resources have the potential to change teachers' practice, but more research is needed as there are still few studies focusing on the ways teachers use these resources in their classrooms (e.g. Trigueros and Lozano 2012).

In this chapter we will analyze primary school teachers' work with technology in their classroom. We are interested in a detailed description of when and how they

use digital resources in their teaching practices and how this relates to different roles they can undertake.

Theoretical Framework

We are interested in investigating what happens in relation to the role of the teacher when technology is introduced in primary school mathematics classrooms. In order to do this, we use an enactivist perspective (Maturana and Varela 1992), which considers that, in the process of living, individuals carry out those actions which are effective or adequate in a given context and that it is this continuous process of successful interaction with the environment that we call 'learning'.

In this way, learning occurs when individuals interact with each other, changing their behaviour in a similar way. In a particular context or location, the participants create together the conditions that will allow actions to be adequate. As members of a particular community interact with each other, patterns of behaviour are created constituting a classroom culture (Maturana and Varela 1992). With these ideas in mind, we are interested in investigating how patterns of effective behaviour emerge in mathematics classrooms as teachers and students use Enciclomedia. In particular, we focus on those patterns related to the role of the teacher.

Learning Mathematics with Computer Tools

From an enactivist perspective, the use of computer tools is part of human living experience since "such technologies are entwined in the practices used by humans to represent and negotiate cultural experience" (Davis et al. 2000, p. 170). Tools, as material devices and/or symbolic systems, constitute an important part of learning, because their use shapes the processes of knowledge construction and of conceptualisation (Rabardel 1999, 2011). When tools are incorporated into learners' activities they become instruments which are mixed entities that include both tools and the ways these are used. Instruments are not merely auxiliary components or neutral elements in the teaching of mathematics, they shape students' and teachers' actions. Every tool generates a space for action, while at the same time imposing on users certain restrictions. This makes possible the emergence of new kinds of actions. When using the tool, teachers' history and context will then determine which actions are undertaken among the ones made possible by the programs.

In this way, we consider teachers as learners who are modifying their actions in an environment that includes specific characteristics and certain tools. The actions teachers undertake define their different roles in the classroom, and we look at them through an instrument of analysis that considers different aspects of the role of the teacher.

The Role of the Teacher and the Different Uses of Technology: An Instrument of Analysis

We propose five different aspects of the teachers' role that we investigate through this work. We are aware that the different aspects overlap and cannot be clearly separated. This classification is used for purposes of theoretical analysis only.

1. ***Role in terms of communication of mathematics***. In mathematics classrooms, students and teachers are in contact with mathematical concepts that are defined by a larger community of mathematicians. Sometimes, these concepts and processes are made available to the students exclusively by the teacher and through textbooks. The inclusion of technology in the classroom often implies that the computer programs become another source of mathematical information. Technology might therefore influence the teachers' role regarding mathematics concepts and procedures by providing a complementary source that the teacher and the students can both comment on and work with. Effective behaviours then might include several forms of interaction with the mathematical content included in the digital programs.
2. ***Role in terms of interaction with students***. The role of the teacher in the context of this study refers to the way in which teachers interact with students and how they manage and regulate what happens in the classroom. Sometimes teachers listen attentively to students and respond accordingly, often modifying what they had planned for a lesson. On other occasions, teachers tend to follow closely a determined path, and respond scarcely to students' questions or interests. The inclusion of technology can influence the way in which the teacher regulates interactions by presenting unexpected situations that might have not occurred without the use of particular programs. Effective behaviours might include allowing students to explore the use of the program and discussing unplanned mathematical problems that might arise while using technology
3. ***Role in terms of validation of mathematical knowledge***. The teacher as a source of validation for correct mathematical procedures and answers is also an important aspect to be examined when technology enters the classroom. Several interactive programs give feedback to the student when an answer is entered. Teachers might discuss answers with students before the program validates them or they might allow students to use the program as a means for validating their own answers.
4. ***Role in terms of the source of mathematical problems***. In mathematics classrooms, the teacher is often the main source of mathematical problems. Even if problems from textbooks are solved, it is the teacher who decides which problems or exercises are to be worked on. Mathematical problems, however, can also emerge from activities in the classroom itself. When technology is used in the classroom, it becomes another possible source of mathematical problems and it can also influence the way in which mathematical problems are selected. Teachers' behaviours might include encouraging students to solve those problems posed by the programs (directly or indirectly), even in those cases

when unexpected uses of technological devices lead to mathematical problems that had not been addressed before and that might not be included in the lesson plan or in the curriculum.

5. ***Role in terms of actions and autonomy of students***. Actions on mathematical objects and tools can be carried out both by teachers and students in the classroom. Sometimes the teacher assumes a more active role, while students mainly listen or copy. At other times students have more autonomy to decide what to do and how to do it. Again, technology may change the dynamics of who does the mathematical actions in the classroom. Teachers' actions might include allowing the students to work with the program and the mathematical problems without much intervention. In other cases, teachers might use the computer to show students certain features or uses of the program before the latter are allowed to explore.

After observing mathematics lessons or videos from lessons involving technology, and having read some of the literature regarding the use of digital programs in the classrooms, we decided that we wanted to explore how the different aspects of the teacher's role related to the particular ways in which technology is used. We considered that looking at both the teacher's role and the use of the programs might give a richer account of the way in which technology is integrated within the classroom culture.

In order to consider different ways of using technology, we use the categories developed by Hughes (2005): "The variation in technology supported pedagogy can be captured through three categories: (a) Technology functioning as replacement, (b) amplifications, or (c) transformation" (p. 281). She defines technology as replacement when it is used in a way that does not change "established instructional practices", that is, "the technology serves as a different means to the same instructional end". Technology as amplifier "capitalizes on technology's ability to accomplish tasks more efficiently and effectively, yet the tasks remain the same"; and technology as transformation may change "teacher's instructional practices and roles in the classroom" (p. 281). From an enactivist perspective, it is important to analyse how the different aspects of the role of the teacher are related to her or his use of technology and how this relation changes through the lessons as he or she takes decisions through the lesson. Taken together we can have a clearer picture of the integration of technology in the mathematics classroom and of which aspects need to be considered to help teachers make innovative and effective uses of technology.

The research questions addressed in this study are:

- How can we describe teachers' effective actions in terms of the different aspects of the role they take when technology is introduced in the classroom?
- How are these aspects of the teacher's role related to their use of technology?
- What kinds of classroom cultures are created by teachers' effective behaviours in terms of the role of the teacher and of the use of technology?
- How are different classroom cultures fostered by different teacher behaviours related to students' learning of mathematics?

Methodology

We decided to investigate the teacher's role in the mathematics classroom when using technology by selecting eleven teachers from five schools in three different states in the country. The states and schools were chosen mainly because we, as participants in a larger study that investigates teaching practices with technology, are in contact with teachers and head teachers from those schools. Three of the schools involved in this study are urban and two of them are semi-rural. Additionally, one of the urban schools is a private school, while the remainder are state schools (see Table 1).

Most grade 5 and 6 classrooms in Mexico have the Enciclomedia project equipment, and some of them have also computer labs with an average of 20 computers. In addition, with the exception of one semi-rural school, all of the schools participating in this project have internet access. The teachers that we selected for this study had at least 1 year of experience using technology in their lessons and had an interest in participating in a research project. We selected them because, from our observations, they each represented a particular group of teachers who use technology in a certain way. The three teachers we selected differed in their background, professional training and experience in teaching mathematics and also in their training experiences on the use of technology.

After observing several lessons and analysing video recordings from the eleven teachers who were initially included in the study, we identified three teachers who were representative of the characteristics of the different groups of teachers, with respect to their experience both as teachers and with the use of technology, and according to the way in which they used digital resources in their classrooms. We decided that, in order to deepen our understanding of the role of the teacher when using technology, we would focus on the analysis of a lesson that can include one or more sessions of these three representative teachers who we will refer to as Gabriel, Juan and Susana.

The Teachers

Gabriel is an elementary school teacher with 30 years of experience and who initially trained as a secondary school mathematics teacher but then decided to work at the elementary level. His use of technology in the classroom started when the

Table 1 The teachers

State	Type of school	Teacher
Distrito Federal	Private	1 teacher
	Public 1	5 teachers
Estado de México	Public 2	2 teachers
San Luis Potosí	Public 3	2 teachers
	Public 4	1 teacher

program Enciclomedia was installed in his school in 2004. He initially received some general training, which consisted only of technical instruction on the use of the software, and has continued with the development of his technological abilities independently. He has not had any training in relation to the teaching of mathematics with digital technologies. Gabriel represents a large group of teachers who are interested in teaching with technology, but who have not received pedagogical training.

Juan is a young primary school teacher with 6 years teaching experience. He has trained himself on the use of technologies; he has never taken any training course, but he is a proficient user of computers and owns a personal computer. During an interview, Juan mentioned that he likes technology because it is helpful to keep track of his records and to search for information. When he uses it in the classroom, it is mainly for teaching mathematics. We selected him because he represents another group of teachers who are proficient users of computers and have recently started to use them in their teaching.

Susana is a teacher-researcher who had been teaching for 5 years in a private urban primary school before she started including digital resources in her lessons. She has been involved in mathematics education for several years and is interested in reading the mathematics education literature. She got involved in 'Enciclomedia's' training workshops from its early stages, and has been using it in her classroom for 6 years. She always has the latest version installed in her computer at home. We selected her because she represented a small group of teachers who were particularly successful in integrating resources in their teaching practice.

Research Tools

In order to study teachers' actions when investigating both the uses they make of the resources available and their role in the classrooms, it is necessary to employ a variety of research tools so that different perspectives are addressed. For this study, we analysed classroom observation notes, video-recordings of teachers' lessons and audio-recordings from interviews. The guides for the observation notes were developed as part of the abovementioned larger study and were very general. Observers described the digital resources used during the lesson and particular incidents they considered important in relation to the use of technology in the classroom. The lessons were video-recorded and teachers were free to decide on the mathematical theme they wanted to teach, the resources they wanted to use and how they wanted to use them. Often they would teach the mathematical topic that was meant to be taught on those dates according to the official programme. During the interviews, which were also carried out as part of the larger project, teachers were asked about their background and training and about the ways in which they worked with technology during their mathematics lessons.

The collected data were reviewed, for this particular study, by all three researchers involved. During a first round we looked at the data together with the aim of

selecting those episodes that would be analysed. For our analysis, we focused on teaching episodes where digital technologies were integrated. We then developed collectively an instrument of analysis to characterise each teacher's practice that we describe below. The purpose of the analysis is to reflect on the dynamics of the use of technology in the classroom, and of the different aspects of the role played by the teacher in relation to the mathematics being taught. In order to make our results reliable, the data coming from the selected teaching episodes were also analysed independently on a second occasion by the three researchers using the same instrument of analysis. Final decisions were the result of comparison and negotiation of the independent outcomes.

Throughout a teaching session, it was possible to identify different time intervals in which a specific kind of activity involving the technology was used to work with mathematics problems. We call each one of these intervals an episode. A matrix is used to describe the dynamics of the selected episode by means of an arrow. A full description of how to interpret the resulting matrices is included later in the chapter. This instrument is useful to compare the dynamics of the lessons of the same teacher and of different teachers and can also be used to make teachers aware of their actions and to reflect on their lessons. In this chapter we use it to compare one typical lesson of the selected teachers.

The comparison of the teachers who represent the selected classes is used to differentiate them in terms of the mathematical activity that is favoured during each interval and possibilities for students to learn the intended mathematical topics. We also use data to determine if certain uses of technology are favoured or more frequently linked to particular aspects of the teacher's role and how it influences mathematical activity.

Results

We analyse in detail the different roles that teachers can play when they use digital resources in their lessons. This analysis provides useful information to help explain why certain approaches are taken by teachers and an indication of the possible learning outcomes (in terms of students' learning) that are associated with the teacher's different roles. For that purpose we have divided the descriptions of the episodes according to the changes in class activity.

Gabriel

For the selected session, Gabriel worked with a chapter from the students' textbook in which, according to official documents, students are expected to "*deduce equivalences between the units of volume and capacity for liquids"* (SEP 2009, p. 155). Gabriel started the lesson (episode 1) using the electronic whiteboard to write some

definitions. He also used manipulatives like a 1 dm^3 glass cube and a plastic bottle. During the whole episode, Gabriel followed closely his lesson plan. He gave explanations and asked some questions, but he did not use the students' responses to review or complete his explanations. He was always in charge of the communication and validation of the mathematical knowledge:

G: *How many centimetres are in a decimetre?*
S_1: *Ten!*
S_2: *A hundred!*
G: And what do you think? A decimetre is equivalent to how many centimetres?
S_4. *To ten?*
G: *To ten what…I cannot hear your, speak louder!*
S_4: *To 10 cm?* […]
G: *How much water can I fit into that cube?*
S_1: *One litre!* S_2: *One half!*
S_4: *A cubic decimetre has a capacity of mmm… litres.*
G: *One litre, because exactly, it was exactly what I could pour here, one, do you agree? Very well.*

(He continues explaining, showing equivalences in capacity using water and writing in the whiteboard).

The interaction with students was limited to the asking of rhetorical questions such as those shown above, to pose some problems from the textbook and to explain them. Students' actions were limited to listening to Gabriel and answering some of his questions, without necessarily reflecting. In this episode the technology was being used as a replacement tool, since Gabriel could have done all of his writing on the blackboard.

Later, Gabriel opened a program (episode 2) called 'Capacity measures' from Enciclomedia project (Fig. 1).

This is an interactive program, which uses the context of a milk factory to invite students to fill containers of different sizes by using smaller containers of different capacities, which are then carried by a truck to the warehouse. With this program it is possible for the students to choose one container from the warehouse and calculate how many of another chosen container would be needed to fill the large

Fig. 1 Capacity Measures

container, prompting a possible class discussion of the comparison of different measurements of capacity. The program includes teaching suggestions and interesting questions to ask while working with the program.

In this episode the teacher shared the communication of mathematical knowledge using the interactive resource, which was a source of mathematical problems. Gabriel chose an activity from the program, asked questions that guided the whole class through the activity, and used the program to verify the results. He remained in control of the computer. His interaction with the students was limited to the posing of questions that students continued to answer, with little evidence of any deeper reflection.

G: *Which of these containers shall we use (referring to the containers in the truck)?*
S: *The big one.*
G: *How many containers are needed to fill up the tank in the warehouse?*
S: *5 1/2* [Gabriel writes the number in the program and they see that when the all the milk is poured the container is not completely full]
G: *Let's see if it works… How many more containers are needed?*
S: *6 ¼, 8 teacher, 11* [Shouting]

Even though some students responded to the questions during the episode, their attitude seemed passive since they were not reflecting on their answers. It can be concluded that in this episode the use of the technology was as an amplifier, since the program showed situations that it would not be possible to illustrate in the classroom. However, the teacher did not exploit fully the possibilities offered by the technology and did not use it as the starting point to further explore the students' understanding.

After working with the program for a while, Gabriel opened the animation 'Metric units of volume' (episode 3), where equivalences and comparisons between different units of volume are illustrated through images of objects such as a swimming pool and a football stadium (Fig. 2).

Students watched the whole animation without interruption. When it finished, Gabriel did not make any comment about it or ask any questions. He left the communication of mathematical knowledge and problems, and the validation of results, to the resource. Students did not have any autonomy to question what was

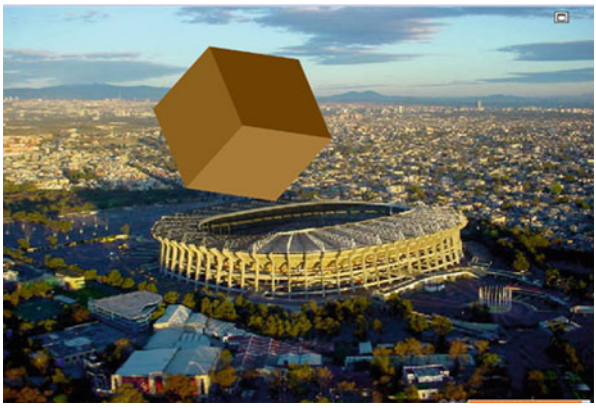


Fig. 2 Metric units of volume

presented to them and there was no possibility to interact with either the program or the teacher. The technology was used in part as replacement, and in part as amplifier, since the animation illustrates situations, which cannot be observed in the classroom, but at the same time it was left completely in charge of the mathematical explanation.

Finally (episode 4), Gabriel went back to the electronic whiteboard. He referred back to the ideas introduced when he started the lesson, but this time he used some of the examples shown in the animation, and used drawings to repeat some of the animation to ask new questions. Again, he expected immediate responses from the students and he did not invite them to think more deeply about the answers:

G: *A cubic decimetre, it is equivalent to what? To a litre. One cubic metre, how many litres can it hold?*
S-G: *A thousand.*
G: *Imagine that you have a cubic decametre. How many litres can it hold?*
S: *Ten thousand? Eight thousand?*
G: *How many cubic metres there are?*
S: *A hundred?*
G: *10 by 10 is 100. And 100 by 10 is 1000. Then a cubic decametre is 1000* m^3.

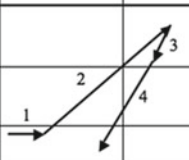
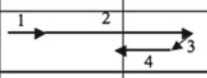
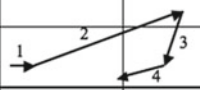
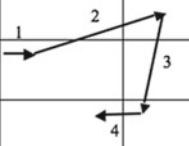
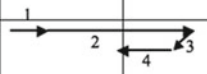
Later, he showed again the same animation as reinforcement of his explanations, but again the students did not make any comment. He finished the lesson by asking students to work on some problems from the textbook. Later, some students went to the whiteboard to write their answers and neither Gabriel nor the other students asked them about their procedures. Gabriel emphasised the formulae that the students needed to know in order to calculate the volume of different containers, the units of volume and the correct responses for the problems. Finally, as an end to the lesson, Gabriel reviewed the relationships between the units of volume and capacity. He asked some questions but, as students did not give the answer he was expecting, he gave the information that he was asking for:

G: How can we calculate the capacity of a container?
G: In order to know how much container can fit, first we calculate its…
[No answer]
G: Volume
G: Here the volume is
S1: 30; S2:50; S3: 60
G: 40 right?

Again he was in control of the communication and validation of the mathematical knowledge, while the source of mathematical problems was the animation and the textbook. There was no real interaction with the students in terms of letting them participate and, although the students had some autonomy as they worked on the textbook problems, they were not able to discuss their ideas. We conclude that, in this final episode, the technology is again used as a replacement since the teacher could have taught the same lesson without the use of the technology, and the animation was not actually discussed.

Table 2 describes the dynamics of Gabriel class. It gives a snapshot of how the different aspects of his role as teacher changed through the lesson and their relation

Table 2 Analysis of Gabriel's aspects of his role as a teacher in relation to the use of technology

Teachers' role / Uses of technology		Replacement	Amplification	Transformation
Communication of mathematics	Students, textbooks, technology and teacher			
	Some other elements of communication			
	Teacher communicates exclusively			
Interaction with students	Modifies plan according to students' participation			
	Listens to students and answers questions but goes back to plan			
	Little interaction with students, follows predetermined plan			
Validation of mathematical knowledge	Multiple sources of validation			
	Teacher as only source of validation			
Which is the source of mathematical problems	Other sources of problems including those coming from digital programs and students themselves			
	Problems from teachers and textbook			
	Unique source of mathematical problems			
Actions and autonomy of students	Students have autonomy to decide what to do and how to do it.			
	Teacher assumes active role, while students mainly listen, copy or answer questions.			

to the use of technology. Each arrow represents an episode by showing a movement regarding, on the one hand, the aspects of the role of the teacher (as denoted by upwards or downwards movements) and on the other, the use of the technological resources (represented by movements towards the left or right). The use of consecutive arrows is intended to give information about how the role and the use of technology change during the lesson. The position of the beginning and ending of each arrow are important in determining how close a teacher's actions are to each aspect of the role or to each use of the technology. For example, a teacher can use technology as a replacement, but his actions can reveal that this use can be closer or further away from using it as an amplifier. The arrow corresponding to that episode will be nearer to the border of the table between these two uses. The same is true for the aspects of the role of the teacher; if the arrow point is closer to a border, it means that most of the actions of the teacher can mainly be considered related to that aspect.

In examining the different aspects of Gabriel's role as a teacher in relation to how he used the technology, the patterns of his actions in the classroom made it

possible to observe his tendencies to maintain control of the class and to follow his teaching plan closely. The use of the whiteboard and the animation did not contribute any substantial change in his actions. It was only when he introduced a more interactive tool that aspects of his role changed slightly as he allowed the program to be the source of mathematical problems and to determine the correctness of answers. Still, during this episode, Gabriel's behaviour did not allow students to explore their ideas further by following unexpected pathways or incorrect answers. For students, effective actions in Gabriel's classroom included guessing and answering by trial and error. Students' answers led to a limited amount of feedback, which came from the interactive program. Often their responses were not followed up as Gabriel would give the correct answer himself. In this context, effective behaviour is unlikely to promote or be conducive to mathematical learning. It seems that even though Gabriel was interested in using technology in the class, he was not able to use it to stimulate students' reflection on the mathematical content he was teaching.

Juan

For the lesson we selected for analysis, Juan taught a chapter from the textbook on "conversion of fractions into decimal numbers and locating them on the number line" (SEP 2009, p. 47). Juan started the lesson (episode 1) by asking his students to draw 5 apples and 3 children, and to share the apples among the children. He interacted with students by means of questions and comments in response to their answers, but he did not ask the students to explain the reasoning behind their answers:

J: How many parts of each apple will be given to each child?
S_1: One whole apple and two thirds.
J: Let's see. Each one will have one apple. I have shared three of the five apples, but I still have 2 more apples. This one and this other apple [signaling his drawings of the apple], how can I divide them?
Students: In three parts.
J: In three. I suppose they have to be equal. [He represents the parts in his drawing of the apples]. Each one will have one part of each apple. So, each child receives, how many?
Students: One whole and two thirds.

He then said:

J: We will see an animation with the computer so we can revise what has been learnt and that it will show us some examples about how certain things can be shared.

Before showing the animation, he reminded students of the names of the elements of a fraction (numerator and denominator) and their meaning. Then the animation 'Fractions' (Fig. 3) from the Enciclomedia project was introduced. It shows several situations in which familiar objects are divided into a given number of equal parts so that they can be shared amongst different numbers of clowns.

Fig. 3 Fractions

Once the animation ended, the teacher asked students whether they liked it and he posed another sharing activity. He did not make further comments that might have made students reflect on the animation content in terms of the lesson's objectives. Throughout the whole episode, Juan was in control of the communication of mathematical knowledge, the main source of problems, in control of the validation of knowledge and making the decisions about what the students had to do. Students did not have autonomy and their participation was limited to answering his questions, although it is possible to observe that they did give careful consideration to their answers. The technology was used as a replacement since it seems Juan only used the animation as a way to review some contents and to motivate students, although it may be considered that it was also used as amplifier since it shows concepts in a contextualised and attractive way, which cannot be done by the teacher himself. However, no reflection was initiated by Juan on what the technology had demonstrated.

Juan then used a worksheet that he had prepared using a word processor that included two sets of problems that presented situations concerning the sharing of donuts (episode 2). One set were problems with proper fractions, while the other involved improper fractions. The worksheet was the sole source of the mathematical problems. Students were asked to write the result using decimal numbers. Juan started with the first set and asked:

J: If we want to convert all these fractions to decimal numbers, is it possible?
Students: Yes!
J: How can we do it? Who can tell me how?
S_1: By dividing.
J: What should I divide? [Showing the first exercise]

A similar dialogue was repeated for each of the fractions, but a different student was selected to work on the whiteboard. In response to a problem where 7 donuts were shared by 3 children a student responded 2 wholes and one third where Juan had expected him to respond 7/3. Juan solved the division sum 7 divided by 3 on the board and wrote 2.33 (see Fig. 4) and gave the fact that the number continues indefinitely.

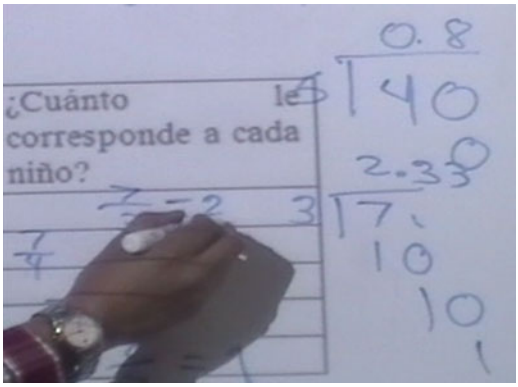


Fig. 4 Juan's solution on the board

When Juan asked the students for an equivalent decimal expression for 7/3, again they gave an unexpected response:

J: And I can continue like this [referring to the division algorithm]. 7/3 is equivalent to…
Students: Two and three tenths.
J: thirty-three, what?
Students: hundredths.
J: And if a write another three?
Students: Thousandths.
J: Then it would be two and three hundred and thirty three thousandths.

When the students had finished the exercises Juan asked for volunteers to give the answers and he verified the answers with the help of the group. When 7/6 appeared, he asked, *What is this number called as a decimal?* However, none of the students could answer and they did not use repeating decimals in their solution.

In this episode there was interaction with students in the form of questions and answers and the communication and validation of mathematical ideas were shared between the teacher and the students, although the teacher had the last word on the validation of their responses. Students participated actively, but they did not have autonomy. The interactive whiteboard and the word processor was used as replacement since the activity on which the group had worked could have been accomplished in the same way using the blackboard, or paper and pencil.

In episode 3 Juan used the interactive Enciclomedia program 'The Number Line' with the group. In this activity the students are asked to find a number between two given numbers that have been chosen randomly within the context of a game (Fig. 5).

Mixed numbers can appear alongside proper and improper fractions. However, numbers can only be entered in the program as mixed numbers and decimal numbers are only accepted by the program when expressed with two decimal places. At a more advanced level, the students are also asked to approximate the location of the number on the line. Four groups of students play the game and, for each correct answer, they are awarded points. Each team is represented by a different colour token, which moves across a board until one of the tokens crosses the finish line. The program gives automatic feedback to the user by indicating whether the answer

Fig. 5 The Number Line

is correct or not. Each time a correct response is given, the program selects the smaller of the two sub-intervals formed on the number line when the number given by the students is introduced. Fractional or decimal numbers soon become a necessary input in order to progress in the game.

Juan explained the goal of the game to the students and chose the intermediate level of the program, which has decimal numbers as the end points of the interval. Hence the technology was the source of the mathematical problems. The teacher selected a student from each team to respond to each activity proposed by the program. There was more interaction between the students in their teams. They participated actively and reflexively in the game. Students compared different numbers and used different operations to make this comparison. Students acted with more autonomy, although the teacher was always in control of the activity as he was the one entering the numbers into the program. However, each time the students chose a wrong number Juan asked the group why it was not correct and helped the students with their explanations. They mainly worked with decimal numbers and hardly used fractions. The validation of knowledge was shared between the interactive program and the teacher, although it was mainly the teacher who justified the feedback given by the program.

At some point they were faced with a situation in which they needed to find a number between 435.36 and 435.37:

J: [After three minutes without a response from the students] Which number have you chosen? What is the whole part?
J: What is the difference between this [.36] and this [.37]?
S_1: One hundredth, isn't it?
J: How do we write it as a fraction?
S_1: 435 and 375 hundredths
S_2: Thousandths

Juan entered a number that lay outside the interval (1/100) so the program marked the answer as incorrect and presented an interval for the next team to play. After all the other teams had participated, the group faced again the interval (435.36, 435.37). This time Juan asked students to use numbers with decimals instead of fractions.

They suggested 435.365 but the program only allowed him to write 435.36. Students commented that the answer was incorrect and Juan tried to use the program to show it was correct but was unable to solve the problem. A student finally found out that the program did not allow the user to write decimal numbers with more than two digits.

J: Here it is again, so your answer is…
S3. Four hundred thirty five point three hundred sixty five.
J: [Enters the number 435.36 into the program] I cannot write such a number.
S3: [After two minutes] If you write that the program will say it is incorrect. If you write point thirty five it will decide it is also incorrect. But there is no more hundredths there [referring to the program]
[Juan continues trying for an other 2 minutes]

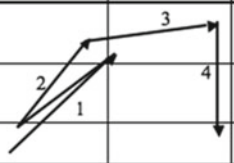
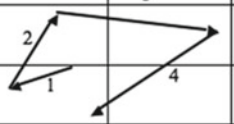
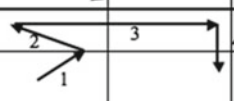
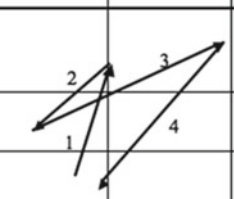
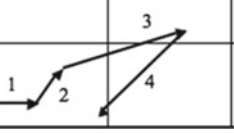
Juan finally closed the program. He did not take the opportunity to discuss further any strategies to find a fraction or decimal number between two decimal numbers where the difference between them is one hundredth. Communication of mathematical knowledge was shared between the program, students and the teacher, although in the last part it was the teacher who communicated this knowledge exclusively.

Throughout this episode technology was used as a replacement and as an amplifier. There was a change in the class dynamics as the interactive program was introduced, and students participated more actively and with more autonomy discussing in their teams which could be the answer and the way to present it since the program gives more points if students use fractions than if they use decimals. However, Juan did not use the resource to challenge students' mathematical knowledge, to help them think on new strategies to find fractions and decimal numbers on the number line, or to guide them in their solution of situations, which could not be solved with the program.

Juan ended the lesson (episode 4) by asking students to work on a textbook activity that asked the students to find fractions and decimal numbers on given number lines. He worked with the whole group by reading each problem and asking questions such as *How do you know? How would you turn 4/5 into a decimal number?* He worked on this last problem on the whiteboard, but wrote an incorrect answer (0.4). Students did not notice the mistake and the lesson ended before they finished the activity. In this episode the teacher was again the source of mathematical problems and was in charge of validation. The interaction with students was in terms of questions and answers and students did not have autonomy. Technology was used as replacement. Table 3 shows the analysis tool in this case.

The different aspects of Juan's role as a teacher in relation with his use of technology show that even though Juan's patterns of actions were guided by his teaching plan, he tried to interact with his students by asking questions and listening to their answers. The use of the animation, his prepared worksheet and the whiteboard did not contribute much to change his actions. However, the use of the program, and particularly the need to divide students in four groups, contributed to a change in his actions. He used the program as a source of mathematical problems and to validate the answers; he was open to giving students more time for discussion before making

Table 3 Analysis of Juan's aspects of his role as a teacher in relation to the use of technology

Teachers' role / Uses of technology		Replacement	Amplification	Transformation
Communication of mathematics	Students, textbooks, technology and teacher			
	Some other elements of communication			
	Teacher communicates exclusively			
Interaction with students	Modifies plan according to students' participation			
	Listens to students and answers questions but goes back to plan			
	Little interaction with students, follows predetermined plan			
Validation of mathematical knowledge	Multiple sources of validation			
	Teacher as only source of validation			
Which is the source of mathematical problems	Other sources of problems including those coming from digital programs and students themselves			
	Problems from teachers and textbook			
	Unique source of mathematical problems			
Actions and autonomy of students	Students have autonomy to decide what to do and how to do it.			
	Teacher assumes active role, while students mainly listen, copy or answer questions.			

their responses and he made them reflect on the reasons behind their answers. For the students, the effective actions in the classroom included the answering of the teacher's questions and, in episode 3, their reflections on how to respond in order to gain more points in the game, with reasons for their answers. Juan is a young teacher eager to use technology, but he did not use it in an effective way to promote students' learning.

Susana

Susana was particularly interested in the teaching of fractions. She developed a teaching sequence involving two interactive programs that involved the use of fractions in different ways. The first program she used is called 'The Balance' (Fig. 6), which shows a problem situation where scales need to be balanced by using

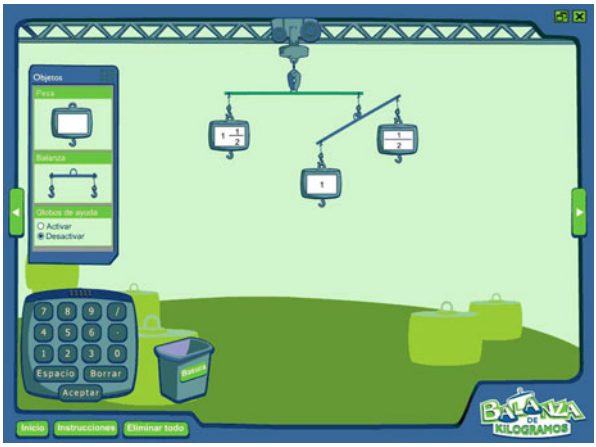


Fig. 6 The Balance

fractions. The program provides the users with automatic feedback that helps them in identifying which parts of the mobile toy are balanced and which are not.

The lesson took place in the computing room, where every pair of students had a computer with access to the program 'The Balance'. In the first part of the session (episode 1), Susana asked the students to use 'The Balance' to compare different pairs of numbers in order to decide which is greater. She started by using whole numbers and then she introduced simple fractions like 1/2 and 1/3 and decimal numbers like 0.5 and 0.05. Students used the program to compare the numbers thus becoming familiar with how the program functioned. Later, Susana taught the group how to build a balance with the program using two different levels of scales and asked them to work with this kind of scales using fractions. She posed different examples for students to solve. There was a group discussion regarding the meaning of equivalent fractions.

In this case, we consider that Susana, 'The Balance' and the students all participated in the communication of mathematics. The program, by imposing a certain problem, was one source of mathematical problems, while Susana acted as another source by asking students to use specific numbers. The interactive program validated the answers by visually showing whether the scales were balanced or not. Students had little autonomy in this part of the lesson, since they would just enter the numbers the teacher suggested in order to compare them, although they did have discussions in which they talked about why they thought one number was smaller than another one. 'The Balance' was mainly used as an amplifier, as it carried out the calculations automatically and showed if the scales were balanced or not.

In a second episode, Susana asked students to build their own mobile toys using different levels and numbers (episode 2). She emphasised to the students that they should make sure that all of the different levels of the mobiles were balanced. In order to do this, students had to add and subtract fractions, and figure out how to divide a fraction into two different equal parts (see Fig. 7).

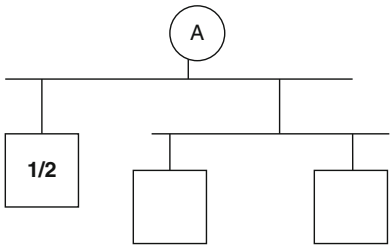


Fig. 7 Problem with the balance

There was a long group discussion in which students talked about how to divide a fraction such as ½ in two equal parts. They used different representations, including drawings of pizzas on the blackboard. They worked with several examples, as after dividing 1/2 by 2 Susana asked about other fractions such as 1/3, 1/4 and later 3/4, 5/8 and so on. In this episode, the students in Susana's class became stronger agents in the communication of mathematics. They even became the source for mathematical problems, since they designed their own mobile toys. The program itself remained the main source for validation, showing every time whether the mobile was balanced or not. Susana adapted the group discussions to the kinds of problems posed by the students and to their explanations. We therefore consider that in the interaction with students she modified her plans according to their participation:

> *I was not planning on discussing how to divide fractions using graphic representations. I thought maybe they would try different options, like trial and error, or adding two numbers, things like that, because I thought they would work with numbers like 2/4, but I was not expecting them to want to come up with an explanation like "the pizzas". When I saw they wanted to do it that way, I was like, okay, let's go ahead with this! (Susana, 01-07-2011)*

The use of technology in this episode became transformational, because, in motivating students to create their own mobile toys and therefore inventing their own mathematical programs, Susana's plans and role were modified:

> *I did not think that students would create such big mobile toys one their own that quickly! It had never happened before when I worked on that particular chapter from the textbook. They were really engaged when using the program (Susana, 01-07-2011)*

We consider that the interactive program alongside the students' ideas and the discussions in the classroom, allowed mathematical learning to happen. In order to act effectively, that is, to balance different kinds of mobile toys that could be constructed using the program, students came up with new explanations and procedures. They had not encountered the partitioning of fractions before, and they developed a procedure that allowed them to solve the problem that was posed by the program as well as the other problems posed by the teacher. During the last episode (episode 3) of the first session, students worked with 'The Balance' in order to solve the problems posed in their textbook (see Fig. 8).

The problems became more difficult. The students finished all of the exercises within the textbook and later they continued to devise their own even larger mobile toys that they had to balance. The discussion on the division of fractions continued.

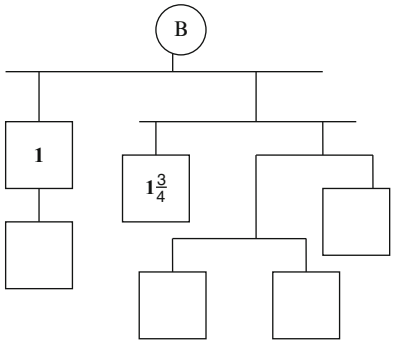


Fig. 8 Problem with the balance

They concluded that, in the case of a division of a fraction by 2, if the numerator was even, it was easy to divide, while if it was odd, more difficult procedures had to be used. They came up with some 'rules' for even and odd numerators, which they wrote on the whiteboard. Later they extended the problem into partitioning fractions in 3, 4 and 5 equal parts and they spent quite a long time discussing different procedures until they came up with a general one (finding equivalent fractions in which the numerator could be divided exactly by the whole number).

In this last episode, the role of the teacher was similar to that of episode 2. The sources of the mathematical problems were the textbook, the program and the students and again Susana allowed the students to formulate their own explanations and procedures, especially in the case in which they had to divide fractions. The use of the technology remained transformational, as students deepened their explanations and justifications.

We consider that, without the visual feedback from the program, students may not have had to conceptualise and develop procedures for the partitioning of fractions. Previously, we had observed that within that particular chapter in the textbook, students would balance mobile toys inadequately, for example by balancing a weight of 1 ½kg with one of 1 kg and one of ½kg placed horizontally. This would balance one level of the mobile but not the second level. 'The Balance' motivated students to find ways of dividing a given fraction into equal parts as it was not effective behaviour to have sections of the mobile toy which were not balanced. Susana had asked the students to balance all sections of the mobile. In the end they were exploring the division of fractions by whole numbers in a general manner.

On the one hand, it is interesting to note here that, in this case, the program became a strong influence on the students' effective behaviours. A different type of program might not have had the same effect. On the other hand, it is important to observe that the way in which Susana used this tool also modified the students' behaviour. The restrictions that she posed and the questions that she asked were important influences on the students' learning. The importance of the way in which the teacher uses the digital tools was confirmed by our observation of a second session in which Susana used 'The Number Line' (see Juan above) in order to deepen students' understanding of fractions. During this session, Susana asked questions

prior to the interactive program providing feedback. Frequently used questions were: *How did you find the number? How do you know that number is between those two?* After the feedback had been given by the program, she asked additional questions such as: *Why is the answer correct/incorrect? How would you find another number between those two?* Students were invited to come to the board in order to illustrate their explanations and justifications. They were allowed to make drawings and to use concrete materials.

During these discussions the use of the program was also transformational. Susana's practice was modified because, as she mentioned, the interactive programs related to fractions available in Enciclomedia invited her to create a teaching sequence related to fractions that she had not previously considered:

> *When I saw that there were several programs that I could use for the teaching of fractions, I decided to create a longer teaching sequence. The textbook chapters which are related to these topics are not sequential, but I decided it would be interesting to explore fractions from different views in consecutive sessions. (Susana, 01-07-2011)*

In this case, the context that the program created in the classroom, together with the teacher and the students, again promoted the students' mathematical learning. Effective behaviour included using known ideas and procedures with fractions to solve mathematics problems that initially, they did not know how to solve. Table 4 shows the analysis in Susana's case. The thicker arrows represent the work with the Number Line.

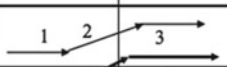
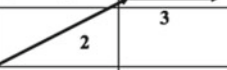
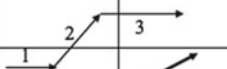
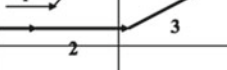
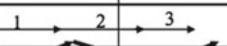
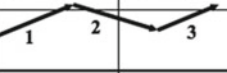
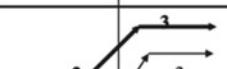
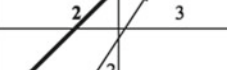
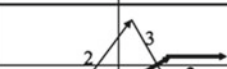
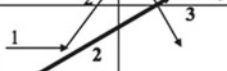
Discussion

A first comparison of the Tables 2, 3 and 4 shows that there is a close interaction between the different aspects of the role of the teacher and the use of technological resources.

Regarding *communication* it can be observed that technology often becomes an additional source of mathematical information in the classroom. Interactive programs, animations and worksheets expose students to mathematical concepts and ideas. However, this mathematical information can promote very different kinds of effective behaviours in the classroom. For example, we observed that the use of interactive resources is more likely to promote the emergence of shared mathematical ideas, compared to the use of animations or activity worksheets based on the textbook where the interactive whiteboard is used to replace traditional paper and pencil activities. When using an interactive program such as the 'Number Line' or 'The Balance', it is possible that mathematical information is communicated also by students as they engage with the problems and share their ideas. Of course, not all uses of interactive programs lead to this form of communication, as we saw in the case of Juan's use of 'The Number Line'. The teacher's behaviour can impose restrictions on what it is possible to do with the program.

Technology also plays a role in promoting *interaction* in the classroom. However, it can be deduced from the data and the information we have about the teachers'

Table 4 Analysis of Susana's aspects of her role as a teacher in relation with the use of technology

Teachers' role / Uses of technology		Replacement	Amplification	Transformation
Communication of mathematics	Students, textbooks, technology and teacher			
	Some other elements of communication			
	Teacher communicates exclusively			
Interaction with students	Modifies plan according to students' participation			
	Listens to students and answers questions but goes back to plan			
	Little interaction with students, follows predetermined plan			
Validation of mathematical knowledge	Multiple sources of validation			
	Teacher as only source of validation			
Which is the source of mathematical problems	Other sources of problems including those coming from digital programs and students themselves			
	Problems from teachers and textbook			
	Unique source of mathematical problems			
Actions and autonomy of students	Students have autonomy to decide what to do and how to do it.			
	Teacher assumes active role, while students mainly listen, copy or answer questions.			

backgrounds, that teacher training, didactical ability and disposition to share the control of the lesson with students and to be flexible regarding the original lesson plan are dominant factors in this aspect. This is exemplified by the way Susana modified her original plan, and managed the use of the resources and the interaction with and between students. Effective behaviours during her lessons included the discussion of mathematical ideas and problems both in small groups and in whole group discussions. Her role as a teacher provided a space in which students could follow their ideas even when they led towards unexpected (to her) places. Gabriel, in contrast, stayed close to his original lesson plan and limited the interaction with students to the use of closed and rhetorical questions which promote effective behaviours such as guessing and trying to please the teacher and do not favour students' mathematical learning.

Validation of mathematical knowledge when using technology appears to be closely related to the design and use of the digital resource, although the teacher can

make use of the resource whereby the validation of knowledge is shared with students. Again, a good example of this possibility to share validation among students, technology and the teacher is illustrated by Susana's use of both 'The Number Line' and 'The Balance'. In this case, effective behaviours include giving reasons for why a specific answer is entered into the program and finding procedures that might lead to correct answers before trying them out in the program.

Regarding the role of the teacher in terms of students' *autonomy*, we observed that certain types of resources are more inviting to explore different mathematical ideas and procedures by students themselves than others. Problems like the ones posed by 'The Balance' or 'The Number Line' can motivate students and promote effective behaviours such as working on their own or with friends to find answers and explanations. Other programs such as animations can promote a more passive attitude from students. This again, is strongly influenced by the way in which the teacher uses the digital resource.

In terms of the uses of technology the tables show that the introduction of digital resources alone is not enough to transform the activity in the classroom. During the same lesson different uses of technology can be observed. The creation of an environment that promotes effective actions, which are related to the learning of mathematics depends on the different aspects of the role of the teacher. It is teachers who make decisions that can change both the use of technology and the dynamics in the classroom so that transformation is possible. A combination of the teacher and the use of the technology can create learning contexts where actions are limited or where they are more conducive to mathematics learning.

The analysis of the use of technology by teachers in this study shows a strong tendency for most of the teachers to limit the use of technology as a replacement and amplifier. Most of the teachers have not received training regarding the didactical use of Enciclomedia's mathematical resources and have not experienced, even as spectators, how the programs can be integrated in lessons which provide a context that promotes learning. This is clearly evidenced by Gabriel who is willing to introduce technology in his classes, but does not exploit the potential of the resources to create an environment where mathematical activity is effective and by Juan who does not use it to challenge the mathematical knowledge of his students. Susana, on the other hand, illustrates how teachers who have been trained in the didactical use of technology can benefit from the different possibilities afforded by resources, can combine them and create classroom contexts where students discuss and reflect on their actions and can learn mathematics. In this classroom culture, the teacher herself can learn.

Mathematical knowledge of teachers can also limit the different aspects of their role in the classroom. Observations show how they are not able to use opportunities presented by the technology's potential or limitations in order to discuss students' strategies. For example, when Juan is faced with the need to explain how to find a number between two given numbers when their difference is one hundredth, and there are students who apparently already know the answer, he did not ask these students how they found the number. When both Gabriel and Juan used animations

in their classroom, they did not ask any question related to the content. The students acted as spectators and in this environment their learning can be inhibited.

Traditional ways of teaching, in which the teacher intends to transfer mathematical knowledge, can become an obstacle to a use of technology that has transformational potential. When teachers stick to their lesson plans and when they keep the control of what mathematical knowledge is communicated, which problems are worked and the validation of students' answers, technology is naturally used as replacement or amplifier. Again, the class environment is such that mathematical learning can be limited.

From the technological resources used by these teachers it can already be observed that there is a variety among Enciclomedia's resources. Although resources can be classified as animations or interactive resources, some interactive programs demand more interactivity than others. In our examples we can observe that the 'Capacity Measures' program used by Gabriel asks for calculations and approximations from the teacher and students, but as we described before, students can effectively guess or use a trial and error strategy in order to select an answer from the options in the program. The responsibility for questioning and inviting students to modify their actions is on the teacher's side. Other resources, such as 'The Number Line's ask for more involvement from the beginning in terms of the mathematics, since trial and error practices do not help in playing the game and are therefore not effective behaviours in this context. Also, the need to locate the selected number on the line requires reflection on the part of the players. The teacher can take this opportunity to discuss procedures and strategies used by students in terms of their being correct or not, or in terms or their being efficient. This discussion can promote effective behaviours such as mathematical reasoning activities and reflection where learning is possible. In our previous description we could observe that Juan does not profit from this opportunity while Susana does. Finally, 'The Balance' exemplifies a very open resource that can trigger both teachers' and students' creativity in order to design a variety of ways of using it for the learning of concepts related to whole numbers, numbers with decimals and fractions. In the selected episode Susana allows the students to design challenging situations that result in interesting opportunities to discuss different properties and operations with fractions.

The differences in the design characteristics of the technological resources can impact differently in the ways teachers use them. Although it is the teacher who can guide activity with resources in the class, some of them can somehow induce changes in some aspects of the teacher's role.

The results that have been analyzed show that the technological resources used by teachers influenced the learning of their students, as they framed the effective actions of both teacher and students. Although there are differences between the technological resources used, as we previously discussed, each of these create spaces for action and at the same time they impose certain restrictions. It is important for the teacher to be aware of the possibilities for action and of the restrictions so that the kinds of actions that promote the learning of mathematics can be fostered.

Conclusions

The data analysis has enabled us to conclude that the characteristics of digital resources used in the classroom have an important influence on the role the teachers play during their lessons. Some technological resources such as 'The Balance' and 'The Number Line' can, by themselves, help the teacher to create a context where the students have greater possibilities for autonomy and where interaction and discussion of mathematical ideas foster effective behaviour, creating an environment for mathematics learning. However this is strongly dependent on the teachers' effective actions, which are influenced by their mathematical knowledge, the experience they have regarding the use of the technological tools and their didactical strategies and practices. It is important to notice that, even when digital resources can promote active participation in the classroom by opening spaces for mathematical exploration, the actions carried out by the teacher, that is, the different aspects of her or his role can inhibit or enhance such effectiveness.

Enciclomedia's resources were conceived as a tool for the classroom. The didactical idea behind their conception was that teachers would develop didactic strategies where the resources could play an important role in helping to provide a dynamical and participative class environment, which can promote students' learning (SEP 2004). As can be concluded from this study, this goal is possible to achieve when teachers have reviewed and studied the programs, have made an open planning for their lessons and, most importantly, have been trained on how to integrate technological resources to their teaching in an effective way. This is the case with regard to Susana. The dynamics of the different aspects of her role in the two sessions provide a good example of how this integration can promote students' learning. For her, technological resources are instruments she can use to replace, amplify or transform situations involved in teaching specific topics and which help create a classroom context where she and her students can propose and discuss mathematical ideas as well as share and develop them.

However, the data shows that developing this expertise is difficult and needs formal training. In our study there was only one more teacher who worked similarly to Susana. Other teachers, who have only received training on the general use of the software, without a hint of how to introduce them into specific lessons, often develop teaching strategies where technology is used as replacement or amplification. In this case there is no real change in the environment they create in their classroom.

Regarding the programs themselves, it would be important to classify Enciclomedia's programs or any kind of digital resources in terms of their possibility to act as tools that can help teachers to: transform the classroom dynamics; work on making the necessary changes so that they include possibilities for exploration; and enable the students to choose scenarios or discuss situations, with and without the teacher. In this kind of environment, effective actions include the reflection on mathematical ideas and procedures that can be conducive to mathematical learning.

While other kinds of resources are not necessarily open, they can be useful as instruments to validate mathematical knowledge. In this case, the different aspects

of the teacher's role such as interaction with students, being the source of mathematical problems, and enabling the shared communication of mathematics and autonomy of the students are fundamental to the creation a productive environment for learning.

Finally, the methodological tool we developed was useful in describing teachers' actions when using technology as it highlights aspects and details of the internal working of classrooms that may otherwise have remained hidden. We believe this instrument might be useful for other researchers and also for teachers who are interested in reflection about their own practice. The matrix also served to highlight the relevant aspects of the role of the teacher with his or her possibility to move towards a transformation use of technology. This does not mean that other uses of technology are not important; each of them can play a part in different moments of the lesson and may complement each other. It is the balance between these uses together with the dynamics of the aspects of the role of the teacher that creates the conditions for effective actions that can be described as mathematical learning.

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